MODELLING OF SPILLING AND PLUNGING BREAKING WAVES IN SPECTRAL MODELS

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Based on the data of field experiments and modeling was revealed that the dissipation of the energy of the high-frequency part of the wave spectrum due to wave breaking should compensate the nonlinear growth of higher wave harmonics, which occurs in different ways both for waves breaking with different types and for different methods of modeling a nonlinear source term. The effect of the dissipative term type used on the estimates of sediment transport is discussed.

Keywords: wave modelling, wave breaking, dissipation, wave spectra

INTRODUCTION

When approaching the shore and with decreasing water depth, the waves transform, becoming more asymmetrical, and then breaking. The process of wave breaking is characterized by significant losses of their energy. Changes in the asymmetry of waves occurring both nonlinear and breaking processes, lead to a change in the higher statistical moments of wave motion, which determine not only the magnitude, but also the direction of sediment flow in the coastal zone (Saprykina, 2020).

There are a number of models that describe with high accuracy the linear and nonlinear transformation of waves over the real bottom topography in the coastal zone. However, dissipation processes have been much less studied. Until now, there is not only a generally accepted and well-tested model of wave energy dissipation in the surf zone, but also an accurate physical description of changes in wave properties during the dissipation process (Mase, Kitano, 2000). In general wave models can be divided into phase resolving and phase averaged. First usually are solved using hydrodynamic equations in time domain and second represent the wave spectrum evolution in frequency domain based on the wave action balance equation (for example, SWAN model). Some phase resolving wave models are solved by spectral methods in frequency domain that provide less time consuming for modelling with comparing time domain simulation. As result of spectral method modelling are the complex amplitudes of wave harmonics (deterministic models (Kirby, Kaihatu, 1996; Madsen, Sørensen, 1993) or modulus of complex amplitudes and biphases (stochastic models (Eldeberky, Madsen, 1999)).

Main difficulties for accurate representation and modeling wave breaking in phase resolving models is to account spatio-temporal occurrence of breaking events that connect with type of wave breaking also. In spectral-type models the main challenge is to describe the effect of wave energy dissipation due to breaking on the frequency structure of waves and wave spectrum shape (Kaihatu, Kirby, 1996).

To assess the effect of wave breaking on the amplitudes of wave harmonics and wave spectrum shape, two approaches are now used:

1) the wave breaking does not change the shape of the spectrum, but only reduces the spectral density of waves by a factor of the same for all frequencies, so-called independent wave energy dissipation (Battjes, Beji, 1992, Beji, Battjes, 1993; Eldeberky, Battjes, 1996);

2) the spectral density during wave breaking decreases depending on the square of the frequency (Kirby, Kaihatu, 1996; Mase, Kirby, 1992).

The concept of uniform or frequency independent wave energy dissipation is often used in modeling the evolution of spectra in phase-averaged models, for example, SWAN.

The combination of two approaches offers an empirical formula that assumes a partial dependence of the dissipation coefficient on the square of the frequency (Kirby, Kaihatu, 1996; Elgar, et al., 1997). The quadratic type of the dependence was shown theoretically in (Mase, Kirby, 1992). However the dependence on the fourth power of frequency was demonstrated in (Bredmose et al., 2004) also.

Based on the data of two field experiments, was showed that in the outer part of the surf zone, the dissipation of wave energy is practically independent of frequency, and in the inner part of the surf zone, either a partially quadratic or selective (at frequencies of the second and third harmonics) dependence of the wave energy dissipation on frequency. It was noted that the type of frequency

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dependence of the dissipation of wave energy inside the surf zone is determined by the wave asymmetry and the bottom slope (Kuznetsov, Saprykina, 2004).

Comparison of the simulation results with the data of laboratory and field experiments (Chen et al., 1997) showed that the exact form of the frequency dependence of energy dissipation is not so important for calculating and modeling wave spectra. But the exact form of the frequency dependence of energy dissipation has a significant effect on the values of the higher wave statistical moments obtained from the elevations of the free surface, which is of great importance for assessing the processes of sediment transport (for example, Antsyferov et al., 2005; Bredmose et al., 2004).

Analysis of waves breaking in the surf zone showed that the energy losses observed in the wave spectrum are primarily the result of nonlinear transfer from the spectral peak to higher frequencies, and therefore energy dissipation occurs mainly in the high-frequency part of the spectrum (Herbers et al., 2000). This indicates that nonlinear processes can play a more significant role in the transformation of the wave spectrum than processes caused by wave breaking. Nonlinear processes of wave transformation also determine the processes of breaking. For example, different amplitude-frequency composition of waves leads to the different types of breaking (Saprykina et al., 2020).

Thus, it can be stated that until now, when describing dissipation processes, there is not only a unified approach and universal dependence for energy dissipation during wave breaking on frequency, but also a clear physical description of changes in wave spectrum during their breaking under the influence of dissipative and nonlinear processes.

The purpose of this work is to study how the dissipation of the spectral energy of waves breaking by different types influences the wave frequency spectrum, how this should be taken into account in modeling and what could be the consequences of using the results of incorrect dissipation modeling.

METHODS

Field experiment

The experiment "Shkorpilovtsy-2007" was carried out on the Black Sea in September-October 2007 on the pier of the Institute of Oceanology of the Bulgarian Academy of Sciences. The length of the pier, specially designed for the coastal zone dynamics researches, is 230 m. The pier has a single line of rare piles minimizing their influence on the wave field measurements. The pier is located in the middle of an almost straight-line section of the coastline of sandy beach. During the experiment, 8 capacitive wire string wave gages (sampling frequency 200 Hz) and 7 resistance wire wave gauges (sampling frequency 5 Hz) were installed. The typical synchronously measured on 15 gauges wave chronograms registered on 27 September 2007 was selected for analysis. The duration of the selected wave records was 20 min. In this day the wave regime practically did not change, and the waves broke in two types. At a distance of 160 - 170 m – wave break by the spilling type, and closer to the shore – by the plunging type (at a distance of 60-70 m). The scheme of the experiment and positions of wave breaking are shown in Fig. 1. A more detailed description of the wave regime on that day is given in (Saprykina et al., 2020, Saprykina, 2020). For the analysis, we selected chronograms of wave transformation between gauges 9 and 12 (spilling breaking) and 1 and 4 (plunging breaking). The wave spectra at the points before and after wave breaking are shown in Fig. 2.

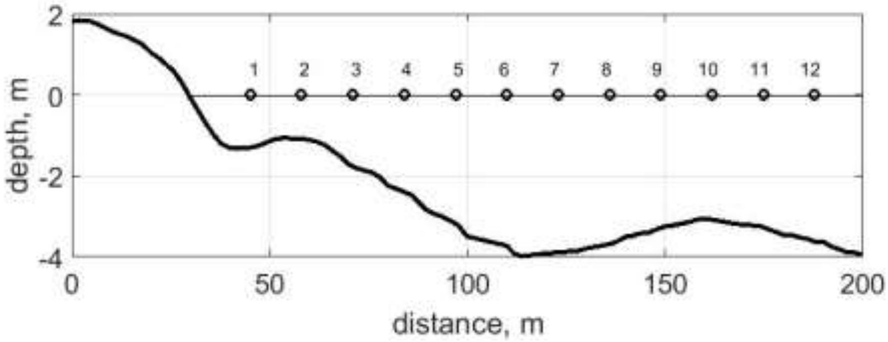
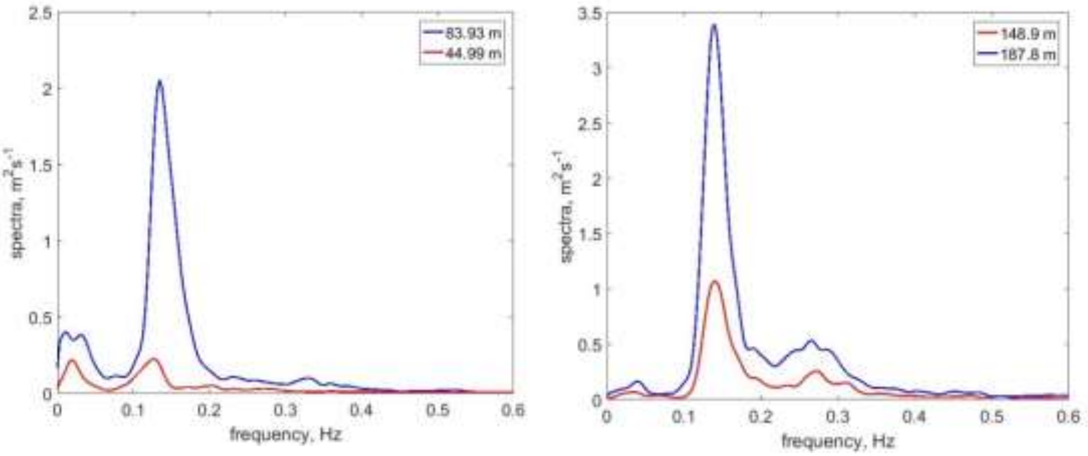


Figure 1. Setup of field experiment and positions of wave breaking.



(a) (b)

Figure 2. Wave spectra before and after wave breaking, field experiment a) plunging breaking waves; b) spilling breaking waves.

Wave energy dissipation coefficient

A technique for estimating the dissipation coefficient as a function of frequency is based on a comparison of the model (without modelling wave dissipation, only wave nonlinearity is taken in account) and experimental (naturally taking into account both wave nonlinear processes and dissipation) wave spectra. It was proposed and described in detail in (Mase, Kirby, 1992).

$$\alpha_n = \frac{S(x_{start} + \Delta x)_{calc} - S(x_{start} + \Delta x)_{meas}}{S(x_{start})_{meas}(2\Delta x)}, \qquad (1)$$

where S is the wave spectrum at the corresponding points, the indices "calc" and "meas" mark the spectra calculated by the model without taking into account dissipation and the experimentally measured spectra, respectively; Δx is the distance of wave propagation, x_{start} is the coordinate of the starting point at which the modelling begin.

With such an estimate, it is assumed that dissipation does not affect the process of nonlinear deformation at a distance Δx, although in fact the rate of generation of new harmonics depends on the amplitudes of interacting harmonics, which can decrease along all considered distance due to dissipation during wave breaking. Thus, in the method used, the energy dissipation of breaking waves does not occur uniformly and constantly, but only once at the end of the considered distance, which does not correspond to the physics of the real continuous process of breaking waves and can lead to underestimated dissipation coefficient. Detailed analysis of the possibility of using such an approach for qualitative assessments of the dependence of the dissipation coefficient on frequency is shown in (Kuznetsov, Saprykina, 2004).

The modeling was carried out using two spectral models: a phase-averaged SWAN model version 41.31AB and a phase-resolving model based on Boussinesq-type equations with improved dispersion characteristics solved by spectral methods (Madsen, Sorensen, 1993).

For the phase-resolving Boussinesq model, the initial complex amplitudes of all harmonics were determined from the Fourier series expansions of the measured free surface elevations. The wave spectra were calculated as quadratic modulus of complex amplitudes with the following moving averaged procedure. Frequency resolution was 0.01 Hz.

For the SWAN model at the input, the spectra constructed from the wave chronogram were set. SWAN model was run on 1D stationary mode with a horizontal discretization of 2 m and in spectral domain the total number of frequencies were set as 38 with minimum and maximum frequencies as 0.04 and 1.4961 Hz. Dissipation terms were disabled including breaking, white capping, turbulence, and bottom friction. No wind is considered during simulations.

The nonlinear transformation of the spectrum in SWAN model was simulated in two different ways: a) using default nonlinear triad source term - the Lumped Triad Approximation model (LTA)

(Eldeberky, 1996; Booij et al., 1999) and b) using the Stochastic Parametric model based on Boussinesq equations (SPB) (Becq-Girard et al., 1999).

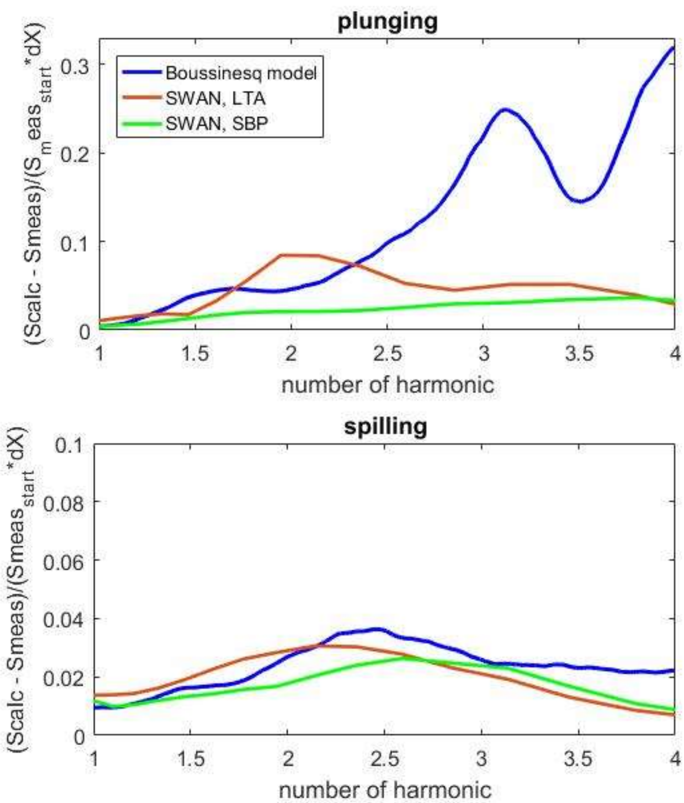
As was shown in (Salmon, et al., 2016) SPB model provides better agreement with the observations including the surf zone.

Evaluation of the dissipation coefficient according to formula (1) was carried out between gauges 11 and 9 for spilling and 4 and 1 for plunging breaking waves. The initial spectra were set from the experimentally measured free surface elevations on gauges 11 and 4, respectively.

DISCUSSION OF RESULTS

Dependence of dissipation coefficient on frequency

The dissipation coefficient (1) calculated for different wave breaking types on the base of two described above wave models are shown on Fig.3. The frequency dependences of the dissipation coefficient presented in this figure differ but can be qualitatively classified as quadratic and selective. It can be seen that the form of the dissipation coefficient for spilling waves on qualitatively level practically coincides (Fig.3b). There is some selectivity of dissipation at frequencies of the 2nd and 3rd harmonics. But since the coefficient itself has small values, it can be assumed that the dissipation of all harmonics occurs approximately uniformly. As was mentioned in introduction this approach is implemented in many wave models. When comparing the dissipation coefficient for spilling and plunging waves, it can be seen that they are different in both models (Fig.3). Plunging waves dissipate more strongly and have a large dissipation coefficient, especially for higher harmonics. If we consider plunging breaking waves, then, in general, we can say that dissipation occurs approximately according to the quadratic law with using Boussinesq and SPB modeling methods. That is second way for modeling of breaking waves suggested in (Kirby, Kaihatu, 1996). But for the SPB method, the quadratic dependence is small, while for the Boussinesq model it is significant. The LTA method still gives a frequency-selective dependence of the dissipation coefficient on the frequency of the 2nd harmonic. Taking into account the fact that for SPB the dependence of the dissipation coefficient on the square of the frequency is small, it can also be considered approximately constant (uniform dissipation).



(a)

(b)

Figure 3. Dissipation coefficients for wave breaking by plunging (a) and spilling (b).

Influence of nonlinearity on wave spectrum shape

As was found the dissipation compensates the effects of linear and nonlinear transformations of the spectrum (Kuznetsov, Saprykina, 2004). Since the nonlinear changes are more important as was noted in the introduction we will consider the influence of nonlinear processes on the spectrum change (Fig. 5).

Note that plunging and spilling breaking waves transform and break over different bottom slopes. For detailing we will simulate the propagation of waves with the "switching off" of the model terms responsible for the nonlinear (triad interactions) transformation of waves. In real conditions it is impossible to strictly carry out such an artificial separation, but in modelling the intensity of manifestation of certain processes in the wave spectra depending on the conditions of their transformation can be demonstrated quite clearly.

Figure 4 shows the results of the influence of nonlinear transformation of waves breaking by different types on wave spectra shape. The relative changes in the spectrum (modelled to initial measured) were estimated in points before and after wave breaking during wave propagation to the coast.

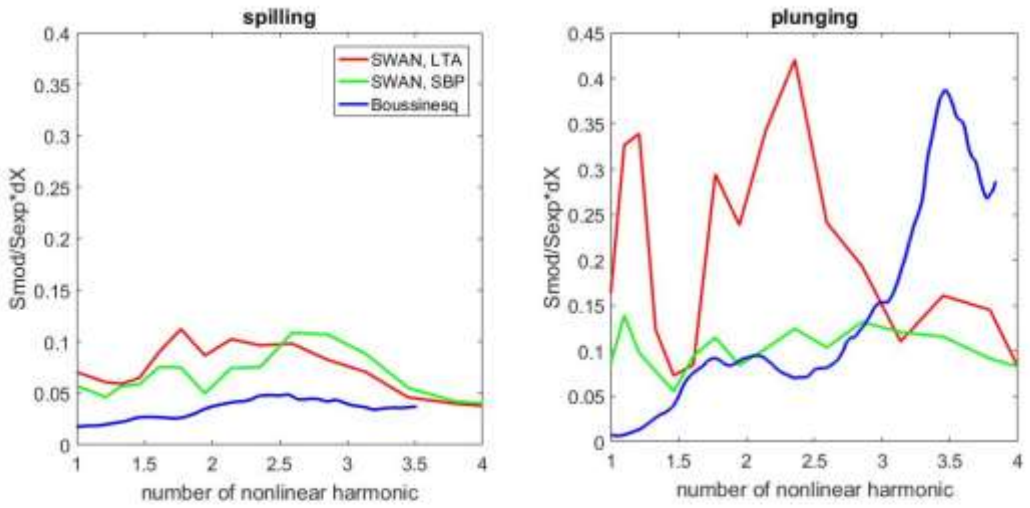


Figure 4. Relative changes of wave spectra without modelling dissipation.

It is clearly seen that dissipation occurs to a greater extent where the spectral energy has increased due to nonlinear processes. It can be seen that due to nonlinear processes during plunging breaking, the energy of the main harmonics is transferred to a greater extent to the higher harmonics, and during spilling breaking, mainly to the frequencies of the second harmonic.

The influence of nonlinear energy transfer within the spectrum during wave transformation on the form of the frequency dependence of the dissipation coefficient was noted in many works. It is assumed that an increase in such transfer leads to a quadratic type of dependence, while a weak transfer leads to frequency independence (Chen et al., 1997; Kaihatu, Kirby, 1996). Thus, the differences in the dependences of the dissipation coefficient on frequency for different models are determined mainly by nonlinear changes arising in the simulation of waves.

It is seen that in the LTA method the nonlinear contribution of the second harmonic is significantly higher, which is associated with the method of parametrization of nonlinear triad interactions. In the SPB method, the nonlinear changes for the 2^{nd} and 3^{rd} harmonics are similar to the Boussinesq model. The Boussinesq model gives a too fast growth of the higher harmonics (for example, 4th) that requires its greater dissipation.

Adjustment model coefficients for SWAN model

To reduce the discrepancy in modeling a tuning of the model coefficients is widely used. Both SPB and LTA nonlinearity source models have adjustable proportionality coefficient named *trfac*, with default values of 0.05 for LTA and 0.9 for SPB models (Swan Team, 2021). Coefficient for LTA model is recently adjusted from its previous default value of 0.8 (Swan Team, 2020). Salmon et al.

(2016) has calibrated the coefficient for 2D applications and recommended a value of 0.52 for LTA model.

The SPB model can be further adjusted by calibrating the parameters *a* and *b* which are related to the broadening of the resonance condition. The default values are recommended as *a*=0.95 and *b*=0.0, but *b*=-0.75 is recommended for one dimensional case. a=1 with default value of *b*=0 and *b*=-0.5 and *b*=-0.75 with the default value of *a*=0.95 are evaluated to calibrate the dissipation coefficient for SPB method. The results are shown in Fig. 5.

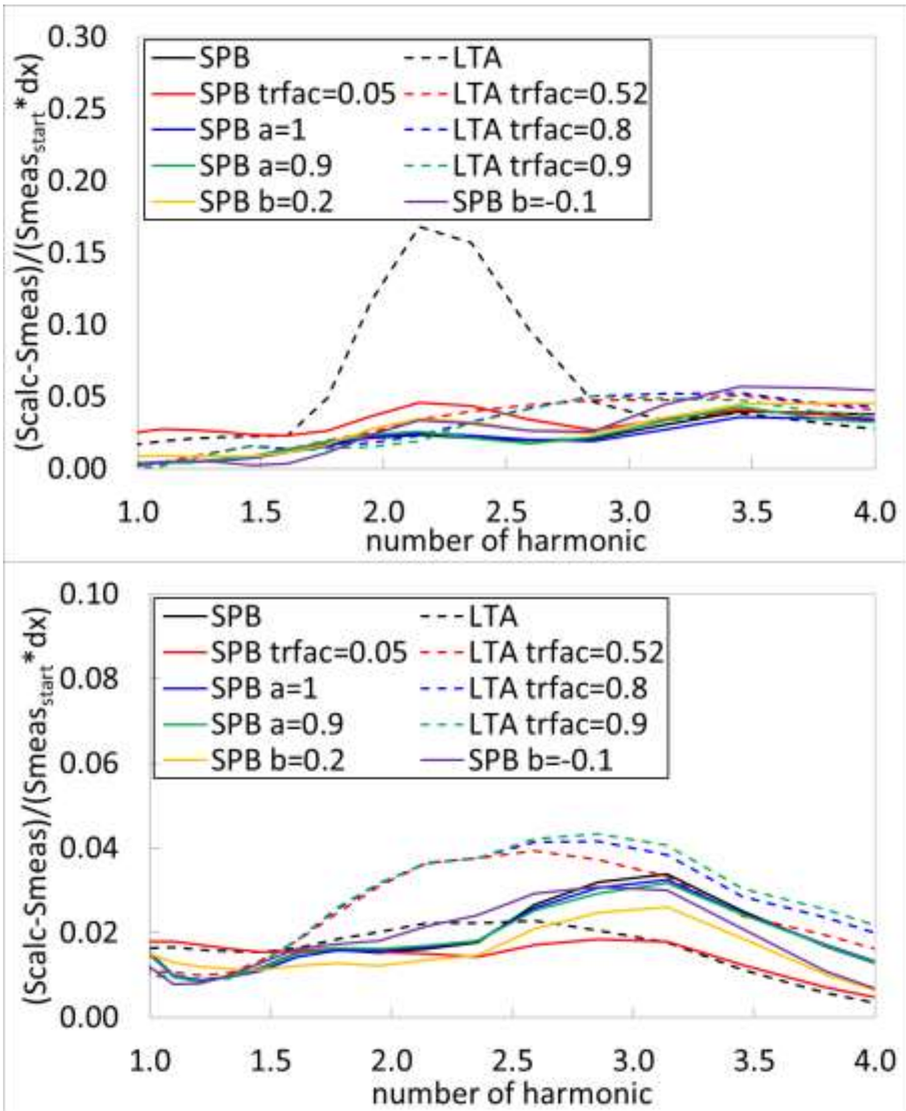


Figure 5. Adjustment of dissipation coefficients for plunging (a) and spilling (b) type wave breaking. In the legend, each parametrization is named after their modified parameters from the default setup. Parameters which are not indicated kept as default.

It is visible that with *trfac*= 0.52 significantly decreases frequency selectivity of wave energy dissipation for LTA method. Dependence of dissipation coefficient on frequency is approximately quadratic and qualitatively less differs from similar for SBP method. Fig. 8 also indicated that almost all SPB model parameterizations provide more uniform dissipation coefficient than the LTA model for both plunging and spilling breaking waves. The applicable range of *a* and *b* coefficients is very narrow in models with “no dissipation”. Model results varied slightly, with the modifications to a and b coefficients whereas impacted significantly from modification of *trfac*. For considered data the results also revealed that SPB model with *trfac*=0.05 parametrization returned most homogenous outputs for both spilling and plunging breaking waves allowing the use of non-frequency dependent energy dissipation formulations (bulk energy dissipation).

Some interesting finding is obtained for plunging breaking by the modification of the model parameters. Extremely greater dissipation is obtained around the first harmonic for *trfac*=0.05 of both SPB and LTA models.

Influence of dissipation coefficient on wave symmetry and sediment transport prediction

On the basis of the complex amplitudes obtained by the phase-resolving spectral model, it is possible to obtain a wave chronogram. This chronogram can be used to estimate bottom relief deformations using simplified models of sediment transport, assuming a linear relationship between free surface elevations and bottom velocity. During wave transformation the wave shape is changed due to nonlinear and dissipation processes. The main factor driving sedimentation is the asymmetry of waves and, accordingly, wave velocities. Using comparison of modeling results with the data of laboratory and field experiments have been shown, that the exact type of frequency dependencies of energy dissipation can be not so important for calculation and modeling of wave spectra, but essential influences on simulation of a free surface elevation and on a values of statistical moments accordingly (Chen et al., 1997). Let us consider how different dissipation coefficients used in modeling can affect estimates of wave symmetry and sediment transport.

The asymmetry of waves is characterized by the third moments of wave motion:

$$S = \frac{\langle \eta^3 \rangle}{\langle \eta^2 \rangle^{\frac{3}{2}}} \qquad (3)$$

$$A = \frac{\langle H(\eta^3) \rangle}{\langle \eta^2 \rangle^{\frac{3}{2}}} \qquad (4)$$

where η - free surface elevation, H – Hilbert transform, the brackets denote a time average.

Skewness (S) characterizes wave symmetry on horizontal axis, asymmetry (A) - wave symmetry on vertical axis. The positive values S correspond to sharp waves crests and the flat trough, negative values A correspond to waves with abrupt forward and flat back fronts.

Consider results of Boussinesq modelling with different type of dissipation coefficient in form:

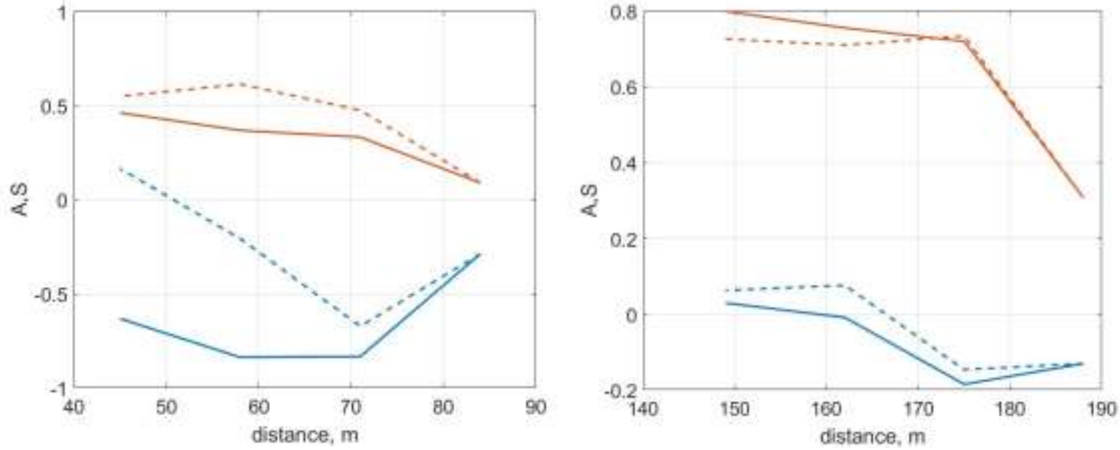
$$\alpha_n = F\beta + (1-F)\beta \frac{f_n^{\,2} \sum_n |A_n|^2}{\sum_n f_n^2 |A_n|^2}, \qquad (5)$$

β – breaking coefficient; A_n - amplitude; f_n – frequency, F – parameter, if F=1, we have uniform dissipation coefficient, independent on frequency (Eldeberky, Batties, 1996); if F=0, we have qudratic depending on frequency dissipation coefficient (Mase, Kirby,1992), n – rank of harmonics.

$\beta = D_{tot} / F_{tot}$, where F_{tot} – total local rate of energy flux per unit width, D_{tot} – total local rate of energy dissipation. D_{tot} was determined from simple probabilistic energy dissipation model (Eldeberky, Battjes, 1996).

In Fig. 6 both moments (3, 4) for wave chronograms obtained on the base of modelling with different dissipation coefficients (uniform and quadratic) for spilling and plunging breaking waves above bottom relief (Fig.1) are shown. At calculation of the moments the infragravity frequencies (smaller than 0.05 Hz) were filtered out.

It is clearly seen that for the spilling breaking waves, when using different dissipation coefficients, the differences in the asymmetry are small. With a uniform dissipation coefficient, the skewness is slightly less, and the asymmetry is slightly larger and has more positive values, which is more consistent with the description of the structure of waves breaking with the spilling type (Saprykina et al., 2020). As for the plunging of breaking waves, the differences are greater. Skewness for model waves with uniform dissipation is greater than for waves with a quadratically dependent frequency. There are significant differences for asymmetry when modeling with a uniform dissipation coefficient. The asymmetry becomes even positive, i.e. the waves have steep backward front and a gentle forward front. The change in asymmetry when modeling waves with a coefficient quadratically dependent on frequency is more consistent with the description of the structure of waves breaking by plunging, when waves constantly have a steep forward front (or negative values of asymmetry) (Saprykina, et al., 2020).



(a) (b)

Figure 6. Asymmetry (blue) and skewness (red) for waves modeling with different dissipation coefficients: uniform (dashedd line) and qudratic (bold line) for plunging (a) and spilling (b) breaking waves in field experiment (Fig.1).

Numerous experimental studies have shown that bottom velocities within a coastal zone are linearly related to the free surface elevations and can be calculated with good accuracy using the linear wave theory. Therefore, we can calculate time series for velocities using wave amplitudes:

$$u_i = \frac{a_i \omega_i}{\sinh(k_i h)} \quad (6)$$

where ω is the angular frequency, k is the wave number, h is the water depth, i is number of wave harmonic, and inverse Fourier transform.

To estimate the cross-shore sediment discharge, Bailard's formula was used (Bailard, 1981):

$$q = \frac{1}{2} f_w \rho \left(\frac{\varepsilon_b}{\tan \sigma} \overline{u|u|^2} + \frac{\varepsilon_s}{W_s} \overline{u|u|^3} \right) \quad (7)$$

where f_w = 0.01 is the coefficient of bottom friction, ρ is the sand density, ε_b = 0.1 and ε_s = 0.02 are the coefficients of turbulent viscosity and turbulent diffusion, respectively, tan *б* = 0.5 is the coefficient of particles' inner friction, where б is the sediment internal friction angle, w_s is the sediments fall velocity, and $\overline{u|u|^2}$ and $\overline{u|u|^3}$ are third and fourth moments of near-bottom velocity, representing entrained and suspended sediments, respectively.

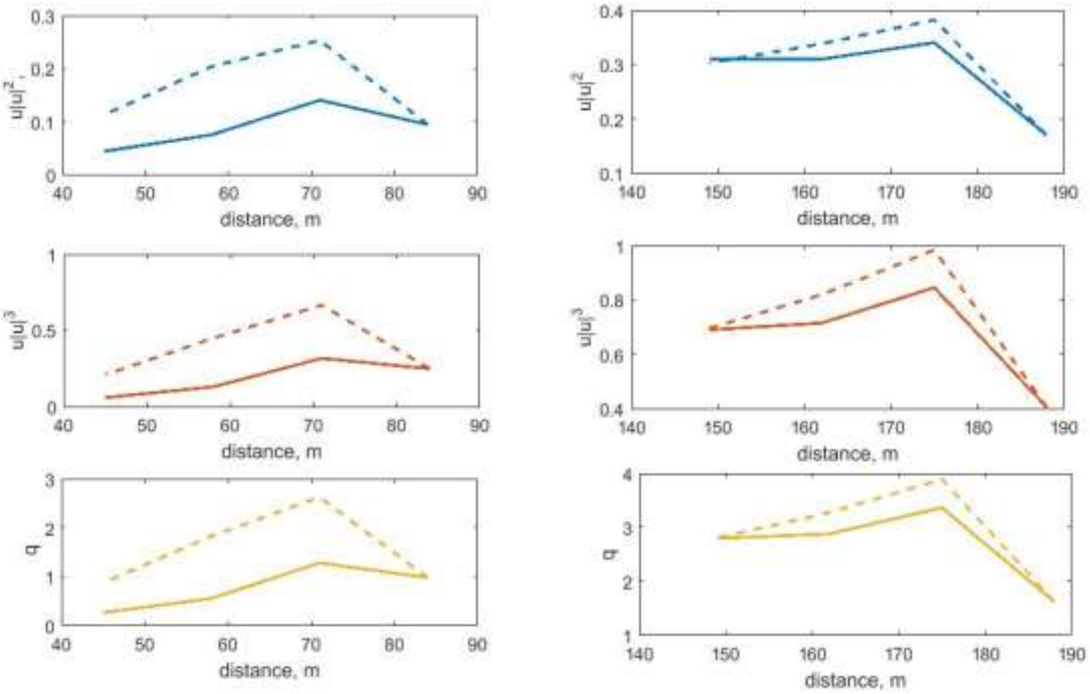
In Figure 7 are presented the third and fourth momentums of velocity and sediments discharge.

As for the estimates of sediment discharge, it can be said that for spilling breaking waves, the differences are small (no more than 3 percent), and for plunging breaking waves, the differences are significant - more than three times. In this case, the results of modeling waves with uniform dissipation are of great importance. That is, when modeling using uniform dissipation, the sediment discharge can be overestimated.

Bottom deformations depend on the sediment discharge gradient and can be calculated as:

$$dq/dx \approx dh/dt \quad (8)$$

where h is the depth. The positive sign (+) corresponds to the increase in depth, i.e., erosion of sea bottom and removal of sandy material, and the negative sign (−) corresponds to the decrease in depth, i.e., the accumulation of sandy material at the bottom.

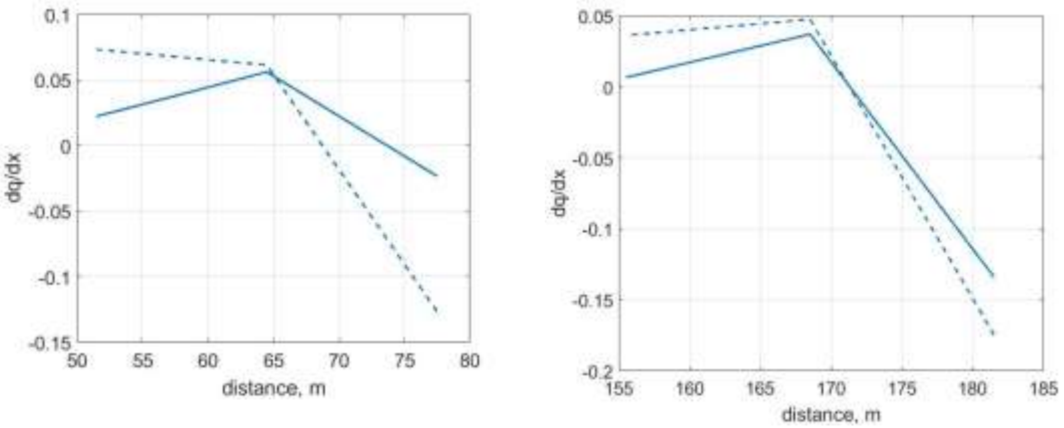


(a) (b)

Figure 7. Third (blue), fourt (red) velocity moments and sediments discharge (yellow) for modeling with different dissipation coefficients: uniform (dashedd line) and qudratic (bold line) for plunging (a) and spilling (b) breaking waves in field experiment (Fig.1)

Bottom deformation estimates are shown in Fig.8. It can be seen that for the spilling waves, the deformations are generally similar. However, we note that with a uniform dissipation coefficient, there is a greater erosion of the top of the outer bar (Fig. 1) and a greater accumulation of material on its back slope than with a dissipation coefficient quadratically dependent on frequency. This is more consistent with the ongoing processes of bottom topography reshaping in this case, a detailed analysis of which was made in (Saprykina, 2020).

For plunging waves, uniform dissipation will result in erosion of the inner bar and transport of sediments down on the slope. With dissipation quadratically dependent on frequency, deformations will lead to bar stability, which corresponds to a detailed analysis of the change in relief under the influence of this wave regime conducted in (Saprykina, 2020)



(a) (b)

Figure 8. Bottom deformations for modeling with different dissipation coefficients: uniform (dashed line) and quadratic (bold line) for plunging (a) and spilling (b) breaking waves in field experiment (Fig.1)

Thus, based on the analysis, it can be concluded that the type of dissipation coefficient used in the simulation effects on the obtained shape of the waves and the corresponding deformations of the bottom topography caused by the action of the waves. The greatest differences will be observed for waves breaking by plunging. For modeling such waves, a dissipation coefficient quadratically dependent on frequency is more suitable.

CONCLUSIONS

Modeling the dissipation of wave energy during wave breaking in the models should be carried out taking into account the compensation of nonlinear changes arising from the use of different nonlinear source terms.

Spilling breaking waves have a dissipation coefficient with frequency selectivity at frequencies of the 2nd and 3rd harmonics, regardless of the type of spectral model and using nonlinear terms. The absence of significant differences is apparently due to the small contribution of nonlinearity to the change in the spectrum, since spilling breaking is characteristic of waves transforming over a gentle slope of the bottom.

For the waves breaking the plunging, the quadratic dependence of the dissipation coefficient on the frequency is characteristic for the SPB method and for the Boussinsq model. For the LTA method with default parameters, plunging breaking waves, as well as spilling breaking waves have a frequency-selective dissipation coefficient at a frequency of the 2nd harmonic.

The accuracy of LTA and SBP methods can be improved by tuning model coefficients. So if parameter *trfac* =0.52, LTA and SBP methods will give on qualitative level a similar results for quadratic frequency dependence of wave energy dissipation for plunging breaking waves.

The type of dissipation coefficient used in the waves simulation influences on the obtained shape of the modeled waves and on predictions of bottom deformations caused by the waves action.

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REFERENCES

Antsyferov S. M., Kosyan R. D., Kuznetsov S. Yu., Saprykina Ya. V. 2005. Physical Grounds for the Formation of the Sediment Flux in the Coastal Zone of a Nontidal Sea. *Oceanology*, 45, S183–S190.

Bailard, J.A. 1981. An energetics total load sediment transport model for a plane sloping beach. *J. Geophys. Res.*, 86, 10938–10954, doi:10.1029/JC086iC11p10938.

Battjes J.A., Beji S. 1992. Breaking waves propagating over a shoal. *Proc. 23rd Int. Conf. Coastal Eng.*, 42-61.

Becq-Girard, F., P. Forget, and M. Benoit. 1999. Non-linear propagation of unidirectional wave fields over varying topography, *Coast. Eng.*, 38, 91-113.

Beji S., Battjes J.A. 1993. Experimental investigation of wave propagating over a bar. *Coastal Eng.*, 19, 151-162.

Booij, N., L.H. Holthuijsen, and M.P. Bénit. 2009. A distributed collinear triad approximation in SWAN, *Coastal Dynamics 2009*, 1-10.

Bredmose H., Schäffer H.A., Madsen P.A. 2004. Boussinesq evolution equations: numerical efficiency, breaking and amplitude dispersion. *Coastal Eng.*, 51, 1117-1142.

Chen Y., Guza R., Elgar S. 1997. Modeling spectra of breaking surface waves in shallow water. *J. Geophys. Res.*, 102, 25035-25046.

Eldeberky Y., Battjes J. 1996. Spectral modeling of wave breaking: application to Boussinesq equations. *J. Geophys. Res*., 101, 1253-1264.

Eldeberky Y., Madsen P.A. 1999. Determenistic and stochastic evolution equations for fully dispersive and weakly nonlinear waves. *Coastal Eng*., 38, 1-24.

Eldeberky, Y. 1996. Nonlinear transformation of wave spectra in the nearshore zone, *Ph.D. thesis*, Delft University of Technology, Department of Civil Engineering, The Netherlands, 203 p.p.

Elgar S., Guza R.T., Raubenhaimer B. et al. 1997. Spectral evolution of shoaling and breaking waves on a barred beach. *J. Geophys. Res*., 102, 15797-15805.

Herbers, T. H. C., Russnogle, N. R., Elgar, S. 2000. Spectral Energy Balance of Breaking Waves within the Surf Zone. *Journal of Physical Oceanography*, 30, 2723-2737.

Salmon J.E., P.B. Smit, T.T. Janssen, L.H. Holthuijsen. 2016. A consistent collinear triad approximation for operational wave models. *Ocean Modelling*, 104, 203-212.

Kaihatu J., Kirby J. 1996. Effects of mode truncation and dissipation on predictions of higher order statistics. *Proc. 25th Int. Conf. Coastal Eng.*, 123-135.

Kirby J., Kaihatu J. 1996. Structure of frequency domain models for random wave breaking. *Proc. 25th Int. Conf. Coastal Eng.*, 1144-1155.

Kuznetsov, S.Y., Saprykina, Y.V. 2004. Frequency-Dependent Energy Dissipation of Irregular Breaking Waves. *Water Resources* , 31**,** 384–392.

Madsen, P.A.; Sørensen, O.R. 1993. Bound waves and triad interactions in shallow water. *Ocean. Eng.*, 20, 359–388.

Mase H., Kirby J. 1992. Hybrid Frequency-domain KDV equation for random wave transformation. *Proc. 23rd Int. Conf. Coastal Eng.*, 474-487.

Mase H., Kitano T. 2000. Spectrum-based prediction model for random wave transformation over arbitrary bottom topography. *Coastal Eng. J*., 42, 111-151.

Saprykina, Y. 2020. The Influence of Wave Nonlinearity on Cross-Shore Sediment Transport in Coastal Zone: Experimental Investigations. *Appl. Sci.*, 10, 4087.

Saprykina, Y.V.; Kuznetsov, S.Y.; Kuznetsova, O.A.; Shugan, I.V.; Chen, Y.Y. 2020. Wave Breaking Type as a Typical Sign of Nonlinear Wave Transformation Stage in Coastal Zone. *Phys. Wave Phenom*., 28, 75–82.

SWAN Team. 2020. SWAN User Manual. SWAN Cycle III Version 41.31A. *Delft University of Technology*, 143 p.p.

SWAN Team. 2021. User Manual SWAN - Cycle III version 41.31AB. *Delft University of Technology*, 152 p.p.