APPLICABLE RANGE OF PERIODICAL WAVE THEORIES UPDATING LE MEHAUTE'S CHART

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INTRODUCTION

Le Mehaute (1976)'s chart (see Fig. 1) has been widely used in coastal engineering community because of its simplicity. For given water depth h, wave height H, and wave period T, users can easily identify the suitable wave theory by calculating h/gT^2 and H/gT^2 (see Figure 1). However, there is a need to update this chart based on newly developed Stokes wave theory.

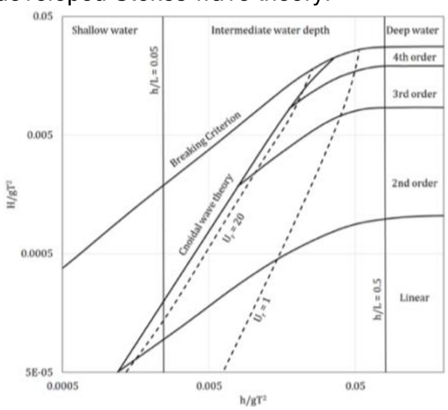

Figure 1. Le Mehaute's diagram redrawn (axis rescaled in terms of dimensionless parameters)

STOKES WAVE

Zhao and Liu (2022) presented an updated 5th order solution for Stokes wave. The free surface profile can be expressed as

$$\eta/a = \underbrace{\cos\theta}_{A_1} + \underbrace{kaB_{22}\cos 2\theta}_{A_2} + \underbrace{k^2a^2(B_{31}\cos\theta + B_{33}\cos 3\theta)}_{A_3} + \underbrace{k^3a^3(B_{42}\cos 2\theta + B_{44}\cos 4\theta)}_{A_4} + \underbrace{k^4a^4(B_{51}\cos\theta + B_{53}\cos 3\theta + B_{55}\cos 5\theta)}_{A_5} + O[(\pi H/L)^6] \quad (1)$$

The coefficients can be found in the same paper. Here, we used the contribution of each order term to update the chart for demarcating the range of Stokes waves. To quantify which order of Stokes wave theory is applicable, we defined a ratio parameter as below and suggest a value of 1% as the criterion.

$$\frac{|A_n|_{max}}{|A_{n,prec}|_{max}} = \frac{|A_n|_{max}}{|A_1|_{max} + |A_2|_{max} + \ldots + |A_{n-1}|_{max}}, n = 2\ldots 5 \quad (2)$$

With the new Stokes wave theory and the clear definition for quantifying each order's contribution, we have an updated chart as in Figure 2. Wavelength L is obtained using non-linear dispersion relationship for given wave conditions. The blue dash dot line indicates wave breaking limit following the equation proposed by Fenton (1990). On the right side of the line with $Ur = (H/L)/(h/L)^3 = 26$, Stokes wave theory is applicable. The limit of H/L for deep water for each order is shown by the numbers above each line. In case users have an even deeper waves than that shown in Figure 2, users only need to calculate H/L and find the respective range for applicable order. The Stokes wave solutions are lower order compatible, i.e., one can use Stokes V solution to calculate the wave parameters in the range of linear waves, but not the other way around.

CNOIDAL WAVE

This paper adopts Fenton (1999)'s 3rd order and 5th order cnoidal wave solution for updating the chart. Fenton's 3rd order solution is applicable in the range between the two lines represented by $m \approx 0.5$ and $m = 0.96$. For higher m values (region above the red dot line), Fenton's 5th order solution is applicable. Alternatively, Clamond (1999)'s solution can be considered.

SOLITARY WAVE

Here we include a red solid line for $m = 0.99999996$, above which the waves can be considered as solitary waves. The criteria recommended is based on the difference between trough elevation and mean water level is less than 5% of h, and 10% of H.

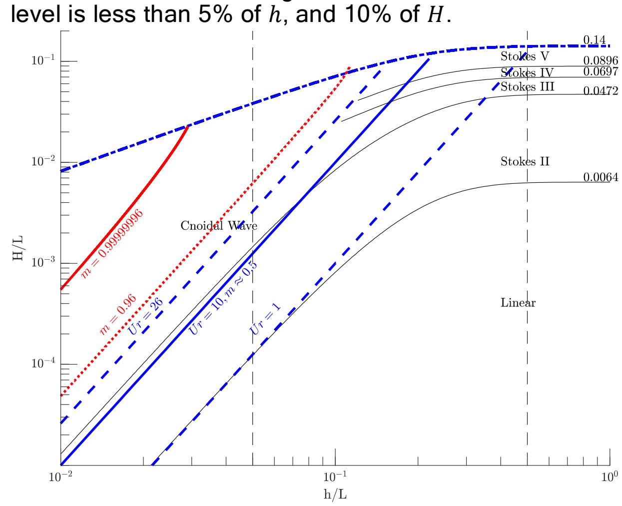

Figure 2. Updated Le Mehaute's diagram - applicable range of various period wave theories

For convenience, the updated figures can be found from https://github.com/KuifengZhao/waveModelSelection.git

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