

# TURBULENCE MODELLING OF INCIPIENT WAVE BREAKING ON A VERTICAL CYLINDER ON A SLOPED BED

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## BACKGROUND AND INTRODUCTION

The extensive use of vertical piles as basic components of coastal structures, e.g. monopile wind turbine foundations, has made the study of wave impact on vertical cylindrical structures in the breaking zone of major practical importance. To investigate the breaking wave forces on a vertical cylinder located on a sloped bed, Reynolds-averaged Navier-Stokes equations (RANS) based turbulence models have been used as closure in most recent computational fluid dynamics (CFD) studies. Qu et al. (2021) evaluated different RANS-based two-equation turbulence models, in their standard and stabilized (Larsen & Fuhrman, hereafter LF18) forms, for simulating the experiment of Irschik et al. (2004), in which waves are breaking on the frontline of a vertical cylinder with its center located at the end of the 1:10 slope. Their results suggest that the stabilized two-equation turbulence model fails to predict the breaking point and peak force accurately, though LF18 have previously demonstrated that such stabilized two-equation turbulence models can accurately predict the breaking point. Qu et al. (2021) likewise suggest that the breaking location is influenced by the choice of turbulence model. This could possibly be due to the large Courant number ( $Co=0.5$ ) used in their simulations, which is specifically investigated in the present work. In addition to the peak force, the secondary load cycle (SLC), which is sometimes linked to 'ringing' i.e. vibrations which appear at natural frequencies of the structure, has also been a focus in some recent studies. To date, the physical cause driving the SLC is still debated. For example, simulations of Paulsen et al. (2014) showed that the occurrence of the SLC is related to the downstream vortex formation. Kristiansen & Faltinsen (2017) suggested that the local rear run-up, which is caused by the pressure due to flow separation, is responsible for the SLC. However, an early study of Grue et al. (1993) reported no flow separation, while still observing the SLC. A recent study of Antoloni et al. (2020) concluded that the SLC may be mainly driven by gravity wave effects rather than vortex formation. Inspired by these recent studies, the prediction of SLC and related flow separation and vortex formation, are also numerically investigated in the present work.

## METHODOLOGY

The present study adopts a Reynolds stress model (RSM), namely the Wilcox (2006) stress- $\omega$  model, as our primary turbulence closure model to simulate the experiment of Irschik et al. (2004). The Wilcox (2006) stress- $\omega$  model has been analysed in the recent work of Li et al. (2022) and proven to be neutrally stable in the potential flow region beneath surface waves, thus it naturally avoids any unphysical turbulence over-

production in the pre-breaking zone (unlike two-equation models in their standard forms). It has also shown good accuracy in predicting surf zone breaking waves in Li et al. (2022) and deep-water wave breaking due to modulational perturbations in Li & Fuhrman (2022). For comparison purposes, the present study also conducts simulations with the stabilized LF18  $k$ - $\omega$  model, as well as with no turbulence model.

## EFFECT OF COURANT NUMBER ON THE BREAKING POINT

To investigate the effect of the Courant number ( $Co$ ), 2D breaking wave simulations are performed with the stress- $\omega$  model and with  $Co$  ranging from 0.025 to 0.3.

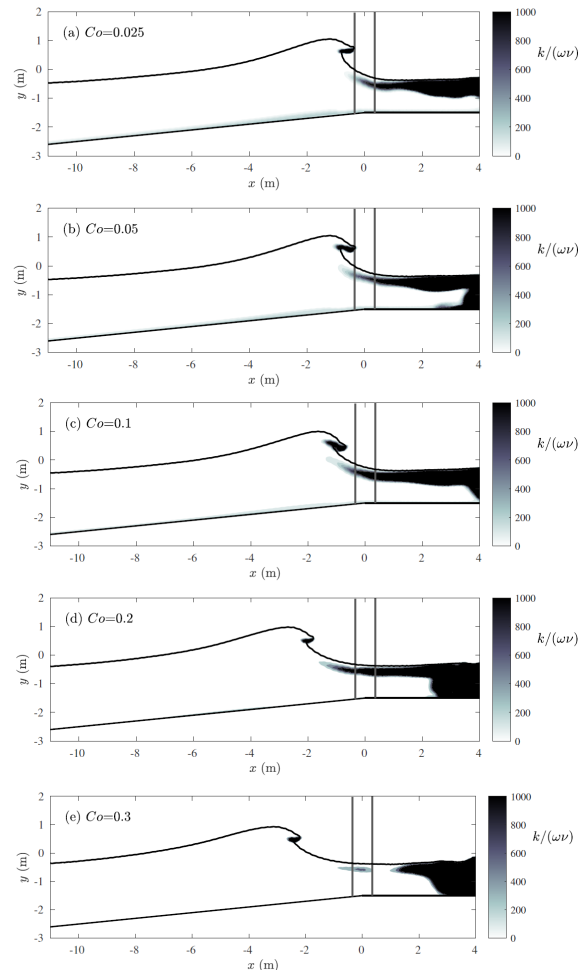


Figure 1 Wave profiles and turbulence levels beneath the surface waves simulated with the Wilcox (2006) stress- $\omega$  model with different  $Co$ .

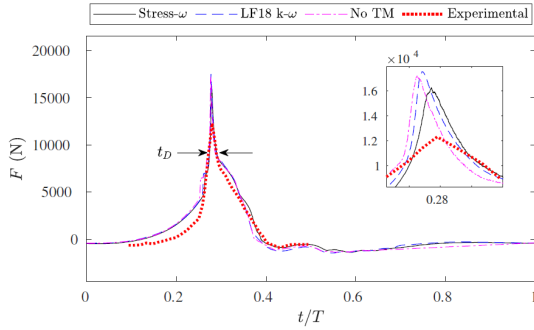


Figure 2 Instantaneous peak force from the numerical predictions and the measured data processed in Choi et al. (2015). Simulations are with  $Co=0.05$  and second-order discretization schemes. TM stands for turbulence model.

It is seen in figure 1 that the breaking point is converged and is on the frontline of the vertical pile with  $Co=0.025$  and  $0.05$  (figure 1a,b), in line with what was observed in the experiment (Irschik et al. 2004). However, as  $Co$  is increased to  $0.1$  and larger, as shown in figure 1c–e, the waves break earlier and earlier. The present study on breaking waves thus demonstrates a similar phenomenon as the study on non-breaking progressive waves of Larsen et al. (2019). Namely, that a large  $Co$  can alter the breaking point due to inaccurate flow kinematics, whereas maintaining  $Co \leq 0.05$  ensures reasonable convergence in the breaking position.

Interestingly, we also find that if a large  $Co$  is used together with a two-equation turbulence model that is unstable in the potential flow region beneath waves (e.g. the standard  $k-\omega$  model, the standard realizable  $k-\varepsilon$  model, which has tendency of turbulence over-production), the two errors could cancel each other, i.e. early breaking due to a large  $Co$  may be delayed due to decreasing wave heights caused by the turbulence over-production and associated unphysical energy dissipation. In this way the waves may coincidentally break on the pile, but the waves would expectedly have both a polluted turbulence field and inaccurate velocity kinematics. This is what we believe occurred in the simulation of Qu et al. (2021); Their simulation with  $Co=0.5$  and the standard  $k-\omega$  SST model showed that their waves broke right on the pile frontline after 40 wave periods, but their domain was clearly polluted by turbulence over-production prior to breaking (see their figure 11d).

## RESULTS ON PEAK FORCE

Figure 2 shows the peak force  $F$  (i.e. the total horizontal force) induced by breaking waves on the vertical cylinder. Numerical predictions with the Wilcox (2006) stress- $\omega$  model, the LF18  $k-\omega$  model, and no turbulence model (all with  $Co = 0.05$ ) are compared to the (filtered) experimental data presented in Choi et al. (2015). The duration of the sharp peak (or slamming force) is  $t_D \approx 0.02T$  (where  $T$  is the wave period). All three turbulence models are seen to predict the slamming duration reasonably in line with the measurement. Another

common feature is that all three likewise predict almost identical rise leading up to the peak force and more or less the same peak value. This is because the peak force is measured right at the onset of wave breaking. From pre-breaking up to the breaking onset, we expect that even with no turbulence model, the peak force should be accurately predicted. Our numerical results are in line with this expectation.

## RESULTS ON SECONDARY LOAD CYCLE

The predicted SLC is compared with the experimental data in figure 3. Figure 3a shows a good match between the stress- $\omega$  model prediction and the measurement of the force magnitude during the SLC. Figure 3b shows an obvious magnitude difference of the force during the SLC between the LF18  $k-\omega$  prediction and the experimental data. The no turbulence model prediction of the force magnitude during the SLC in figure 3c is somewhere in between the numerical results in figure 3a and 3b. We have stated earlier that the accurate peak force prediction is essentially independent of the turbulence model utilized. However, as the SLC is generated after wave breaking and is associated with the flow around the cylindrical body, turbulence models are expected to have a more significant effect on the SLC predictions. Figure 4 shows the vorticity field at  $z=-D/2$  at  $t/T=0.49$ , when the SLC reaches its local peak (see figure 3). When comparing figure 4a and 4b, an earlier flow separation and a stronger vorticity field is predicted with the stress- $\omega$  model (figure 4a), which in turn causes larger magnitude negative pressure on the lee-side during the flow reversal. This generates a stronger suction force on the lee-side and raises the total horizontal force up in figure 3a, which matches much better with the measurement.

## CONCLUSIONS

The present work has focused on the turbulence modelling of incipient wave breaking on a vertical circular pile on a sloped bed. In the present study, we have first investigated the effects of the Courant number ( $Co$ ) on the breaking point. It has been shown that the peak force on the vertical cylinder is not affected by the choice of turbulence model as long as the turbulence model is stable and the simulations are converged (i.e. with sufficiently small  $Co$ ). Specifically, up to the onset of breaking where the incipient peak force occurs, accurate predictions can be achieved even without a turbulence closure model. However, the prediction of the secondary load cycle (SLC) requires proper turbulence modelling, as the process is post-breaking and involves turbulence production and lee-side flow separation. Overall, the Wilcox (2006) stress- $\omega$  model has been proved capable of providing good accuracy for both breaking wave induced peak force and the secondary load cycle.

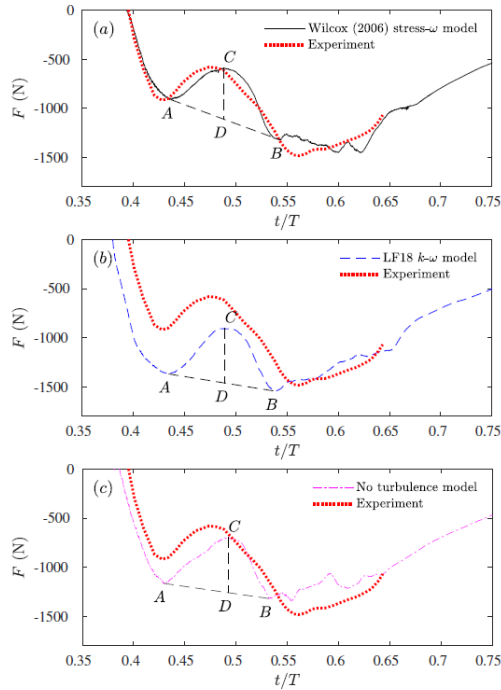


Figure 3 Secondary load cycle from numerical predictions and the experimental measurement.

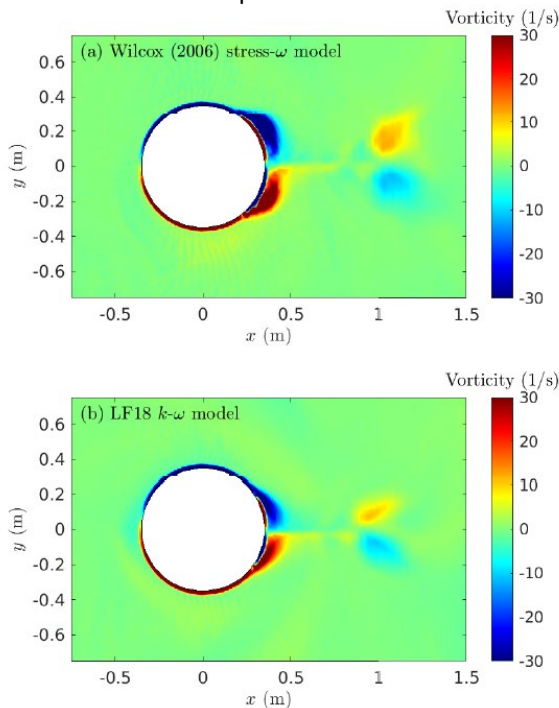


Figure 4 Vorticity field computed with (a) the Wilcox (2006) stress- $\omega$  model and (b) the LF18  $k$ - $\omega$  model at  $z = -D/2$  and time instant of  $t/T=0.49$  i.e. the local peak of the SLC.

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