ANALYTIC SOLUTION FOR THE VELOCITY FIELD AROUND SUBMERGED PERMEABLE BREAKWATERS

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INTRODUCTION

In this paper, an analytic solution to the scattering of water wave by submerged permeable breakwaters in infinite-depth water is presented. The analytic solution of the velocity field around the permeable breakwater under the linear wave train in deep water condition is sought by formulating a nonhomogeneous Riemann-Hilbert problem. To deal with the nonlinearity due to the permeability of the plates, a perturbation method is applied in terms of a small parameter representing permeability. We present the leading order solution (impermeable breakwaters) and the first order solution (permeable breakwaters) of the spatial velocity field. The validity of the leading order solution was verified through comparison with the solutions suggested by Evans (1970). Then, the impermeable and permeable breakwater's reflection and transmission coefficients are compared in order to investigate the effects of permeability. The analytical solution presented in this research can be used for designing breakwaters, such as calculating the force applied to the breakwaters, and for validation of numerical simulations or experimental results.

FORMULATION

Assume that the fluid has infinite depth and is incompressible, inviscid, and irrotational. Under a small amplitude linear wave train, let N permeable plates occupy the intervals L_n: $x = 0, -b_n < y < -a_n$, where $n = 1, 2, 3, \cdots, N$. From the potential wave theory, there exists a velocity potential $\Phi(x, y, t) = \Re_j\{\phi(x, y)e^{-j\omega t}\}$, and the spatial potential $\phi(x, y)$ should satisfy the Laplace equation. As we need the spatial potential to be bounded in all domain to solve the Laplace equation, we should define the boundary conditions on the free surface, at $x \rightarrow \pm\infty$, at $y \rightarrow -\infty$, and lastly, on the plates.
First, there will be combined free surface boundary condition along the free surface. Since we assumed the small amplitude wave, the linearized combined free surface boundary condition is used, expanded around $y = 0$ and removed the higher-order terms. Next, there are the radiation boundary conditions for the scattered wave traveling toward $x \rightarrow \pm\infty$. The reflected wave propagating to $+\infty$ will be superposed with the incident wave, and the transmitted wave will solely travel outwards towards $-\infty$. Also, the fluid velocity should vanish as $y \rightarrow -\infty$. Thus, the first derivative of the spatial potential with respect to x and y tend to be zero. Fluid velocity components are bounded everywhere, but except at the edges of the plates. The velocity may be unbounded but permit a mild singularity. Here we assumed the mild singularity to make the singularity integrable.
The remaining boundary condition is that on the permeable plates. Following Taylor's (1956) method, we assumed that the pressure difference between both sides of the plates makes the flow through the barriers. Then, substituting the pressure term with the linearized Bernoulli's equation, the horizontal velocity through the plate is represented by the difference with the time derivative of the total wave on both sides of the plates. However, since the plate boundary conditions are expressed in terms of the velocity potential itself, the problem becomes nonlinear. Thus, we introduce the perturbation method to seek the permeability effect on the wave scattering problem.
Introducing a small parameter $\varepsilon = \kappa\omega/\nu Dk$, the perturbed solution takes the form as $\phi = \phi_0 + \varepsilon\phi_1 + \cdots$. Substituting the perturbed spatial velocity potential into the boundary conditions we defined, we can arrange the terms in ascending order with respect to ε. Now, the boundary conditions separate to the leading order and the first order.

RESULTS AND DISCUSSION

Introducing the complex potential $w(z) = \phi + i\psi$ and considering the reduced potential defined by $W(z) = dw/dz + ikw$, $W(z)$ can be extended into $y > 0$ by Schwarz's reflection principle. From the boundary conditions, the problem of determining $W_0(z), W_1(z)$ becomes a typical homogeneous and nonhomogeneous Riemann-Hilbert problem, respectively. Deriving the solution of the Riemann-Hilbert problem for the plane with cuts distributed along a straight line, we can find the general form of the solution, with some unknown constants. These unknown constants are determined from the zero-circulation assumption around the plates and the radiational boundary condition at $x \rightarrow \pm\infty$. Then, the complex potential $w_0(z)$ and $w_1(z)$ is obtained by integrating the reduced potentials.
To find out how the first order solution had a correction effect on the leading order solution, the expressions for reflection and transmission coefficients are derived. For further discussion, numerical computation of the solution should be presented so that the effect of the permeability of the plate can be qualitatively examined. Experiments are ongoing in order to validate the analytical solution.

ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government(MSIT) (No. NRF-2020R1F1A1070390).
This work is supported by the Korea Agency for Infrastructure Technology Advancement(KAIA) grant funded by the Ministry of Land, Infrastructure and Transport (Grant 22DPIW-C153746-04).

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