ON THE BEHAVIOR OF A TETHERED CYLINDER ARRAY UNDER IRREGULAR WAVES

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INTRODUCTION

Anthropogenic pressure combined with natural forcing contributes both to a worsening of the environmental quality of coastal areas, and to the triggering of erosion dynamics. Nowadays, over 44% of the world's sandy beaches are persistently eroding (Luijendijk, 2018). An interesting option for coastal wave attenuation can be provided by tethered floating breakwaters (see *e.g.* Agerton, 1976 and Seymour, 1979). This type of structure can be designed to be effective, while determining a very small impact on the environment (Dai, 2018). In principle, a regular lattice of reversed pendula (*i.e.* the breakwater) can be seen as an approximation to a mechanical *metamaterial* with interesting properties related to its periodic structure.

Metamaterials are engineered structures designed to interact with waves and to manipulate their propagation properties, *e.g.* their dispersion relation (Hussein, 2018). For example, dispersion curves can display zero or negative slope, leading to the formation of *bandgaps*, *i.e.* ranges of frequencies where wave propagation is forbidden (*i.e.* the waves become evanescent). These concepts have been applied with success in many different fields (see *e.g.* Lu, 2009 and Brulè, 2020).

In fluid dynamics, so far, most of the work has focused on the interaction between gravity waves and bottom periodic structures (Davies, 1984). In the case of periodic structures, results show the existence of bandgaps which can be related to a Bragg-scattering mechanism. Similar wave blocking mechanism has also been observed for a single fixed immersed cylinder (Grue, 1992). Manipulation of bandgaps has also been discussed for an array of fixed floating pontoon breakwaters (Ouyang, 2015).

The tethered floating breakwater we are studying has been designed as a finite lattice of submerged inverse pendula (Figure 1). The objective is to determine the efficiency of a regular two-dimensional lattice of spherical pendula. To isolate the one-dimensional behavior, a single array of reversed cylindrical pendula anchored to the bottom and excited by long crested waves has been tested. Numerical simulations and experiments with regular waves in a wave flume have shown that bandgap formation can be linked to dissipation around the single pendulum natural frequency (De Vita, 2021 and Lorenzo, 2022). The enhanced motion at similar frequencies increases wave turbulence decaying into heat losses. From the collective point of view, regular wave flume results also indicate that the wave attenuation depends on the reflective capabilities of the pendula array and how this in turn depends on the geometry of the metamaterial (Lorenzo, 2022).

The following question is then to which extent is this wave attenuation mechanism applicable in more realistic sea conditions. Understanding the non-linear behavior due to the metamaterial nature of the device is not an easy task (Patil, 2021). We thus decided to test the response of a known configuration of the device to the excitation of long-crested irregular waves.
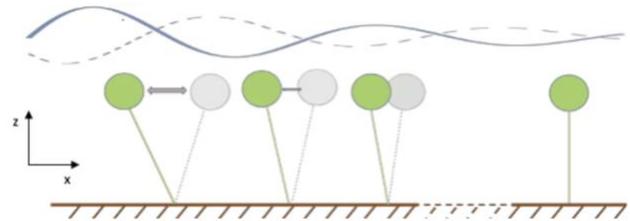
Figure 1 - A lattice of reversed pendula acting as a wave attenuator

EXPERIMENTAL SETUP AND METHODS

The aim of the present work is to understand the properties of the metamaterial in an irregular wave environment. Therefore, the extension of the array should be as large as possible, in order to observe the asymptotic properties of the system. In practice, the limitations imposed by a long wave flume and regular wave tests allowed to test up to 11 cylinders (Lorenzo, 2022). This configuration (see Figure 2) has thus been selected for testing in the irregular wave flume.

Figure 2 - Experimental setup: 11 cylindrical pendula with a mutual distance L = 0.35 m

Cylinders of 82 mm diameter were assembled using commercial pvc pipes (3 mm thick) and are placed with their 98 cm-long axes parallel to the wave crests, attached with 2.4 mm steel cables at 2 cm above the bottom. Each cylinder is provided with two bracing cables to avoid oscillations along its axis and to damp any possible rotations around the vertical axis. The cylinders are fully

submerged. Water level h is set to 0.45 m, with a fixed distance between the still water level and the top of the cylinders at rest equal to 0.02 m. The natural frequency of the pendula is estimated to be about 0.6 Hz.

Irregular wave tests are performed at the ICEA Dept. of the University of Padova (Italy) in a 36 m long, 1.0 m wide and 1.3 deep wave flume. Reflections are minimized due to the wavemaker active wave absorption and a parabolic beach. Two arrays made of 4 resistance-type wave gauges are used to measure the surface elevation in the offshore and nearshore areas. Lastly, incoming and reflected waves are separated with a classic linear least squares method (Zelt, 1992).

The repetition cycle of the irregular waves is set to reproduce about 512 peaks, aiming to replicate JONSWAP spectra with the same standard peak enhancement factor γ of 3.3. We run 30 tests spanning the available frequency range with peak periods going from 0.67 to 2.00 s, repeating them with three different seeds and maintaining the same bulk steepness H_s / λ of 0.025, with H_s the significant wave height and λ the wavelength corresponding to the spectrum peak.

RESULTS AND DISCUSSION

Figure 3 shows incident (S_i), reflected (S_r) and transmitted (S_t) wave spectra for a single test. In this example, the 11-cylinder configuration is forced by a JONSWAP spectrum with a frequency peak at 0.6 Hz and significant wave height of 0.08 m. The loss-gain line is deduced for each frequency using a simple energy equilibrium $S_d = S_i - S_r - S_t$. Negative values indicate that some energy has been transferred from the incident field to the reflected wave or to the transmitted one, or to both. The beach-reflected energy is also depicted and found to be negligible.

Note that the peak spectrum coincides with the single pendulum natural frequency. A strong energy loss close to 0.6 Hz can be expected, as observed during regular wave experiments (Lorenzo, 2022). While the transmitted spectrum has almost the same form of the incident one, the shape of the reflected spectrum is almost flat with a broad peak around 1.6 Hz.
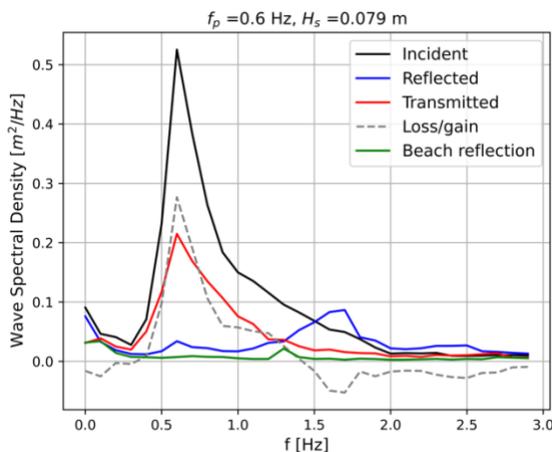
Figure 3 - Effects of the configuration in Figure 2 forced by a JONSWAP spectrum with frequency peak at 0.6 Hz

The reflection maximum appears at a frequency which is not a multiple of the incident peak frequency, so we can exclude that it is caused by higher order water wave interactions. If the distance between two cylinders is taken as half of a wavelength, we see that this wave corresponds to 1.49 Hz. So, it is likely that the collective response of the pendula causes a net energy transfer to the reflected spectral tail. Again, we note that no sign of such dynamics was seen on the transmitted signal. From this fact, one can infer that the device geometry induces a bandgap close to a Bragg-like scattering frequency, as observed using regular waves with small amplitudes (Lorenzo, 2022). Moreover, in the reflection peak interval, the reflected spectral density is evidently higher than the incident one.

The conclusion that naturally follows is that the energy within the bandgap is not only prevented from propagating inside the metamaterial, but also that some of this energy is transferred to other frequencies, as a result of the interaction with the pendula array, and then radiated backwards. These aspects can be observed in all the tested sea conditions, except for the ones in which the incident energy is concentrated close to the device response frequency. An example of this is given in the following figure 4. Here, the reflection bump is slightly shifted to lower frequencies where most of the incident energy is reflected back and almost no energy is transmitted.
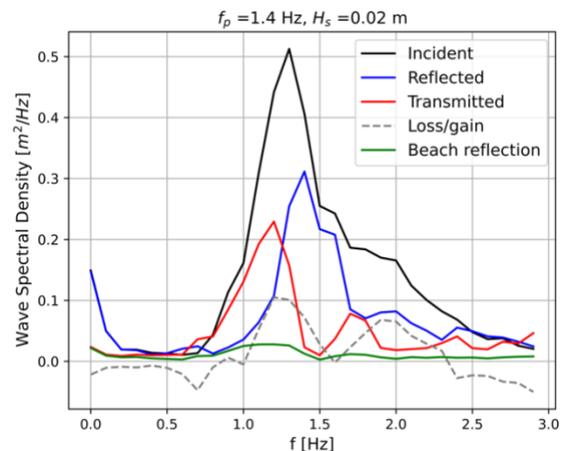
Figure 4 - Effects of the configuration in Figure 2 forced by a JONSWAP spectrum with frequency peak at 1.4 Hz

Let us finally consider a mean frequency (*e.g.* $f_{2,0} = 1/Tm_{0,2}$) as an indication of the distribution of the spectral energy in the frequency domain. Figure 5 depicts the computed mean transmission and reflection frequencies vs. the incident mean frequency. Clearly, the transmitted mean frequencies match the incident ones, while the reflected ones are mostly concentrated in the range between 1.50 and 1.75 Hz. In practice, the reflection mean frequency turns out to be almost independent of the forcing. This strengthens the idea that the spatial geometry of the pendula lattice is the main factor determining the metamaterial non-linear response.
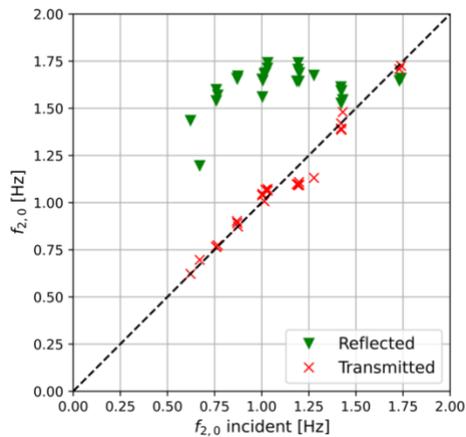
Figure 5 - Mean frequency $f_{2,0}$ for the reflected and transmitted energy as function of the mean frequency $f_{2,0}$ for the incident spectra.

CONCLUSIONS
Tethered floating breakwaters, designed as a lattice of reversed pendula, can be an effective solution to mitigate the effects of coastal erosion. With respect to rigid structures like groins and rubble mound breakwaters, this type of structure can prevent scour-and-deposit effects with a consequent lower impact on water circulation. In addition, being completely submerged, they are not visible along the coast. In this work, exploiting the concept of metamaterial wave control, the two-dimensional configuration of the device (made of an array of reversed cylindrical pendula) has been forced by long-crested irregular waves, closest to real conditions at sea.
Wave flume results show how the non-linear response of the device seems to be forced by the reflection mechanism. Indeed, a net energy transfer to the reflected spectral tail has been observed, because of the collective response of the pendula. Moreover, the independence of the reflection mean frequency from the incident spectra underlines how the mutual distance between the pendula (*i.e.* the geometry of the metamaterial) seems to determine the non-linear behavior. Transmitted energy on the beach side of the device, instead, follows the trend of the incident spectrum, but with a net energy attenuation.

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