HIGH ORDER NONLINEAR WAVE INTERACTIONS FROM DEEP TO FINITE WATER DEPTH WITH BOTTOM TOPOGRAPHY CHANGE

Zuorui Lyu, Kyoto University, lyu.zuorui@gmail.com
Hiroaki Kashima, Port and Airport Research Institute, kashima-h2w7@p.mpat.go.jp
Nobuhito Mori, Kyoto University, mori@oceanwave.jp

INTRODUCTION

In recent years, freak wave/rouge wave has become an important problem in science and engineering. Modulational instability is considered to be an important factor leading to freak wave in the wave evolution of deep water, and Janssen (2003) defined Benjamin-Feir index (BFI) to reflect it. Mori and Janssen (2006) gave the occurrence probability of freak waves based on a weakly non-Gaussian theory, and distribution of wave height is determined by skewness and kurtosis of surface elevation to a considerable extent in deep water.

According to observational record, freak wave has not only been found in deep water in the ocean, but also been observed in shallow water and coastal areas. In the process of water wave entering continental shelf, water depth is changing with mild slope after a long distance propagation. This study focus on investigating how water depth affect skewness and kurtosis in the high order nonlinear wave evolution from deep water to finite water depth in two-dimension.

THEORY OF WAVE EVOLUTION IN 2D

Lots of studies have been accomplished about Nonlinear Schrödinger (NLS) equation considering bottom topography change. Zeng and Trulsen (2012) gave the Monte - Carlo simulation in 1D space. Here we extend NLS equation considering bottom effects into 2D. With the hypothesis of irrotational, inviscid and incompressible flow with free surface, we establish a coordinate system $Oxyz$ with origin O at quiescent water surface. Plane Oxy coincides with quiescent water surface and Oz vertically points upwards. The bottom located at $z = h(x, y)$. Wave potential is $\Phi = \Phi(x, y, z, t)$ and surface elevation is $\zeta = \zeta(x, y, t)$ where t represents time. Φ, ζ satisfy Laplace and boundary equations. We suppose at the initial point $t = 0$, a progressive wave at surface satisfies

$$\zeta|_{t=0} = A(x, y)\exp\{\mathrm{i}(kx - \omega t)\} + \mathrm{c.c} + O(\varepsilon^2) \qquad (1)$$

where $A(x, y)$ is wave amplitude in first harmonic, k is wave number, ω is circular frequency, g is gravity acceleration, ε is a small positive constant (i.e. wave steepness). To make the depth varies slowly and explicit, multiple scale variables are introduced as:

$$\tau = \varepsilon\left[\int^{x} \frac{dx}{c_g} - t\right], \qquad \xi = \varepsilon^2 x, \qquad \eta = \varepsilon y. \qquad (2)$$

where we allow slowly change of bathymetry in x and y with the different order of ε. After the method of multiple scales, we can give the evolution equation of progressive wave in the form of modified Nonlinear Schrödinger equation (mNLS-2D):

$$\mathrm{i}\mu A + \mathrm{i}\frac{\partial A}{\partial \xi} + \lambda\frac{\partial^2 A}{\partial \tau^2} + \gamma\frac{\partial^2 A}{\partial \eta^2} = \upsilon|A|^2 A \qquad (3)$$

where coefficients $\mu, \lambda, \gamma, \upsilon$ are functions of k, h, ω. There is new term in left hand side of Eq.(3) with coefficients γ.

NUMERICAL MODELING

Eq. (3) can be solved by taking a Fourier transform to τ and η in temporal domain because of change of bottom shape. Then Eq. (3) becomes an ordinary differential equation about ξ. With initial data at $\xi = \xi_0$, surface elevation about at each spatial step in wave evolution process can be determined. Figure 1 gives the result of mean kurtosis of surface elevation at different initial BFI value from 200 samples in 1-D Monte - Carlo simulation, where the wave phase is considered to be random. Kurtosis reflects nonlinear four-wave interaction, and BFI represents modulational instability of wave train.Figure 2 gives the surface elevation when $x = 60L_{x0}$, where L_{x0} is wave length in x direction.

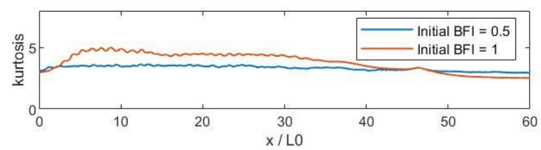

Figure 1 - Spatial evolution of kurtosis on sloping topography in 1-D modeling

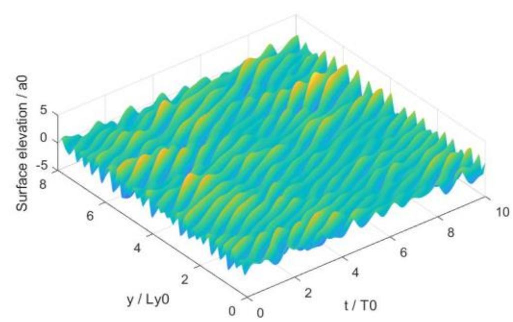

Figure 2 - Surface elevation at $x = 60L_{x0}$ in 2-D modeling

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