FREE AND BOUND WAVES IN THE COASTAL ZONE: FIELD, LABORATORY AND NUMERICAL EXPERIMENTS

Sergey Kuznetsov[1], Yana Saprykina[1], Margarita Shtremel[1]

Traditionally we look on storm waves as on periodic in space and time motion. Main feature of wave transformation in the coastal zone over inclined bottom is the higher harmonic generation that occurs so fast (during the one wavelength) that we can't consider the wave motion as periodic in space. Spatial measurements of the wave profile are difficult in the coastal zone and are replaced by point chronograms of free surface elevations and then interpreted using traditional wave theories to calculate the practically important the wave energy and speed of wave propagation. These interpretations very often lead to paradoxical results, such as unexpected spatial fluctuations of the wave energy and propagation velocities, which are usually interpreted as shortcomings in the calibration of gauges, but our experience told that is the result of simultaneous existing of free and bound waves. Many researchers consider the free and bound second and higher harmonics of waves on the intermediate water as "parasitic" or "spurious" and try to avoid it in laboratory and in numerical experiments, but second harmonic are practically important. We will try to figure out what is the matter on the basis of field, laboratory and numerical experiments consider the waves in both time and space domains.

Keywords: free and bound waves; nonlinear wave transformation; waves at intermediate depth; coastal zone

INTRODUCTION

Nonlinear wave transformation in the coastal zone over a sloping bottom leads to significant changes in the wave shape - a change in the slopes of the front and back sides of the wave and relative changes in the heights of the crests and depths of the wave troughs, i.e. to a change in the symmetry of the waves about the horizontal and vertical axes. Changes in the waveform are reflected in the wave spectrum as changes in the ratios between the amplitudes of the first and second harmonics of the waves, as well as changes in the phase shift between them. Many researchers in numerical and laboratory experiments with coastal waves consider the higher harmonics of waves as parasitic and try to get rid of them by "correctly" setting the initial conditions as described, for example, in the works (Madsen, Sorensen, 1993 and Pierella et al., 2021). However, the second harmonics really exist and are very important for the dynamics of the coastal zone, since they significantly affect the sediment transport and the breaking of waves. In a simplified way, it is possible to represent the elevations of the free surface in the coastal zone of the sea as the sum of the first and second wave harmonics

$$\zeta(x,t) = A_1\cos(\omega_1 t - k_1 x) + A_2\cos(2\omega_1 t - 2k_1 x + \varphi),$$

where A is the amplitudes, φ - phase shift, ω and k – frequency and wavenumber, t and x – time and distance.

As shown in the journal paper (Saprykina et al., 2020) the phase shift between the first and second harmonics is responsible for the type of wave breaking over a sloping bottom:

$\varphi = 0$ – Spilling,
$\varphi = -\pi/2$ – Plunging.

According to the Bailard's formula (Bailard, 1981) simplified by Stive (Stive, 1986), the cross-shore component of the sediment discharge, directed towards the shore, is expressed as

$$q \approx Au_1^2 u_2\cos(\varphi) + Bu_1^3 u_2\cos(\varphi),$$

where A and B are the constants dependent on sediment parameters, u are the amplitudes of the harmonics of the near-bottom water particles velocity, and φ is the phase shift between these harmonics.

If the amplitude of the second harmonic $u_2 = 0$ or the phase shift is $\varphi = \pm\pi/2$, then $q = 0$, and under the action of the countercurrent there will be a constant erosion of the bottom of the coastal zone and the removal of sediments into the open sea. If $\varphi = 0$, then the wave component of the sediment discharge is directed toward the coast and the bottom sand will be accumulated near the coast,

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[1] Shirshov Institute of Oceanology, Russian Academy of Sciences, Nakhimovskiy pr., 36, Moscow, 117997, Russia

i.e. the second harmonic is responsible for the safety of the coast. The influence of harmonic parameters on the change in the bottom topography has been demonstrated in many works, for example (Saprykina, 2000, Shtremel et al., 2022).

For practical problems of the operation of beaches and technical structures, estimates of changes in the coastal zone of wave parameters are required, which should be determined based on spatial changes in the elevations of the free water surface by estimating the wave spectra in the wave number domain.

A rigorous analytical theory of wave transformation still does not exist, and researchers and coastal engineers are forced to use semi-empirical estimates. But the estimation of harmonic amplitudes and phase shifts in the coastal zone of the sea is complicated by the nonlinearity of the process of wave transformation over an inclined bottom. Spatial measurements of the wave profile are practically impossible in the coastal zone and are replaced by point measurements of elevation chronograms, which are then interpreted using traditional wave theories to calculate the energy and speed of wave propagation. Since changes in the shape and spectrum of waves in the coastal zone occur rapidly, at distances of shorter lengths, the wave motion in space becomes non-periodic, and the application of the concepts of the linear theory of the wave number or wavelength leads to paradoxical estimates of wave parameters, such as spatial fluctuations of the "visible" wave energy (Saprykina, Kuznetsov, 2008) and anomalous dispersion of the second harmonic (Kuznetsov, Speransky, 1994) observed in field and laboratory experiments and confirmed by numerical simulation (Kuznetsov, Saprykina, 2021).

We will figure out what is the matter on the basis of full-scale field, laboratory and numerical experiments. On the basis of field and laboratory experiments on the propagation of waves over an inclined bottom, we will show that the second harmonics of the waves really exist and periodically exchange energy with the first harmonics in space. On the basis of numerical experiments, we will show that space-periodic fluctuations of the amplitudes and phases of the first and second harmonics are caused by the simultaneous existence of free and bound wave harmonics.

PHYSICAL EXPERIMENTS

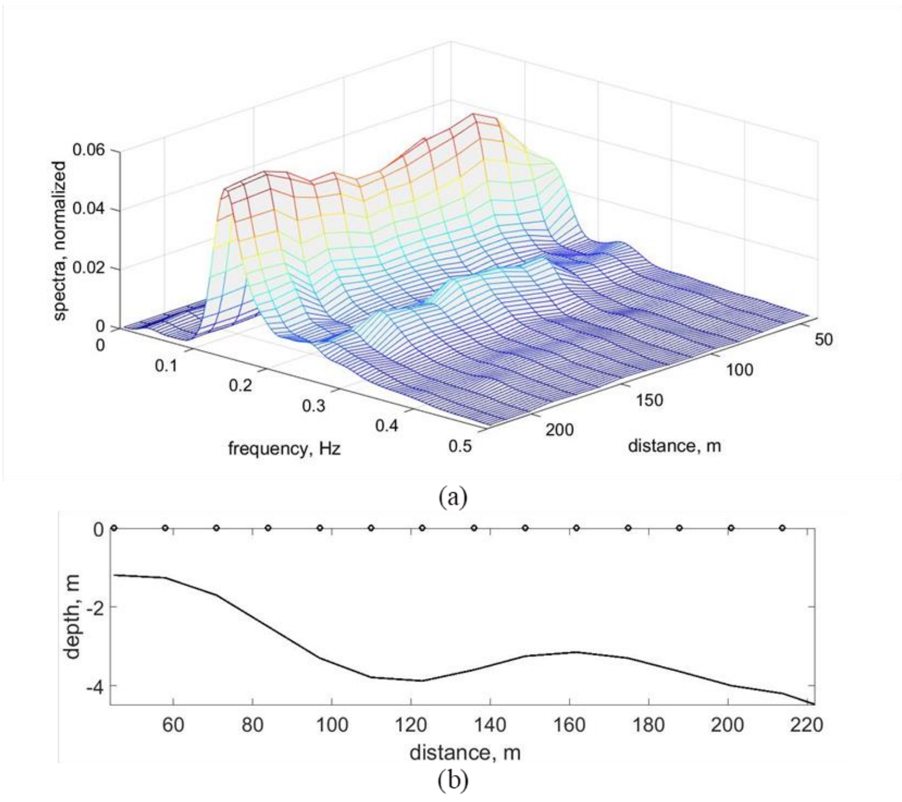

Figure 1. Spatial fluctuations of wave spectrum over the inclined bottom and gauges position in field experiment Shkorpilovtsy 2007.

Field Experiment

Field experiment Shkorpilovtsy 2007 (Black sea, near Varna town) was performed by 15 capacitance and resistance types wire gauges installed equidistantly in depth range 4.5 – 0.5 m and synchronously acquired at 5 Hz frequency during the hour as shown at Fig.1.

At the same Figures we can see the spatial periodic fluctuations of the frequency spectrum of waves over an inclined bottom. So we can decide – yes, there are spatial beats of amplitudes of first and second frequency harmonics in nature and it is not an artificial result of wrong signal on the wave maker or at initial boundary condition in numerical experiments.

Laboratory Experiment

Laboratory experiment was performed in Tainan Hydraulic Laboratory, Taiwan, 2015. During the experiments the initially monochromatic waves 20 cm in height and 5 s in period generated at distance 0 m and were measured synchronously by 21 capacitance type wire gauges acquired by 20 Hz frequency (Fig.2). The water depth at the wave maker (left side of the picture) was 1.3 m. Spectrum evolution looks quite similar to field conditions.

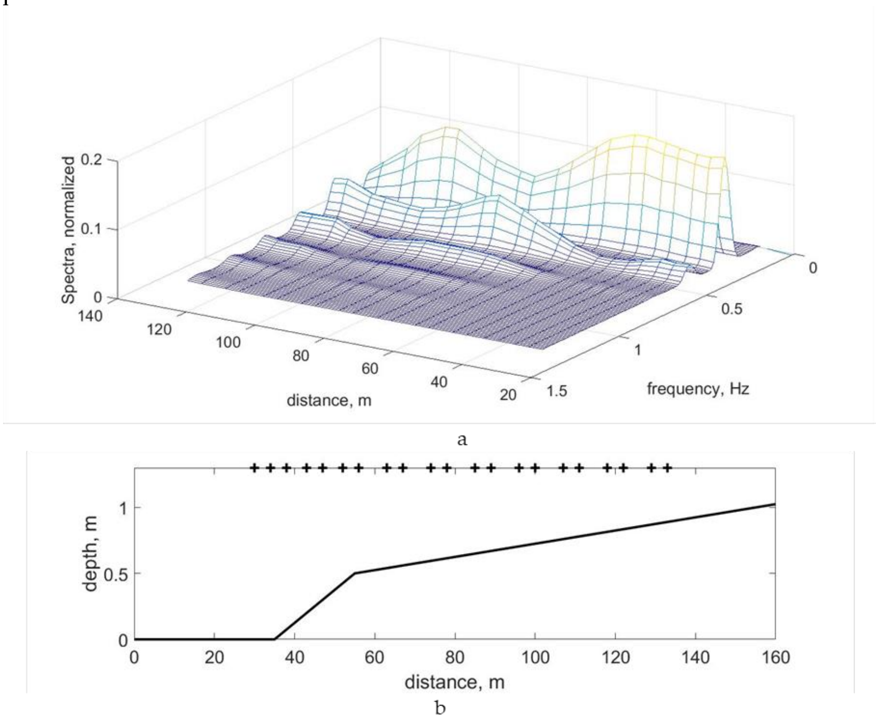

Figure 2. Antiphase spatial fluctuations of amplitudes of first and second frequency harmonics over the inclined bottom and gauges position in the laboratory experiment Tainan 2015.

NUMERICAL MODELLING

We cannot estimate the wavenumbers spectrum with sufficient accuracy due to the small number of points for measuring waves in space both in field and laboratory experiments, so will use the numerical modelling of wave transformation over the constant depth for the condition worse coinsided with laboratory experiment.

Frequency Domain Modelling

Madsen and Sorensen's (1993) Boussinesq-type model of wave propagation was solving in the spectral domain for complex harmonic amplitudes evolution. Having carried out the inverse Fourier transform of the complex amplitudes with addition of linear phases, we obtain a two-dimensional array of elevations of the free surface of the water. According to one dimension - changes in elevations in space, according to another - in time.

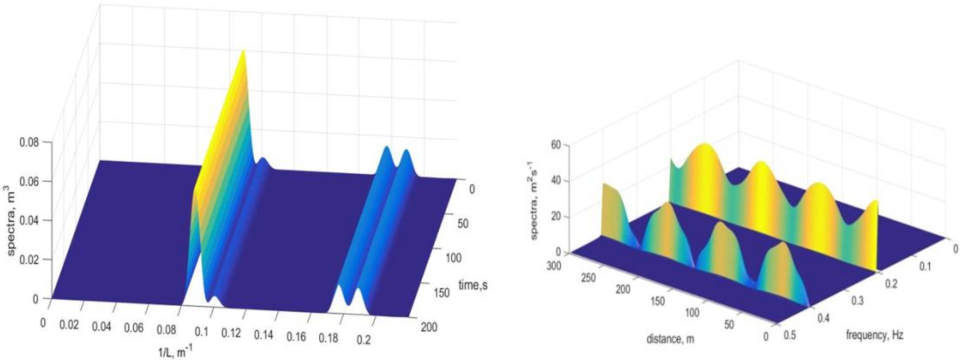

Figure 3. Frequency and wavenumber spectra of initially monochromatic waves over the constant depth calculated by frequency domain model.

Spectra of model waves in the frequency and in the wavenumber domains modelled over the constant water depth 0.5 m with a length of 300 m, initially monochromatic waves with a height of 0.1 m and a period of 5 s, are shown on Fig.3.

Two dimensional *k-f* spectrum for the same case, received by two dimensional Fourier transform of a two-dimensional array of elevations of the free surface of the water, is shown on Fig.4.

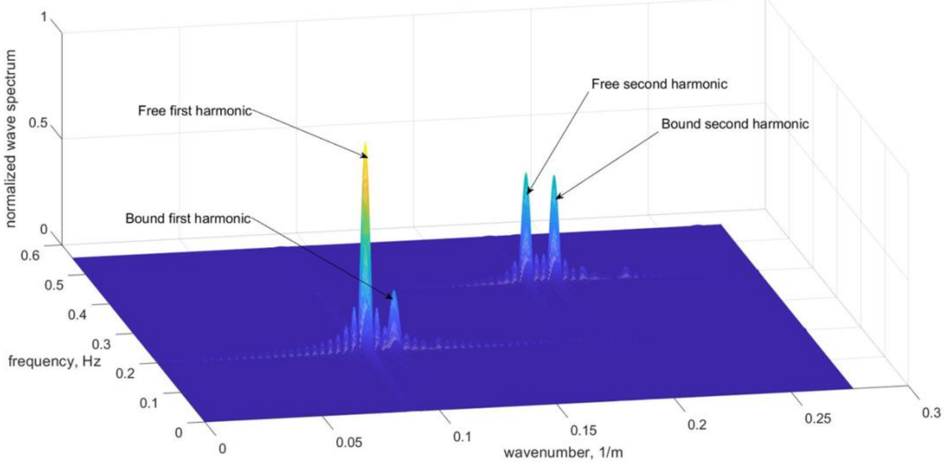

Figure 4. Two dimensional k-f spectrum corresponding to the spectra of Fig.3.

On the spectra we clearly had seen the peaks corresponding to the free and bound waves of first and second harmonics. The values of the peak frequencies are shown on Table 1.

Table 1. Wavenumbers and frequencies of harmonics

Wave harmonics	Wavenumber, 1/L, L – wave length (1/m)	Frequency, Hz
First free (k_1, f_1)	0.0885	0.2
First bound (k_2–k_1, f_1)	0.0976	0.2
Second bound ($2k_1$, $2f_1$)	0.1766	0.4
Second free (k_2, $2f_1$)	0.1889	0.4

On the base of wavenumber spectra a linear filtration the waves corresponding to all wave-numbers free and bound harmonics (Table 1) using the direct and inverse Fourier transform was carried out. It can be seen that fluctuations of the amplitudes of both the first and second harmonics at distance (in the spatial domain) are determined by the interference (or by the summation) of waves of wavenumbers of free and bound wave harmonics (Fig. 5). The maximum amplitude of the frequency second harmonic corresponds to the superposition of two wavenumbers of second harmonic in phase (Fig. 5, distance 40 m), and the minimum - to their change in antiphase ((Fig.5, distance 80 m). Similarly, but in opposite phase realized the interference of two first harmonics. At the initial moment of time (t=0) both of second harmonics in wavenumber space have equal amplitudes and are in antiphase that corresponds theoretical assumption in (Madsen, Sørensen, 1993).

Thus, the observed spatial beats or energy exchange between the first and second nonlinear harmonics are the result of the interference of free and bound waves with same frequencies, but different wavenumbers. There is no real exchange of energy between nonlinear harmonics. But there are periodic fluctuations in the wave shape corresponding to the beats periods.

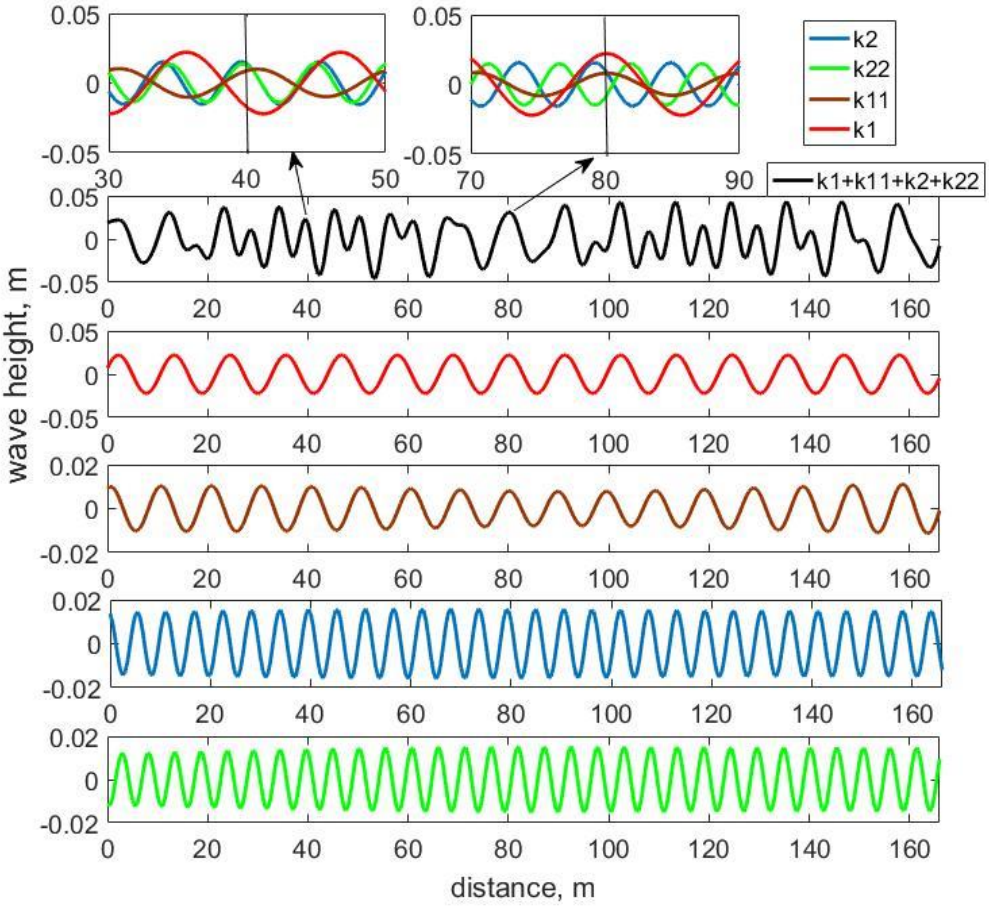

Figure 5. Demonstration of interference of waves corresponding free and bound waves of wave harmonics of model monochromatic wave propagated over constant depth 0.5 m.

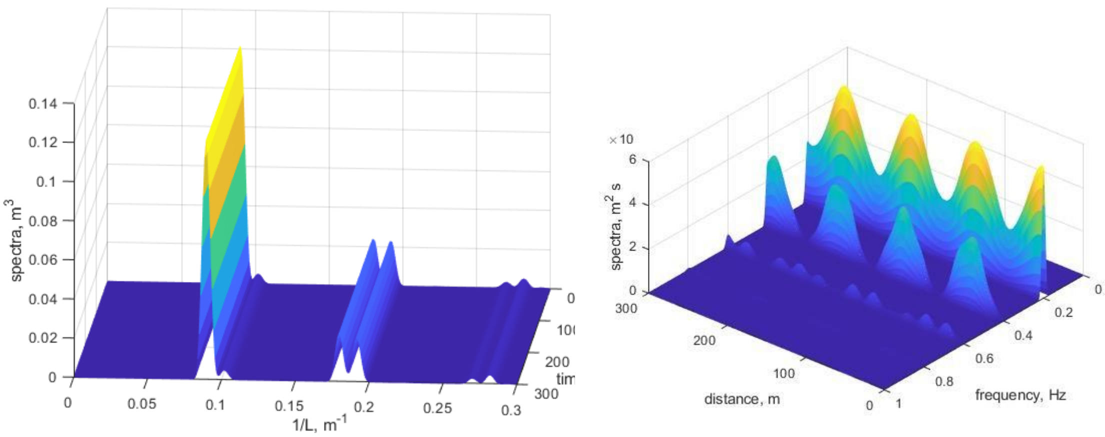

Figure 6. Frequency and wavenumber spectra of initially monochromatic waves over the constant depth calculated by time domain model.

Time Domain Modelling

The frequency model of Madsen and Sorensen was originally built on the interactions of harmonics at multiple frequencies, so we decided to check the simulation results against the independent SWASH model (Zijlema et al., 2011), in which higher harmonics occur naturally without pre-assigned frequencies.

The results are shown in Fig. 6 and fully correspond to the results obtained by the Madsen and Sorensen’s model.

SPATIAL FLUCTUATIONS OF WAVE ENERGY

The simultaneous existence of free and bound waves results in spatial fluctuations of significant wave height along the way of propagation as shown on Fig. 7 for field and laboratory conditions. To save the wave energy flux the speeds of propagations of second harmonics also fluctuate around the speed of first harmonic that expressed as biphase fluctuation shown on the lover part of Fig.7. Fluctuations of the biphase calculated by bispectral analysis mean that the second harmonic is shifted forward relative to the first, then backward, which means that its phase velocity is sometimes greater, sometimes less than the phase velocity of the first harmonic.

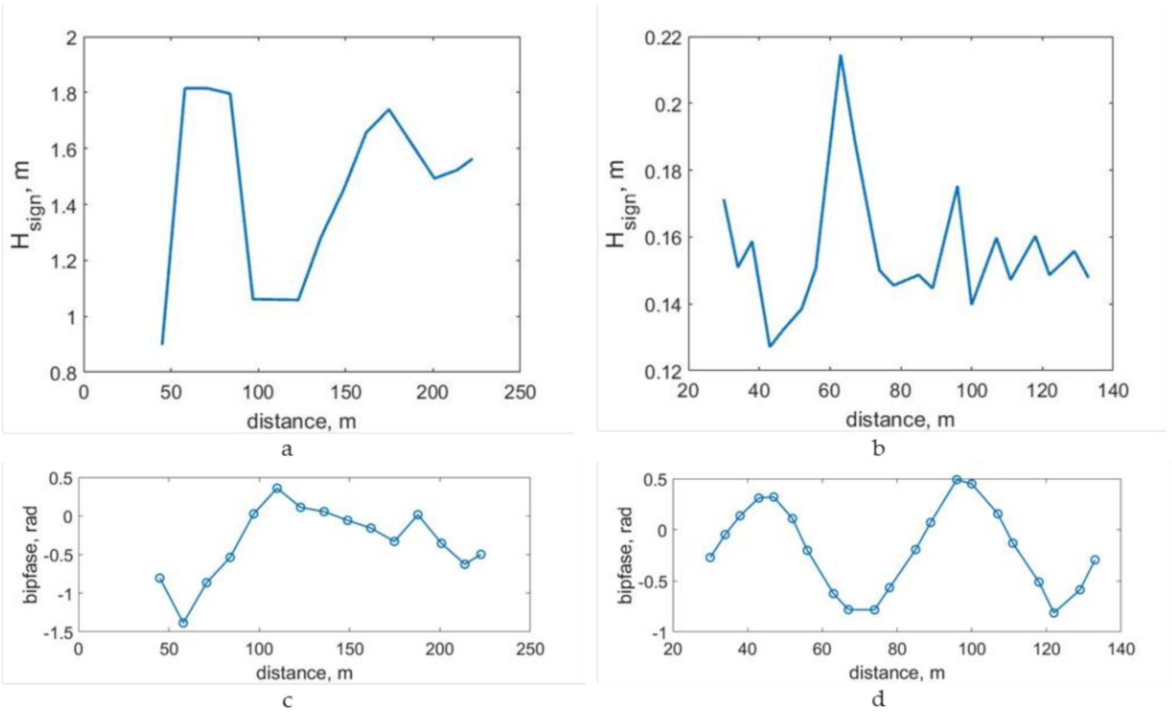

Figure 7. Spatial fluctuations of significant wave heights and phase shift between the first and second harmonics in field and laboratory experiments.

To prove the experimental data the spatual fluctuation of significant wave heights was calculated by SWASH model (Zijlema et al., 2011) for the initially monochromatic waves propagated over the constant depth in the conditions similar to field experiment: depth 3 m, period of 8 s, wave height – 1.4 m. The wave height spatial fluctuation is significant, but less than for the field conditions of irreguliar waves, as shown on Fig. 8.

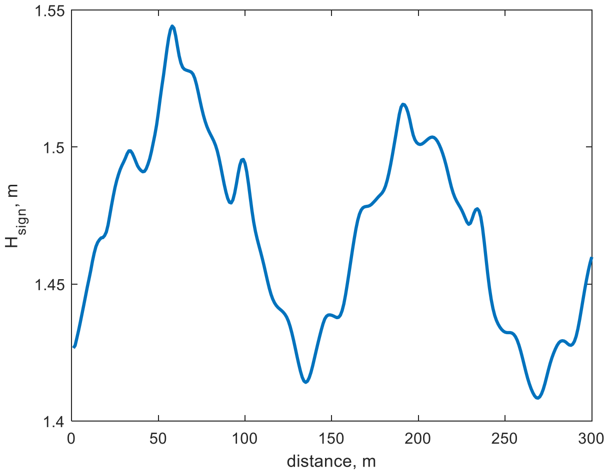

Figure 8. Spatial fluctuations of significant wave heights of initially monochromatic waves over the constant depth calculated by time domain model SWASH.

CONCLUSIONS

1. We prove the simultaneous existing of free and bound waves that arise naturally in field conditions. When analyzing the spatial evolution of waves in the frequency domain, the effect of periodic energy exchange and changes in the phase shift between the first and second wave harmonics are observed. When considering the wavenumber domain, the free and bound waves of both the first and second harmonics with constant in space amplitudes appear, and all spatial fluctuations of the wave parameters are caused by interference of these four harmonics.

2. Due to the simultaneous existence of free and bound waves with the same frequency but different wave numbers (wavelengths), we cannot use the concept of wavelength associated with the frequency harmonics of waves in simplified models of the coastal zone of the sea.

3. The use of a one point wave chronograms leads to paradoxical results: spatial fluctuations in the wave energy and wave energy flux estimations, and the anomalous dispersion of the second harmonic.

4. So we need to taking in the account existence both free and bound waves in the engineering practice.

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