ON THE PROBABILITY OF UNIDIRECTIONAL NONLINEAR EXTREME WAVES IN THE PRESENCE OF WAVE REFLECTION

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INTRODUCTION

Extreme waves are known to appear in different water depth regimes, and their focusing mechanisms have been intensively studied over the last decades. This experimental study aims to improve understanding of rogue wave statistics when wave reflections are at play. A series of unidirectional JONSWAP wave trains have been generated in a wave flume while varying the water depth and beach inclination slope. The data collected near the beach installation suggests a general decrease in extreme wave probability with the increase of the beach inclination, thus, with the increase of wave reflection. In deep-water, the numerical simulations based on the coupled nonlinear Schrödinger equation, which accounts for the presence of reflective waves, show a very good agreement with the experimental data. The irregular wave measurements in the presence of wave reflection indicate that the decrease in probability of exceedance, which correlates with the decrease in the value of kurtosis, is due to the weakening of third-order effects. When the water depth is decreased to a finite depth regime, we can observe an increase of extreme events, which we attribute to the change of the wave breaking threshold. However, no substantial rogue waves have been observed below the dimensionless depth kh value of 1.36 . Then again, nonlinear effects always remain relevant and at play in all cases considered.

METHODOLOGY

The analytical framework adopted in deep-water is based on the weakly nonlinear approach for water waves, which accounts for interaction between incident wave dynamics ψ_1 and reflective motion ψ_2. The wave coupling can be described by the following linked framework, also known as the coupled nonlinear Schrödinger equation (CNLSE):

$$i\left(\psi_{1_x}+\frac{1}{c_g}\psi_{1_t}\right)+\beta\psi_{1_{tt}}+\mu\left(|\psi_1|^2-2|\psi_2|^2\right)\psi_1=0,$$
$$i\left(\psi_{2_x}-\frac{1}{c_g}\psi_{2_t}\right)+\beta\psi_{2_{tt}}+\mu\left(|\psi_2|^2-2|\psi_1|^2\right)\psi_2=0. \tag{1}$$

Here, $c_g=\omega/(2k)$ is the value of group velocity, $\beta=-k/\omega^2$ the dispersion, and $\mu=-k^3$ the nonlinearity parameter, see (Gramstad and Trulsen, 2011). The laboratory experiments were conducted in a $30\ \mathrm{m}$ wave flume with $25\ \mathrm{m}$ of effective wave propagation distance, i.e., excluding the beach installation, see Figure 1.

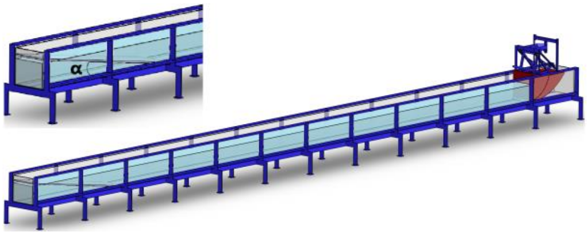

Figure 1 - Wave flume with a piston-type wave maker and variable linear beach inclination. The inclination angle is denoted by α.

The water depth and slope inclination have been varied to allow a variety of cases for completeness of the experimental study. The wavemaker has been programmed to generate time-series, which satisfy the representative unidirectional JONSWAP sea state (Hasselmann et al. 1973) with a peakedness parameter $\gamma=6$. The statistical analysis has been performed on the wave measurements collected by a resistive wave gauge placed $2.5\ \mathrm{m}$ away from the beach installation.

RESULTS

We have performed 70 experimental runs in total and applied our statistical analysis on the time-series, which includes at least 1200 wave periods. An example of deep-water exceedance probability variation of crest heights normalized by the value of significant wave height $H_s=0.06\ \mathrm{m}$ with a selected peak frequency of $1\ \mathrm{Hz}$ is shown in Figure 2.

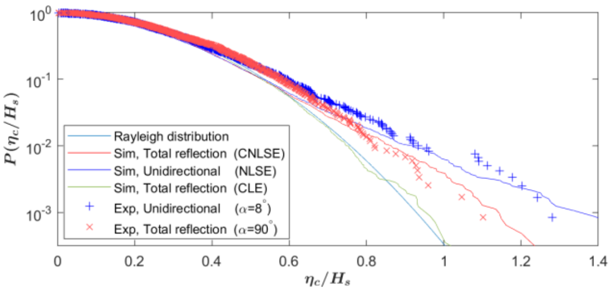

Figure 2 - Example of exceedance probability of deep-water extreme events for the unidirectional ($\alpha=8°$) and full reflection ($\alpha=90°$) cases. Solid lines: expectations from linear (Rayleigh and coupled linear Schrödinger equation (CLE); for $\mu=0$) and nonlinear theory (NLSE and CNLSE). Crosses: experimental values.

Our simulations show that despite the weakening of third-order nonlinear effects, these remain relevant even for the case of standing waves (full wave reflection). This is also confirmed for all cases explored, particularly, when the constant water depth is decreased to satisfy finite water depth regimes. The extreme wave statistics results for a fixed generated significant wave height value $H_s = 0.045\ \mathrm{m}$ and a peak frequency of $1.2\ \mathrm{Hz}$ while varying the water depth, are shown in Figure 3.

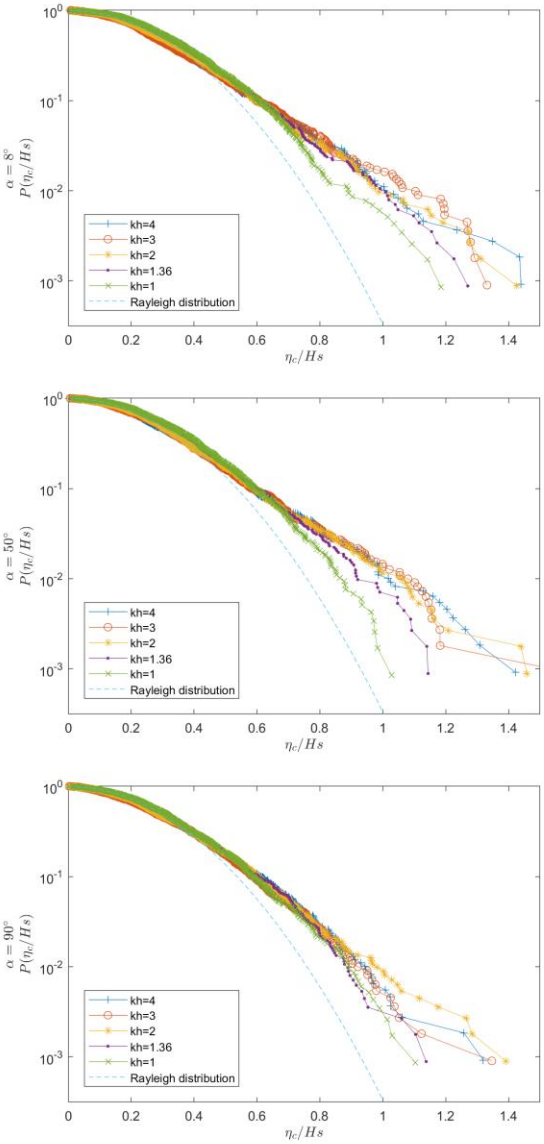

Figure 3 – Variation of the exceedance probabilities by changing the water depth and the reflective beach angles. The angles adopted are $8°, 50°, 90°$, respectively, representing the unidirectional, intermediate reflection, and full reflection scenarios at different water depth regimes.

We can clearly note that all experimental extreme probabilities deviate from linear expectations and overwhelmingly decrease with the depth decrease or the reflection increase, *except* in finite water, i.e., the case of $kh = 2$. We conjecture that this is due to the change in the associated wave breaking threshold. The experiments also underline the fact that modulation instability is not active for $kh < 1.363$ through the absence of any major extreme event occurring in such configurations.

CONCLUSION

We have performed an experimental study to quantify extreme wave statistics when unidirectional waves are reflected from an artificial beach installation. The overall results indicate that the CNLSE simulations show a very good agreement with the experimental statistics in deep-water. This confirms the fundamental role of nonlinearity in such wave processes. A fact also underlined in finite depth regimes. When $1.36 < kh < 2$ we observe a clear increase of extreme events in the presence of counterpropagating waves due to the increase of the wave breaking threshold.

REFERENCES

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