A CLOSED-FORM SOLUTION FOR INTERACTIONS BETWEEN WAVES AND AN ARRAY OF FISH NET CAGES

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ABSTRACT

Understanding the multi-cage interaction mechanism in fish cage arrays and the hydrodynamic behaviour of cage structures will help engineers explore the feasibility of offshore fish farming. In this study, a semi-analytical method is developed to simulate the wave response of an array of cylindrical fish cages with arbitrary layouts and dimensions, including the wave interference effects among the multiple cages. By eigenfunction expansions, the general solution of the present research problem is expressed in the form of a Fourier-Bessel series, and the corresponding particular solution is obtained by introducing relevant boundary conditions and the least-squared method. The simulation results show that significant wave effects occur at the top of the cages, and the dynamic response of each cage has obvious phase differences. Parametric studies on the wave forces show that the wave force is zero at a specific wave frequency and the wave forces are decreased under high porosity nets.

INTRODUCTION

Aquaculture plays an important role in the supply chain of animal protein intake. From the statistical data provided by "The State of World Fisheries and Aquaculture (2018)", the output of aquatic products present a growth rate of 840% from 1950 to 2016, in which marine aquaculture products accounted for approximately 69% of 170 tons per year. For the future tendency, the global output of marine aquaculture production will increase to 74 million tons with a growth rate of 155% from 2020 to 2050 according to the predicted data from DNV (2021).

As a result of heavily contested nearshore spaces and the environmental deterioration due to nearshore fish farming, there is a strong push to move fish farms offshore where there are large pristine water columns for larger space and better waste dispersion. The exploration of offshore marine aquaculture has also promoted the upgrade of the new technology for fish cage systems. Norway and China have led the way in designing, constructing and deploying innovative ultra-large fish cage designs such as Ocean Farm 1, Havfarm 1 and Shenlan 1 and 2. However, existing numerical models covering all scale ranges of fish cages are currently a challenge due to the huge computational costs. On the other hand, multi-cage systems offer superior advantages in equipment installation, maintenance and productivity, compared with traditional single cages. Nevertheless, the mechanical analysis and layout design of fish cage arrays are more complicated, especially for the interaction phenomenon among multiple cages. The layout and hydrodynamic conditions of the cage array directly determine the wake interaction induced by the individual cages, resulting in a stronger blocking effect (Klebert et al. 2013). Therefore, it is essential to develop an advanced hydroelastic analysis for fish farms as an alternative.

A few scholars have proposed basic semi-analytical solutions to investigate the interaction between waves and net cages (Su et al., 2015; Mandal and Sahoo, 2016). Selvan et al. (2021) then incorporated and extended their work to multi-cage systems, where Kagemoto and Yue (1986) interaction theory is adopted to describe the phenomenon of wave interference among multi-body structures. However, in the aforementioned analytical solutions, the net chamber was oversimplified and modelled as a one-dimensional beam or string. Flügge (1973) proposed an elastic constitutive relation for cylindrical shell structures, and this theory was introduced in the fluid-structure interactions of cylindrical shell structures (Guo et al., 2017; Ji et al., 2019). Ma et al. (2022) combined these theoretical studies with the wave-porous structure interaction theory to derive an upgraded semi-analytical solution for the interaction between waves and single cages.

Although the abovementioned studies offer developed hypotheses and a wealth of information for elucidating the hydrodynamic response of net cages to waves, the development of hydroelastic models for fish cage arrays still faces several difficulties. Firstly, the governing equations for the cage deformation of fish cage arrays require modification, where the cage can deform in different degrees of freedoms, especially when the waves are incident at various angles. Secondly, there are currently no realistic parametric investigations of fish cage arrays in high-energy environments, such as deep-water conditions. Therefore, it is desired to resolve the two engineering scenarios.

In this study, a hydroelastic solution based on the small-amplitude wave theory and shell-membrane theory is proposed to investigate the multi-body interactions and the dynamic response of an array of fish cages in waves. The fish cage net is treated as a porous membrane, and the penetration flow through the cage interface is governed by Darcy's Law. Graf's Additional Theorem is introduced to realise the coordinate conversion among the local cylindrical coordinate systems at each cage. Through eigenfunction expansions, this hydroelastic solution can be described as a Fourier-Bessel series, and the unknown constants in the particular solution are derived by utilising the boundary conditions and the least-squared approximation.

METHOD OF MODELLING

In a global coordinate system ($x-o-y$), incident waves with a frequency ω propagate in the direction with an angle β between the x-axis, in which a series of submersible cages numbered from $j=1,2,\ldots,NC$ are considered, and they have radiuses a^j, submerged depths ${d_1}^j$ and cage heights ${d_2}^j$. Local coordinate systems ($r^j-\theta^j-z^j$) are located at the centre of each cage. In the local coordinate system of k^{th} ($j=k$) cage, the cylindrical coordinate of any point is ($r^j-o^j-\theta^j$), and (R^{jk}, α^{jk}) is the polar coordinate of the centre o^k for the k^{th} cage in the local coordinate systems of other cages ($j=1,2,\ldots,NC$, $j\neq k$). Therefore, the relationship between different coordinate systems satisfies Graff's Addition Theorem:

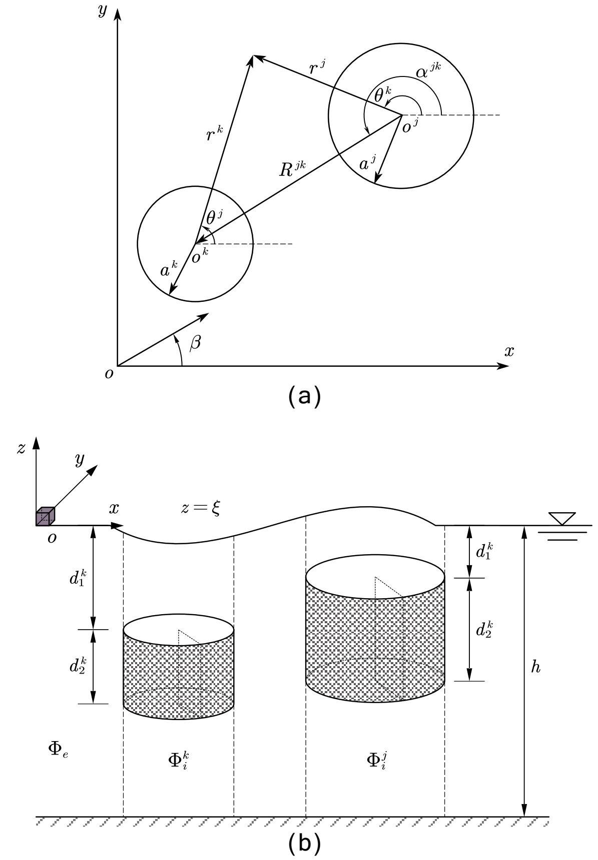
$$C_m(\kappa r^j)e^{\mathrm{i}m(\theta^j-\alpha^{jk})} = \sum_{l=-\infty}^{+\infty} C_{m+l}(\kappa R^{jk})J_m(\kappa r^k)e^{\mathrm{i}l(\pi-\theta^j+\alpha^{jk})} \quad (1)$$

where C_m is any Bessel function.

The flow field is expressed by the linear potential flow theory, where the fluid is assumed to be inviscid and irrotational. Herein, the flow velocity is the gradient of the velocity potential which is governed by the Laplace equation. In the external region of all cages, the velocity potential is denoted as Φ_e, and ${\Phi_i}^j$ is the velocity potential in the inner region ($r^j<a^j$ and $0<z<h$) of each cage. Moreover, the top edges ($z=-{d_1}^j$) of the cages are assumed in a clamped condition, and their bottom ends at $z=-({d_1}^j+{d_2}^j)$ are in a traction-free condition. The definition of coordinates and parameters of the cylindrical cage array system is illustrated in Fig. 1.

For the structural domain, the net cage is equivalent to a perforated cylindrical shell whose displacement components are driven by the wave pressure drop $-\Delta p^j$ include the axial displacement u^j, circumferential displacement v^j and radial displacement w^j (Fig. 2). The elastic constitutive law between the displacements and the shell stress N_z and N_θ can be found in Flügge (1973). By substituting the elastic constitutive law into the motion equation of the shell element which satisfies Newton's Second Law, the governing equation for the displacement component can be obtained. Additionally, the permeable flow through the cage interface satisfies linear Darcy's Law.

Since the above governing equations are all linear differential equations, the superposition principle can be applied. Therefore, the general solution can be expressed as a Fourier-Bessel series by eigenfunction expansions. A closed-form solution is obtained by matching the boundary conditions and the least-squared method.



(a)

(b)

Figure 1 - Coordinate and parameter definition of cylindrical cage array system, (a) top view, (b) side view.

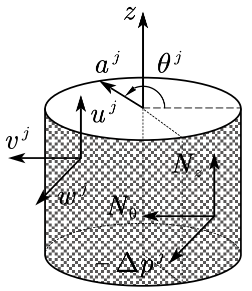


Figure 2 - Displacements and stress in a cylindrical shell.

RESULT DISCUSSION

As an example of hydroelastic response, a 1x3 array of fish cages is shown in Figure 3, where the incident wave propagates in 45°. It can be seen that the significant wave actions and the corresponding structural dynamic response mainly concentrate at the top ends of the cages due to the surface wave effects. Therefore, submersible cages are recommended for the fish farm exposed to high-energy environments. Additionally, because of the wave interference effect caused by the interaction of the scattered waves due to individual cages, the wave response of each cage has obvious phase differences.

In addition, this paper also conducts parametric studies on several pertinent factors in the fish cage design. In Fig. 4a, when the normalised wave frequency $g/(\omega^2 h)$ is about 0.136, the wave forces F^j acting on the cages are all zero. As seen in Fig. 4b, as the porous resistance parameter τ^j of the fish cage net is increased, the magnitude of the wave force on each cage will decrease. This means the wave loads are suppressed under high porosity nets. Moreover, the rear cages withstand minor wave action due to the porous resistance effect of the front cages.

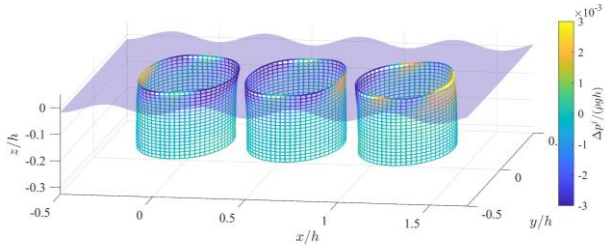
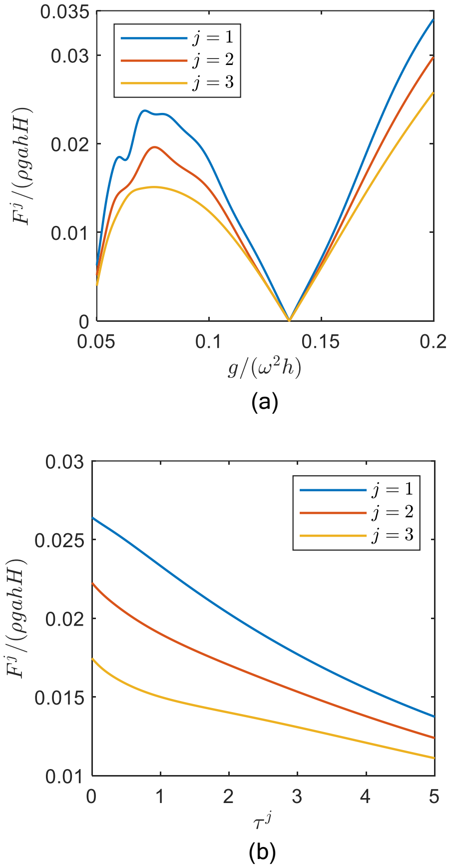


Figure 3 - Hydroelastic response of a 1x3 array of net cages in waves.



(a)

(b)

Figure 4 - Wave forces F^j acting on a 1x3 array of net cages versus: (a) normalised wave frequencies $g/(\omega^2 h)$, (b) porous resistance parameters τ^j.

CONCLUSIONS

The present study investigates the wave interference effects and the dynamic response for an array of fish net cages through a semi-analytical solution based on the potential flow theory and the shell-membrane theory. In accordance with the numerical results, it is found that the wave loads on the net cage in the array are relatively minor under high porosity nets. The wave forces acting on the cages will vanish at the normalised wave frequency $g/(\omega^2 h)$ = 0.136. In addition, the net cage shows severe dynamic response near the free water surface. Therefore, the submersible cage facilitates the avoidance of strong wave action in extreme storm conditions. This study will provide offshore fisher industry to carry out feasibility studies of offshore aquaculture systems in the future.

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