DAMPING OF FINITE AMPLITUDE SOLITARY WAVES IN A FLUME

Yufei Wang, National University of Singapore, yufei.wang@u.nus.edu
Philip Li-Fan Liu, National University of Singapore, philip.liu@nus.edu.sg

INTRODUCTION

Solitary wave is a permanent wave when the dissipation is ignored. Many analytical solutions have been developed for finite amplitude solitary waves. In additional to the perturbation solutions, closed form solutions are also available (e.g., McCowan 1891, Clamond and Fructus 2003, being denoted as CF from hereon), which are more accuracy, especially for larger amplitude solitary waves which were discussed in Wang and Liu (2022). All the closed solutions satisfy the Laplace equation, bottom boundary condition and kinematic free surface boundary condition exactly. However, the dynamic free surface boundary conditions are only satisfied approximately. Wang and Liu (2022) showed that the CF's closed form solution matched very well with experimental data and Tanaka's (1986) numerical solutions, which is based on the boundary integral method solving the complex potential function.

In Wang and Liu (2022) 's laboratory experiments, solitary waves are slowly damped along the wave flume, which can be attributed to the energy dissipation inside the boundary layers on the bottom and sidewalls. Keulegan (1948) first derived an approximate solution to describe the wave damping in the wave flume for small amplitude solitary wave. The basic idea of calculating the damping effect is that the rate of energy dissipation inside the boundary layers, (D) must be the same as the rate of wave energy loss, (dE/dt).

In this work, the same principle of energy balance is adopted for calculating the wave damping for finite amplitude solitary waves in a wave flume. The closed form solutions provided in CF are employed here for analyses.

NUMERICAL MODEL

In the present paper, the wave flume has a rectangular cross section, where the water depth is h and the width of the flume is B. The parameter $\gamma = B/h$ represents the shape of the wave tank. For a finite amplitude solitary wave, the wave amplitude is denoted by H. The parameter $\alpha = H/h$ defines the nonlinearity of the solitary wave. As the wave propagates down the flume, transient viscous boundary layer flows are developed along the bottom and two sidewalls. The closed form solutions in CF provide the free steam velocity components at the outer edges of the respective boundary layer and the wave energy in the potential flow region. The velocity components in the boundary layer has been calculated accordingly. In estimating the energy loss inside the sidewall boundary layers, two important features are considered in the analyses: 1) The wetted areas above the still water level (see the red area in Figure 1), and 2) both the horizontal and vertical velocity components and their profiles in the water column.

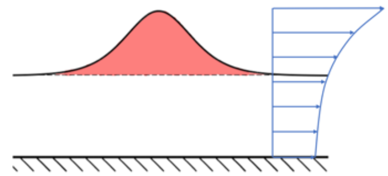

Figure 1 - Sketch of solitary wave and its horizontal velocity distribution (blue lines) under the wave crest (not to scale).

During the calculation of D, the linearized boundary layer equations are used here as in Keulegan (1948), i.e., the convection terms are neglected. The energy balance equation is derived by equating the rate of energy dissipation inside the boundary layers, (D), and the rate of wave energy loss, (dE/dt), which can be solved numerically by a time marching scheme.

EXPERIMENTS

The damping numerical solutions have been compared with experiments conducted in NUS Hydraulic Laboratory using the long stroke flume with a piston type wavemaker. The dimension of the wave flume is 36m x 0.9m x 0.9m.

As shown in Figure 2, six capacitance wave gauges were used to measure the wave height along the flume. The distances between the six wave gauges and the initial position of the wave maker has been shown in Table 1. The sampling rate of the wave gauge is 200 Hz. As described in Wang and Liu (2022), to ensure the solitary wave has been fully developed in the measuring area, the first wave gauge need to be installed 9m away from the initial position of the wavemaker's paddle. To minimize the effect from the slope, the last wave gauge was installed 5m away from the end of the slope.

To study the influence of the γ and α value on the damping effect, eighteen sets of experiments has been conducted. The experiments were conducted with 0.18m, 0.3m and 0.45m water depth (γ =5, 3, and 2, respectively). For each water depth, the six different initial amplitudes were measured within the range of $0.1 < \alpha < 0.65$. For each case, the experiments were repeated for five times.

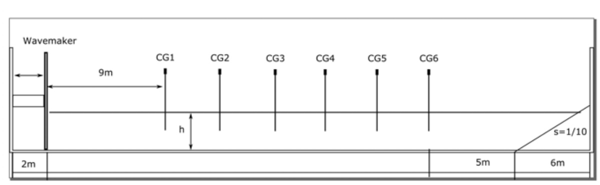

Figure 2 - A sketch of the long stroke wave flume (Note: CG denotes capacitance gauges.) .

Wave Gauge	CG 1	CG 2	CG 3	CG 4	CG 5	CG 6
Distance	9.3 m	11.3 m	14 m	17 m	20 m	23 m

Table 1 - the distance between the capacitance wave gage and the initial position of the wave paddle.

RESULTS

Figure 3 shows the comparisons among the present numerical model simulated all the three water depths of our laboratory experiments. The models were set that the started amplitudes of the solitary wave is $\alpha = 0.75$. Then, the solitary wave propagates to α near zero, as the study is focus on the finite solitary wave. The solitary wave damps faster as γ increasing and α increasing.

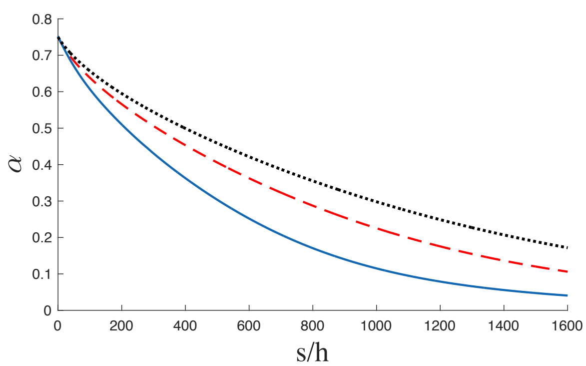

Figure 3 - Calculated α values at different locations with initial α values. Different lines represent difference conditions: blue solid line is h=0.18m (γ=5); red dash line is h=0.3m (γ=3); black dotted line is h=0.45m (γ=2). s is the distance. s=0 stands for where the numerical model is started.

In the result section, the experimental results of h=0.3m (γ=3) are discussed as an example. The trend is very similar for other water depths experimental results. Figure 4 shows the comparisons among the present numerical solutions, Keulegan's solutions and five-runs averaged experimental data for h=0.3m (γ=3). The initial α value is calculated from the first wave gauge. From the largest to lowest amplitude solitary wave, the initial α values are 0.63, 0.54, 0.45, 0.35, 0.26 and 0.17, subsequently.

For the first three wave gauges, Keulegan (1948) and the present model are all agree with experimental data well. As the wave propagating longer distance, the differences becomes more obvious. Compared with experimental data, Keulegan's solutions tend to underestimate the damping effect, especially for the finite amplitude waves.. The new damping solutions agree well with the measurements for all the amplitudes. The trend is very similar for other water depths experimental results.

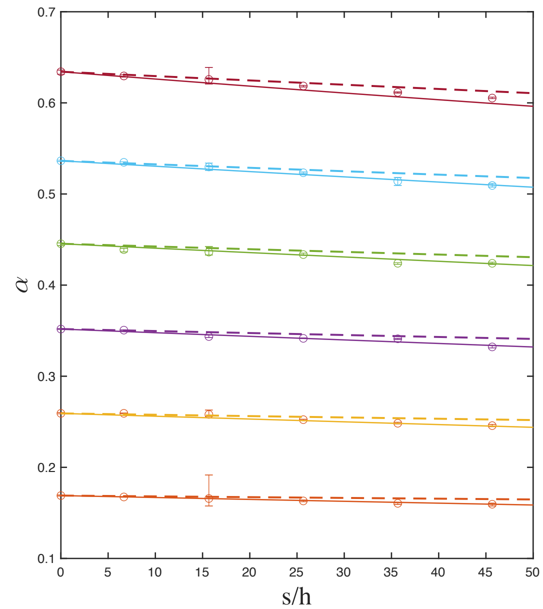

Figure 4 - Measured and calculated wave heights, $\alpha = H/h$, at different distance, s, from the wavemaker for $h = 0.3m$. The solid lines denote new damping solutions; the dashed lines represent Keulegan's solutions; the circles with error bar are the experimental data. From top to bottom, The different line's colors represent difference initial α values. The initial α value is 0.63, 0.54, 0.45, 0.35, 0.26 and 0.17, subsequently.

CONCLUDING REMARKS

Overall, the new numerical model with CF's solution provides good estimation on the solitary wave attenuation especially for a narrower wave tank and finite amplitude waves.

As in the numerical model's assumption, the convection terms (i.e., the nonlinear terms) are neglected as it has been seen as a small value. For majority cases, it could provide accurate estimation. However, for a very narrow wave tank with a large amplitude, the nonlinear terms might have more obvious influence. Especially for the area near the wave crest, the horizontal velocity and its gradient product could be large. The next step is to determine the influence of the nonlinear terms on the solitary wave damping estimations. The accuracy of the solitary wave damping model in a flume could be further improved.

REFERENCES

Clamond & Fructus (2003): Accurate simple approximation for the solitary wave, Comptes Rendus Mécanique 331, 727-732 (2003).

Keulegan (1948): Gradual damping of solitary waves, Journal of Research of National Bureau of Standards, 40 (6), 487.

McCowan (1891): VII. On the solitary wave, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 32 (194), pp. 45-58.

Tanaka (1986): The stability of solitary waves, The Physics of Fluids, AIP, vol. 29 (3), 650.
Wang & Liu (2022), On finite amplitude solitary waves, submitted for publication.