STABILITY OF RUBBLE MOUND STRUCTURES UNDER OBLIQUE WAVE ATTACK

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INTRODUCTION

Stability formulae for armour layers of rubble mound breakwaters are generally developed for perpendicular wave attack and do not include effects of oblique waves. Waves usually attack breakwater obliquely as the sea wave is three dimensional. Several studies have been performed to investigate the effect of wave angle (β) on the armor stability. Galland (1994), Yu et al. (2002), Wolters and Van Gent (2010) and van Gent (2014) performed laboratory experiments to consider effects of oblique waves on the stability of armour layers. They performed tests with long-crested and/or short-crested waves on rock and concrete armours.

As a result, they proposed a reduction factor (γ_β) in the required armor size. This reduction factor has been found to be a function of ($cos^x\beta$). Table 1 shows the suggested wave obliquity reduction factors for rock armor stability.

Table 1 – Wave obliquity reduction factor for rock armor size from various studies

References	Formula		
Galland (1994)	$\cos^{0.25}\beta$		(1)
Yu et al. (2002)	$\cos^{1.157}\beta$		(2)
Wolters and Van Gent (2010)	$\cos^{1.1}\beta$		(3)
Van Gent (2014)	$(1-c_\beta)\cos^2\beta+c_\beta$	c_β=0.42 for short crested	(4)
		c_β=0.35 for long crested	

The variation of these factors vs. wave angle is shown in Figure 1. As seen, Galland (1994) indicates a lower influence of wave obliquity than other studies. In the range of $0 <\beta <50$, the magnitudes of wave obliquity reduction factors by Yu et al. (2002), Wolters and Van Gent (2010) (WV coefficient) and Van Gent (2014) (VG) are very similar. For $\beta > 50$, Yu et al. (2002) and WV predicts a much higher influence of the wave obliquity on armour size of rock slopes than Van Gent (2014).

It should be mentioned that the formula proposed by Yu et al. (2002) is compatible with Hudson (1958) stability formula, i.e.:

$$N_s=(K_D \cot\alpha)^{1/3} \qquad (5)$$

Where N_s is the stability number, α is the structure front angle and K_D is the stability coefficient. Van Gent (2014) one is compatible with Van Gent et al. (2003) (VSK) rock armour stability formula, i.e.:

$$N_s=8.4P^{0.18} N_w^{-1/10}S_d^{1/5}\xi_{m-1,0}^{-1/2}(H_s /H_{2\%}) \qquad (6a)$$

if $\xi_{m-1,0} <\xi_C$ or $\cot\alpha \geq 4$

$$N_s=1.3P^{-0.13}N_w^{-1/10}S_d^{1/5}\xi_{m-1,0}^{P}(H_s /H_{2\%})\cot\alpha^{0.5} \qquad (6b)$$

if $\xi_{m-1,0} \geq\xi_C$ or $\cot\alpha<4$

With $\xi_c= (6.46P^{0.31}\tan\alpha^{0.5})^{1/(P+0.5)}$.

where P is the permeability, S_d is the damage level, N_w is the number of waves, $\xi_{m-1,0}$ is Iribarren no using $T_{m-1,0}$ (the spectral mean energy period). H_s is the significant wave height and $H_{2\%}$ is average of the highest 2% of incident waves. One of the most recent formulas for the estimation of rock stability number is Etemad-Shahidi et al. (2020), hereafter EBV:

$$N_s=3.9C_pN_w^{-1/10}S_d^{1/6}\xi_{m-1,0}^{-1/3} \qquad \text{if } \xi_{m-1,0} \geq 1.8 \qquad (7a)$$

$$N_s=4.5C_pN_w^{-1/10}S_d^{1/6}\xi_{m-1,0}^{-7/12} \qquad \text{if } \xi_{m-1,0} <1.8 \qquad (7b)$$

where $C_p = [1+(D_{n50c}/D_{n50})^{3/10}]^{3/5}$ is the coefficient of permeability. D_{n50c} and D_{n50} are the median nominal size of core and armour material, respectively.
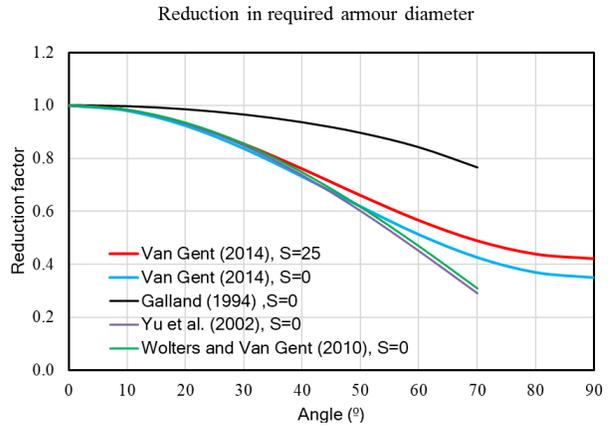

Figure 1 – Comparison of methods describing the influence of oblique waves on armour size

The aim of this study is to find an appropriate and compatible reduction factor for EBV stability formulae. For this purpose, Van Gent (2014) data set (170 records) have been used for the development of a new reduction factor and Yu et al. (2002) data set (70 records) have been used for its evaluation. Tests with very low damage level ($S_d <$ 2) and very high damage level ($S_d > 12$), which are not relevant to the practice, were excluded first.

In total, 77 records were selected for further processing. It should be noted that for the tests with directional spreading, the amount of directional spreading is described by S, where S=0 corresponds to long-crested waves.

RESULTS AND DISCUSSION

First, Galland (1994), Yu et al. (2002), Wolters and Van Gent (2010) and Van Gent (2014) reduction functions in combination with the EBV stability formula have been evaluated. Then, an attempt is made to find an appropriate and compatible reduction factor for EBV stability formula. Figure 2 shows the comparison of observed and predicted stability numbers using existing reduction factors. As seen, the Van Gent (2014) one is more appropriate compared to other reduction factors. As discussed before, the influence of oblique wave by Yu et al. (2002), Wolters and Van Gent (2010) is higher for $\beta > 50$.
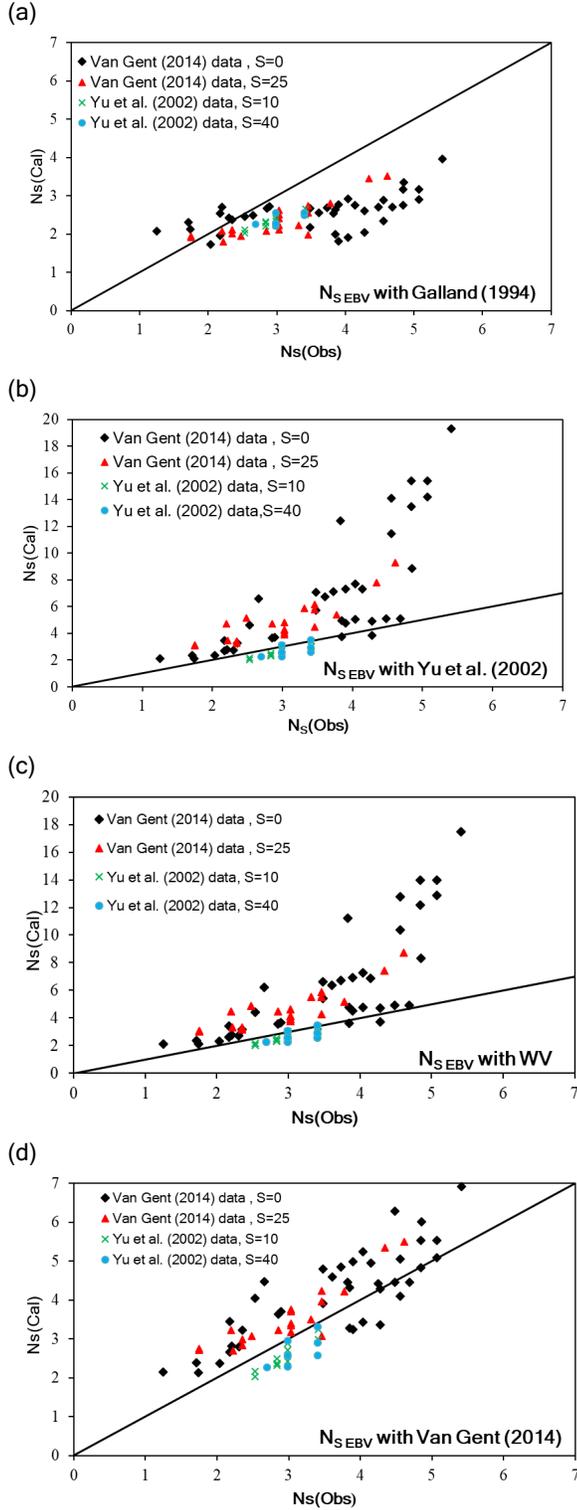
(a)

(b)

(c)

(d)

Figure 2 - Comparison between measured and predicted stability number using (a) EBV with Galland (1994), (b) Yu et al. (2002), (c) Wolters and Van Gent (2010) and (d) Van Gent (2014) reduction factors.

Next, it was attempted to derive an improved reduction factor for applications in combination with the EBV stability

formulae. Figure 3a shows the $f(\beta)= Ns_{EBV}/Ns_{Measured}$ versus β. As seen $Ns_{EBV}/Ns_{Measured}$ is scattered. For example, $Ns_{EBV}/Ns_{Measured}$ is between 0.4 and 0.8 for β=60º. Moreover, some records for relatively small wave angles and long-crested waves (S=0) result in reduction factors larger than 1 which is not physically justifiable. As seen, the data points at β=0 are mostly above 1. This is not because of using $N_{S\,EBV}$ to estimate the stability, as the issue also exists when using other stability formulas. For example, in Figure 3b the VSK stability formula has been applied and data points at β=15 are also more than 1.
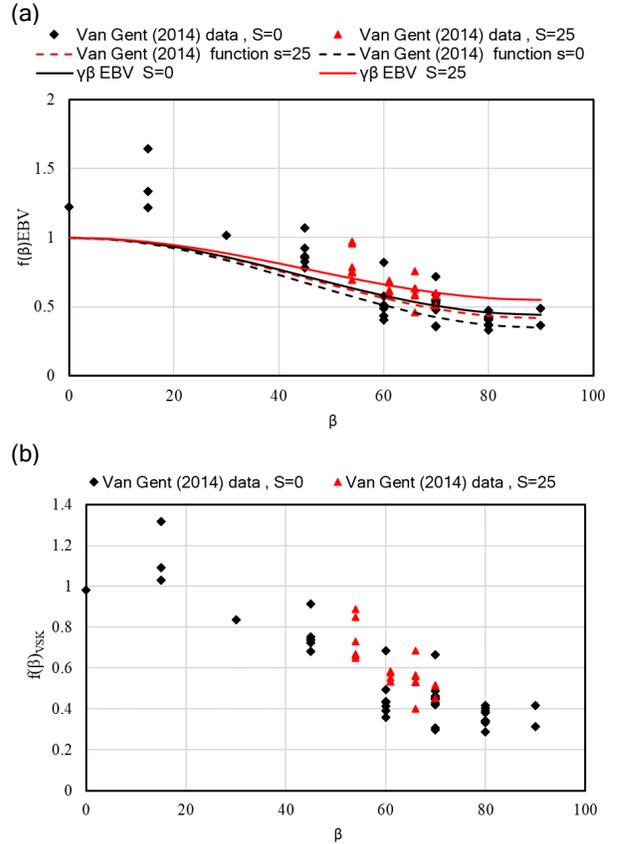
(a)

(b)

Figure 3 - Variation of f(β)EBV versus β, (b) Variation of f(β)VSK versus β

To derive a more accurate reduction factor for application in combination with the EBV stability formula, the wave obliquity reduction function by Van Gent (2014) can be used with a modified c_β value. Using Van Gent (2014) approach, the optimal c_β values were found to be 0.54 and 0.44 for short and long crested waves, respectively. Therefore, the reduction factor for EBV stability formula can be proposed as:

$$\gamma_{\beta\,EBV} = (1-c_\beta)\cos^2\beta + c_\beta \qquad c_\beta=0.54 \text{ for short crested} \qquad (8)$$
$$c_\beta=0.44 \text{ for long crested}$$

Figure 3a shows the comparison of Van Gent (2014) and reduction factor calibrated for the EBV formulae versus β. As seen, using of modified c_β for EBV indicates a lower influence of wave obliquity than what suggested by Van Gent (2014) (which is suggested for application in combination with another stability formula).

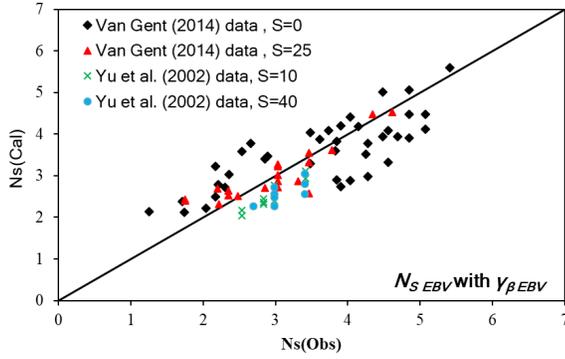Fig. 4 shows the comparison between the measured and

Figure 4 – comparison between measured and predicted (by EBV) stability numbers using the new reduction factor.

predicted stability numbers using the new wave obliquity reduction factor. As seen, the scatter in the data is reduced. The performances of the various formulas were also evaluated quantitatively using accuracy metrics such as the normalized bias (*NBias*), the scatter index (*SI*) and correlation coefficient (CC), defined below:

$$NBias = \frac{1}{n} \frac{\sum_{i=1}^{n}(p_i - m_i)}{\overline{mi}} \times 100 \tag{9}$$

$$SI = \frac{\sqrt{\frac{1}{n}\sum_{i=1}^{n}(p_i - m_i)^2}}{\overline{mi}} \times 100 \tag{10}$$

$$CC = \frac{\sum_{i=1}^{n}(p_i - \overline{p_i})(m_i - \overline{m_i})}{\sqrt{\sum_{i=1}^{n}(m_i - \overline{m_i})^2 (p_i - \overline{p_i})^2}} \tag{11}$$

where p_i and m_i denote the predicted and measured values, respectively. The number of measurements is n and the bar denotes the mean value.

Tables 2 and 3 displays the accuracy metrics of EBV stability formula using Van Gent (2014) and the new reduction factor for Van Gent (2014) and Yu et al. (2002) data set, respectively. As seen, the calibration of the coefficient in Eq.8 results in negligible bias when using Van Gent (2014) data.

Table 2 - Accuracy metrics of $N_{S\ EBV}$ using the new and Van Gent (2014) wave obliquity reduction factors; Van Gent (2014) data

	$N_{S\ EBV}/\gamma_{B\ VG}$	$N_{S\ EBV}/\gamma_{\beta\ EBV}$	$N_{S\ EBV}/\gamma_{\beta2\ EBV}$
NBias	16.4	-0.05	-0.05
SI	24	17.7	17.7
CC	0.83	0.80	0.80

Table 3 Accuracy metrics of $N_{S\ EBV}$ using the new and Van Gent (2014) wave obliquity reduction factors; Yu et al. (2002) data

	$N_{S\ EBV}/\gamma_{B\ VG}$	$N_{S\ EBV}/\gamma_{\beta\ EBV}$	$N_{S\ EBV}/\gamma_{\beta2\ EBV}$
NBias	-13	-16.0	-15.7
SI	15	16.9	16.6
CC	0.77	0.82	0.81

As discussed by Yu et al. (2002) and Van Gent (2014), unidirectional (long-crested) or multidirectional (short-crested) type of wave can affect the stability number differently. They concluded that the effect of directional wave spreading on armour unit stability is that the directional spreading leads to a lower influence of oblique

wave attack. Hence, the effects of wave directionality were reanalyzed. Experiments by Yu et al. (2002) included only tests with S=10 and S=40 as a measure for the amount of directional spreading. The lack of sufficient and comprehensive data makes it difficult to accurately resolve the effect of the amount of directional spreading. However, based on the available data, a linear function can be proposed for a unified estimation of c_β as function of S (spreading) as:

$$\gamma_{\beta\ 2EBV} = (1 - c_\beta)\cos^2\beta + c_\beta \qquad c_\beta = 0.44 + 0.004S \tag{9}$$

This means that c_β varies linearly between 0.44 for S=0 to 0.6 for S=40. This implies that the more the spreading, the less the effect of wave obliquity, which makes sense. The accuracy metrics of this reduction factor ($\gamma_{\beta2\ EBV}$) for different data sets are shown in the last column of Tables 2 and 3.

SUMMARY AND CONCLUSION
One of the most recent formulae for estimating the stability of rock-armoured slopes is Etemad-Shahidi et al (2020). The aim of this study was to develop a suitable wave obliquity reduction factor for the EBV stability formulae. Hence, the influence of oblique waves on the stability of rock armour layer has been investigated based on the available data set. Data records of Yu et al. (2002) and Van Gent (2014) with damage levels in the range of $2 \le S_d \le 12$. These studies show that the influence of oblique waves on the stability of rock armour layers is significant, and the required armour size can be reduced compared to the perpendicular wave attack case. This effect can be considered as the reduction factor γ_β for the required armour size. All available γ_β formulas were evaluated in combination with the EBV stability formulae using different data set, and it was concluded that Van Gent (2014) approach is more accurate than others. Based on this approach, an appropriate and compatible reduction factor for EBV stability formula has been proposed, which quantifies and includes the effect of directional spreading explicitly. It was concluded that the result will improve slightly by using new wave obliquity reduction factor.

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