HYDROSTATIC STABILITY EXPLORATION ON FLOATING STRUCTURES USING MACHINE LEARNING

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SUMMARY
A hydrostatic stability analysis is an important first step in designing floating structures. Most of the currently available commercial software is limited to hydrostatic stability curves. Current research tries to address this limitation, by developing a framework which couples numerical hydrostatic stability analysis based on potential energy minimization, with a machine learning (ML) model based on genetic programming (GP). In this way, potential energy functions are efficiently obtained. The resulting analytical formulations offer a wider understanding of the hydrostatic stability of floating structures.

INTRODUCTION
Rapid urban population growth combined with the increase in climate temperature are increasing the risk of coastal hazards and the demand for living space for the urban population [Vardy, 2017]. Among many solutions proposed, one of the most ambitious ones is the floating city, which is deemed suitable to tackle the unavoidable increase in sea level [Wang, 2011]. Hydrostatic stability analysis is an important first step in designing any floating structure. There are multiple ways to perform a hydrostatic stability analysis of a floating structure, however, one of the most efficient ways to do this is by using the potential energy minimization [Neves, 2011]. Since the mass of the structure can be assumed to be constant regardless of the floatation orientation, the potential energy (PE) of the structure can then be conveniently represented as being proportional to the vertical distance between the center of buoyancy and the center of gravity [Neves, 2011]. A free-floating body will move to an orientation that results in the least amount of potential energy (and thus called a stable position). Figure 1 illustrates that less energy is required for the long bar with a square cross-section to float in an orientation such that the longitudinal axis is parallel to the water plane; much more energy is required when the bar is perpendicular to the waterline. The potential energy needed for the long bar to stay at various positions relative to the water line is illustrated in the graph in Fig. 1. Generating the potential energy surface of a floating body offers a wider understanding of its hydrostatic stability.

The objective of the current research is to develop functions representing the potential energy surface of floating structures of any shape using a novel approach that combines the following: (1) modeling software called Rhinoceros® [McNeel, 2022], with Grasshopper network to determine the potential energy surface, and (2) machine learning (ML) models based on genetic programming to develop the function. Grasshopper, which is a visual programming language within Rhinoceros®, has been chosen due to its parametric modeling capabilities. It is expected that is novel framework will help designers have a better understanding of the hydrostatic stability of floating coastal structures in the design process.

POTENTIAL ENERGY GENERATION
Although a simple cuboid is used to illustrate the concept in Figure 1, an algorithm is developed to perform a hydrostatic stability analysis of a three-dimensional floating structure of any geometric complexity. Due to its parametric modeling tools, Grasshopper has been used to build the algorithm, which works as follows: rotate the floating object through all possible rotations (i.e., orientations); for each rotation, an iterative algorithm is used to determine the location of the waterline given the weight to buoyancy force ratio (***r***) of the body, as illustrated in Figure 2; once all iterations are run, a potential energy surface can be generated for the floating structure.

The developed network was validated using a theoretical derivation of 2D rotation of square shape floating body rotated around the x-axis denoted as θ. For the square shape, the analytical equations for potential energy depend on whether one or two corners are immersed in water as shown in Fig. 3 [Abolhassani, 2004].

If two corners are immersed:

$$PE_a(m) = a\left[\frac{1}{2}(1-r)\cos(\theta) + \frac{1}{24r}\sin(\theta)\tan(\theta)\right] \tag{1}$$

If one corner is immersed:

$$PE_b(m) = \frac{1}{6}a\left[3\cos(\theta) + 3\sin(\theta) - 4\sqrt{r}\sqrt{\sin(2\theta)}\right] \tag{2}$$

where a is the length of the side of the square.

A square cross-section with side $a = 100\ m$ is constructed in the Grasshopper network where r is varied from 0 to 0.5, and θ from 0 to 90 degrees to generate the potential energy dataset. The results have demonstrated a perfect match with the theoretical equations, i.e., 0% error. An example from the network is shown in Fig. 1 where the red dotted curve represents the potential energy curve of the square cross-section for $r = 0.1$.

SYMBOLIC REGRESSION
The next step involves using an ML algorithm called genetic programming (GP) to solve a symbolic regression (SR) problem using the collected data points, i.e., potential energy surface. The goal of SR is to find a

closed-form mathematical model for the potential energy surface. To solve the SR, GP works by heuristically searching over a very large space of functions to find the best fitting function. In this research, the open-software HeuristicLab [Wagner, 2014] is used to perform SR.

Considering a displaced volume of 10%, i.e., $r = 0.1$, the dataset used to validate potential energy generating is used to validate the SR model. The dataset randomly is split into 80% training, and 20% testing and the Mean Squared Error (MSE) is used as a fitness metric. The chosen genetic programming settings are as follows: population size = 1000, mutation probability = 15%, suggested maximum tree length = 10, and suggested maximum tree depth = 6. Refer to Wagner [2014] for parameters definition. The generated functions are as follows:

If two corners are immersed:

$$PE_a(m) = c_1 + c_2 \tan(c_3\theta) \tag{3}$$

If one corner is immersed:

$$PE_b(m) = c_4 + c_5 \cos(c_6 - \sin(2\theta)) \tag{4}$$

where $c_1 = 45$, $c_2 = 1.63768$, $c_3 = 1.1705$, $c_4 = 51.095$, $c_5 = 7.3992$, $c_6 = 15.336$.

The MSE for the two corners immersed and one corner immersed are $5 \times 10^{-6}\, m$ and $5 \times 10^{-3} m$, respectively. The generated functions match the theoretical equations with no overfitting problem.

Therefore, this novel framework can be used to generate potential energy surfaces and find a corresponding closed-form analytical model.

FLOATING BREAKWATER APPLICATION

Floating breakwaters with circular cross-sections are commonly researched in literature. Consider a circular long bar (radius $R = 10\, m$, and length $L = 100m$) as shown in Fig. 4. Two parameters are considered: θ from 0 to 90 degrees, and r from 0 to 0.5, generating 225 data points for the potential energy. SR is performed on the potential energy dataset using the same parameters from the previous section, resulting in the following closed-form function:

$$PE(m) = c_7 + \frac{1}{c_8 r + c_9} \tag{5}$$

Where $c_7 = -14.528, c_8 = 2.276 \times 10^{-2}, c_9 = 4.189 \times 10^{-2}$. The MSE for this function is 0.12.

An important observation is that θ is specified as an input, but it has been eliminated in the generated function for the circular cross-section. This can be explained by the fact that, given constant displacement, the immersed volume does not change its shape as θ is varied. Therefore, the resulting PE function is expected to be a function of r only as predicted by the above equation.

CONCLUSION

The paper presented and validated a framework based on parametric modeling and machine learning to determine a closed-form analytical solution for potential energy of simple floating bodies. Future work will consider solving SR for the three-dimensional behavior of more complex floating bodies.

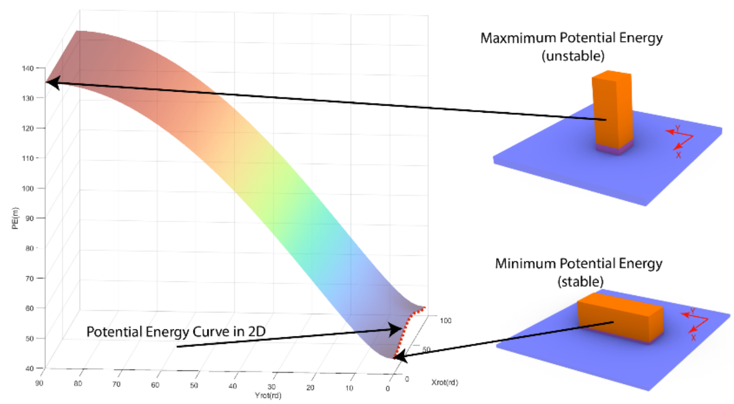

Figure 1 - Potential energy surface for a long bar (orange) with a square cross-section and W/BF of 0.1 where blue represents water.

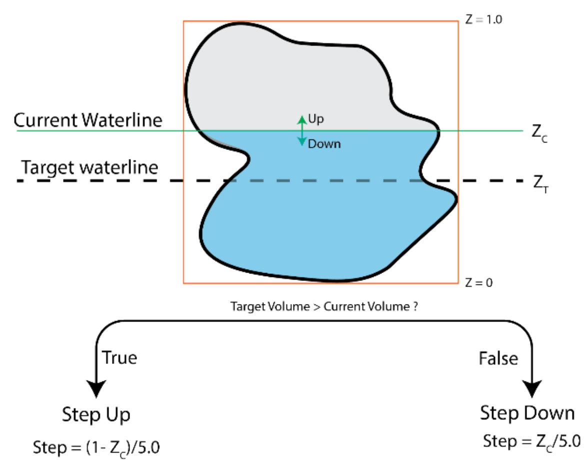

Figure 2 - Bouncy solver used to determine the location of the waterline at a given orientation.

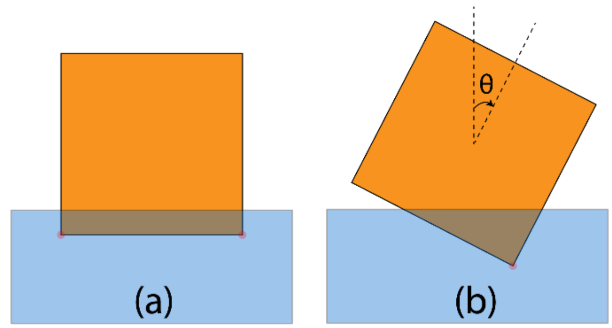

Figure 3 - Square cross-section floating configuration for (a) two corners immersed, (b) one corner immersed.

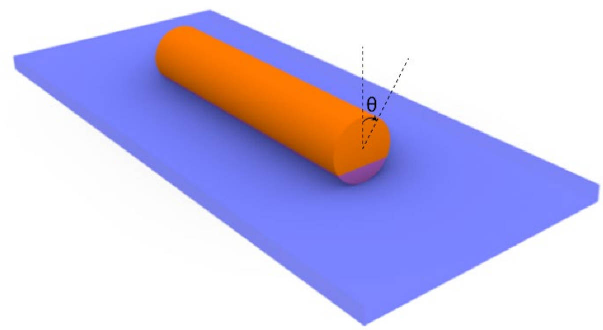

Figure 4 – Floating long bar with a square cross-section.

REFERENCES

Abolhassani (2004). On the stability of floating bodies. *arXiv preprint physics/0411244*.

McNeel, et al. (2022): Rhinoceros 3D, Version 6.0. *Robert McNeel & Associates, Seattle, WA*.

Neves, Marcelo Almeida Santos, et al. (2011): Contemporary ideas on ship stability and capsizing in waves.

Vardy, Mark, et al. (2017): The intergovernmental panel on climate change: challenges and opportunities." *Annual Review of Environment and Resources* 42: 55-75.

Wagner et al. (2014) Architecture and Design of the HeuristicLab Optimization Environment. In Advanced Methods and Applications in Computational Intelligence, Topics in Intelligent Engineering and Informatics Series, Springer, pp. 197-261. 2014

Wang, and Tay (2011): Very large floating structures: applications, research and development. *Procedia Engineering* 14: 62-72.