MAXIMUM MOMENTUM FLUX FOR STABILITY ANALYSIS OF MODEL AND PROTOTYPE BREAKWATERS

Indra FW Jayewardene[1]*, Aliasghar Golshani*[2] *and Ed Couriel*[3]

None of the established formulae for breakwater armour stone stability, Hudson (1958) or van der Meer (1987), explicitly account for the water depth at the toe of the structure. More recently, Hughes (2004), Melby and Hughes (2004), Melby and Kobayashi (2011) have developed equations for stability and runup utilising the concept of wave momentum flux which explicitly accounts for the water depth of the wave probe(s) in close proximity to the structure.

The equations are adapted to three forms; namely (1) linear, (2) extended-linear and (3) non-linear. In the paper linearity is assessed by using the Ursell number at each probe depth. Also, due to the placement of probes at various depths in MHL's 2D wave flume it is possible to correlate the linearity of the wave measurement for the same time series and subsequently test the appropriateness of the momentum flux equation applied for assessment of stability and runup. The stability and runup data from 43 2D physical model tests where stability was previously assessed using van der Meer's and Hudson's equations are assessed using the momentum flux equations and an evaluation of the results has been made. It was found that the estimation of notional permeability and selection of the use of the plunging or surging formulae was critical to obtaining a closer match between measurement and prediction. The equations were also utilised in conjunction with numerical models to evaluate the armour size for repair of two breakwater heads in South Camden Haven and Bellambi. The maximum momentum flux equations were found to perform satisfactorily at these locations where the Ursell numbers were found to be high and the waves non-linear.

Keywords: maximum momentum flux, stability, impermeable slope runup, notional permeability

Brief History of the Development of Stability Equations

Traditionally, breakwater armour stone stability has been assessed utilising Hudson's equation (Hudson 1959) and van der Meer's (vdM) equations (1987). The structure of the vdM equations is as follows:

$$\frac{H_s}{\Delta D_{n50}} = 6.2\, P^{0.18} \left(\frac{S}{\sqrt{N}} \right)^{0.2} \xi_m^{-0.5} \qquad (1)$$

(for plunging waves where $\xi_m < \xi_{mc}$)

$$\frac{H_s}{\Delta D_{n50}} = 1.0\, P^{-0.13} \left(\frac{S}{\sqrt{N}} \right)^{0.2} \cot g\, \alpha^{0.5}\, \xi_m^{P} \qquad (2)$$

(for surging waves where $\xi_m > \xi_{mc}$)

$$\xi_{mc} = \left[6.2\, P^{0.31} \sqrt{\tan \alpha} \right]^{\frac{1}{P + 0.5}} \qquad (3)$$

where:

P = notional permeability factor
S = damage level = A_e / D_{n50}^2
N = number of waves (storm duration)
$\xi_{m = Hs}$ / tan α / $(Sm)^{0.5}$, A_e=damage area
D_{n50} =diameter of 50% armour, α=slope angle

The rock manual (2007) recommends using van der Meer (1987) for deep and moderate shallow water conditions and Van Gent et al. (2004) approach for shallow water conditions as a general application procedure. The rock manual provides rough definitions for deep and shallow water that have been utilised in the calculations carried out when making comparisons of design armour size for breakwater head repair at South Camden Haven and Bellambi breakwaters. The vdM equations have stood the test of time despite their discussed shortcomings in the literature. Notional permeability is defined in Figure 1. The clear quantitative demarcation between plunging wave is included in this

[1] Manly Hydraulics Laboratory, Department of Environment and Planning, Manly Vale 2093, Sydney, Australia
[2] Manly Hydraulics Laboratory, Department of Environment and Planning, Manly Vale 2093, Sydney, Australia
[3] Manly Hydraulics Laboratory, Department of Environment and Planning, Manly Vale 2093, Sydney, Australia

formulation. Hudson's equation (1959) in its standard form and its variant that includes (S) as referenced in the rock manual (2007) is indicated below as equations (4) and (5).

$$H_{sig} / \Delta D_n = (K_D \operatorname{Cot} \alpha)^{1/3} \quad (4)$$

where:

H_{sig} = design significant wave height at the structure
Δ = relative mass density
K_D = coefficient of damage
D_n = nominal diameter of M_{50} armour
α = angle of breakwater slope (obtained from cross sections in Appendix A)

The variation to Hudson's equation using Broderick and Ahren's Sd was utilised where damage was assessed by a profiler instead of photographic methods.

$$\frac{Hs}{\Delta D_{n50}} = 0.7(K_D cot \propto)^{1/3} {S_d}^{0.15} \quad (5)$$

As discussed by Hughes (2004) both these sets of equations do not explicitly account for water depth at the toe of the structure. Hence, this paper applies the equations proposed by Hughes (2004) and Melby and Hughes (M&H 2004) to three separate sets of stability tests (T1–T3) performed at Manly Hydraulics Laboratory (MHL) to obtain a better understanding of the proposed equations. The Ursell number (HsLm 2/h3) is utilised to obtain the degree of non-linearity of the wave time series at the probe selected to measure the wave height and wave period. The structure of the maximum wave momentum flux equations (non-dimensional) proposed by Hughes (2004) follows.

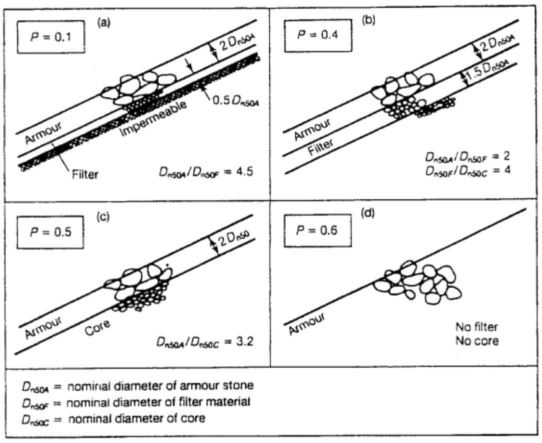

Figure 1. Estimation of notional permeability of a structure (Shore Protection Manual 1984, Vol. 2)

Using linear wave theory:

$$\left(\frac{M_F}{\rho g h^2}\right)_{max} = \frac{1}{2}\left(\frac{H}{h}\right)\frac{\tanh kh}{kh} + \frac{1}{8}\left(\frac{H}{h}\right)^2 \times \left[1 + \frac{2kh}{\sinh 2kh}\right] \quad (6)$$

Using extended linear wave theory:

$$\left(\frac{M_F}{\rho g h^2}\right)_{max} = \frac{1}{2}\left(\frac{H}{h}\right)\frac{\sinh[k(h+H/2)]}{kh\cosh(kh)} + \frac{1}{8}\left(\frac{H}{h}\right)^2 \times \left[\frac{\sinh[2k(h+H/2)]+2k(h+H/2)}{\sinh 2kh}\right] \quad (7)$$

Using non-linear wave theory and Fourier approximations

$$\left(\frac{M_F}{\rho g h^2}\right)_{max} = A_0\left(\frac{h}{gT^2}\right)^{-A_1} \quad (8)$$

where

$$A_0 = 0.6392\left(\frac{H}{h}\right)^{2.0256} \quad (9)$$

$$A_1 = 0.1804\left(\frac{H}{h}\right)^{-0.391} \quad (10)$$

where maximum momentum flux is given by

$$M_F(x,t) = \int_{-h}^{\eta(x)} (p_d + \rho u^2)\mathrm{d}z \quad (11)$$

A generalised equation for measured momentum flux could be given as

$$N_m = \left(\frac{(M_F)_{max}}{\rho_w g h^2}\frac{K_a}{\Delta}\right)^{1/2}\frac{h}{D_{n50}} = \text{function}(\theta, P, S, N_z, s_m, R_c/H_{m0}), \quad (12)$$

The LHS of equations (6), (7) and (8) quantifies the measured value for maximum momentum flux for a given test time series. Equation 12 gives the measured stability number for the time series. The resulting predicted stability number equation developed by M&H (2004) is given by equation (13) for plunging waves

$$N_m = 5.0(S/N_z^{0.5})^{0.2}P^{0.18}\sqrt{\cot\theta} \qquad s_m \geq s_{mc} \quad (13)$$

and transitions into the surging wave equation (14) if the criteria for wave steepness is not exceeded.

$$N_m = 5.0(S/N_z^{0.5})^{0.2}P^{0.18}(\cot\theta)^{0.5-P}s_m^{-P/3} \qquad s_m < s_{mc} \quad (14)$$

The critical wave steepness is given by equation (15)

$$s_{mc} = -0.0035\cot\theta + 0.028 \quad (15)$$

S_{mc} denotes the wave steepness where the plunging wave equation transitions to the surging wave equation. The RHS of equations (13) and (14) provide the basis for the predicted value of the stability number based on the input parameters .

The paper estimates measured stability number (Tables 2a and 2b) for linear, extended-linear and non-linear wave trains and compares the values with predicted estimates for the M&H equations (plunging and surging, Table 2c) and for vdM (plunging, Table 2c) equations. This is due to the lack of clear quantitative demarcation of the two wave conditions provided in the M&H equations (13 and 14) unlike the vdM equations (3).

The expression for stable stone size using the M&H equation is given by equation (16) as indicated by Melby et al (2011).

$$N_m = \left(\frac{(M_F)_{\max}}{\rho_w g h^2 \Delta}\right)^{1/2} \frac{h}{D_{n50}}.$$

The parameter a_m is estimated as

$$a_m = \frac{1}{5P^{0.18}\sqrt{\cot\theta}} \qquad s_m \geq s_{mc} \qquad \text{(plunging waves)} \tag{16}$$

$$a_m = \frac{s_m^{P/3}}{5P^{0.18}(\cot\theta)^{0.5-P}} \qquad s_m < s_{mc} \qquad \text{(surging waves)}$$

with

$$s_{mc} = -0.0035\cot\theta + 0.028.$$

Brief History of the Development of Runup Equations

Irregular wave runup equations have, in a manner similar to the stability equations, evolved in time. A more recently accepted equation for runup on an impermeable smooth slope is provided in USACE (2006) as follows.

$$\frac{R_{u2\%}}{H_{m0}} = 1.65 \cdot \gamma_b \cdot \gamma_f \cdot \gamma_\beta \cdot \xi_{m-1.0} \tag{17}$$

The equation developed by Hughes (2004b) is:

Plunging/spilling waves ($H_{mo}/L_p > 0.0225$): (18)

$$\frac{R_{u2\%}}{h} = 4.4(\tan\alpha)^{0.7}\left[\frac{M_F}{\rho g h^2}\right]^{1/2}$$

for $1/5 \leq \tan\alpha \leq 2/3$

The paper provides estimates for runup on an impermeable surface utilising maximum wave momentum flux using a wooden board in one half width of the flume side by side with rock armour (Figure 3c, Table 3).

The Physical Model

Flume Layout Example data from three physical models completed at MHL (T1, T2 and T3) are utilised to test the veracity of these equations. Probes P1 to P3 (Figure 2) were utilised to obtain estimates for the wave reflection coefficient. Probes P4 and P5 were utilised for estimating the incident wave utilised for maximum wave momentum assessment. Since the water depth at each probe is known probes P4 and P5 were utilised to compare wave data from the same generated wave time series with differing Ursell numbers with the efficacy of the respective capabilities to describe damage using the momentum flux equations. Whilst Model T1 and T3 utilised five probes, Model T2 only utilised four probes.

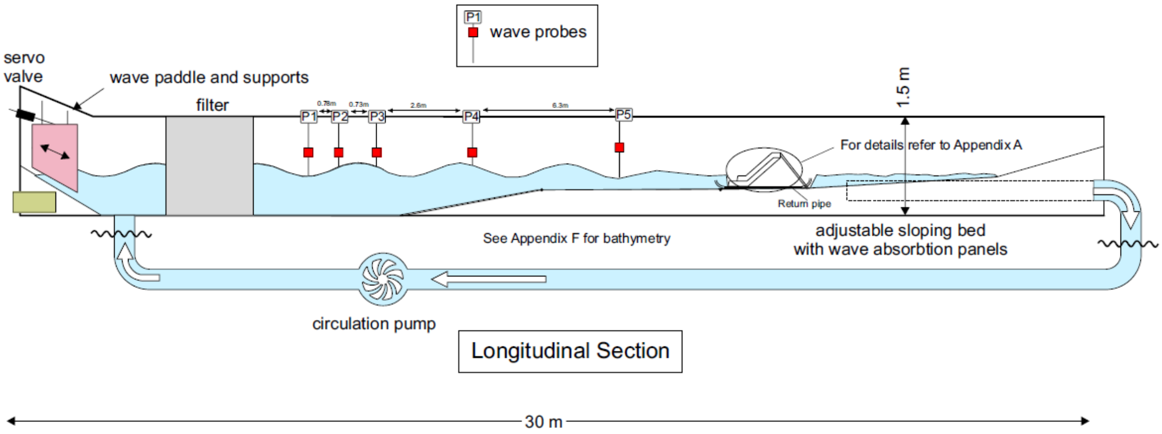

Figure 2 indicating relative positions of probe P4 and P5 at differing depths and measuring waves of differing linearity (Ursell number)

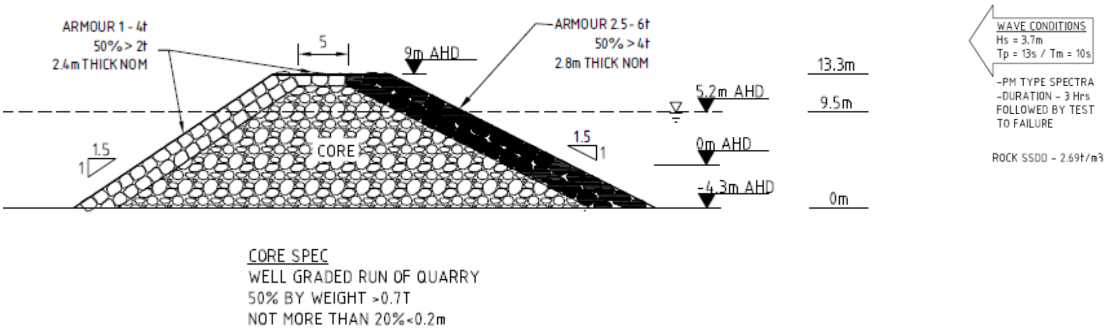

Figure 3a Test cross-section (T1)

Figure 3b Model T1 Section A2 after test 10 – Hs = 4.56m, Tp = 14.07s, damage 9.5%

Figure 3c comparing wave runup on permeable (rhs rock) and impermeable structure (lhs wood)

Table 1a Examples of damage as estimated by Hudson's and van der Meer's methodologies – Model T1				
Test No.	Description	Damage (Hudson)	Damage (S_d) (van der Meer)	Damage description
1	External breakwater A1	Less than 1%	Negligible	Negligible
2, 3, 4, 5, 6	External breakwater A1	Less than 1%	Negligible	Negligible
9	External breakwater A2	4.1%	4.64	Initiation of damage
10	External breakwater A2	9.5%	7.28	Significant damage however no secondary armour was visible. Hence deemed to be permissible
13	Internal breakwater B	Less than 1%	Negligible	Negligible
14	Internal breakwater B	3%	Negligible	Negligible

Table 1b vdM Damage criteria S (Shore Protection Manual 1984)			
Slope	*Initial damage	*Intermediate damage	*Failure (under layer visible)
1:1.5	2 (0-5%)	3–5 (5-10%)	8 (>15%)
1:2	2 (0-5%)	4–6 (5-10%)	8 (>15%)

*Hudson's criteria for damage in parentheses

Test Objectives and Limitations

Test Results for Stability Table 2a, 2b and 2c include tabulated measured and predicted momentum flux and the predicted value of momentum flux based on damage measured to the structure. Tests where negligible damage was recorded were not considered for the comparison as Melby and Kobayashi (2011) indicate that prediction is very variable in such instances. In cases where S values were not obtained and damage was only asessed by the Hudson methodology an equivalent S value was determined utilising Table 1(b). USACE (2006) indicates that parameters such as the Ursell number (HL^2/d^3) are utilised to categorise the non-linearity of the wave condition and appropriateness of the wave theory, the variables H, d and L representing wave height, water depth and wave length respectively. The degree of non-linearity is indicated by the Ursell parameter. Ursell numbers (HL^2/d^3) greater than 26 are associated with non-linear wave conditions (USACE 2006). During previous investigations MHL has successfully utilised non-linear theory to reconcile force measurement where linear theory proved erroneous (Melby and Hughes 2004).

Table 2a Examples of test cross-sections and damage results							
Test series-Test section (Test number)	Probe number	Wave height (Hs), Wave period (Tm)	Number of waves/test	Estimated notional permeability used in equation	D_{n50} (m)	Damage level parameter $S_d=A_e/D_{n50}^2$	Ursell Number ($H_sL_m{}^2/d^3$)
T1-A1 (1–6)*	5		6400 to19500	0.4	1.46	Negligible	4 to 20
T1-A2 (9)	5	3.86, 9.87	6474	0.4	1.16	4.6	24
T1-A2 (10)	5	4.56, 9.94	12856	0.4	1.16	7.3	28
T1-B (13)	5	3.27, 11.06	5777	0.4	0.72	Negligible	38
T1-B (14)	5	4.18, 11.06	11554	0.4	0.72	2.4	49
T1-A1 (1–6)*	4		6400 to19500	0.4	1.46	Negligible	1 to 4
T1-A2 (9)	4	4.26, 9.87	6474	0.4	1.16	4.6	4
T1-A2 (10)	4	5.07, 9.94	12856	0.4	1.16	7.3	5
T1-B (13)	4	3.69, 11.06	5777	0.4	0.72	Negligible	6
T1-B (14)	4	5.2, 11.06	11554	0.4	0.72	2.4	8
T2-A (1)	4	2.92, 8.08	7048	0.4	1.09	1.22	15
T2-A (2)	4	3.27, 8.48	14097	0.4	1.09	1.3	17
T2-A (3)	4	3.83, 8.95	21145	0.4	1.09	3.91	18
T3-A (1)	5	3.32, 9.1	1100	0.4	0.58	2	154
T3-A (2)	5	3.71, 9.1	2200	0.4	0.58	4	136
T3-A (3)	5	3.41, 9.1	3300	0.4	0.58	7	168
T3-A (4)	5	3.89, 9.1	4400	0.4	0.58	8	138
T3-A (5)	5	3.44, 9.1	4400	0.4	0.58	4	137
T3-A (1)	4	3.36, 9.1	1100	0.4	0.58	2	47
T3-A (2)	4	4.05, 9.1	2200	0.4	0.58	4	39
T3-A (3)	4	3.45, 9.1	3300	0.4	0.58	7	52
T3-A (4)	4	3.87, 9.1	4400	0.4	0.58	8	47
T3-A (5)	4	3.44, 9.1	4400	0.4	0.58	4	43
T3-B (1)	5	3.15, 9.1	1100	0.4	0.58	4	125
T3-B (2)	5	3.21, 9.1	2200	0.4	0.58	6	112
T3-B (3)	5	3.34, 9.1	1100	0.4	0.58	4	133
T3-B (4)	5	3.61, 9.1	2200	0.4	0.58	7	125

Table 2a Examples of test cross-sections and damage results							
Test series-Test section (Test number)	Probe number	Wave height (Hs), Wave period (Tm)	Number of waves/test	Estimated notional permeability used in equation	D_{n50} (m)	Damage level parameter $S_d=A_e/D_{n50}^2$	Ursell Number ($H_sL_m{}^2/d^3$)
T3-B (5)	5	3.36, 9.1	1100	0.4	0.58	7	134
T3-B (6)	5	3.25, 9.1	2200	0.4	0.58	8	113
T3-B (7)	5	3.13, 9.1	1100	0.4	0.91	3	125
T3-B (8)	5	3.04, 9.1	2200	0.4	0.91	6	106
T3-B (1)	4	3.8, 9.1	1100	0.4	0.58	4	49
T3-B (2)	4	3.73, 9.1	2200	0.4	0.58	6	44
T3-B (3)	4	3.45, 9.1	1100	0.4	0.58	4	45
T3-B (4)	4	3.61, 9.1	2200	0.4	0.58	7	43
T3-B (5)	4	3.6, 9.1	1100	0.4	0.58	7	46
T3-B (6)	4	3.37, 9.1	2200	0.4	0.58	8	40
T3-B (7)	4	3.79, 9.1	1100	0.4	0.91	3	49
T3-B (8)	4	3.79, 9.1	2200	0.4	0.91	6	45

* All negligible damage tests are categorised together

Table 2b Examples of critical parameters utilised for selection of plunging equations for each test series (T1, T2, T3)							
Test series-Test section (Test number)	Probe number and water depth	Wave height (Hs), wave period (Tm)	Wave steepness at probe (Sm)	Wave steepness (S_{mc}) to delineate plunging from surging formulae (M&H)	M&H Eq 6, 7, 8 measured stability number range from linear to non-linear - for plunging wave	M&H Eq 14 predicted stability number - for plunging wave	M&H Eq 151 predicted stability number - for surging wave
T1-A2 (10)	5	4.56, 9.94	0.0469	>0.0227	3.59–4.44	3.69	4.06
T1-A2 (10)	4	5.07, 9.94	0.0390	>0.0227	4.64–5.04	3.65	4.85
T1-B (13)	4	3.69, 11.06	0.0322	>0.0227	4.55–5.60	2.61	4.42
T2-A (3)	5	3.83, 8.95	0.0444	>0.0227	3.12–3.68	2.52	3.23
T3-A (4)	5	3.89, 9.1	0.0660	>0.021	4.72–7.00	3.93	4.37
T3-A (4)	4	3.87, 9.1	0.0523	>0.021	5.55–7.28	3.93	4.48

Table 2c Examples of critical parameters utilised for selection of plunging equations for each test series (T1, T2, T3)								
Test series-Test section (Test number)	Probe number and water depth	Eq 6 linear - measured	Eq 7 extended-linear - measured	Eq 8 non-linear - measured	Eq 14 predicted stability number - for plunging wave	Eq15 predicted stability number - for surging wave	vdM Eq 1 predicted stability number - for plunging wave	Ursell Number
T1-A1 (1–6)*	5	2.23–2.60	2.52–2.93	2.68–3.25	1.94–2.16		1.14–1.34	4 to 20
T1-A2 (9)	5	3.26	3.59	3.89	3.61	4.14	2.00	24
T1-A2 (10)	5	3.59	4.03	4.44	3.69	4.06	2.13	28
T1-B (13)	5	4.55	4.98	5.60	2.61	4.42	1.37	38
T1-B (14)	5	5.25	5.89	6.76	2.91	4.37	1.63	49
T1-A1 (1–6)*	4	2.88–3.61	3.03–3.91	2.79–3.78	2.25–2.50	3.79	1.25–1.41	1 to 4
T1-A2 (9)	4	4.22	4.52	4.31	3.58	4.83	2.18	4
T1-A2 (10)	4	4.64	5.04	4.93	3.65	4.85	2.32	5
T1-B (13)	4	6.23	6.62	6.23	2.56	3.40	1.46	6
T1-B (14)	4	7.51	8.19	8.12	2.86	3.69	1.78	8
T2-A (1)	4	2.49	2.72	2.85	2.23	2.85	1.24	15
T2-A (2)	4	2.73	3.00	3.184	2.11	2.67	1.18	17
T2-A (3)	4	3.12	3.45	3.68	2.52	3.23	1.43	18
T3-A (1)	5	4.08	4.96	6.08	3.42	4.14	2.10	154
T3-A (2)	5	4.49	5.50	6.70	3.66	4.06	2.29	136
T3-A (3)	5	4.18	5.08	6.22	3.94	4.42	2.42	168
T3-A (4)	5	4.72	5.77	7.00	3.93	4.37	2.47	138
T3-A (5)	5	4.28	5.17	6.30	3.42	3.79	2.10	137
T3-A (1)	4	4.98	5.64	6.46	3.42	4.26	2.14	47
T3-A (2)	4	5.60	6.50	7.53	3.66	4.09	1.97	39
T3-A (3)	4	5.07	5.76	6.61	3.94	4.53	2.28	52
T3-A (4)	4	5.55	6.36	7.28	3.93	4.48	2.33	47
T3-A (5)	4	5.10	5.79	6.61	3.42	3.89	1.98	43

Table 2c Examples of critical parameters utilised for selection of plunging equations for each test series (T1, T2, T3)

T3-B (1)	5	4.19	4.89	5.87	3.93	3.46	2.36	125
T3-B (2)	5	4.35	5.04	5.99	3.97	3.19	2.38	112
T3-B (3)	5	4.35	5.12	6.15	3.93	3.42	2.39	133
T3-B (4)	5	4.68	5.52	6.59	4.10	3.15	2.53	125
T3-B (5)	5	4.37	5.14	6.18	4.39	3.55	2.68	134
T3-B (6)	5	4.38	5.09	6.05	4.21	3.71	2.53	113
T3-B (7)	5	2.65	3.09	3.71	3.71	3.43	2.22	125
T3-B (8)	5	2.67	3.08	3.65	3.97	3.68	2.35	106
T3-B (1)	4	5.78	6.48	7.15	3.93	3.36	2.33	49
T3-B (2)	4	5.81	6.47	7.07	3.97	3.59	2.33	44
T3-B (3)	4	5.46	6.06	6.63	3.93	3.30	2.27	45
T3-B (4)	4	5.70	6.33	6.89	4.10	3.55	2.38	43
T3-B (5)	4	5.60	6.24	6.85	4.39	3.37	2.57	46
T3-B (6)	4	5.47	6.03	6.53	4.21	3.91	2.41	40
T3-B (7)	4	3.67	4.11	4.54	3.71	4.36	2.19	49
T3-B (8)	4	3.73	4.16	4.56	3.97	4.25	2.34	45

* All negligible damage tests are categorised together

Test Results for Runup

Table 3 Results for runup on an impermeable slope utilising the USACE (2006) equation

Measured H_s, (probe)	WL (m AHD)	*Eq 18 linear	* Eq 18 extended linear	* Eq 148 non-linear	Eq17*USACE (2006)	*Measured runup
1.71 (p5)	0.05	3.74	4.28	5.37	5.01	3.25
0.78 (p5)	-0.19	2.06	2.23	2.77	7.15	1.45
1.63 (p5)	-0.19	3.28	3.81	4.88	4.89	2.05
1.24 (p5)	0.21	3.32	3.64	4.41	6.04	3.15
0.73 (p5)	0.65	3.08	3.21	3.62	8.24	3.65
1.34 (p4)	0.05	4.27	4.53	4.92	5.65	3.25
0.6 (p4)	-0.19	2.51	2.59	2.68	8.18	1.45
1.31 (p4)	-0.19	3.91	4.17	4.58	5.48	2.05
1.01 (p4)	0.21	3.88	4.05	4.24	6.66	3.15
0.54 (p4)	0.69	3.36	3.43	3.43	9.48	3.65

Figures 4a, 4b and 4c together with a simple error analysis based on difference between the three formulations and the predicted value indicate that: (a) the result estimated by the linear version (4) of the maximum momentum flux equation estimated the model results more accurately than the other three equations; (b) Figure 4d indicates that the linear version of Hughes formulation for runup on an impermeable slope (2004b) was more accurate than the USACE (2006) version. This was probably due to the reduced wave heights and therefore the non-linear influence of a greater percentage of breaking waves.

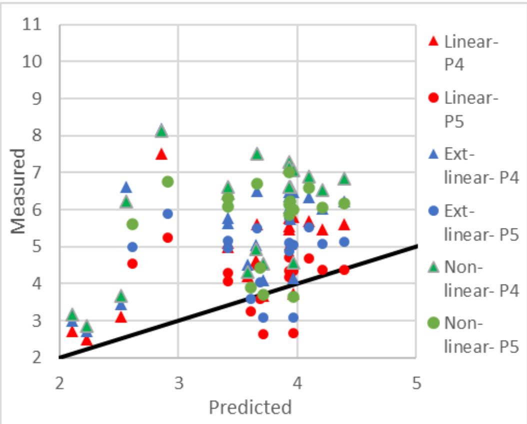

Figure 4a Graph of predicted vs. measured maximum momentum flux for all time series

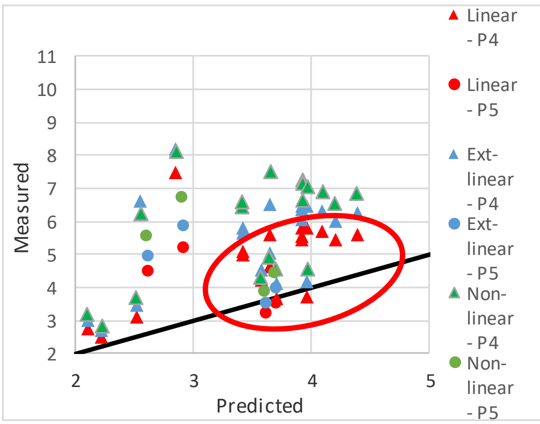

Figure 4b Graph of predicted vs. measured maximum momentum flux for Ursell Number <50 (more linear)

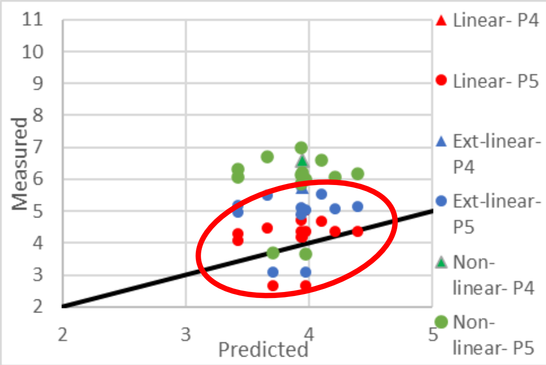

Figure 4c Graph of predicted vs. measured maximum momentum flux for Ursell Number >50 (more non-linear)

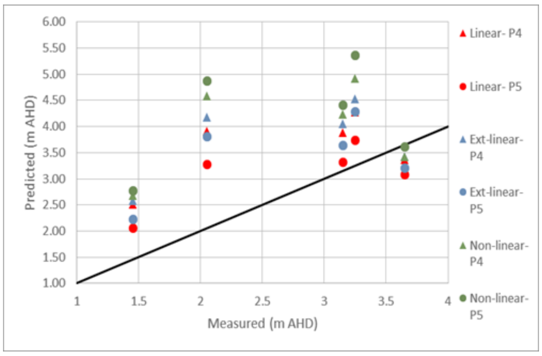

Figure 4d Graph of runup vs. measured runup on an impermeable surface using the maximum momentum flux equations)

Sensitivity Analysis of Maximum Momentum Flux Equation Parameters

For stability

Two parameters influence the predicted maximum flux parameter assessment: P, the notional permeability which in these series of tests was assessed to be P=.4 as advised in vdM (1987) for a two-layered structure using a geotextile and secondary armour in the model; and S_{mc}, the criteria utilised to assess the average of the time series wave steepness which in turn determines whether the plunging equation (14) or the surging equation (15) is utilised to predict stability. The sensitivity of P to the predicted stability for both a plunging wave and surging wave is indicated in Table 4. Time series T1-A2 (10) was selected due to the lower value of steepness (Table 2b).

Table 4 Variation of predicted stability number for plunging and surging wave (H&M) with notional permeability

Time series S_m = .0332 S_{mc} = .0227	P (notional permeability)	Predicted stability number (plunging wave Eq 14)	Predicted stability number (surging wave Eq 15)
T1-A2 (10)	0.3	3.47	4.42
T1-A2 (10)	0.4	3.65	4.85
T1-A2 (10)	0.6	3.93	5.49

For runup on an impermeable slope

Hughes runup equation is sensitive to the depth of water in front of the structure and therefore degree of wave breaking. The wave heights used were comparatively smaller for these tests. This is indicated in the accuracy of the linear equation in comparison to the extended and non-linear equation. For the conditions tested and small sample, runup based on the linear momentum flux equation appears to be more accurate than the USACE (2006) formulation.

Applicability to prototype design

The maximum momentum flux equations were applied to two training walls on the NSW coastline in conjunction with the use of numerical models, namely South Camden Haven breakwater and Bellambi breakwater.

Figure 5a Damaged South Camden Haven breakwater head armoured with 6T to 8T armour, January 2021

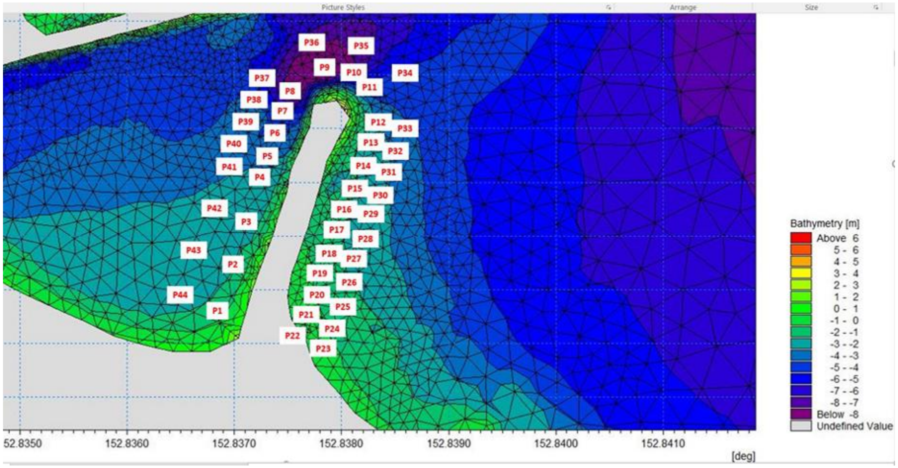

Figure 5b Spectral model utilised to obtain design wave height data from indicated locations for head design

The assessment of the South Camden Haven breakwater indicated the following:

- The offshore wave data buoy indicates 28 events that have exceeded a significant wave height of over 4 m since the June 2016 inspection.
- The well-accepted DHI Spectral Wave (SW) model was utilised to simulate 50-year (with and without SLR) and 100-year (with and without SLR) ARI conditions at the breakwater from NE, E, SE and S and obtain the wave height in order to estimate the design rock armour requirements for the head, trunk and root of the ocean side for the proposed repair.
- Given that at the location of wave height measured, the waves can be considered to be appreciably non-linear to the extent that if the maximum momentum flux equations were utilised to assess design armour of rock armours, the non-linear version would be deemed the most applicable. The Ursell number varied from 30 to 364 (where Ursell number >26 is considered to be non-linear).
- The repaired structure had little or no seconndary armour placed on it. Hence, a P=0.5 was assumed for the design.

Table 5 indicates that the non-linear form of the maximum momentum flux equation used in conjunction with the SW model indicated the design rock armour to be 8–10 tonnage. The damaged head had previously contained 6–8 tonne rock armour.

Table 5 Results from South Camden Haven head rock armour design (Ur>200 50Yr ARI +SLR design using formulae for Ursell numbers>300 K_D=3 p=0.5 and S=2, initiation of damage)

Design methodology	Design armour size (tonnes)
Hudson equation	13.7
van der Meer/van Gent (shallow water)	9.4
van der Meer (deep water)	9.0
Maximum flux methodology (linear)	3.5
Maximum flux methodology (extended linear)	6.4
Maximum flux methodology (non-linear)	8.9

Figure 5c South Camden Haven head repaired utilising 8T to 10T rock armour, November 2022

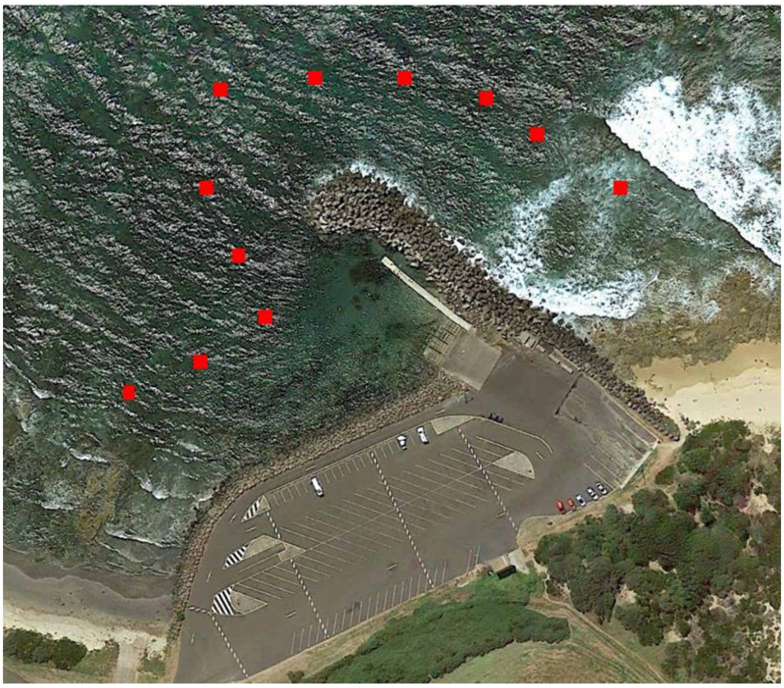

Figure 6 Bellambi breakwater undamaged for 40 years using Hanbars of 12T. Locations and depths at which 100-year ARI wave heights were assessed using DHI Boussinesq model

The assessment of the Bellambi breakwater indicated the following:

- The structure had little or no core material and there had been high wave transmission through the structure as the core had been washed away after its construction in 1978. Hence a P=0.6 was assumed for the design.
- An underwater inspection indicated that only two 12 T Hanbars of the original 400 Hanbars had been displaced, although the Port Kembla buoy which measured offshore waves indicated that over the past 40 years broken waves from over 50 offshore storms of Hs greater than 5m had impinged on the breakwater, indicating that the structure was over-designed.

- This breakwater armour design was utilised to calibrate the H&M equations for high porosity and minimum damage after weathering over 50 storms of over 5m Hs and remaining undamaged, with the displacement of only two of the 400 units. Table 6 indicates that the non-linear form of the maximum momentum flux equation used in conjunction with the BW (Boussinesq) model indicated a very similar 8T tonnage to the Hudson equation, thereby confirming the structure armour was conservatively designed.
- Given that at the location of wave height measured the waves were very non-linear with an Ursell number of 352 (where Ursell number >26 is considered to be non-linear) the design value from the extended linear or non-linear methodology is given greater weight and provides further verification regarding the proven stability of the Bellambi breakwater over 40 years since its construction.

Since numerous model tests and repairs have been carried out at MHL a well established K_D=6 for Hanbar units (Jayewardene 2009) was utilised for the design.

Table 6 Design armour sizing breakwater head for Hanbar armour for the 100-year ARI event using formulae for Ursell numbers>300 K_D=6, p=0.6 and S=2, initiation of damage

Expression	Hanbar head design (tonnes)
Hudson	Hanbar 8.1 Rock 16.2
Melby and Hughes (linear)	Hanbar-2 Rock 3.9
Melby and Hughes (extended linear)	Hanbar 2.5 Rock 5.0
Melby and Hughes (non-linear)	Hanbar 4.2 Rock 8.3

DISCUSSION

Stability in 2D Breakwater Model and Prototype Breakwaters

- The linear formulation of the maximum momentum flux equation appears to depict stability number more accurately for the stability tests carried out for Ursell numbers less than 50.
- The linear and extended linear formulation provides greater accuracy for time series with Ursell numbers greater than 50 in comparison to those under 50.
- Although not recommended a notional permeability value of 0.5 would have resulted in closer agreement between measured and predicted value of stability number.
- Use of the maximum momentum flux equations for prototype repair of breakwater heads for two breakwaters on the NSW coastline
- Only two units of the placed 400 units of Hanbar armour at Bellambi have been displaced over a period of 40 years. The offshore data buoy at this location has recorded over 50 storms with significant wave heights greater than 5m at this location, hence it could be concluded that the breakwater is over-designed with 12T Hanbar armour.
- The non-linear form of M&H equations predicted that 4.2 T Hanbar concrete armour instead of the existing 12T armour would suffice for an over-designed breakwater at Bellambi which had been virtually undamaged after weathering over 40 Hs> 5.0m storms for 40 years.
- The non-linear form of the M&H equation resulted in 8.9T armour rock for South Camden Haven and was less conservative than the result given by Hudson's equation. The Ursell numbers for waves at the head varied from 30 to 364, hence the applicability of the non-linear form.
- No clear quantitative criteria are provided in the demarcation of the use of M&H plunging and surging wave unlike in the vdM formulation (1987).

Runup

For the limited number of tests carried out on an impermeable slope the Hughes linear wave runup formulation was more accurate than the USACE formulation.

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Notation

Greek Symbols

- α beach or structure slope
- Δ armor unit immersed relative density
- η instantaneous sea surface elevation relative to still water level
- η_s instantaneous sea surface elevation relative to the sea floor
- ξ local Iribarren number (surf similarity parameter)
- ξ_o deepwater Iribarren number
- π mathematical Pi
- ρ mass density of water
- ω circular wave frequency [$=2\pi/T$]
- g gravitational acceleration
- h water depth from bottom to the still water level
- H_{mo} zeroth-moment wave height related to the area beneath the spectrum
- H_s significant wave height for irregular wave train
- k wave number [$=2\pi/L$]