

PHYSICS-INFORMED DEEP LEARNING OF NEARSHORE WAVE PROCESSES

Qin Chen¹, Nan Wang¹ and Zhao Chen¹

The paper introduces the *NWnets*, a physics-informed deep learning model for reconstructing nearshore wave fields and mapping bathymetry. The physics encoded into the deep neural networks are the wave energy balance equation and dispersion relation. Insights into the model capability are gained through application of the *NWnets* to a laboratory experiment of wave transformation over a circular shoal. If the bathymetry and discrete measurements of wave height are available, the *NWnets* model is capable of simulating nearshore wave transformation. Moreover, the extended *NWnets* can be used for depth inversion if the bathymetry is unknown. Two methods for simultaneously estimating water depths and surface waves are presented. If surface wave number and limited wave height measurements are available from remote sensing platforms, the first method employs wave numbers and scarce measurements of wave height as training data. The second method utilizes scarce wave height and limited water depth measurements as training points to reconstruct bathymetry and wave fields. The results show that both methods are capable of simultaneously mapping the bathymetry and waves when the locations of training points are appropriately distributed.

Keywords: Depth inversion; physics-informed neural network; deep learning; wave transformation; bathymetry mapping; wave field reconstruction

INTRODUCTION

Numerical models solving the conservation laws of fluid motion and field observations from in-situ or remote sensing platforms have played an important role in understanding nearshore processes. Data assimilation techniques allow for the combination of field data with physics-based models to further our understanding of coastal processes and improve forecast skills of numerical models. In the past several years, a new approach that integrates the conservation laws with observations using scientific machine learning or deep learning (DL) has emerged. One of the new methods is the Physics-informed (or guided) Neural Network (PINN). Compared with conventional machine learning models, PINNs encode prior scientific knowledge (e.g. conservation laws) during the training process of the neural networks, which constrains and compensates for the insufficiency or scarcity of field observations in many applications (Karniadakis et al., 2021; Jin et al., 2021; Wang et al., 2022). Models based on PINNs have the potential to provide accurate predictions consistent with the physical laws even with inaccurate or missing data as an input (e.g. Chen et al. 2021). The objective of this study is to demonstrate the suitability and utility of PINNs for coastal research by introducing the nearshore wave nets (*NWnets*) for reconstruction of nearshore wave fields, inversion of wave height and bathymetry from remotely sensed data, and discovery of new knowledge (parameterizations) from physics-guided deep learning of laboratory or field observations.

The paper is organized as follows. First, we introduce the conservation law encoded into the neural network as well as the algorithm and model setup for the PINNs. Next, the performance of the *NWnets* is demonstrated by modeling wave transformation over a three-dimensional circular shoal based on measurements from a laboratory experiment. After that, we present results of simultaneously mapping bathymetry and reconstructing nearshore wave fields using the extended *NWnets* followed by a discussion on future research of PINNs and concluding remarks on this study.

METHODOLOGY

Energy Balance Equation for Wave Transformation in the Nearshore

In principle, all conservation laws of mass, momentum, and energy that govern the nearshore wave processes can be encoded into DL models. In this study, we focus on integral wave parameters such as significant wave height and mean wave direction, and directional distribution of wave energy of a stationary wave field. Wave shoaling, refraction, and depth-limited breaking are the dominant processes in the nearshore. Thus, the governing equations encoded into the fully connected neural networks are the wave energy balance equation and dispersion relation. The effect of amplitude dispersion (i.e., nonlinear dispersion relation) on depth inversion and wave field reconstruction is also included. For water waves, the energy balance equation can be expressed as:

$$\frac{\partial ec_{gx}}{\partial x} + \frac{\partial ec_{gy}}{\partial y} + \frac{\partial ec_{g\theta}}{\partial \theta} + d_w = 0 \quad (1)$$

¹ Department of Civil and Environmental Engineering, and Department of Marine and Environmental Sciences, Northeastern University, 360 Huntington Ave., Boston, MA, 02115, USA.

where e is the wave energy density in each directional bin, c_g is the group velocity, θ represents the angle of incidence with respect to the x -axis, and d_w is the dissipation of energy density caused by wave breaking. The Janssen and Battjes (2007) formulation for wave breaking is used to compute d_w . In addition to the linear dispersion relation, the nonlinear dispersion relation proposed by Kirby and Dalrymple (1986) is utilized to examine the effects of amplitude dispersion on depth inversion and wave field mapping. Details about the governing equations encoded in the deep neural network can be found in Chen et al. (2023).

Physics-Informed Neural Networks (PINN)

A novel composite PINN model is developed to find the solutions to the phase-averaged and depth-integrated conservation law, i.e. energy balance equation (EBE), for mapping nearshore bathymetry and wave transformation. The corresponding residuals from the EBE are used as restraints for the training of the *NWnets* to generate physically consistent predictions. Moreover, the *NWnets* are also constrained to fit the available measurements scattered in the computational domain. Because the wave energy density function varies in the horizontal plane and the directional space, while wave number only changes in x and y space, composite neural networks are utilized.

Figure 1 shows the schematic representation of the algorithm for simultaneous mapping of nearshore bathymetry (depth inversion) and wave field. The loss function consists of two parts. The first part corresponds to the collocation points (i.e., residual loss), where the physical constraints are imposed to encourage that the energy balance equation and dispersion equation are satisfied. In general, the collocation points can be grid points or random points inside the computational domain (Lu et al., 2021), and the former one is chosen in this study. Automatic differentiation is used to calculate the partial derivatives in the residual expression (Kissas et al., 2020). The second part encourages the outputs of PINNs to match wave parameters obtained from field or lab observations (i.e., measurement loss). Therefore, the total loss function for mapping nearshore bathymetry and waves is the summation of the residual loss of encoded physics and the measurement loss owing to the errors between the model prediction and the observations. Minimizing the total loss function leads to the solutions that nearly satisfy the energy balance equation and match the measurements.

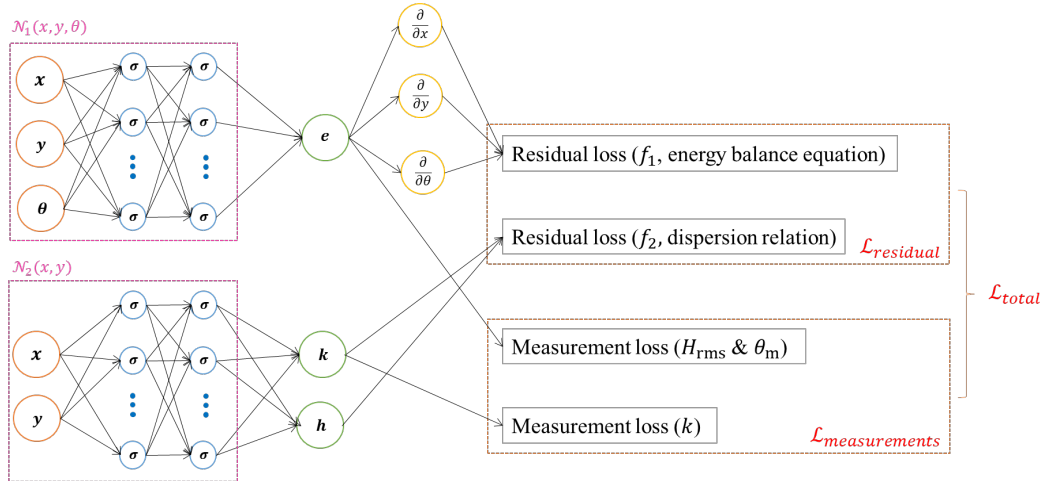


Figure 1. A schematic representation of *NWnets* for reconstructing nearshore wave field and bathymetry.

PINN models have been developed to solve a wide range of scientific problems recently, such as fluid flows (Jin et al., 2021; Sun et al., 2020; Gao et al., 2021), cardiovascular flows (Kissas et al., 2020), vortex-induced vibrations (Raissi et al., 2019), and reconstruction of surface wave field (Wang et al., 2022). By infusing the governing equations into the deep neural networks, PINNs can bridge the gap between ML-based methods and scientific computations and deduce solutions involving partial differential equations.

We assess the performance of the *NWnets* for solving the EBE and dispersion relation by reconstructing the wave field over a submerged circular shoal. Wave measurements from Chawla and Kirby (1996) are used for training, validating, and testing. We apply relatively simple feedforward neural networks without additional regularization. Hyperbolic tangent is used as the activation function. All the networks are initialized with Xavier initialization. Normalization is carried out to

keep the input and output between -1 and 1, and the EBE is also normalized by the same factor. Thus, network weights and biases have values of similar magnitudes, and the negative impact of the large difference between various parameters is avoided. The influence of network structures on the performance of the *NWnets* is tested. Details can be found in Wang et al. (2022).

Experimental Design for Mapping Nearshore Bathymetry and Waves

Recent advances in remote sensing technologies capable of observing a broad spatiotemporal range of geophysical parameters enable real-time monitoring of the nearshore, such as video cameras, radar, infrared, and LiDAR on shore-based towers or airborne platforms (e.g. Wilson et al., 2014). In this study, we take a simplified approach by using synthetic model data instead of actual remotely-sensed data to demonstrate the model performance of PINN. We assume that the free surface parameters derived from remote sensing data are sufficiently accurate in this work. Two methods are proposed to map the bathymetry and wave field based on the availability of field observations.

Method A: We assume that the surface wave celerity (or wave number) and limited wave height measurements are available from various remote sensing platforms. Then, the bathymetry and wave fields are determined by the inverse PINNs developed with wave number and scarce wave height measurements as the training data. The performance of this method is investigated by solving the depth inversion problem over a three-dimensional (3D) barred beach with the simulation data from XBeach (Roelvink et al., 2009). Furthermore, we examine the effects of amplitude dispersion (i.e., nonlinear dispersion relation) on depth inversion and wave prediction using waves over an alongshore uniform barred beach as an example.

Method B: When the wave number or wave celerity data are unavailable, PINN models can still be utilized to simultaneously map the bathymetry and reconstruct wave fields if wave heights and water depths at limited locations are partially known. The second method uses the scarce measurements of wave height and water depth to train the PINNs for mapping nearshore bathymetry in the absence of wave celerity or wave number measurements. The model performance is investigated by solving the depth inversion problem over the 3D barred beach under field conditions at Duck, NC, USA. Although this test case is the same as the one used for examining Method A, the training data and network structures are different from those in Method A.

RESULTS

Reconstruction of Wave Field Over a Circular Shoal by *NWnets*

A series of laboratory experiments on wave propagation over a circular shoal were carried out by Chawla and Kirby (1996) in a directional wave basin at the University of Delaware. In this study, test case 4 of the laboratory experiments with the directional random wave input is utilized as the testbed to examine the performance of *NWnets*.

Figure 2 shows the model results produced by the *NWnets*. The top panel illustrates the modeled spatial distribution of H_{rms} normalized by the incident wave height over the circular shoal in the Chawla and Kirby's (1996) physical experiment when all of the 126 wave measurements are employed as the training data. It is seen that the reconstructed wave field captures not only wave focusing and defocusing by the shoal, but also the combined effect of wave refraction and diffraction, or wave interference. By contrast, if a quarter of the measurements are used as the training data, as shown in the bottom panel, although the focusing and defocusing of wave energy is captured well by the model, wave diffraction or wave interference is absent in the modeled wave field because the wave energy balance equation does not include wave diffraction. This suggests that the *NWnets* could be used to discover the missing mathematical terms in the energy balance equation to account for wave interference if sufficiently dense wave measurements are available.

Figure 3 shows the comparisons between the PINN-simulated and experimental data over the circular shoal. Solid lines represent the outputs from the *NWnets*, and circles are the measurements in the laboratory. The experimental data used for training and validating the PINN models are shown by the red- and black-filled circles, respectively. The hollow circles represent the testing data for the model. When half of the wave height measurements are used for training (i.e., 63 training data), good agreement was found between the experimental and simulated wave heights. The simulation accuracy decreases when a quarter of the wave height measurements (i.e., 31 training points) are used as training data, but the *NWnets* can still capture the focusing and defocusing of wave energy caused by the bathymetric variations. Unsurprisingly, a better simulation performance can be obtained with more experimental data employed as training points. Note that the water depth is known in this test

case. The only training data are wave height measurements. No wave number or wave celerity is required for the training of the *NWnets* when only reconstruction of the wave field is desired.

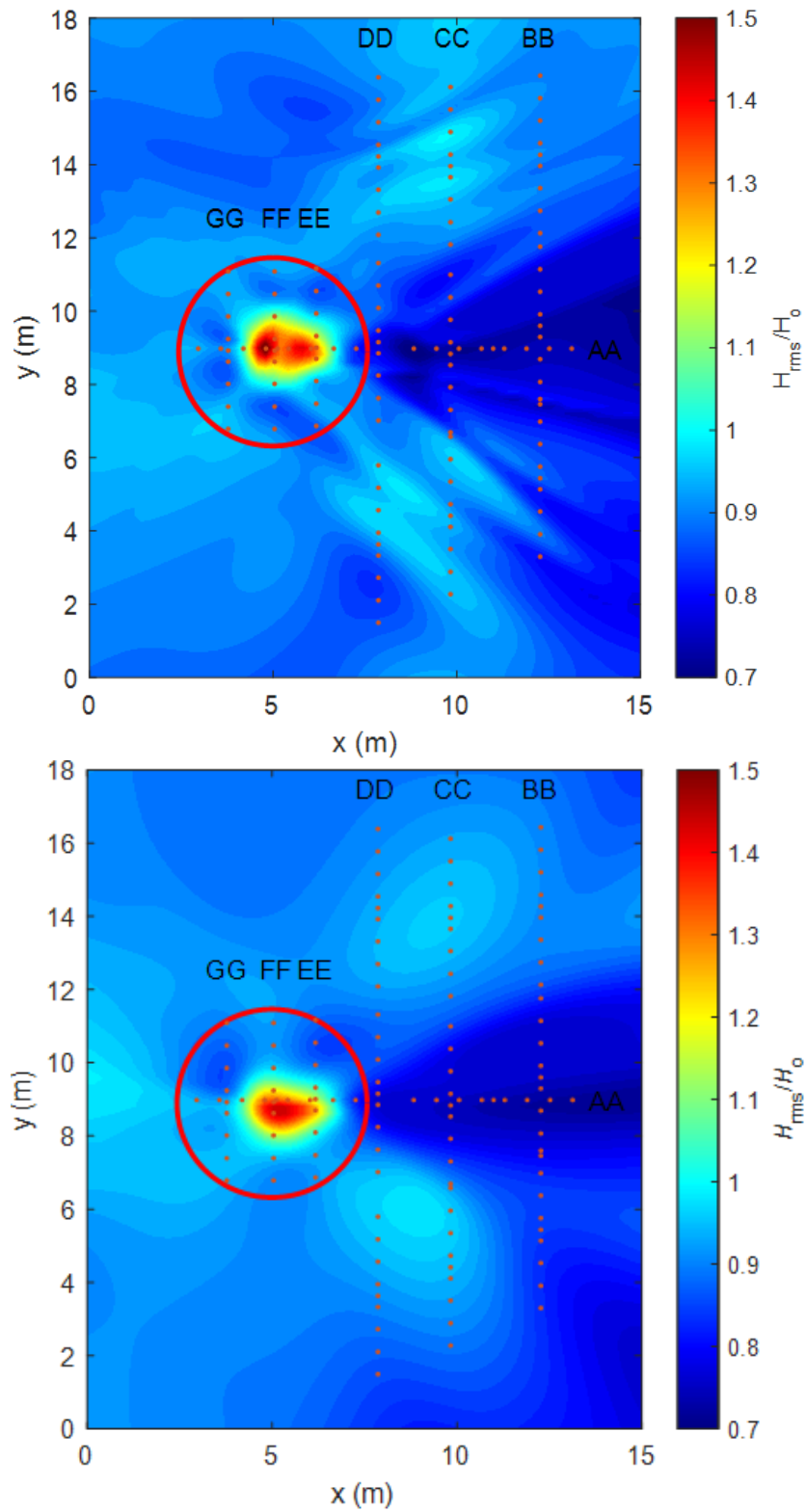


Figure 2. Reconstructed wave fields over a submerged shoal (red circle) by the *NWnets*. Left: using all the measurements (red dots) from Chawla and Kirby (1996) as training data; Right: using 1/4 measurements as training data. Waves propagate from left to right.

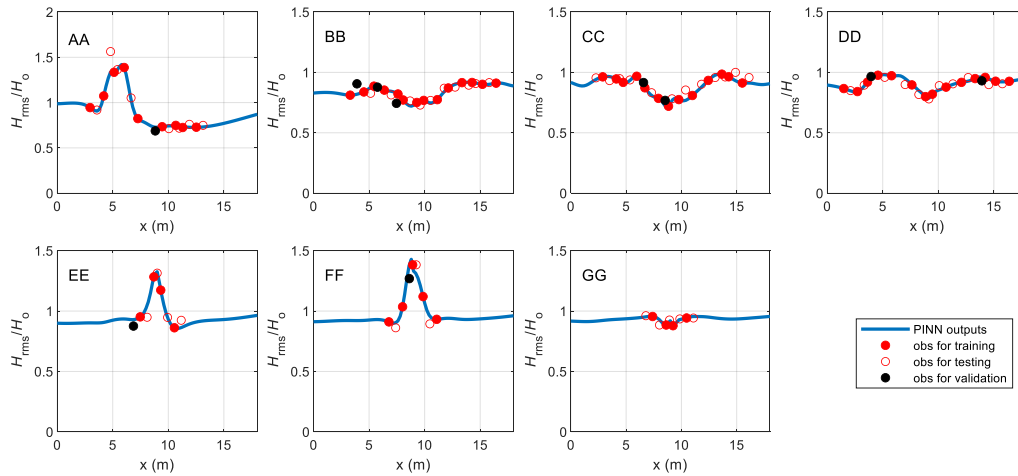


Figure 3. Comparisons of experimental and PINN-simulated wave heights with half of the wave height measurements used as training data.

Simultaneous Mapping of Bathymetry and Waves on a Barred Beach with Method A

We use the beach profile survey data from the US Army Corps of Engineers Field Research Facility (FRF) at Duck, NC to construct the 3D barred beach for testing. The wave condition offshore of the barred beach is set as $H_{rms}=1$ m with a peak wave period of 8 s. The peak wave period remains constant over the entire computational domain. The computational domain extends from $x = 0$ to 980 m in the cross-shore direction and from $y = 20$ to 480 m in the alongshore direction (FRF coordinates) with a resolution of 10 m. The resolution of directional spreading of waves is set to be 10° in both XBeach and PINN models, and the lower and upper directional limits are defined as -90° to 90° , respectively. A total of 4653 collocation points are uniformly distributed from $x = 0$ -980 m and $y = 20$ -480 m to constrain the deep learning for generating physically consistent predictions. The outputs from XBeach and PINNs are compared to determine the feasibility of using PINNs to estimate water depth and reconstruct wave fields over a 3D barred beach with Method A.

Figure 4 shows the comparison between the PINNs and XBeach outputs. The contour plot in the left panel depicts the simulation error of PINN-predicted water depth with the white dots showing the locations of H_{rms} training points. It can be observed that PINNs have good skills for estimating water depths with small errors (maximum error = 1.6%). The 3D plot in Figure 4 presents the PINN-simulated wave heights which is in good agreement (RMSE = 5 mm) with the numerical results from XBeach. Overall, the developed PINN model has a promising ability to simultaneously estimate water depths and reconstruct wave fields over a 3D barred beach with known wave numbers and scarce wave heights (synthetic data from XBeach) applied as the training data.

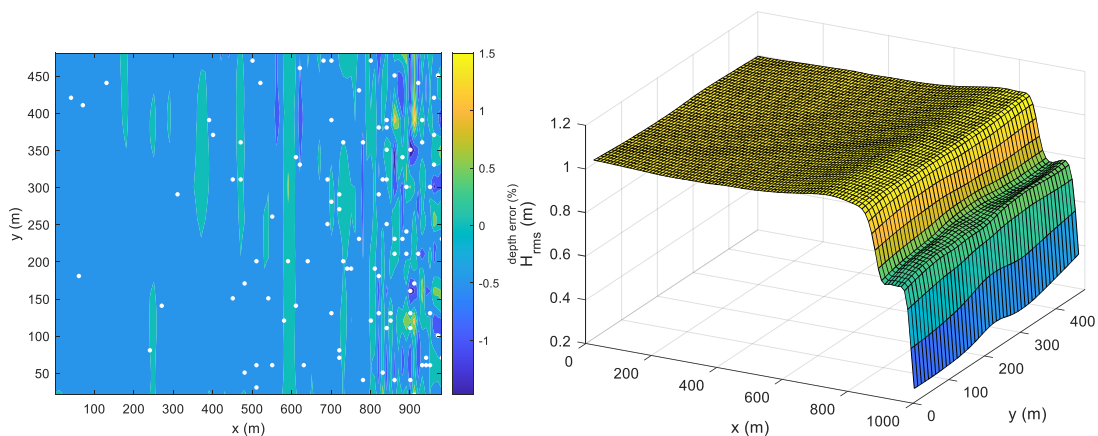


Figure 4. Comparison between the XBeach and PINN outputs over the barred beach. Left: contour plot shows the simulation errors of the PINN-predicted depth. The white dots represent the locations of wave height training points; Right: spatial variation of the PINN-predicted wave heights.

Effects of Amplitude Dispersion on Depth Inversion

One advantage of using PINNs to estimate nearshore bathymetry from remotely sensed data is that nonlinear dispersion relation can be embedded in the model. To test the PINNs' ability to account for the effect of wave nonlinearity in depth inversion (e.g. Grilli, 1998; Yoo et al., 2011), we use the nonlinear dispersion relation instead of the linear dispersion relation to reconstruct the wave field and estimate the bathymetry over an alongshore uniform barred beach with known wave numbers. Fig. 5 (left) shows that the PINN outputs are in good agreement with the numerical solutions to the EBE with the nonlinear dispersion relation embedded in the model. Such synthetic data are called "reference data." We also estimate the nonlinear wave field with the linear PINN model. It can be seen that the skills of the PINN model encoded with the linear dispersion relation deteriorate in the surf zone (Fig. 5-right). In other words, the PINN model embedded with the linear dispersion relation is not capable of learning the effect of nonlinear waves on wave propagation. This finding indicates that selecting an appropriate physical constraint in PINN is crucial for solving depth inverse problems and reconstructing wave fields with sufficient accuracy. For field applications where the observed wave number vectors are strongly influenced by wave nonlinearity, it is expected that the PINN model embedded with the nonlinear dispersion relation will give a more accurate estimate of water depth in the surf zone than do existing methods.

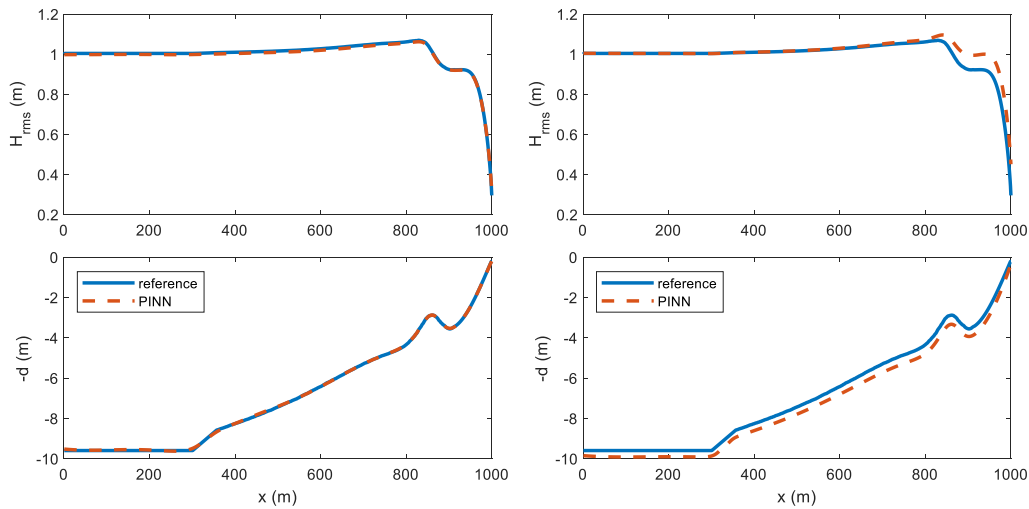


Figure 5. Comparisons between reference and PINN-simulated wave height and water depth with nonlinear dispersion relation (left) and linear dispersion relation (right).

Simultaneous Mapping of Bathymetry and Waves on a Barred Beach with Method B

In field experiments or monitoring programs, nearshore bathymetric data are routinely collected along multiple cross-shore transects spaced about 50 m apart, such as the Field Research Facility (FRF) in Duck, NC. Normally, these surveys are then interpolated linearly in the cross-shore and alongshore directions to obtain the bathymetry of the entire area of interest, which is then used as an input to physics-based numerical models (e.g., Chen et al., 2003) to simulate the nearshore wave processes. To reconstruct the bathymetry and map the wave field with Method B, we use the measured data of water depth along the cross-shore transects and wave heights scattered between adjacent transects as training points. The resolution of measured bathymetric data in the alongshore and cross-shore directions is set as 50 m (or 110 m) and 10 m, respectively. The measured wave height locations are randomly selected inside the domain with more data placed nearshore (i.e., $x = 800-980$ m). The PINN-predicted wave height and water depth are then compared to the true data to examine the performance of Method B. In this study, the synthetic data from XBeach are used as the "observational data" for demonstration purposes. Testing of the inverse PINN models against real field observations will be carried out in future studies.

In this study, we examine the performance of Method B by estimating the water depth based on measured bathymetry along cross-shore transects and wave heights scattered between adjacent transects, an analog to the long-term nearshore surveys at FRF in Duck, NC. Figure 6 shows the spatial distributions of the simulation errors in the PINN-predicted water depths. The results indicate that bathymetry and wave height (not shown) can be well estimated by the developed PINN models

with measured water depths along a limited number of cross-shore transects and scarce wave height measurements applied as training data (without wave number data). Compared to linear interpolation, the inverse PINN models provide a more accurate way to determine the water depths over the entire domain, because small-scale bathymetry changes between the cross-shore survey transects can be captured by the limited wave height measurements used to train the inverse PINN model.

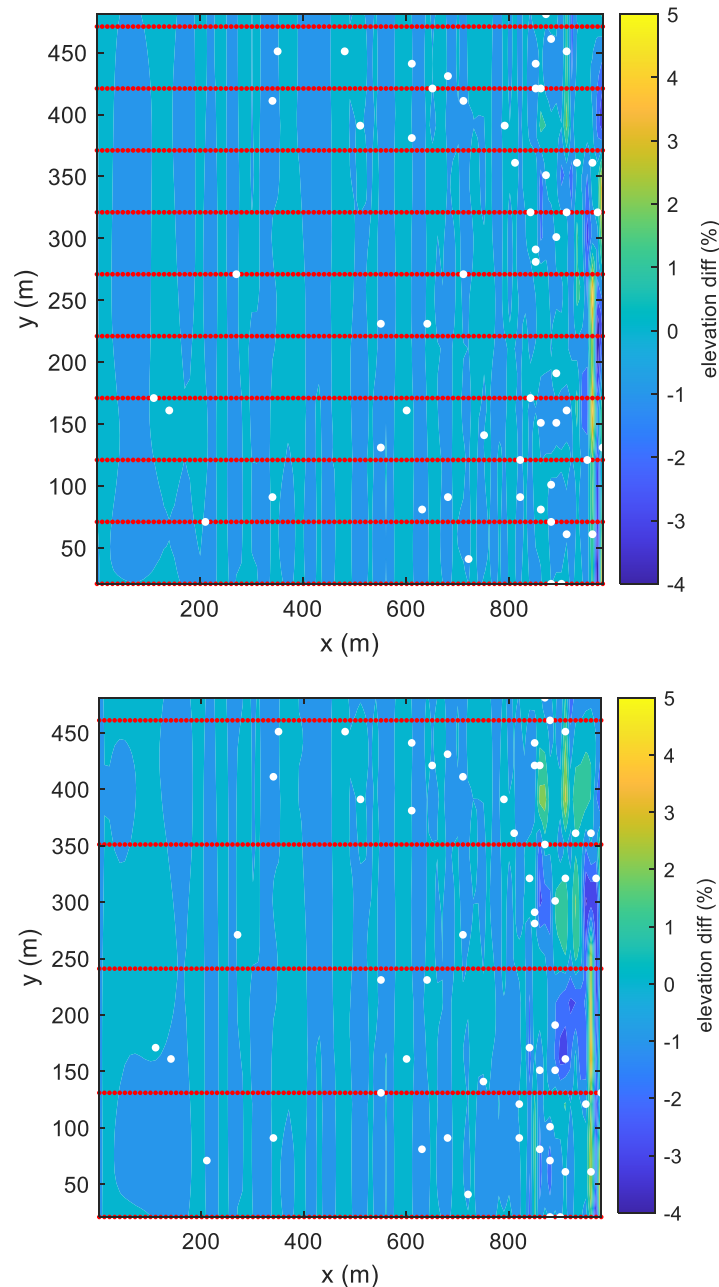


Figure 6. Simulation errors of the PINN-predicted water depths with 10 cross-shore transects of 50m apart (top) and 5 cross-shore transects of 110 m apart (bottom) as training data. The red and white dots represent the locations of the training points for water depth and wave height, respectively.

CONCLUSIONS

We introduce the *NWnets*, a composite physics-informed neural network deep learning model for nearshore research. The physics encoded into the deep neural networks are the wave energy balance equation and dispersion relation. Insights into the capability of PINNs are gained through application of the *NWnets* to a well-documented laboratory experiment of wave transformation over a submerged circular shoal. If the bathymetry and discrete measurements of wave height are available, the *NWnets*

model is capable of reconstructing a complex wave field. The nearshore wave processes, such as wave shoaling, refraction, diffraction, and depth-limited breaking, are well captured by the *NWnets* even wave diffraction is absent in the energy balance equation. This is attributed to the large amount of wave height data available for training, suggesting the potential of PINN to discover missing physics in the encoded equation (e.g. energy balance equation) from measurements.

To extend the *NWnets* for depth inversion if the bathymetry is unknown, we introduce two methods to simultaneously estimate the water depth and wave field. Assuming surface wave number and limited wave height measurements are available from various remote sensing platforms or synthetic data from a physics-based model, the first method employs wave numbers and scarce measurements of wave height as training data. The second method utilizes scarce wave height and limited water depth measurements as training points to reconstruct bathymetry and wave fields. The results show that both methods are capable of simultaneously mapping the bathymetry and wave fields when the locations of scarce training points are appropriately distributed.

Although physics-informed deep learning models are promising, more studies are definitely needed to test the performance of the *NWnets* under field conditions. Furthermore, other conservation laws, such as the continuity and momentum equations, ought to be encoded into deep neural networks trained with remotely-sensed phase-resolving data to further our understanding of nearshore wave processes.

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