

# LONG WAVES FORCED BY SYMMETRIC AND ASYMMETRIC WAVE GROUPS

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## INTRODUCTION

Two solutions have been proposed for forced waves by wave groups. Figure 1 illustrates the two interpretations proposed by Longuet-Higgins and Stewart (1962), LHS1962, and Mei (1989) for steady groups. These indicate a forced wave oscillating either around or solely below the mean water level for the short waves, respectively. The existence of forced waves as either pure depressions or as oscillations about the still water level has significant implications in terms of shoaling behaviour and the generation of free long waves. With a bound wave with both positive and negative components then shoaling (an increase in amplitude in shallower water) does not necessarily require radiation of free waves to ensure continuity of volume, but with a bound wave of pure depression, then an increase in amplitude of that depression requires radiation of free waves to ensure continuity (Nagase and Mizuguchi, 1996, Nielsen and Baldock, 2010). Radiation of those free waves then changes the shape of the total infragravity wave during shoaling and may also influence the commonly observed lag between incident bound waves and the incoming short-wave envelope.

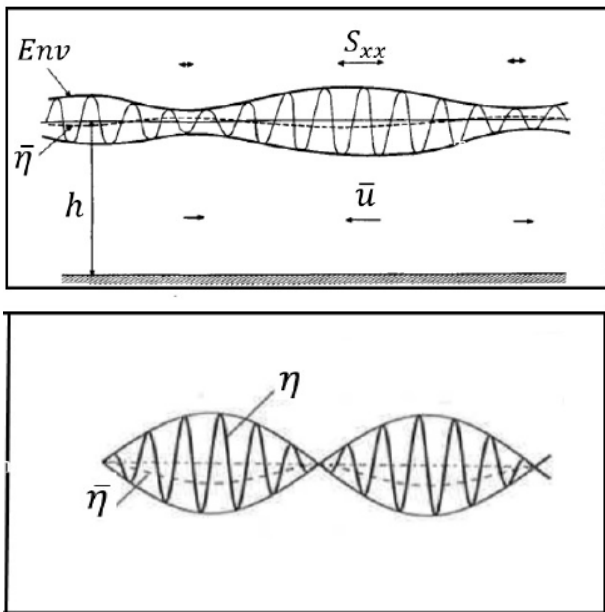


Figure 1. Illustration of the bound wave by Longuet-Higgins and Stewart (1962) with positive and negative part (a, top) and by Mei (1989) with a purely negative wave (b, bottom).

Moura and Baldock (2019) showed that models comprising a bound wave and radiated free waves show the apparent growth of the long wave at rates less than that proposed by LHS1962, which has been observed in many data sets, and theoretically analysed by alternate means. With this model approach, the forced waves (not

the total long wave) follow the equilibrium solution of Longuet-Higgins and Stewart, and the shoaling of the forced wave part is not frequency dependent. However, we note that radiation of free waves during shoaling has yet to be verified in observations. The analysis of shoaling conditions is complicated by the asymmetry of the short-wave groups, and consequent asymmetry of the forced long waves. If bound waves are not pure depressions, then long waves forced by asymmetric wave groups over horizontal bottoms should clearly indicate which of the two models for the forced wave is most appropriate. Here, we show that pure long wave depression can be generated by Gaussian wave groups, and that asymmetric forced long waves, with a leading crest and following depression, are generated by triangular / Gaussian groups.

## EXPERIMENTS AND RESULTS

The experimental examine the long waves generated during the propagation of wave groups of various shape. The experiments eliminate breakpoint generation and are performed with small amplitude short waves in a 35m long wave flume over a horizontal bottom to minimise non-linearity and energy transfers during shoaling. The water depth is 0.24m, which is intermediate and shallow. We consider the long waves generated during the propagation of wave ‘packets’ various shape, including individual and pairs of bichromatic wave group ‘packets’, Gaussian wave group ‘packets’, wave ‘packets’ with triangular short-wave envelopes, and asymmetric wave ‘packets’ with a Gaussian envelope on one side and a triangular envelope on the other. For convenience however, we will refer to all such short-wave trains as groups. The same wave groups were generated using both 1st order and 2nd order wave generation. Results from first and second order wave generation are contrasted, and the long wave surface elevation is plotted with the predictions of 2<sup>nd</sup> order theory from LHS1962.

Figure 2 illustrates the long wave forced by a pair of bichromatic wave groups similar to that in figure 1b. The oscillation about the still water level is clear, in accordance with LHS (1962). For this pair of groups, the gradient of the wave envelope is finite at the ends of the group, as indicated in figure 1. This contrasts with a Gaussian envelope, where the gradient of the envelope approaches zero at the ends of the group. For the bichromatic group, the 2<sup>nd</sup> order solution shows a leading and trailing hump, which is matched in the data when using second order wave generation. The trailing hump is partially dispersive. With first order wave generation the forced long wave is initially cancelled by the free waves generated by the wavemaker, consistent with the initial conditions assumed by Nielsen and Baldock (2010), which then evolves to a total long wave which has a phase lag compared to the short-wave group and the 2<sup>nd</sup> order theory (figure 2c).

$f_1=0.5\text{Hz}$ ,  $f_2=0.6\text{Hz}$ ,  $a_1=a_2=0.01\text{m}$ , non-dispersive group.

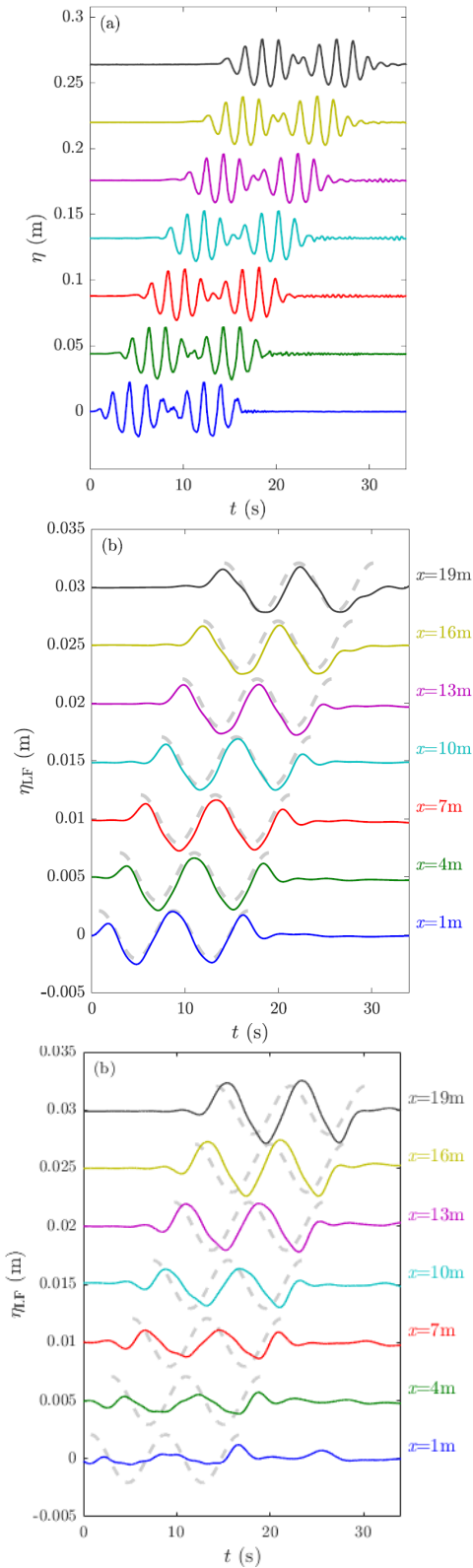


Figure 2. A pair of bichromatic wave packets (top), resulting long waves, with 2<sup>nd</sup> order (middle) and 1<sup>st</sup> order (bottom) generation and 2<sup>nd</sup> order theory (grey dashed lines),

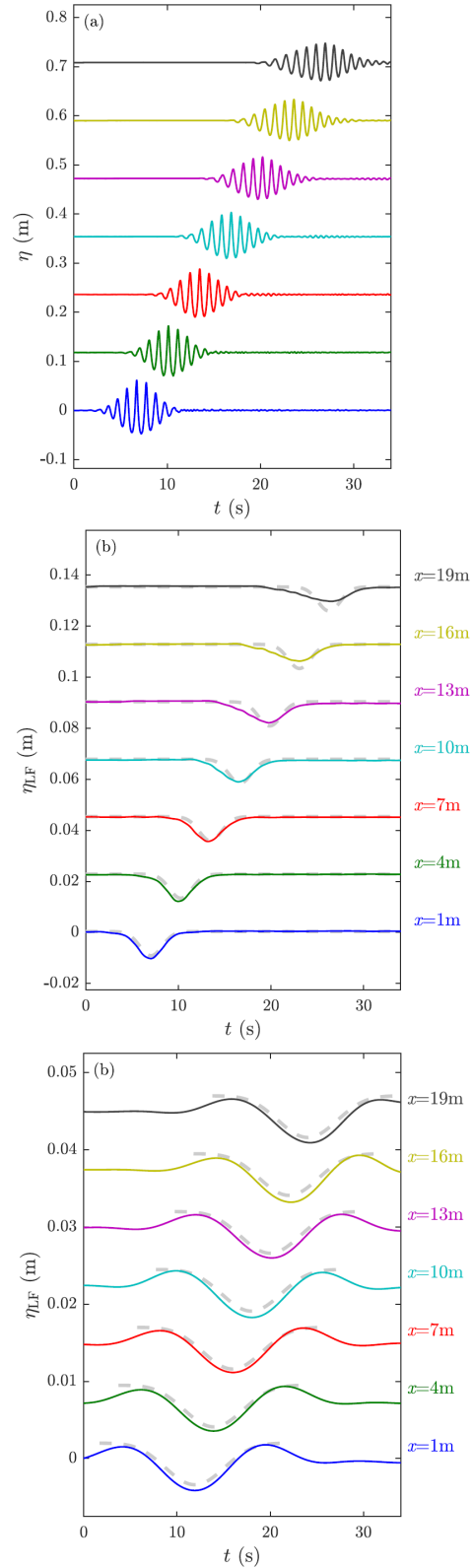


Figure 3. A Gaussian dispersive packet (top), resulting long waves, and long waves from a triangular wave packet (bottom), and 2<sup>nd</sup> order theory (grey dashed lines).

Thus, the *I*-wave shaped total long wave resulting from the Nielsen and Baldock (2010) model, which is not in antiphase with the forcing, is to some extent an artefact of the assumed initial condition.

Figure 3 (top and middle) illustrates the short-waves and corresponding long waves (2<sup>nd</sup> order generation) for a single Gaussian wave packet, where the gradient of the short-wave envelope asymptote to zero at the ends of the wave packet, with corresponding zero radiation stress gradients at those locations. Therefore, the resulting long wave is a pure depression, below the still water level and mean level of the short-waves. Note that the 2<sup>nd</sup> order theory indicated is for the initial wave packet, which is slightly dispersive in this case. The same case with 1<sup>st</sup> order generation gives an asymmetric long wave with a leading positive crest and lagging long wave trough.

The lower panel in figure 3 illustrates the long waves for similar conditions as the top panels, but in this case for a single triangular short-wave packet. This has a very similar shape to the Gaussian short-wave packet, but as for the bichromatic packets in figure 2, the gradient of the wave envelope is finite at the ends of the group, so the 2<sup>nd</sup> order solution shows a leading and trailing hump, which is again matched in the data. Consequently, single wave packets can generate long wave of pure depression or long waves that oscillate around the mean short-wave level.

Given the data shown, it is concluded that wave packets with finite gradients of the wave envelope at the edges or ends of the packet will generate forced long waves with crests of positive elevation at the edges and between the groups (and troughs of negative elevation below the centre of the packet), as in figure 1a. Gaussian wave groups will generate long waves of pure depression, provided correct 2<sup>nd</sup> order generation is used. In a numerical model the initial conditions must then match the forcing and not be a flat initial water surface and free long waves. The flat initial water surface condition in Nielsen and Baldock (2010) corresponds to 1<sup>st</sup> order generation conditions.

The presentation will show other wave packet shapes, notably asymmetric wave packets with a triangular-Gaussian form, which generates an asymmetric forced wave. This aims to mimic the asymmetric radiation stress gradients generated as a wave packet propagates over a slope. These conditions generate an asymmetric forced wave that remains in antiphase with the wave envelope at both ends of the group (Baldock, 2006). Further work includes RANS modelling of these wave packets with the OpenFoam solver.

## REFERENCES

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