3D VELOCITY FIELDS WITH 2DH NUMERICAL STABILITY:
A 3D ANALYTICAL-NUMERICAL MODULE FOR 2DH NUMERICAL MODELS

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INTRODUCTION

This research consists of the mathematical development of a three-dimensional (3D) analytical-numerical *module* that can be installed on vertically-averaged (2DH) numerical *models*. Designed to combine the advantages of both 3D and 2DH hydrodynamic models, it simulates a wide range of coastal natural water bodies of free surfaces and irregular geometries. Examples include estuaries, rivers, bays, lakes, and lagoons.

METHODOLOGY

Assuming a parabolic profile of turbulent viscosity and a predictor-corrector scheme applied to 2DH variables, a *preliminary* (predicted, analytical) logarithmic profile of horizontal velocity is used for the calculation of an *adjusted* (corrected, analytical-numerical) realistic 3D velocity field (u,v,w). Through a process of continuous synergetic interdependence, both the 3D *module* and the 2DH *model* feed on and enhance one another at every time step of a simulation in a cycle of mutual contribution that increases its accuracy. The following terms of the Navier-Stokes horizontal equations are considered in full three-dimensionality: local acceleration, non-linear advective accelerations, Coriolis acceleration, barotropic and baroclinic pressure gradients, and vertical shear stresses. All 2DH variables are allowed to vary in time and space (x,y,t) and all boundary conditions (dynamic and kinematic, bottom and surface) are analytically satisfied.

PRELIMINARY RESULTS

Figure 1 illustrates an example of application of this 3D analytical-numerical *module* embedded in a 2DH numerical *model*: a simulation of the velocity field of a meandering river featuring a non-linear helical flow (only horizontal components of velocity shown in this figure).

DISCUSSION

This 3D module can simulate non-linear hydrodynamic circulation and transport of passive scalars on a wide range of coastal natural water bodies of free surfaces and irregular geometries as long as: (1) their pressure fields are predominantly hydrostatic, (2) their density gradients are predominantly horizontal, and (3) they vary in time and space continuously, smoothly, and without sharp gradients. However, the robustness of its mathematical structure provides notable advantages. For instance, the type of flow illustrated by Figure 1, responsible for riverine coastal erosion and meander migration, cannot be simulated by 2DH or 1D models alone. On the other hand, 3D fully numerical models are often numerically unstable and take a significant amount of the modeler's time to be set. This 3D analytical-numerical *module*, however, when embedded in 2DH numerical *models*, provides velocity fields that match the ones of 3D fully numerical models and a numerical stability that matches the one of 2DH numerical models. Besides, this research originated a comprehensive collection of original analytical and semi-analytical formulas that can be readily used out of the context of computational modeling. In the context of field campaigns, for example, one of its analytical formulas can be used to calculate the bottom stress caused by a propagating tidal wave - or even to investigate how this stress changes for different surface roughness conditions (which, in turn, can be related to surface wave conditions).

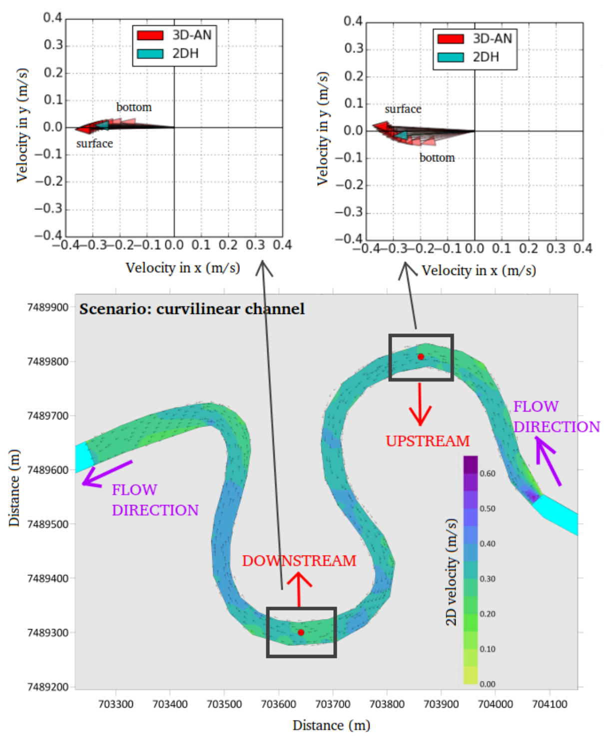

Figure 1 – Velocity field of a meandering river featuring a non-linear helical flow (only horizontal components shown).

NEXT STEPS AND EXPECTATIONS

Currently being implemented on SisBaHiA, a numerical model with both 3D and 2DH capabilities developed by the Federal University of Rio de Janeiro (1st in Americas for Ocean Engineering, in Brazil), future aims include comparing this 3D *analytical-numerical module* against 3D *fully numerical models* for simulation time (expected to be shorter) and accuracy (expected to be similar).

REFERENCES

Araujo, Rosman (2019): An analytical-numerical module to calculate vertical profiles of velocity in free surface flows (in Portuguese), Rio de Janeiro, UFRJ/COPPE.
Rosman (2019): Technical Reference of SisBaHiA (in Portuguese), Rio de Janeiro, Fundação COPPETEC.