ORBITAL VELOCITIES DUE TO BICHROMATIC-BIDIRECTIONAL WAVES

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The paper describes the orbital velocity pattern generated by Bichromatic-Bidirectional waves (Bi-Bi waves), and investigate how the forced (long period) wave decays along the depth. Experiments conducted at a wave basin proved that the use of ADVs provided much better results than free surface elevations, due to the higher sensitivity of the instrument, as compared of the free surface measurements.

Keywords: Bichromatic-Bidirectional waves; second order forced waves; cross waves; wave basin experiments.

INTRODUCTION

Bidirectional seas are a common feature at many locations worldwide, which may result from a coherent source (e.g., wave diffraction or reflection) or from different energy sources (e.g., uncorrelated storms, superposition of local wind seas and swell). Sea states like the one pictured in Figure 1 have been often observed and recorded along the coast of Rio de Janeiro and Espírito Santo States, where many harbors and a significant portion of the Brazilian offshore oil exploitation are located (e.g., Pereira et al. 2017). How should coastal and ocean structures be designed to resist to such waves? How would beach morphology respond? How to estimate littoral drift under such conditions? These are important questions which must be addressed now by coastal engineers, since neither a clear answer is available, nor the theoretical basis (partially available) has been used for most practical design purposes.

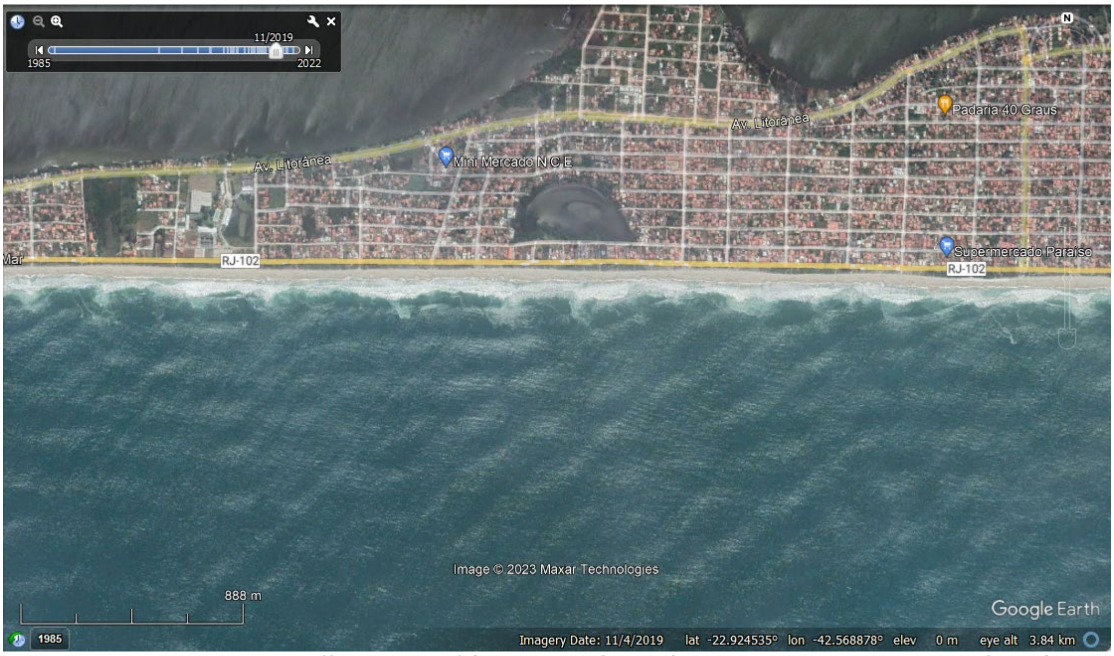

Figure 1. Bidirectional waves off the coast of Saquarema, State of Rio de Janeiro, Brazil (image from Google Earth on 11/4/2019.)

Groundbreaking laboratory experiments on the formation of rip currents have been conducted by Dalrymple (1975) and Dalrymple and Lanan (1976), where bidirectional waves were generated by placing a reflective wall inside a wave basin. These experiments have been reproduced in a larger wave basin at the Institute of Hydraulic Research of the Federal University of Rio Grande do Sul, however in

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a movable bed model (Puhl et al. 2022). The complex pattern of sand ripples, bars and beach cusps are shown in Figure 2, thus emphasizing the need for further investigation about the hydrodynamic patterns associated with bidirectional waves.

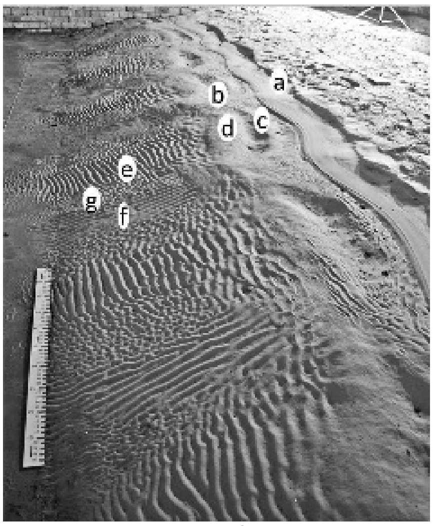


Figure 2. Sand ripples, beach cusps, bars and bumps formed by bidirectional monochromatic waves in a movable bed model. (Photograph by E. Puhl)

Direct measurement of orbital velocities with ADVs offer several advantages to the coastal engineer, in order to characterize wave patterns, to quantify forces or bottom stresses, and to estimate sediment transport, as compared to direct measurements of free surface elevation followed by the application of transfer functions, and finally estimating the desired quantities such as wave orbital velocities (Neves et al. 2012).

The most basic example of a complex sea state is a Bichromatic-Bidirectional (Bi-Bi) wave system. However, generating such waves in a basin poses many challenges, both at the wave maker and at the control of wall reflection. Further, the analysis of orbital velocities associated to Bi-Bi waves requires more advanced mathematical tools, instead of Fourier analysis applied separately to each velocity component. Three-dimensional Hilbert-Huang Transform (3D HHT) or Wavelets are examples of such methodologies, which can benefit from the wealth of information (frequency, magnitude, and direction) provided by ADVs (Moura et al. 2010). Souza e Silva et al. (2018) extended this multi-frequency, adaptative methodology (Multidimensional HHT) for analyzing simultaneous Bi-Bi wave signals, recorded in a wave basin by an array of 6 ultrasonic wave gauges (USS) and another array of 5 ADVs.

The advantages of the velocity over the surface measurements became evident during the experiments, because second-order effects could be more clearly identifiable. The paper extends the investigation on second order subtractive (low frequency) and additive (high frequency) effects on orbital velocities of Bi-Bi waves, looking at the vertical profile of the second order forced waves.

THEORETICAL BACKGROUND

Souza e Silva et al. (2022) expanded the theory of Bi-Bi waves, as initially proposed by Sharma and Dean (1981), and obtained the expressions for second order mean water level, orbital motion (mass transport), and radiation stress tensor. Souza e Silva (2019) presented a detailed version of mathematical development and described an experiment on a wave basin to validate the theoretical findings.

The angle difference between the primary waves plays a very significant role on the second order quantities. In Figure 3, the wave number $\mathbf{k_b}$ is kept fixed at horizontal position, while the wave number $\mathbf{k_a}$ may occupy any position along the black circle. The red circle represents the subtractive interference wave number (Figure 3(a) for the vector $\mathbf{k_a} - \mathbf{k_b}$) or the additive interference wave number (Figure 3(b) for the vector $\mathbf{k_a}+\mathbf{k_b}$). Depending on the frequencies and orientation of the primary waves, the difference wave number may be shorter or longer than the primary wave numbers. It may happen that the interference wavelength be too long, hence $\| \mathbf{k_a}-\mathbf{k_b} \|$ be small, and the depth decay of second order subtractive interference may be very small, as if it were a shallow water wave, even though the interference frequency does not correspond to a shallow water wave, since the interference waves do not obey dispersion relation. A similar effect may happen with the additive wave number (Figure 3(b), bottom right).

Hence, second order forces on piles and pipelines may have a lower (higher) frequency contribution, which may not usually be accounted for, but which would interfere for instance with the pattern of vortex shedding. A similar discussion holds for second order bottom stresses and sediment dynamics, which became evident in Figure 2.

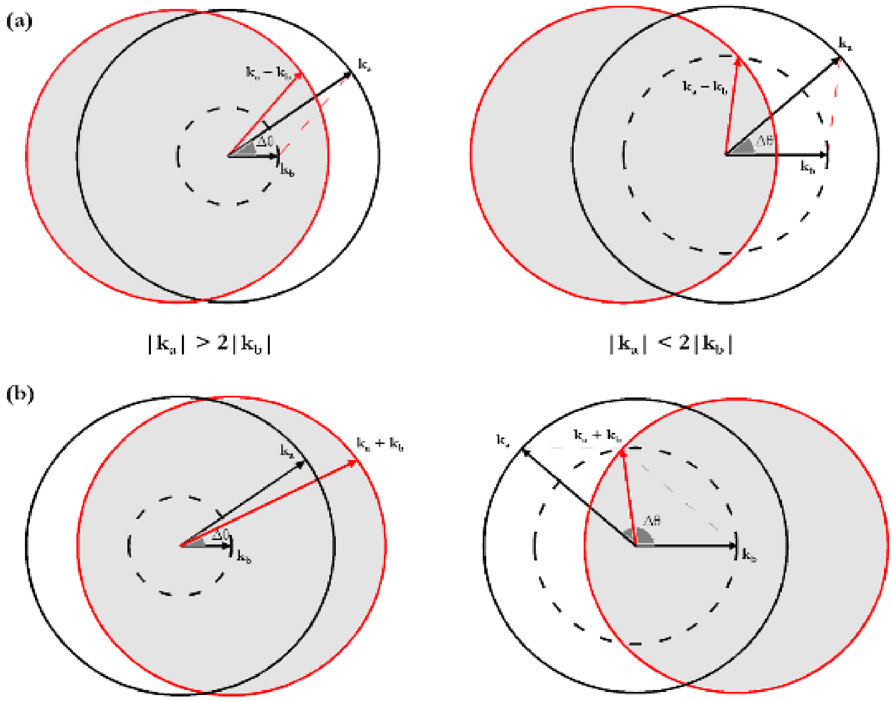


Figure 3. Resulting wave number vector for second order interaction (red circle): (a) subtractive interference; (b) additive interference.

Full expressions of second order wave orbital velocities and induced forces can be obtained in Sharma and Dean (1981). Souza e Silva et al. (2022) show the pattern of bottom orbital velocity for a Bi-Bi wave, thus raising the question on how to characterize the effective shear stress under a Bi-Bi wave system. At this point, it is noteworthy to emphasize the need to correctly describe the radiation stress pattern and the mean water level (wave set-up and set-down) under Bi-Bi waves (Souza e Silva 2022), and to introduce these quantities on long term responses of natural environments or man-made structures.

EXPERIMENTAL SET-UP

Experiments were held at the wave basin of the Ludwig Franzius Institut of Hydraulic, Estuarine and Coastal Engineering / Leibniz University Hannover where Bi-Bi waves were generated. The basin was 30 m × 15 m and equipped with a serpent type wave maker with 72 individual paddles, with active wave compensation routines capable of generating Bi-Bi waves (Figure 4). Full experimental set-up is described elsewhere (Souza e Silva et al. 2018, 2022). A total of 271 tests were simulated: water depth of 0.60 and 0.75 m; wave heights ranging from 0.05 to 0.16 m; periods ranging from 1.1 to 3.0 s; angle differences between primary waves of 0°, 10°, and 30°.

Two arrays of six ultrasonic sensor wave gauges (USSs) measured the free surface elevation according to CERC-6 recommendations (Davis and Regier 1977), and one array of five ADVs measured the orbital velocities. The ADVs were positioned either at 0.35 m or at 0.50 m below the free surface.

Among the large number of experiments, where different frequencies and directions were tested, the present article focuses on those (few) experiments which had velocity measurements at both elevations, as shown in Table 1, in order to investigate the orbital velocity decay along the vertical direction. Water depth was kept at 0.75 m for all tests.

Test duration was very precisely controlled in order to avoid the contribution of reflected waves from the walls, even though the basin was equipped with passive wave absorbers.

Figure 4. Wave basin at Ludwig Franzius Institute of Hydraulic, Estuarine and Coastal Engineering, Leibniz University of Hannover, generating Bi-Bi waves.

Table 1. Selected experimental conditions.

Test	Z (m)	H_a (m)	T_a (s)	θ_a (deg.)	H_b (m)	T_b (s)	θ_b (deg.)
T8-D5a	-0.35	0.13	2.1	15	0.13	2.1	-15
T8-D5b	-0.50						
T8-B2a	-0.35	0.10	1.3	10	0.16	1.7	0
T8-B2b	-0.50						
T8-B3b	-0.35	0.10	1.3	30	0.16	1.7	0
T8-B3c	-0.50						

RESULTS

Table 2 summarizes the wave lengths for each primary wave and for the subtractive interference wave. This is the depth decay scale for particle velocities. It is evident that the forced interference wave would be apparently at shallower relative depth (compared to the primary waves) and the decay along the water column for orbital velocities and pressure attenuation would be weaker than that predicted for the primary waves.

Table 2. Relative water depth for interference waves

Test	L_a (m)	L_b (m)	$L_{[a-b]}$ (m)	h/L_a	h/L_b	$h / L_{[a-b]}$
T8-D5	5.04	5.04	9.74	0.149	0.149	0.077
T8-B2	2.52	3.81	6.87	0.298	0;197	0.109
T8-B3	2.52	3.81	4.66	0.298	0.197	0.160

Even though these second order effects might be smaller than the first order quantities of the primary waves, the second order interference wave would be stronger than the second order effects of the primary waves, as $L_{[a\text{-}b]} >> L_a/2$ or $L_b/2$. On the other hand, the interference period $T_{a\text{-}b}$ would be given as

$$T_{a-b} = \frac{T_a T_b}{|T_a - T_b|} \qquad (1)$$

which gives a value of 5.53 s.

Considering this experiment might correspond to a hydraulic model on scale 1:25 (prototype water depth 19m, primary waves with periods 6.5 s and 8.5 s, and heights 2.5 m and 4.0 m, respectively), these conditions and equations indicated that a 27.65s oscillation might be recorded at the bottom by pressure transducers or ADVs.

This is certainly in the region of infragravity waves, which might remain unnoticed if the focus were on the rather strong wave conditions. As a matter of fact, long period oscillations in coastal environment may lead to structural fatigue, pollutant residual transport, and offshore sediment transport, among other effects.

Figure 5 shows an example of the 3D hodograph (3D plot of the orbital velocity) for experiments T8-B2 at the two elevations, conditions given at Table 1. The theoretical decay on the vertical velocity along the depth is more significant than the actual measurements. However, the theoretical decay on the horizontal velocity was better explained.

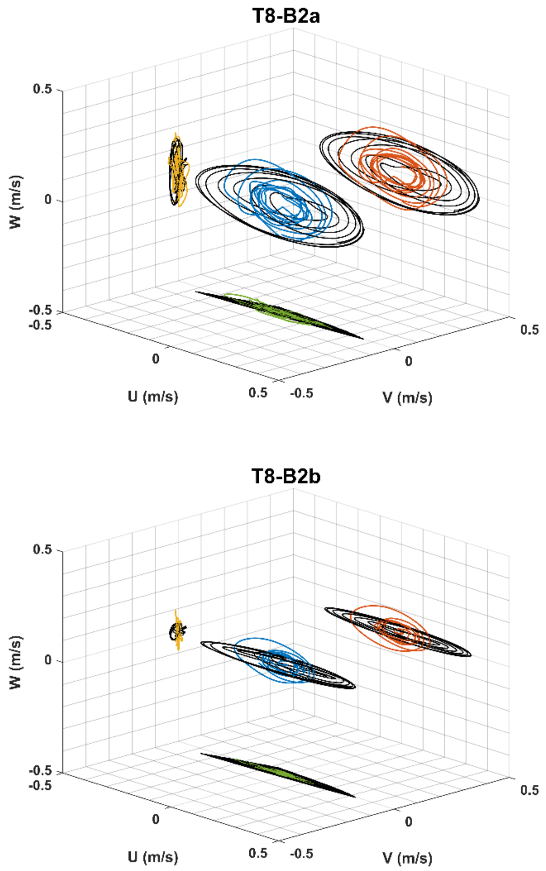


Figure 5. Hodograph of the orbital velocities for experiment T8-B2 at (a) z=-0.35m and (b) z=-0.50m. Black lines correspond to theoretical expressions.

Results for the total magnitude of orbital velocity (maximum value minus negative minimum value) are given in Table 3.

The ratio between the total magnitude of orbital velocities (maximum value minus negative minimum value), for Case T8-B2, at elevations z=-0.35 and z=-0.50 m are given in Table 4. The theoretical values (Sharma and Dean 1981) show a much stronger decay on vertical velocities, even though the predicted vertical velocity were much weaker than the measured velocity.

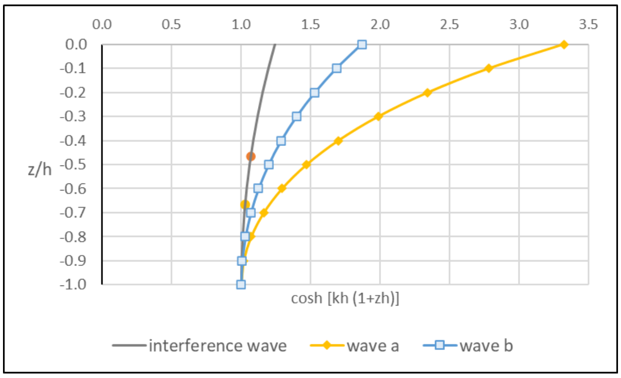


Figure 6. Depth attenuation of orbital velocities. The two dots indicate the elevations 0.35m and 0.50m below water surface.

Results from Table 3 indicate that the theoretical expressions (Sharma and Dean 1981) underestimate the vertical component of the orbital velocities (as seen on the U-W plane), but the horizontal velocities are well predicted. However, the actual velocity decay (ratio at the two elevations) is higher than predicted, and even higher than estimated by linear wave theory for the primary waves.

Table 3. Total magnitude of orbital velocity.

Elevation below free surface	Measured / Theoretical	U (m/s)	V (m;s)	W (m/s)
-0.35 m	Measured	0.494	0.050	0.330
	Theoretical	0.734	0.066	0.301
-0.50m	Measured	0.371	0.028	0.182
	Theoretical	0.681	0.073	0.073

Similar analysis was conducted for Case T8-B3, where the theoretical results provided better adjustment to the measurements at higher elevation. However, at lower elevation, the theoretical expressions largely underestimated the measurements, mostly the vertical component, as seen on Figure 7.

Table 4. Vertical decay ratio for orbital velocities

Case T8-B2	U	V	W
wave a (linear)	1.283	--	1.762
wave b (linear)	1.128	--	1.671
subtractive (2^{nd}.order)	1.040		
Sharma & Dean	1.077	0.909	4.111
measurements	1.333	1.752	1.820

DISCUSSION

Experiments in wave flumes with bichromatic waves do indicate the appearance of second order subtractive interference motion (e.g., Conde et al. 2014; Padilla and Alsina 2018). However other interference frequencies between higher harmonics also appear, as observed by Souza e Silva (2019) and elsewhere in the literature.

Introducing a second horizontal dimension (wave basin), the dynamics change completely (Sands 1982) as seen in Figure 8. However, few physical model experiments have been conducted with Bi-Bi waves in a wave basin, bringing attention to the innovative characteristics of the investigation by Souza e Silva (2019). As the main purpose of these experiments was to investigate the ability of an array of ADV sensors to identify second order waves, as well as the development of an adaptive methodology of analysis, the number of tests with measurements at two simultaneous depths was too short.

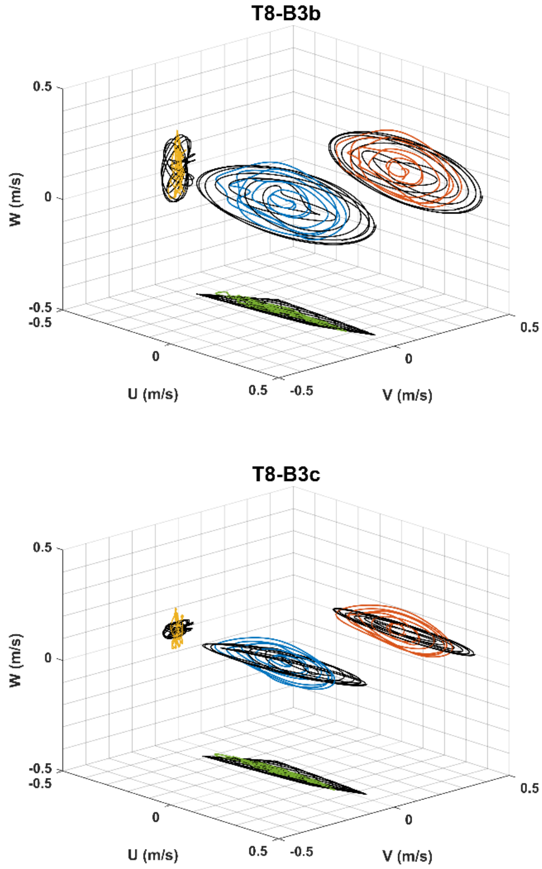
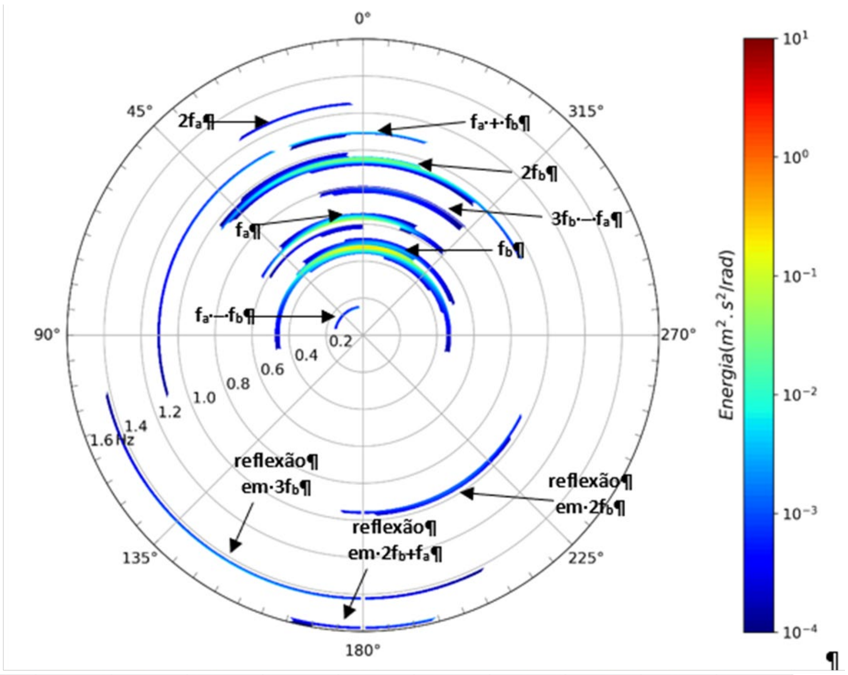


Figure 7. Hodograph of the orbital velocities for experiment T8-B3 at (b) z=-0.35m and (c) z=-0.50m. Black lines correspond to theoretical expressions.



	Ha	Ta	La	Da	Hb	Tb	Lb	Db	h	z
T2-E4	0.10m	1.6s	3.27m	5 deg	0.16m	2.1s	4.63m	-5 deg	0.60m	0.35m

Figure 8. Interference between harmonics of Bi-Bi wave: 10 degrees between primary directions, Ta=1.6s and Tb=2.1s. Reflected waves are identified at $2f_b$, $3f_b$, f_a-f_b, and $2f_b$+f_a; harmonics are identified at $2f_a$, $2f_b$, f_a-f_b, f_a+f_b, and $3f_b$-f_a.

Therefore, another set of laboratory experiments should be designed in order to investigate the actual effect of slow vertical decay of second order motion, investigating wider angle differences between primary waves. As a matter of fact, this slow motion might have influence on bottom shear stresses, sediment movement, ripple formation, and eventual long period fatigue of submarine structural elements.

It is undisputed that Bi-Bi waves generate two-dimensional wave groups (Herbers et al. 1994, 1995[a], 1995[b], 1997). There is also enough evidence that the continental shelf functions as a resonance box for infragravity waves (Smit et al. 2018). The main challenge consists of identifying the correct way of observing the phenomenon, as well as of developing adequate analyzing methodology. The use of arrays proved to be essential on identifying infragravity waves (Herbers 1994). It does not seem reasonable that a single point measurement would be able to characterize the two-dimensional pattern of Bi-Bi waves, even though it might be able to isolate low frequencies. As a thought-provoking suggestion, the same array set-up of ADVs used by Souza e Silva (2019) should be tested in the field. The distance between instruments would depend on local wave conditions, based on a Froude scale relation to the experiments at the Leibniz University Hannover. Certainly, it would not be an easy task to position and synchronize all instruments, but arrays of wave gauges have already been used in the past. Analyzing 3D-velocities, however, would be an innovative methodology, especially applicable to locations subject to bichromatic and/or bidirectional wave trains.

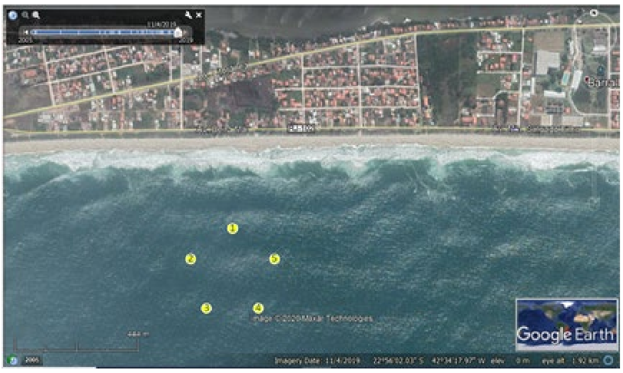


Figure 9. Proposed field experiment to identify Bi-Bi waves: array of ADVs (or ADCPs) and bottom pressure sensors.

CONCLUSION

Experiments with Bidirectional-Bichromatic (Bi-Bi) waves were conducted, at very controlled conditions, on a wave basin. The use of ADVs to describe the wave proved to be very illustrative about the actual kinematics and should be further investigated as an engineering tool. Results showed that the horizontal velocities are better described by theory than the vertical velocities. The presence of stronger vertical velocities at deeper elevation indicates that long period, second order oscillations would also be present. This might have significance on structural design and offshore near breaking sediment transport. Actual orbital velocities present weaker decay along the depth, indicating that stronger vertical velocities might exist closer to the bottom.

The analysis based on an adaptative method (Empirical Mode Decomposition) applied to an array of ADVs proved very efficient and it is suggested that a field experiment be designed using a similar scaled up array.

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