THE OCCURRENCE OF EXTREME WAVE HEIGHT IN A TWO-DIMENSIONAL RANDOM WAVEFIELD IN COASTAL AREA

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INTRODUCTION

As the water waves come to the coastal area, the variation in dispersion relation due to the spatial inhomogeneity brings about the deformation of the wave shape. With the local bathymetry effect, the propagation of the wave experiences a complicated evolution process, including the reflection, wave shoaling and breaking, etc. Considering the directional spreading effect and wave refraction becomes indispensable when it comes to a two-dimensional (2D) problem in the random wavefield. Mori et al. (2011) indicated that the distribution of maximum wave height in deep-water is decided by the directional dispersion.

In this research, we aim to discuss the occurrence of maximum wave height in the 2D wave evolution in the coastal area. It helps to estimate better extreme events in the prevention work of coastal hazards. To simulate the 2D propagation, we develop a nonlinear model as an extension work of Lyu et al. (2021), which considers the high order harmonic interactions of irregular waves before the breaking in surf zone.

THEORETICAL MODEL

For a 2D wavefield with finite water depth h, the propagation of the random wave can be expressed as the surface elevation η in a 2D space-time (2D+T) form:

$$\eta(x,y,t) = \varepsilon A\exp\{\mathrm{i}[(k_x x + k_y y) - \omega_0 t]\} + \mathrm{c.c} + O(\varepsilon^2), \quad (1)$$

where ε is the wave steepness, k_x, k_y and ω_0 are the wave numbers and angular frequency, $A(x,y,t)$ is the phase-independent amplitude. Based on the hypothesis of the four-wave interactions, the evolution of A can be given by a modified Nonlinear Schrödinger type equation:

$$\mathrm{i}\mu A + \mathrm{i}\frac{\partial A}{\partial x} + \lambda\frac{\partial^2 A}{\partial t^2} + \gamma\frac{\partial^2 A}{\partial y^2} = \upsilon |A|^2 A, \quad (2)$$

where coefficients $\mu, \lambda, \gamma, \upsilon$ are functions of k_x, k_y, ω_0, h. μ is related to the derivative of h, so Eq. (2) can describe the wave evolution on a varying depth.

Different from the ordinal treatment of the spectral wave modeling, in this model, we integrate Eq. (1) from offshore to onshore, assuming periodic boundary conditions in time. At the initial condition at $x = x_0$, we give the Fourier amplitude $\hat{A}$ as the Gaussian distribution:

$$\hat{A}(\omega, x_0, \theta) = \frac{\varepsilon}{2\pi\sigma_\omega\sigma_\theta} \times e^{-\frac{1}{2}\left[\left(\frac{\omega-\omega_0}{\sigma_\omega}\right)^2 + \left(\frac{\theta-\theta_0}{\sigma_\theta}\right)^2\right] + \psi}, \quad (3)$$

where $\theta = \arctan k_y / k_x$ represents the wave direction of different wave components, and θ_0 is the initial wave principal direction. σ_θ,σ_ω are spectral bandwidth of θ, ω, respectively. ψ gives a random phase.

NUMERICAL RESULT

In Figure 1, we give the transient surface elevation of one sample of oblique random wave on a 30×30L_0 (L_0: wavelength on k_x) over a flat bottom. The waves propagate with a dispersion range based on the principal direction $\tan\theta_0 = 0.2$. With the Monte Carlo simulation (MS) over an uneven bottom, the exceeding probability of maximum wave height $P_H(H_{\max}/\eta_{\mathrm{rms}} > 8)$ at different σ_θ and kh is given in Figure 2. As the σ_θ increases from 0.3 to 0.5, P_H monotonically drops and the degree of decline decreases from deep to shallow water. It indicates that the directional spreading has a dispersion effect on the occurrence of extreme events, and the extent of this effect is related to the water depth.

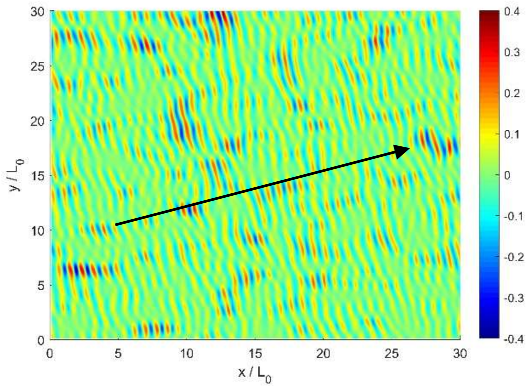

Figure 1 - Transient surface elevation η at $t = 40T$ with $\tan\theta_0 = 0.2$, $\sigma_\theta = 0.3$ over a flat bottom $kh = 5$

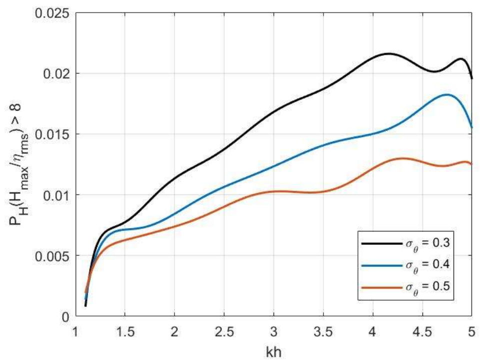

Figure 2 - Exceeding probability of $H_{\max} > 8\eta_{\mathrm{rms}}$ by MS

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