COASTAL ENGINEERING

PROCEEDINGS OF SIXTH CONFERENCE

ON

COASTAL ENGINEERING

GAINESVILLE, PALM BEACH, AND MIAMI BEACH
FLORIDA

DECEMBER 1957

Edited by

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PROFESSOR OF HYDRAULIC ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKELEY

PUBLISHED BY

COUNCIL ON WAVE RESEARCH
THE ENGINEERING FOUNDATION

1958

Council on Wave Research
Building 159
Engineering Field Station
University of California
Richmond, California

Lithographed in the United States of America
San Francisco, California

ACKNOWLEDGMENTS

This conference was sponsored jointly by the Council on Wave Research and the University of Florida. The National Science Foundation assisted by making a grant to pay the travel expenses of some of the foreign authors. Appreciation is expressed to the University of Florida Coastal Engineering Laboratory and to the Jacksonville District of the Corps of Engineers for photographs supplied to illustrate the cover and the section title pages of this publication.

CONTENTS

CONTENTS

CONTENTS

CONTENTS

CONTENTS

CONTENTS

Canaveral Harbor

Part 1

WIND, WAVES, AND WIND TIDES

Daytona Beach

CHAPTER 1

WINDS AND PRESSURES IN HURRICANES

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The hurricane is one of the most dangerous and at the same time one of the most interesting weather phenomena with which the citizens of the United States have to deal. We have more hurricanes on our southern and eastern seaboards than any other major continental area, except Southeast Asia (1, 2). These storms of course are very familiar to certain Island countries: Cuba, Haiti and the Dominican Republic, Japan, the Philippines. Mexico and Australia receive their share also.

Under Congressional directive the Weather Bureau in collaboration with other Federal Agencies and universities is at the present time pursuing an expanded program of research on hurricanes, termed the National Hurricane Research Project (3). The most spectacular phase of the program are the reconnaissance flights of three especially instrumented Air Force aircraft assigned to the Project base at West Palm Beach, Florida. These planes, especially during the hurricane season just ended, have gathered some very excellent data on the structure of hurricanes, superior in detail and reliability to anything available before. However, the aircraft for safety reasons do not fly near the surface, and the surface winds and sea-level air pressures are the factors of immediate concern to coastal engineers. The stress of surface winds produces surges and waves in bodies of water, and the sea-level pressure deficit in the inner part of a hurricane permits a corresponding rise in the water level by hydrostatic equilibrium principles. It will be some time, perhaps several years, before more refined models of the surface layers of hurricanes can be deduced by indirect analysis from the West Palm Beach data.

This paper will summarize some of the pertinent information available up to now on surface winds and pressures and will make no further reference to the West Palm Beach investigations. There will also be a brief treatment of rainfall in hurricanes which is of interest to coastal engineers because of flooding produced by rain-water behind seawalls and in estuaries where a flood can result from an abnormal tide or an abnormal stream flow or both at the same time.

ENERGY SOURCE

We shall pass over how a hurricane starts except to say that it is always over warm ocean waters and always in an area of squally weather and take a look at what makes it keep going once it has started. Descriptions of the formation and life cycle of hurricanes are given by Tannehill (4), Dunn (5), and Rhiel (6). The key to hurricane

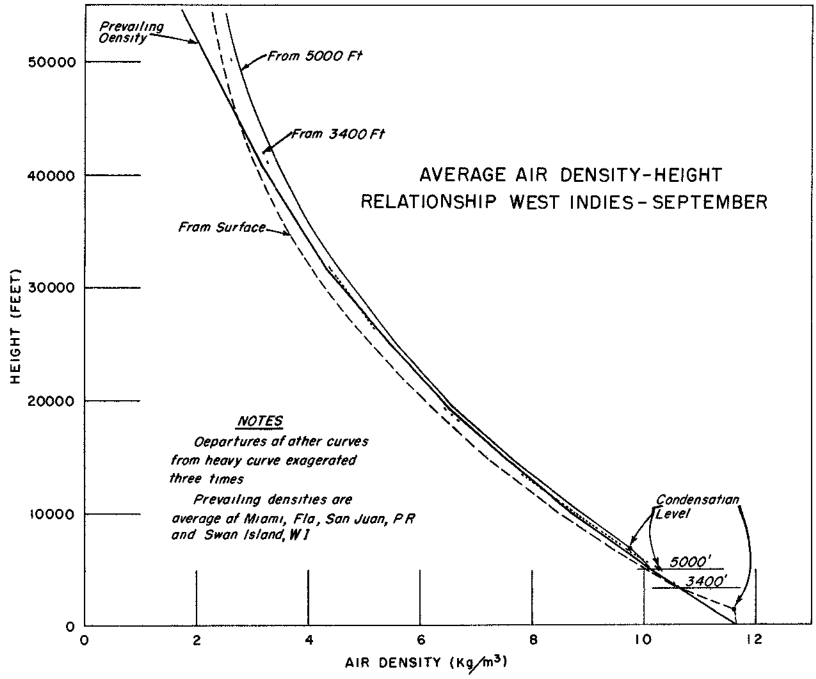

Fig. 1

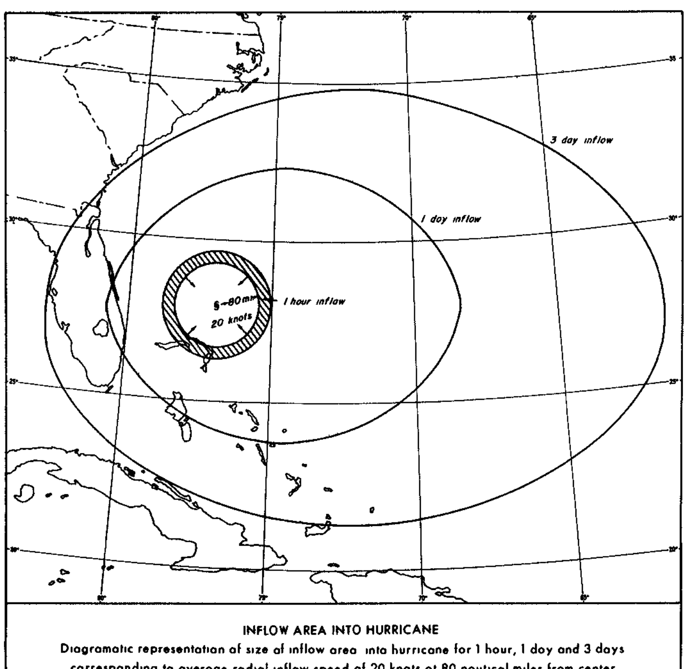

Fig. 2. Schematic diagram of inflow area into a hurricane.

energy is the vertical distribution of air density, which is, in turn, related to temperature and moisture content. The solid curve of figure 1 shows average air density vs. height in September in the West Indian tropics, based on atmospheric soundings at Miami, San Juan, P. R., and Swan Island (7). At the first glance it looks like a stable arrangement; every layer of air has a smaller density than the layer immediately beneath and therefore will tend to float on top of it. Next we must consider how air behaves when it changes elevation. Rising air encounters progressively lower pressure and therefore cools adiabatically. There is one standard well known rate of cooling for unsaturated air. There is a different rate for saturated air because the release of latent heat decreases but does not overcompensate the adiabatic cooling. Now let us consider what will happen if a square mile or so of a layer of air near the ocean surface should somehow be lifted through its environment. The density this lifted surface air would acquire at each level is shown by the curve with long dashes in figure 1. The density decrease is first along the unsaturated adiabatic rate to the condensation level and then along a standard curve for saturated air. It is seen that this air at about 3000 feet will become less dense than its environment (heavy solid curve); it will then of course continue to rise spontaneously to great heights, perhaps over 40,000 feet. Similar curves of the density changes associated with lifting are shown for a 3,400-ft parcel, which contains very little of this potential relative density decrease (called latent instability) and for a 5,000-ft parcel which contains none. When a hurricane forms, layers of air rather close to the surface of the ocean break through the slightly stable layer above to the level when they can rise spontaneously. Once this great chimney of convection (perhaps 50-100 miles in diameter) has been established the hurricane will maintain its vigor and will continue to drift so long as it can feed on air of the proper vertical density distribution. The air processed by the hurricane is drawn from a large area. This is shown in figure 2. The figure shows schematically the size of area required to feed air to a hurricane for an hour, a day, and for three days. The figure is based on an average component of the surface wind of 20 kts toward the storm center at a distance from the center of 80 miles. This is a moderate value of the inflow rate. Another way of emphasizing the vast low-level indraft and high-level outflow needed for hurricane maintenance is to look at the distortion of a cube of saturated air several miles on a side required to produce one, two, and five inches of rain. This is shown in figure 3.

The latent instability necessary for hurricane formation requires a strong heat and moisture source at the bottom of the atmosphere, occasioned by strong insolation and warm sea-water. On the other hand, the stabilizing subsidence, or settling, of the atmosphere that is always found in large high pressure areas must be absent. These factors limit the seasons and places that hurricanes may

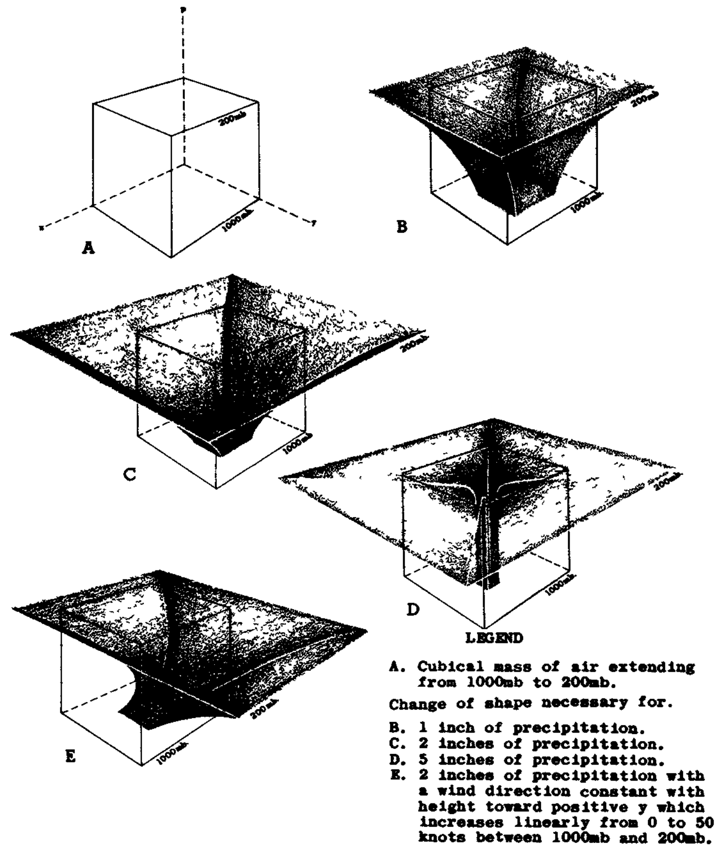

Fig. 3. Change of shape of cubical mass of air necessary for various rainfall amounts.

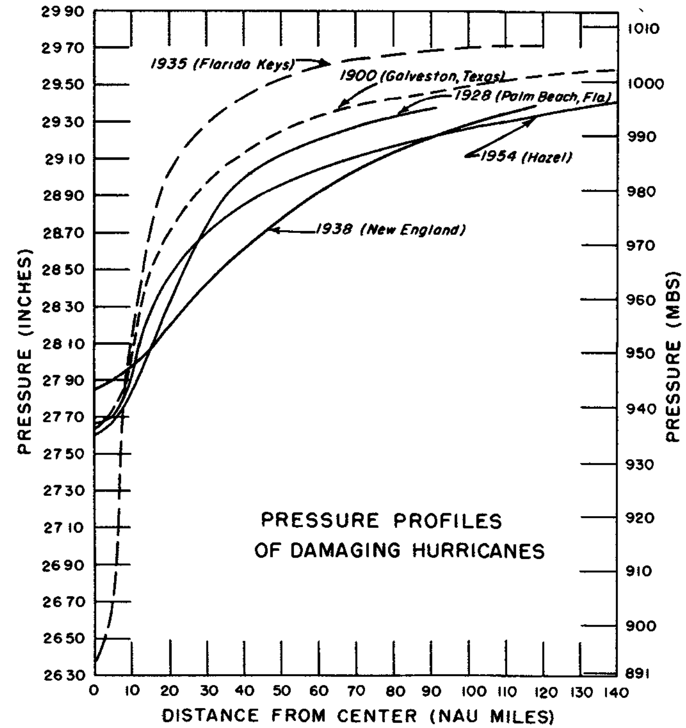

Fig. 4. Pressure profiles of damaging hurricanes.

form. The United States hurricane season is mid-June to mid-October, approximately. In this connection it should be noted that the highest water temperatures in the hurricane genesis areas of the Atlantic and Gulf of Mexico lag behind the most intense insolation of June and July and do not occur until August or early September.

If relatively dry air is drawn into a hurricane, the storm must weaken. If it moves over extensive land it will die because among other things the friction upsets the delicate balance between the vast indraft required of the surface layers and the compensatory upper outflow; if it turns into a more northerly latitude it may acquire an additional energy source namely, a non-homogeneous horizontal density distribution; colder air sinking on one side of the storm and warmer air rising on the opposite side. Usually in such cases there will be a temporary increase in energy followed by a decrease or at least a dispersion of the concentrated kinetic energy (winds) of the hurricane.

PRESSURES

In discussing winds and the pressures in hurricanes, we prefer to present the pressures first because the data are easier to handle. Hurricanes are not quite circular but are frequently considered so for purposes of analysis of the pressures. Figure 4 shows radial profiles of sea level atmospheric pressure for several famous hurricanes. In the center of the storm the pressure can fall to values 6 to 10% or more below normal atmospheric pressure. The tremendous horizontal pressure forces acting inward in a hurricane are balanced in largest measure by the centrifugal force, to a lesser degree by friction and by the coriolis force from the earth's rotation; and then, of course, there is not a complete balance of forces; the inward-moving air parcels are accelerated.

Frequently it is convenient to have an analytical expression that approximately describes the usual shape of a hurricane pressure profile. Such an expression is:

$$P = P_o + (P_n - P_o)\ e^{-R/r} \qquad (1)$$

where P is the pressure at radius r, P_o the minimum pressure at the center of the storm, P_n the pressure "outside" the storm, (theoretically at $r = \infty$) and R is a characteristic radius. An alternate form is

$$P = P_n - (P_n - P_o)\ (1 - e^{-R/r}) \qquad (1a)$$

Equation (1) has been found empirically to fit a good many hurricanes fairly well (8). One application of this expression by the authors of this talk and their colleagues was to estimate systematically the central pressures of all hurricanes affecting the United States since 1900 by plotting all observed pressure data at the appropriate distance from the center of the hurricane on a radial profile, such as those of figure 4, and then extrapolating into distance zero by fitting a curve described by the formula (9). The accumulated frequenc of central pressures of hurricanes affecting the eastern United Stat Coast obtained in the manner just described is shown in figure 5, se arated into regions. All of the central pressures are at the point storm center entering the coast.

These regions are the Florida Keys, the Florida Peninsula, th Texas Coast, The Gulf Coast from the Texas-Louisiana border to Apalachee Bay, and the Atlantic Coast from Georgia to just south of Cape Hatteras, and the Atlantic Coast from Cape Hatteras northward. Clea ly the Florida Keys experience the lowest pressures in hurricanes. the other curves there is a suggestion of a latitudinal variation wit lowest pressures in the south but the data do not demonstrate this conclusively.

There is a considerable range in the reliability of the centr pressure estimates on which figure 5 is based. The factor having mo influence on the reliability of the central pressure estimate for an individual storm is the length of radius from the storm center to th closest pressure observation.

WINDS

Relation of maximum wind to central pressure

The central pressure of a hurricane is a convenient though so what approximate index of the strength of the storm. An expression for the maximum wind in a hurricane which has some both empirical and theoretical support is of the form (9), (10), (11), (12):

$$V_x = K\sqrt{P_n - P_o}, \qquad (2)$$

where P_o is the central pressure; P_n the pressure outside the storm; the highest wind speed. With wind speed in knots and pressures in mi libars, average values of K in the August 1949 hurricane at Lake Okee chobee were 7.5 for off-shore winds averaged over 10 minutes, 9 for o shore winds, and 14 for peak on-shore gusts (12).

Lake Okeechobee 1949 hurricane

Perhaps the best detail of surface wind observations in a hurr cane anywhere was in the hurricane of August 26-27, 1949 which crosse Lake Okeechobee, Florida. The Corps of Engineers, charged with the d sign, construction, and operation of protective levees around this Lake, for data gathering purposes established a special meteorologica network in the area (13). There are seven autographic wind and pressure stations on the shore of the Lake, which is some 30 miles across at the time of the 1949 hurricane there were three such stations moun ed on navigation--light pylons out over the water surface of the Lake These data have been intensively studied by the Weather Bureau and th

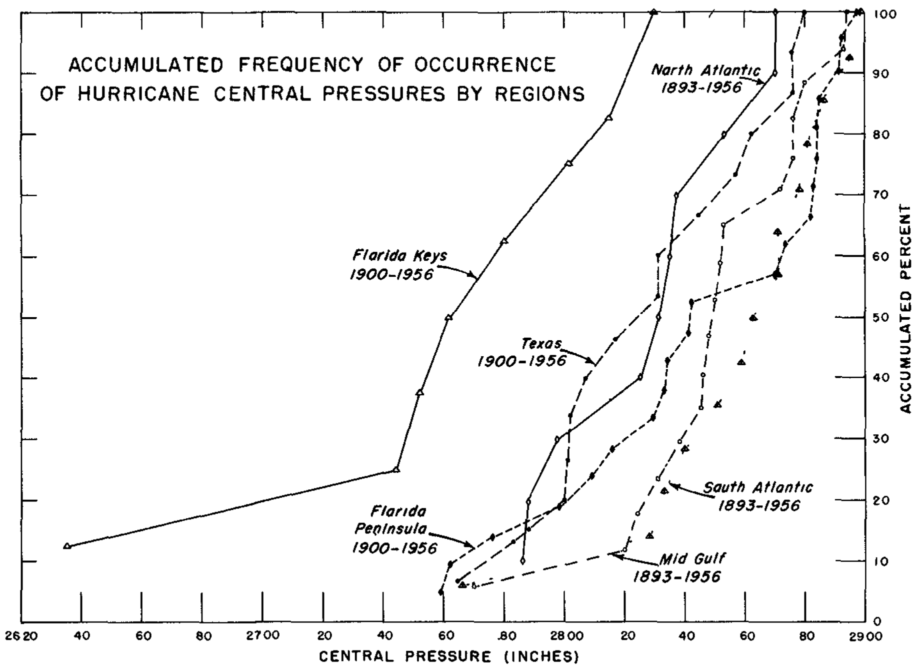

Fig. 5. Accumulated frequencies of hurricane central pressures by regions.

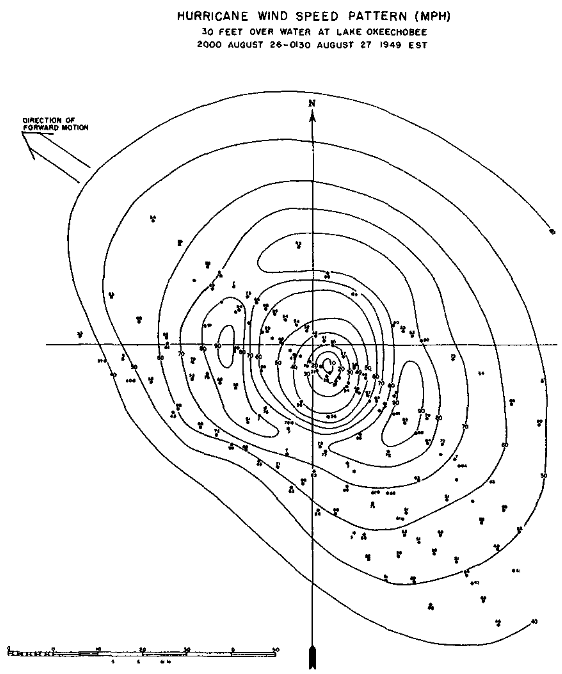

Fig. 6. Hurricane wind speed pattern, August 26-27, 1949.

Corps of Engineers in connection with a design study by the latter agency. Figure 6 shows a map of the wind speeds in the 1949 hurric as it crossed the Lake. This is a composite picture of all 10-minut average wind speeds during a total elapsed time of about 5 hours. Each speed is plotted at its bearing and distance relative to the c ter of the hurricane. Adjacent data do not always match in part be cause of inaccuracies of factors used to adjust speeds to a common frictional surface, but also because the wind field of a hurricane i far from unvarying. Superimposed on the overall wind pattern depict ed here by the solid isopleths are gust-type variations of all kind of scales from a few yards and fractions of seconds of time up to 1C or 20 miles and twenty or thirty minutes. Radar has revealed that there are spiral bands in hurricanes where the rainfall is heavier than other places. The wind directions and speeds may show a slight discontinuity at these bands. Note that the band of highest wind speeds is not a smooth symmetrical circle; the most extreme speeds a generally found somewhere on the right side of the storm though ther seems to be considerable variation from one storm to another as to the exact bearing from center of the highest speeds.

Applications of 1949 hurricane data

The average radial profiles of wind speed in the same Lake Okeechobee hurricane are shown in figure 7. From this diagram illus trates several things: First, the curves show the typical general shape of the wind profile for a severe hurricane, gradual rise in speed from the outskirts to a maximum value at some point at a radiu of about 22 miles in this case and then a sharp decrease in speed to almost calm conditions at a center of the wind circulation. Second-ly, the curves show the variation of hurricane speeds over different kinds of frictional surfaces. The lowest three curves are for 10-minute average speeds at Lake Okeechobee, respectively for off-land winds at the shore, off-water winds, and over-water winds. The last is from the speeds measured at the pylons in the Lake several miles from shore. Empirical factors from these three curves have been use for adjusting hurricane wind speeds measured over land along much of the coast of the United States to over-water values. We have been able to make a few supplementary wind speed comparisons of limited a plication; these include comparing Nantucket Island, Mass. with Nan-tucket Light Ship, the Friendship International Airport at Baltimore with speeds at the Chesapeake Bay Bridge, and the New Orleans Weathe Bureau Office with speeds at the Huey Long Bridge. There are a few other pairs of nearby stations that have not been fully exploited data-wise yet.

The third deduction from figure 7 is the relation of gusts to sustained speeds. The two top curves are smooth plots of the highes point in each 10-minute interval on a wind speed trace from the Dine pressure-tube type of anemometer. The gust defined in this way aver aged about 1.4 of the sustained 10-minute average speeds. Probably the most satisfactory way to analyze hurricane wind fields for en-gineering purposes is to make the basic analysis in terms of sustain ed average speeds over 5, 10 or 15 minute intervals. For building design appropriate gust factors are applied to these mean speeds.

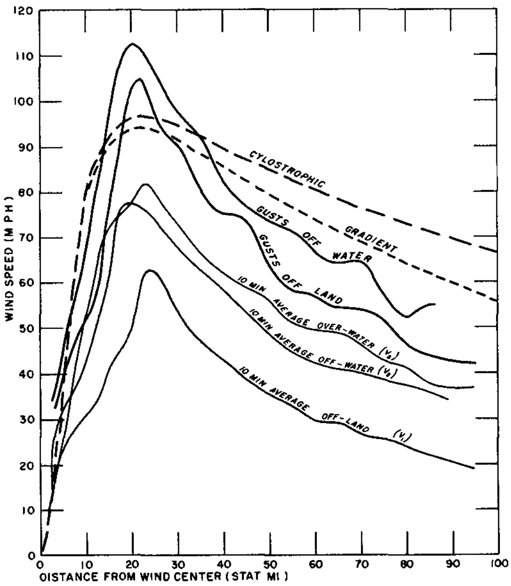

Fig. 7. Hurricane wind speed profiles August 26-27, 1949.

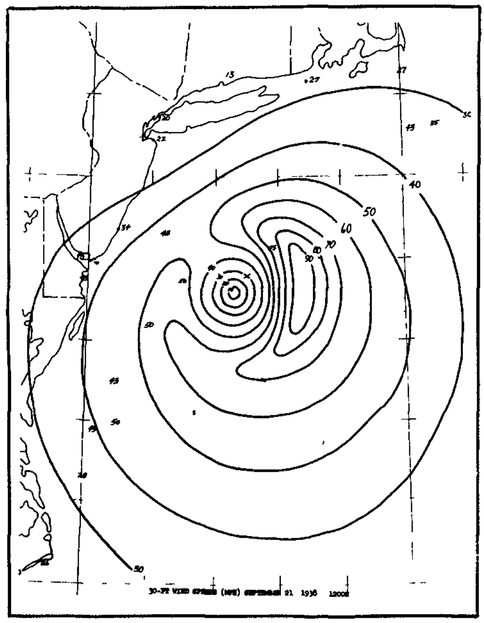

Fig. 8a

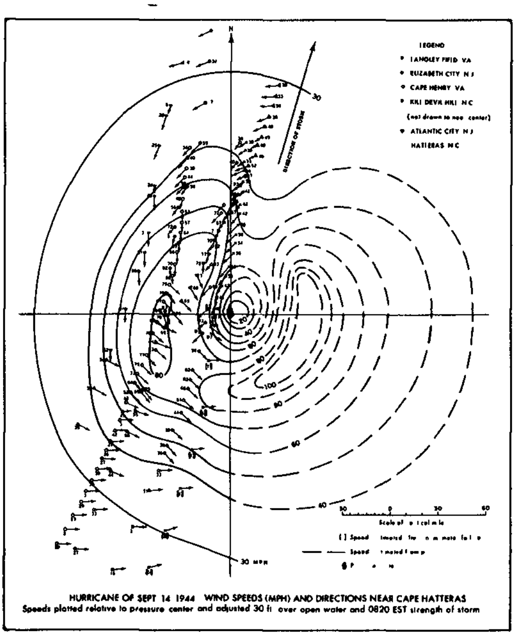

Fig. 8b

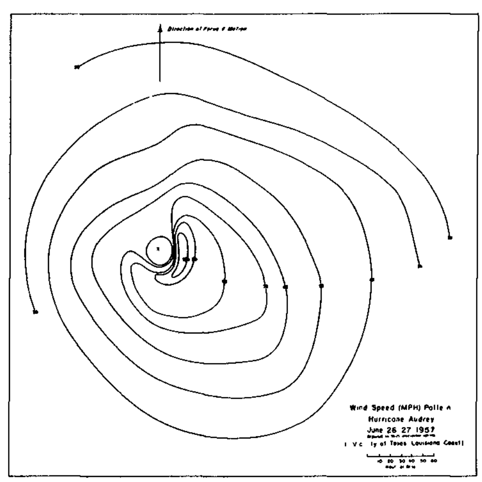

Fig. 8c

Fig. 8. Hurricane wind fields over the sea.

A fourth deduction of engineering usefulness from figure 7 is the relation of the actual winds to theoretical winds computed from the pressure field. The dashed curve is the so-called gradient wind which is the speed necessary for the pressure gradient force, centri ugal force, and coriolis force from the earth's rotation to be in bal ance. Wind fields in other hurricanes have been reconstructed by com puting the pressure fields from pressure observations and then comput ing the gradient wind and reducing to actual wind by empirical facto derived from this diagram. The Hydrometeorological Section of the Weather Bureau has reconstructed the wind fields in a number of hurri canes along the coast for the Corps of Engineers (/14/for example). The purpose is for a check-out of procedures for computing the hurri- cane surge in the various coastal reaches from the winds. In almost none of these reconstructed hurricanes, even for so recent hurricane as Hazel of 1954 and Audrey of 1957 has there been much wind data available over the water surface itself where the wind fields are nee ed and considerable reliance has had to be placed in estimating winds from pressure by the Lake Okeechobee empirical factors and in adjust- ing winds at land stations. A few of these wind fields for certain famous hurricanes are depicted in figure 8.

The great size of hurricanes warrants emphasis. As can be seen from figure 8 a typical hurricane is hundreds of miles across; it also extends several miles vertically. A hydrogen bomb is small compared to the total kinetic energy of a hurricane.*

Trajectory method for hurricane wind models

There are few wind data on the right side of the Lake Okeechob hurricane; it is here that the highest winds in the storm are thought to occur. Composite patterns of the wind flow in hurricanes obtained by combining data from a number of storms by Hughes (15) and others give good pictures of the nature of the flow in the outer parts of the storm but are lacking in the detail necessary for surge studies in the zone of maximum winds near the center. To refine our empiri- cal wind model from the Lake Okeechobee hurricane, especially with respect to the asymmetry of the wind field, at present we are experi- menting with synthetic wind fields constructed by a trajectory method Starting with low-speed winds on the outskirts of a hurricane, the ac celerations of the air are computed from the estimated forces (real and apparent): pressure gradient, centrifugal, coriolis force, and friction. Horizontal motion at 30 feet is assumed and the work is

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*The kinetic energy of the winds of a typical hurricane has been estimated as about 5 x 10^{26} ergs at any one time. The mechanical equivalent of the energy released by a bomb equal to 10^7 tons of TNT is about 1/1000 of this.

restricted to this anemometer level. The resulting trajectories of air parcels are computed. The friction is the so-called eddy stress and incorporates the transfer of momentum from one level to another by turbulence. Empirical values of the friction are being developed by comparing computed trajectories in the same Lake Okeechobee hurricane with trajectories reconstructed from the data. It is interesting to note that at the 30-foot level the effects of stress have a component not only opposite to the mean wind but also a component normal to the direction of the mean wind that is almost as large. This is due in part to the fact that the effects of the turbulent components of the wind are not linear and do not cancel out.

Variation of wind with height

The variation of wind speed with height is directly applicable to building construction rather than to work with bodies of water but is indirectly used in the latter instance when observed wind speeds at various anemometer levels are adjusted down to the standard 30-foot surface. The best available data on the variation of wind speed in hurricanes up to heights of several hundred feet were obtained at the Brookhaven Laboratory wind tower on Long Island where winds at three or four levels were measured with laboratory-calibrated anemometers in hurricanes Carol and Edna of 1954 (16). At the top of the tower at 410 feet winds almost up to a hundred miles an hour were observed in Carol. The variation of wind speed with height in the two hurricanes was about the same and is depicted in figure 9. The Brookhaven curves should tentatively be regarded as showing the extreme of the variation with height. The increase of speed with height at Lake Okeechobee in an October 1953 tropical storm, as measured by the Jacksonville District of the Corps of Engineers (13), was relatively smaller. Other data at lower speeds from other wind tower sites also indicate that Brookhaven has a relatively large wind speed increase with height. It is assumed that the surface there is dynamically relatively rough.

Wind direction

The overall average anemometer-level wind direction in a hurricane at sea is about 25° to 35° to the left of a tangent to a circle drawn about the storm center. There are variations in this angle from quadrant to quadrant and between storms. This vast indraft or convergence compensates vast updrafts, strongest in the region of maximum winds outside the eye. There are turbulent departures from the mean direction at all times in all parts of the storm which obscure the systematic variations of this angle of deflection which probably exist from front to rear and left to right. It is hoped that our trajectory studies will yield a more refined model of the wind direction in hurricanes, especially the variation from one quadrant to another.

WEAKENING OVER LAND

The next topic is weakening of hurricanes over land surfaces. This is of importance to coastal engineers concerned with inland bodies of water such as Lake Okeechobee, Lake Pontchartrain, Chesapeake Bay or even New York Harbor. At New York City the extreme surge presumably would be associated with a hurricane moving inland over New Jersey and having some over-land trajectory before reaching the latitude of New York.

The critical hurricane path for the worst surge for the places named, of course, is not a track along the shortest distance to the sea, but rather a track that will give the longest duration of winds from the critical direction.

It is a common observation that the winds decrease markedly as a hurricane moves inland. We should distinguish carefully between tw different effects in this connection. First, for a storm of a given intensity of pressure gradient, the surface wind will be less over th land surface than on the open coast because of the greater impedance of the surface roughness. The other effect is a weakening of the pressure gradient itself. If only the former effect dominates, then a hurricane approaching Lake Okeechobee or Lake Pontchartrain could be expected to regain its over-water vigor over the Lake.

Survey of a large number of hurricanes suggest the following. First, that the decrease in intensity of hurricanes over land is partly a frictional effect, and partly because the hurricane frequently encounters drier air which is less favorable for its maintenance. Over the Florida Peninsula, where in general during hurricane weather the air will be just about as humid as over the sea, there may be ver little decrease in the overall intensity of a storm, only the immediate surface-layer winds decrease. There are individual variations. In 1930 a small-diameter but very intense hurricane was very destructive in the Dominican Republic. The center passed directly over the Island including some rather rugged terrain (4). The storm did not amount to much after leaving the Island. Two years earlier the center of an intense hurricane passed directly over the Island of Puerto Rico, and then continued but little diminished, if any, to the Florid coast and produced the famous Palm Beach and Lake Okeechobee disaster. This of course was a larger hurricane and a smaller island with lower mountains than in 1930. The Florida 1947 hurricane was the most seve of the last decade in this area. It passed close to Miami (over Ft. Lauderdale), diminished in intensity a little as it crossed the Flori Peninsula, but then regained its strength over the Gulf, passed over New Orleans with great intensity. Some average empirical factors for weakening of hurricanes over land have been developed and are listed in Table 1.

RAIN

The final consideration for hurricanes is rain. The release o latent heat and rain is a necessary and always present feature of the storm in the tropics though it will not necessarily rain heavily in every quadrant of the storm during every hour. To further augment the flooding risks from hurricane rains it not infrequently happens, especially with hurricanes moving up the Eastern Seaboard, that there are rather heavy rains a day or so in advance of the arrival of the actual storm circulation itself and therefore the rain immediately associated with the hurricane may fall on already swollen streams. This combination prevailed, for example, in the September 1938 hurricane in New England and more recently in hurricane Diane of 1955. The Diane flood, of course, was further augmented by a previous hurricane only ten days before.

Tropical storms of less than hurricane intensity on the average give almost as much rain as hurricanes. Extensive hurricane rainfall statistics have been compiled by two of our collaborators. A few samples will be shown.

A typical isohyetal pattern for a storm moving up the East Coast, is shown in figure 10 and three similar isohyetal patterns for East Texas--Louisiana hurricanes in figure 11 (17). Time distributions of heavy hurricane precipitation are illustrated in figure 12. These are for the heaviest 12-hour precipitation at a U. S. Weather Bureau recorder station within 30 miles of a hurricane center on the middle Atlantic U. S. coast. These were suggested as prototypes for an interior drainage design problem behind a sea wall at Norfolk, Va. Figure 13 shows the penetrations of hurricane rains inland. This envelopes all hurricanes since 1900 excepting a 1915 Texas hurricane and Hazel of 1954. Both of these joined up with fronts and became sort of combined frontal and tropical storms. Both carried 5" isohyets much farther inland than shown here.

Table 1

AVERAGE FACTORS FOR REDUCING HURRICANES FOR FILLING OVER LAND

Time (hours)	Adjustment ratio for wind speed
T (at coast)	1.00
T + 1	0.93
T + 2	0.88
T + 3	0.85
T + 4	0.82
T + 5	0.80
T + 6	0.78
T + 7	0.76
T + 8	0.74

The factors in Table 1 will yield speeds for portions of the storm that are still over water. Further reductions would be required to obtain the speeds over land.

Based on observed pressure changes in eleven hurricanes that entered the United States and equation (2).

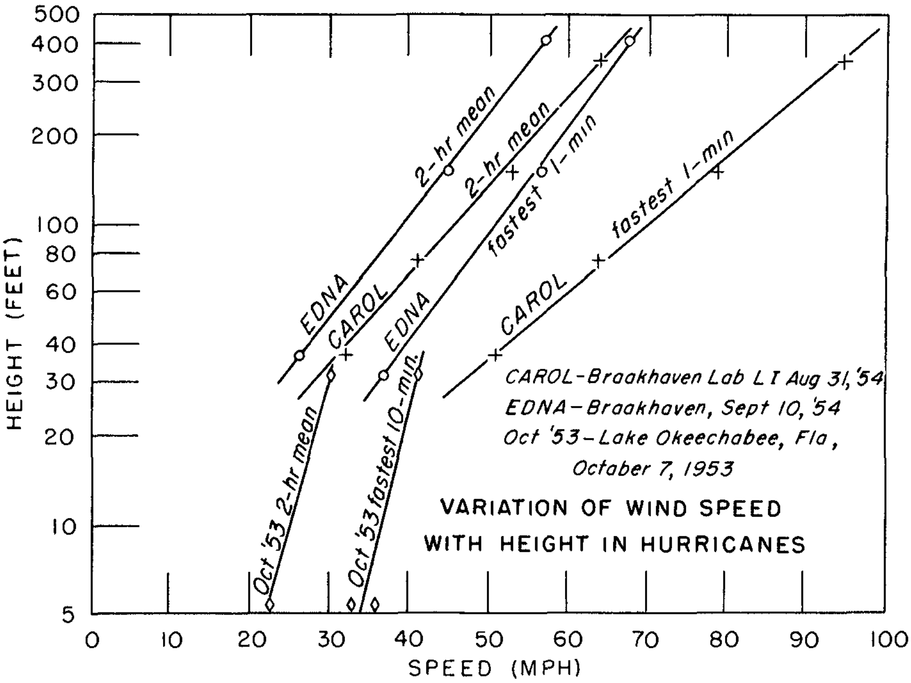

Fig. 9. Variation of wind speed with height in selected hurricanes.

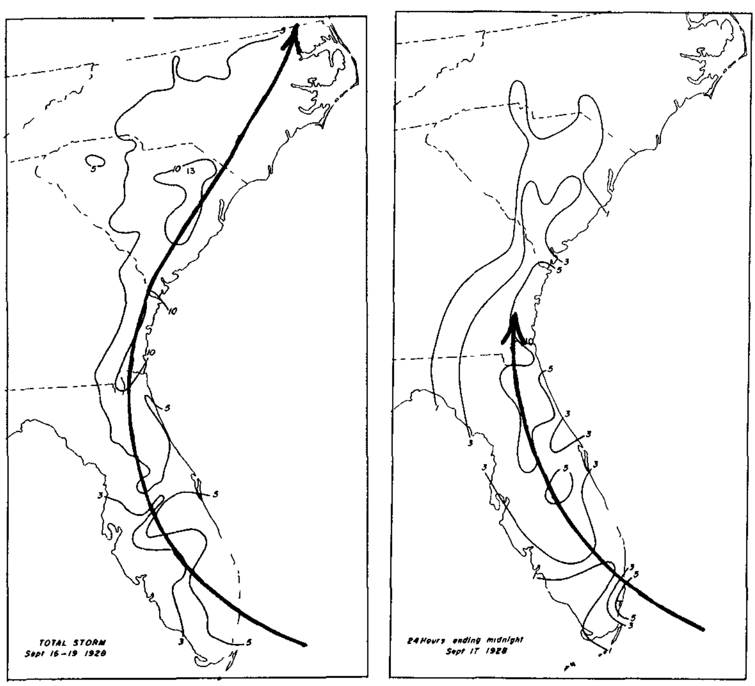

Fig. 10. Typical isohyets (inches) for East Coast hurricane.

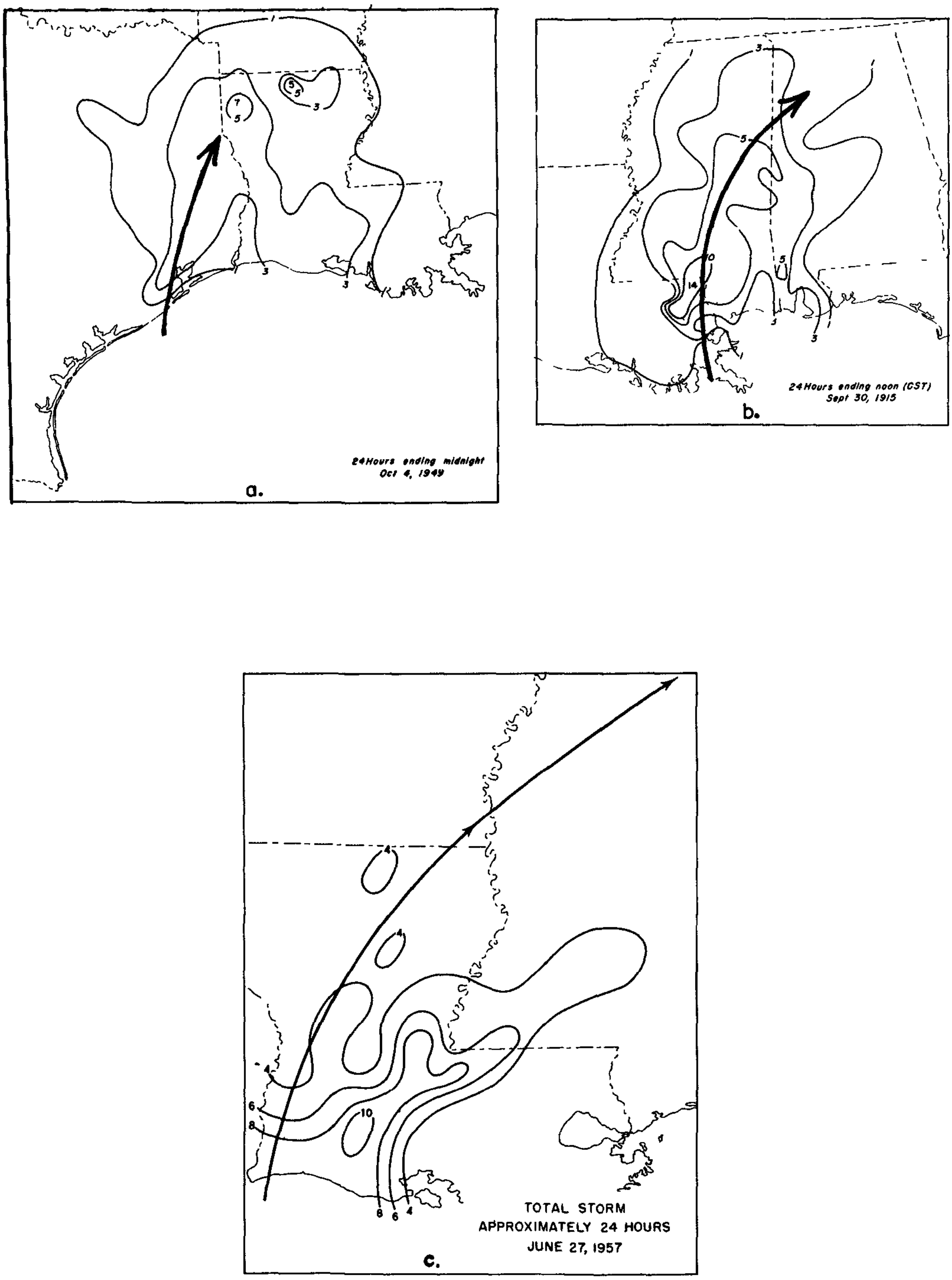

Fig. 11. Typical isohyets (inches) for Gulf Coast hurricanes.

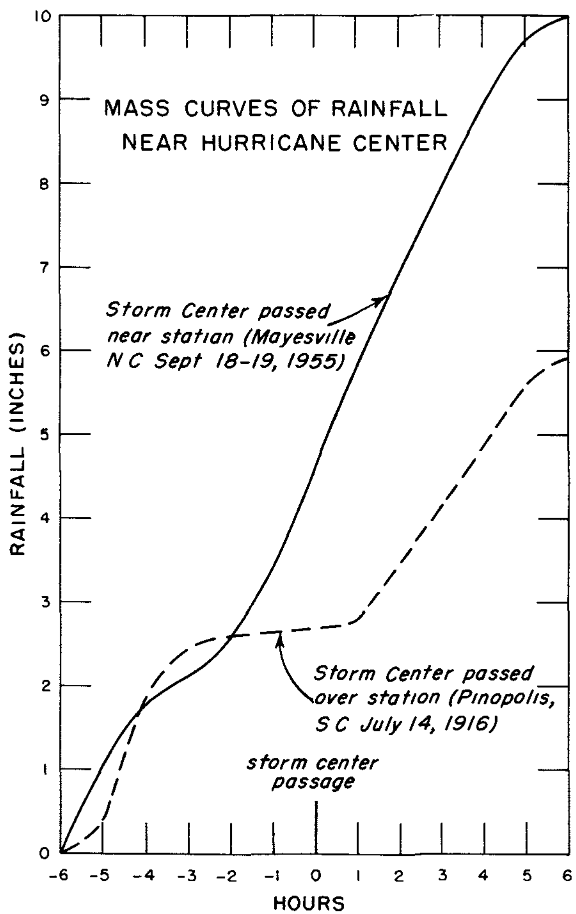

Fig. 12. Typical mass curves of rain-

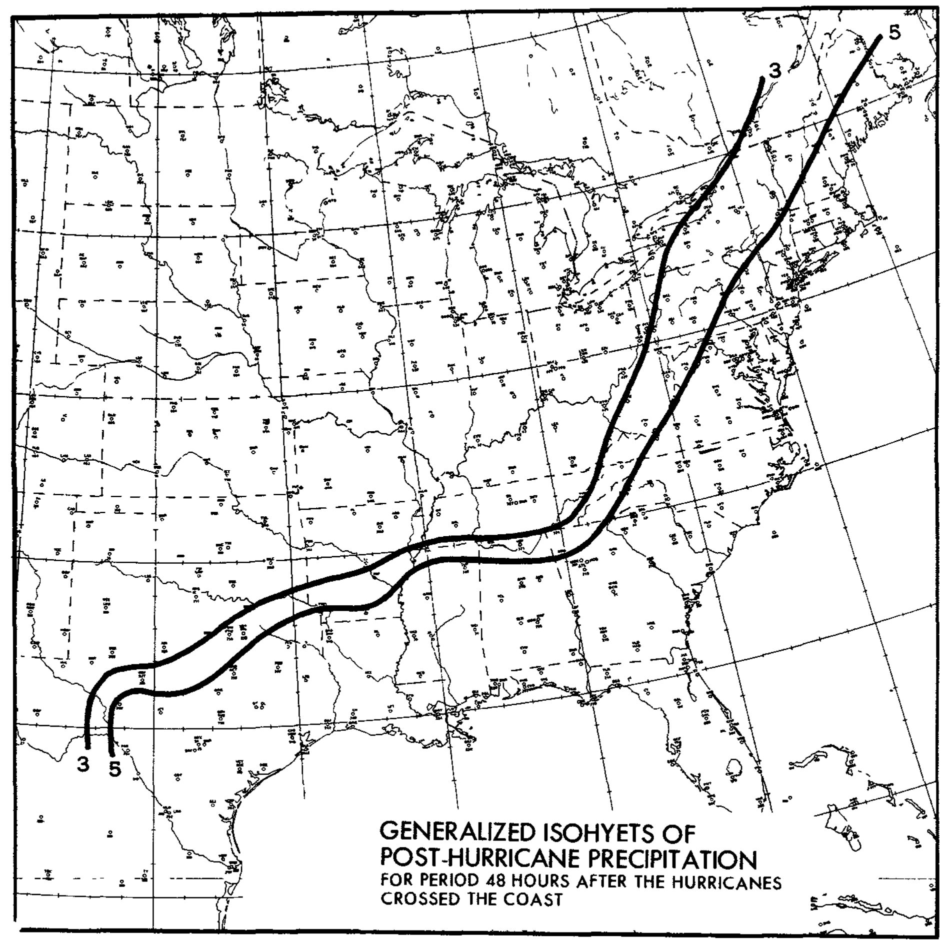

Fig. 13

WINDS AND PRESSURES IN HURRICANES

SUMMARY

Hurricanes are vast, somewhat circular storms that originate only over ocean areas at seasons of strong insolation and warm water temperature, but which frequently move over land. The immediate driving force to the winds is the horizontal pressure gradient directed toward the low pressure always present at the center of the storm. The ultimate source of energy in the tropics is the potential density difference between air near the surface, continually warmed and moistened from below, and at higher levels. Horizontal density gradients are a supplementary source of energy for storms moving into middle latitudes. The mere presence of these energy sources are not sufficient to initiate a hurricane. There are other necessary conditions not discussed in this paper.

The extreme minimum pressure in a hurricane in the vicinity of the United States was 26.35 inches (892 mb) in the Florida Keys in 1935. Pressure experienced in various reaches of the mainland coast have ranged down close to 27.50 inches (931 mb). Empirical relations between pressure gradients and winds have been developed that are useful in reconstructing winds over the sea in past hurricanes that have caused important surges.

The well-known typical wind speed pattern is for a small region of light or near-calm winds to be encircled by a band of very strong winds, tapering off more gradually to moderate winds at some scores of miles, or even a hundred miles or more, from the center. There are variations with azimuth from direction of storm motion as well as along a radius, with the strongest winds generally on the right side. Very detailed wind hurricane observations were obtained at Lake Okeechobee, Florida on August 26-27, 1949. From these data empirical relationships have been developed of the comparative strength of winds off-land at a shore, onshore, and over open water that have been useful in reconstructing other hurricanes over the sea.

In assessing reported winds in hurricanes care must be taken to distinguish between sustained winds averaged over several minutes and peak gusts. The relation of the latter to the former, depending on exact definition of a gust and other circumstances, is about 1.4 or 1.5. The gust factors are applied for building design according to standards in that branch of engineering. The variation of wind speed with height over a moderately rough land surface was measured at Brookhaven, Long Island in hurricanes Carol and Edna of 1954.

Hurricane winds diminish over land. Always present is the increased surface friction as compared with the sea, which slows down the anemometer-level winds. Usually present also is a weakening of the pressure gradients in the storm.

Rainfall is an inherent and necessary part of hurricanes. Some typical rainfall patterns were shown.

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CHAPTER 2
HURRICANES AND HURRICANE TIDES

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Most of the maximum tides of record between Cape Hatteras, N.C., and Brownsville, Tex., have been produced by tropical cyclones, or, as they are generally known in the United States, hurricanes. Some of the highest tides of record northward along the coast from Cape Hatteras to Cape Cod have been produced by hurricanes. From time to time our "northeasters", which are extra-tropical storms, may also cause millions of dollars of damage along the Atlantic coast between Miami, Fla., and Eastport, Me.

The Atlantic hurricane is identical with the Pacific typhoon and the tropical cyclone of the Indian and South Pacific Oceans. The term "hurricane" is defined as a storm of tropical origin with a cyclonic wind circulation (counter-clockwise in the northern hemisphere) with winds of 75 mph or more. However, in popular terminology, any winds of 75 mph or more are often described as hurricane winds.

FORMATION

Tropical cyclones develop in essentially homogeneous warm moist tropical air with no fronts or temperature and moisture discontinuities. The exact nature of the physical processes involved in the formation of hurricanes is not definitely known. However, there appears to be a number of meteorological conditions essential for tropical storm formation: (1) comparatively warm water 80-81°F or higher; (2) a pre-existing wind or pressure perturbation; (3) some outside influence which will intensify this disturbance, and (4) a type of wind flow in the high troposphere which will permit ready removal of the excess air and heat to other regions outside the hurricane area. These conditions are frequently present during the hurricane season but not necessarily in the proper relationship, and hurricane formation is relatively rare. It must be admitted meteorology does not yet have a complete answer to the problem of hurricane formation.

Hurricanes form only in those oceans and in those seasons in which sea surface temperatures are the highest. Here the accumulation of latent and sensible heat in the atmosphere reaches its maximum. The energy for the intensification of an ordinary disturbance in the tropics into a hurricane comes from the release of energy in the form of latent heat (latent heat of condensation) during the precipitation process.

Frequently in the early stages of development and even for a few days after reaching hurricane intensity, the hurricane may be quite small, almost a pinpoint on the usual weather chart. As it becomes older, it also becomes larger, although it may not, and indeed usually does not, become any more intense. The most intense hurricane of record on land, the Labor Day storm on the Florida Keys in 1935, with a central pressure of perhaps 892 mbs or 26.35 inches, was quite small and had a path of destruction only 35 to 40 miles wide. The largest Atlantic hurricanes may have damaging winds over an area some 400 to 500 miles wide and full hurricane winds 300 miles wide. The average diameter of hurricane winds is perhaps 75 to 100 miles.

FREQUENCY

The number of tropical storms (winds of 40 mph or higher) has averaged about 8 per year for the past 75 years, 9 per year for the last 40 years, and 10 per year for the last 20 years. During the past 70 years, the largest number of tropical storms noted in any one year was 21 in 1933. In 1914 only one tropical storm was reported and that was not of full hurricane intensity. About 58% reach full hurricane intensity and on the average only about two storms per year bring hurricane force winds to the coastline of the U. S.

AREAS OF DEVELOPMENT

Easterly waves, in which Atlantic hurricanes frequently develop, may move more than 2000 miles before any indication of intensificatior can be detected. Even after the transition from stable to unstable conditions has begun, a period of 3 to 6 days may be required for the initial vortex circulation to grow to full hurricane intensity. Durir this period the wave may travel an additional 1000 to 2000 miles. Where should it be said the hurricane formed? Where the easterly wave first began to intensify, where the tropical storm reached hurricane intensity or perhaps at some other point in its life history?

The approximate positions where tropical storms reached hurricane intensity during the period 1901-1957 have been plotted on Fig. 1. Only those storms where this point could be estimated with reasonable accuracy have been used. It can be seen that the density of hurricane formation increases steadily from the extreme eastern Atlantic to Long tude 56°. It is noted that almost no tropical storms reached hurricai intensity between Hispaniola and South America in the Caribbean but elsewhere in the tropical and sub-tropical Atlantic south of Latitude 30°, hurricanes are about as likely to develop in one place as anothe For many years textbooks have described the doldrums as the area where most hurricanes develop. This is certainly not true if the position where hurricane intensity is reached is considered as the place of development. Indeed, many tropical storms attain hurricane intensity in the area where the trade wind has the greatest strength and persisten

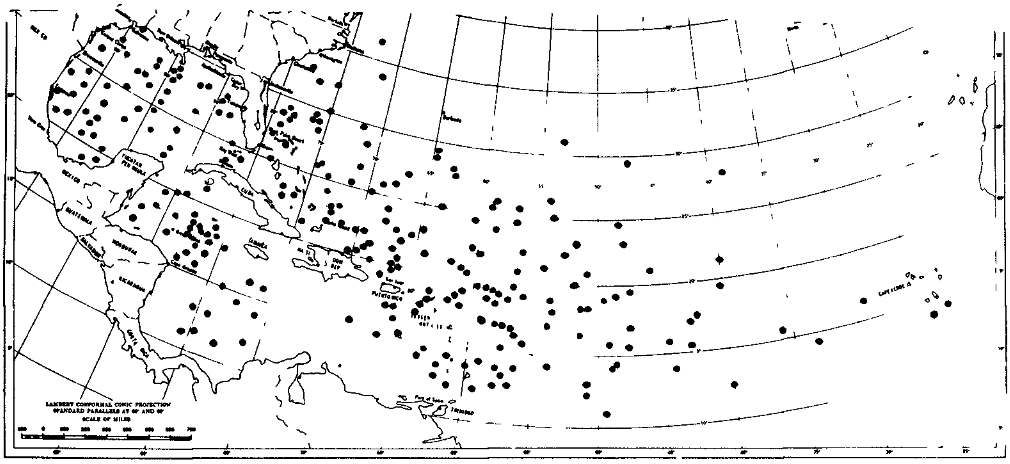

Fig. 1. Locations where Tropical Storms reached hurricane intensity, 1901-1957.

TABLE I

1. July 1896
2. Sept. 28, 1896
3. Oct. 8, 1896
4. Aug. 2, 1898
5. Aug. 31, 1898
6. Oct. 2, 1898
7. Aug. 1, 1899
8. Aug. 17-18, 1899
9. Oct. 30, 1899
10. Sept. 8, 1900
11. July 10-11, 1901
12. Aug. 14, 1901
13. Sept. 11, 1903
14. Sept. 14, 1904
15. Oct. 17, 1904
16. June 17, 1906
17. Sept. 17, 1906
18. Sept. 27, 1906
19. Oct. 18, 1906
20. July 30-31, 1908
21. Aug. 31, 1908
22. July 21, 1909
23. Sept. 20, 1909
24. Oct. 11, 1909
25. Oct. 17, 1910
26. Aug. 11, 1911
27. Aug. 28, 1911
28. Sept. 13, 1912
29. Sept. 3, 1913
30. Aug. 16, 1915
31. Sept. 4, 1915
32. Sept. 29, 1915
33. July 5, 1916
34. Aug. 18, 1916
35. Oct. 18, 1916
36. Sept. 28, 1917
37. Aug. 6, 1918
38. Sept. 14, 1919
39. Sept. 21, 1920
40. June 22, 1921
41. Oct. 25, 1921
42. Sept. 15, 1924
43. Oct. 20, 1924
44. Nov. 30, 1925
45. Dec. 2, 1925
46. July 28, 1926
47. Aug. 25, 1926
48. Sept. 18, 1926
49. Sept. 20, 1926
50. Aug. 7-8, 1928
51. Sept. 16, 1928
52. June 28, 1929
53. Sept. 28, 1929
54. Sept. 30, 1929
55. Aug. 13, 1932
56. Sept. 1, 1932
57. July 30-31, 1933
58. Aug. 4, 1933
59. Aug. 23, 1933
60. Sept. 4, 1933
61. Sept. 5, 1933
62. Sept. 16, 1933
63. June 16, 1934
64. Sept. 2, 1935
65. Nov. 4, 1935
66. July 31, 1936
67. Aug. 7, 1940
68. Aug. 11, 1940
69. Sept. 23, 1941
70. Oct. 6, 1941
71. Oct. 7, 1941
72. Aug. 30, 1942
73. July 27, 1943
74. Aug. 1, 1944
75. Oct. 19, 1944
76. Aug. 27, 1945
77. Sept. 15, 1945
78. Sept. 17, 1945
79. Sept. 17, 1947
80. Sept. 19, 1947
81. Oct. 11, 1947
82. Oct. 15, 1947
83. Sept. 21, 1948
84. Oct. 5, 1948
85. Aug. 26, 1949
86. Oct. 4, 1949
87. Sept. 5, 1950
88. Oct. 18, 1950
89. Aug. 30, 1952
90. Aug. 13, 1953
91. Sept. 26, 1953
92. Oct. 15, 1954
93. Aug. 12, 1955
94. Aug. 17, 1955
95. Sept. 19, 1955
96. Sept. 23-24, 1956
97. June 27, 1957

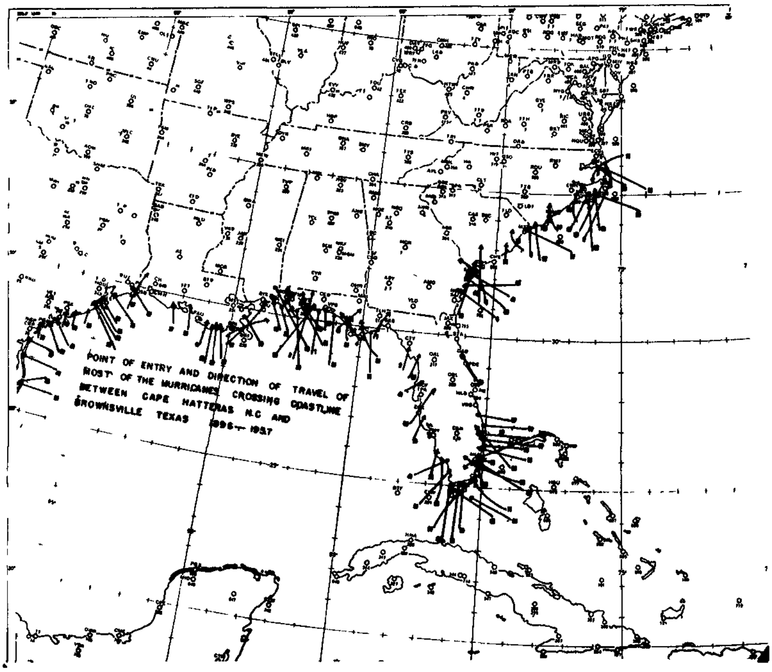

Fig. 2. Point of Entry and Direction of Travel of most of the hurricanes crossing the coastline between Cape Hatteras, N. C., and Brownsville, Tex., 1896-1957 (Number at beginning of arrow refers to number of storm in Table I.)

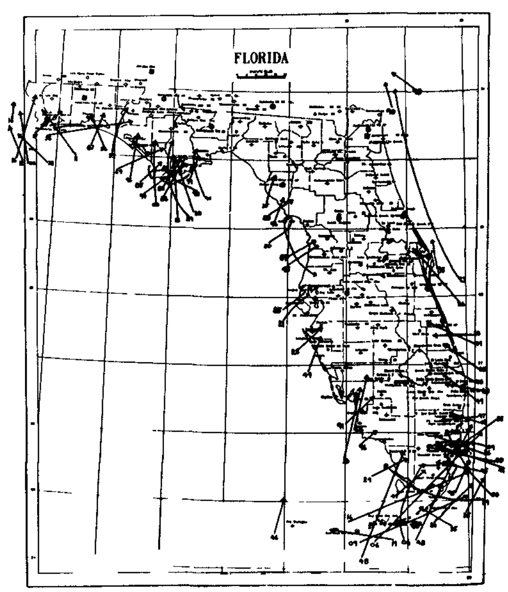

Fig. 3. Point of Entry and direction of travel of all Tropical Cyclones giving hurricane winds in Florida, 1885-1957 (Number at beginning of arrow indicates year of storm).

SUSCEPTIBLE COASTAL AREAS

The points of entry and the direction of travel of each hurricane which has crossed the U. S. coastline from Cape Hatteras, N.C., to Brownsville, Tex., from 1896 to 1957 are shown on Fig. 2. The dates of these storms can be found in Table I. All sections from Palm Beach, Fla., southward and along the entire Gulf coast are subject to hurricane visitation from 1 in every 7 years to 1 every 20 years or more on the average. The remainder of the South Atlantic coast is visited less frequently. Hurricanes are comparatively rare north of Cape Hatteras and especially so from north of the Virginia Capes to New York City. However, New England is occasionally subject to major hurricanes and was frequently struck by these storms between 1938 and 1955.

The points of entry and the direction of travel of all Florida hurricanes from 1885 through 1957 are shown in Fig. 3. The sections with highest frequency are the extreme southern portion of the Florida peninsula and the panhandle section on the Gulf coast. The hurricane frequency is very low on the northeast Florida coast. The reason for the low frequency is that the coastline is parallel to the normal storm track and if the storm recurves to the extent that it misses the southeast coast, it will also miss the northeast coast. This section is more susceptible to the fall and winter northeasters. The apparent low frequency on the Gulf coast between Cedar Keys and St. Marks is not believed real. This area is very sparsely settled and the exact point where many of the centers actually reached the coastline is not known, and there has been a tendency to place the centers too close to the regular Weather stations with the lowest pressure.

Of the 74 Florida hurricanes occurring during the past 75 years, 31 are known to have been attended by damaging tides. However, many of the centers made landfall in relatively uninhabited areas and the exact storm tide is unknown. It is estimated that a 6' storm tide occurs somewhere along the Florida coast on the average at least once every two years and probably more often.

LIFE SPAN OF HURRICANES

The average life span of a hurricane is about 9 days. August storms normally last the longest or about 12 days. The factors which determine the lifetime of a hurricane are the time and place of origin and the general circulation features existing in the atmosphere at the time of occurrence. Very few hurricanes dissipate while they remain over tropical or sub-tropical waters unless some abnormal feature of the wind flow pattern surrounding the storms acts to bring cold or dry air into the hurricane circulation.

Obviously those storms that develop in the Cape Verde region in August and September, when the semi-permanent Azores-Bermuda HIGH is at its greatest intensity, will have the longest life spans, since they normally travel westward for several thousand miles before re-curving northward and eventually northeast or eastward around the western and northern sides of the HIGH. One hurricane has been tracked for over a month. This year (1957) hurricane Carrie was picked up, already a hurricane of great intensity, on September 6. In the wave stage it can be tracked back to near the African coast on the 2nd, Fig. 4. It was still of hurricane intensity on September 22nd as it moved through the Azores. It finally became extra-tropical and eventually moved across the British Isles.

AVERAGE DAMAGE AND FATALITIES

In this century in the United States alone, at least 12,322 persons have been killed by hurricanes, or an average of over 200 per year. During this same period hurricanes have also caused at least 3 billion dollars of damage in this country, or over fifty million dollars per year. It is estimated that over 90% of all fatalities were from drowning and about 75% of all damage was from hurricane induced sea action or floods. The rapid continuing growth of population and construction along vulnerable coastal areas is increasing potential casualties and property losses from tropical storms. If occasional catastrophic property losses are to be avoided, better coastal zoning and scientific coastal engineering are necessary.

THE WIND FIELD

The mean wind field for the lowest 1000 meters around the center has been calculated by Miller(1) for a large number of observations in some twenty hurricanes. The wave heights (over the open oceans) depen upon the wind velocity, the length of time the wind operates upon the wave, and the fetch or distance over which the wind has blown in a rel tively straight path. It can be seen from Fig. 5 that the highest win occur in the right semi-circle, and also that the winds operate upon t waves there for the greatest length of time in the direction in which the storm is advancing. Thus the largest waves and swells are generat in the right semi-circle. These move faster than the storm and may mo many hundreds of miles out ahead of the center and eventually reach th coastline. The direction from which these swells approach the coast i determined by the storm's direction of motion and its bearing from the place of observation at the time the swells were generated. Lines or zones of convergence can also be seen in this composite picture, which in individual hurricanes may form or dissipate or rotate a considerabl distance around the center within a few hours. Although there is some difference of opinion among storm surge specialists, it is not believe these convergence lines have any significant effect on storm tides.

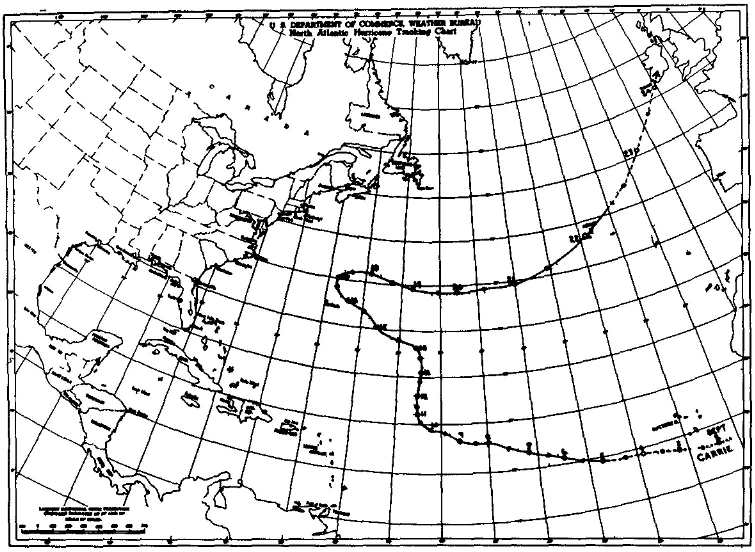

Fig. 4. Track of Hurricane Carrie, 1957.

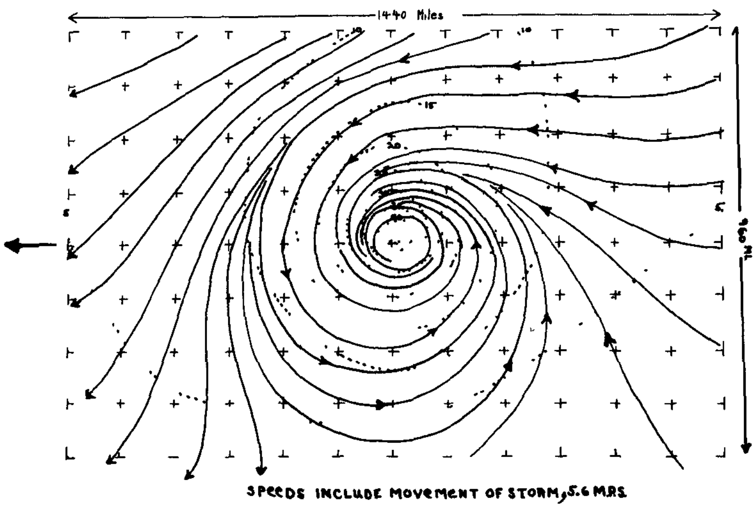

Fig. 5. Miller's Mean Wind Field 0-1 km Layer Movement of Storm.

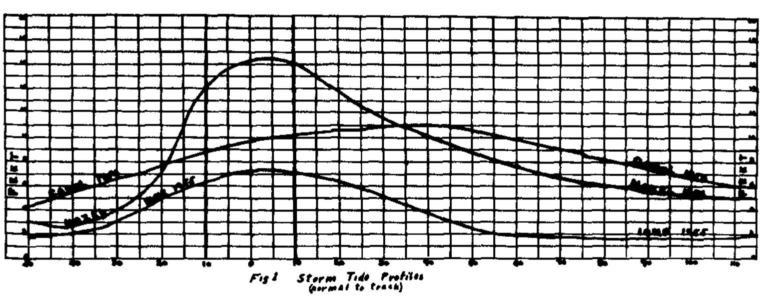

Fig. 6. Profile of Carol, Hazel, Ione.

At the center of the storm's circulation is the 'eye' of the hurricane. Formerly the eye was defined as the central portion of th storm where the winds were light and variable and the skies partly cloudy with no precipitation. In the classical hurricane, a cumulonimbus type cloud, or 'wall' cloud, extending to 30 to 40,000 feet o more tightly encloses this relatively calm area and within five miles the wind may increase from 15 to as much as 125 mph depending upon th intensity of the storm. However, from radar it is evident that the diameter of the precipitation eye is often much larger than the wind eye. The precipitation eye is occasionally 40 to 60 miles in diamete while at the same time the wind eye may be only 15 miles across. The complicated relationships between the size of the eye and the maturit and intensity of the storm are beyond the scope of this paper except to say that extremely high tides are rare in hurricanes with large wind eyes; i.e., wind eyes with diameters in excess of 30 miles.

ENERGY CONSIDERATIONS

A tremendous amount of energy is released in a hurricane through the process of condensation which has been estimated by some meteorol gists as the equivalent of several hundred atomic bombs per minute. About 15 to 20% of this energy is needed to drive the wind circulatio of the storm(2). A large portion is necessary to maintain convection in the hurricane, where the atmosphere is very close to the moist adiabatic. Only about 2% is used to overcome the effects of surface friction(3). While the hurricane is over water, waves and swells are formed by the frictional action of the winds on the surface of the water. These result in a dispersal of energy from the storm in all directions.

Energy both in the form of sea action and maximum winds concentrates the hurricane's destructive forces along the immediate coastline. Friction and often other factors tend to increase the atmospheric pressure at the center of the storm diminishing the pressure gradient and consequently the maximum winds near the center as the en tire storm circulation moves over land. The energy which the sea receives from the wind is dispersed radially from the storm. Part of the energy directed toward the coast is used to raise the water level over the continental shelf before the main wind system of the storm arrives at the shore. The energy arriving at the coast becomes progressively more concentrated until it reaches a maximum in the form o wind, storm tide and storm waves with the arrival of the storm's central area.

The rise in the ocean level induced by meteorological condition should not, strictly speaking, be called a 'tide' since that term implies a periodic rise and fall of the level of the oceans. Since i seems likely the term 'storm tide' and 'hurricane tide' will continu in popular usage within the foreseeable future they will be used in this paper interchangeably with the more technically correct 'storm surge'. Definitions of the storm surge and the storm tide and discussion of the equations of motion governing storm surge generation have been discussed by Harris(4).

The storm tide, or meteorological tide, resulting from hurricanes can often be described as having two stages. The first is the 'forerunner' which is a slow rise due to the shoreward transport of water by shoaling swells and waves irrespective of local wind direction. The rate of this rise in sea level varies as the concentration of energy radiated from the storm through the water. The second is the 'surge' which is usually a more rapid rise caused by direct transport of water by hurricane winds and sometimes believed to be intensified by a gravity wave possibly produced by a shoaling of the 'inverted barometer' wave. Dr. I. M. Cline, Meteorologist in Charge of the Weather Bureau Office at Galveston at the time, reported a rise of 4 feet in as many seconds at about the time of lowest pressure during the famous Galveston hurricane of 1900 and there have been similar observations in other hurricanes. The rate of the storm tide rise near and a short distance to the right of the center of hurricane Audrey, 1957, was about 1.5 feet per hour along the immediate Gulf coast for the 4 or 5 hours preceding the arrival of the center but there was no authentic evidence of a bore or very rapid rise there.

Several outstanding storm tides, all in connection with hurricanes, have occurred along the Atlantic and Gulf coasts in this century, namely: Galveston 1900 and again in 1915; Tampa Bay in 1921; Miami 1926; Palm Beach and Lake Okeechobee 1928 and again in 1949; the Florida Keys in the Labor Day storm 1935; New England, particularly Narragansett Bay 1938 and again in 1954; Hazel, south of Wilmington, N.C., 1954, and Audrey, Cameron, La., 1957. This list does not include all the outstanding storms with high tides since 1900. The maximum reported tides of all these storms averages 12.5 feet above mean low water.

Of the 24 best documented storm tides along the coast of the Gulf of Mexico, the maximum storm tide heights averaged 10.3 feet with a range between 5 and 15 feet. The average maximum reported height of 14 fairly well documented storm tides of the Atlantic coast was 9.7 feet with a range between 3 and 15.5 feet. This group does not include some entering the Florida peninsula where the average height of 15 major storm tides between 1900 and 1955 was 9.8 feet, MSL. The number of documented storm tides is not great enough to attach much significance to the differences between the averages for the various sections given above but because of the predominately shallow coastal waters of the Gulf of Mexico and the concavity of the coastline, a higher average might be expected there. Very high storm tides will occur at the heads of bays and estuaries, particularly when the storm center moves inland on a course at an angle of 90° or less to the coast line (right quadrant).

In Fig. 6, three storm tide graphs with height plotted as a function of distance normal to the track of the storm center clearly shows the maximum tide occurring at or immediately to the right of the center. This slopes rapidly down to about the level of the pre-storm tide height or the height of the 'forerunner' and then very slowly diminishes with distance along the coast. It is obvious that a forecast of storm tide levels must be based on an accurate forecast of the point of entry of the storm center, which, unfortunately, is not always possible.

The present methods of forecasting the hurricane tide are largely empirical, and perhaps the one by Conner, Kraft and Harris(5) is the most widely used. The basic tide producing capacity of the storm is assumed to be indexed by its minimum central pressure. Other modifyin factors such as (1) slope of the continental shelf; (2) shape of the coast (concave or convex); (3) coastal topography and presence of bays estuaries, etc., which tend to accentuate convergence or divergence of ocean currents, must be evaluated qualitatively.

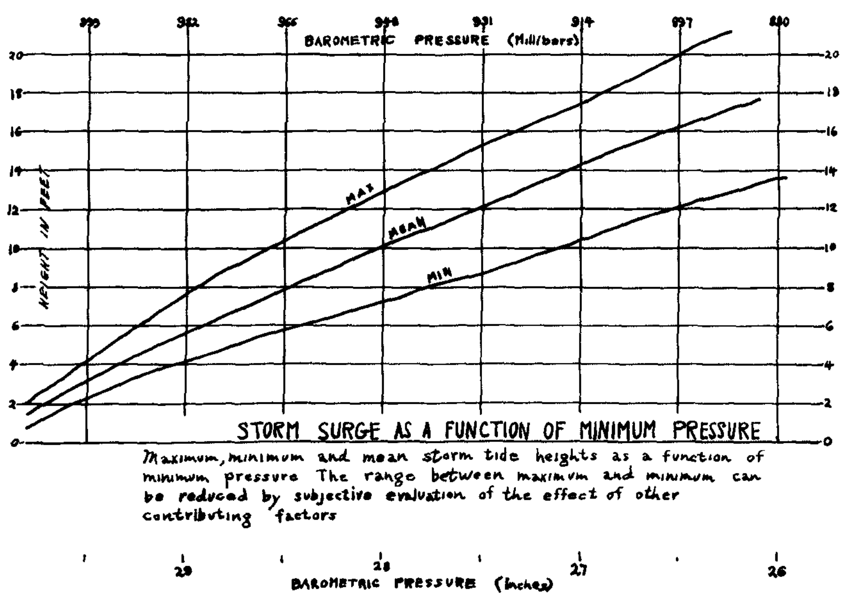

Fig. 7. Storm Surge as a Function of Minimum Pressure (After McGehee).

In Fig. 7, tide heights are plotted as a function of lowest pressures observed within a group of Florida and South Atlantic storms. This results in a graph with considerable scatter. However, two lines can be drawn, one representing the maximum and the other the minimum tide heights produced by storms with the same central pressure. A tide height is forecast which is a value between the maximum and minimum as determined from a subjective evaluation of the modifying factors described in the preceding paragraph. The central pressure of a hurricane is usually, but not always, known. Probably hurricane Audrey of this year was intensifying rapidly as she reached the Louisiana coast and her minimum pressure was not available to the forecaster. It is well known there are other important factors which contribute to the total storm height. Mention of these is omitted since at the present time there is no known method of evaluating their contribution.

A scientific analysis of a hurricane tide presents manifold difficulties. Construction of a laboratory model would present several difficult if not insoluble problems. The moving short radius of curvature with proportionate pressure distribution of the hurricane wind field probably defies duplication. And, in nature, the quantitative contribution of the numerous factors determining the total storm tide have proven impossible to evaluate separately up to this time.

In conclusion, I would like to acknowledge the very considerable contribution of Mr. William McGehee, storm surge forecaster at the Miami Hurricane Forecast Center, in the preparation of this paper.

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CHAPTER 3

REVISIONS IN WAVE FORECASTING: DEEP AND SHALLOW WATER

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ABSTRACT

During the past six years since the latest revisions in wave forecasting (Bretschneider 1951) were made, much information has become available such that another revision is in order. An abundance of published (and unpublished) accounts of wave generation and decay in both deep and shallow water from various sources, as well as new ideas in the art of wave forecasting, are used in this revision. Deep water wave forecasting relationships, relationships for the generation of wind waves in shallow water of constant depth, and techniques for forecasting wind waves over the Continental Shelf are included in this paper. Forecasting hurricane waves is also discussed, from the engineering design point of view. The concept of significant wave is still retained as the most practical method in wave forecasting to date. The significant period has definite significance in that the wave energy is propagated forward at a speed approximately equal to the corresponding group velocity.

The graphical approach (Wilson 1955) for moving fetches and variable wind vectors is discussed, and is the best approach for forecasting waves. Without Wilson's graphical technique it is difficult for any two forecasters supplied with the same meteorological data to obtain the same degree of verification, or determine whether the forecaster or the forecasting relationships are in error. It is quite possible that by use of this technique further revisions in wave forecasting are possible.

The problem of wave variability is discussed, and the distribution functions are given. A short summary of the wave spectra (Bretschneider 1958) used in connection with the revisions is also given.

When the present forecasting relationships are applied to sections of the world, other than that from which the basic data were procured, it is recommended that atmospheric stability factors be taken into account. This essentially involves a slight modification or calibration of the forecasting relationships and techniques, prior to general use in the area of interest.

INTRODUCTION

Ordinary gravity waves have been classified by Munk (1951), and are ocean waves having periods between 1.0 and 30 seconds. This range of periods is included in the spectrum of ocean waves, which at the lower limit are capillary waves having a period less than 0.1

second and the upper range transtidal waves having a period of 24 hours and greater. Waves having a period between 0.1 and 1.0 second are called ultragravity waves. Hence, gravity waves in general include both ordinary gravity and ultragravity, although the latter are affected by surface tension. When discussing ocean waves in this paper, the meaning is intended to be these gravity waves.

Deep water waves are defined as gravity waves unaffected by the depth of water. For all practical purposes deep water waves have wave lengths equal to or less than twice the water depth. All other waves are considered as waves in shallow or intermediate water depths.

When forecasting waves use is made of the term significant wave. The significant wave height is the average of the heights of the highest one-third waves in a wave train or wave record. The mean period of the significant wave is termed the significant period. Sometimes the significant wave is defined for the convenience of wave record analysis according to Wiegel (1953): A length of wave record is selected, usually of 20 minutes duration; the high groups of waves are selected to determine a mean period, called the significant period; the length of record in seconds is divided by the significant period to obtain a wave number; one-third of this number is the number of waves to consider in determining the significant height, beginning with the highest wave. Both of the above definitions give almost the same results, the latter being a time saver in the analysis of wave records.

Wave forecasting may be classified into three general groups, (a) ordinary deep water wind waves and swell; (b) wind waves and swell in shallow water; and (c) hurricane wind waves and swell, deep or shallow water. The above cases are discussed in this report, although none are completely understood at present.

By ordinary wind waves is meant waves generated by stationary or slowly moving storms, having more or less constant wind speed and direction. More knowledge is available on ordinary deep water wind waves than on any of the other above two topics. Wind waves in shallow water of constant depth is fairly well established, at least semiempirically. Hurricane wind waves in deep water are least understood, chiefly because of the lack of data, both on winds within hurricanes as well as the complex nature of the sea. The transformation of sea into swell and the decay of swell is partly understood, at least physically, although suitable wave theory and data are lacking for an accurate description of this phenomenon.

FORECASTING DEEP WATER WAVES

Little literature was available prior to World War II on wave generation and decay. During the war advance knowledge of wave activity was required for areas where amphibious landings were planned. This problem was first attacked by Sverdrup and Munk (1947), who combined classical wave theory with available data to obtain semiempirical or "semitheoretical" wave forecasting relationships.

This was the first great advance in the art of wave forecasting, and i known as the significant wave method of forecasting, sometimes referre to as the Sverdrup-Munk-Bretschneider or SMB method. Actually the B in the above method deserves little credit at the most, since it becam attached through a simple revision (Bretschneider 1952) of the origina work by Sverdrup and Munk (1947), which already had experienced its first revision by Arthur (1947). These revisions were based on additional wave data not available at the time of the original work of Sverdrup and Munk.

At present two main schools of thought exist for deep water wave forecasting, (a) the significant wave method, mentioned above, and discussed in more detail in the present paper, and (b) the wave spectr method, discussed by Pierson, Neumann and James (1955), referred to as PNJ method. Both the SMB and the PNJ methods have certain advantages and certain disadvantages, since neither method has realized the perfection desired in the art of wave forecasting. The desired perfection of either the SMB or PNJ method might be attained in the near future by use of the graphical approach to wave forecasting as given by Wilson (1955), supplemented with additional wave data for calibration purposes.

Two other methods of forecasting waves may also be mentioned, (a) the method of Darbyshire (1955), and (b) the method of Suthons (1945), both European methods. The Suthons method is the least familiar but similar in techniques and principle to the SMB method. Darbyshire method is based on the development of a wave spectrum, quite different than the Neumann (1953) wave spectrum. An important consideration is that each of the four methods is based essentially on wave data, and hence each must give forecasts as accurate as the data from which the particular method was derived. It is foolish for PNJ to evaluate the Darbyshire method using PNJ data, just as it is for the present author to evaluate the PNJ method using SMB data, and vice versa. That is to say each method has inherent characteristics associated with the procurement of data. A very objective verification study of the above four methods was made by Roll (1957), and the general conclusion was that each method works better for the particular region from which the principal data were obtained. Perhaps, even better verification might have been obtained provided the individual contributors made the forecast, each using their own methods and techniques.

GENERATION OF DEEP WATER WAVES BY WIND OF CONSTANT SPEED AND DIRECTION

The present revisions are based on the SMB technique. As shown by Johnson (1950) and perhaps others, one may arrive at the generating parameters by use of the PI-theorm (Buckingham 1914) and dimensional analysis. These parameters are:

$$\frac{gH}{U^2} = f_1 \left[\frac{gH}{U^2} \quad , \quad \frac{gt}{U} \right] \tag{1}$$

$$\frac{C_o}{U} = \frac{gT}{2\pi U} = f_2 \left[\frac{gF}{U^2} \; , \; \frac{gt}{U} \right], \quad \text{where} \tag{2}$$

$H = H_{1/3}$ = significant wave height, feet
C_o = wave speed in deep water, feet/second
$T = T_{1/3}$ = wave period, seconds
g = acceleration of gravity, 32.2 feet/second^2
U = wind speed in feet per second
t = duration of wind in seconds
F = fetch length in feet

Equations 1 and 2 are for constant wind speed and direction.

Figure 1-A is a revision of the Fetch Graph of Bretschneider (1952), originally revised from Sverdrup and Munk (1947). The upper limits of the generating parameters, corresponding to a fully developed sea are obtained by use of the wave spectrum proposed by Bretschneider (1958), a short abstract of which is given at the end of this paper. These upper limits are reached when

$$\frac{gT_{1/3}}{2\pi U} = \frac{g\bar{T}}{2\pi U} = 1.95 \tag{3}$$

$$\frac{gH_{1/3}}{U^2} = .282 \tag{4}$$

$$\frac{gF}{U^2} \approx 6 \times 10^5 \quad \text{where} \tag{5}$$

$T_{1/3}$ = significant wave period, $\bar{T}$ = mean wave period, and

$H_{1/3}$ = significant wave height.

The original curves of Sverdrup and Munk (1947) places the upper limit corresponding to $gF/U^2 = 10^5$, those revised by Bretschneider (1952) at $gF/U^2 = 4 \times 10^4$, and those utilized by Pierson, Neumann and James (1955) at $gF/U^2 = 10^5$. It can be seen that all previous investigations underpredict the parameters for a fully developed sea. A fully developed sea for moderate to large wind speeds can never be experienced on this world. However, about 90 percent of the fully developed sea is reached at $gF/U^2 = 10^5$, but the last 10 percent of the generation takesplace over a very much longer fetch length. For low to moderate wind speeds, storms moving across the Pacific Ocean

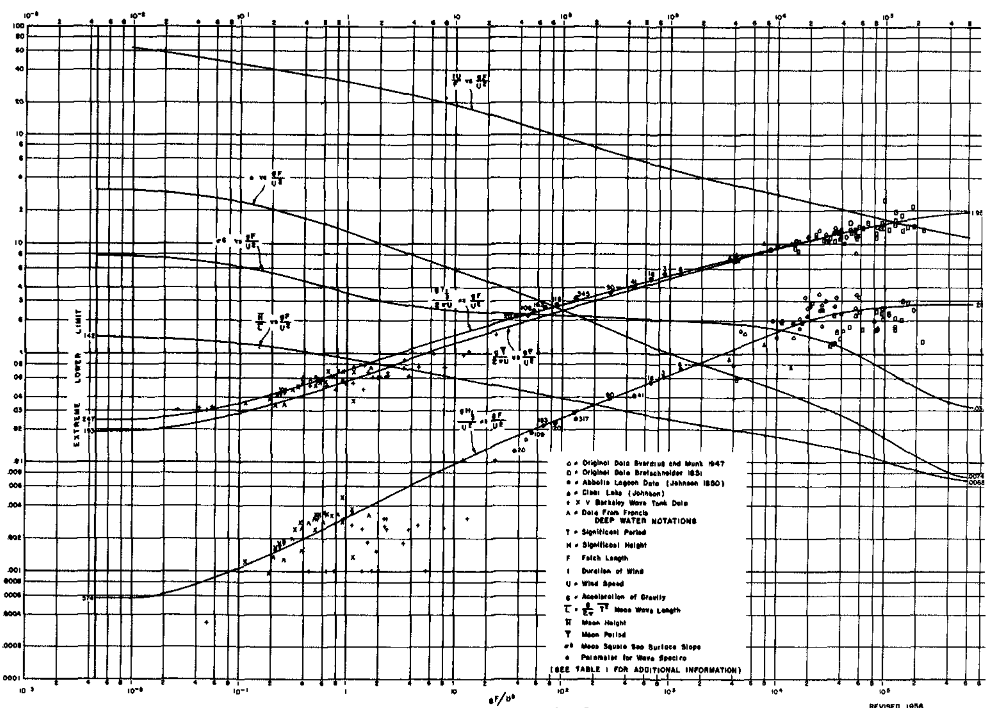

Fig. 1A. Fetch graph for deep water.

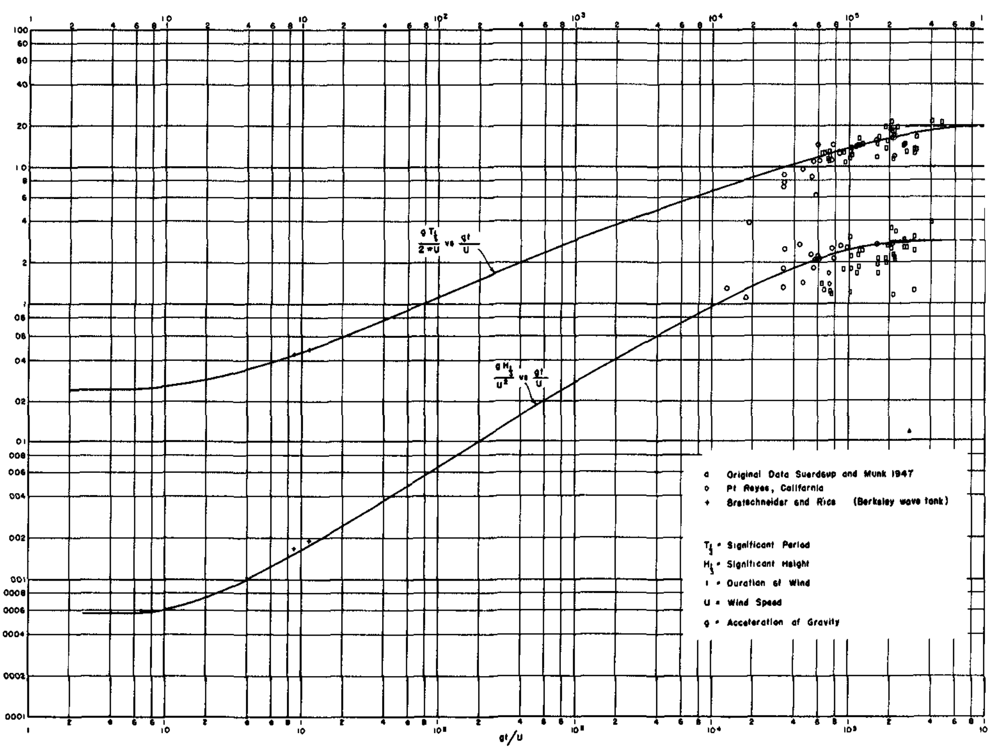

Fig. 1B. Duration graph.

might generate a fully developed sea.

The lower limits of the fetch graph are obtained from Bretschneider (1958):

$$\frac{g\bar{T}}{2\pi U} = .0193 \qquad (6)$$

$$\frac{gT_{1/3}}{2\pi U} = .0244$$

$$\bar{H}/\bar{L} = \frac{1}{7}$$

$$\bar{L} = \frac{g\overline{T^2}}{2\pi}$$

$$\overline{T^2} = 1.078(\bar{T})^2 \qquad (7)$$

$$\frac{g\bar{H}}{U^2} = .000357$$

$$\frac{gH_{1/3}}{U^2} = .000572$$

$$\frac{gF}{U^2} = .0046$$

Using the upper and lower limits given above, and the available wave data, Figure 1-A, the Fetch Graph, was constructed. The curve of $g\bar{T}/2\pi U$ is a first approximation for mean wave period, based on meager data and the asymtopic limits given above. This curve may need a slight revision when more data become available.

The Duration Graph, Figure 1-B, can be constructed from the Fetch Graph and the considerations following. The duration of time required for generation depends on the fetch distance traveled and the group velocity appropriate to the most energetic waves. It is shown by Bretschneider (1958) that the band of waves having a period very nearly equal to the significant period are the most energetic. This is as should be expected. For very low values of gF/U^2 the significant period is equal to the period corresponding to the band of waves having maximum energy, and for a fully developed sea the significant period is 1.027 times the period corresponding to the band of waves having maximum energy. Thus it must be emphasized, that the signific period has definite significance, the significance being that it represents very closely the period appropriate to the band of waves having maximum energy, and hence may be used to determine the duratio time required for wave generation.

The general form of $F = C_g t$ (fetch distance is equal to group velocity times time) can be applied in differential form $dt = \frac{1}{C_g} dF$, where C_g, the group velocity is a variable and increases with time an distance. In parametric form the expression becomes:

$$\frac{gt}{U} = \int \frac{U}{C_g}\, d\,(gF/U^2), \text{ where} \qquad (8$$

$$\frac{C_g}{U} = \frac{1}{2}\left[\frac{gT_{1/3}}{2\,U}\right]\left[\frac{T_{op}}{T_{1/3}}\right], \text{ where } T_{op} \qquad (9$$

corresponds to the optimum period around which is concentrated maximun energy. $T_{op}/T_{1/3}$ varies from 1.0 at $gF/U^2 \rightarrow 0$ to 1.027 at $gF/U^2 = 6 \times 10^5$, with maximum of 1.0375 at $gF/U^2 \cong 200$.

Numerical means and Figure 1-A were used to establish the Duratic Graph, Figure 1-B. Figures 1-A and 1-B were used to construct the forecasting curves presented in Figure 2. Table I summarizes the fina selected values of generation parameters. The parameters α, σ^2 and ϵ a discussed later in the paper with respect to the wave spectra.

Short Fetches and High Wind Speeds. For short fetches and high wind speeds, it was shown by Bretschneider (1957) one might use the following formulae as first approximations:

$$H = .0555\sqrt{U^2F} \qquad (10$$

$$T = .50\sqrt[4]{U^2F}\text{ , and} \qquad (11$$

$$\frac{F_{min}}{t_{min}} = .57\sqrt[4]{U^2F} = 1.14\,T, \qquad (12$$

These equations will result in slightly different values of H and T than those obtained from Figure 2, and should therefore be used with caution. However, these equations become quite useful when discussing hurricane waves later in this paper. In the above equations:

TABLE I

SUMMARY OF DEEP WATER WAVE GENERATION PARAMETERS

$\frac{gF}{U^2}$	$\frac{gt}{U}$	$\frac{tU}{F}$	$\frac{gH_{1/3}}{U^2}$	$\frac{gT_{1/3}}{2\pi U}$	r	$\frac{T_{1/3}}{\bar{T}}$	$\frac{g\bar{H}}{U^2}$	$\frac{g\bar{T}}{2\pi U}$	$\frac{\bar{H}}{\bar{L}}$	$\frac{\bar{H}}{L}$	α	σ^2	ϵ	$\frac{H_{1/3}}{T_{1/3}^2}$
.01	.63	63	.000574	.0247	.998	1.2645	.000359	.0195	.1394	.1397	3.07	.766	.464	.767
.02	1.14	57	.000611	.0258	.995	1.2637	.000382	.0204	.1355	.1361	2.90	.728	.465	.748
.04	2.06	51.5	.000738	.0288	.992	1.2629	.000461	.0228	.1309	.1316	2.71	.685	.466	.725
.06	2.92	48.6	.000867	.0316	.985	1.2613	.000542	.0251	.1270	.1282	2.55	.650	.467	.708
.08	3.7	46.3	.000957	.0334	.981	1.2602	.000598	.0265	.1257	.1272	2.50	.640	.468	.699
.10	4.5	45.0	.00105	.0353	.979	1.2598	.000656	.0280	.1235	.1251	2.41	.620	.474	.687
.20	8.0	40.2	.00143	.0425	.965	1.2566	.000894	.0338	.1155	.1179	2.11	.553	.479	.645
.40	14.1	35.5	.00195	.0521	.949	1.2526	.001219	.0416	.1040	.1071	1.71	.460	.487	.585
.60	20.3	33.9	.00235	.0591	.936	1.2497	.001469	.0473	.0969	.1005	1.48	.408	.494	.548
.80	25.7	32.1	.00269	.0646	.927	1.2474	.001681	.0518	.0925	.0965	1.35	.377	.497	.525
1.0	31	31.0	.00301	.0695	.916	1.2450	.001881	.0558	.0885	.0928	1.24	.352	.510	.508
2.0	54	27.0	.00430	.0869	.878	1.2357	.002688	.0703	.0803	.0860	1.02	.311	.547	.464
4.0	94	23.5	.00610	.108	.827	1.2231	.003813	.0883	.0722	.0794	.824	.274	.572	.426
6.0	129	21.5	.00743	.124	.790	1.2141	.004644	.102	.0659	.0739	.686	.260	.596	.393
8.0	160	20.0	.00855	.137	.762	1.2071	.005344	.113	.0618	.0703	.603	.249	.016	.371
10.0	192	19.2	.00951	.147	.740	1.2017	.005944	.122	.0590	.0678	.550	.242	.632	.359
20.0	306	15.3	.0129	.179	.671	1.1845	.008063	.151	.0522	.0621	.431	.232	.677	.328
40.0	488	12.2	.0175	.215	.590	1.1636	.01094	.185	.0472	.0583	.352	.228	.713	.309
60.0	654	10.9	.0208	.240	.546	1.1524	.01300	.208	.0444	.0559	.311	.221	.736	.294
80.0	792	9.9	.0232	.261	.513	1.1437	.01450	.228	.0412	.0527	.268	.219	.750	.278
100	920	9.2	.0255	.279	.486	1.1367	.01594	.245	.0392	.0507	.243	.218	.766	.267
200	1520	7.60	.0337	.337	.400	1.1355	.02106	.297	.0352	.0469	.196	.212	.792	.242
400	2440	6.10	.0441	.403	.322	1.0918	.02756	.369	.0300	.0416	.142	.205	.819	.221
600	3300	5.50	.0522	.453	.275	1.0794	.03263	.420	.0273	.0386	.118	.202	.832	.208
800	4056	5.07	.0583	.486	.243	1.0705	.03644	.454	.0261	.0374	.108	.199	.840	.201
1000	4800	4.80	.0641	.519	.220	1.0640	.04006	.488	.0248	.0359	.0972	.198	.846	.194
2000	8000	4.0	.0841	.618	.160	1.0469	.05256	.590	.0223	.0330	.0786	.194	.863	.179
4000	13,800	3.45	.1110	.735	.102	1.03005	.06938	.714	.0201	.0294	.0638	.186	.875	.167
6000	18,960	3.16	.130	.816	.079	1.0232	.08125	.797	.0189	.0289	.0564	.178	.882	.159
8000	23,760	2.97	.145	.872	.062	1.0183	.09063	.856	.0183	.0281	.0529	.170	.885	.155
10,000	28,100	2.81	.157	.924	.052	1.0154	.09813	.910	.0175	.0270	.0484	.163	.888	.1550
20,000	48,200	2.41	.195	1.10	.027	1.0080	.1219	1.091	.0151	.0235	.0360	.142	.896	.131
40,000	82,000	2.05	.234	1.28	.010	1.003	.1463	1.276	.0133	.0208	.0279	.113	.901	.116
60,000	112,800	1.88	.253	1.39	.006	1.0018	.1581	1.388	.0121	.0190	.0231	.0962	.902	.107
80,000	140,000	1.75	.264	1.49	.002	1.0006	.1650	1.4891	.0110	.0173	.0191	.0815	.903	.0969
100,000	168,000	1.68	.270	1.54	.001	1.0	.1688	1.54	.0105	.0165	.0174	.074	.904	.0928
150,000	228,000	1.52	.277	1.67	0	1.0	.1731	1.67	.00916	.0143	.0133	.0583	.905	.0809
200,000	286,000	1.43	.279	1.74	0	1.0	.1744	1.74	.00850	.0134	.0114	.0500	.906	.0751
300,000	393,000	1.31	.281	1.84	0	1.0	.1756	1.84	.00766	.0119	.0093	.0410	.907	.0676
400,000	496,000	1.24	.282	1.90	0	1.0	.1763	1.90	.00721	.0113	.0082	.0370	.908	.0637
500,000	595,000	1.19	.282	1.93	0	1.0	.1763	1.93	.00699	.0110	.0077	.0345	.909	.0617
600,000	702,000	1.17	.282	1.95	0	1.0	.1763	1.95	.00685	.0107	.0074	.0337	.910	.0604

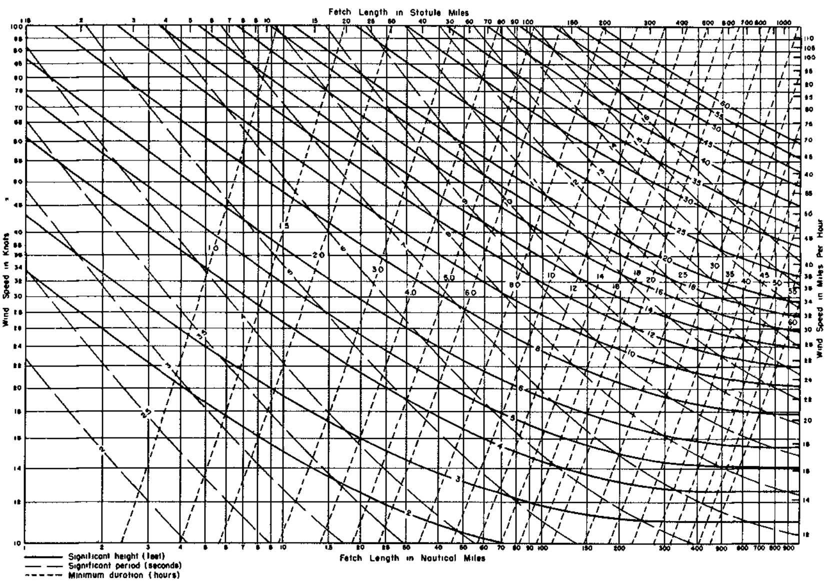

Fig. 2. Deep-water wave forecasting curves as a function of wind speed, fetch length, and wind duration.

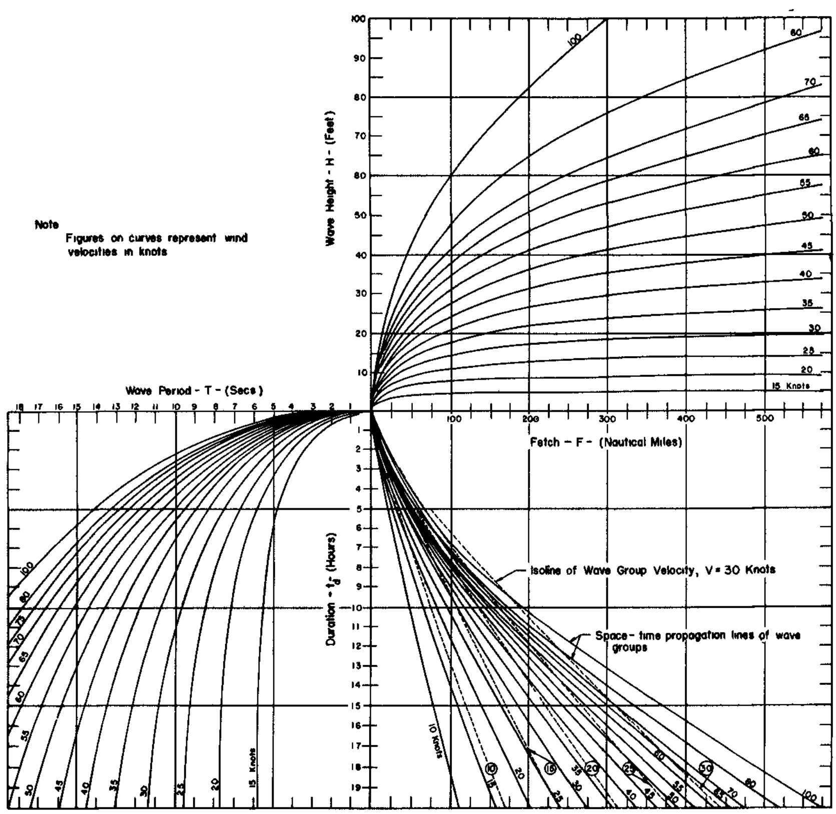

Fig. 3. Relationships for forecasting significant heights and periods of waves in deep water under wind of particular velocity, duration, and fetch.

H = significant height in feet
T = significant period in seconds
U = wind speed in knots
F = fetch length in nautical miles
F_{min} = minimum fetch length in nautical miles
t_{min} = minimum duration in hours

The value of F to use in equations 10 and 11 must be equal to or less than F_{min} obtained from equation 12.

GENERATION OF DEEP WATER WAVES BY VARIABLE WIND SPEED AND DIRECTION AND MOVING STORMS

If the wind field is not too irregular and the movement of the storm is fairly slow, then Figure 2 can be used to advantage. Methods of techniques used are given by Kaplan (1953). However, when the variables are ill-defined then the graphical method proposed by Wilson (1955) must be used, the method of which also applies for winds of constant speed and direction. Figure 3, wave forecasting curves used by Wilson (1955), is somewhat different than the revised curves of Figure 2. Whether Figure 3 or Figure 2 is more accurate under steady wind conditions is difficult to say. The important thing is that Figure 3 can be used readily for ill-defined storm situations whereas the use of Figure 2 becomes somewhat awkward, even with the additional techniques proposed by Kaplan (1953). Figure 4 is a typical profile of a variable wind field taken from Wilson (1955). Figure 5 is a typical example taken from Wilson (1955) in the application of the graphical technique. In explaining Figure 5 the following is quoted from Wilson (1955):

"Wind of Variable Velocity in a Variably Moving Wind System of Finite Fetch

"Dispensing now with the restriction of a uniform wind velocity, U, but retaining the concept of uniformity along closed contours of a space-time wind field, it becomes possible to represent a wind system that has both variable wind velocity and variable speed of forward (or rearward) progression by a wind-field of closed contour lines whose intervals apart represent equal increments of wind velocity U.

"Figure 5 shows such a wind-field with contours of wind velocity at 5-knot intervals from 20 to 40 knots. Superimposed thereon at an arbitrarily selected point 0 in space and time is the HtFt diagram with H(F), $F(t_d)$ and $T(t_d)$ curves drawn in for the same 5-knot intervals of wind velocity U from 20 to 40 knots.

"The problem now is one of determining the history of the height and period growth of the waves originating at the point O.

"In relation to the wind-field the origin O is seen to be at a point where the wind velocity would be of the order of 21 knots. Waves originating at O would be obliged to follow a space-time path somewhere along the belt of propagation lines forming the relationships $F_U(t_d)$. It is clear that the actual path of the waves must initially be along a line intermediate between the propagation lines for U = 20 and U = 25 knots as far as a, the intersection point with the 25-knot wind-field contour. Along the path Oa the waves would be under the influence of winds ranging from 21 to 25 knots so that, to all intents and purposes, Oa can be regarded as the propagation line for U = 23 knots.

"Over the same interval of time the growth in significant period of the waves will follow the line Ob (Figure 5), equivalent to the curve $T_U(t_d)$ for U = 23 knots.

"Having arrived at a, the waves pass into the next incremental wind zone over which wind velocity rises from 25 to 30 knots. Their further space-time path from a to e must be at a rate (or group velocity) appropriate to the average wind of U = 27½ knots, but the propagation rate must start off from a at the same slope as the line Oa has at a.

"To ensure that the group velocity shall remain the same at the transition, it becomes necessary to trace a line bc at constant period and locate a point c intermediate between $T_{25}(t_d)$ and $T_{30}(t_d)$. The condition of constant period ensures constant wave group velocity since group velocity is directly proportional to wave period under deep water conditions. By drawing the abscissa cd, the point d is found intermediate between the curves $F_{25}(t_d)$ and $F_{30}(t_d)$. An imaginary propagation line $F_{27\frac{1}{2}}(t_d)$, drawn through d would now have the same slope as the curve Oa at a. To find ae, therefore, it is only necessary to transcribe, as it were, a piece of the $F_{27\frac{1}{2}}(t_d)$ curve from d, parallel to itself, and add it to the curve Oa at a. By this means the point e is established.

"In the same sense, by transcribing a portion of the curve $T_{27\frac{1}{2}}(t_d)$ from c, parallel to itself, and adding it to the curve Ob at b, the point f can then be located (via ef), marking the further growth in period of the waves, bf, under the influence of the 25 to 30-knot wind.

"This procedure may be followed consistently to trace the actual space-time path of the waves, Oaekosw, through the wind-field and to give the history of the period growth of the waves, Obflptx. It will be noted

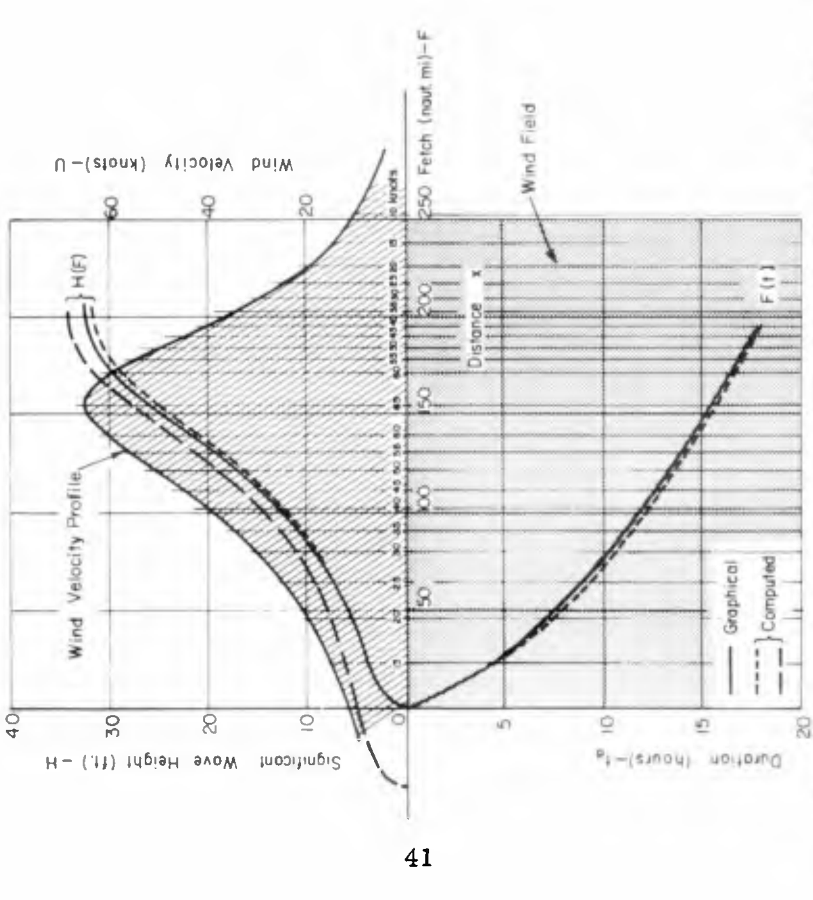

Fig. 4. Comparison of computed and graphically determined heights and travel distances of waves generated within a stationary hurricane.

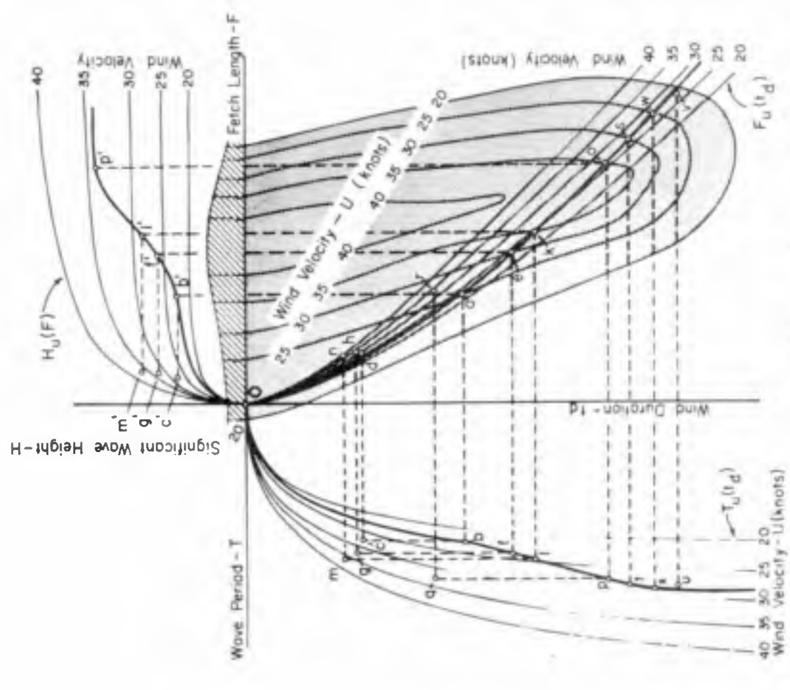

Fig. 5. Significant heights and periods of waves generated in a variably moving wind system of variable wind velocity.

that the same method applies in the zone of declining wind velocities as in the zone of increasing velocities. Thus the portion os of the wave propagation curve is drawn parallel to $F_{32\frac{1}{2}}(t_d)$ at r, the wave group velocity at O and r being different in the declining 35 to 30-knot wind zone from that at h and k in the increasing 30 to 35-knot wind zone.

"The graphical charting of the corresponding growth in significant height of the waves follows essentially the same procedure as described above. The curve Ob' follows the $H_{23}(F)$ isoline as far as b', which is the intersection point with the ordinate drawn through a. Further increase in height of the waves in the next incremental wind zone (U = 25 to 30 knots) must continue at a rate appropriate to $H_{27\frac{1}{2}}(F)$, starting, however, at the same height as at b'. Accordingly b'f' is drawn, parallel to $H_{27\frac{1}{2}}(F)$ at c', to give the intersection point f', with the ordinate drawn from e on the propagation line.

"The final curve of significant wave height follows the line Ob'f'l'p' and tapers off to a maximum value which is maintained to the end of the wind field. In the same way the curve of significant wave period, Obflptx, is found to taper off to a maximum value of wave period."

The above quote from Wilson (1955) should be sufficient to understand the practical applications of the graphical technique, but if more details are required the reader is referred to the paper by Wilson (1955). Experience and practice of course are necessary to perfect one's techniques in the above method.

DECAY OF WAVES IN DEEP WATER

When waves leave a generating area and travel through an area of calm or lighter winds a transformation takes place. The significant height decreases and the significant period shifts to the longer period waves, resulting in an increase in the mean period (significant period) of the significant height. To understand the meaning of the above statement one must refer to the spectrum of waves, and the original work of Barber and Ursell (1948). Although one forecasts the significant height and period, it must be remembered that the sea is actually made up of a spectrum of waves, with varying amplitudes and frequencies or periods. The work of Putz (1952) and that of Longuet-Higgins (1952) show the distribution of heights about the mean height, and Putz (1952) shows the distribution of periods about the mean period. The theoretical distribution of heights by Longuet-Higgins is known as the Rayleigh type distribution based on a narrow spectrum, and the agreement is surprisingly good with the distribution obtained by Putz (1952) based on the analysis of wave records. Thus the significant height can be related to the distribution of all heights in the spectrum, and the significant period can be related to the distribution of all periods in the spectrum, when the significant and mean wave periods are nearly equal. The joint relationship between the individual heights and periods has not yet been established satisfactorily. An unsuccessful attempt to determine the joint distribution was made by Bretschneider (1956), except for the special case of zero correlation. A revision of the work on the joint distribution is presently under way. This, together with the wave spectrum derived therefrom, should be valuable in future studies in regards to the decay of waves. Remarks on this matter are discussed later.

However, the physical behavior of the decay of waves in the form of either joint distribution or wave spectrum can be visualized accordingly: first, the heights of long period waves are reduced proportionately less than the shorter period waves for the same decay distance. Furthermore, the long period waves travel faster than the shorter period waves and hence will be dominating. With respect to decay distance the height will decrease and the period increase for the significant waves. Consider now a fixed decay distance with respect to time. First will be noticed the arrival of long period swell, and some time later the shorter period waves begin to arrive, which in effect causes a decrease in significant period with respect to time. In case of deep water waves the decrease in significant period with respect to time will continue, but the lowest value of this significant period can never be less than the original,(generated) significant period. Thus, with respect to distance, there will be a space time history of the joint distribution or the spectrum of waves, and a space time history of the significant waves. The work of Barber and Ursell (1948) shows this to be true. The same discussion applies to the mean height and the mean period, wherein certain low period waves are completely filtered out.

General expressions for the decay of waves can be written as follows:

$$\frac{H_D}{H_F} = f_1\left[\frac{D}{gT_D^{\,2}}, \quad \frac{gF}{U^2}, \quad \frac{F}{W}\right] \tag{13}$$

$$\frac{T_D}{T_F} = f_2\left[\frac{D}{gT_D^{\,2}}, \quad \frac{gF}{U^2}, \quad \frac{F}{W}\right] \tag{14}$$

where H_F = significant height at end of fetch
H_D = significant height at end of decay
gF/U^2 = generating parameter
$D/gT_D^{\,2}$ = decay parameter
F/W = ratio of fetch length to fetch width

Exact relationships for equations 13 and 14 cannot be obtained due to lack of proper wave data. In most cases F/W does not vary much, and its effect, if any, is lost in the scatter of wave data. By empirical means relationships were found (Bretschneider 1952) for wave decay as follows:

$$\frac{D}{H_D} = f_1 \left[\frac{D}{H_F} \; , \; \frac{D}{F} \right] \text{, and} \tag{15}$$

$$\frac{D}{gT_D{}^2} = f_2 \left[\frac{D}{gT_F{}^2} \; , \; \frac{D}{F} \right] \tag{16}$$

The empirical relationships were transformed into practical curve for forecasting the decay of waves. This is shown in Figure 6. It must be emphasized that Figure 6 still requires revisions, based on more suitable wave data. In the development of Figure 6 wave data included that from the Pacific Ocean, both from the northern and southern hemispheres. Waves from the southern hemisphere had decay distances from 4,000 to 6,000 miles, and those from the northern hemisphere had decay distances from 50 to 3,000 nautical miles. The waves in the fetch were forecast by use of the generation graphs. Twenty-four hourly weather maps of the southern hemisphere and twelve hourly weather maps of the northern hemisphere were used to obtain wind speeds and fetch lengths. Twenty-four hourly maps are never very satisfactory.

Perhaps a properly calibrated wave spectrum method needs development for prediction of waves in the decay zone. In either case more reliable wind and wave data are required to obtain accurate decay relationships.

DECAY OF DEEP WATER SWELL OVER SHALLOW BOTTOM

Figure 6 presents curves for obtaining the decayed wave height and period for deep water. The swell may have advanced hundreds of miles and in some cases such as southern swell, several thousand miles The greatest rate of decay takes place over the first few hundred miles, after which the rate is not so great. However, if the swell advances into shallow water, the rate of decay may again increase due to dissipation of wave energy by bottom friction and percolation in the permeable sea bottom. Putman and Johnson (1949) have developed a dissipation function for bottom friction and Putman (1949) presents a dissipation function for percolation in a permeable sea bottom.

Using the above-mentioned dissipation functions, Bretschneider and Reid (1954) have obtained a number of solutions and present a number of nomographs for determining the change in wave height (or change in wave energy) due to bottom friction, percolation and refraction, for swell traveling over a shallow bottom. Since it is

REVISIONS IN WAVE FORECASTING: DEEP AND SHALLOW WATER

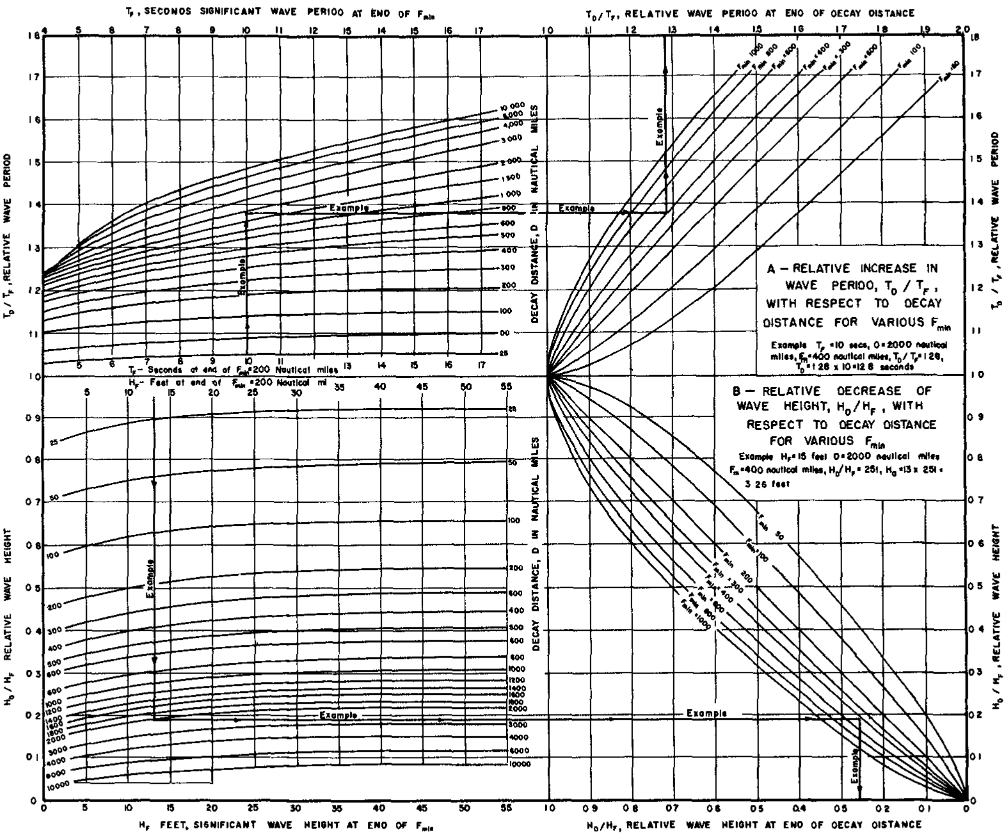

Fig. 6. Forecasting curves for wave decay.

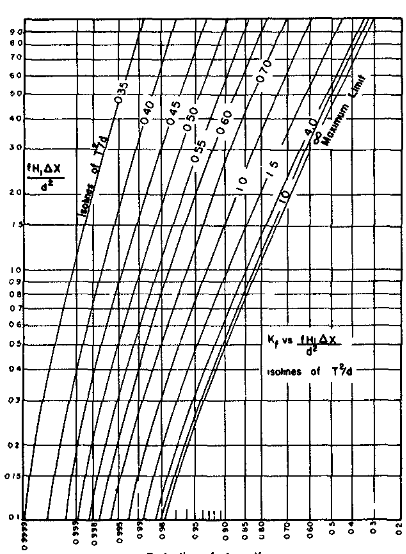

Fig. 7. Relationship for friction loss over a bottom of constant depth.

difficult to isolate the individual effects of percolation and bottom friction, an overall bottom friction factor can be used.

The actual height of swell traveling over an impermeable bottom, water of constant depth and no refraction is obtained from

$$H_{Ds} = K_f \, K_s \, H_{Do}, \text{ where}$$

H_{Do} is the deep water decayed height. K_f is the reduction factor to take into account wave energy loss due to bottom friction. K_f may be obtained by use of Figure 7. K_s is the shoaling coefficient and is given in terms of H_o/H_o', as a function of d/L_o in tables by Wiegel (1954). Figure 8 of the present paper gives K_s as a function of T^2/d.

At this point consider an example of a wave forecast, given:

U = 30 knots
F = 400 nautical miles
t_d = 36 hours, duration of wind
D_o = 600 miles in deep water
D_s = 50 miles decay in shallow water
d = 50 feet constant depth over the 50 miles in shallow water
f = .01 bottom friction factor

From Figure 2 for $U = 30$ knots, $F = 400$ nautical miles, $t_d = 36$ hours, read $H_F = 17.0$ feet and $T_F = 12.1$ seconds, $F_{min} = 400$ nautical miles. From Figure 6 read $H_D/H_F = .43$ and $T_D/T_F = 1.22$. Thus the decayed wave height and period at the end of the deep water section respectively, are

$$H_{Do} = 17.0 \times .43 = 7.3 \text{ feet}$$
$$T_D = 12.1 \times 1.22 = 14.8 \text{ seconds}$$

Over the last 50 miles further decay is possible by using $D = 650$ miles instead of 600 miles. However, this will be much smaller than the reduction due to bottom friction, and does not combine linearily. Now also, the significant period will shift back to the lower periods since waves having longer periods are first affected by bottom friction. At present this factor also will not be considered.

Figure 7 is used to obtain the reduction factor K_f.

$$\text{Compute} \quad \frac{fH_1\Delta X}{d^2} = \frac{.01 \times 7.3 \times 50(6080)}{50^2} = 8.9$$

and $$\frac{T^2}{d} = \frac{(14.8)^2}{50} = 4.38$$

From Figure 7 read $K_f = .38$
From Figure 8 read $K_s = 1.05$

Thus, the 7.3-foot wave height, after traveling 50 miles over a shallow depth of 50 feet, will be

$$H_{Ds} = 7.3\ (.38)(1.05) = 2.9 \text{ feet.}$$

The general procedure, however, since the Continental Shelf is not a flat bottom, is to segment the traverse between two orthogonals, each segment assuming a mean water depth. In this manner refraction can also be taken into account. If more detailed information is required one might decay elements of the joint distribution or spectrum of waves and thereby obtain also the change in significant wave period. The primary purpose of the above discussion is to show that a numerical process is possible to obtain wave height reduction in shallow water due to bottom friction, percolation, and refraction.

FORECASTING WIND WAVES IN SHALLOW WATER

Less information is available on wind waves in shallow water than for deep water. This is true in regards to both theory and available data. The first information on this subject is given by Thijsse (1949), based on laboratory data. Additional data and relationships were brought forth by Dr. Garbis Keulegan of the National Bureau of Standards, although never published to the knowledge of the present author. The U. S. Corps of Engineers, Jacksonville District (1955), performed an extensive field investigation on wind, waves, and tides in Lake Okeechobee, Florida. Based on the hurricane wind and wave data from Lake Okeechobee, and some ordinary wind wave data from the shallow regions of the Gulf of Mexico, Bretschneider (1954) was able to establish a numerical procedure for computing wind waves in shallow water taking bottom friction into account. A friction factor of $f = .01$ appears satisfactory. Presently these techniques are used for the Continental Shelf, but may require further calibration when more wind and wave data are available.

GENERATION OF WIND WAVES OVER A BOTTOM OF CONSTANT DEPTH

If $d/T^2 < 2.5$ feet/sec^2, then the waves effectively "feel bottom" and the depth and bottom conditions enter as additional factors with respect to the heights and periods of waves which can be generated. The effect of frictional dissipation of energy at the bottom for such waves limits the rate of wave generation and also places an upper limit on the wave heights which can be generated by a given wind speed and fetch length.

The following expression for the reduction in height of waves traveling over an impermeable bottom of constant depth without refracti is obtained from Bretschneider and Reid (1954).

$$H = H_1 \left[\frac{f H_1 \phi_f \Delta X}{K_s T^4} + 1 \right]^{-1} \qquad (1$$

where H = the final height at X
H_1 = original height at $X = X_1$
$\Delta X = X - X_1$, the horizontal distance of wave travel in feet
f = friction factor (dimensionless), a characteristic of the bottom
T = wave period in seconds
K_s = shoaling factor, given as H/H_o' in tables by Wiegel (1954), also given in Figure 8

$$\phi_f = \frac{64\pi^3}{3g^2} \left[\frac{K_s}{\sinh 2\pi d/L} \right]^3 , \ \sec^4 \text{ft}^{-2} \qquad (1$$

d = depth of water in feet

$L = g/2\pi\, T^2 \left(\tanh \frac{2\pi d}{L}\right)$, wave length in feet

g = acceleration of gravity in feet/sec^2

Equations 18 and 19 are based on consideration of waves of small steepness and therefore represent only an approximation for waves near the maximum steepness.

The solution of Equations 18 and 19 is given in Figure 7, where $K_f = H/H_1$.

Figure 1, the deep water wave forecasting relationships, in effect represents the generation of wave energy in deep water as a function of F, U, and t, since the energy is proportional to H^2; whereas Figure 7 represents the dissipation of wave energy due to bottom friction. Figure 1 and equation 18 were combined by a numerical method of successive approximation to obtain relationships for the generation of waves over an impermeable bottom of constant depth. Best agreement between wave data and the numerical method was obtained when a bottom friction factor f = .01 was selected. Perhaps a "calibration friction factor" is a more appropriate term, since it would take into account other influential factors not normally included in the friction factor term. Figures 9 and 10 are the results of these computations. Actually the curve of gT/U versus gd/U^2 is based on the wave data, whereas the curves of gH/U^2 versus gd/U^2 and gF/U^2 are based on the numerical computations. The curves of these figures are not too much different from those presented by Thijsse and Schijf (1949). Figure 11, based on Figure 9, gives wave forecasting curves for shallow water of constant

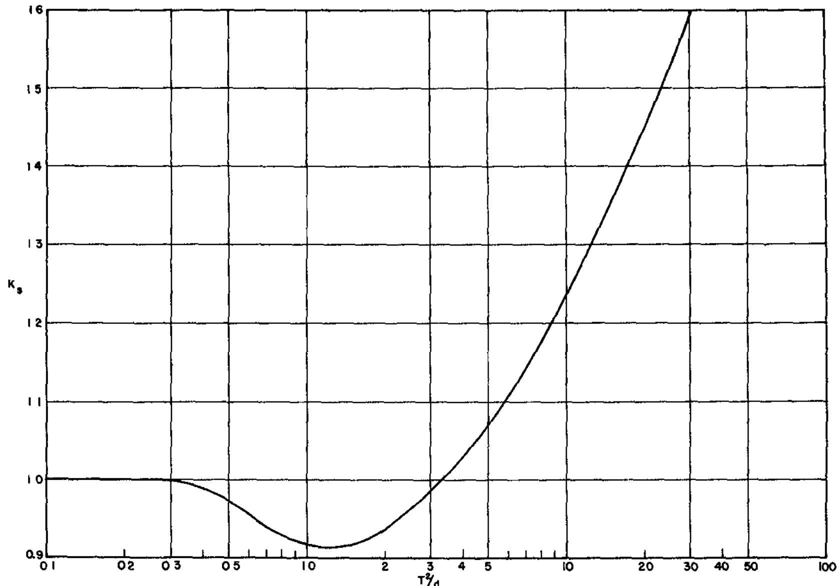

Fig. 8. Shoaling coefficient K_s vs T^2/d.

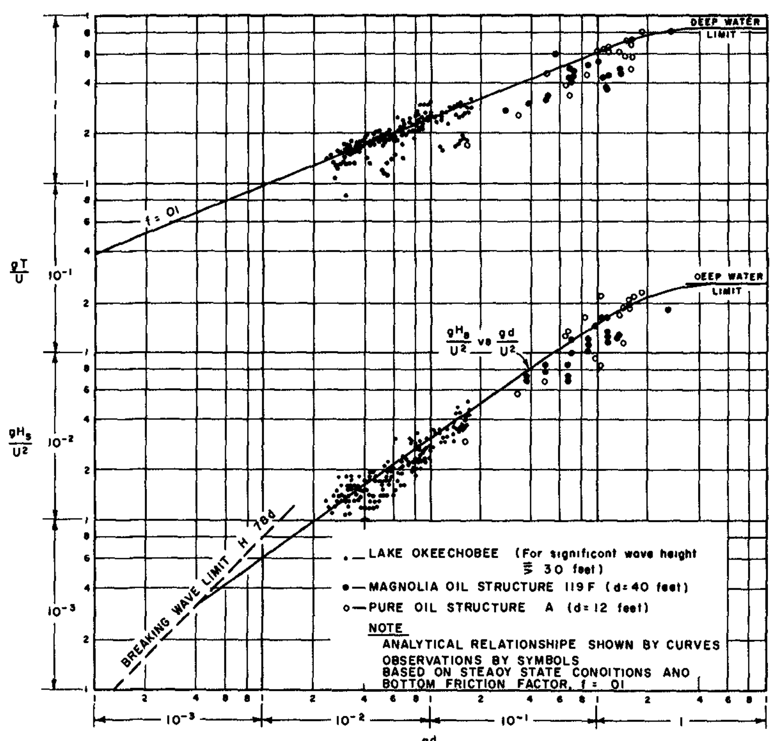

Fig. 9. Significant wave height and period as functions of constant water depth and wind speed.

depth, and unlimited wind duration and fetch length. Figure 10 may be used when both the fetch length as well as the depth are restricted.

The important fact from the above material, however, is the establishment of a numerical procedure for computing wind waves in shallow water of constant depth which can be verified by use of wave data. This procedure can be extended to a bottom of constant slope, wherein the bottom is segmented into elements, each element having a mean depth assumed to be constant.

FORECASTING WAVES OVER THE CONTINENTAL SHELF

In general, for any locations on the Continental Shelf, three special cases for wind-wave generation exist: (a) winds blowing parallel to the coast, (b) winds blowing from land to sea, and (c) winds blowing from sea to land. These are discussed below.

(a) Case I - Winds Blowing Parallel to the Coast - In this case, except where very irregular bottom topography exists, the best approach is to use the flat bottom relationships, Figures 9, 10, or 11, as the case may be. Wherever wave data are available, however, it is recommended that a calibration be made of the forecasting curves. When refraction becomes important, the numerical method must be used.

(b) Case II - Winds Blowing From Land to Sea - In a report by Bretschneider and Thompson (1955) it was shown that for most offshore winds, waves are generated which do not feel the bottom, at least for the Gulf of Mexico. This is probably true for other Continental Shelf areas, and the reason is that the fetch length, increasing seaward, is generally limiting. As the fetch length gets longer the wave period gets longer, but the water depth becomes greater. This would indicate that for offshore winds, one may use the deep-water forecasting curves, Figure 2. However, for the cases where the Continental Shelf is long and relatively flat, Figure 10 might be used, or perhaps the numerical method.

(c) Case III - Winds Blowing From Sea to Land - This is perhaps the most complex situation for wave generation, and no one set of generalized curves can be developed similar to those for Cases I and II. However, forecasting curves have been worked out by Bretschneider (1956) for various sections of the Gulf of Mexico. In general, each section or location has a different bottom profile leading shoreward from various directions. In some cases refraction must also be considered, and hence the numerical method must be used. The numerical method is calibrated by use of hurricane wind wave data from Lake Okeechobee, Florida and a limited amount of ordinary wind wave data from the Gulf of Mexico, and it appears that a bottom friction factor of $f = .01$ is also applicable to the Continental Shelf. It must be emphasized, however, that when more wave data become available for the Continental Shelf, a refined calibration of the method should be made.

In regards to wave generation by onshore winds, there are two conditions to consider. First, the initial deep-water waves generated may be propagated shoreward as swell under the continued influence of the generating winds; and second, regeneration of wind waves is

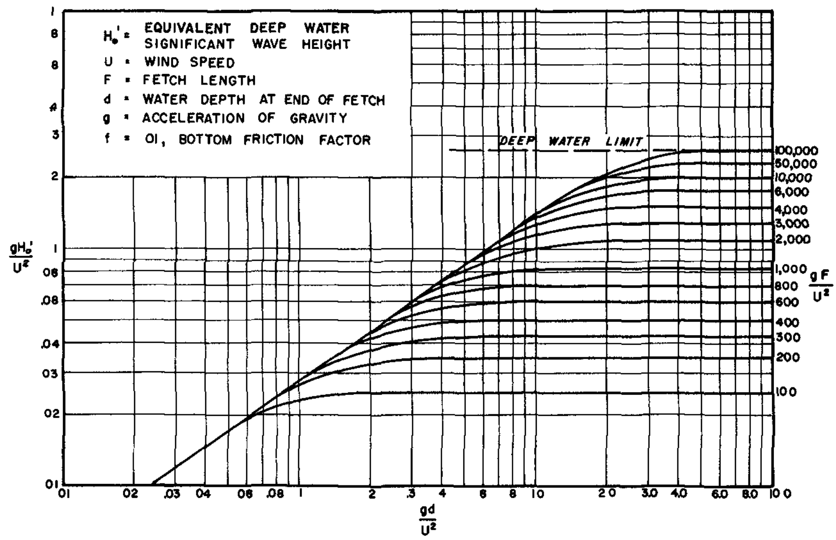

Fig. 10. Generation of wind waves over a bottom of constant depth for unlimited wind duration represented as dimensionless parameters.

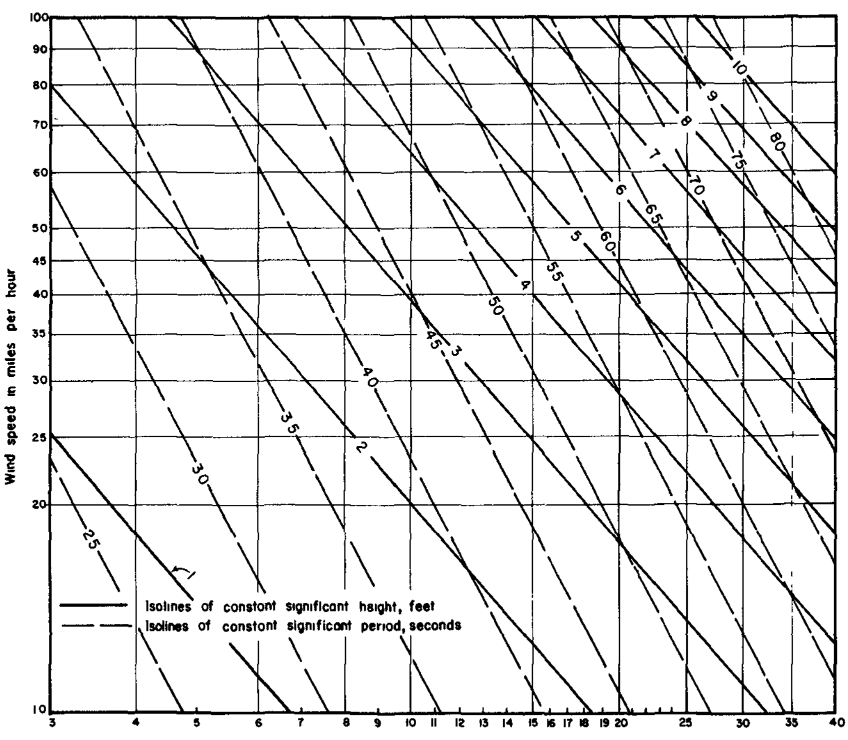

Fig. 11. Wave forecasting relationships for shallow water of constant depth.

constantly taking place all along the fetch over the Continental Shel Swell will feel bottom far from shore and commence losing energy at a early stage, whereas wind waves with shorter periods continue to grow and do not feel bottom until they are sufficiently large and are near ing the coast. In deep water one would observe the largest wave heig and longest periods, and in the shallow water both the significant wi wave heights and periods will have decreased, although a presence of swell might be noted. In the breaker zone one would observe both swe and wind waves. Hence, the shift of significant waves in the spectru will be toward the lower periods, opposite to that for deep water dec where the shift was toward the longer periods.

Steps in numerical procedure for computing wind waves generated up the Continental Shelf from deep water shoreward are as follows:

(a) A wind speed is selected, and a graph of H_o and T versus fetch length is computed from Figure 2.

(b) The minimum fetch length F_{min} is selected correspondin to the wind speed, actual fetch length and minimum duration. H_o and ' are determined at $F = F_{min}$. The deep water wave length is computed fro $L_o = 5.12\ T^2$ and the waves will begin to feel bottom at a depth $d = L_o/2$. This is the initial point from which to begin computations.

(c) The bottom profile along the fetch toward the location of interest is determined. The traverse is segmented into at least 10 to 15 equal increments ΔF, of about 10 to 5 miles or less in leng each, depending on the bottom slope and width of the Continental Shelf. Figure 12 is a schematic diagram illustrating the procedure.

(d) An average depth, d_{ave}*, is determined over each increment (Figure 12).

(e) A deep water wave height, H_o, and wave period, T, is determined at the beginning of the first increment of ΔF using deep water relationships.

(f) This value of H_o is then assumed to travel over the increments ΔF as swell, taking bottom friction into account. This is done by use of Figure 7. The quantity $\frac{fH\Delta X}{d^2}$ is determined, using

*It is assumed that wind set-up has been computed and is included in the depth. For high winds and shallow water, the wind set-up or storn surge must first be computed before wave computations begin.

bottom friction factor ($f = .01$); $H_1 = K_r K_s H_o$, where K_s is the shoaling coefficient obtained from Figure 8; K_r is the refraction coefficient over the increment ΔF; $\Delta X = 6080 \Delta F$, where ΔF is in nautical miles; $d = d_{ave}$, average total water depth over the increment ΔF; and T^2/d is computed using the average significant period over the increment and the average depth, d_{ave}. K_f is read from Figure 7 and the actual significant height at the end of the increment ΔF is equal to $H_s = H_o K_f K_s K_r$.

(g) An equivalent deep-water wave height H_o' is obtained from $H_o' = K_f H_o$. (20)

(h) Using H_o' and Figure 2 an equivalent deep water fetch length F_e' is obtained. For the case of regeneration of wind waves one also obtains an equivalent deep-water period, T_o.

(i) An equivalent deep-water wave height is determined at the end of the second increment for $F = F_e' + \Delta F \leqq F_{min}$, using Figure 2. For the case of regeneration of wind waves one also obtains an equivalent deep-water period.

(j) With the average wave height $1/2(H_o' + H_{02}')$ steps f, g, h, and i are repeated. (This gives the swell height when the wave period T is held constant; T is given at the beginning of the first increment for U at $F = F_{min}$). Using the average of the periods $1/2(T_{01} + T_{02})$ steps f, g, h, and i are repeated. This gives the regenerated wind-wave height. The above procedure is used for all except the last increment or until the waves break, whichever occurs first. The last increment of ΔF cannot be treated by the above method since here the bottom slope increases too rapidly and the surf zone is experienced. The procedure can be used for depths from deep water to about 20 feet, but has been applied up to depths of 12 feet, when the winds are not too great. Figure 13 is a typical example of wind-wave forecast for a 26-knot wind. Note, the last increment must be treated as surf.

The above procedure can also be set up on a high speed computer, and for a particular area one could determine a family (or families) of forecasting curves similar to Figure 13. This would be desirable, once sufficient data are available for a refined calibration.

If the wind is variable in speed and direction, the graphical method of forecasting waves in deep water by Wilson (1955) might be extended to include shallow water computations. This can become quite involved, but could be programmed on a high speed computer.

FORECASTING WAVES GENERATED BY HURRICANES

The problem of forecasting hurricane waves in deep water is somewhat handicapped by lack of adequate hurricane wind and wave data. In this respect the Japanese (Orakuwa and Suda 1953 and Unoki and Nakano 1955), and others have been doing a great deal of work on the study of winds, waves and swell in typhoons, which are somewhat similar to the hurricanes.

The most satisfactory tool for predicting hurricane waves is the graphical method by Wilson (1955). This method has been used by Wilso (1957) to compile hurricane statistics in deep water for the Gulf of Mexico.

Three of the most important fundamental differences between gener tion of wind waves under hurricane conditions and that of normal wind waves are as follows: (a) winds within a hurricane are not constant in speed; (b) winds within a hurricane are circular in direction as oppos to straight line; and (c) the hurricane moves over waves generated at various angles of direction to the path of the storm. This fact may cause pyramidal waves formed by two trains approaching at a wide angle each other.

Figure 14, a typical wind field for a standard project hurricane off the Texas Gulf coast, was constructed by the Hydrometeorological Se the U. S. Weather Bureau (1957). The wind field will be slightly different for a similar hurricane off the east coast of the United Stat The fact that the wind speed varies in direction and speed poses no difficult problem, since in equations 10 and 11 one may replace U^2F by

$$U^2F = \int_{x_1}^{x_2} U_x^2 \, dx \qquad ($$

U_x is the component of wind in some arbitrary straight line direction. In case of a moving hurricane the space-time distributions $(U_x{}^2)t$ can be used. The integral can be evaluated numerically. The graphical method of Wilson (1955) can also be used, in which case a space-time wind field is determined. Figure 4 from Wilson (1955) shows a comparison between the numerical and the graphical methods.

It is interesting to note that the maximum value of the significant height is slightly afront the peak wind. If the hurricane moves at a moderate speed, steady state may not necessarily exist, and the peak height and peak wind speed may coincide.

There are occasions when a general knowledge of hurricane waves is important, and little time is available to perform the work involved in the graphical approach by Wilson (1955). The following material is presented to obtain significant waves within a hurricane for a slowly moving model or standard project hurricane in the Gulf of Mexico, such as might be used for design purposes.

By taking various cross sections of a hurricane wind field one may obtain wind distributions, similar to that given in Figure 4. Applying the numerical formula 20 and equation 10, one obtains the significant height distribution. This could also have been done by Wilson's (1955) graphical method. Based on a few theoretical model or standard project hurricanes moving at a slow to moderate speed in the Gulf of Mexico, wave distributions were computed similar to that shown in Figure 4. It was found that if these distributions were expressed in the dimensionless form $H\,1/3/(H\,1/3)_{max}$ versus r/R, the

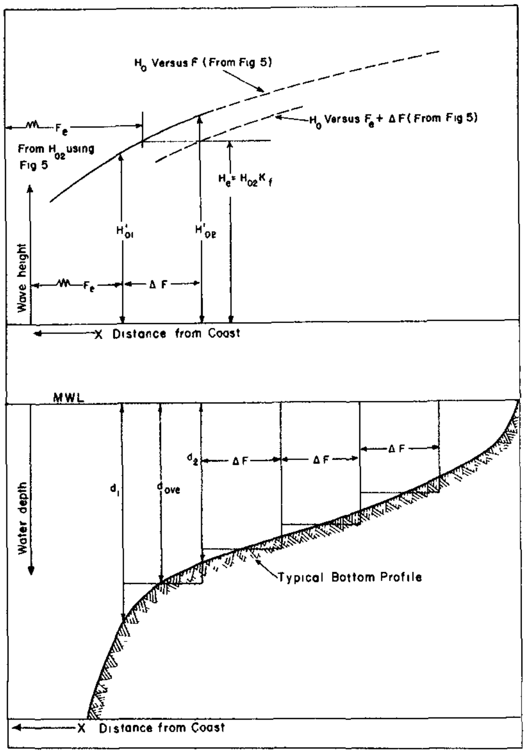

Fig. 12. Schematic diagrams illustrating procedure for computing wind waves in shallow water.

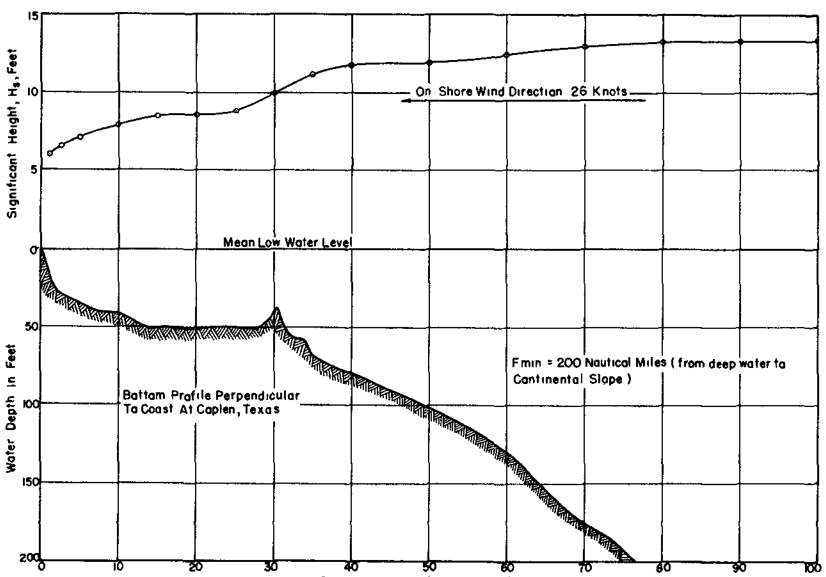

Fig. 13. Wind-wave forecast for a 26-knot wind blowing perpendicular to coast at Caplen, Texas.

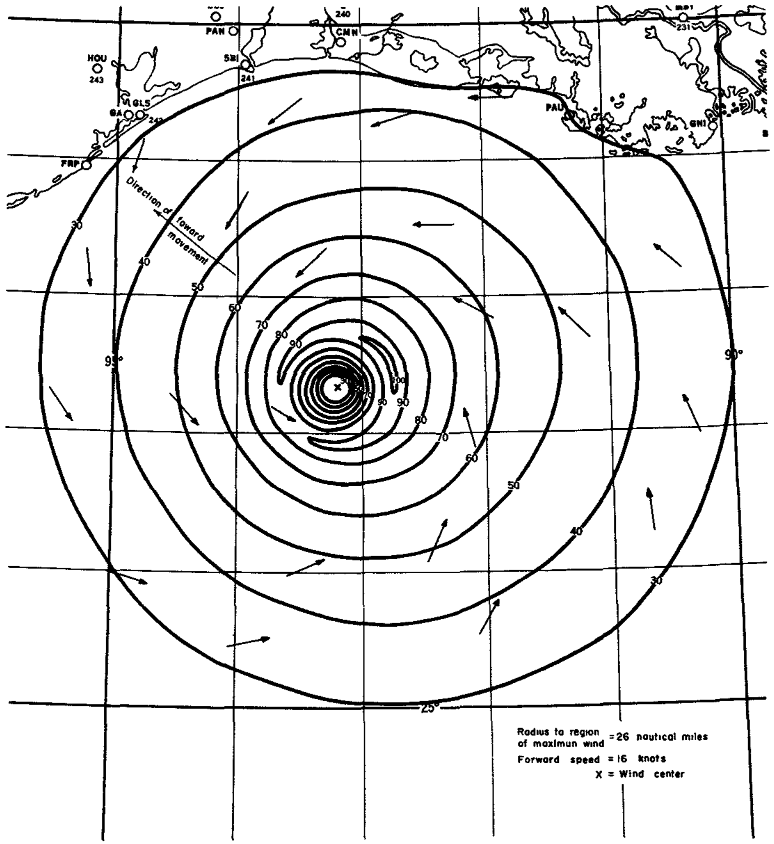

Fig. 14. Wind speed (MPH) at 30 feet over water for standard project hurricane, Galveston, Texas area.

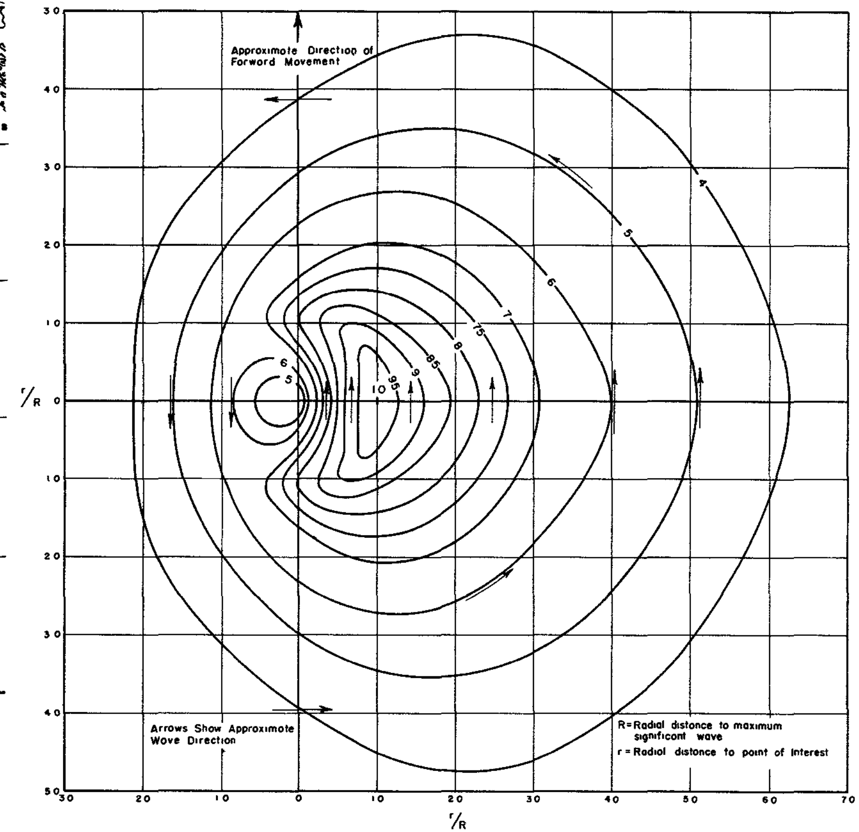

Fig. 15. Isolines of relative significant wave height for slowly moving hurricane.

results of the hurricanes investigated could be represented by a single family of curves. $H_{1/3}$ is the significant height at any position in the hurricane and $(H_{1/3})_{max}$ is the maximum value of the significant height in the hurricane. R is the radial distance to maximum wind and r the radial distance to any coordinate in the wave field. The results of these computations are given in Figure 15. Deviations from heights obtained by use of Figure 15 are less than ±5 percent obtained by the numerical method, and hence are within the same degree of accuracy as might be obtained by the graphical method of Wilson (1955).

In the use of Figure 15 it is only necessary to predict the maximum value of the significant height which might be generated by a design hurricane. A reasonable estimate of the significant period may be obtained from

$$T = \sqrt{H/.22} \tag{22}$$

Based on the analysis of thirteen east coast hurricanes, Bretschneider (1957) obtained a simple formula for obtaining the maximum value of the deep water significant height and period that might be generated by a hurricane under steady state conditions. These formulae are as follows:

$$H_o = 16.5\, e^{R\Delta P/100} \left[1 + \frac{\alpha .208\, V_F}{\sqrt{U_R}}\right] \tag{23}$$

$$T_s = 8.6\, e^{R\Delta P/200} \left[1 + \frac{\alpha .104\, V_F}{\sqrt{U_R}}\right], \text{ where} \tag{24}$$

$H_o = (H_{1/3})_{max}$ = maximum value of the significant height in feet

T_s = period of significant wave, seconds

R = radius of maximum wind, nautical miles

ΔP = atmospheric pressure reduction from center of hurricane in inches of mercury

V_F = forward speed of hurricane

α = percent effectiveness of V_F to be added to the wind field of a stationary hurricane to obtain the wind field of the moving hurricane. For slowly moving hurricanes, $\alpha = 1.0$

U_R = maximum wind in knots at R for stationary hurricane

TABLE II

JOINT DISTRIBUTION OF H AND T FOR ZERO CORRELATION

Number of waves per 1,000 consecutive waves for various ranges in hei and period.

Range in Relative Height	RANGE IN RELATIVE PERIOD $T/\bar{T}$										
$H/\bar{H}$	0-.2	.2-.4	.4-.6	.6-.8	.8-1.0	1.0-1.2	1.2-1.4	1.4-1.6	1.6-1.8	1.8-2.0	0-2.0	Accumulat
0- .2	.03	.50	2.05	4.86	7.68	8.09	5.31	1.92	.34	.03	30.81	30.81
.2- .4	.10	1.41	5.81	13.78	21.76	23.92	15.05	5.44	.98	.07	88.32	119.13
.4- .6	.14	2.06	8.54	20.23	31.95	33.65	22.10	7.99	1.44	.11	128.21	247.34
.6- .8	.16	2.40	9.91	23.48	37.08	39.06	25.65	9.27	1.67	.12	148.80	396.14
.8-1.0	.16	2.40	9.92	23.51	37.13	39.11	25.69	9.28	1.67	.12	148.99	545.13
1.1-1.2	.15	2.14	8.87	21.02	33.19	34.97	22.96	8.30	1.49	.11	133.20	678.33
1.2-1.4	.12	1.74	7.21	17.07	26.96	28.40	18.65	6.74	1.21	.09	108.19	786.52
1.4-1.6	.09	1.30	5.37	12.72	20.09	21.16	13.90	5.02	.90	.07	80.62	867.14
1.6-1.8	.06	.90	3.72	8.82	13.93	14.67	9.64	3.48	.63	.05	55.90	923.04
1.8-2.0	.03	.48	1.99	4.72	7.45	7.85	5.15	1.86	.33	.03	29.89	952.93
2.0-2.2	.03	.42	1.72	4.09	6.45	6.80	4.47	1.61	.29	.02	25.90	978.83
2.2-2.4	.01	.18	.76	1.80	2.84	2.99	1.97	.71	.13	.01	11.40	990.23
2.4-2.6	.01	.09	.39	.93	1.47	1.55	1.02	.37	.07		5.90	996.13
2.6-2.8		.04	.18	.43	.67	.71	.47	.17	.03		2.70	998.83
2.8-3.0												
0-3.0	1.09	16.06	66.44	157.46	248.65	262.93	172.03	62.16	11.18	.83		
Accumulative	1.09	17.15	83.59	241.05	489.70	752.63	924.66	986.82	998.00	998.83		

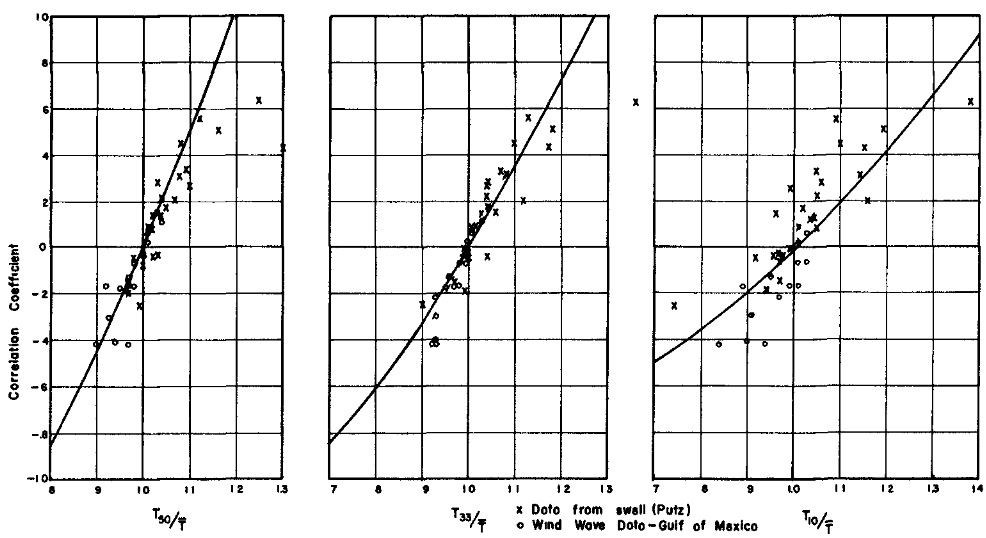

Fig. 16. Period ratio of highest 50 percent, 33 percent, and 10 perce wave height versus correlation coefficient.

WAVE VARIABILITY

Although one forecasts the significant height and period, it may be desirable to predict the distribution of heights and also the periods. The work of Putz (1952) and that of Longuet-Higgins (1952) are quite useful in this respect. It was shown by Bretschneider (1957) that the distribution of wave periods squarred as well as the wave heights can be represented very closely by the Rayleigh type distribution, as utilized by Longuet-Higgins (1952) for wave heights. The accumulative form of the distribution function for heights is given by

$$P\left[H\right] = 1 - e^{-\frac{\pi}{4}\left[\frac{H}{\bar{H}}\right]^2}, \text{ where} \tag{25}$$

$\bar{H} = .625\, H_{1/3}$, and may be obtained from forecasting relationships presented before.

The accumulative form of the distribution of periods is given by

$$P\left[T\right] = 1 - e^{-.675\left[\frac{T}{\bar{T}}\right]^4}, \text{ where} \tag{26}$$

$\bar{T}$ may be obtained from Figure 1-A.

Equations 25 and 26 were found to apply approximately for the swell as well as wind waves in deep or shallow water. In case of very long swell, however, agreement is not always satisfactory, except when a long record is used.

When $g\bar{T}/2\pi U = gT_{1/3}/2\pi U$, according to Figure 1-A, the stage of generation is in zero correlation with respect to H and T^2. In this case the joint distribution function is the direct product of the marginal distributions, whence

$$P\left[H,T\right] = P\left[H\right] \cdot P\left[T\right]. \tag{27}$$

Table II presents the number of waves per 1,000 consecutive waves that may be expected to fall within various ranges of heights and periods. This is for zero correlation only.

It can be seen from Figure 1-A, if the relationship of $g\bar{T}/2\pi U$ is approximately correct, that zero or near zero correlation begins at a moderate stage of generation and persists to the limit of the fully developed sea, and equation 27 is quite applicable. However, when $\bar{T} \neq T_{1/3}$ then correlation exists and equation 27 does not apply, except perhaps approximately for low correlations. As mentioned earlier, the equation given by Bretschneider (1957) for the joint distribution is not correct, except for the special case of zero correlation, and therefore should be used accordingly.

For decayed waves, the marginal distribution functions, equations 25 and 26, still apply, at least approximately, since they are in close agreement with the relationships given by Putz (1952) based on the analysis of 25 records of ocean swell. As soon as waves begin to decay the correlation rotates to positive values of the correlation coefficients, the larger the decay distance the larger the positive correlation. Hence, equation 27 does not apply, and thi phase of the problem has not been established to date.

WAVE SPECTRA

Although the wave spectra is not intended to be part of this paper, it seems appropriate to include results of some recent studies on which certain revisions in this paper are based. When, and only when $\bar{T} = T_{1/3}$, the period spectrum is given by Bretschneider (1958) according to

$$S_{H^2(T)} = \frac{\alpha g^2 T^3}{(2\pi)^4} e^{-.675 \left[\frac{T}{\bar{T}}\right]^4} \tag{28}$$

The corresponding frequency spectrum is given by

$$S_{H^2(\omega)} = \alpha g^2 \omega^{-5} e^{-.675 \left[\frac{g}{\bar{T}\omega}\right]^4} \text{, where} \tag{29}$$

$$\omega = \frac{2\pi}{T}, \quad \frac{g\bar{T}}{2\pi U} = f\left[\frac{gF}{U^2}, \frac{gt}{U}\right] \text{, and}$$

$$\alpha = 3.437 \left[\frac{g\bar{H}}{U^2}\right]^2 \left[\frac{2\pi U}{g\bar{T}}\right]^4 = 16\pi^2 \left[\frac{\bar{H}}{\bar{L}}\right]^2 \tag{30}$$

The above relationships evolve directly from the joint distribution function for wind generated waves when zero correlation exists. For large ω equation 29 becomes

$$S_{H^2(\omega)} = \alpha g^2 \omega^{-5} \text{, which is exactly that form given} \tag{31}$$

by Burling (1955) based on very accurate measurements. The form of equation 31 has also been proven by Phillips (1957) from an entirely different approach by use of the definition of the energy spectrum and dimensional analysis, a priori reasoning.

For the case of a fully developed sea

$$\frac{g\bar{T}}{2\pi U} = 1.95 \text{ and } \frac{g\bar{H}}{U^2} = .625 \left[.282\right] = .172, \text{ according to}$$

Figure 1-A, from which one obtains the minimum value of

$$\alpha_{min} = 7.4 \times 10^{-3}, \text{ which is in agreement} \qquad (32)$$

with the value reported by Burling (1955). Actually, the value of 7.4×10^{-3} was used together with

$g\bar{T}/_{2\pi U} = 1.95$ obtained from the period spectrum to arrive at the value of $g\bar{H}/_{U^2} = .172$, corresponding to $gH_{1/3}/_{U^2} = .282$.

The mean wave steepness for zero correlation evolves from the joint distribution function according to

$$\overline{\left[\frac{H}{L}\right]} = \frac{\pi}{2}\,\frac{\bar{H}}{\bar{L}} \qquad (33)$$

The mean square sea surface slope σ^2 for zero correlation evolves from the period spectrum according to

$$\sigma^2 = \alpha\left[\ln\frac{g\bar{T}}{2\pi U} - \ln\frac{g\,T_{min}}{2\pi U} - .0671\right] \qquad (34)$$

If the spectrum is composed of waves generated from the lowest value of $gF/_{U^2}$ to the value of $gF/_{U^2}$ as imposed by F, t, and U, one might assume that $\frac{g\,T_{min}}{2\pi U} = .0193$ and $g\bar{T}/_{2\pi U}$ a function of $gF/_{U^2}$. This would result in a maximized value of σ^2 according to

$$\sigma^2 = \alpha\left[\ln\frac{g\bar{T}}{2\pi U} + 3.88\right] \qquad (35)$$

The value of $\frac{g\,T_{min}}{2\pi U} = .0193$ corresponds to the lowest possible period in the spectrum that might be generated by a wind speed equal to the critical wind speed of 6 meters per second, assuming this period is equal to .074 seconds as governed by the capillary limit. From actual measurement, however, using measuring instruments which attenuate a portion of the high frequency components, $\frac{g\,T_{min}}{2\pi U}$ may be considerably larger, in which case σ^2 measured will be lower than tha given by the above equation.

The non-dimensional spectral width parameter, ϵ , as defined by Williams and Cartwright (1957) evolves from the spectrum according to:

$$\epsilon = \sqrt{1 - \frac{\pi}{4}\frac{a}{\sigma^2}} \qquad (3\text{(}$$

Since a and $\frac{g\bar{T}}{2\pi U}$ are both functions of gF/U^2 , the mean square sea surface slope σ^2 and the spectral width parameter ϵ are functions of gF/U^2 . From actual measurements if the high frequency components are attenuated ϵ will be measured less than that predicted by use of equation 36.

Equation 28 for the period spectrum or equation 29 for the frequency spectrum can be used together with the forecasting relationsh to obtain the corresponding wave spectra for all cases of generation where $\frac{g\bar{T}}{2\pi U} = \frac{gT_{1/3}}{2\pi U}$.

When $T_{1/3} \neq \bar{T}$ then the correlation parameter enters the problem. Where r is the correlation coefficient between H and T^2, the following equations are presented by Bretschneider (1958):

Period Spectrum

$$S_{H^2}(T) = \frac{a\,g^2T^3}{(2\pi)^4}\,\frac{\left[1 - r + .925r^2\left(\frac{T}{\bar{T}}\right)^2\right]^2}{1 + .273r^2}\, e^{-.675\left[\frac{T}{\bar{T}}\right]^4} \qquad (37$$

Frequency spectrum

$$S_{H^2}(\omega) = a\,g^2\omega^{-5}\,\frac{\left[1 - r + .925r^2\left(\frac{2\pi}{\bar{T}\omega}\right)^2\right]^2}{1 + .273r^2}\, e^{-.675\left[\frac{2\pi}{\bar{T}\omega}\right]^4} \qquad (38)$$

Mean sea steepness

$$\left[\frac{H}{L}\right] = \frac{\pi}{2}\,\frac{\bar{H}}{\bar{L}}\left[1 - r\left(1 - \frac{2}{\pi}\right)\right] \qquad (39)$$

Mean square sea surface slope

$$\sigma^2 = \frac{\alpha}{1 + .273r^2}\left[(1 - r)^2\left(\ln \frac{g\bar{T}}{2\pi U} - \ln \frac{gT_{min}}{2\pi U} - .0671\right) + r - \left(\frac{\pi - 1}{\pi}\right)r^2\right] \quad (40)$$

Spectral width parameter

$$\epsilon = \sqrt{1 - \frac{\alpha}{\pi\,\sigma^2}\left[\frac{.5708\,(1 - r)^2 + 1}{1 + .273r^2}\right]^2} \quad (41)$$

Ratio of significant period to mean wave period

$$T_{1/3}/\bar{T} = \sqrt{1 + .6r} \quad (42)$$

Ratio of mean period of the highest p- percent waves to the mean wave period

$$T_p/\bar{T} = \sqrt{(1 - r) + r\,\frac{\eta e^{-\pi/4\,\eta^2} + 1 - \Phi_p}{e^{-\pi/4\,\eta^2}}} \quad (43)$$

where $\eta = \frac{H}{\bar{H}}$

$$\Phi = \frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}\,du \quad (44)$$

$$u^2 = \pi/4\,\eta^2$$

Figure 16 shows the comparison between theory and wave data for the ratio of the significant period (T_{33}) to the mean wave period $\bar{T}$. Ratio of the mean period of the highest 50 percent waves to the mean wave period and the highest 10 percent waves to the mean wave period are also given.

All of the above equations will result in answers to the same degree of accuracy as the forecasting curves if used for predictions. However, if measured values of $\overline{H}$, $\overline{T}$, and r between H and T^2 are used, quite satisfactory results should be obtained, assuming the record is sufficiently long and representative.

SUMMARY AND CONCLUSIONS

The present paper presents the latest revisions in wave forecasting based on the significant wave method. It is emphasized that the significant period as well as the significant height has definite significance. Three special classes of wave forecasting are discussed: (a) forecasting deep water wind waves and swell, (b) forecasting shallow water wind waves, and (c) forecasting hurricane waves. It is recommended that in using the techniques discussed, that wind and wave data, where available, be used to improve one's technique as well as a possible refinement in calibration. There are certain conditions under which one might use the wave spectra, and a summary of useful formulae are presented. The greatest hope for future revisions in either the significant wave method or the spectrum method rests with the procurement of more and better data and the utilization of the graphical method for forecasting waves.

ACKNOWLEDGMENTS

The author extends appreciation to the staff of the Beach Erosion Board for making available the time and personnel required in the preparation of this paper. Appreciation is also extended to the California Company of New Orleans, Louisiana, who permitted the use of its standard project hurricane wave fields used to develop the generalized hurricane wave field, Figure 15, of this paper. Appreciation is extended to Professor J. W. Johnson and Mr. R. R. Putz of the University of California, for the use of the tabulated wave data of the twenty-five records of ocean swell.

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CHAPTER 4
HURRICANE WAVE STATISTICS FOR THE GULF OF MEXICO

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ABSTRACT

This paper contains the results of a statistical hindcast study of the heights and periods of significant waves generated by hurricanes in the Gulf of Mexico in the period 1900 to 1949. Results are presented in a series of polar plots of frequencies of occurrence of waves of given height and period at deep-water (100 fathoms depth) stations at different bearings offshore from five coastal stations (Brownsville, Tex., Gilchrist, Tex., Burrwood, Miss., Apalachicola, Fla., Tampa, Fla.).

Analysis was conducted by selecting a sample of 9 hurricanes and hindcasting by graphical moving fetch techniques, wave heights, periods and arrival times along eleven approach-directions to the five coastal stations for one storm, and from two to three approach directions for the remaining eight storms. Maximum heights and periods were correlated with hurricane characteristics (pressure, radius of maximum winds, forward velocity and direction). From the correlation the sample was increased by an additional 23 hurricanes whose characteristics were known. Heights and periods plotted against frequencies of occurrence gave mainly normal probability distributio Finally taking account of the total number of tropical storms occurring in the Gulf of Mexico in 50 years and the incidences of waves from various directio at the five stations, the chances of occurrence of full hurricane waves were evaluated.

1. INTRODUCTION

As part of a general statistical study of ocean wave heights and periods covering a period of three years at stations off the United States coast of the Gulf of Mexico, a separate analysis was undertaken of the wave conditions arising from a selection of the more severe hurricanes occurring in the Gulf in the first half of the present century. The method used in hindcasting the waves was specially developed to handle the intricacies of a moving fetch and variable wind [Wilson, 1955].

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* Contribution from the Department of Oceanography and Meteorology, Agricultural and Mechanical College of Texas, Oceanography and Meteorology Series No. 100.

In the 44 years from 1900 to 1943 Tannehill [1944] records a total of some 112 tropical storms as having invaded the Gulf of Mexico. Of these only 66 comply with the criterion of a central pressure less than 29.00 ins., qualifying them for consideration as full hurricanes [Myers, 1954]. From a group of 34 of these, listed by Myers, a selection was made of 10 of the most severe ones for purposes of detailed study.

2. SELECTION OF HURRICANES FOR HINDCAST STUDY

The choice of hurricanes was based primarily on their potential for generating storm waves, without regard to capacity for raising storm tides. The criterion used was the magnitude of 'wave energy index', E, [Reid, 1955] defined as

$$E = (\Delta p) R \tag{1}$$

where R is the radial distance from the hurricane center at which maximum winds are encountered and Δp is the anomaly of pressure from normal at the storm center; that is

$$\Delta p = p_n - p_o \tag{2}$$

p_n being normal pressure at a large distance from the hurricane eye and p_o the minimum central pressure. Values of R, p_n, p_o, Δp, and E for the selected hurricanes are given in Table I.

Table I: Characteristics of Selected Gulf of Mexico Hurricanes

Date	Place	p_n (ins. merc.)	p_o (ins. merc.)	Δp (ins. merc.)	R (naut. mi.)	E (n.mi. ins.)
Sept. 8, 1900	Galveston, Tex.	29.78	27.64	2.14	14	30.0
Aug. 16, 1915	Velasco, Tex.	29.57	28.14	1.43	32	45.8
Sept. 29, 1915	New Orleans, La.	30.14	27.87	2.27	29	65.8
Aug. 18, 1916	Santa Gertrudis, Tex.	30.77	28.00	2.77	35	96.9
{ Sept. 9, 1919	Dry Tortugas, Fla.	29.73	27.44	2.29	15	34.3 }
{ Sept. 14, 1919	Corpus Christi, Tex.	29.54	28.65	0.89	75	66.8 }
June 22, 1921	Houston, Tex.	30.03	28.38	1.65	17	28.0
Aug. 13, 1932	E. Columbia, Tex.	30.11	27.83	2.28	12	27.4
Sept. 5, 1933	Brownsville, Tex.	30.24	28.02	2.22	30	66.6
{ Sept. 17, 1947	Hillsboro, Fla.	29.83	27.76	1.09	19	20.7 }
{ Sept. 19, 1947	New Orleans, La.	29.70	28.61	1.06	28	29.7 }
Oct. 4, 1949	Freeport, Tex.	30.13	28.88	1.25	28	35.0

Choice of the above hurricanes was also conditioned by their tracks across the Gulf, shown in Fig. 1. Other hurricanes returning larger E values were ruled out because their paths were generally unfavorable to development of onshore waves. The hurricane of August 1916 had ultimately to be discarded because of a lack of adequate synoptic data near its center.

3. STATION POINTS AND APPROACH DIRECTIONS

Wave hindcasts were undertaken along particular approach directions to five coastal stations along the United States shores of the Gulf of Mexico. These station points and the approach directions are illustrated in Fig. 2(a) and are defined in Table II below.

Table II: Locations of Reference Wave Stations

Station Symbol	Latitude	Longitude	Vicinity
A	25° 55' N	97° 09' W	Brownsville, Texas
B	29° 30' N	94° 30' W	Gilchrist (near Galveston), Texas
C	29° 03' N	89° 20' W	Burrwood (southwest pass), Miss
D	29° 35' N	85° 00' W	Apalachicola, Florida
E	27° 55' N	82° 51' W	Tampa, Florida

In the results that will be quoted hereafter the deep-water offshore stations referred to will be those points marking intersections of the 100 fathoms dep contour with the various approach directions. These station points are defin more specifically in Table III hereunder:

Table III: Locations of Deep-Water Offshore Wave Stations

Station	Approach Direction	Bearing	Location at 100 fathom depth	
			Latitude	Longitude
A	AA_1	SE	25° 21' N	96° 26' W
	AA_2	E	26° 00' N	96° 19' W
B	BB_1	S	27° 51' N	94° 27' W
	BB_2	SE	27° 57' N	92° 41' W
C	CC_1	SW	28° 37' N	89° 47' W
	CC_2	S	28° 39' N	89° 20' W
	CC_3	SE	28° 48' N	89° 07' W
D	DD_1	SW	29° 03' N	85° 53' W
	DD_2	S	28° 10' N	84° 49' W
E	EE_1	W	27° 45' N	85° 11' W
	EE_2	SW	26° 21' N	84° 23' W

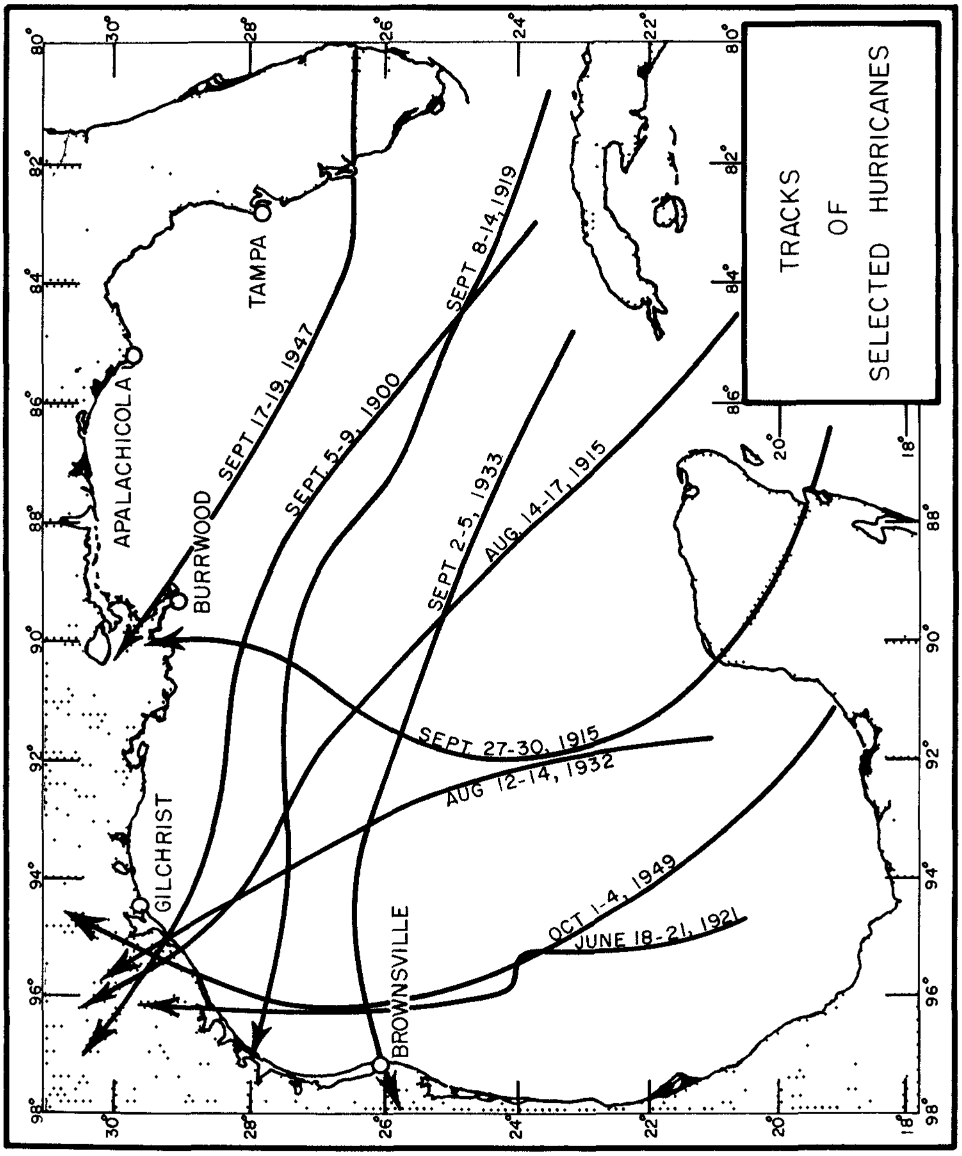

Fig. 1. Tracks of Particular Hurricanes (1900-1949) in the Gulf of Mexico, Selected for Wave Hindcasting.

4. PREPARATION OF SYNOPTIC WEATHER MAPS

The graphical wave forecasting procedure for moving variable-wind fetches [Wilson, 1955] is dependent upon preparation of adequate space-time wind-fields delineating wind strengths along the lines of approach of the wave to the different stations. To obtain these it was necessary to estimate surfac wind velocities over the entire Gulf area prevailing during the different synoptic weather situations. The rough reticulation system shown in Fig. 2(a was adopted to assist this procedure, the circled points of the grid network being selected as locations for determining wind speeds.

Synoptic weather data were obtained from the daily historical weather maps of the Weather Bureau.* In general the isobar contours at 5 mb interv given on these maps were rather poor fits to the observational data from sh stations and ships and it was found necessary to re-draw the maps entirely insert additional contours at 1 mb intervals. In addition it was considered necessary to interpolate intermediate 12-hourly maps to give an adequate pi ture of the time changes in the wind. Typical examples of the isobaric chart constructed on this basis, for the case of the Galveston hurricane of Septem 5-9, 1900, are shown in Figs. 3 and 4.

Found to be generally representative of all the hurricanes studied was an ellipticity of isobars round the storm centers revealing lesser pressure gradients on the left hand sides than on the right in the direction of motion. Having regard to the nature of the data, reasonable accord was found betwee observed wind directions and those indicated by the isobars.

The pressure patterns were used exclusively to determine surface wi velocities, and observational data from ships and shore stations were used merely as checks and controls for minor modification when necessary. No attempt was made to define isobars for pressures below about 995 mb near the hurricane centers; lack of information in these areas militated against t

5. DETERMINATION OF HURRICANE CHARACTERISTICS OVER THE OCE

A formula for the rate of change of pressure, p with radial distance r from a hurricane center has been evolved by the Hydrometeorological Secti of the Weather Bureau (HMS/WB) from studies made of various hurricanes the times of their crossing of a coastline [Myers, 1954], namely

$$\frac{dp}{dr} = (\Delta p)\,\frac{R}{r^2}\, e^{-R/r} \tag{3}$$

* The Weather Bureau (Office of Climatology) was unable to supply more da at the time this study was in progress.

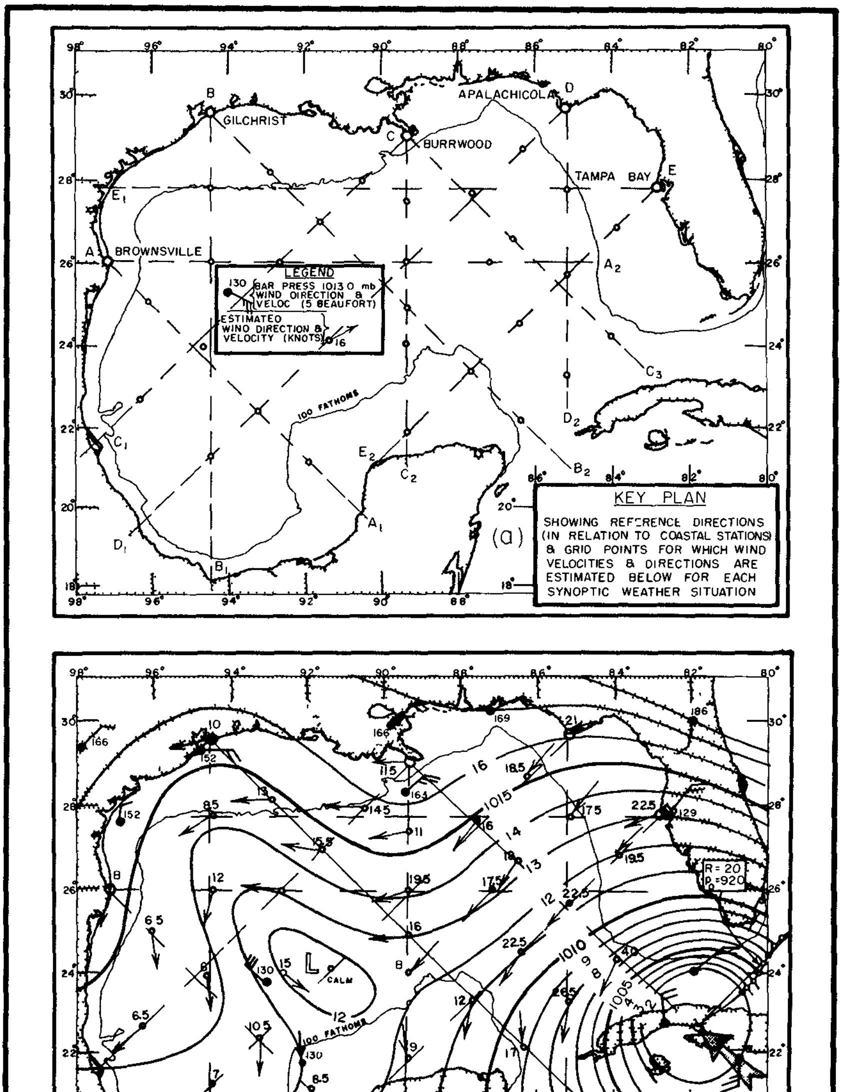

Fig. 2. (a) Key Diagram of Reference Stations, Approach Directions, Reticulation System and Codes for Synoptic Maps of the Gulf of Mexico.
(b) Synoptic Map of Gulf of Mexico Defining Entrance to the Gulf of the Hurricane of Sept. 5-9, 1900.

As in Eq.(1), the principal characteristics of a hurricane here are its parameters R and Δp. A natural question immediately arising is whether R and Δp are sensibly variable during the life history of a hurricane after the latter has acquired its so-called 'maturity'. It seemed implicit in the synoptic maps that changes from intensification and mitigation were proceeding continuously in all the storms during their progression. This was borne out also by the different values of R and Δp computed by HMS/WB for those hurricanes which made two land crossings (eg, 1919 and 1947 hurricanes, Table I).

The initial problem posed then was to evaluate R and Δp for a hurricane during its transit over water. Upon these quantities depended the magnitudes of the important winds near the hurricane centers.

Profiles of pressure through the storm centers in the direction of motio at different times were plotted, as in Fig. 5(a), from information in the synopti maps. The supposition then made was that, if these profiles obeyed the law o the integrated Eq.(3)

$$p = p_o + (p_n - p_o)e^{-R/r}, \tag{4}$$

the unknown elements p_o and R at any given time could be evaluated by maki the equation fit two points on each profile, (p_1, r_1) and (p_2, r_2). Fig. 5(b) illustrates the graphical method used in solving the two simultaneous equatio obtained in this procedure. On trial it was found that p_o values thus derived were much too high to be valid and it was obvious therefore that the actual pressure profiles were not conforming adequately to the theoretical pattern c Eq.(4).

The final attack on this problem was made by use of a series of auxilia polar 'spiral' diagrams such as Fig. 6 giving for a specific value of p_o (900 mb in this example) and a p_n value of 1020 mb, spiral isobars of press p applicable to different radial-line values of R. These diagrams were computed from Eq.(4) and drawn to the same scale as the synoptic maps of the Gulf (Figs. 1 and 2) so as to be superimposable on the pressure pattern of the hurricane in the plan sense.

The spiral diagrams afforded a trial and error method of finding some radial direction in the storm along which the pressure distribution would accord most satisfactorily with Eq.(4). To achieve the optimum agreement, a particular spiral diagram (such as for p_o = 910 mb) would be overlaid on a synoptic map so that its center coincided with the apparent hurricane center at the time considered (Fig. 7). The diagram would then be rotated about th center until the intersections between the isobars on the two charts most nearly lined up in a radial direction as shown in Fig. 7.

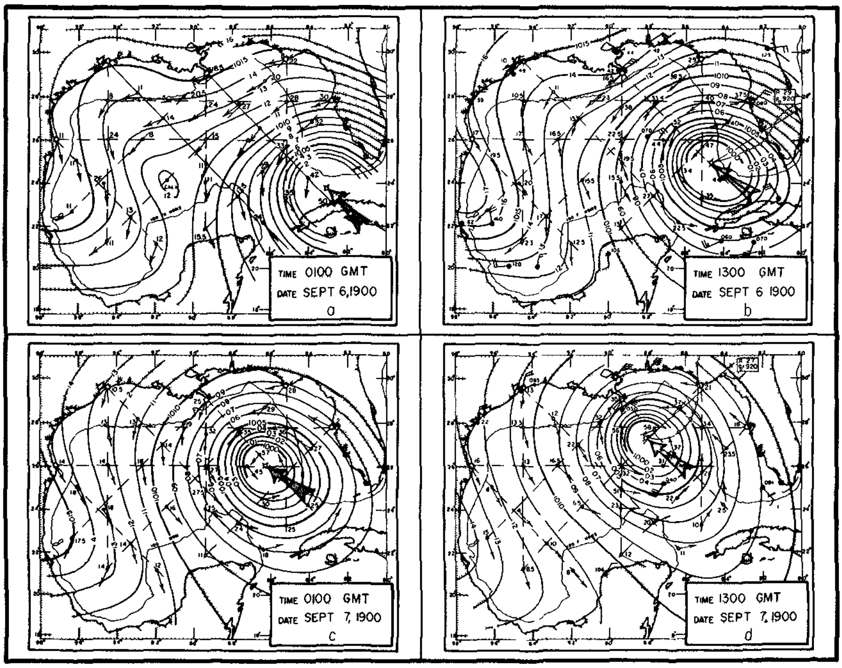

Fig. 3. 12-Hourly Synoptic Maps for Gulf of Mexico. (a-d) Hurricane of Sept. 6-7, 1900.

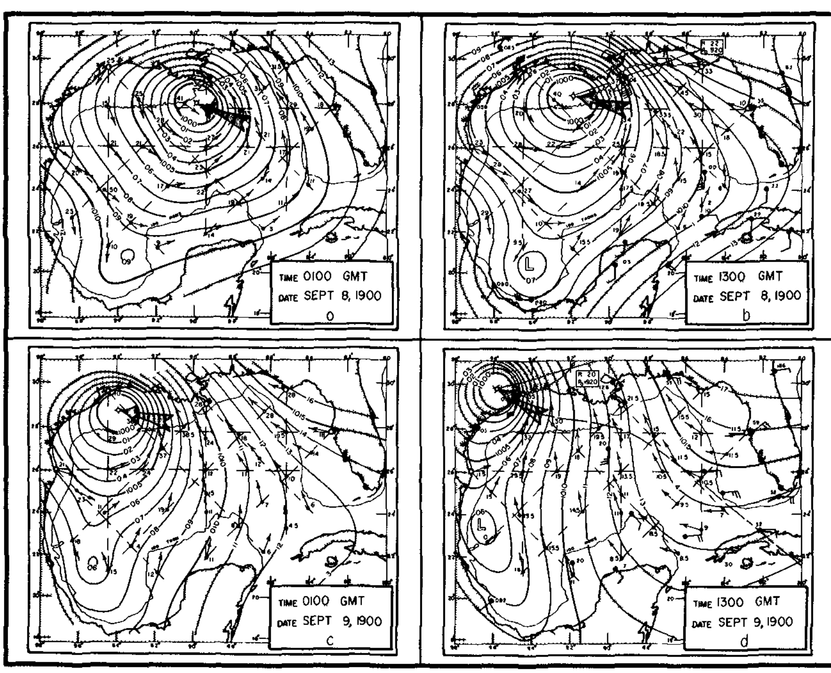

Fig. 4. 12-Hourly Synoptic Maps for Gulf of Mexico. (a-d) Hurricane of Sept. 8-9, 1900.

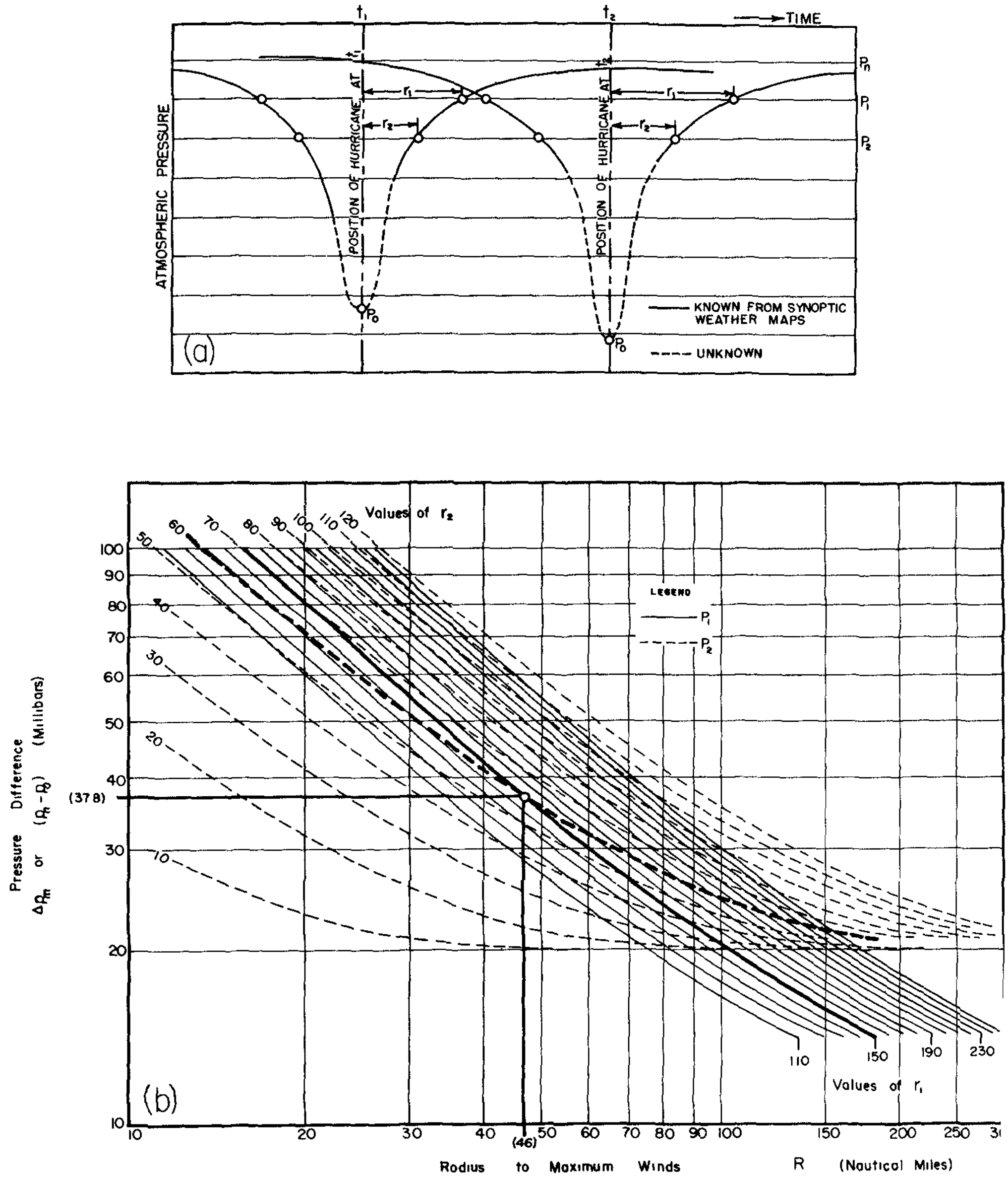

Fig. 5. (a) Schematic Diagram of Pressure Profiles of Hurricane at Different Times along Travel Path. (b) Example of Graphica Solution of Two Simultaneous Equations in Δp and R. [Frc pressure profile (e.g., Fig. 5 (a)) p_1 = 1005 mb, r_1 = 150 mi., p_2 = 995 mb; r_2 = 60 n. mi., whence solution Δp = 37.8 and R = 46 n. mi.]

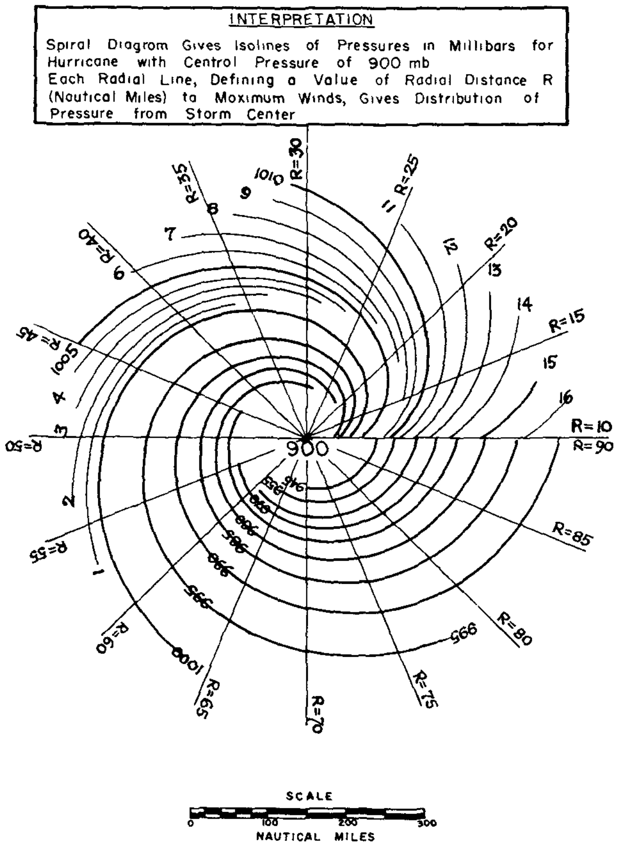

Fig. 6. Spiral Diagram of Radial Pressure Distributions from Hurricane Center for Central Pressure of 900 mb.

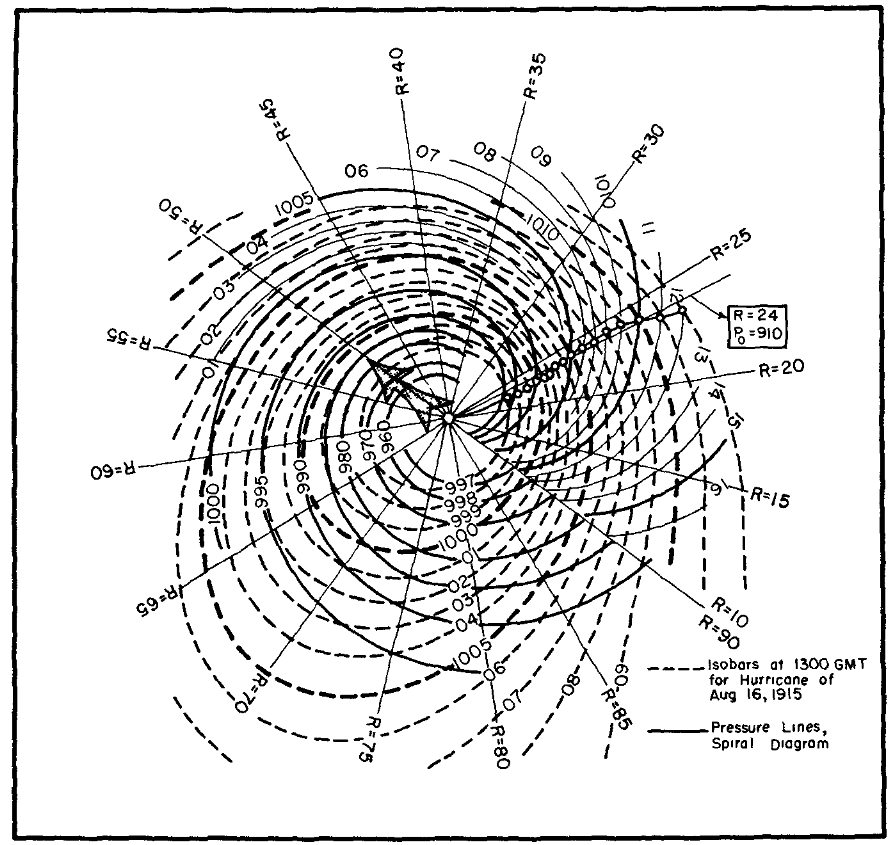

Fig. 7. Example of Best-Fit Determination of p_o and R for a Hurricane over the Ocean. Spiral Diagram for p_o = 910 mb gives nearest approach to straight (radial) line-concurrence of pressure distributions at R = 24 n.mi., Hurricane of 1300 GMT, Aug. 16, 1915.

It was found that by applying several spiral diagrams to each map a condition of best fit could usually be found, permitting definition of central pressu p_o and radius, R, to maximum winds. In almost all the storms thus treated reasonable agreement between the actual pressure distribution and the theoretical could be established only on the right hand sides in the direction of travel Values of p_o and R found in this way appeared rational and in reasonable accord with the values cited by HMS/WB in Table I. Plotted as functions of time as in Fig. 8, some idea was available of the continuous changes taking place in a hurricane along its path at sea.

6. DETERMINATION OF SURFACE WIND VELOCITIES OVER THE OCEAN

Surface wind velocities at the various points of the reticulation system shown in Fig. 2(a) were obtained from the pressure patterns (eg. Fig. 2) by various graphical aids devised from the fundamental equation of cyclonic gradient flow in a moving cyclone. This, according to Holmboe [1945], is:

$$K_{hs} U^2 + (2\Omega \sin \phi + \frac{\partial \psi}{\partial t}) U - \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \tag{5}$$

where K_{hs} is the horizontal curvature of the streamlines in a circular cyclon streamline pattern, U the horizontal wind velocity above the friction layer, Ω the angular velocity of the earth, ϕ the latitude of the point considered, ψ th horizontal angle of wind vector, positive counterclockwise from some fixed reference such as the west-east direction, ρ the density of the air, p the pr sure and r the radius from the center to the point considered.

For a hurricane moving with velocity V, Eq.(5) reduces to

$$\frac{U^2}{r} + \frac{UV}{r} \sin \Theta + 2\Omega U \sin \phi = \frac{1}{\rho} \frac{\partial p}{\partial r} \tag{6}$$

where Θ now defines the angle of bearing at the center of the point considere positive counterclockwise with reference to the direction of travel of the stor

For large r the solution of Eq.(6) approximates the geostrophic wind equation

$$U = U_g = \frac{\frac{1}{\rho} \frac{\partial p}{\partial r}}{2 \quad \sin \phi} \tag{7}$$

but near the storm center the full Eq.(6) is involved and its solution for U m be designated the gradient wind U_G. It is possible to resolve this solution f U_G in the form:

$$U_G = U_c [\sqrt{\gamma^2 + 1} - \gamma] \tag{8}$$

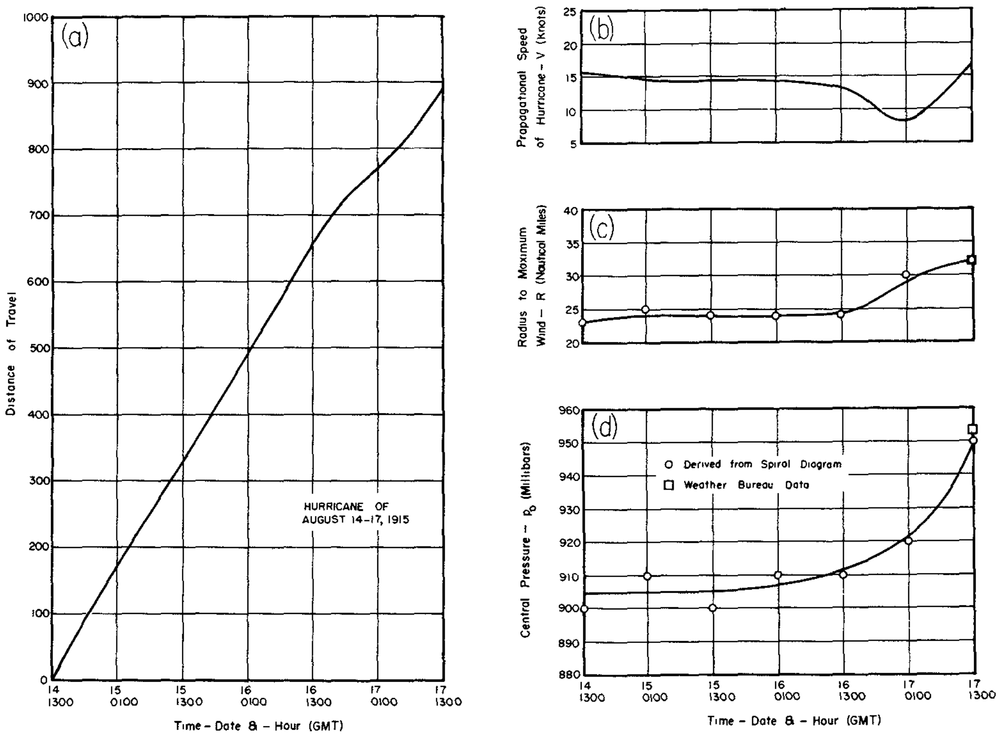

Fig. 8. Time-Functions of Characteristics, Hurricane of Aug 14- 17, 1915. (a) Distance of travel, (b) Propagational speed, (c) Radius R to Maximum wind, (d) Central pressure, p_o.

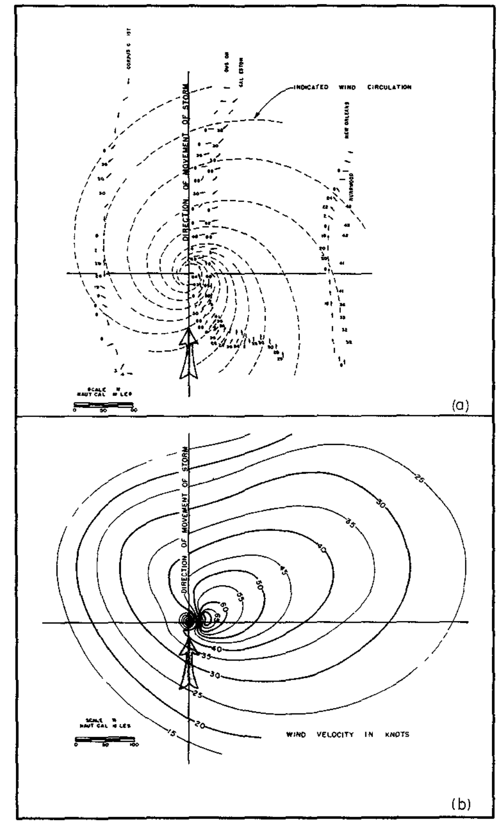

Fig. 9. Distribution of Surface Wind Velocity in the Hurricane of Aug. 17, 1915. (a) Data from 5 stations (adapted from Cline 1926, p. 82). (b) Contours of wind velocity derived from (a).

where U_c, the cyclostrophic wind, is defined as

$$U_c = \sqrt{\frac{r}{\rho}\,\frac{\partial p}{\partial r}} \tag{9}$$

and

$$\gamma = \frac{1}{2}\left(\frac{V \sin \Theta}{U_c} + \frac{U_c}{U_g}\right) \tag{10}$$

The problem of determining the surface winds was fraught with knowing the ratio of the surface wind, U_s, to the geostrophic or gradient wind (having regard to atmospheric stability), and the degree of inclination of the wind to the isobars. The assumption was made that sea-air temperature differences within the ambit of a Gulf hurricane would be small enough to be taken as zero Allowances for curvature in estimates of geostrophic wind, U_g, were made in amounts used in current wave forecasting practice [Beach Erosion Board, 195

Since the whole system of estimating surface wind velocities from the isobars involves many approximations it was considered sufficient to insert wind directions on the maps with deflection angles in the neighborhood of 18° (based on formulae of Haurwitz [1941] and Holmboe [1945]) with some decreas on near approach to the hurricane centers as suggested by HMS/WB [Myers, 1954]. Allocated directions were modified here and there to accord with ship or shore observations.

For situations in which a hurricane was close to the coast it was possibl to make use of the wind records from several coastal stations in the vicinity according to the information and method given by Cline [1946] as illustrated in Fig. 9(a). Such data when contoured for wind velocity, as in Fig. 9(b), gave useful information on the distribution of wind magnitudes in a storm and serve as a boundary-check on the wind velocities evaluated from the isobars.

7. CONSTRUCTION OF SPACE-TIME WIND-FIELDS

Surface wind velocities, U_s, found for the different points of the grid-network, are shown in the sample Figs. 2 to 4. The resolved components of these along the direction lines toward the five coastal stations A to E were determined. A component directed toward the coast was taken as positive; negative, if directed away from the coast.

The task of compiling wind-fields [Wilson, 1955] for all 11 directions (Table III) for all 9 hurricanes (Table I) was beyond available resources. It was decided therefore to treat all directions for just one hurricane, that of Aug. 14-17, 1915, and select only two or three directions for each of the remaining 8 selected hurricanes.

Fig. 10 is typical of space-time plots of wind velocity components (contoured at intervals of 2.5 knots) along the two directions for the hurricame of Aug. 14-17, 1915. Stippled areas define positive zones of wind directed shoreward, in contradistinction to offshore winds in negative (white) zones. A characteristic feature of these diagrams when the storm-track crosses a direction line (cf. Figs. 1 and 2) is a peak and trough formation flanking each other across a nodeline as shown in Fig. 10 (Burrwood). Fig. 10 (Gilchrist), on the otherhand, is typical of a condition in which the hurricane is travelling along or nearly parallel to the approach-direction.

In compiling the wind-field at the peaks and troughs in the neighborhood of the crossing point of the storm over the direction line, it was found necessary to use an estimate of wind distribution such as Fig. 9(b) in order to obtain the magnitudes and positions in space and time of the maximum positive and negative wind components.

It cannot be gainsaid that the method of estimating surface wind velocities from gradient wind speeds is subject to appreciable error. However, by the very nature of the procedures involved in compiling the space-time wind-fields, these errors, which are likely to be both plus and minus, are subject to considerable smoothing from the act of contouring the diagrams and by the influence of adjacent observations upon each other.

8. GRAPHICAL HINDCASTING OF WAVE CHARACTERISTICS AND ARRIVAL TIMES

The graphical procedure of conducting a deep-water wave hindcast for a variable wind, moving fetch has been described elsewhere [Wilson, 1955] and therefore need not be repeated here. Starting points in the wind fields from which wave propagation lines were run graphically toward the deep-water limits (Table III) were chosen by judgement so as to give the largest possible end-result of significant wave height and period. In Fig. 10 (Gilchrist), for example, starting points are all located along what is virtually the line of advance of the hurricane center or the node-line,demarcating positive and negative wind zones, being so chosen as to give the longest possible wave propagation lines falling within the stippled (positive) zone and passing through the region of high wind velocities, near the right-hand corner of the wind-field. The propagation lines from each starting point curve downward and to the right in Fig. 10. Also radiating from starting points are height lines H (upward and to the right) and period lines T (downward and to the left). Where the propagation lines intersect the contour of 100 fathoms depth (dash-line), the heights and periods attained by waves in the available time and distance from their origin are indicated by figures. The wave arrival times, of course, are given by the time-ordinates of these intersection points.

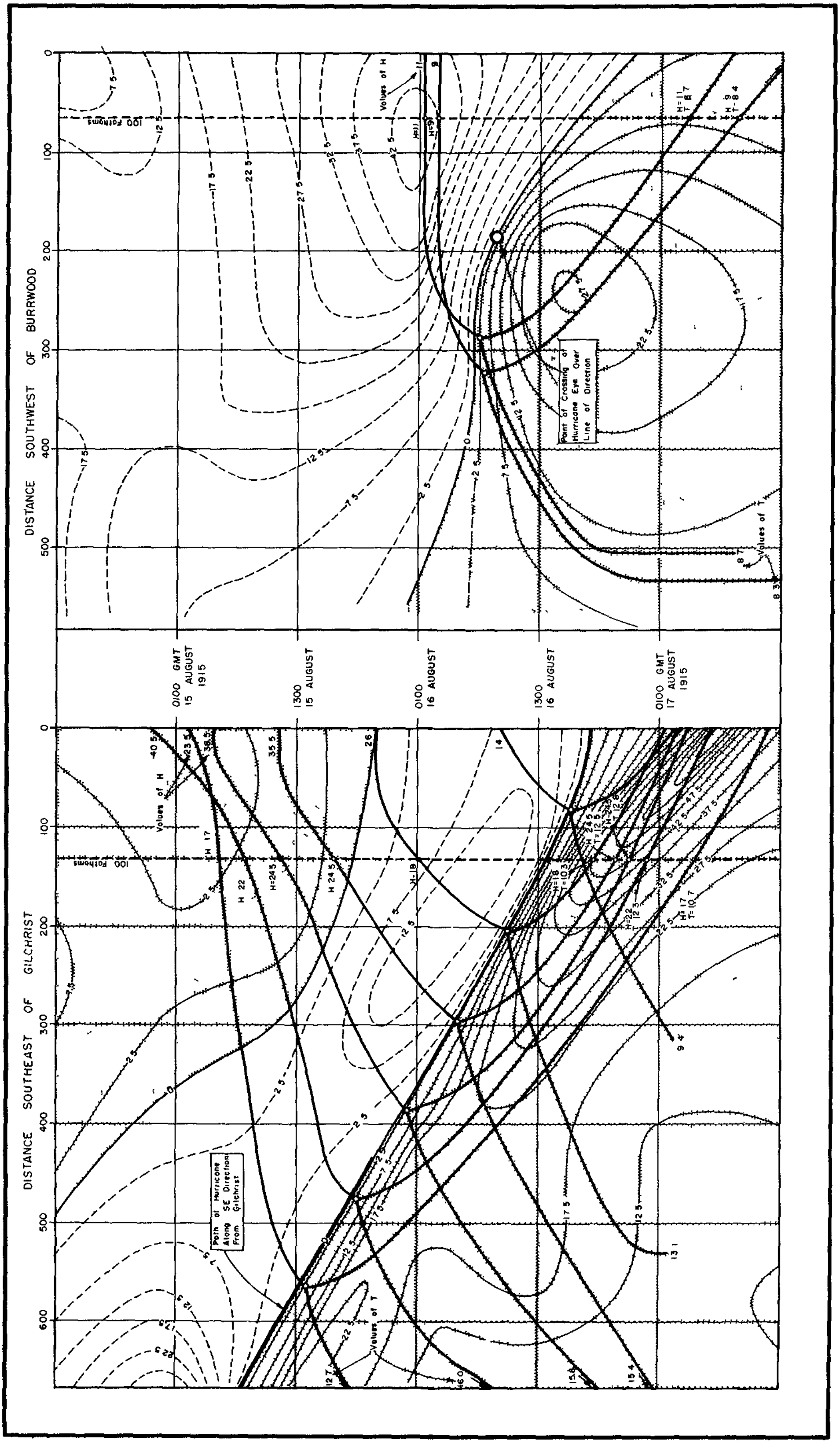

Fig. 10. Space-Time Wind-Fields along Direction Lines: (Left) BB_2, Southeast of Gilchrist, (Right) CC_1,

With certain wind-fields almost the entire area was negative, making incidence of onshore waves virtually impossible. In such cases no attempt was made to apply the graphical procedures.

Typical results from conducting the hindcasts are portrayed in Fig. 11. Significant wave heights, H, and periods, T, are plotted against arrival times and envelope curves drawn in to embrace the plotted points and indicate the overall growth and decay of H and T with time.

The significant wave heights, such as obtained in Fig. 11, would be subject to considerable reduction upon the waves reaching the coastline, as a result of wave energy losses sustained through friction and refraction over the continental shelf. These modifications were not allowed for in this study.

9. BASIS FOR CORRELATING SAMPLE HEIGHTS WITH HURRICANE CHARACTERISTICS

While the wave energy index, E, of Reid [1955], Eq.(1), is an adequate indication of the wave generating capacity of a stationary hurricane in a wide expanse of ocean, it fails to take into account the differing lengths of fetch in variable directions resulting from forward movement of the hurricane. In order to make use of the sample values of H and T derived from the detailed study of the nine selected hurricanes in any generalization of hurricane wave statistics it was necessary to correlate the wave heights and periods determined with some more satisfactory index of each storm's directional wave generating potential, with due regard to the storm's idiosyncrasies in crossing a given tract of water.

This problem may be approached by reverting to the fact, pointed out by Reid and Bretschneider [1953], by Reid [1955] and again by Bretschneider [1956], that, for hurricane conditions, the dimensionless parameters $\frac{gH}{U^2}$ and $\frac{gF}{U^2}$ are statistically related by an equation which approximates to

$$\frac{gH}{U^2} = 0.0026 \left(\frac{gF}{U^2}\right)^{\frac{1}{2}} . \tag{11}$$

This implies that

$$H \propto U\sqrt{F} . \tag{12}$$

It is possible to show from Eq.(6) that maximum wind velocity in a hurricane is proportional to $(\Delta p)^{1/2}$, whence from Eq.(12)

$$H \propto \sqrt{(\Delta p)\, F} . \tag{13}$$

This result is the basis of Eq. (1) if the fetch is regarded as stationary and proportional to R, the radius to maximum winds. However, the circumstance of the movement of a hurricane along some arbitrary path may be expected seriously to detract from the validity of Eq.(1) since F is then largely a function of that path.

To illustrate this, the application is shown in Fig. 12 of an HtFT forecasting diagram [Wilson, 1955] to the idealized windfield of a hurricane. A uniform (root mean square) wind velocity, U_r, is considered to prevail over an effective fetch, ab, of length 4R representing a wind velocity profile through the hurricane. For such a uniform wind it is sufficient to consider only those forecasting curves $H_U(F)$, $T_U(t_d)$ and $F_U(t_d)$ which specify significant wave height, period and fetch, as functions of the indicated variables applicable at the constant wind velocity U_r.

The hurricane is assumed to be crossing coastline A at a time when three-quarters of its effective fetch, aO, is over water. Along a particular direction line leading to coastline B_1, the storm is assumed to advance, in the first instance, at velocity V_1, resulting in the space-time wind-field shown stippled behind the line of advance ac. Waves originating as ripples at O propagate along Oc until they leave the wind area at C with maximum height H_1, and period T_1, given by points d and e.

In a direction along which the extent of ocean may be AB_2, at right angles to the true line of advance of the storm, there can obviously be no forward advance of the wind system ($V_2 = 0$), with the result that the wind-field in this case covers an area directly below ab in Fig. 12, behind the line af. In consequence the waves which start from O now leave wind domination at f with maximum height H_2 and period T_2 corresponding to points a and g.

Finally, in yet another direction, giving an extent of ocean AB_3, the wind although blowing in the direction of B_3, may be receding due to the recession of the storm at velocity V_3 along the line ah. Waves originating at O and travelling along oh thus pass out of the wind at h with maximum height H_3 and period T_3, as given by points k and 1.

These several examples serve to show the importance of the actual fetch lengths Od, Oa, and Ok on the wave height and period - fetch lengths which depend on the movement of the storm and the particular direction being considered as well as upon the basic (stationary storm) fetch, somewhat arbitrarily taken as 4R. In practice then, F in Eq.(13) may be conside to be some function of R and ΔF where ΔF corresponds to such increm of fetch as ad or ak in Fig.12.

In place of E of Eq.(1) as a criterion of the wave generating capacity a hurricane, the parameter $\sqrt{(\Delta p)(2R + \Delta F)}$ was therefore adopted for

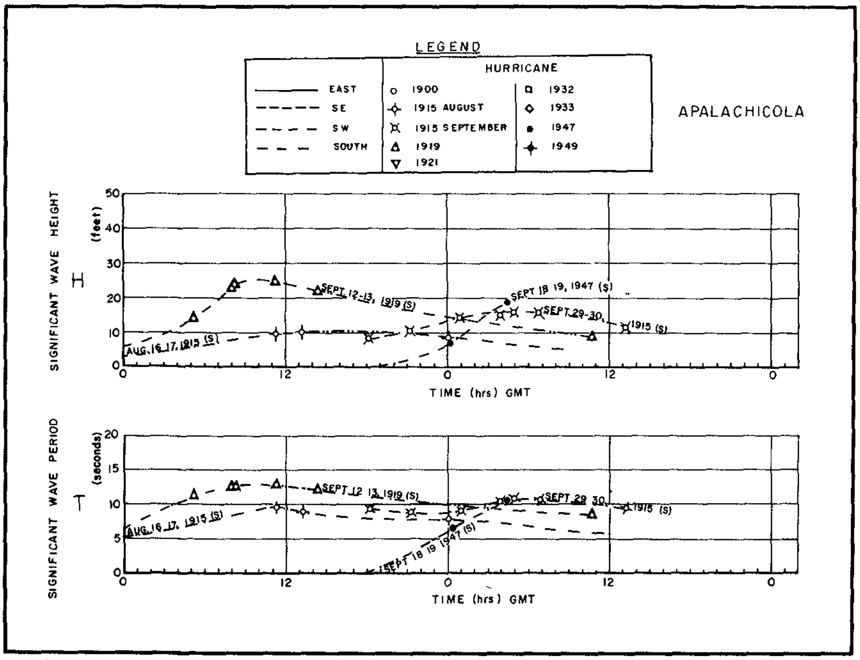

Fig. 11. Envelope Curves of Maximum Significant Wave Height and Period as Functions of Arrival Time at Deep-Water (100 Fathoms) Stations off Apalachicola.

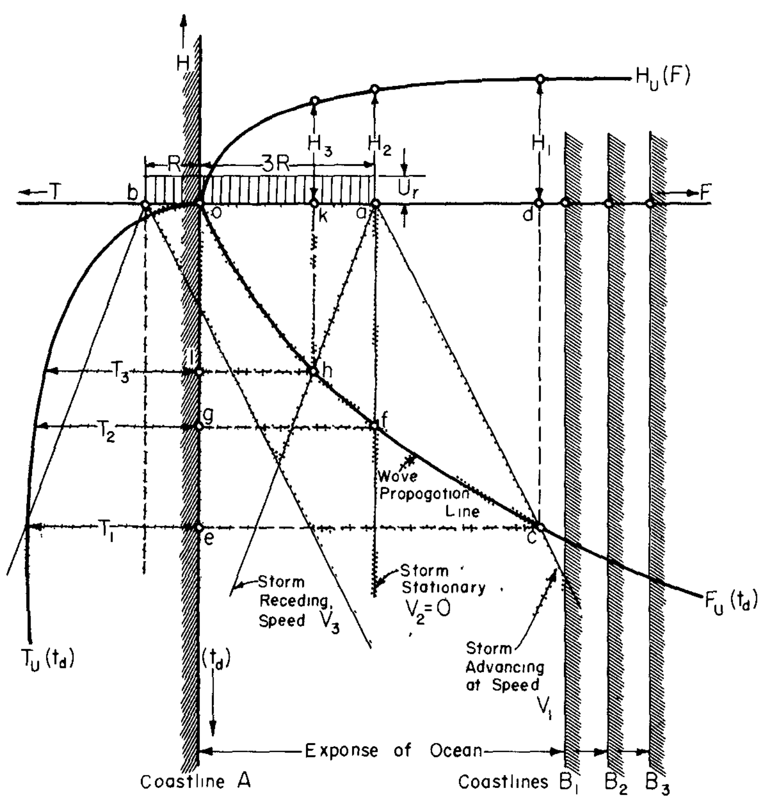

Fig. 12. Schematic Diagram of Directional Effect on Hurricane Fetches or Wind Fields.

purposes of seeking a constant of proportionality in the statement of Eq.(13). The aim here was to make it possible for wave height H to be determined for additional storms (other than the selected nine), for which the characteristics (Δp) and R were known. Since particulars of the fetches for these additional storms, in the absence of detailed analysis, would have to be estimated, the basis of estimation that would have to be applied in deriving ΔF, was determined from experience with the nine hurricanes analyzed (for which the true fetches were known).

Values of F (= $2R + \Delta F$) assigned in this way were incorporated into the parameter $\sqrt{(\Delta p)\, F}$ and plotted against maximum significant wave heights, H, obtained from the envelope curves, such as Fig. 11. The expected linear relationship evolves in Fig. 13(a).

10. BASIS FOR CORRELATING SAMPLE PERIODS WITH HURRICANE CHARACTERISTICS

It has been shown [Wilson, 1955] that the statistical deep water relationship between the ratio of wave phase-velocity, c, to wind velocity, U, and the parameter $\frac{gF}{U^2}$ can be fitted satisfactorily by an equation of the form:

$$\frac{c}{U} = 1.40 \tanh \left\{ \frac{4.36}{100} \left(\frac{gF}{U^2} \right)^{\frac{1}{3}} \right\} . \tag{14}$$

For the same (hurricane) conditions prescribed in deducing Eq.(11), the value of the hyperbolic tangent in Eq.(14) approximates to the angle, thereby simplifying the expression to

$$\frac{c}{U} \propto \sqrt[3]{\frac{gF}{U^2}} . \tag{15}$$

Since $c \propto T$ for deep water conditions and $U_{max} \propto \sqrt{\Delta p}$ the expression (15) further resolves to

$$T \propto \sqrt[3]{F \sqrt{\Delta p}} . \tag{16}$$

Determination of the constant of proportionality in Eq.(16) provided the means of finding T for all hurricanes not analysed whose characteristics R and (Δp) were known. The best-fit regression line in Fig. 13(a) was used to advantage in determining more refined values of F to be used opposite values of H and their corresponding values of maximum T, as found from the envelope curves (such as Fig. 11). In this way the parameters $\sqrt[3]{F\sqrt{\Delta p}}$ were computed and plotted against T for the nine selected hurricanes. Again a satisfactory regression line was obtained and the underlying principles confirmed.

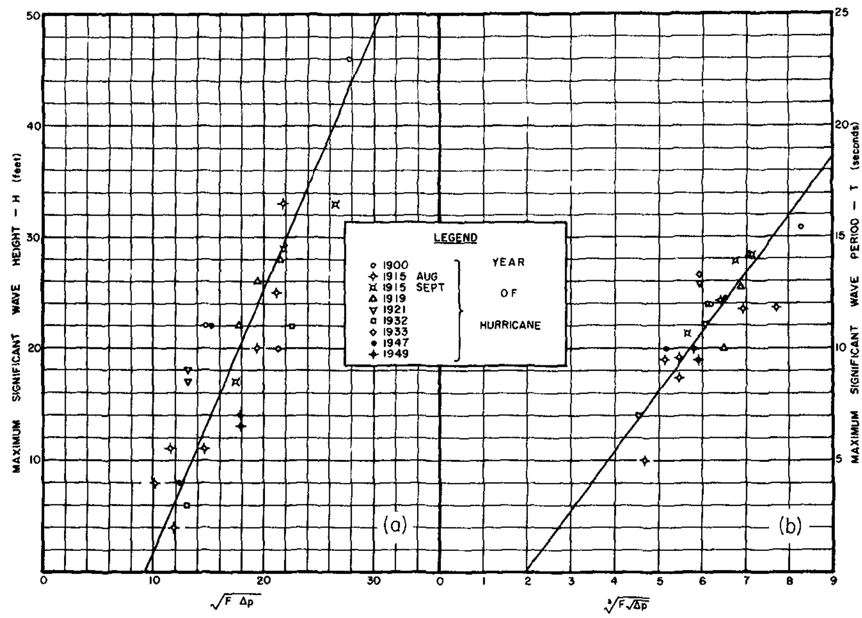

Fig. 13. Correlation of Hindcast Wave Heights and Periods with Parameters Involving Hurricane Characteristics and Movements.

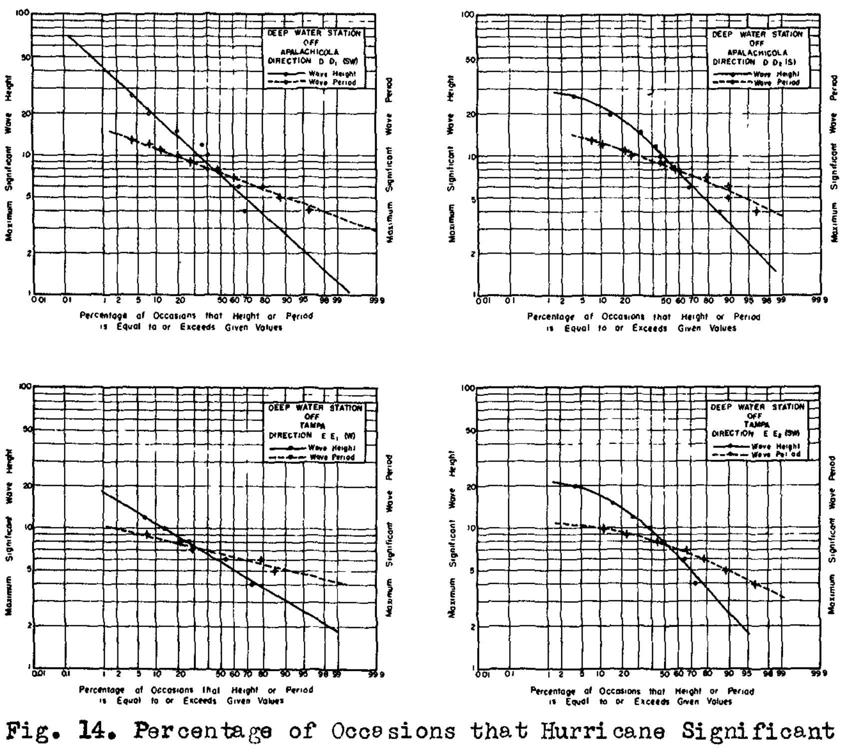

Fig. 14. Percentage of Occasions that Hurricane Significant Wave Heights and Periods Equal or Exceed a Given Value along Directions: DD_1, DD_2 (Apalachicola); EE_1, EE_2 (Tampa). Height in feet; period in seconds.

11. EXTENDING THE SAMPLE OF SIGNIFICANT WAVE HEIGHTS AND PERIOD

From the regression lines of Figs. 13 it was possible without further detailed hindcasts of hurricane waves to increase the sample 'population' of wave heights and periods in the eleven directions bearing on the five stations A to E The mechanism for doing this was simply to estimate the applicable fetches F in the various directions for an additional 23 hurricanes whose characteristics R and Δp were known. The values of F were judged on the same basis as had been done for the selected hurricanes with due regard to the tracks followed by the storms in relation to the eleven directions to the shore stations. The charts of hurricane tracks in the Gulf of Mexico from 1901 to 1943, given by Tannehill [1944], were found invaluable for this purpose.

The judgement of F was necessarily subjective; for this reason the estimations were made only by one person (the author) on the strength of experien gained in handling the windfields in the worked cases. It may at least be said th the estimated values of F for the 23 storms cited were derived in comparable fashion to the values of F adopted for the 9 selected storms. It is a fair conclusion also, since the parameters $\sqrt{F\ \Delta p}$ and $\sqrt[3]{F\sqrt{\Delta p}}$ for the 9 hurricanes comply with the theoretico-empirical trends in relation to H and T respectively, that the same trends will be obeyed by these parameters as found for the 23 additional storms.

The resultant statistics were plotted as the percentage of occasions that hurricane waves equalled or exceeded stated heights or periods at the deep water limit along the various approach directions to the shore stations. Fig.14 is a sample of these plots for Apalachicola and Tampa. In quite a number of cases the points, plotted on log probability paper, conformed well to straight line (normal) distributions. Best-fit regression lines, drawn through the plotte points, may be considered to have improved still further the adequacy of the sample 'populations' upon which the further statistics are based. Data equivalent to the above have been tabulated in Tables AI and AII (Appendix A).

12. FREQUENCY OF OCCURRENCE OF HURRICANE WAVES OF GIVEN SIGNIFICANT HEIGHT OR PERIOD

As remarked earlier, Tannehill [1944] has recorded and charted the tracks of some 112 tropical storms which entered the Gulf of Mexico between 1900 and 1943. Exclusive of the 9 selected, and 23 additional, full hurricanes already considered, the balance of these storms were examined for their capacity to generate waves in the several approach directions to the shore stations A to E, taking into consideration the tracks followed. It was possibl to determine when shoreward generation of waves in the various directions could, or could not, have taken place.

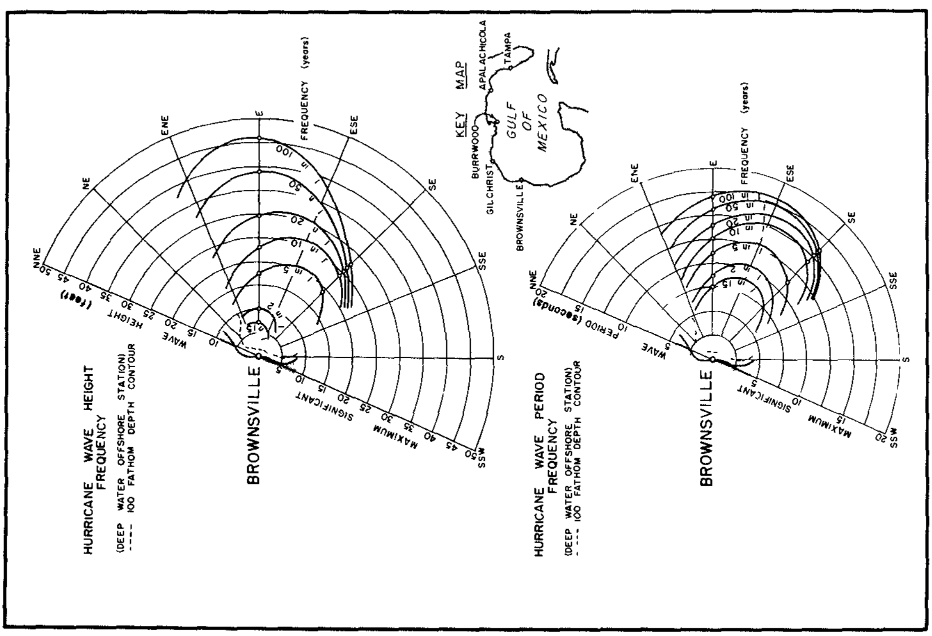

Fig. 15. Frequencies of Occurrence of Hurricane Waves of Specific Significant Heights and Periods at Deep-Water (100 Fathoms) Stations off Brownsville, Texas.

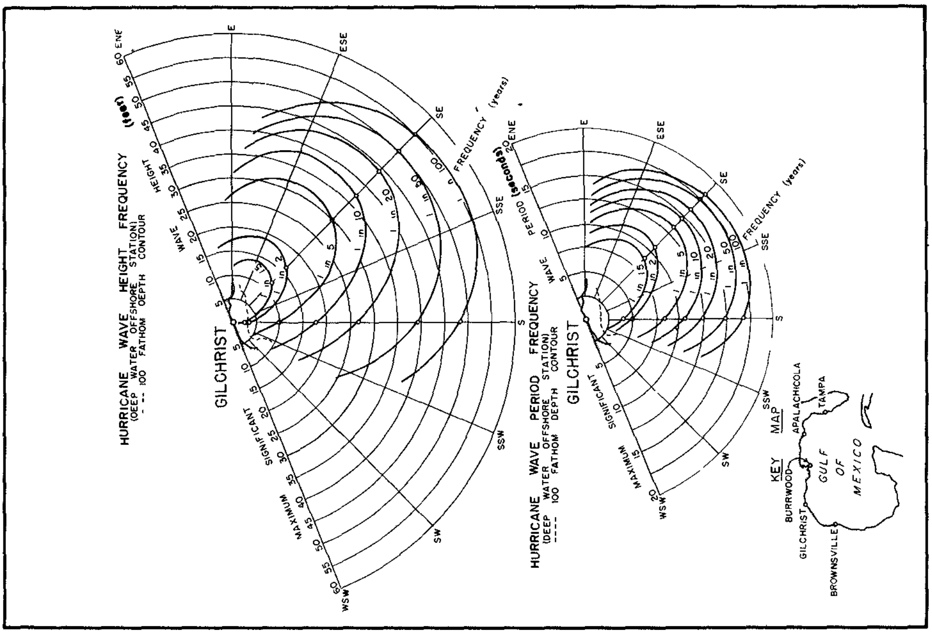

Fig. 16. Frequencies of Occurrence of Hurricane Waves of Specific Significant Heights and Periods at Deep-Water (100 Fathoms) Stations off Gilchrist, Texas.

From this it was possible to add up the total number of incidences, N, that tropical storm waves had been experienced from a given direction for all 112 storms referred to. However, of these 112 storms only 66 have been rated as worthy of being retained in the category of hurricanes so that 66/112 of the number of incidences, N, of tropical storm waves in 44 years in any of the chosen directions will represent the frequency of occurrence of hurricane waves in this length of time.

Denoting 1 in n years as the equivalent of this frequency, then

$$n = \frac{44}{\frac{66}{112} N} \quad \text{or} \quad \frac{74.6}{N} \text{ years} . \tag{17}$$

(Values of N and n are given in Table AI, Appendix A). Further, if f be the percentage of hurricane wave occasions for which H (or T) equals or exceed: a certain value (such as specified by the regression lines in Fig. 14), then it may be expected that hurricane waves of this height (or period) will be experienced once in $\frac{100n}{f}$ years. If this be written as once in m years, whe m has successive values 1, 1.5, 2, 5, 10, 20, 50, 100 years, then

$$f = \frac{100n}{m} \% . \tag{18}$$

Tabulation of values of f is given in Tables A III (Appendix A).

The final step in the compilation of hurricane wave statistics involved interpreting values of f in terms of the corresponding significant wave height H (or periods T) by reading from the regression lines such as Fig. 14. Table AIV and AV (Appendix A) list the applicable values of H and T respectivel

To condense the results into easily comprehensible form, polar diagra of the frequency (1 in m years) of hurricane (significant) wave heights and periods are presented in Figs. 15 to 19. These are based directly on the dat: of Tables AIV and AV. To take Fig. 17 as an example, the two polar diagr therein give isolines of the frequency (1 in m years, where m is successive 1, 1.5, 2, 5, 10, 20, 50, and 100 years), upon polar co-ordinates of which concentric semi-circles (or radius values) represent magnitudes of significa wave height (or period) and radial lines (or bearings) represent approach directions toward the shore station at Burrwood, Miss. The contours of frequency have been interpolated from the plotted points for directions CC_1, CC and CC_3 so as to cover all directions from which waves of any consequence might be expected.

It may be inferred from the frequency curves of significant wave height for Burrwood (Fig. 17), to continue the example, that the chances of getting 35 ft. high deep-water significant waves from the south is 1 in 100 years.

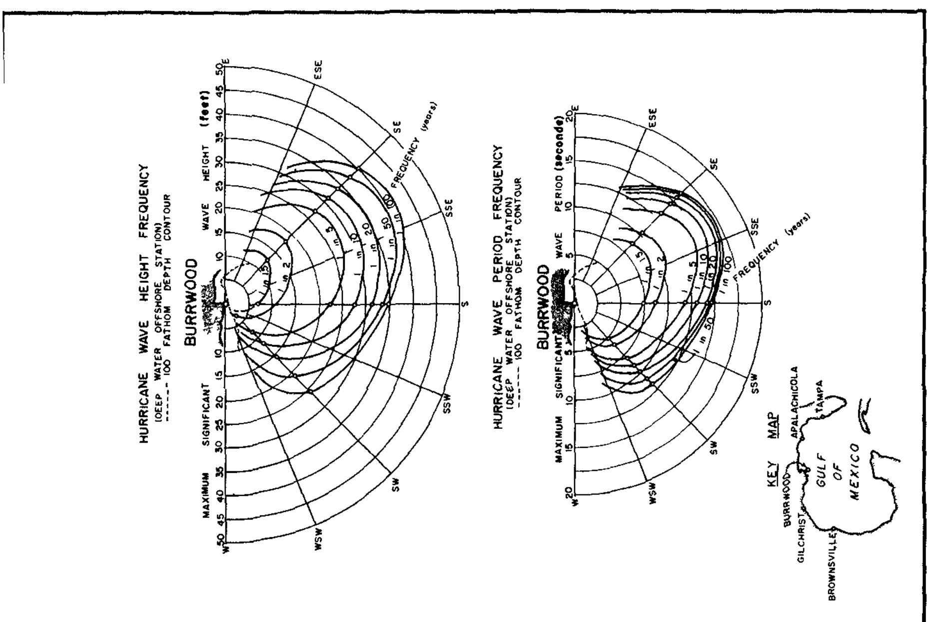

Fig. 17. Frequencies of Occurrence of Hurricane Waves of Specific Significant Heights and Periods at Deep-Water (100 Fathoms) Stations off Burrwood, Mississippi.

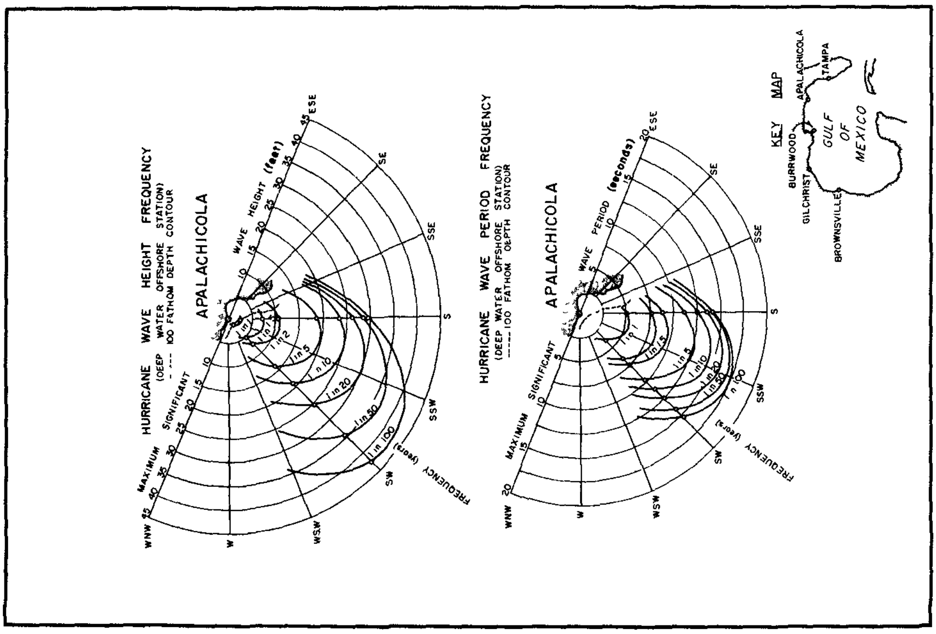

Fig. 18. Frequencies of Occurrence of Hurricane Waves of Specific Significant Heights and Periods at Deep-Water (100 Fathoms) Stations off Apalachicola, Florida.

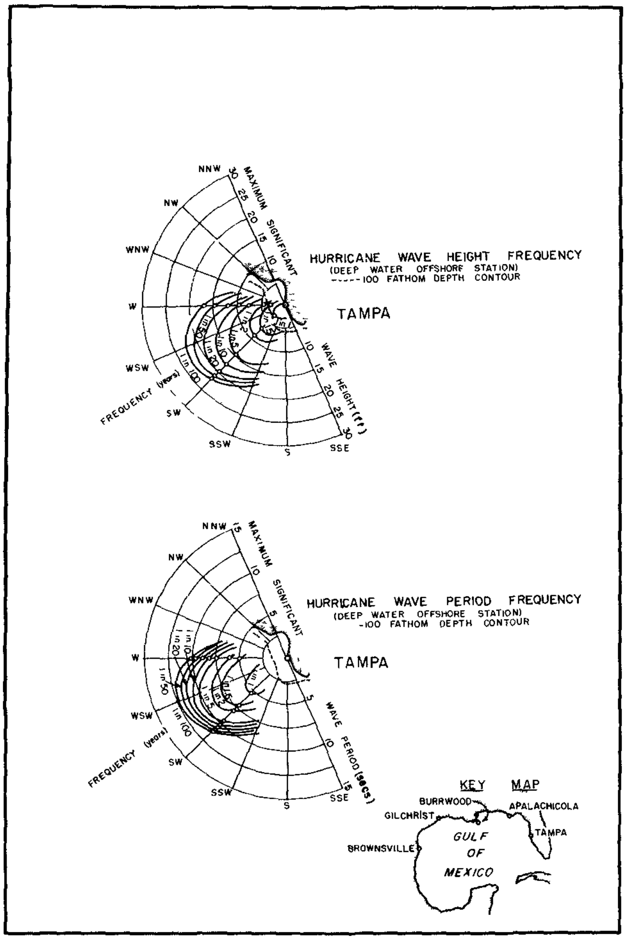

Fig. 19. Frequencies of Occurrence of Hurricane Waves of Specific Significant Heights and Periods at Deep-Water (100 Fathoms) Stations off Tampa, Florida.

From SSE, however, the chances are 1 in 20 years. The highest waves of all are likely to come from a direction between SE and SSE and may be as much as 42 feet once in 100 years or as high as 28 feet once in 5 years. Once in 2 years waves as high as 18.5 feet may be expected from the south-east. The significant wave periods corresponding to these latter heights would be 17 secs once in 100 years or 14 secs once in 5 years from the direction between SE and SSE; 10.4 secs once in 2 years from the SE.

It should be noted that the quoted frequencies refer strictly to waves generated in the Gulf by full hurricanes and do not preclude the possibility of existence of waves of comparable magnitudes generated by frontal storm systems which do not classify as hurricanes or tropical storms.

13. CONCLUSIONS

Comparing the polar frequency diagrams for the five stations (Figs. 15 to 19) it is found that Gilchrist (Galveston area) has the expectation of highest hurricane waves. At frequencies of once in 2 years and oftener, however, wave heights are as great at Burrwood as at Gilchrist and, therefore, along the intermediate coastline. Waves of considerable height may be expected from the SE near Gilchrist and from the SSE to SE near Burrwood at somewhat rare intervals. Once in 5 years significant wave heights in these deep water areas will reach about 30 feet; once in 2 years about 19 feet.

The comparative vulnerability of the five (deep-water) stations to hurricane waves may be listed in the following order:

1. Gilchrist, Texas
2. Burrwood, Mississippi
3. Brownsville, Texas
4. Apalachicola, Florida
5. Tampa, Florida

Brownsville and Apalachicola, in the above, actually have about equal susceptibilities. Tampa, it can readily be seen, is well protected from hurricane wave attack by virtue of its position in the Gulf in relation to the tracks usually followed by hurricanes.

14. ACKNOWLEDGEMENT

Submission of this paper, which is a shortened and refined version of Technical Memorandum No. 98, recently published by the Beach Erosion Board, Corps of Engineers, U.S. Army, is made possible through the courtesy of that body. The assistance of Codie S. Wells in the work of hindcasting the hurricane waves is gratefully acknowledged.

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APPENDIX A

TABLE A 1

PERCENT OF OCCASIONS THAT HURRICANE WAVE HEIGHTS EQUAL OR EXCEED GIVEN VALUES IN VARIOUS DIRECTIONS

No. of Occasions H ≥ Value Below	Approach Direction										
	AA_1	AA_2	BB_1	BB_2	CC_1	CC_2	CC_3	DD_1	DD_2	EE_1	EE_2
2	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
4	75.0	88.9	67.7	94.1	70.6	90.0	87.5	68.0	85.7	73.3	73.0
6	75.0	83.3	41.7	94.1	47.0	80.0	79.2	64.0	67 9	53.3	65.4
8	75.0	61.1	33.3	87.3	41.2	65.0	79.2	40.0	60.7	26.7	50.0
10	75.0	61.1	33 3	76.5	29.4	60.0	75.0	40.0	46.4	13.3	38.4
12	75.0	50.0	25.0	64.7	11.8	50.0	70.9	36.0	42.9	6 7	26.9
15	75.0	27.8	25.0	58.8	5 9	45.0	62.5	20.0	32.1	0.0	15.4
20	50.0	16.7	16.7	41.2	5.9	30.0	45.8	8.0	14.3	-	3.8
27	0 0	5.5	8.3	35.3	0.0	5.0	20 8	4.0	3.6	-	0.0
35	-	0.0	0.0	5.9	-	5.0	4.2	0.0	0.0	-	-
45	-	-	-	5.9	-	0.0	0.0	-	-	-	-
Total No. of Occasions of Waves in 44 Years from 112 Tropical Storms N	27	70	46	69	50	70	73	77	80	55	86
Equivalent No. of Occasions of Full Hurricane Waves from 66 Hurricanes in 44 Years (N x 66/112)	16	41	27	41	29	41	43	45	47	32	51
Frequency of Occurrence Once in n years n	2.75	1 07	1.63	1.07	1.52	1.07	1.02	0.98	0.94	1.37	0.86

TABLE A II

PERCENT OF OCCASIONS THAT HURRICANE WAVE PERIODS EQUAL OR EXCEED GIVEN VALUES IN VARIOUS DIRECTIONS

No. of Occasions T ≥ Value Below	Approach Direction										
	AA_1	AA_2	BB_1	BB_2	CC_1	CC_2	CC_3	DD_1	DD_2	EE_1	EE_2
4	100.0	100.0	100.0	100.0	100.0	100.0	100.0	96.0	96.4	100.0	96.3
5	100.0	100.0	100.0	100.0	94.0	95.0	95.8	88.0	89.3	85.7	88.9
6	75.0	94.5	66.7	93.7	82.5	90.0	91.7	80.0	89.3	78.0	77.8
7	75.0	61.1	50.0	87.5	47.0	75.0	83.4	60.0	78.5	28.6	66.8
8	75.0	61.1	33.3	81.3	23.5	55.0	79.2	48.0	57.2	21.4	44.4
9	75.0	50.0	25.0	68.7	23.5	50.0	75.0	28.0	46.4	7.1	22.2
10	75.0	38.9	25.0	68.7	0.0	40.0	66.7	20.0	25.0	0.0	11.1
11	75.0	33.3	8.3	50.0	-	35.0	50.0	12.0	21.4	-	0.0
12	50.0	11.1	8.3	25.0	-	25.0	29.2	8.0	10.7	-	-
13	0.0	5.5	8.3	25.0	-	5.0	12.5	4.0	7.1	-	-
14	-	5.5	8.3	18.7	-	0.0	8.3	0.0	0.0	-	-
15	-	5.5	0.0	6.3	-	-	8.3	-	-	-	-
16	-	0.0	-	6.3	-	-	4.2	-	-	-	-
17	-	-	-	0.0	-	-	0.0	-	-	-	-

TABLE A III

PERCENTAGE FREQUENCIES OF OCCURRENCE (f) OF HURRICANE WAVES IN VARIOUS DIRECTIONS OF HEIGHT (OR PERIOD) GREATER THAN OR EQUAL TO A GIVEN VALUE

Occurrences of Full Hurricane Waves 1 in m Years	Approach Direction										
	AA_1	AA_2	BB_1	BB_2	CC_1	CC_2	CC_3	DD_1	DD_2	EE_1	EE_2
1	275.0	107.0	163.0	107.0	152.0	107.0	102.0	98.0	94.0	137.0	86.0
1.5	183.0	71.0	92.0	71.0	101.0	71.0	68.0	65.0	63.0	91.0	57.0
2	138.0	53.0	81.0	53.0	76.0	53.0	51.0	49.0	47.0	68.0	43.0
5	55.0	21.0	33.0	21.0	30.0	21.0	20.0	20.0	19.0	27.0	17.0
10	27.5	11.0	16.0	11.0	15.0	11.0	10.0	10.0	9.0	14.0	9.0
20	13.8	5.3	8.1	5.3	7.6	5.3	5.1	4.9	4.7	6.8	4.3
50	5.5	2.1	3.3	2.1	3.0	2.1	2.0	2.0	1.9	2.7	1.7
100	2.8	1.1	1.6	1.1	1.5	1.1	1.0	1.0	0.9	1.4	0.9

TABLE A IV

SIGNIFICANT WAVE HEIGHTS OF HURRICANE WAVES IN VARIOUS DIRECTIONS CORRESPONDING TO FREQUENCIES (f) OF TABLE A III

Occurrences of Full Hurricane Waves 1 in m Years	Approach Direction										
	AA_1	AA_2	BB_1	BB_2	CC_1	CC_2	CC_3	DD_1	DD_2	EE_1	EE_2
1	-	-	-	-	-	-	-	1.5	2.6	-	3.0
1.5	-	7.1	1.3	11.5	-	7.5	12.8	5.3	7.2	3.0	6.8
2	-	9.8	2.3	16.0	3.6	11.4	18.4	7.3	10.4	4.6	9.0
5	19	17.5	9.6	30.0	8.4	23.0	27.5	13.8	18.7	7.7	14.8
10	24	23.0	17.5	37.0	12.0	28.3	31.2	19.2	23.5	9.8	17.5
20	25.5	29.6	26.5	44.0	15.8	32.0	34.0	25.5	26.2	12.0	19.7
50	26.4	39.0	39.0	50.0	21.5	34.0	37.7	34.6	28.5	14.8	21.2
100	27.5	46.0	48.0	55.0	26.0	35.0	40.0	42.5	29.5	17.0	21.6

TABLE A V

SIGNIFICANT WAVE PERIODS OF HURRICANE WAVES IN VARIOUS DIRECTIONS CORRESPONDING TO FREQUENCIES (f) OF TABLE A III

1	-	-	-	-	-	-	-	3.9	5.0	-	5.4
1.5	-	7.6	4.0	9.0	-	7.4	8.8	6.6	7.7	5.0	7.2
2	-	8.7	5.0	10.5	6.1	8.7	10.3	7.5	8.8	6.0	8.0
5	11.3	11.1	8.4	13.4	8.0	11.8	13.2	9.8	11.2	7.4	9.6
10	14.3	12.6	10.6	14.7	9.0	13.1	14.6	11.2	12.6	8.2	10.1
20	15.3	14.0	12.5	16.0	9.9	14.2	15.4	12.6	13.6	8.8	10.5
50	15.8	15.7	14.9	17.4	11.0	14.6	16.0	14.3	14.2	9.6	10.8
100	16.0	16.9	16.8	18.2	11.8	14.7	16.1	15.4	14.3	10.2	11.0

CHAPTER 5
THE HURRICANE SURGE

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ABSTRACT

The landfall of a hurricane is generally accompanied by an increase in the tide level by four to fifteen feet above the normal value for the time and place at which the storm crosses the shoreline. This difference between the observed tide and that predicted from astronomical considerations is called the storm surge. The hydrodynamic theory of these disturbances has not been worked out in sufficient detail to permit a satisfactory theoretical approach to the storm surge prediction problem. Hence, the best guide to the probable behavior of future hurricane surges is believed to be the study of the effects of past hurricanes on sea leve

A gradual rise in tide level above predicted values may begin more than 24 hours before the storm makes its nearest approach to the station. Occasionally, the tide falls below normal for many hours during the approach of the storm. A rapid rise generally begins about the time gale winds associated with the hurricane are first experienced. The peak surge at any location along the shore usually occurs within an hour or tw after the nearest approach of the storm to the station. The maximum surg generally occurs somewhat to the right of the storm track, and the zone o extremely high water usually extends further to the right than to the lef of the storm track. The fall in water level after the storm is more rapi than the rise in areas with good drainage, but in marshland a week or mor may be required for the water level to return to normal.

The maximum height of the storm surge along the open coast is clearl a function of the storm intensity, but this factor alone is not sufficien to explain more than half of the observed variance in the reported peak storm tides. Topographic effects, such as the funneling of water in converging bays can alter the amplitude of the surge by a factor of two in a distance of only a few miles. This fact, together with the tendency of severe hurricanes to destroy the tide gages near their centers, and the difficulty of eliminating the effects of surface wind waves in interpreti high water marks, make it difficult to determine exactly the nature of pa storm surges.

THE HURRICANE SURGE

1. INTRODUCTION

The approach of a hurricane to the shore is accompanied by an increase in the tide level above the normal value. For storms which barely qualify as hurricanes, or which do not cross the coast, this increase may be no more than four feet. For the more intense hurricanes the tides may rise more than fifteen feet above normal.

The effects of storms and normal tides on sea level are almost independent along the open coast. Thus, it is convenient to consider these two effects separately. The storm surge is defined as the difference between the actual tide as influenced by a meteorological disturbance and the tide which would have occurred in the absence of the meteorological disturbance. The term storm tide is used to describe the observed tide at a time when this is significantly affected by meteorological factors.

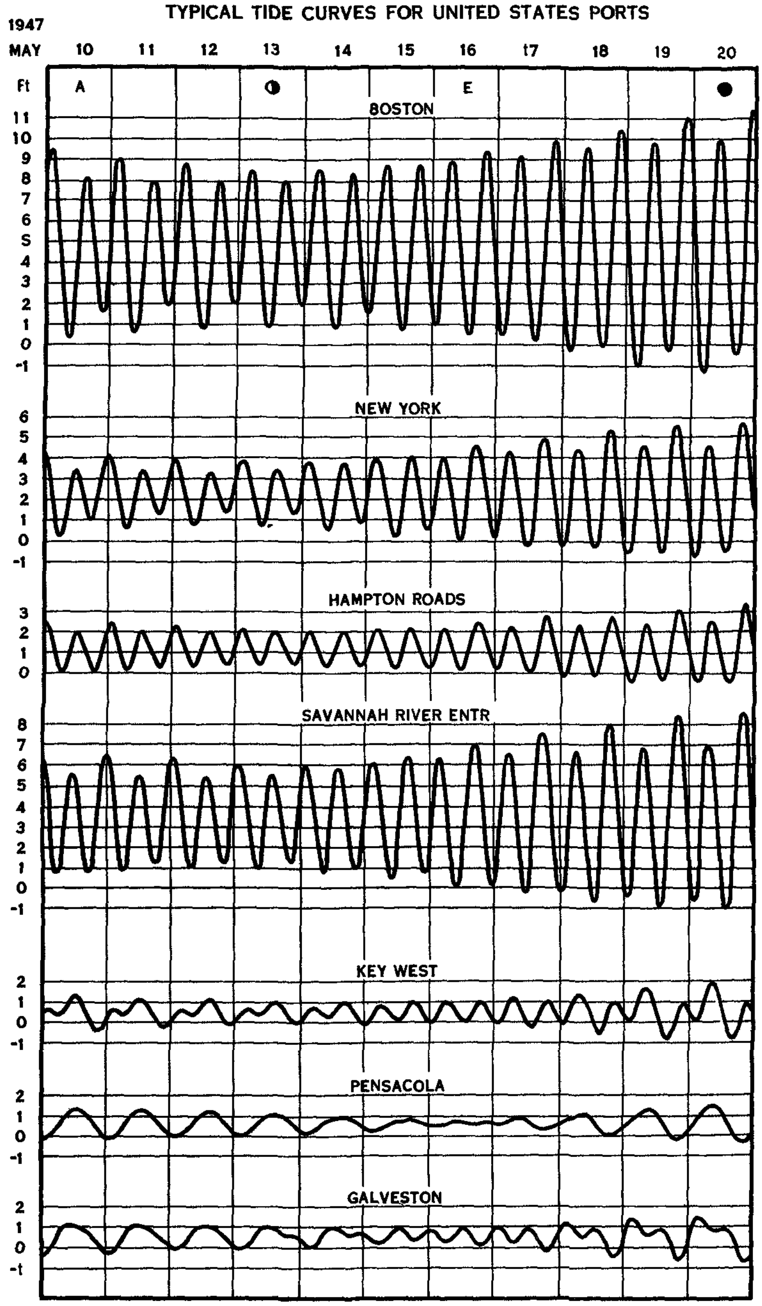
The practical importance of a given storm surge will depend on the stage of the normal tide at the time of the storm tide and on the elevation of the land in the region of the storm. Waves and swell, with periods of only a few seconds, add greatly to the damages caused by flooding in regions that are inundated by storm tides. The effects of these short-period waves also interfere with the collection of data on the actual tide elevations during a storm and considerably handicap all studies of actual storm tides. Thus, it is necessary to consider the normal tides, the short-period waves, and the elevation of the land near the coast in any discussion of the practical importance of storm surges.

2. THE NORMAL TIDE

Before entering into a discussion of the tide abnormalities caused by hurricanes, it is worthwhile to describe the major features of the normal behavior of the tide.

The normal tide is a regular quasi-periodic rise and fall of the level of the sea having periods of approximately 12.5 and 25 hours. The principle cause of the tide is the difference between the gravitational attraction of the sun and moon for the waters of the earth and for the solid earth. Meteorological factors such as land and sea breezes, the annual cycle of atmospheric pressure and wind systems also play a role in the normal tide. Tidal theory can describe completely the nature of the astronomical forces producing the tide. Theory cannot predict the manner in which the sea will respond to these forces. Practical tide predictions are based on an analysis of observed water level variations in the region for which the tide predictions are desired. Consequently, the published predictions of the normal tide include climatic as well as astronomical effects on sea level.

The amplitude of the diurnal and semidiurnal oscillations of sea level and the interval between successive high and low waters vary with the phase and declination of the moon, and the distance between the moon, the sun, and the earth in such a way that it is not satisfactory to think of the tide



Lunar data max S declination, 9th, apogee, 10th, last quarter, 13th, on equator, 16th, new moon, 20th, perigee, 22d, max N declination, 23d

Fig. 1. Typical tide curves for United States ports., (from U.S.C. & G.S. Tide Tables East Coast).

as a regular periodic oscillation having some mean range. Figure 1, taken from the Coast and Geodetic Survey Tide Tables, shows a number of typical tide curves. In general, the tide range is greatest at about the time of full moon and new moon and least at the time of first and third quarters, and greater at perigee when the moon is nearest the earth than at apogee when the moon is at its greatest distance from the earth. In the Gulf of Mexico the greatest range occurs when the moon is near maximum declination and the least range occurs as the moon crosses the equator.

In the United States, and in most other countries, tide predictions are usually made by using an analog computer to sum the series:

$$h = \bar{h} + \sum_{n=1}^{M} B_n \text{ Cos } (A_n t + D_n) \qquad (1)$$

where: h is the predicted sea level at any time.

$\bar{h}$ is the height of mean sea level above the datum plane. The datum plane used along the Atlantic and Gulf coasts of the United States is mean low water.

B_n is the amplitude of the n'th constituent of the tide. (Determined from observations and modified by theory).

A_n is the speed of the n'th constituent of the tide, usually expressed as degrees per hour. (Determined from theory).

D_n is the phase displacement or epoch of the n'th constituent as of 0000 January 1 of the year of the predictions. (Determined in part from theory and in part from observations).

M is the total number of constituents considered.

t is the time, usually expressed in hours since 0000 January 1.

Equation (1) is designed for the computation of the effect of astronomical forces in causing the sea level to rise above or fall below its long term mean value. Local mean sea level varies from year to year because of changes in the proportion of onshore and offshore winds, (DeVeaux - 1955), and perhaps also because of crustal movements of the earth, the melting and reformation of glaciers, the average temperature of the water, and a host of other causes. If an adjustment is made to the constant term, $\bar{h}$ so that it equals the observed mean sea level for the year of predictions, the predictions will usually agree with observations within about 0.2 ft. at the times of high and low tide. Equally good agreement will be found at any phase of the tidal cycle at most stations along the open-coast. At inland stations, the agreement will not be as good near mid-tide as at the time of high and low tide.

Hurricane Surge

The 1958 Tide Tables, East Coast North and South America, contains daily predictions of high and low water for 28 locations along the Atlanti and Gulf coasts of the United States. A satisfactory estimate of the astr nomical tide at any time for many of the tide stations near the open coast can be obtained by fitting a sine curve to the published predictions of hi and low water as described in Table 3 of the Tide Tables. This procedure is often unsatisfactory in rivers, canals, and long bays such as Long Isla Sound. In such regions the tide curve may depart markedly from a sine cur A direct summation of equation (1) will improve the predictions in such cases, but even this will not always be satisfactory.

Most users of the tide tables are primarily interested in the time an height of the tide at high and low waters. An approximation to this data, valid for almost all purposes, can be determined for several hundred additional locations by using the table of differences published in the Ti Tables. It must be remembered, however, that these differences are average and that some of them are based on much less than the optimum amount of da Consequently, the approximate prediction of high and low water elevations obtained in this manner may sometimes differ from the true astronomical ti by several tenths of a foot, and the time may differ by as much as an hour This difference in the time of high and low water may lead to a difference of one or two feet in the estimated height of the tide at some specified time between high and low water.

3. THE STORM SURGE

The high winds and low pressures associated with hurricanes usually lead to significant anomalies in the tide. A gradual rise in the tide level often begins more than 24 hours before the storm makes its nearest approach to the station. Occasionally, the tide falls below normal for many hours during the approach of the storm. A rapid rise generally begir about the time gale winds associated with the hurricane are first experienced. The peak storm surge usually occurs within an hour or two after tl storm makes its nearest approach to the station. In areas with good drair age conditions, the fall in the tide level is generally more rapid than tl rise and the tide often drops below normal for a few hours after the storr passes. In marshlands and other areas with poor drainage, many days may l required for the water levels to return to normal. The first storm surg peak is sometimes followed by a series of resurgences. The second storm surge peak, occurring several hours after the storm has passed, may be as high as the first. If the first storm peak occurs near the time of norma low water, and the second coincides approximately with normal high water, this second peak will be the more important (Redfield and Miller - 1955, Munk - 1955). Fortunately, the resurgences are not prominent in the regi of hurricane landfall but they may be important along the east coast of t United States for storms moving approximately parallel to the coast.

The storm surge moves through long bays, such as Long Island Sound, as a progressive wave. Consequently, the peak surge at the head of the b may occur many hours after the peak storm conditions(Harris, 1957a). The storm surge as a function of time at the tide station nearest the point of landfall is shown for four hurricanes in Figure 2, taken from Harris (1957b).

Analogues showing the effects of past storms are essential to an understanding of the hurricane surge. Several papers of this type have been published within the past few years.

Cline (1920 and 1926) gives a record of the observed tide at one or more stations and the highest observed tide or storm surge at a number of points for many Gulf of Mexico hurricanes. Redfield and Miller (1955) give similar data for several Atlantic Coast storms. Hubert and Clark (1955) give data on the peak storm tides or peak storm surges associated with 16 Atlantic and Gulf hurricanes, including most of the data previously published by Cline. Zetler (1957) has given an exhaustive tabulation of the peak storm surge recorded by the Coast and Geodetic Survey tide gage in Charleston,S.C., during a great many hurricanes. Harris (1957a) gives the time history of the surge at all Coast and Geodetic Survey tide stations affected by eight hurricanes. Additional data on past storms are being collected by the United States Army Corps of Engineers and many coastal Weather Bureau offices as well as the Central Office of the Weather Bureau. It is hoped that a more exhaustive collection of the records of the tides during the past hurricanes can be published within the next few years. The data contained in the above reports indicate that maximum storm surge heights usually occur somewhat to the right of the storm center and the region of above normal tides generally extends farther to the right of the storm center than to the left. Profiles of the storm surge along the open coast, for four hurricanes taken from Harris (1957b) are shown in Figure 3. Deviations in this pattern are occasionally produced by local topography.

4. HYDRODYNAMIC THEORY OF STORM SURGES

The complete equations of motion governing storm surge generation have never been solved in closed form. The existing solutions have all been derived by idealizing the problem in some way. An analytic solution can be obtained most readily by assuming that the sea is a rectangular lake of constant depth on a non-rotating earth, and the solution obtained in this way will have a close resemblance to the true solution in a great many cases. For other problems it may be better to assume that the sea is unbounded, or that the sea has only one boundary and the depth increases at a constant rate as one leaves the shore. The simplest solutions are obtained by assuming that flow can take place in only one horizontal direction. Solutions obtained in this way sometimes give excellent results for the storm surge generated in a long narrow lake or for the advection of a storm surge in a river or other narrow channel. Even in the great majority of cases in which flow is not one-dimensional, the one-dimensional equations reveal many of the major factors involved in storm surge generation.

If a steady wind blows parallel to the axis of a narrow channel long

Hurricane Surge

enough for equilibrium conditions to develop, the differential equation fc the slope of the free surface may be expressed as

$$\frac{\partial h}{\partial x} = \frac{\rho_a}{\rho_w} \frac{\gamma^2 V^2}{g(H+h)} \tag{2}$$

where: h is the height of water surface above the equilibrium plane.

x is the distance along the axis of the channel, with the wind blowing toward positive x.

ρ_a is the density of the air.

ρ_w is the density of water.

γ^2 is the wind stress coefficient, approximately 2×10^{-3}.

V is the wind speed.

H is the equilibrium depth of the water when no wind is blowing.

g is the acceleration of gravity.

A derivation is given by Keulegan (1953).

The total storm surge height, frequently called set up, in this sim case can be obtained by integrating equation (2) from a fixed boundary, o from a position at which, due to a low value for V or a high value for H the slope is virtually zero.

Although this simple situation rarely exists in nature, this equatic does serve to show that the slope of the free surface is related directly to a power of the wind speed and inversely to the total depth of water. This suggests that a given wind condition will produce a slightly lower s surge if it occurs at high tide than if it occurs at low tide. This dedu ion is supported by observations, Schalwijk (1947). Equation (2) also indicates, that when other conditions are equal, the highest surges will occur in regions in which the wind has a long fetch over relatively shal water. This also is generally supported by the observations, but suffici data to establish this empirically for hurricane conditions over open wat are not available.

Several empirical studies have shown a reasonably good fit between t slope of the water surface and some power of V different from 2 (Hellstr 1953, Darbyshire and Darbyshire 1956). The factor γ^2 is related to surfa roughness of the water; that is, to the wave height and wave velocity re tive to the wind. Neumann (1948) has suggested that this would lead to a decrease in γ^2 with increasing wind velocity and therefore to an expone

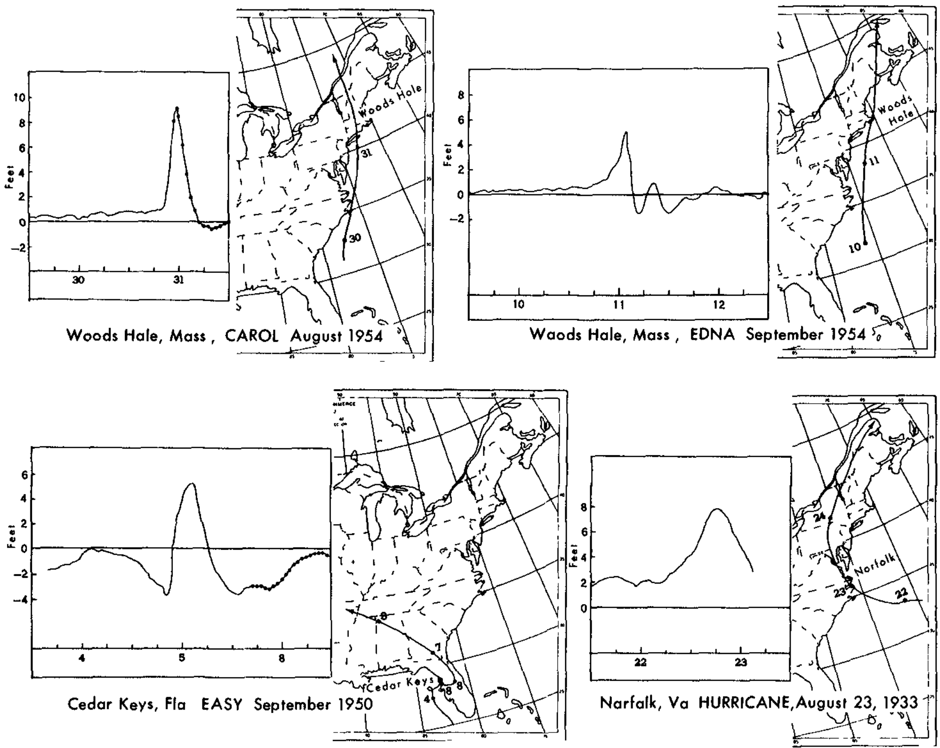


Fig. 2. The storm surge, as a function of time in the region of hurricane landfall.

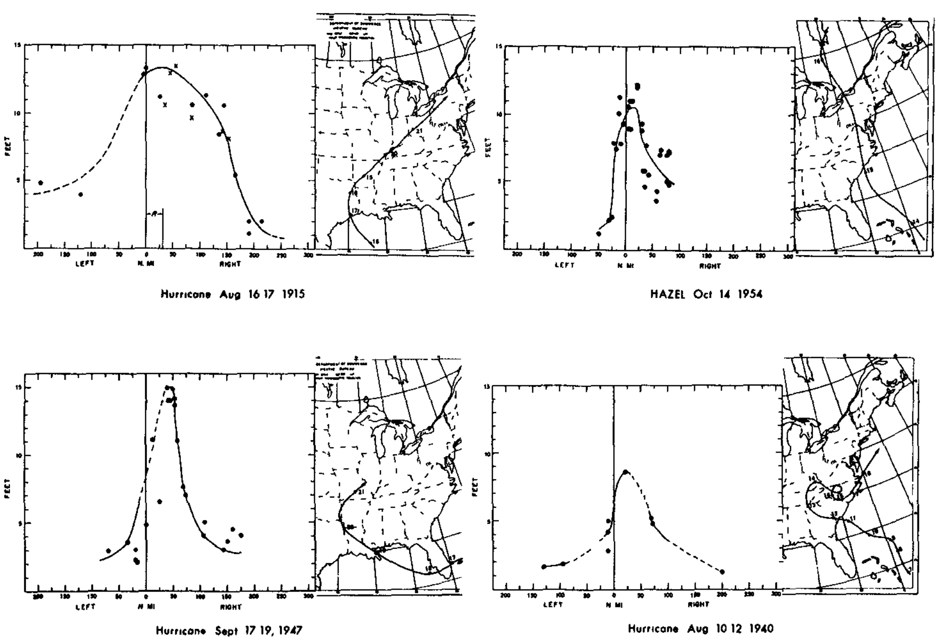


Fig. 3. Storm surge profiles along the coast.

of less than 2. Reid and Wilson (1954),in a study relating surge height to wind speed, have shown that an exponent dirrerent from 2 may arise, even though γ^2 is assumed to be constant, because of the laws governing V and H in the particular case. It is also likely that the data developed in man empirical studies, involving only a few cases and a restricted range of velocities, will fit a linear law as well as a square law.

Although equation (2) sometimes gives a valid representation of the storm surge on a lake or bay, it is necessary to consider several other factors in order to explain the storm surge which develops along the open coast. The finite size of the storm is clearly important. If the storm were stationary and no flow parallel to the shore were possible, the wind effect at any point on the shore would be a function only of the wind stres seaward of that point, and the profile of the peak storm surge values woulc approximately coincide with the profile of the wind stress component perpendicular to the shore. If flow parallel to the shore should occur, the peak would be somewhat flattened. These alongshore currents due to the gradient in the water elevation, brought about by variations in wind streng would reduce the water level below its equilibrium value near the peak and lift it above its equilibrium at some distance to either side of the peak.

The component of wind stress parallel to the shore would also generat an alongshore component of the ocean current. The Coriolis force acting o an alongshore current would cause an increase in water level to the right of the current.

The decrease in pressure in this stationary storm will be exactly com pensated for by an increase in water level of 13.2/12 ft. of water for eac inch of mercury.

All of the above effects will also be present in a moving storm, but in many cases the disturbing forces will not last long enough to permit th equilibrium value to be obtained. In others, the effects of resonance wil lead to the development of heights above the equilibrium value.

The presence of these dynamic factors is well illustrated in Figure 4 the storm surge curve for Galveston, Texas during hurricane AUDREY, June 1 The dashed line in the bottom of the figure gives the observed water level obtained by plotting hourly values. The solid line gives the storm surge obtained by subtracting the predicted tide height from the observed values The onshore component of the wind is shown by the solid line in the upper part of the figure, and the component of the wind parallel to the shore is shown by the dashed line. This component is considered positive when the shore is to the right of the wind vector. Notice that the wind had an off shore component during most of the period of increasing storm surge. Alth the wind data used here are those observed at the Weather Bureau Airport station in Galveston, they are representative of the true winds for at lea 50 miles on either side of Galveston during this storm. This figure indic

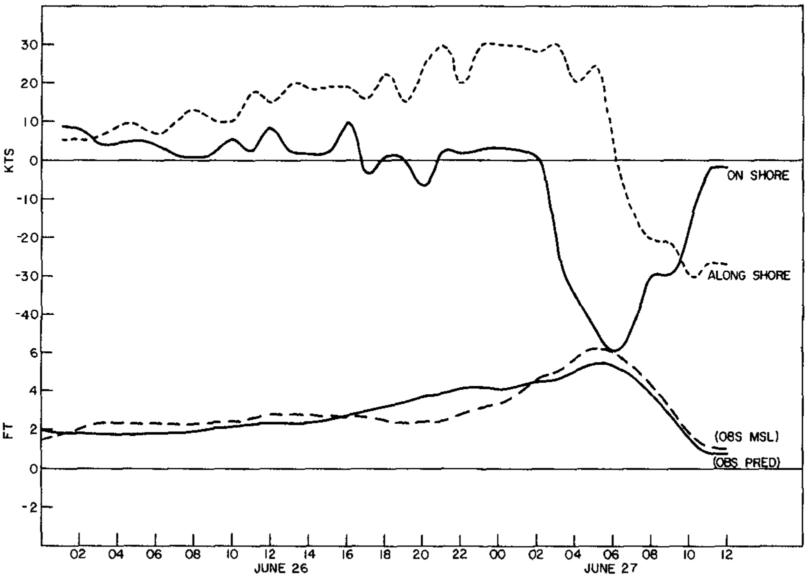
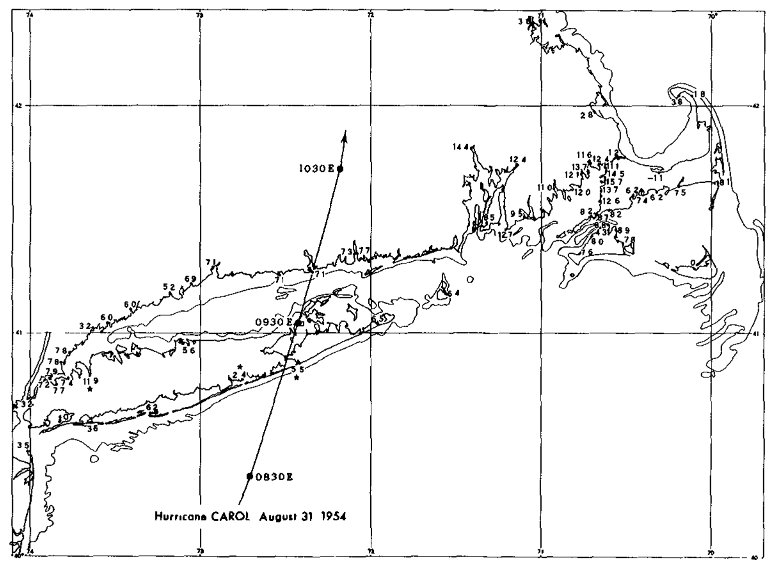


Figure 4. Tide and wind records at Galveston, Texas during hurricane AUDREY, Bottom: Dashed line - tide height above mean sea level, solid line - difference between observed and predicted tide height. Top: Dashed line - alongshore component of wind velocity, solid line - onshore component of wind velocity.



igure 5. The storm surge associated with the landfall of Hurricane CAROL, 1954. The storm surge, observed minus predicted tide, is shown in feet and tenths. The peak storm surge coincided in time with the normal high tide at most recording tide stations. It is assumed that this also holds true for high water data. It was not possible to obtain tide predictions for a few locations. The peak storm tide above mean sea level is shown for these stations, and they are indicated by a star.

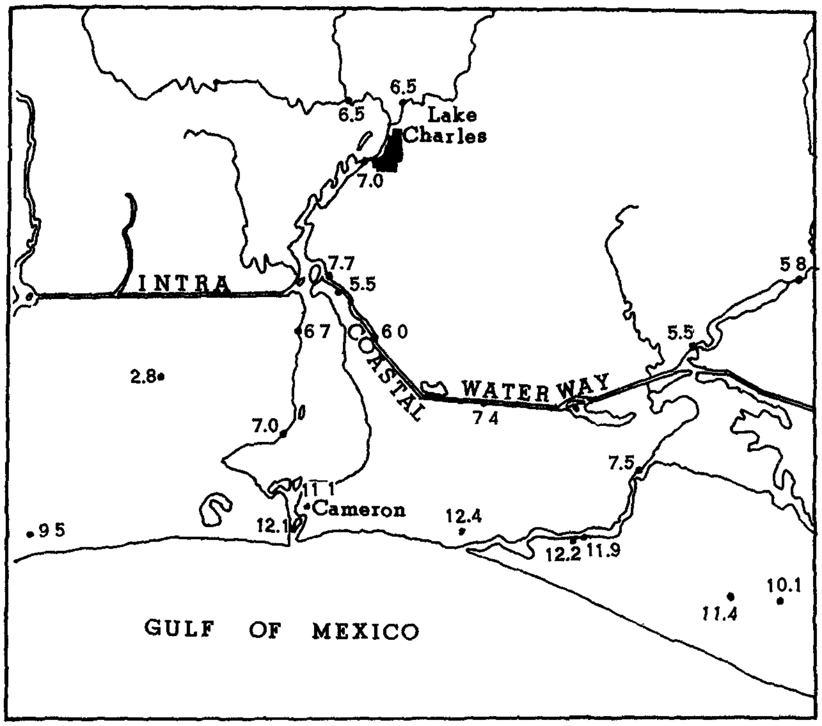


Fig. 6. Storm tide elevations in the vicinity of Cameron, La., produced by Hurricane AUDREY, June 1957. All elevations are expressed in feet above mean sea level. Data obtained from U. S. Army Corps of Engineers, New Orleans District, and State of Louisiana, Department of Public Works.

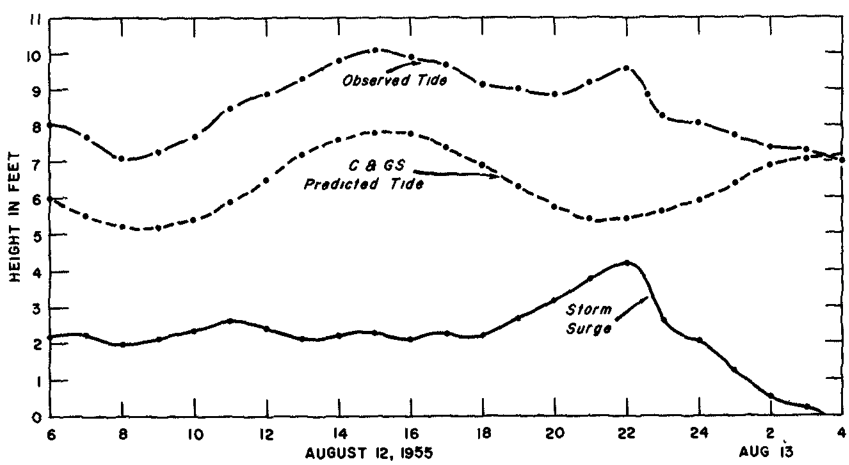


Fig. 7. Observed, predicted, and meteorological tides at Little Creek, Norfolk, Va., for Hurricane CONNIE, August 1955.

clearly that the water level at Galveston was not controlled by the onshore winds. The increase in water level may have been due to the Coriolis force or to the boundary effects of the Louisiana and Texas coasts, or to a combination of these effects.

Freeman, Baer, and Jung(1957) have proposed a system for computing the effects of the Coriolis force due to the alongshore component of the wind, and the direct effect of the onshore component of the wind in piling up water against the shore. This system, which is based on fundamental principles, does predict surge heights which are well within the proper order of magnitude. However, it does not take into account all of the dynamic effects which we believe to be important and requires a more detailed specification of the wind field while the storm is still at sea than is possible to give at the present time.

5. THE EFFECTS OF LOCAL TOPOGRAPHY

It is often useful to think of the storm surge as a wave-like disturbance of the sea surface in which the wave length is of the order of a hundred miles and the period between 8 and 24 hours. Considered in this way, it is easy to see that the surge height experienced on the open shore can be greatly modified as it moves through a bay or a river. The amplitude of the disturbance will frequently double within a distance of only a few miles as it progresses into a bay with converging shorelines. Likewise the height of the disturbance may be decreased near the middle of a wide bay with only a narrow connection with the sea. Figure 5, taken from Harris (1957b) shows the effects of local topography on the storm surge produced by Hurricane CAROL.

If the storm tide at the coast rises above the top of barrier islands, the water will flow directly inland with little regard to the natural drainage channels. In these cases the peak tide levels are to be expected a short distance inland from the natural coast, with levels sloping downward inland. This peak occurs landward of the natural shore because the presence of the submerged coastline has little effect on the slope of the free surface of the water. The slope downward at points farther inland occurs because the inertia of the water and the effects of friction as the water flows over vegetation prevents the transport of enough water inland to maintain an equilibrium between the moving storm and the slope of the free water surface. This effect is shown by the record of peak tides produced by Hurricane AUDREY, Figure 6. No land remained above water south of the intracoastal water at the height of the storm tide. In some areas the storm tide extended far beyond this canal and in others the spoil banks, formed in building the canal, served as dikes to impede the northward flow of water. These dikes had to be breached at several places to permit the land to drain after the storm.

The direct effect of wind stress over an enclosed or semi-enclosed body of water is to pile up water at the leeward end of the basin. The effect is nearly independent of the advection of the surge from the open sea into the basin. A wind blowing toward the head of a bay will serve to increase the surge height at the head of the bay. A wind blowing toward the sea wil tend to decrease the water level at the head of the bay but it may not ove come the effects of the progressive surge.

6. STORM SURGE OBSERVATIONS

The best storm surge records are obtained from a continuously recording tide gage at a site for which the observations have previously been analyzed for astronomical tide predictions, for it is only in this case th the storm surge can be accurately determined. Unfortunately, there are ma large gaps in the tide gage network and the peak storm tide is not often experienced at a recording tide gage. The peak storm tide can often be determined by an inspection of the coast soon after the passage of a sever storm. High water marks may be located inside buildings flooded by the storm, and sometimes in natural or artificial basins whose connections wit the sea are good enough to permit the passage of the tide but too tortuous to permit the passage of the high seas prevalent on the outercoast. The peak storm surge, however, cannot always be obtained from these data. The reason for this is shown in Figure 7 adapted from Hoover (1957). The peak storm tide occurred about 1500 EST, near the time of normal high tide, and indicated a storm surge of 2.3 feet, however, the peak storm surge of 4. feet occurred about 2200 EST, near the time of normal low water.

7. EMPIRICAL FORECASTING AIDS

Since the dynamic models of storm surge generation are either greatly oversimplified or too complicated for ready use in the field, and since in either case they are rather uncertain, it is necessary to consider empirical correlations between other, more easily observed, hurricane parameters and the associated storm surge. Accounts of two such studies, Conner, Kra and Harris (1957) and Hoover (1957), have been published in the Monthly Weather Review recently. Figure 8 shows the correlation between the minimum pressure, as determined by the methods described by Myers(1954), and the highest reported storm tide along the coast of the Gulf of Mexico or estimated highest storm surge along the coast of the Atlantic Ocean. This is a revision of a similar figure published by Conner, Kraft, and Harris (1957). Improved data were obtained for a few storms, and data have been added for many more storms. The change in the prediction equation is not believed to be significant. An effort was made to eliminate the effects of local topography, as discussed in section 4 above, as far as possible Some of the storm surge data may be subject to the uncertainty indicated in figure 7. The correlation between the central pressure and the peak

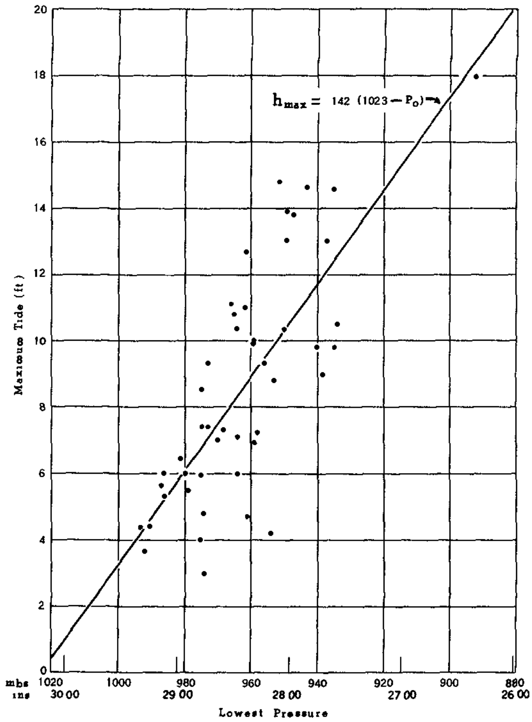


Fig. 8. Storm surge height, as a function of the central pressure in the storm.

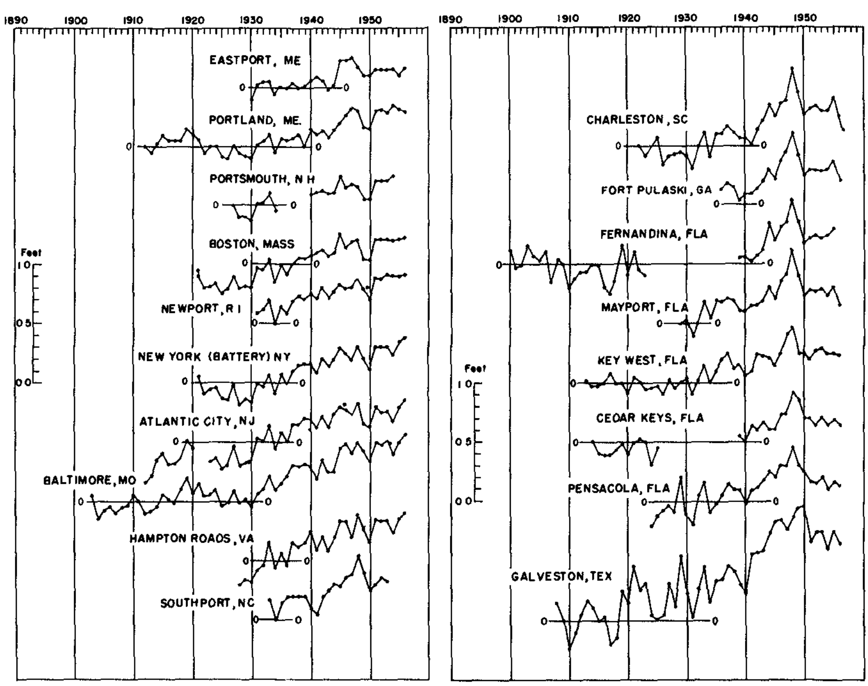


Fig. 9. Yearly sea level, Atlantic and Gulf Coast, (from NHRP Report 7)

storm surge on the Atlantic Coast, or the peak storm tide in the Gulf of Mexico, is about 0.75 indicating that approximately half of the total vari· ability in the peak storm surge height on the open coast can be explained by variations in the intensity of the storm. A further analysis of the data, not shown here, suggests that the storm surge is higher in regions in which the continental shelf is flattest. The data do not indicate any clear cut relation between the peak storm surge and the speed of the storm or the pressure at the edge of the storm.

8. WAVES AND SWELL

In the immediate neighborhood of the coast, and in the flooded region the most damaging aspects of the hurricane are the short-period waves and swell. These may be prominent along the coast many hundreds of miles from the storm. Quantitative forecasts of the waves associated with hurricanes and other storms are desirable. However, the character of the swell and breakers reaching the shore at any location depends to a great extent on t local topography and may vary widely over short distances. The wave heigh in the open sea is a function of the wind speed, the fetch (the length of the region in which the wind direction is essentially constant), and the duration (the length of time the wind blows over the fetch). Near the coa the bottom topography becomes important and may dominate the other factors in hurricane conditions. The most important factor limiting the wave heig in the flooded region may be the depth of the water. Studies of waves bre ing in shallow water indicate that the maximum wave height, trough to cres will rarely exceed 0.78 times the still water depth (Munk 1949). The stil water depth referred to here is the depth of water as averaged over severa wave periods. For example, if the storm tide reaches a level of 8.0 ft. M in a region in which the land elevation is 3.0 ft. MSL, the water depth w be 5.0 ft. and the maximum wave height will be approximately 4.0 ft.

The wave run-up along a sloping beach may be somewhat higher than th indicated above. Detailed studies by coastal engineers show a lower limi ing wave height at many coastal locations. However, in the absence of su studies, it is recommended that waves of the limiting height shown above, be assumed in making plans for hurricane preparedness. Warnings of high waves should be included in warnings of hurricane storm surge conditions but quantitative forecasts of the wave height to be expected under these conditions are not warranted at the present time.

9. LAND ELEVATION AND DATUM PLANES

The end product of a storm surge forecast is the decision to recomme or not recommend special preparations for the safety of life and property in exposed places. This decision depends not only on the height of the water level, storm surge plus normal tide, but also on the elevation of t land and the type of exposure of the place in question.

The The most extensive collection of land elevation data available in the United States is that given in the quadrangle maps published by the United States Geological Survey. These maps, with a horizontal scale of from 0.5 to 2.6 inches to the mile, give contours of land elevations with intervals ranging from one foot in some parts of Texas to 20 feet in some sections of New England, and many spot elevations. The contour analysis is generally accurate to within 25 per cent of the contour interval at the time of preparation of the maps.

The datum of reference for elevations on most of the charts published since 1930, is the "Sea Level Datum of 1929". This datum plane is based on tide observations for periods of various lengths from 1875 to 1924 at 26 stations in the United States and Canada, (Harris and Lindsay-1957). The sea level datum used on the map may differ from the local mean sea level as obtained from local observations by as much as 0.5 ft. at many locations and by more than 1.0 ft. in a few places.

Additional land elevation data may be obtained from bench mark descriptions published by the Coast and Geodetic Survey, the Geological Survey, the Corps of Engineers, U.S. Army, state, city, and county engineering offices. The Sea Level Datum of 1929 is the most widely used plane of reference in these bench mark descriptions, but other planes are used by some agencies. It is generally possible to relate these local datum planes to the Sea Level Datum of 1929. The Coast and Geodetic Survey has established local tidal datum planes, based on local water level observations in many coastal regions. Spirit level connections have been made between many of these tidal bench marks and the geodetic lines of the Sea Level Datum of 1929.

The annual mean sea level is not constant, but varies from year to year. The actual sea level, relative to the land, along most of the Atlantic and Gulf coasts is higher now than when the 'Sea Level Datum of 1929" was established. However, the trend toward rising sea levels prominent between 1928 and 1948 appears to have been arrested along much of the coast. In many areas the current trend is toward lower sea level, see figure 9, taken from Harris and Lindsay. Since the cause of the rising sea levels of the first part of the century and of the falling sea levels of the past decade are unkrown, extrapolation of the observed trends either forward or backward in time is not recommended. Hydrographic charts of the Atlantic and Gulf of Mexico waters are based on a mean low water datum. This is also the datum used in the published tide predictions in this area, and is the datum most widely used in many coastal communities.

Mean low water is defined as the average elevation of all low waters in the tide cycle. Hence it depends on both mean sea level and the tide range. The range of the tide is greatly affected by local topography and may change by several feet within a few miles. Navigation improvements and the reclamation of land also affect the range of tide. Since mean low

Hurricane Surge

water is subject to all of the factors which affect the range of tide as well as to those which affect mean sea level, it is not well suited as a reference plane for land elevations. Mean sea level will not often vary by as much as a tenth of a foot in a hundred miles along the open coast, but may vary by as much as a foot in a hundred miles in an estuary or rive Sea Level Datum of 1929 will not vary by a measurable amount within a simi lar distance. Mean low water, on the other hand, may vary by more than 2.(ft. within a distance of five miles.

If mean low water or any other datum is more widely understood locall than mean sea level, it may be desirable to use the local datum in local forecasts in order to be understood by the public. However, the use of me sea level or Sea Level Datum of 1929 is to be encouraged whenever this is practicable.

10. SUMMARY

Tides as much as four to fifteen feet above normal frequently accompany hurricanes as they move inland. This difference between the normal and observed water levels, called the storm surge, is correlated with the central pressure of the storm. It is also greatly affected by local topography and by many other factors not yet well understood. The practical importance of the storm surge depends on the phase of the normal tide at the time of the storm surge and on the land elevations in the vicinity of the storm surge as well as on the magnitude of the storm surge itself.

Although present knowledge of the processes by which storm surges are generated by hurricanes is not great enough to permit forecasting the elevation of the storm tide with the precision desired, meteorologists, engineers, and other public officials can contribute greatly to public safety during hurricanes if they are able to combine a knowledge of local topography and normal tide characteristics with a familiarity of the effec of past hurricanes on sea level.

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CHAPTER 6

HIGH-WATER PROBLEMS ON THE DANISH NORTH SEA COAST

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With reference to the use of high-water frequency curves, which have been suggested by Wemelsfelder as an aid to fix the maximum flooding level, an attempt will be made in the following to estimate how far certain special geographical and meteorological conditions may be expected to influence the shape of the frequency curves for different localities. The investigation concerns a particular point on the Danish North Sea coast compared with the Dutch coast, but its principles may possibly be of interest in a wider sense.

INTRODUCTION

THE NORTH SEA.

The North Sea forms a kind of bay of the Atlantic, being at its north-western end connected with the great depths of that ocean, while its other connection with the Atlantic, through the English Channel, is so narrow and shoal that its influence on the water level is small. It is a very shallow sea (see the map, Fig.1, which also shows the main features of the bottom topography) particularly its southern part, which is just where adjoining territories in England, Germany and Denmark as well as - and predominantly - in Holland comprise very low-lying tracts. Many inundation catastrophes have, therefore, hit these regions through the ages, and dikes have been built at many places to safeguard against floods.

FREQUENCY CURVES

For the purpose of planning dikes and certain other maritime constructions, a determining maximum flooding level for each individual place must be fixed, and the establishment of this level was formerly based upon the knowledge of "the highest possible flood" gained by experience from inundations actually occurred during historical times, possibly with a certain safety margin added. Such experience may, however, be rather doubtful, and usually covers only a comparatively short span of years.

In order to provide a better survey of this problem P.J. Wemelsfelder (see [1] in "References") in 1939 suggested the application of a "frequency curve" produced by plotting in a system of co-ordinates the values of high water against the frequencies

(i.e. the average annual number of times according to available statistical material) with which a water level equal to or higher than the value under consideration has occurred at the place in question. When a semi-logarithmic system was used (the frequencies being represented in the logarithmic scale) the curve proved to be very nearly rectilinear (see Fig. 2, in which however, the lower curve is drawn on the basis of gales under special meteorological conditions only), and extrapolation of the curve gives an impression of the frequency with which specified extraordinarily high water levels may be expected to occur, or rather the probability of their occurring.

ASTRONOMICAL AND METEOROLOGICAL TIDES

That part of the change of level which is due to astronomical causes, the tides, is fairly exactly known in the case of the North Sea [2] . It consists of standing oscillations, maintained by the oceanic oscillations of the Atlantic in the north, but superimposed on these main longitudinal oscillations are smaller transverse oscillations due to the earth's rotation, the aggregate result being a circling tidal surge travelling southwards along the coast of Great Britain and - with greatly decreasing amplitude - northwards along the west coast of the Danish peninsula of Jutland. As to the influence of meteorological conditions on the water level much greater uncertainty reigns, and these are, therefore, the only ones which are considered when recording frequency curves, the known values of the astronomical tide being deducted in advance.

The influence of meteorological conditions on the water level has been studied in detail in various countries where flood warning services for threatened areas have been set up, and diverse methods of preparing these warnings on the basis of theoretical investigations combined with the utilization of statistics have been developed, for instance in Great Britain by R.H. Corkan [3] , in Holland by W.F. Schalkwijk [4] , in Germany by G.Tomczak [5], and in Denmark (where so far no warning service has been established) by J. Egedal [6] .

EXTRAPOLATION OF FREQUENCY CURVES

Generally. After the great inundation catastrophe that hit England and Holland i 1953, and during which the water level considerably exceeded heights previously recorded within historical times, the application of frequency curves has particularly come to the fore. It must be admitted that the utilization of a frequency curve embodying and arranging, almost automatically, all the existing actual observations, seems more attractive than relying only on the highest known water level within an arbitrary, pretty short, period, but the difficulty appears when the curve is to be extrapolated. Its upper part, being provided by very few observations, must be highly unreliable, and it also appears to be a fact that in Holland, where observations for more than 50 years are available, curves recorded separately for the individual decades

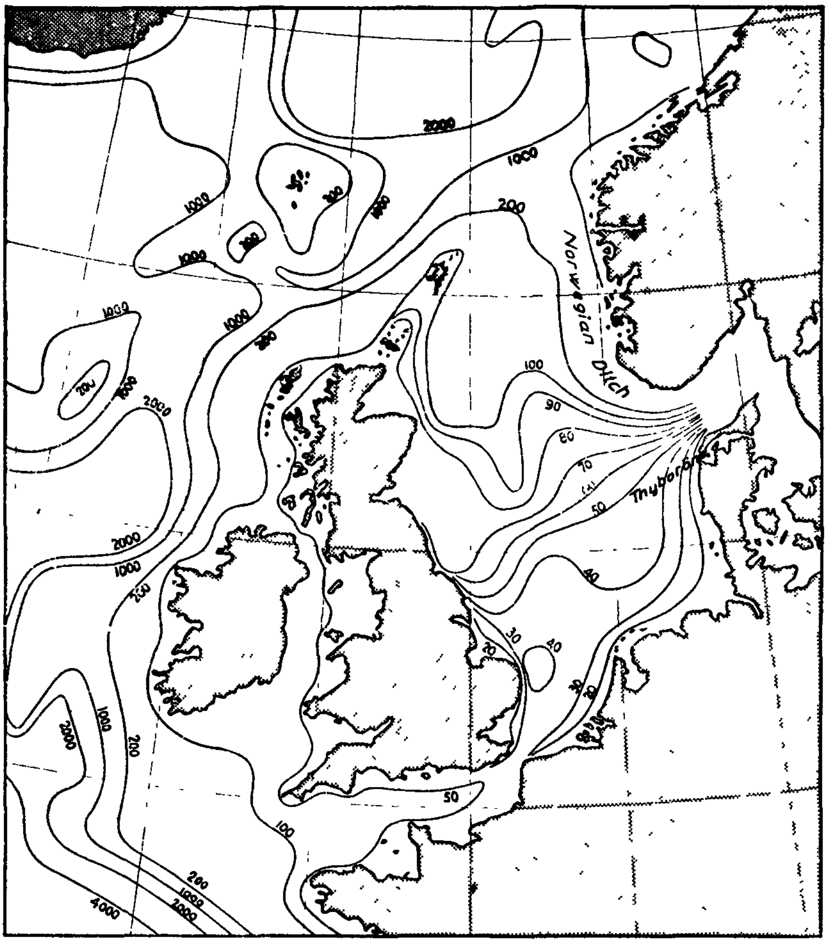


Fig. 1 Depths of the North Sea (in meters)
(After Schalkwijk)

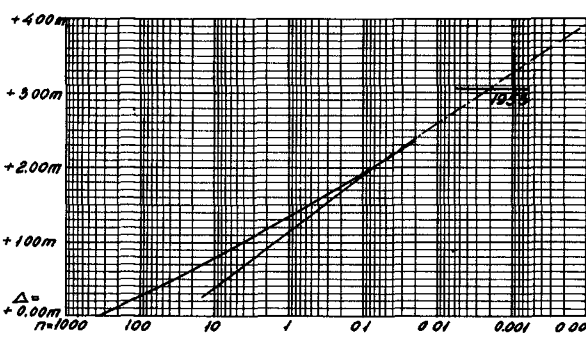
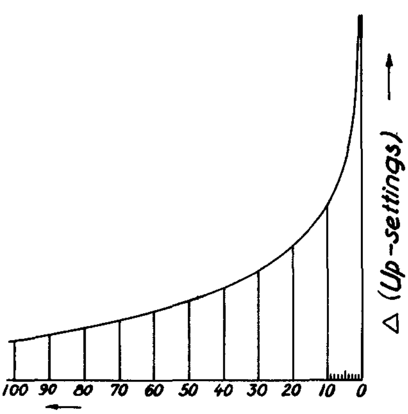


Fig. 2 Frequency curves for Hook of Holland



Frequency of upsettings ≥ Δ

Fig. 3

differ greatly at their upper ends [7] . Further, it must be borne in mind that a rectilinear extension of the curve in the semi-logarithmic system has been chosen arbitrarily among many possible extpolations, and that this straight line, when transferred to a non-logarithmic system, gives a curve asymptotically approaching the y-axis (see Fig.3), thus involving the inherently unreasonable result that the height of the wind-effects tends to ∞ when the frequency approaches nil. This conforms badly to the common and "natural" conception that the water level must be approaching a certain limit when the frequency tends to nil, and after all it is th minute topmost fraction of the curve, where it grows asymptotic (and consequently improbable) which is "enlarged" by the logarithmic system and utilized. Already when advancing the method, Wemelsfelder was, indeed, aware of this, but he adduced that a minor extrapolation must be justifiable, as the limit of what is physically possible would probably not thereby be exceeded, and that even appreciable extrapolation does not lead to quite absurd figures (for the frequency 10^{-6}, i.e. once per one million years, he mentions a wind-effect of 5.9 m at Hook of Holland).

In Holland - To be sure, much discussion and deliberation ha taken place in Holland as to how the curve should be extrapolated [7] , but agreement has gradually been reached there on a rectilinear extrapolation, which seems to yield probable results in the case of Holland within that section of the curve which it is found reasonable to deal with, and it is thus assumed that a maximum flo level off Holland must at any rate be very high.

In Germany - Also in Germany rectilinear extrapolations of the frequency curves have been used when contemplating the height dikes in the marshlands [8] , but here it is only a matter of a small extrapolation (up to 10^{-2}), as - unlike in the case of Holland - the tracts to be protected are agricultural areas and not important, densely populated industrial ones.

CONDITIONS AT THE DANISH NORTH SEA COAST

In Denmark there are in the southern part of Jutland marshlands with conditions much like those in the adjoining regions of Germany. However, the question of maximum high water is also of importance for certain contemplated constructions at Thyborön in the northern part of Jutland (see Fig. 1), but here water level observations are only available for a comparatively short period, and some uncertainty exists as to how the frequency curve should be extrapolated at this place, where conditions are very different from those prevailing off Holland and Germany. While the mean rang of the tide varies between 1.3 and 3.7 m off Holland and is about 1.5 m at the Danish-German boundary, it is only 0.4 m at Thyborön, which is situated almost off the transition between the particularly shallow southern part of the North Sea and its somewhat deeper northern part.

SCOPE AND PRESUPPOSITIONS

In the following an attempt has been made to realize to what extent various geographical and meteorological conditions may influence the course of the frequency curves off Holland and at Thyborön respectively. Only a qualitative investigation in broad outline has been undertaken for the purpose of establishing the probable mutual relation between the two curves, but a lucid exposition of the problem has been aimed at. Moreover, the investigation has been confined to dealing with the stationary state directly generated by homogeneous fields of wind over the entire North Sea (here called the "wind-effect"), disregarding barometric effect, phenomena of inertia, additional quite local effects, seiches, etc., as well as disturbances generated outside the North Sea. As to these last mentioned "external effects", to which great weight is attached in Britain, it must be justifiable to presume that their amplitude - just as that of the tide - will have decreased much when they reach Thyborön.

METHODS

Schalkwijk concerning the North Sea - In his treatise [4] Schalkwijk surveys how the problem of wind-effect has been treated previously, and uses the formula:

$$\Delta\zeta = \frac{a\ V^2\ L\ \cos\psi}{H} \qquad (1)$$

in which $\Delta\zeta$ is the height of the wind-effect in cm over a stretch of L km with a depth of H m and a velocity of wind of V m/sec., a is a constant which Schalkwijk fixes at 0.032, and ψ is the angle between the direction of the wind and the direction of a line connecting the two points between which the difference in water level is sought.

The introduction of cos ψ is due to the fact that the formula was originally established for a canal, and in his investigations concerning the Dutch North Sea coast Schalkwijk only applies it as follows: He seeks out that direction of wind (about 15° west of the longitudinal axis of the North Sea, which is about NNW-SSE) which causes the highest wind-effect at the point under consideration, and thereafter, as far as certain secondary elements are concerned, he calculates the effect of winds from other directions by means of the cosine of the angular deviation, while for the main element he uses an empirical function of the angular difference, the effect proving to be not quite symmetrical about the direction which causes maximum. A special section of the treatise deals with the connection with the Atlantic.

As an introduction Schalkwijk had considered the comparatively simple case of the wind blowing across an enclosed sea, and he finds (as previously Ekman [9]) that in this case an inclination of the surface arises which - in spite of the effect of the earth's rotation - very closely corresponds to the direction of the wind (i.e. is directed against it). This result holds good when the state has become stationary, while - during its development - the contour lines of equal water level of the surface are turned somewhat to the right.

Hellström concerning the Baltic Sea - The influence of the wind on an enclosed sea may thus be treated fairly simply, and this has for instance been done by B.Hellström in an investigation into the wind- effects in the Baltic Sea [1o] , the comparatively narrow outlets to the North Sea through the Danish straits being disregarded. Hellström uses a formula corresponding to the one mentioned before, but in a slightly different notation, viz.:

$$\frac{dz_0}{dx} = \frac{\varkappa\ k}{\gamma(z_0 - z_1)} \tag{2}$$

where $(z_0 - z_1)$ = the depth of water. x = the longitudinal coordinate, k = the tangential pressure of the wind, γ = the density of the water, and $\varkappa$ is a dimension-less coefficient, which is fixed at 1.5. If the ratio k/V^2 is assumed to be constantly o.ooo213, while it is supposed by Hellström to be slightly varying according to the velocity of the wind and γ is assumed to be 1ooo kg/m³, we obtain:

$$\frac{dz_0}{dx} = \frac{3.2\times10^{-4}\,V^2}{1000\ (z_0 - z_1)} \tag{3}$$

i.e. exactly the same as (1) when the inclination is taken to be dimensionless, and this form will be used in the following estimatory calculations.

Hellström proceeds as follows: For each direction of wind the mean depth for a number of cross-sections at right angles to the direction of the wind is calculated, which results in a mean longitudinal profile in the direction of the wind, and by applying the formula the inclinations of the surface corresponding to a number of points in the longitudinal profile are found. The mut al positions of the longitudinal section of the surface with incl nations varying according to the depth at each point and the calm water surface are now fixed so as to comply with the requirement that the aggregate amount of water must remain constant, and the contour lines of the surface are then drawn as straight lines at right angles to the direction of the wind and with the longitudin section as directrix, quite irrespective of the very irregular shape of the sea with islands, bays, etc. It is evident that a gr number of equalizing currents must arise before the surface can adjust itself in this position, but apparently they come to pass approximately within the same time as it takes for the inclinatio to develop. Also minor corrections are computed in order to take into account local bottom topography, but all things considered t results correspond pretty well with actual observations. It shoul however, be remembered that the depths of water in the Baltic Sea are comparatively small, averaging about 6o m, and that the great depths (maximum 46o m) are confined within few areas of negligibl extent.

COMPARISON BETWEEN CONDITIONS OFF HOLLAND AND DENMARK

ENCLOSED SEA

In order to get an impression of the importance of the uneven depths of water in the North Sea, let us first consider an enclosed sea, shaped as a square 600 x 600 km (see Fig. 4 a), one half of which is 40 m deep, while the other half is 80 m, as indicated by the cross-section shown in Fig. 4 b. For the sake of simplification the "sea" is provisionally supposed to be oriented due north-south, but actually it is intended to represent the southern two-thirds of the North Sea, highly simplified, and the point A would thus correspond to a point on the Dutch coast and B to Thyborön. A velocity of wind of 29 m/sec. (i.e. on the lower side of force 12 by Beaufort's scale), would give an inclination of the surface, according to (3) of

$$\frac{dz_0}{dx} = \frac{3.2\times10^{-7}\times29^2}{40} = 0.67\times10^{-5} \text{ or } 200 \text{ cm in } 300 \text{ km}$$

For the northern part half that figure, viz. 100 cm in 300 km will be found. If the two halves were separated, a northerly wind would cause the water level to take up the position shown by dotted lines in Fig. 4 b (the differences of level being much exaggerated) and indicated in Fig. 4 a by contour lines in connection with shading of high-water areas, but when they are inter-connected the water surface will follow the full-drawn oblique line in Fig. 4 b, as also indicated in Fig. 4 c, and in order to bring about this result there must, besides other currents, pass a current from the northern to the southern half as indicated by arrows in Fig. 4 d. A westerly wind would cause different inclinations in the two halves if they were separated (see Fig. 5 a), and Fig. 5 b shows a section along the "partition wall" with both inclinations represented by dotted lines, but when no such separation exists, the water must adjust itself by the mean depth, i.e. with an inclination of

$$\frac{dz_0}{dx} = \frac{3.2\times10^{-7}\times29^2}{60} = 0.45\times10^{-5} \text{ or } 135 \text{ cm in } 300 \text{ km}$$

as indicated in Figs. 5 b and 5 c, and to make the water assume this inclination, a circling current, as indicated in Fig. 5 d, is required.

Figs. 6 a , b,c and d give a corresponding representation of the effect of a north-westerly wind; Fig. 6 e shows a longitudinal section in the direction of the wind (along the diagonal) in which, besides the actual depths, also the mean depths of cross sections at right angles to the direction of wind are shown (as a shaded line). Accordingly inclinations of the surface would arise varying by these mean depths, as indicated in Figs. 6 e and 6 c, and this requires both an inflow from the north and a circling current as shown in Fig. 6 d (both currents, however, somewhat weaker than in the above cases); the two kinds of currents may also be added and will then give the result shown in Fig. 6 f. (This result will also be borne out when Fig. 6 b is considered).

South-westerly winds will give quite analogous conditions w: a circling current in the same direction as for NW,but now combin(with a northward outflow, as shown in Figs. 7 a - f.

All the Figures 4 -7 apply equally to winds in the respecti' opposite directions, when high and low waters are interchanged in Figs. a, b, c and e (the zero line remaining fixed), and the current arrows are inverted in Figs. d and f. On the basis of 4 c,5 (6 c and 7 c curves have been drawn in Fig . 8, which for this par cular velocity of wind show variations in the water level at poin A and B when the wind veers round the entire compass. (If the basin were supposed to represent the North Sea, it would only nee(to be turned $22\frac{1}{2}^{\circ}$ counter-clockwise, i.e. the compass points shou be shifted in relation to the curves as shown for N in brackets under the figure, but until further the orientation due north-sou will be retained in these reflections).

OPEN SEA

If we now try to infer what conditions would be like, when is no longer a question of an enclosed sea, it is evident that th results will be far less exact, and that much depends on an estim. te.

So, let us suppose that the northern third of the North Sea id added, in which the depth of water is again considerably great averaging perhaps about 160 m, i.e. near the limit of the depth t which the influence of strong winds will penetrate, and this basi further is supposed to be connected at its northern end with the lantic, the depth of which (at least 1000 m) in this connection m be regarded as being infinite, so that the wind cannot here produ inclinations (although certainly currents) of any importance.On t southern, shallow areas the wind will try to create an inclinatio corresponding to the depth of water there, while at the northern edge the water level must nearly remain constant. So Figs. 9,10, and 12 have been drawn in such a way that the course of the conto lines in the southern third is retained as in the case of an encl sed sea, while in the middle third they are modified slightly, so as to pass through the northmost part, which acts as an intermediate link, into the almost unchangeable water level at the nor ern end.

In case of northerly wind (Fig.9) water must thus flow in from the north until the entire water level has been raised as in cated in Fig. 9 a, and in case of westerly wind (Fig.10) - in ord to maintain the inclination of the surface in the southern part - circling current must flow in the same manner as in the enclosed sea, in this case without any raising of the water level as a who In case of north-westerly wind (Fig. 11) both effects of current will arise, but in a somewhat lesser degree and consequently with a somewhat slighter raising of the entire water level. In the cas of southerly, easterly and south-easterly winds corresponding opp site effects will prevail, and finally, Fig. 12 shows the situati during south-westerly (and north-easterly) winds.Under the condi

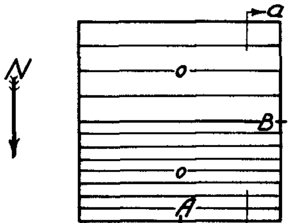


Fig. 4a

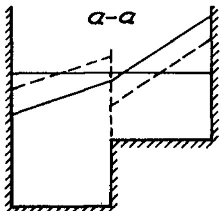


Fig. 4b

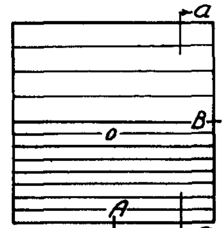


Fig. 4c

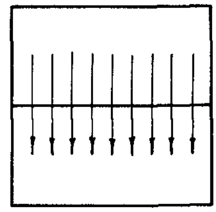


Fig. 4d

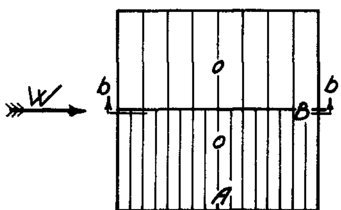


Fig. 5a

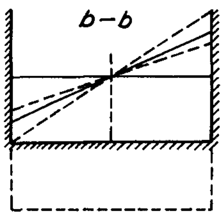


Fig. 5b

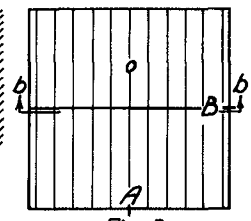


Fig. 5c

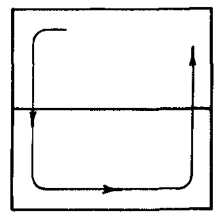


Fig. 5d

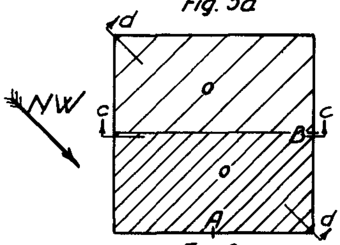


Fig. 6a

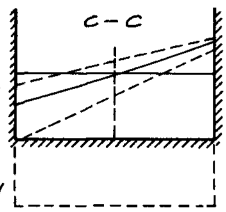


Fig 6b

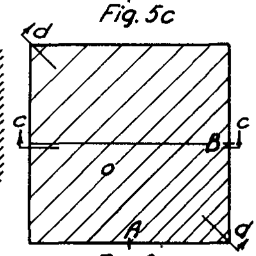


Fig 6c

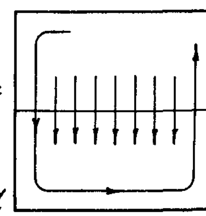


Fig 6d

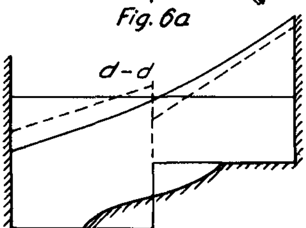


Fig. 6e

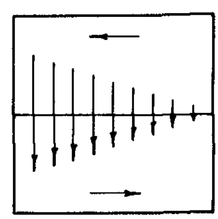


Fig 6f

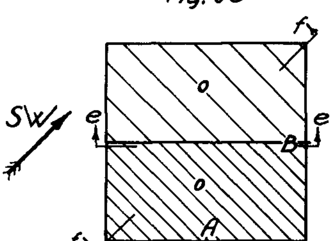


Fig. 7a

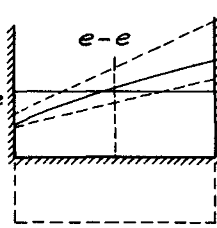


Fig 7b

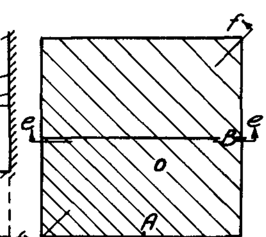


Fig. 7c

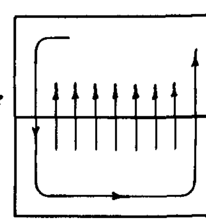


Fig. 7d

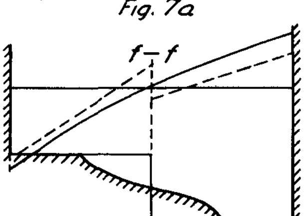


Fig 7e

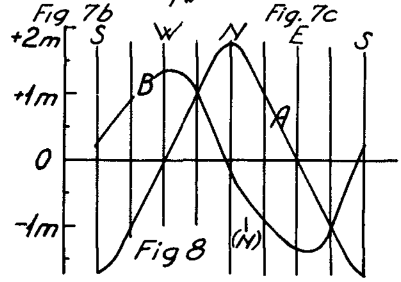


Fig 8

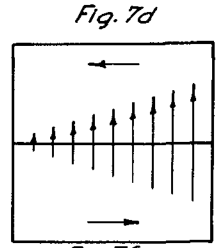


Fig 7f

tions of an open sea the same effects of current as in the case of the enclosed sea will assert themselves, but whereas the southward (and the corresponding northward) flow only take place while the situation is developing, the circling current must continue as long as the situation persists. The water levels at points A and B might now be read from Figures 9-12, but the same result will be obtained by adding to the curves from Fig. 8, which have been traced in thin lines in Fig. 13 oriented by the compass points written under the figure, the dotted curve I, which shows the raising or lowering of the entire water level in the southern part, which movements - according to what is said above - are zero for west and east and maximum (1.25 m , corresponding to the figures calculated above) for north and south. The result is the two curves A_1 and B_1 in heavy lines in Fig. 13.

The earth's rotation - When regard is to be paid to the effects of the earth's rotation one might reason that as its influence on the surface inclinations in the case of an enclosed sea is minimal, it should be sufficient to include a calculation of the additional effects of the Coriolis-forces on the currents shown in Figs. 4 d- 7 d. For the circling currents we would thereby get a curve shaped as the dotted curve II in Fig. 13, the currents circling counter-clockwise (i.e. those caused by winds from westerly directions) raising the water level along the shore, and clockwise currents lowering it, and curve II would have to be added both to the A_1- and the B_1-curve. The currents flowing north-south would similarly give an additional curve as III, which only was to be used on the B_1-curve; as mentioned above, the latter current only flows while the situation is developing. Even if an estimate may well be formed as to the force of the said currents, there is nevertheless uncertainty in estimating their effects on the water levels, and further these effects depend on the depths at the respective places. In connection with the previously-mentioned turning of the whole system by $22\frac{1}{2}^{\circ}$ in order to make it correspond to the orientation of the North Sea, we will therefore now instead of adding the curves II and III merely turn the system by a further 15°, as this will give almost the same effect, including that the maximum effect off Holland is caused by a direction of wind of 15° west of NNW as stated by Schalkwijk. In Fig. 13 these turnings have been made by shifting the compass points to the positions noted above the figure, which positions when related to the heavy curves A_1 and B_1 should thus give an approximate picture of the combined effects off Holland and at Thyborön (The B_1-curve will be seen to be unmistakably lower than the A_1-curve, while its position does not quite agree with experiences from Thyborön, but this - which is of no great importance for the deliberations that follow below - is probably due to the very summary regard paid to the earth's rotation. Incidentally, in a later section this question will be given a little more consideration).

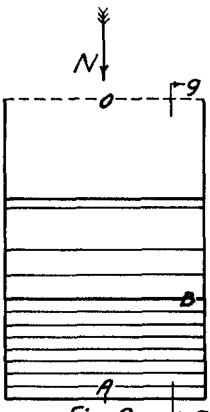


Fig. 9

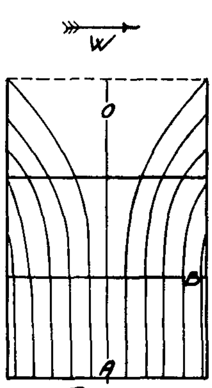


Fig 10

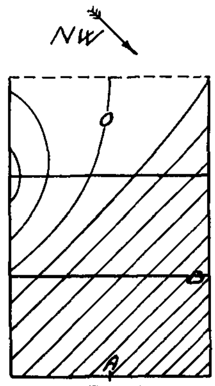


Fig. 11

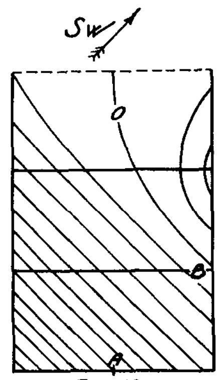


Fig. 12

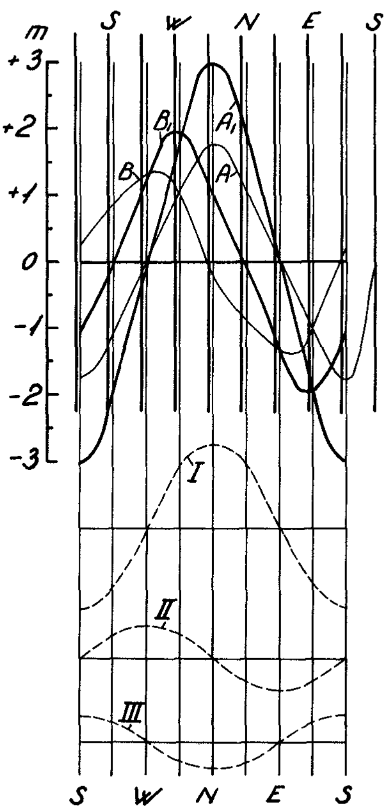


Fig. 13

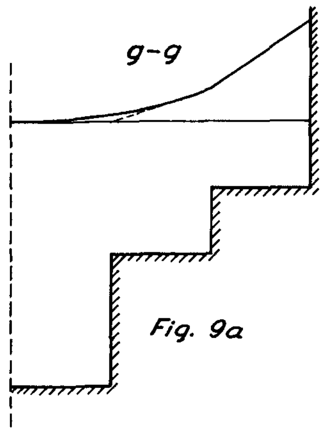


Fig. 9a

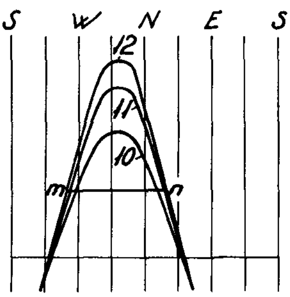


Fig. 14a

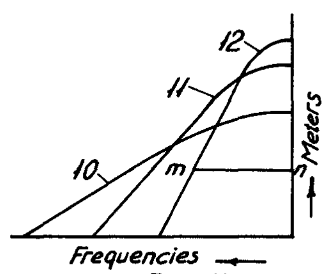


Fig 14b

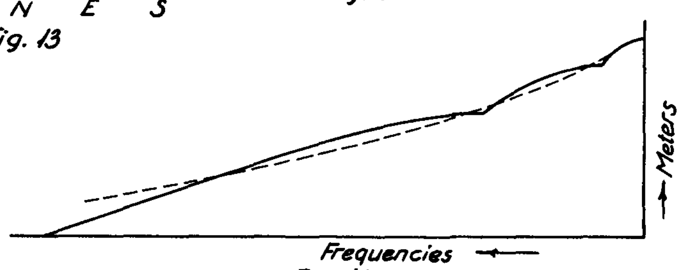


Fig. 14c

INFLUENCE OF BOTTOM TOPOGRAPHY ON THE FREQUENCY CURVE

As mentioned before, the curves have been traced for a definite velocity of wind (approx.Beaufort 12), but the corresponding curves for other velocities will be found from (3) by simply reducing it in the proportion$(V_{11}/V_{12})^2$, i.e. for force 11 in the proportion ($27/29)^2 = 0.87$, for force lo in the proportion $(23/29)^2 = 0.63$, and in Fig. 14 a the high water section of the A_1-curve has accordingly been traced for these three velocities of wind.

If, until further, we now look at the 12- curve only, and imagine that all directions of wind are equally frequent, then the line segment, m - n, cut off by the 12-curve from a horizontal lin representing a certain height of water will be proportional to (and consequently constitute a measure for) the frequency with which this particular or still greater heights of water occur, and by inserting this segment as in Fig. 14 b we get a frequency curve for wind force 12 only, subject to the assumptions mentioned. Similarly, in Fig. 14 b frequency curves for other velocities of wind might be sketched in, but here with the use of different scales corresponding to the greater frequency of these velocities (in Fig. 14 b only a slightly larger scale has been used, althoug the 11-curve should, in fact, have been drawn with abscissas lo ti mes and the lo-curve with abscissas loo times as big), and by adding up the abscissas for all velocities of wind the aggregate fre quency curve would be obtained; in Fig. 14 c such an addition is shown for the three curves traced, and its wavy shape, which is du to the discontinuous division of velocities of wind, has been smoothed out by the dotted curve. Had a similar plotting of the B_1 - curve been made, it is apparent that we should have got a fre quency curve starting from the same point on the x-axis, but lying lower throughout than the A_1-curve.

INFLUENCE OF WIND FREQUENCIES ON THE FREQUENCY CURVE

However, the various directions of wind do not occur with equal frequency, and the distribution of frequency on the directions of wind also varies according to the velocities of wind. Thi will be found (on the basis of Danish statistics published in [1] for wind force 3 the relative distribution shown in Fig. 15-a of the frequency for various directions of wind on the North Sea, whi le Fig. 15 c shows the distribution at wind force lo (a scale lo times as large as in Fig. 15 a having been used in Fig. 15 c). Th the two curves selected are, in fact, representative of the very powerful winds and the weaker(but very frequently occurring) win will be seen from Fig. 16, in which corresponding curves for all the wind forces 2 -12 have been traced together in a semi-logarith mic system (with the frequencies plotted logarithmically as ordi tes against the respective directions of wind), and it will be se that south-westerly and westerly winds are the most frequent ones but that north-westerly winds more frequently occur with very gre velocity.

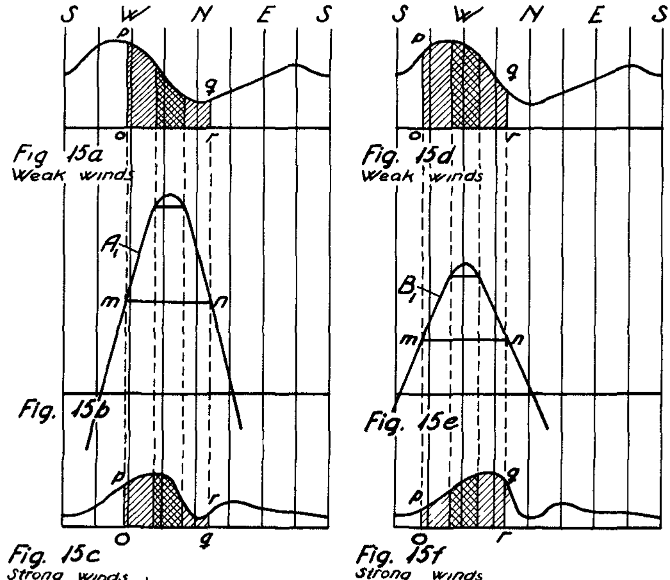


Fig 15a Weak winds

Fig. 15b

Fig. 15c Strong winds

Fig. 15d Weak winds

Fig. 15e

Fig. 15f Strong winds

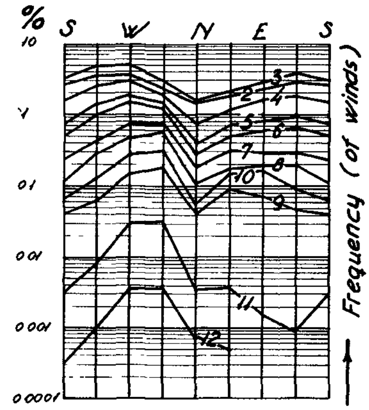


Fig. 16

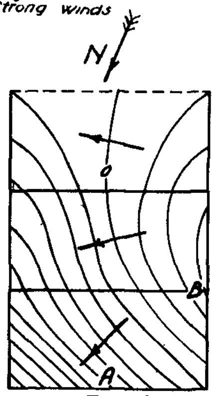


Fig 17

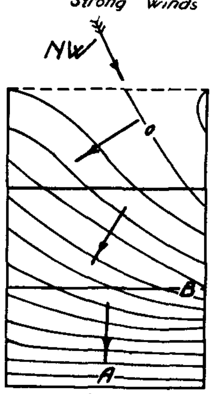


Fig. 18

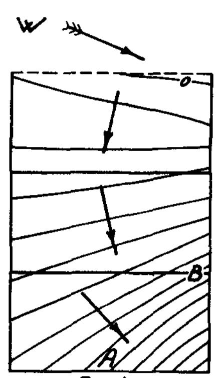


Fig. 19

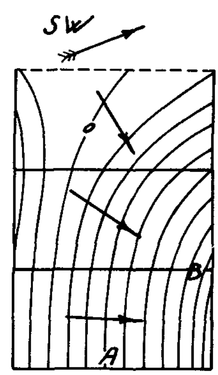


Fig. 20

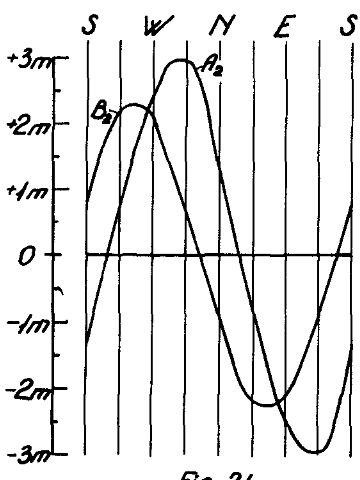


Fig. 21

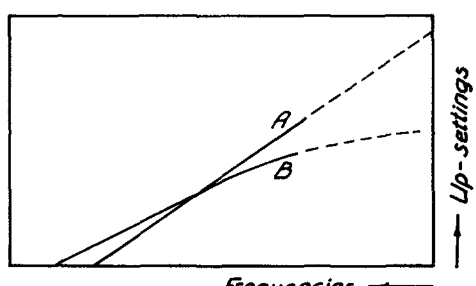


Fig. 22

The water level curves A_1 and B_1 previously found for Holland and Thyborön respectively, have been plotted separately in Figs. 15 b and 15 e, but now they are not supposed to apply to wind force 12, but alternately - using different scales - to the wind forces 3 and 10. While the segment (m-n) cut off by the water-level curve was used above as a measure for the frequency of a water level $\geqq$ the one under consideration, this frequency may now be regarded as being represented by the shaded area o-p-q-r cut off on the frequency distribution curve for winds (see e.g.Fig. 15 a) off the said segment m-n. It appears that for weak winds greater high-water frequency is clearly found at Thyborön than off Holland,while for strong winds the difference is only negligible (and, if anything, rather opposite). When the high-water frequency curves for the individual velocities of wind are again supposed to be added up into a single curve it will be seen directly that the Thyborön curve must begin with greater frequencies for low water levels than th Dutch one, and as it should be less steep, taken as a whole, than the latter, they must intersect. It will also be seen that by plotting the wind frequency distribution curves semi -logarithmically a in Fig. 16, a high-water frequency curve of the same character as actually used will be obtained.

INFLUENCE OF THE EARTH'S ROTATION ACCORDING TO EKMAN'S THEORY

As already mentioned, the earth's rotation has only been considered very summarily above, but there may be reason to mention a few effects of this phenomenon, which seems to approximate the whole representation closer to actual facts.

V. Walfrid Ekman, who has developed the fundamental theory o the influence of the earth's rotation on ocean-currents caused by wind-effects [9] , shows in a later work [12] that currents cause by pressure gradients (i.e. generated, for instance, by an inclination of the surface and reaching right down to the bottom) in addition to the usual deflection owing to Coriolis-force will be further deflected when flowing across areas with varying depths of water, this deflection being cum sole (that is to the right on th northern, and to the left on the southern hemisphere) when shallower water is encountered, and contra solem when it enters deeper water. In consideration of the above-mentioned circling currents this phenomenon would, if anything, during westerly winds tend to reduce up- settings at Thyborön, while it would increase them at t northern part of the English coast. During the gale in February, 1953, the maximum of up-settings at the English coast was, as a matter of fact, according to [13] , reached at "The Wash", where i is quite possible that the water may have been pressed up owing to the sea growing shallower.

According to Ekman's theory a wind-generated current will at the surface be directed 45° cum sole from the direction of the win when the depth is ∞ . The angle of deflection increases regularl with the depth, while at the same time the velocity decreases.

Supposing the current-velocities at various depths are represented by vectors, and the end points of these vectors are projected on a horizontal plane, a logarithmic spiral will be obtained, and the resultant of all these vectors (i.e. the average velocity of the total flow) is directed 90° cum sole from the direction of the wind. The influence of the wind ceases, practically speaking, at a certain depth, D, Ekman's "depth of frictional influence", depending on the velocity of the wind and representing the layer of water stirred up by the wind, and according to [4] Palmén fixes the value of D as:

$$D = 35 + 5.4\ V \qquad (4)$$

At depths smaller than D the deviation of the total flow from the direction of the wind will decrease, thus being (according to Ekman):

about 70° when the depth $d = 0.5\ D$
- 30° - - - $d = 0.25\ D$
- 10° - - - $d = 0.1\ D$

If on this background we examine how the wind will act on our tripartite model of the North Sea, the value of D for V= 29m/sec. being $35 + 5.4 \times 29 \sim 190$ m, the depths of the three different sections of the model should correspond to about 0.20 D, 0.40 D and 0.85 D respectively, and consequently for this velocity of wind the current-directions shown by the arrows in Figs. 17-20 are obtained.

However, it must be remembered that these currents generated directly by the wind, are only one factor among many determining the inclinations of the surface, these inclinations being the result of interaction between the said wind-currents, gradient-currents (also influenced by the earth's rotation), the course of the coast lines, etc., but merely by considering the directions of the wind-currents - which are, after all, the very root of the matter - we obtain an impression of how SW (and perhaps in an even higher degree SSW) must give the highest water level at Thyborön, as is also shown by experience. Based on an estimate, contour lines have been drawn in Figs. 17 - 20 in a similar manner as in Figs. 9-12, but now with regard paid to the various directions of wind-currents. It must,however, be admitted that the result according to the above is bound to be rather arbitrary, which is the reason why these figures have not been used as a basis for the foregoing investigations concerning the frequency curves. From the Figs. 17-20 water level curves for points A and B have been drawn in Fig. 21, and they seem to correspond quite well to actual conditions. For lower velocities of wind a rather more pronounced deflection of the directions of currents might be expected as in these cases the value of D decreases.

It may be added that Ekman's theories are not altogether undisputed. They simplify matters by assuming a constant " eddy viscosity" from surface to bottom, and theories have been advanced which take into consideration that this is hardly true, but H.U. Sverdrup writes in 1946 in [14] that "Ekman's classical theory appears to give satisfactory approximation, especially because no observations are as yet available by means of which the results of a refined theory can be tested".

INFLUENCE OF THE NORWEGIAN DITCH

Above has only been discussed how the up-settings arise, and it will have been seen that they reach the highest levels in the southern part of the North Sea. If we now assume that the wind actions cease (or diminish) after great quantities of water have bee pressed up in the south, this water will seek back towards the nort and on its way it will be deflected sum sole by the earth's rotation and consequently preferably seek towards the west coast of Jutland (the deflection, however, being counteracted to some extent by the increasing depth of water). That this actually does happen is illustrated by a series of curves (reproduced from J.R. Rossiter in [15]) depicting the variation in up-settings at various places during the gale in 1953. The highest up-setting was reached at Harlingen, Holland, (about 3.4 m), while at Hanstholm, some 50 km north of Thyborön, which was not directly affected by the gale, a high-water accurred about 9 hours later which, to be sure, was only about 1 m, but which in return was of longer duration.

Moreover, the up-setting will fall most quickly where the water, on account of a large sectional area of the current (i.e. great depth of water), can flow away quickly, and it will therefore seek towards the deep ditch running along the southern and western coast of Norway right up to the Atlantic. This Norwegian Ditch ("Norske Rende") is deepest off the southern point of Norway, where it reaches a depth of more than 500 m with more than 50 km between the 100 m contour lines; further north its depth decreases while in return its width increases. A calculation shows that with a head loss in water level of 1 m along the 600 kr stretch from the southern point of Norway to the deep water of the Atlantic, this ditch can carry a quantity of water in the magnitude of 14 x 10^{10} m^3 per hour, and even if the upper layers (here estimated at 150 m), which are subject to influence by the wind, are left out of the calculation, the same head loss in water level will give a rate of flow of about 4 x 10^{10} m^3 per hour. In the latter case the velocity will at its highest (and only for a relativ ly short distance) be about 1.5 m /sec. ~ about 3 knots, i.e. no in any way improbable, and in consideration of the fact that the total excess of water in the North Sea during the gale in 1953 has been calculated by Rossiter (as quoted in [13]) to be about 43 x 10^{10} m^3, it will be seen that the outlet through the Norwegia Ditch must exercise a very great influence on the water-level conditions. Irrespective of influences of the wind on the surface, th ditch will be capable of carrying back to the Atlantic large quant ties of water pressed into the North Sea, and in doing so offset all very great fluctuations of the water level in its vicinity, thus also at Thyborön. It may be added that owing to the earth's rotation the surface in the ditch will endeavour to develop a transverse inclination, the water level being raised on the right and lowered in the left looking in the direction of the current, and this transverse inclination, which - according to Ekman - is more pronounced for great depths of water than for shallower water, will likewise contribute to offsetting both high and low waters off the coasts of northern Jutland. The fact that high water in the North Sea finds a certain outlet round the north of Jutland to replenish the Danish home waters, will have a similar effect. However, the case is far from being simple, the current in the Norwegian Ditch also being affected by many other circumstances, such as outflow from the Baltic Sea, variations in the temperature and salinity of the water, etc., but it must be justifiable to expect that the proximity of the ditch must cause a downward bend of the upper part of the frequency curve for Thyborön and an increased number of moderate high waters.

SUMMARY

The results found in the foregoing may be summarized as follows:

I. The main features of the bottom topography of the North Sea being so that Thyborön is situated almost off the transition between the particularly shallow southern and the somewhat deeper northern part, cause the frequency curve throughout to be less steep for Thyborön than for the sea off Holland.

II. The fact that south-westerly and westerly winds as a whole occur most frequently, whereas the north-westerly winds are those which most frequently occur with great force, involves that the frequency curve for Thyborön starts with greater frequencies than does the Dutch curve.

III. The presence of the deep Norwegian Ditch, which provides an outlet for all great accumulations of water in the southern part of the North Sea, causes the same effect as in II, viz. an increased number of moderate high waters at Thyborön, while the very proximity of this deep ditch counteracts the formation of extraordinarily high floods at Thyborön and thus causes a downward bend of the upper part of the frequency curve.

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In view of this one would expect the mutual relation between the frequency curves for Holland and Thyborön roughly to be like that of the curves A and B in Fig. 22.

CLOSING REMARKS

As already mentioned, only the direct influences of wind hav been dealt with above, while many secondary circumstances have bee disregarded, including the familiar fact that the astronomical and the meteorological high water cannot directly be superimposed on each other, as the changes in the height of water will cause mutual influences between them. Further the above-mentioned circling currents may be expected to cause some loss of energy, thereby diminishing the up-settings, and another point is that the formula used for the wind-effect should possibly be modified when D is exceeded. It will thus be understood that the results found can only represent certain main features concerning the frequency curves.

Finally, it must be pointed out that the "derived" type of frequency curve with a maximum value corresponding to a wind-velocity of 29 m/sec. does not, of course, correspond to actual conditions, as the wind may be much stronger (cf. the fact that Beaufort 12 just designates wind- velocities higher than 29 m/sec. even if, as a rule, the strongest winds may probably be supposed only to prevail over a limited area. However, as van Veen has said [7] , a gale can blow "harder than hard and longer than long", and the very purpose of the frequency curve is by its extrapolation to provide a well-founded estimate concerning extreme cases, but - especially where the period of observations is short - there will be every possible reason to supplement direct observations by a closer study of geographical and meteorological conditions, thereby widening our knowledge of actual circumstances and improving our understanding of the phenomena which occur.

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CHAPTER 7

WIND TIDES ON LAKE OKEECHOBEE

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INTRODUCTION

Determination of wind tides and wave action is an essential step i the design of flood-control and navigation projects and of structures near large bodies of water which may be subjected to hurricane winds. I 1948, the Corps of Engineers initiated a program to collect wind-tide ε wave data on Lake Okeechobee. Basic data collected and investigations made under that program have been published as a series of project bull tins, "Waves and Wind Tides in Inland Waters, Lake Okeechobee, Florida, and in a summary report, "Civil Works Investigation CW-167, Waves and Wind Tides in Shallow Lakes and Reservoirs." Data on wind velocities, wind tides, and waves have been collected under that program during six hurricanes and many minor storms. In this paper an attempt is made to summarize the results of the wind-tide studies and outline the procedur developed for computing wind tides on Lake Okeechobee.

DESCRIPTION OF AREA

Lake Okeechobee is a large, shallow body of fresh water in souther Florida. The lake is nearly circular in shape and has an area of about 730 square miles. The lake bottom is saucer-shaped, with the deepest part at about mean sea level. Bottom composition is chiefly sand, shel. muck, and rock. Aquatic grasses and marsh vegetation cover the western portion of the lake. The entire southern half of the lake and a small section at the north end are inclosed by levees. The crown elevation o existing levees ranges from 32.5 to 37 feet. The lake has a drainage area of 5,500 square miles. The lake is now regulated between the limi of 12.5 and 15.5 feet above mean sea level insofar as hydrologic conditions permit; during flood periods, lake levels often rise several feet above the scheduled stage.

The network of gages from which records have been collected is show on figure 1 along with the topography of the lake bottom. The network c gages includes 14 recording water-level gages, 5 wave staffs, 12 statior where wind speed and direction are measured about 32 feet above the wate surface, and 7 stations where barometric pressures are recorded. Two of the lake stations have additional anemometers near the water surface so that the wind gradient can be measured. Data collected from this networ of gages have been used to evaluate unknowns in the wind-tide formula an develop procedures for computing wind tides.

WIND-TIDE FORMULA

The term "wind tide" is used to describe the changes in water surface caused by action of the wind on the water. Wind tides may be composed of both static and dynamic tides. Dynamic tides, or seiches, occu

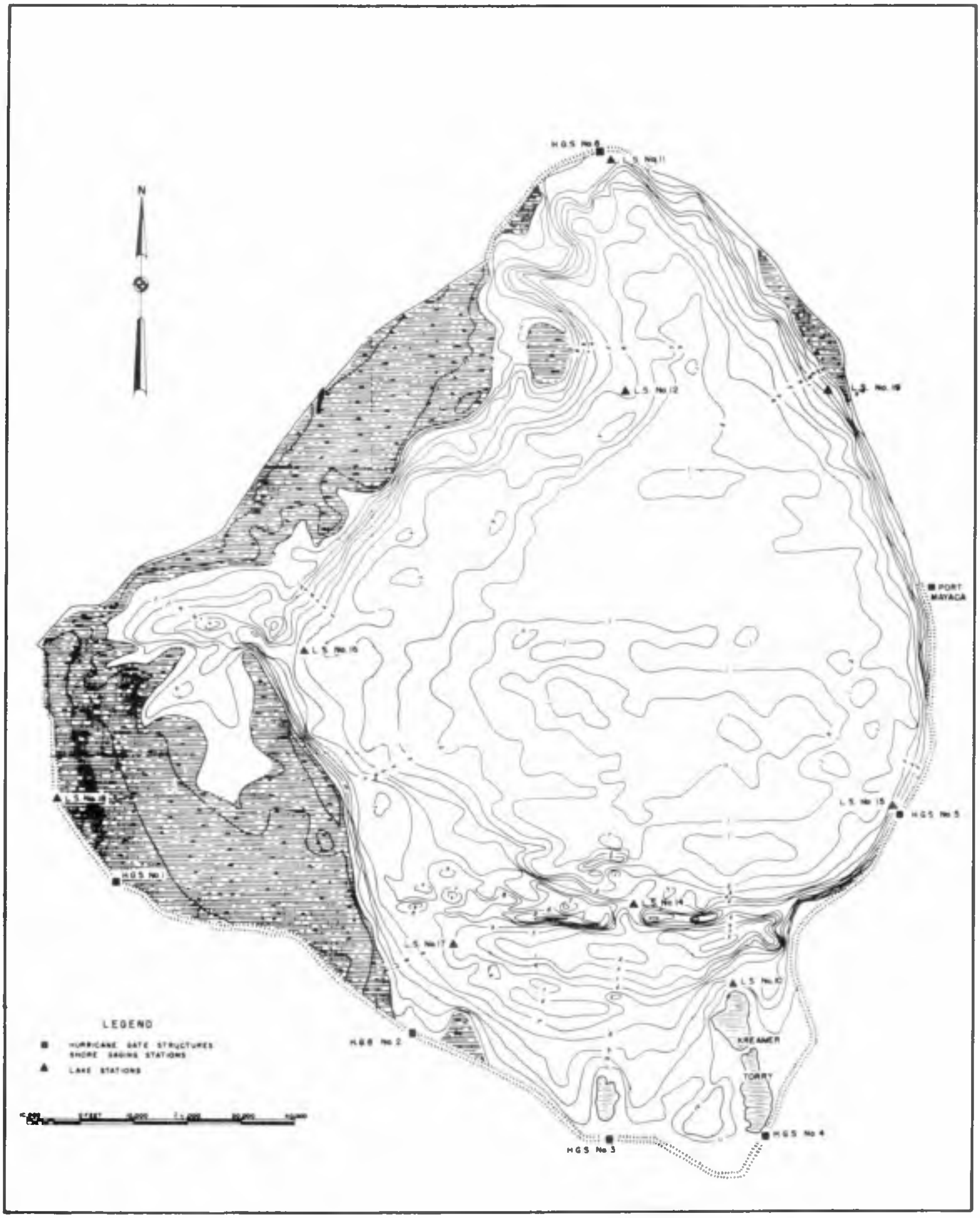


Fig. 1. Topography and gage locations, Lake Okeechobee, Florida

when the momentum of the water carries it beyond the position of stat equilibrium, and oscillations result. The magnitude of the seiches ap pears to be a function of depth, fetch, and bottom characteristics. Seiches of about 1 foot have been recorded on Lake Okeechobee, but the do not appear to have a significant effect on major wind tides on this lake. Analyses of wind tides in this paper pertain solely to static tides.

The basic wind-tide equation used by Hellstrom and others can be written as the differential of the change in water-surface slope with distance:

$$\frac{dh}{dx} = \frac{\lambda T_s}{\gamma (D+h_s)}$$

Terms and symbols used in the wind-tide equations in this paper are defined as follows:

h_s	Setup above mean water level (ft.)
F	Fetch distance (ft.)
D	Water depth (ft.)
γ	Specific weight of water (62.4 lb./ft.^{3})
V	Wind speed (ft./sec.)
T_s	Tangential stress on water surface (lb./ft.^{2})
T_b	Tangential stress on the bottom (lb./ft.^{2})
λ	A dimensionless parameter, defined as $\lambda = T_s/T_b+1$
N	A dimensionless factor, related to the ratio h_s/D
P	A planform factor used to evaluate the effects of converging and diverging shorelines

The basic equation integrates to

$$h_s = \frac{\lambda T_s F}{2 \gamma (D+h_s)}$$

for a rectangular channel of uniform depth. With the introduction of a planform factor P to account for the effect of converging and diverging shorelines and of Keulegan's N factor to replace h_s on the right of the equation, the basic wind-tide formula becomes

$$h_s = \frac{N \lambda T_s F P}{2 \gamma D}$$

FACTORS AFFECTING WIND TIDES

The principal variables in the basic equation for computing static wind tides are fetch, depth, and shear stress. In some cases, planform and barometric pressure should be considered.

Depth and fetch are physical features of a body of water and can be determined from hydrographic maps. In irregularly shaped lakes of varying depth, the lake should be divided into increments and the average depth determined in each increment so that variations in depth can be taken into consideration. In most cases, the fetch is measured as the straight-line distance across the water surface. However, when there is an appreciable curve in the wind streamlines, the fetch should be measured along the curved line.

Surface shear, T_s, is the tangential stress exerted by the wind on the water surface. Since it is very difficult to obtain a direct measurement of this force over rough water, the surface shear is usually computed from measurements of the wind gradient above the water surface. Von Karman and Prandtl found that the wind profile is logarithmic and that the surface shear can be computed from the equation

$$T_s = \rho_a V^2$$

$$\text{where } V = \frac{k_o V}{2.3 (\log z - \log z_o)},$$

and k_o is a coefficient which Von Karman found to have a value of 0.4, ρ_a the density of air, V the wind velocity, z the distance above the water surface at which the wind velocity was measured, and z_o a roughness parameter obtained by projecting the wind velocity to the boundary layer. Unfortunately, very few reliable measurements have been obtained to establish the velocity gradient above a water surface during unusual storms or hurricanes. Multiple anemometer stations were not established on Lake Okeechobee until 1953 and the highest velocity recorded since that time was 60 feet per second. Anemometers on lake station 14 are now operated about 5.5, 13.5, and 32 feet above the normal water surface. Direct computation of T_s using data collected at that station showed considerable scatter, which made it difficult to extrapolate the data to high wind velocities. Plotting data observed at the 32-foot level against that at the 5.5-foot level indicated the linear relationship shown on figure 2. The points plotted as (.) on figure 2 are data recorded on October 9, 1953, when air and lake temperatures were about equal and rain was falling. The data plotted as (x) were obtained in February and March 1954 after cold air masses had moved across the lake. The air temperature was from 1° to 17° F. colder than the water temperatures and the air was relatively stable. This relationship was used to compute values of T_s for wind velocities measured at the 32-foot level. The resulting shear curve, which has the equation $T_s = 4.39 \times 10^{-6} V^2$, is shown on figure 3. For comparison, figure 3 also shows a shear curve computed by Sibul from wind-velocity measurements in a wind-actuated model at the University of California with winds extrapolated to the 30-foot level by using a logarithmic relation. The close agreement between the curves obtained from

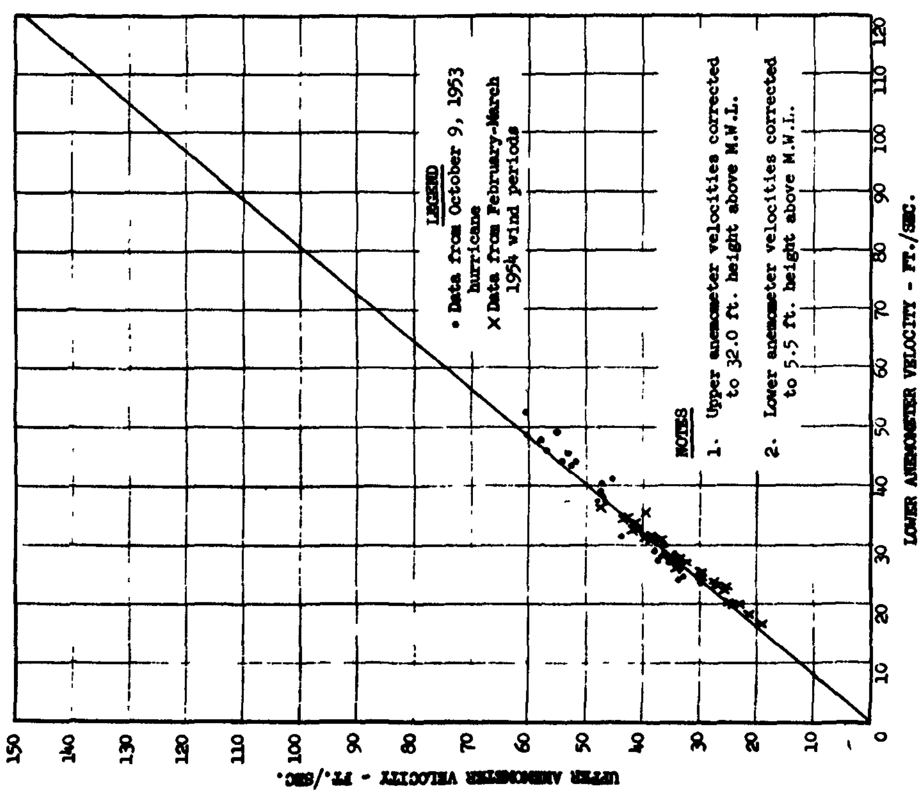


Fig. 2. Relation between wind velocity and elevation of anemometers

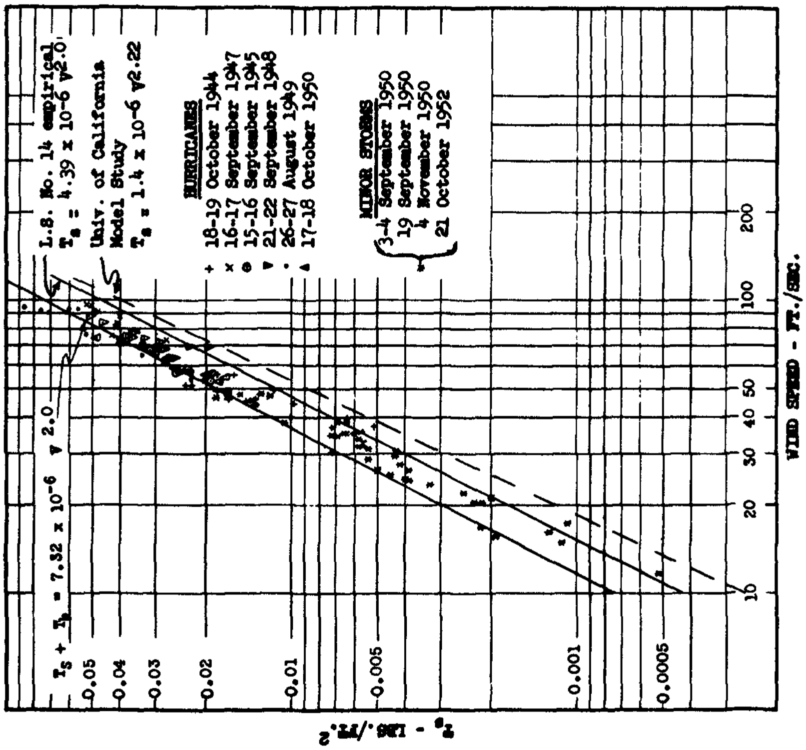


Fig. 3. Shear-stress curves

model and field measurements indicates that data from wind-actuated models can be used in analyzing wind-tide problems.

Total shear stress, λT_s or $T_s + T_b$, has been determined from laboratory studies by Hellstrom, Keulegan, and Sibul. The model data indicated that T_b varies from zero to a value greater than T_s. However, definite relationships between T_b and bottom roughness, depth, and other factors have not been determined for conditions similar to those found in Lake Okeechobee. Therefore, values of λT_s were obtained by solving the wind-tide equation for λT_s using the basic data collected at Lake Okeechobee during six hurricanes and many minor storms. Basic data are contained in the project bulletins referred to above. The shear stresses needed to reproduce observed wind tides, which ranged from 1 to 10 feet, are plotted on figure 3 against lakewide average wind velocities at the 32-foot level. The values shown were obtained from the wind-tide equation using average depths and velocities over the whole lake without a correction for planform. The scatter of data indicates the value of λ may vary from 1 to 2, and the average value is about 1.66. Some of the scatter in the data could be eliminated by computing the wind tides by a step-integration procedure in which the effect of variations in depth and wind velocity is considered. Computations using step-integration procedures indicated λ may have values of about 1.2 near the center of the lake and about 2 near the edges where the bottom is sloping and considerably rougher. Additional studies are needed to determine how bottom shear varies with depth, bottom roughness, water turbulence, and other factors. Until such studies are completed, the shear-stress curve shown on figure 3--which has the equation $\lambda T_s = 7.32 \times 10^{-6} V^2$--will be used to compute wind tides in Lake Okeechobee. Tickner has found that the wind tides in the model at the University of California can be doubled when window screen is stapled to the bottom to simulate bottom roughness.

Planform.--A planform factor P is used when the center of gravity of a body of water is not at the midpoint of the fetch. For a triangular lake, the factor varies from 0.67 when the wind tide is occurring along a side of the triangle to 1.33 when the tide occurs at an apex. If the shoreline forms an approximate trapezoid, the planform can be obtained from the formula

$$P = \frac{2}{3}\left(\frac{2\, b_o + b}{b_o + b}\right)$$

where b_o is the width of the windward shore and b is the width of the leeward shore.

N is a variable, derived by Keulegan, based on the ratio of setup to depth. With its use, wind tides with either exposed or nonexposed bottom may be computed. Variations in the values of N with h_s/D are obtained from a curve through the following points:

h_s/D	N	h_s/D	N
0.01	1.00	1.00	0.75
0.10	0.97	1.50	0.58
0.30	0.92	2.00	0.44
0.50	0.89	4.00	0.19
0.70	0.85	10.00	0.04

Atmospheric pressure.--When a hurricane passes over a large body c water, the reduction in atmospheric pressure near the center of the storm causes the water level to rise. When there is a difference in th pressure at the point where the wind tide occurs and the average pressu over the lake, a correction of 1.14 feet of water for each inch of mercury is applied.

PROCEDURES FOR COMPUTING WIND TIDES

Wind tides can be computed on lakes or reservoirs where variations in bottom, shorelines, and wind velocity are small, using lakewide aver ages and the basic wind-tide equation. However, where there are large variations in any of these factors, more accurate results can be obtain by breaking the lake up into sections. Two integration methods have be used on Lake Okeechobee. In the cross-sectional method, the lake is di vided into cross sections perpendicular to the wind streamlines. Then, the average bottom elevation, fetch, and wind velocity over each cross section are determined and the water-surface profile across the lake co puted. The integration procedure is started at the approximate center (gravity of the lake. A setup or setdown across the section to be computed is assumed and values of P and N obtained and substituted in the wind-tide equation along with values of F and λT_s. The setup is computed and compared with that originally estimated. If the assumed setu does not agree with the computed, new assumptions are made and the proc· ess repeated until satisfactory agreement is obtained. Then, similar computations are made for the next cross section. When the wind-tide profile has been completed, the volume of water above the normal level i the setup portion of the lake is determined and compared with the volume removed in the setdown end. If the volumes do not balance, an adjustmer must be made in the location of the node line and the entire integratior process repeated until a volume check is obtained.

Another method, suggested by Hunt, divides the lake into a number c segments. The number of segments required depends on variations in size of lake, depth, and wind speed. The wind-tide profile is computed for each zone in the same manner as described for cross sections, beginning at an assumed node line. However, with a cross section divided into a number of segments, variation in the water-surface profile in different

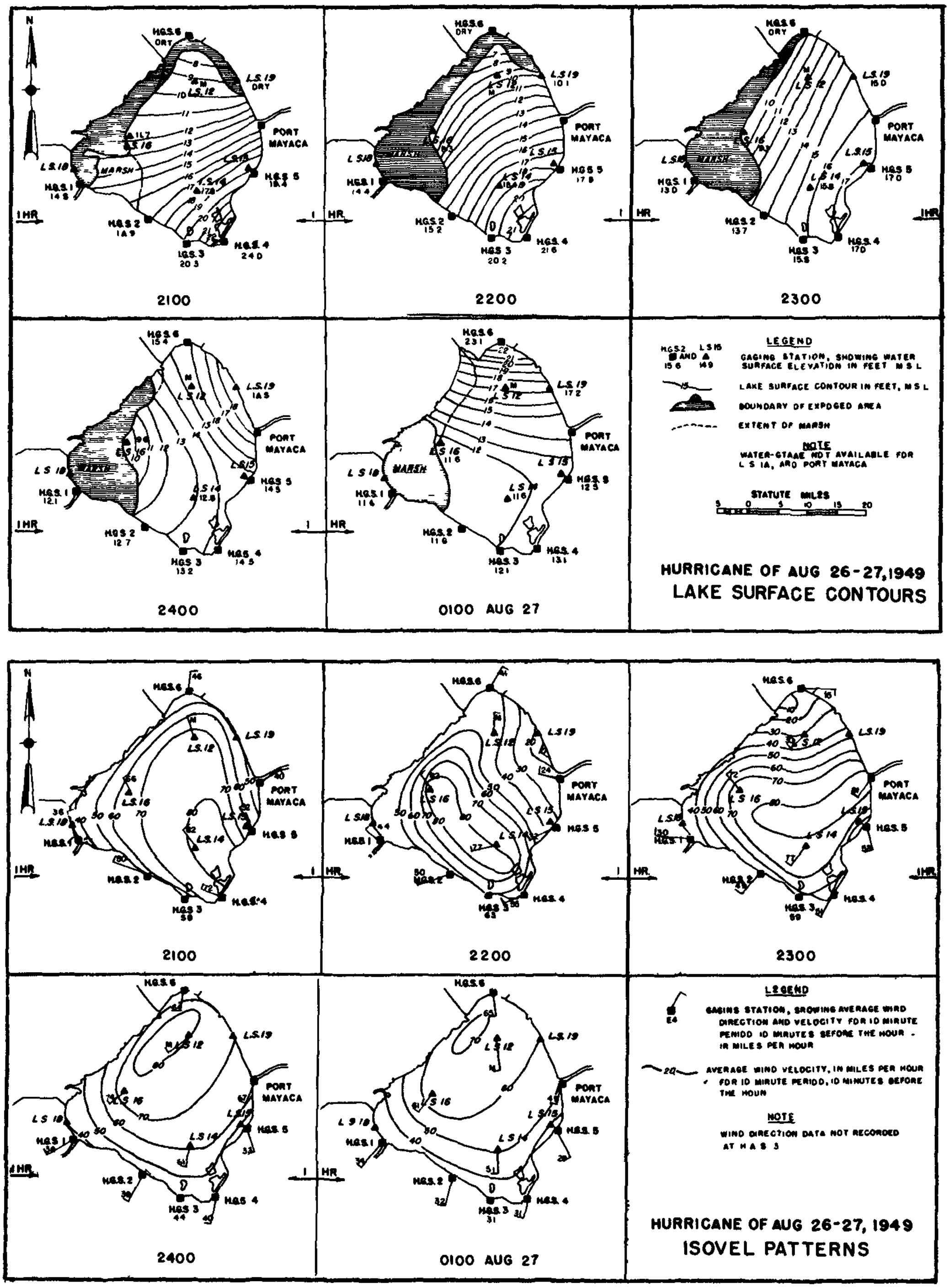


Fig. 4. Water-surface contours and wind velocities during 1949 hurricane

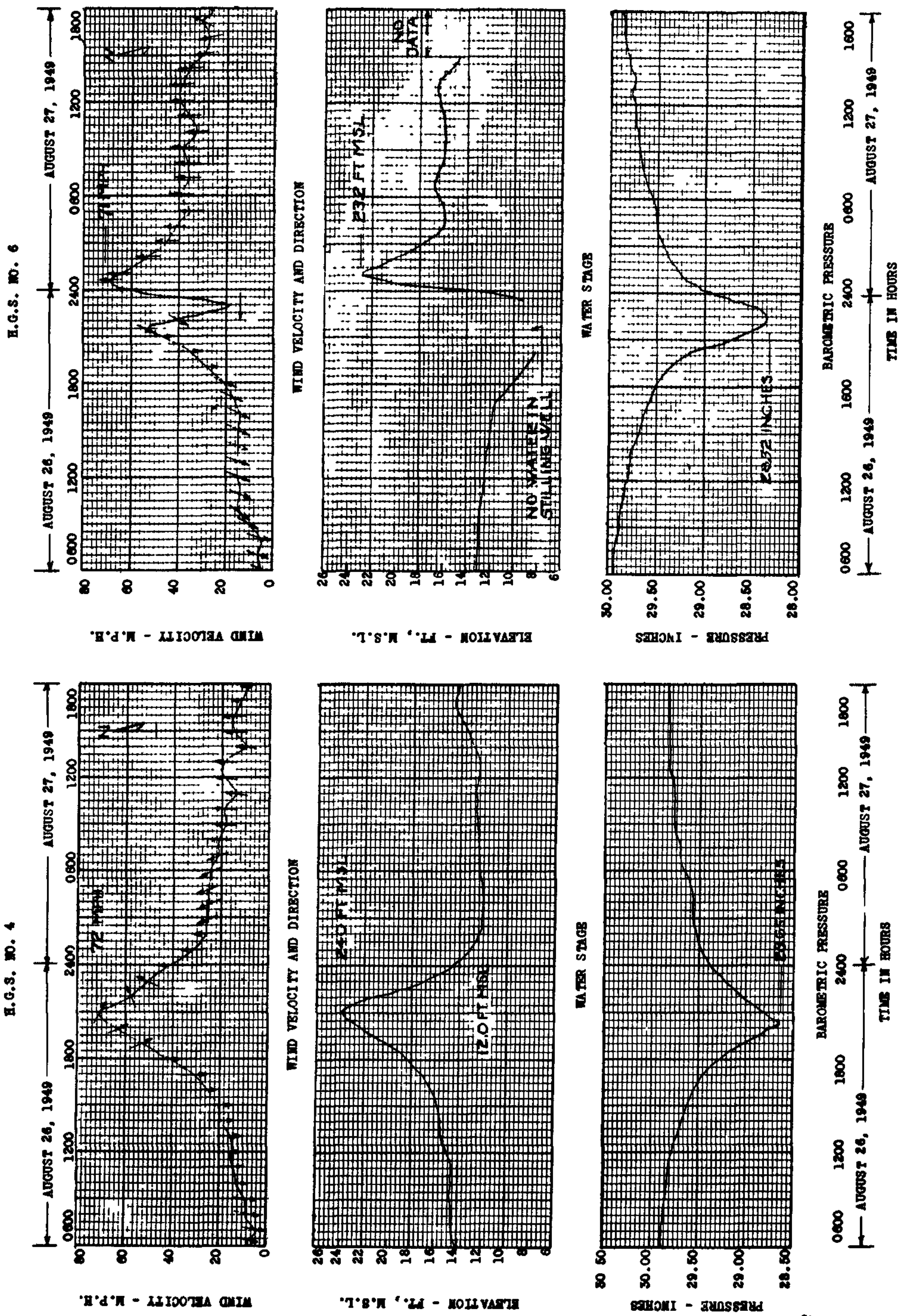


Fig. 5 Meteorological data at key stations

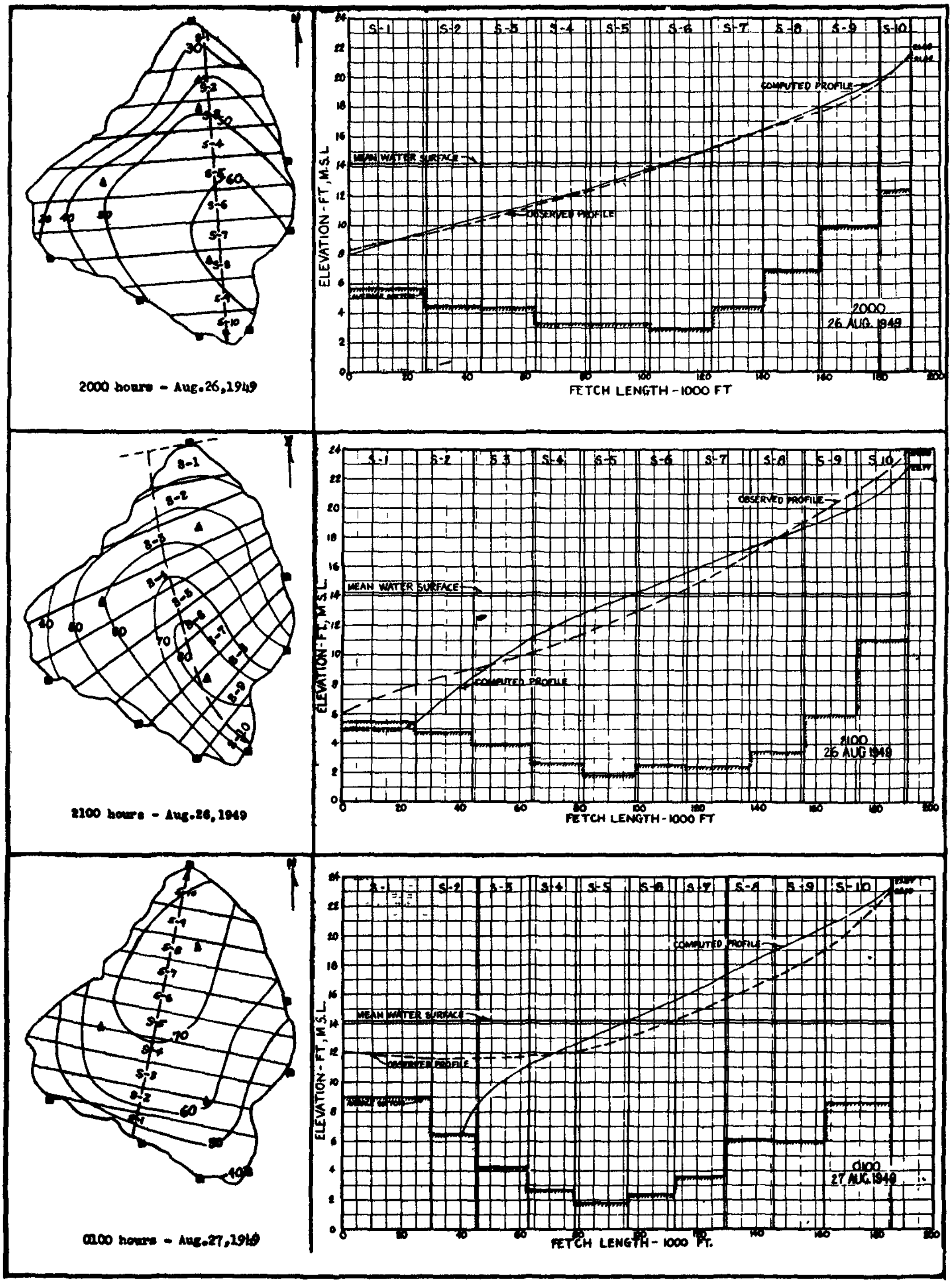
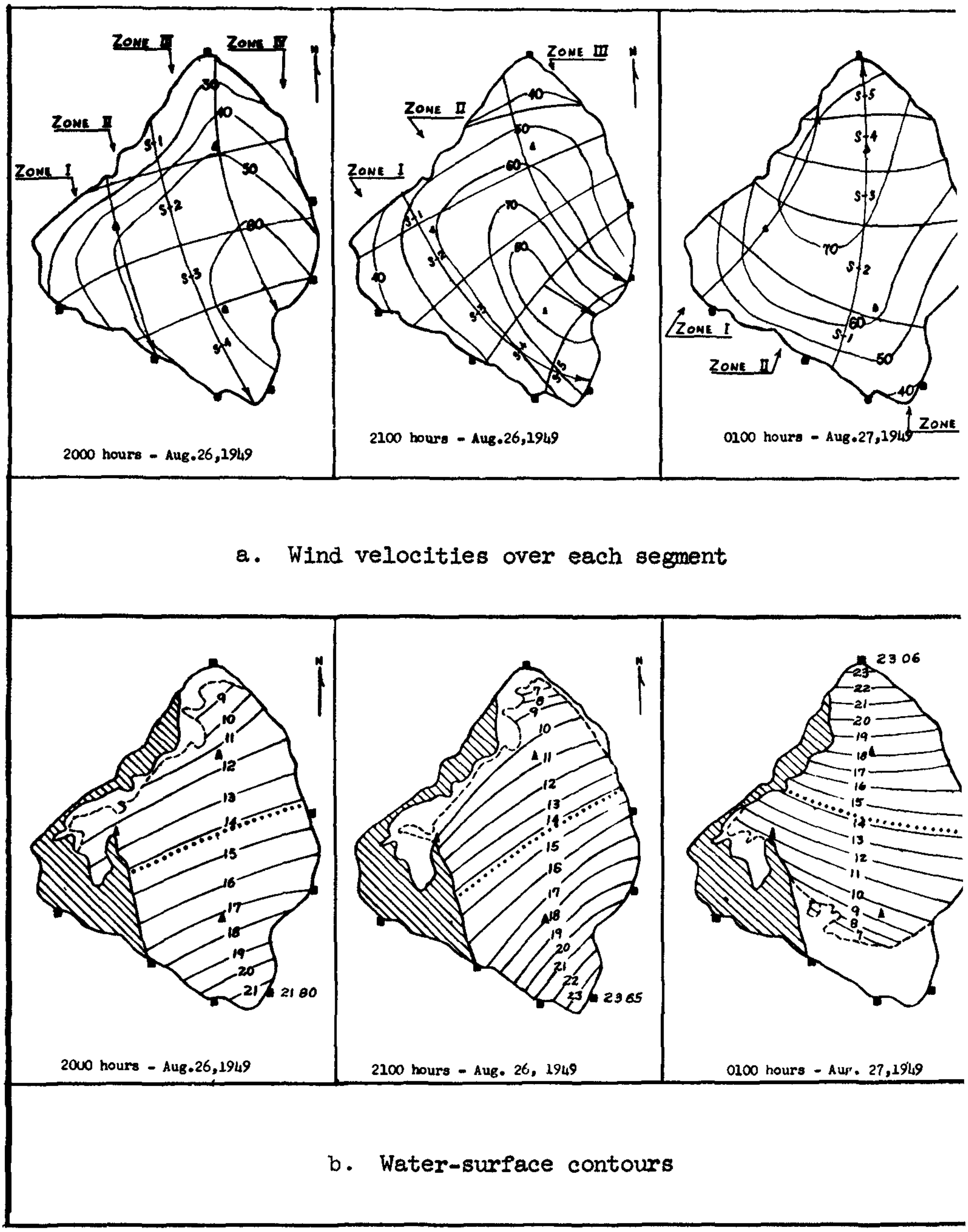


Fig. 6. Wind-tide profiles computed by cross-sectional method



a. Wind velocities over each segment

b. Water-surface contours

Fig. 7. Water-surface contours computed by segmental method

parts of the cross section may exist and adjustments between adjacent zones are required. Where variations in water levels occur perpendicular to the wind streamlines, Manning's formula is used to compute the flow to the adjacent segments. The procedure is one of trial and error and computations are continued until reasonable balances are obtained. After satisfactory water-surface profiles are obtained for each segment, water-surface contours are constructed and the total volume in the lake under the water-surface contours determined. If the volume does not check, an adjustment is made in the nodal line and the entire procedure repeated.

WIND TIDES DURING 1949 HURRICANE

On August 26, 1949, a very severe hurricane came off the Atlantic Ocean and entered Florida near West Palm Beach. It continued on a northwesterly path and the center passed over the northern edge of Lake Okeechobee. Isovel patterns over the lake for the period from 9 p.m. on August 26 to 1 a.m. on the 27th are shown on figure 4 along with water-surface contours for the same period. Graphs on figure 5 show variations in wind velocities, water levels, and barometric pressures at stations where maximum wind tides were recorded. Wind tides computed by both the cross-sectional and segmental integration methods are shown on figures 6 and 7, along with maps of the zones used in computing wind-tide profiles.

SUMMARY

Although much remains to be learned about wind tides and tangential shear stresses, the empirical curves and procedures described here can be used to compute wind tides on Lake Okeechobee with considerable accuracy. Additional studies are needed to determine relationships between wind velocities and shear stresses. It is hoped that sufficient data can be collected on Lake Okeechobee or other bodies of water subjected to hurricane winds to permit verification of all factors in the wind-tide equation and development of accurate procedures for determining wind tides on all bodies of water.

ACKNOWLEDGMENTS

The data presented and the analyses thereof were accomplished in the Jacksonville District Office, Corps of Engineers, under the general supervision of Edwin W. Eden, Jr., Chief of the Planning and Reports Branch, Engineering Division. Personnel assisting in the study included T. E. Haeussner, E. H. Miller, and others in the Meteorology and Regulation Section. Appreciation of the Corps of Engineers' permission to use this material is acknowledged.

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CHAPTER 8

WATFR WAVES DUE TO A LOCAL DISTURBANCE

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Abstract - A model investigation of the characteristics of waves generated by a local disturbance was made in order to obtain comparison with the theories of UNOKI and NAKANO (1953) and KRANZER and KELLER (1955).
The two-dimensional model for the case of initial local elevation or depression of uniform height of the water surface showed that certain wave characteristics such as phase periods and "interference" pattern could be described reasonably well within certain limits of water depth and height and extent of the disturbed area. Beyond those limits the leading part separated from the generated wave pattern as a solitary wave or a more complicated wave system. For a certain range of conditions the leading part was preceded by a bore during the first portion of the travel.

INTRODUCTION.

The mathematical studies on the properties of surface waves generated by a local disturbance is a classical problem. It finds its application in the prediction on the wave motion resulting from an underwater seismic disturbance (tsunami) and it has recently come into prominence in the description of the wave motion resulting from an artificial near-surface, or underwater, explosion.
The mathematical description is dependent on a rather schematized assumption of the initial conditions. Most of the derived equations have never been physically checked in a systematic way as to their reliability in practical applications.

The generation of surface waves due to a local disturbance can be considered under three groups:

A. Initial elevation or depression of the surface without initial velocity.

B. Undisturbed surface with an initial distribution of a surface impulse.

C. Undisturbed surface with an initial distribution of a submerged impulse (underwater explosion).

According to these groups PRINS (1956) tabulated the authors, mentioning initial conditions, wave equations and wave characteristics. In the groups A and B the mathematical solutions are summarized from the authors LAMB (1945), UNOKI and NAKANO (1953), KRANZER and KELLER (1955) and PENNEY (1950), in group C from KIRKWOOD and SEEGER (1950) and FUCHS (1952).

Only a few model investigations on the wave performance due a looal disturbance are available. BRYANT (1950) compared the theor of PENNEY (1950) with experimental observations. Experiments on the wave generation by a surface impulse were done by JOHNSON and BERME (1949). A laboratory study of gravity waves generated by the moveme of a submerged body was done by WIEGEL (1955).

Prototype data are given in a comprehensive description in " EFFECTS OF ATOMIC WEAPONS" (1950) and on tsunami by UNOKI and NAKAN (1953).

In this paper some results of two-dimensional model investig tions for the case of initial local elevation or depression of unif height will be given. In the experiments the heights of the elevati or depression, the extent of the disturbed area and the water depth were varied. A comparison will be made with the theories of UNOKI a NAKANO (1953) and KRANZER and KELLER (1955). The leading part under oonditions not fitting the theories will be discussed.

EXPERIMENTAL EQUIPMENT AND PROCEDURE.

The model investigations were carried out in a flume one foo wide by sixty foot long. On the one end of the channel was placed a air-tight box of plexiglas with a sliding front wall, in which the water level could be elevated or depressed by means of a decreased increased pressure in the air compartment. By pulling the slide upwa in the shortest possible time it was possible to develop a free elevated or depressed area of uniform height with all the water particl effectively at rest, figure 1. The back wall of the box was conside to oause a total reflection and hence is the axis of symmetry of the system. At the opposite end of the channel a wave absorber was installed.

The extent L and the height Q of the elevations or depression and the water depth h were varied by steps:

h = 2.3, 0.5, 0.35 and 0.2 foot.

L = 1/3, 1 and 2 foot.

Q = $\pm$ 0.1, $\pm$ 0.2 and $\pm$ 0.3 foot.

The vertical movement of the water surface η was recorded as function of time simultaneously at five places along the ohannel wit a six-channel Brush Electronic Co. recorder by using parallel wire resistanoe wave gages (WIEGEL 1956), figure 2. On the same record th zero time was fixed by a resistance gage inside the box near the fro which indecated the drop, or rise, of the inside water level after pulling the slide. The charts present an x,t-plane, in which are projeoted the η,t-histories of the five gages, figure 4.

The movement inside the box and in its vioinity was filmed. T first stage of generation is shown in figure 3.

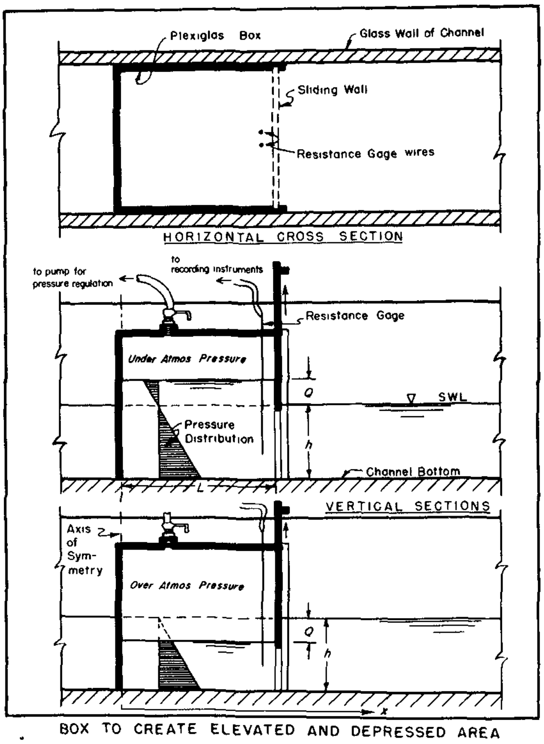

Fig. 1. Box to create elevated and depressed area.

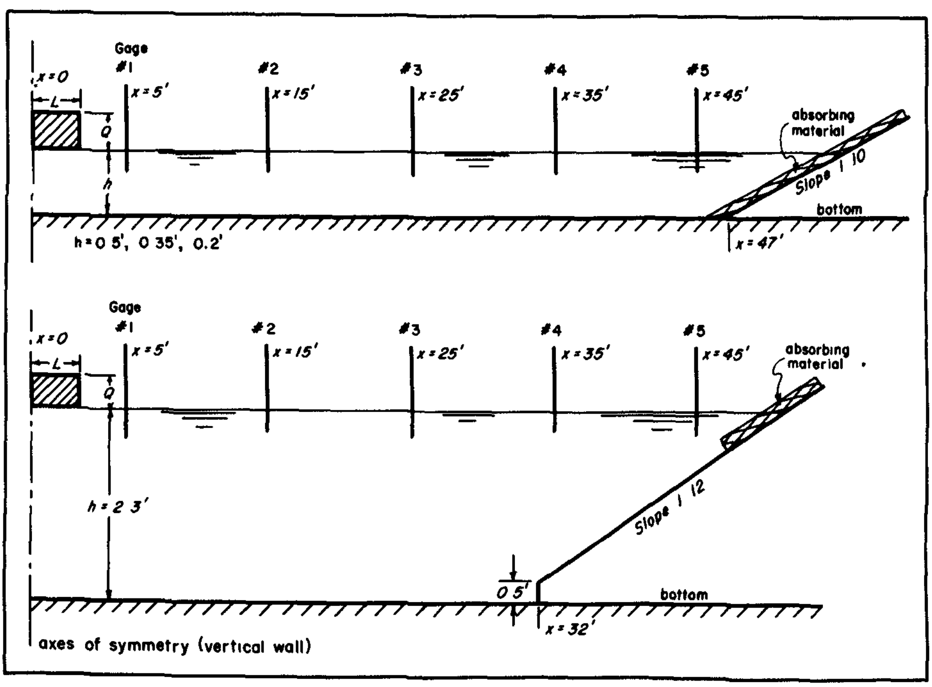

Fig. 2. Arrangement of model tests.

MATHEMATICAL DESCRIPTION AND COMPARISON WITH THE EXPERIMENTS.

The formulation of the problem in regard to the model invest gation was to describe the water surface elevation with respect to undisturbed water level as a function of place and time when a disturbance consisting of an uniform elevation or depression with a constant height Q extending over a length L in a two-dimensional system of water depth h was released.

In general $\eta = f(Q, L, h, x, t)$ (

Two cases given in the literature are applicable to the give conditions. One is the solution of the Cauchy-Poisson wave problem for the case of an initial elevation to a finite area in water of infinite depth by UNOKI and NAKANO (1953) in their approach for the description of tsunami waves. The other case is the description of surface waves produced by explosions in water of finite depth by KRANZER and KELLER (1955). A summary of their results adapted to th model investigations is given by PRINS (1956).

The simple form of the UNOKI-NAKANO equation illustrates in the best way the characteristics of the generated waves.

The equation $$\eta = \frac{4Q\, x^{\frac{1}{2}}}{\pi^{\frac{1}{2}} g^{\frac{1}{2}} t} \sin\left(\frac{gt^2}{4x} \cdot \frac{L}{x}\right) \cdot \cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right) \quad (\text{:}$$

for $x \gg L$,

is composed of two periodic systems.

The phase period is defined by $\cos\left(\frac{gt^2}{4x} - \frac{\pi}{4}\right)$.

so that $$T = \frac{4\pi x}{gt}, \quad \lambda = \frac{8\pi x^2}{gt^2} \quad \text{and} \quad C = \frac{2x}{t} \left(= \sqrt{\frac{g\lambda}{2\pi}}\right) \quad (\text{:}$$

As the period T is dependent on x and t, we have to deal with dispersive waves. The phase period is independent of the height Q and extent L of the initial disturbance.

The term $\sin\left(\frac{gt^2}{4x} \cdot \frac{L}{x}\right)$ represents a relatively long periodic syste defining the amplitude variation of the waves with phase velocity. I will be referred to as the amplitude envelope term, with the qualities:

$$T_1 = \frac{4\pi x}{gt} \cdot \frac{x}{L}, \quad \lambda_1 = \frac{8\pi x^2}{gt^2} \cdot \frac{x}{2L} \quad \text{and} \quad C_1 = \frac{x}{t} \quad (4$$

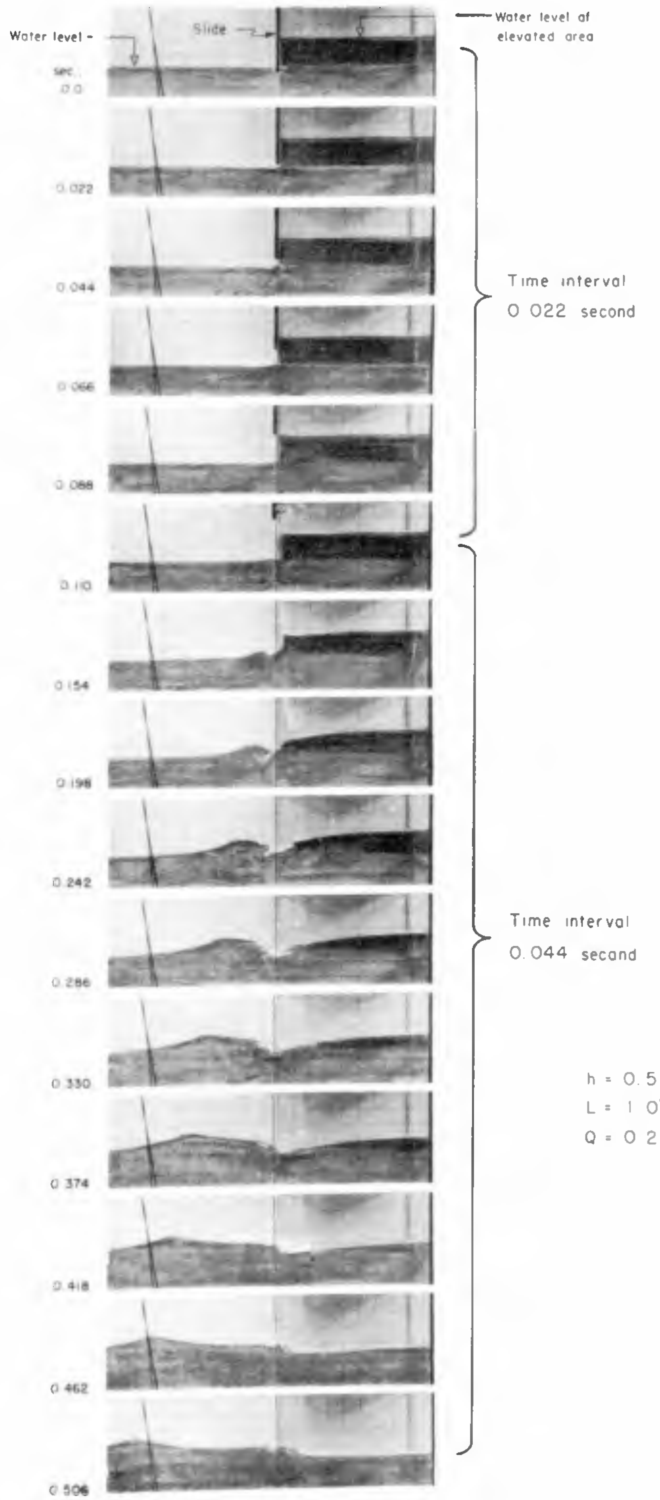

Fig. 3. Photographs showing early stages of wave generation.

The period T_1 also shows dispersive properties. It is inversely proportional to the extent L of the initial elevation.

At a certain position (x, t) the envelope velocity is one-ha: the phase velocity. At infinite depth the group velocity is definec likewise, so that

$$C_1 = \tfrac{1}{2}C = C_{gr} = \frac{x}{t} \; . \qquad ($$

This equation can be written $x = C_{gr} t$ (6) which according 1 KRANZER and KELLER is also valid for water of finite depth h, if tl following expression is introduced:

$$C_{gr} = \tfrac{1}{2}\left[1 + \frac{4\pi h/\lambda}{\sinh 4\pi h/\lambda}\right] (\tanh 2\pi h/\lambda) \cdot C_0 \qquad ($$

in which $C_0 = \dfrac{g}{2\pi} T$. (

The formula $x = C_{gr} t$ shows that a wave reaching a oertain place X with period T can be considered as a wave of constant lengt propagating with its group velocity from the origin to X.

The KRANZER-KELLER equation also shows relations with a dependency on the water depth h giving similar wave characteristics

The experimental data are obtained as η, t-curves at a fixed place. There is a definite distinotion with respect to the leading part of the wave pattern, dependent on the ratios Q/h and L/h.

Within the limits $Q/h < 0.18$ and $L/h < 0.9$ the leading part had oscillatory charaoteristics and a reasonable agreement of the experimental data and the theory was found.

It was found that:

1. The generated wave pattern was of a dispersive character.
2. The variation of height and extent of the initial disturbanoe di not affect the phase velocities.
3. The phase periods were found to agree with the theory of KRANZER and KELLER, and for "deep water" with the theory of UNOKI and NAKANO.
4. The wave pattern shows an interference phenomenon, figure 4.
5. The position of the zero-points of the amplitude envelope curve was found to be well expressed by the theory of UNOKI and NAKANO and fairly well by the theory of KRANZER and KELLER.
6. The leading part of the wave pattern for the case of an initial elevation showed exactly the negative performance of the waves generated by an initial depression ($\eta_{elev} = -\eta_{depr}$), figure 6.
7. The amplitudes were found to be directly proportional to the height of the initial disturbance.
8. In considering the energy distribution within the wave pattern, together with the area of disturbance, it was noticed that when

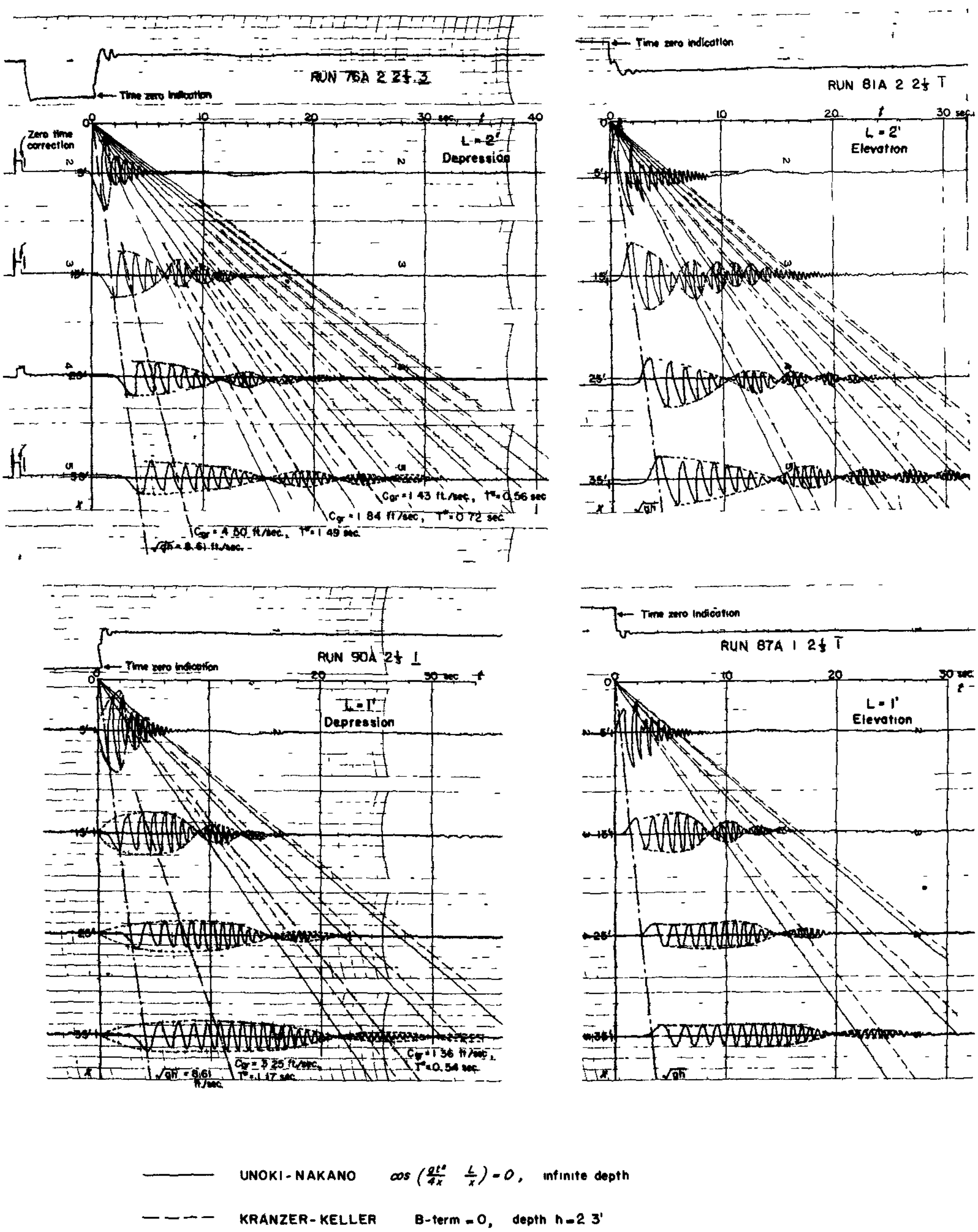

Fig. 4. Amplitude envelope zero-points, h = 2.3 ft.

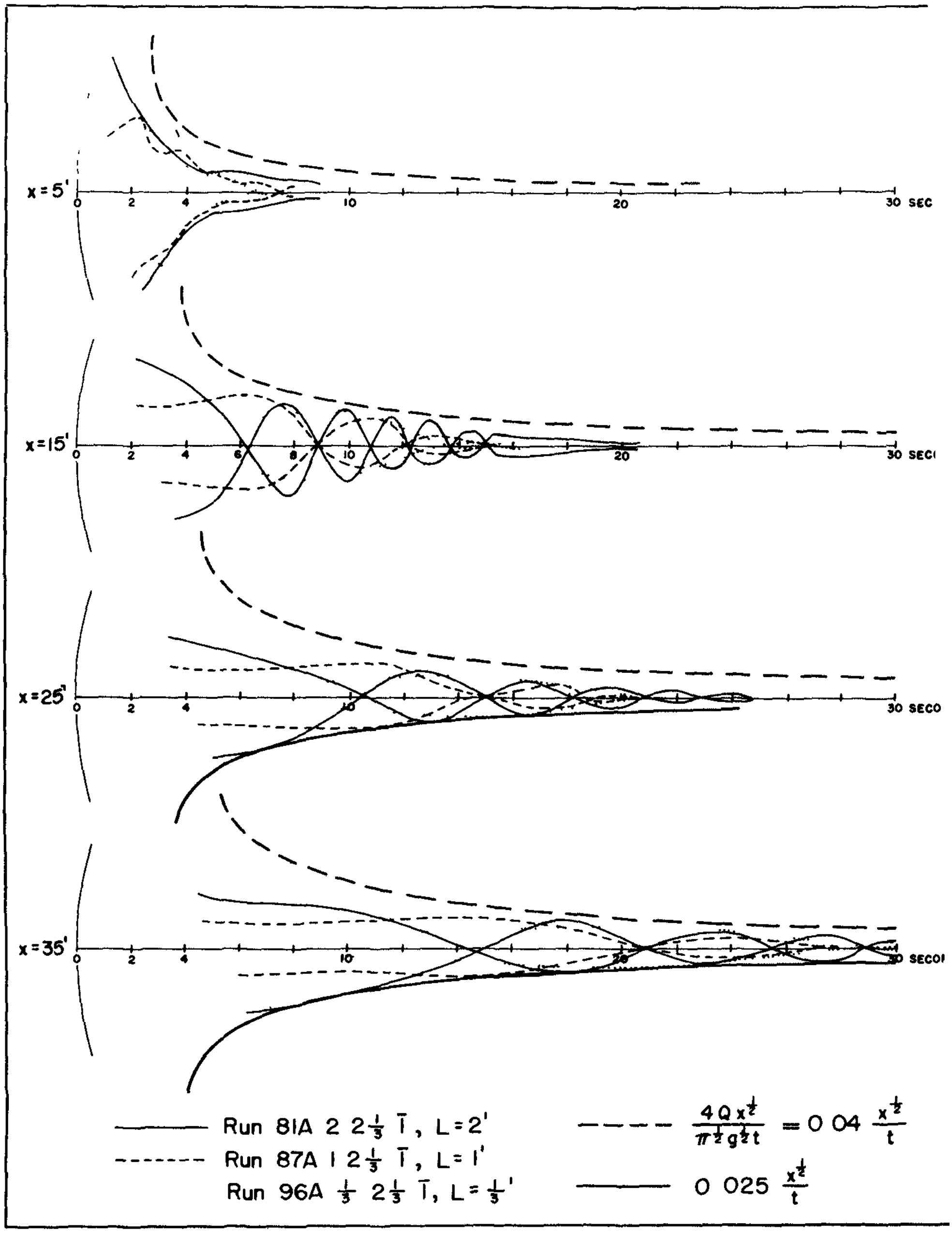

Fig. 5. Amplitude envelope curves with constant Q and variations of

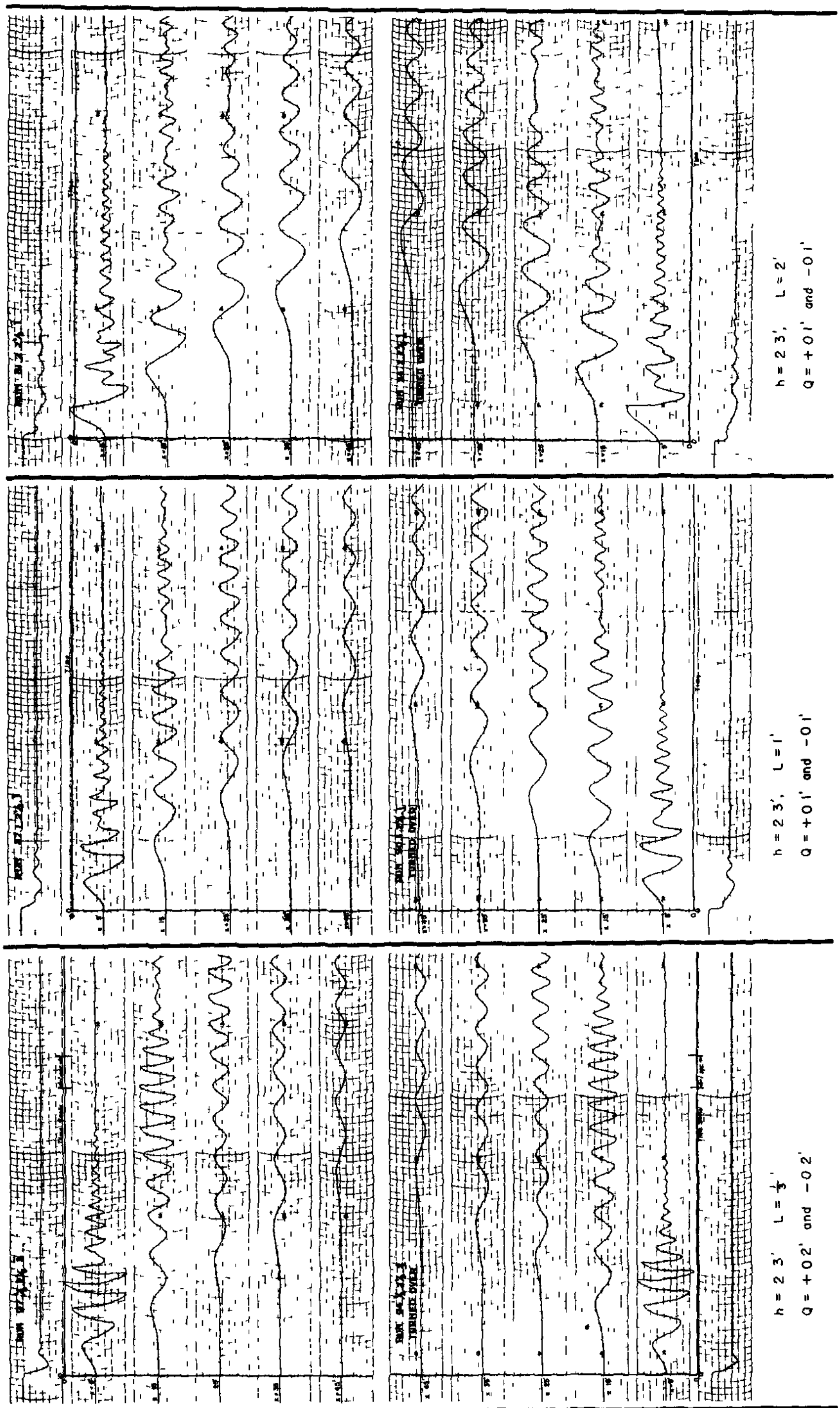

Fig. 6. Comparison of wave patterns produced by elevated and depressed areas.

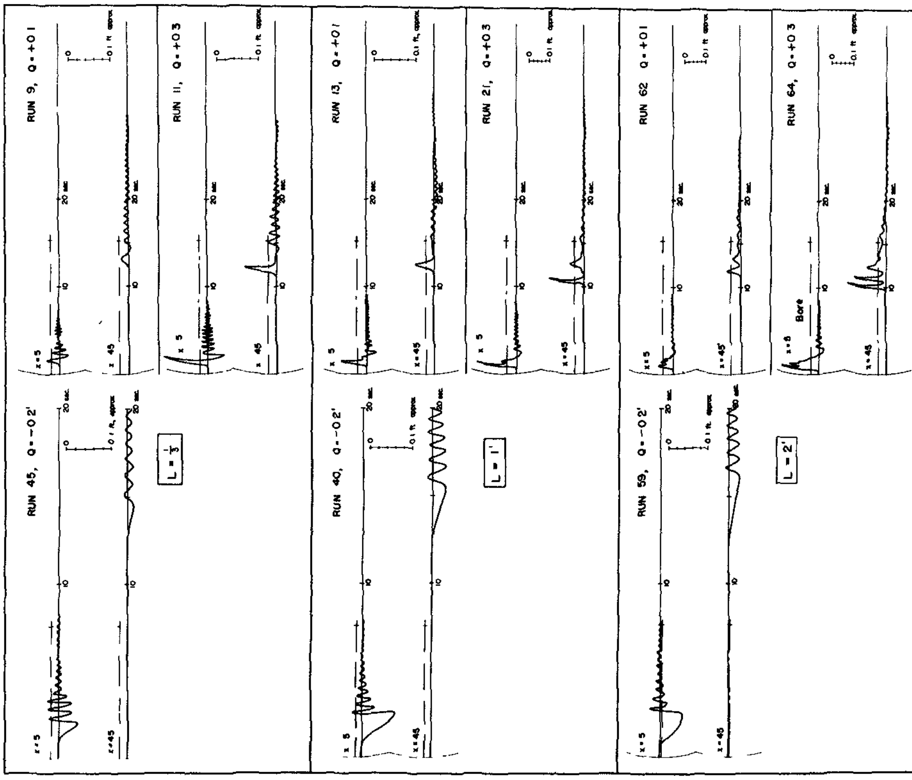

Fig. 8. Leading parts of the wave patterns at depth h = 0.25 ft

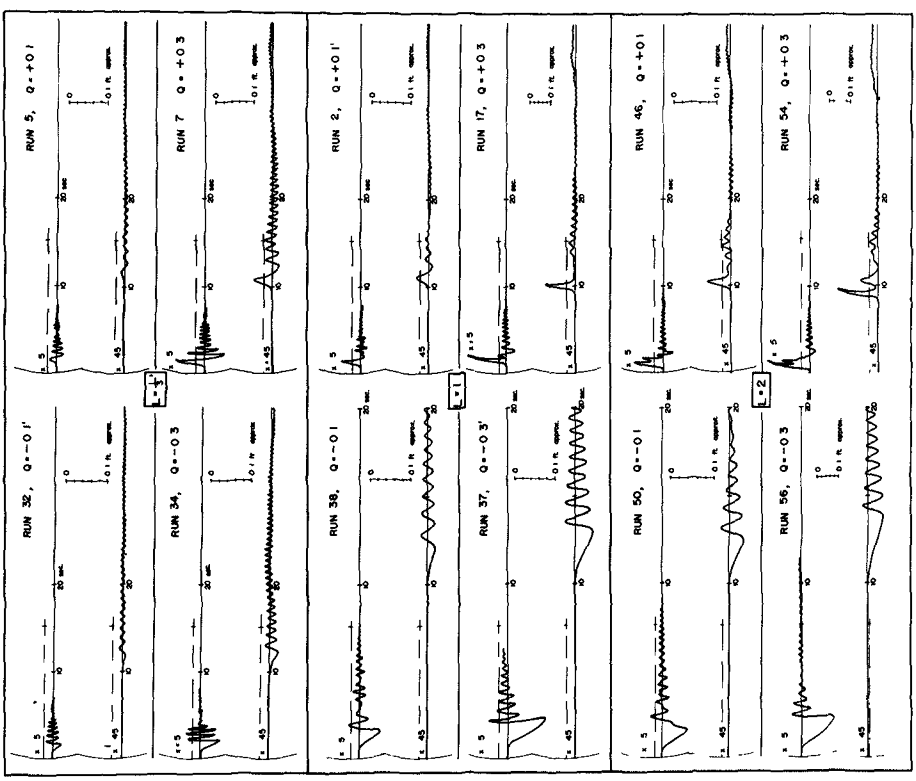

Fig. 7. Leading parts of the wave patterns at depth h = 0.50 ft

the disturbance was extended vertically, the character of the wave pattern was maintained with the amplitudes being proportional to the vertical change of disturbance. When the disturbance was extended horizontally, the wave pattern changed its appearence, but the amplitudes did not exceed those of the common envelope curve, which was independent of the variation of L, figure 5.

9. The wave amplitudes of the experiment in general were found to be smaller than the theoretical values.

CHARACTERISTICS OF THE LEADING PART WITH VARIATION OF Q/h AND L/h.

With respect to the dependency of the leading part on the ratios of Q/h and L/h four types could be distinguished:

1. Leading wave with oscillatory wave characteristics being part of the dispersive wave pattern (O), figure 6.
2. Leading wave with solitary wave characteristics with respect to its velocity of propagation followed by a trough connecting it with the dispersive wave pattern (ST), figure 7.
3. Leading wave being a single wave with solitary wave characteristics, separated from the dispersive wave pattern by a more or less flat part at the still-water-level (SS), figure 7 and 8.
4. The leading part being of complex form, which, while traveling outward, breaks up into a few waves with solitary wave characteristics, separated from the dispersive wave pattern (CS), figure 9.

For certain conditions either a bore (B) or a top bore (TB) occurred which did not seem to affect appreciably the type of leading wave.

In figure 10 the types of the leading part as a function of Q/h and L/h are given schematically, based upon the available model data for the case of an initial elevation.

In the following paragraphs some characteristics of the leading part are discussed more extensively.

In the case of waves with oscillatory characteristics the characteristics of the leading part are in close accordance with the above mentioned theories. Complete similarity is found between the performance due to an initial elevation and an initial depression, figure 6. The amplitude is direct proportional to the height of the disturbance, figure 11.

With increasing Q/h and L/h in the case of an initial elevation the leading wave tends to the solitary wave form and finally reaches the stage where its shape closely approximates the Boussinesq wave form, and it propagates with the celerity
$C = \sqrt{g(h + \eta)}$. Due to its larger velocity of propagation it separates from the wave train. The wave train still agrees reasonably well with the theory.

There is a region, which depends mainly on the ratio Q/h, where

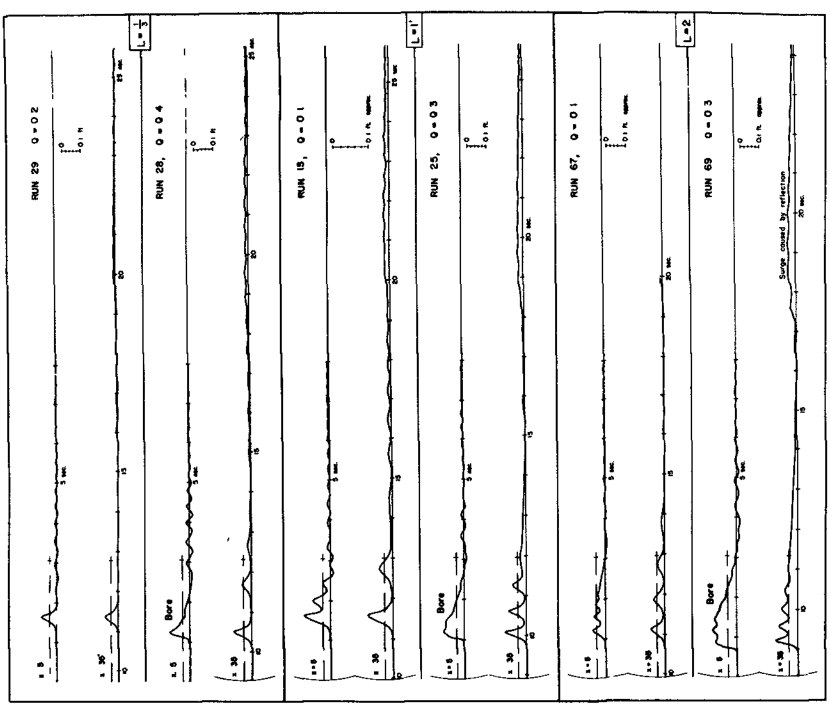

Fig. 9. Leading parts of the wave patterns at depth = 0.2 ft.

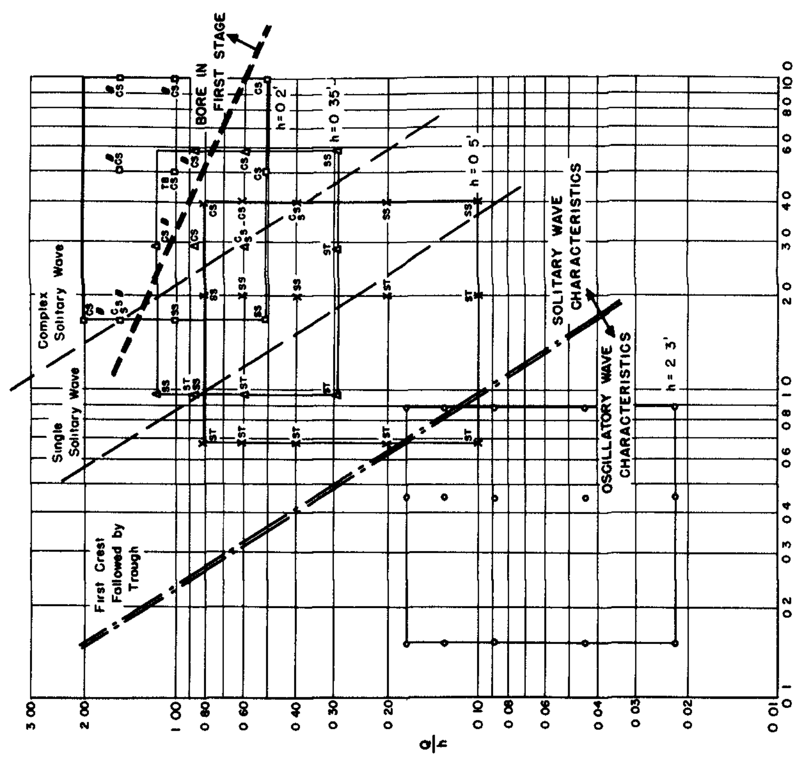

Fig. 10. Relationship between $\frac{L}{h}$, $\frac{Q}{h}$ and the characteristics of the leading wave in the case of

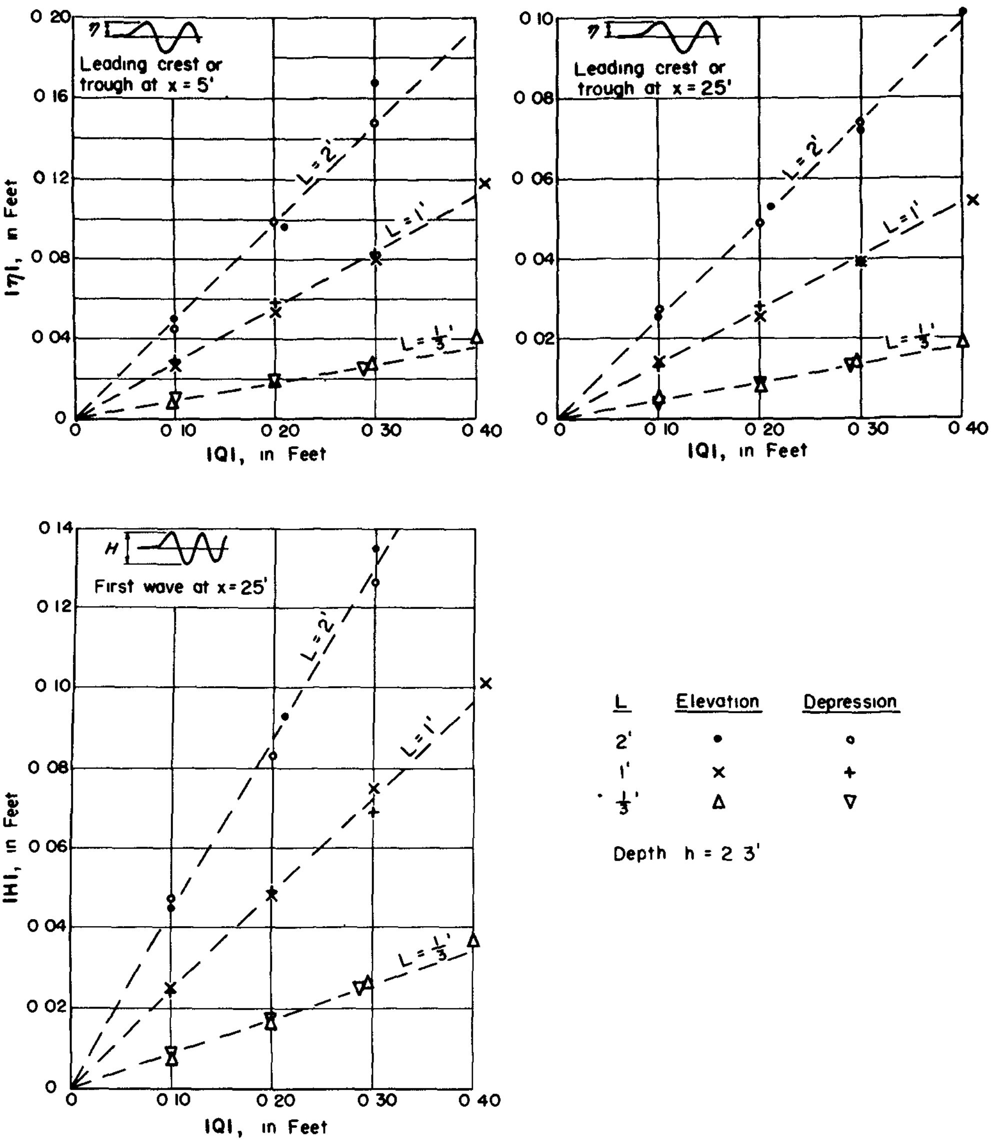

Fig. 11. ηQ - relation at constant x, t, and L.

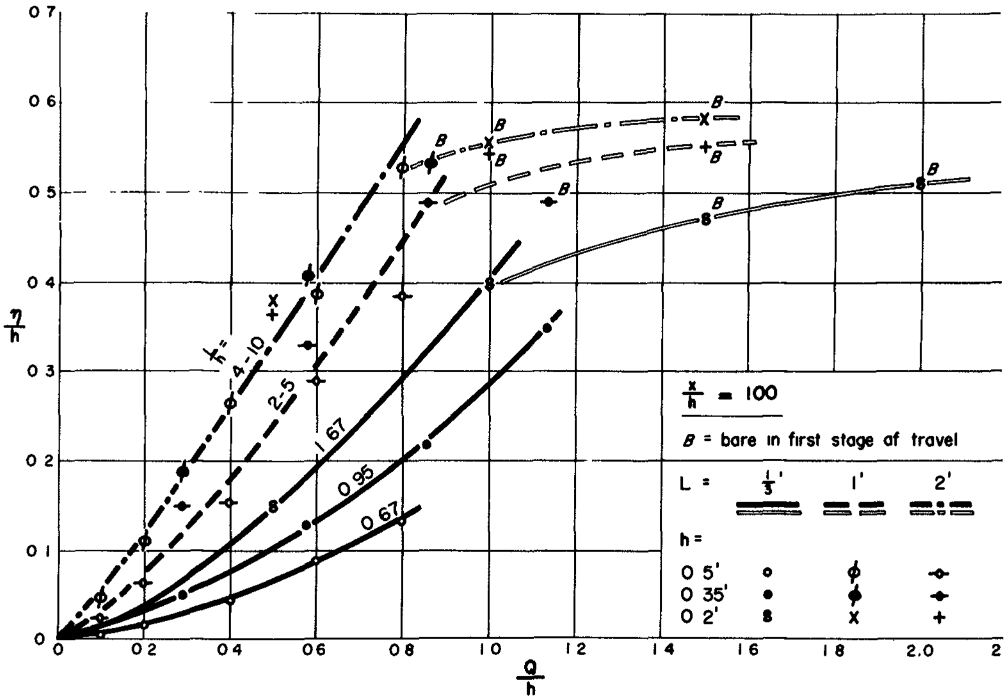

Fig. 12. η, Q - relation at x/h = 100 of leading part with solitary wav characteristics.

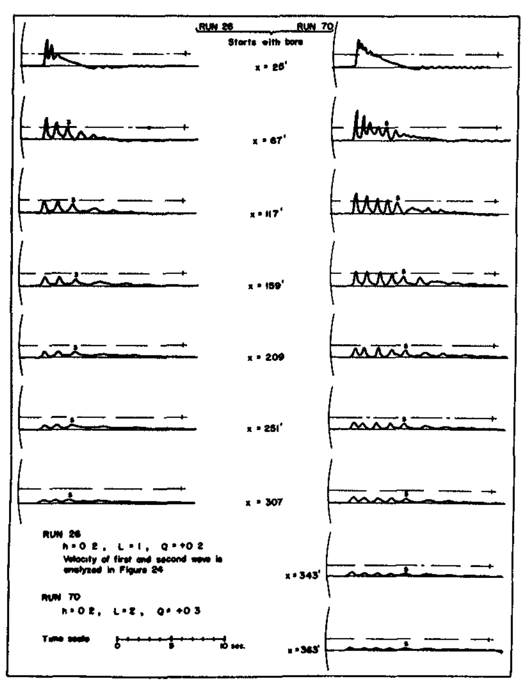

Fig. 13. "Complex Solitary Wave" group.

during the first stages a bore occurs. (Considering the initial conditions, it could be expected that every wave would have to start with an unstable front. As can be seen from the photographs on figure 3, this assumption is false as the zone of instability (the vertical front at t = o) is not propagated away from its zone of occurrence. The mechanism of generation is not analysed).

It was evident that the formation of a bore would limit the maximum possible height (amplitude η) of the leading wave, and that if the leading wave had the characteristics of a solitary wave the ratio η/h would always be smaller than the theoretical maximum value of 0.78. As Q/h can have any value, the affect of Q/h on η/h was studied. An illustration of this is given in figure 12. The height η is dependent on Q (for deep water it is directly proportional to Q) and on L through the form of the amplitude envelope curve. For L = 1' and 2' the data of the three water depths were combined in one curve. It can be seen that as soon as a bore occurred the height was cut down.

In the case of the "complex solitary wave" (CS) the first stage consisted of a serrated elevated mass of water, of which the back side gradually sloped down to about still water level, followed by a dispersive wave pattern. On traveling out from the origin the serrations (waves on top of the elevated water mass) became bigger and their troughs approached the still-water-level. They formed a group of waves with about equal heights, the number of which remained constant during their travel, so that there was no dispersion. With increasing time there was a considerable attanuation, but the troughs never went below still-water-level. It appeared that each individual wave of the group had the characteristics of a solitary wave. Figure 13 illustrates this type of leading wave pattern.

CLOSING REMARKS.

Although the model investigations were made primairily to check the theories cited, the regions beyond the validity of these theories produced quite interesting data which may be valuable in interpreting some hydraulic phenomena.

It may give an explanation concerning the difficulties met in the generation of a pure solitary wave starting from an elevated area. According to this study the solitary wave forms a portion of a dispersive wave pattern.

It may give an indication on the characteristics of the waves of shorter period originated from a breaking wave.

Further more it may explain the breaking up into a wave group of a long wave on reaching a sudden shaol.

Acknowledgement - The investigations described in this paper have been conducted under sponsership of the Office of Naval Research and were carried out by the author while a Fulbright Research Scholar at the Wave Research Laboratory of the Institute of Engineering Research, University of California, Berkeley.

The author wishes to express his appreciation to Professor J.W. Johnson for enabling the investigations and to R.L. Wiegel for his suggestions and critiques.

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CHAPTER 9

WHAT WE ARE DOING ABOUT SEISMIC SEA WAVES

by

Rear Admiral H. Arnold Karo, Director
Coast and Geodetic Survey
U. S. Department of Commerce

I am glad to have this opportunity to participate in this section of the Sixth Conference on Coastal Engineering. My remarks to the distinguished group of engineers and scientists assembled here today will emphasize a phase of the work of the Coast and Geodetic Survey of the United States Department of Commerce pertinent to one of the problems encountered in coastal engineering.

I should like to mention the observance of our 150 years of service to the Nation, and that we are nearing the end of our year-long celebration of our Sesquicentennial year. In this long history of the Bureau in surveying and charting the coastal waters of the United States and possessions, our work has always been closely associated with the type of engineering activities with which you are identified.

Successive hydrographic and topographic surveys of our coastal regions furnish basic data essential in studying changes in coastline and adjacent underwater topography; geodetic surveys provide the framework upon which all other survey work is based, and serve in developing construction plans for large-scale improvement projects; our geomagnetic work is being intensified during the International Geophysical Year in support of United States participation with some 60 nations; our tidal surveys and seismological investigations will be emphasized in discussing the subject selected for detailed consideration on this occasion.

The seismic sea wave is one of the forces of nature that has demonstrated on many occasions its ability or potential for great destruction. Earthquakes spawn these destructive seismic sea waves and the science of seismology is therefore basic to the engineering and related measures required in establishing effective safeguards.

The Coast and Geodetic Survey is uniquely qualified to carry on this type of activity. The solution which we have developed for establishing effective safeguards involves the use of seismographs, tide gages, seismic sea wave detectors, and an effective communications system to alert potential disaster areas. Rapid dissemination of due warning to inhabitants of the potential danger areas is also essential to an effective system.

The seismic sea wave, or tsunami, the Japanese term often usec to identify this phenomenon of nature, demonstrated its great capacity for destruction as recently as March 9, 1957. Traveling as fast as a jet airplane, the huge series of waves struck the Hawaiian Islands with a blow having an intensity proportional to the displacement which occurred on the earth's crust 2200 miles away in the Aleutian Trench neε Amukta Pass. For the first time we were provided a first hand, eye witness account of a major sea wave as seen from a submarine resting in a small harbor in the path of the surging sea. There was never a more graphic example of nature's potential for violence.

The USS WAHOO was lying to for the week end in Nawiliwili Harbor on the Island of Kauai in the Hawaiians. This small port is on the protected side of Kauai for a wave approaching from the north. The first indication to the crew of the WAHOO of the arrival of the killer wave was a draining of the harbor, lowering the water level approximately five feet. This lasted about one or two minutes and was followed immediately by the first wave, which refilled the harbor in less than a minute.

Although the currents during the initial wave did not reach alarming proportions, the rapid shift of water was disquieting to the crew, to say the least. After a slight pause the water rapidly receded a second time, but came roaring back in a few moments as the second wave hit. This caused a change in water level of about ten feet in less than a minute.

The tremendous rise in water increased the harbor current to great proportions. The water rose relentlessly, submerged buoys, and set up a giant whirlpool action measuring about 300 yards in diame in the harbor basin. The water receded just as quickly as it had arriv with a rush which emptied the channel, accelerating the whirlpool action. The currents around the edge of the jetty on the inboard end of the channel increased to an estimated 18 knots, which resulted in the channel becoming a boiling mass of foaming water.

Cross currents generated in the main channel continually fought the main ship stream assisted by a vast number of whirlpools and eddi In areas where the current was strongest the water was sucked away from the beach so as to expose land actually lower than the level of the main body of water. Subsequent waves of varying degrees of intensity acted in exactly the same manner. The period between the waves in the basin was irregular with short intermittent periods of relief, but after the initial onslaught the channel outside the jetty was a mass of cross currents which did not completely subside until several hours after the last wave had receded. The tide continued to surge inside the port for some time after the actual sea wave action stopped. These surges were rather gradual, continually decreasing in force, although it was six or seven hours before the harbor calmed down to normal conditions.

This first-hand account verified the fact that a small harbor with a narrow funneling channel, although not directly exposed, is particularly vulnerable to sea waves of this type. While the seismic sea wave in the open sea is relatively small in height, the rise of water in a harbor will vary tremendously, depending upon physiographical conditions. We have known that height at point of origin or at some intermediate stage of its travel does not indicate the extent or power of a sea wave.

The unbelievable current speeds generated in restricted channels in just a matter of minutes and the fact that a current under these conditions will reverse direction completely in less than two or three minutes at the peak of intensity, were verified by this first-hand account. Those who had this unique experience described the appearance of the wave as it entered the harbor as very similar to rapids in a river, having many eddy currents and considerable turbulence.

The entire perimeter of the north Pacific Ocean is an active seismic area, making the Hawaiians and other islands of the Pacific especially vulnerable. In fact, it is possible for seismic sea waves to reach Hawaii from all points of the compass. Since 1819, about 40 such waves have been recorded, although many of them resulted in very little damage.

The last truly great disaster in the Hawaiian Islands occurred on April 1, 1946, from the major seismic sea wave which was generated by a tremendous crustal movement along the northern slope of the Aleutian Trench south of Unimak Island. The violent earthquake setting off this movement was recorded on seismographs all over the world. The devastating wave originated at the epicenter of the earthquake simultaneously with the seismic disturbance.

The series of destructive waves thus generated raced across the entire Pacific Ocean. Scotch Cap Lighthouse located on the western extremity of Unimak Island about 93 miles from the origin of the disturbance and towering 92 feet above high water was completely destroyed and five men were killed. The waves were recorded along the shores of North and South America, reaching Valparaiso, Chile in a little over eighteen hours. They were even recorded feebly in Sydney, Australia, 6700 miles away. They rushed southward at an average speed of 490 miles per hour, reaching the shores of the Hawaiian

Islands in about four and a half hours.

The coastal areas were submerged by a wall of water rising mor than fifty feet above the high-water shoreline, and rushed inland for more than a quarter of a mile. This was the most destructive of all waves in the history of the Islands. The loss of life reached 173; many more persons were seriously injured, and property damage amounted to 25 million dollars. The waves struck on all sides of the Islands, but their size and violence on reaching shore varied greatly from place to place, depending on the topography and other local features.

This variation ranged from two feet or less to a maximum of 55 feet. The violence of the onslaught ranged from gentle rises and falls almost entirely free from turbulence, to raging torrents. The variatic in height reached by the water was determined by the following factors (1) position of the coast in relation to the direction of wave origin; (2) shape of the island; (3) submarine topography; (4) presence or absence coral reefs; (5) exposure of the coast to storm waves; and (6) shoreline figuration and topography.

Variation in the turbulence of the waves as they rose over the coast was related partly to the varying height of water. This was due to the fact that the greater height of water piled against the shore resulted in a steeper hydraulic gradient with increased rapidity of inland flow of water. This was particularly true in areas where the water surged over a barrier. Likewise, greater depth of water on shore resulted in a steeper gradient and more violent runback to the ocean duri wave troughs. Generally, in places where the water rose more than twenty feet the effects of the surge were violent.

In general, the waves were much more severe on the north shor facing the oncoming waves. The receding waters produced strong bac water which caused most of the violent damage. The sea at the time was running moderately heavy and in some exposed areas wind-produc storm waves came in on the crests of the seismic sea waves to increase considerably their destructive force.

Well developed reefs or shoal areas reduced markedly the intensity of wave assault on shores thus protected. The northern coast of Oahu, for instance, has a better developed reef pattern than the northe coast of any of the other islands. Therefore, the average height reacl by the water along the northern coast of Oahu was correspondingly les than along the northern shore of any other of the larger islands, excer Lanai which is well protected on the north by Molokai. Reefs are especially effective in reducing the size and intensity of the waves where channels or lagoons of greater depth lie between the shoalest part of the reef and the shore. The great barrier reef of Australia, together with the shielding islands of Melanesia to the northeast, afford a large measure of protection to the Australian coast. Similar protection is produced by other effects, however, such as in the case of a small bay on Hawaii whose entrance is nearly blocked by a ridge of lava rising nearly to or just above sea level. Hydrographic survey data by the Coast and Geodetic Survey are essential in studies of this type.

Shoreline protective measures such as breakwaters, if properly constructed, provide a measure of protection. The configuration of shoreline has a marked effect on the sea wave pattern. Waves are always largest at the heads of large funnel-shaped bays. In fact, coastal characteristics such as those existing at the Bay of Fundy greatly magnify ordinary tidal fluctuations.

Submarine topography plays a large part under such conditions due to the fact that the great wave length of seismic sea waves extends the movement of the water from the surface all the way to the ocean bottom. As the waves enter shallow water, interference of the bottom with wave motion causes an increase in the heights of the waves, a lessening of their speed, and a steepening of the wave fronts. We know that submarine ridges and valleys projecting from shore in the direction of wave origin are important factors in determing the height and violence of waves. The advance of waves moving toward shore parallel to the axial direction of the submarine ridge is retarded along the ridge more than in the deeper water on either side of it. Thus, the wave front becomes concave toward the head of the ridge and a large amount of wave energy builds up.

The speed of the waves is computed by the formula of the square root of the acceleration of gravity, multiplied by the depth of water. In oceanic depths of 2000 fathoms, the speed of the wave is about 370 knots, at 5000 fathoms they travel at about 580 knots, while at 10 fathoms the speed is greatly reduced to 26 knots.

The period of a seismic sea wave is more difficult to determine accurately. The tide gage record can seldom be read accurately to the minute, and in many cases the uncertainty covers several minutes. Moreover, the waves set up reflections or interference which cannot be accurately predicted. The period of the initial wave may vary from a few minutes to an hour or more. This diversity of periods is, of course, a confusing feature of the seismic sea wave.

Following the 1946 disaster in the Hawaiian Islands, the Coast and Geodetic Survey organized the Seismic Sea Wave Warning System for detecting such waves immediately following their inception, and reporting them to potential disaster areas in the Pacific. This is a cooperative undertaking, involving seismological observers for detecting and reporting large earthquakes in the Pacific area; tide stations located throughout the Pacific for detecting and reporting the resulting sea waves; a central station in Honolulu for receiving and evaluating reports, and alerting the central military and civil agencies, and rapid communications service between all stations and Honolulu.

Upon being alerted to the existence of a wave, the respective agencies put into operation previously developed plans for warning the civilian communities, shipping, and military bases. This warning system functioned for the first time to warn of the potentially destructive wave of November 4, 1952. Tide gage readings at Honolulu on this occasion reached a higher level than was recorded in 1946, but the sea wave itself was not as destructive. In no cases did the waves of 1952 roll or dash up to the heights reached at some places by the great waves of 1946. No lives were lost in 1952 and property loss was relatively small, which is of course attributable in large part to the successful operation of the Seismic Sea Wave Warning System. Last March, Secretary of Commerce Sinclair Weeks commended the Survey upon the successful operation of the system in giving sufficient advance warning to avert any loss of life following the great seismic sea wave of March 9th.

The system includes four Coast and Geodetic Survey seismological stations, each equipped with visible recorders and an alarm system to alert the observer. These stations are located at Fairbanks and Sitka, Alaska; Tucson, Arizona; and Honolulu. An additional observer is assigned to the Honolulu station which is the nerve center of the system and controls the operation of the system during an actual alert.

Additional seismological data are furnished by the University of California, California Institute of Technology, the United States Navy, Peruvian Huancayo observatory, the Japanese Central Meteorological Observatory, and the Manila observatory. Sixteen tide stations located throughout the Pacific are utilized to determine if a wave has been generated by a particular earthquake. Seismic sea wave detector have been installed at selected stations in the system as a precaution against delay in giving the alarm. This device alerts the tide observe to the arrival of large waves having the period of seismic sea waves. He can then examine his tide record and if necessary initiate a warnin message to Honolulu. Some of the Coast and Geodetic Survey stations are operated with the assistance of the Navy, Air Force, Civil Aeronautics Administration, University of California, and the government of American Samoa. All stations are serviced regularly by the Burea with the exception of one station located in the Canal Zone.

The all-important communications services are furnished by the Navy, Army, Air Force, Civil Aeronautics Administration, Samoa, and the Peruvian Civil Aviation Corporation. Others at various places have made important contributions to the success of this undertaking. For example, the sheriff of Del Norte County in California makes available telephone service between his office and the Naval Communications station in San Francisco and the tide observer at our Crescent City tide station. Valuable assistance through unselfish cooperation of many others too numerous to mention, insures the successful operation of the warning system.

In the initial planning of the system, it was realized that a graphic method of rapidly determing the time of arrival of a wave in the central Pacific was an urgent requirement. To meet this need the Survey constructed a special chart for the Pacific area from which the travel time to Honolulu of a seismic sea wave could be readily determined. This chart consisted of a series of more or less concentric circles overprinted in red on a specially constructed chart of the Pacific Ocean. The encircling lines were spaced to represent hourly and half-hourly distances from Honolulu in sea wave travel time. In order to visualize the probable paths of wave travel, broken lines are made to radiate from Honolulu on great circle courses. Travel times were computed along the great circle paths in various directions from Honolulu. Each path was subdivided into segments of 120 nautical miles, and the travel times were computed in hours for each section.

In computing the times along the paths it was found that in some instances great circle courses passed over units of land or covered areas of lesser depth than in adjacent areas. Since a seismic sea wave advances more rapidly in deep water it was necessary to compute travel times along combinations of arcs of two or more intersecting great circles. This factor explains the irregularities in time curves which necessitated computation of additional paths to fully develop the bathymetric pattern.

At the request of Civil Defense authorities of the Pacific coast states, the warning system was extended this year to include predictions for the shores of California, Oregon, and Washington. This extension of the warning system required publication of five additional travel time charts which have been issued for selected areas along the Pacific coast.

Of particular interest in coastal engineering is the design of structures to withstand tremendous wave assault, and in the design and development of protective coastal installations. Structural damage has been at a minimum where buildings have been constructed of reinforced concrete. Damage has always been heaviest to the buildi using light construction as is customary in the Hawaiian Islands. Dam to roads and railroads has been mostly of a nature common in any floo The foundations of bridges are particularly vulnerable, and extensive damage has been reported in the past. Piers withstand the waves well and most breakwaters survive with slight damage. Seawalls, when we constructed, are usually not materially damaged.

Coastal erosion due to sea waves is in general not as easy to determine as the damage to structures. Beaches are subject to some damage, as might be expected, however all beaches are subject to change during ordinary storms. This is especially true of the Hawaiia Islands where the distinct difference between wind and current directions during the stormy season results in considerable change, conforming to a pronounced annual cycle. Accurate surveys of the shoreline and intensive development of the adjacent coastal waters are essential in planning and developing protective measures.

The Coast and Geodetic Survey has been engaged in making the intensive topographic, hydrographic, and other surveys basic to this problem for well over one hundred years. The vast reservoir of precise facts which we have accumulated concerning the coastal regions of our country increases in importance with each successive decade. Additions are being constantly made to this storehouse of knowledge, both in quantity and quality, through intensive application of modern electronic techniques in refining and extending our basic operations.

In conclusion, I should like to re-emphasize the contributions of the Coast and Geodetic Survey to the seismic sea wave problem. Through intelligent utilization of our wealth of coastal data, we are able to supply the answers to the basic questions, "What causes a seismic sea wave?", "How fast does it travel?", and "Where does it originate?". This rather specialized application of our knowledge and resources is in keeping with our fundamental policy of safeguarding human life and property at sea, on land, and in the air.

CHAPTER 10

AN EXPERIMENTAL INVESTIGATION OF DRIFT PROFILES IN A CLOSED CHANNEL

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INTRODUCTION

The first order theory of water waves states that water particles move in closed orbits; that this was not exactly so, and that there was a slow drift of the water in the direction of wave propagation, was first realised by Stokes(1) who proposed a solution of the horizontal drift or mass transport for waves in an inviscid fluid. Recently the effect of the viscosity has been investigated by Longuet-Higgins.(2) Experimental measurements have been made by Caligny,(3) the U.S. Beach Erosion Board,(4) Bagnold,(5) and previously by the Hydraulics Research Station. The evidence from these sources is incomplete.

In the experiments described in this paper measurements of the mass transport at all levels were made with progressive waves, varying in length from about 1½ to 50 ft and in water between 6 and 20 in. deep.

SUMMARY OF RESULTS

It was found that the drift near a horizontal bed was invariably in the direction of wave propagation and quantitatively in good agreement with that predicted by Longuet-Higgins. The drift at the surface was in the direction of wave propagation in every experiment but one, and the return flow took place near the centre of the fluid. In deep water the theory of Stokes predicted the drift velocities. When the ratio of depth to wave-length lay within a certain range defined by $.7 < kd < 1.5$ the drift velocities at all depths were in agreement with those calculated from Longuet-Higgins "conduction solution" which however is not strictly applicable in the present observations.

It was found that the drift on the seaward and shoreward faces of a submarine bar, that had both faces at slopes of 1 in 20 was insignificantly different from the drift on a horizontal bed; except when the waves were spilling over the bar. When the waves were spilling, the direction of the drift at the bed on the inshore face was reversed: the drift was then towards the crest of the bar from both sides.

THEORETICAL WORK

Table of Symbols

H	wave height
a	wave amplitude H/2
d	still water depth
λ	wave length
k	$2\pi/\lambda$
T	wave period
σ	$2\pi/T$

ν kinematic viscosity
y distance from the surface, measured downwards from still water leve
μ y/d
U drift velocity, taken as positive when the motion is in the directi of wave propagation. Subscripts indicate the value of μ where the velocity is measured, e.g. U_0 is the drift velocity at the surface.

Stokes' theory assumes irrotational waves of small but finite heigl propagated in a perfect inviscid fluid. It gives a drift velocity in deep water of

$$U = a^2 \left(\frac{2\pi}{\lambda}\right)^{3/2} g^{\frac{1}{2}} e^{-4\pi y/\lambda}$$
$$= a^2 \sigma k e^{-4\pi y/\lambda}$$

thus $U_0 = a^2 \sigma k$

All drift velocities are positive; and the condition of no net flow across a section, which is assumed to prevail in a narrow channel, is no satisfied. However, if the axis is shifted arbitrarily, the condition c be satisfied, and this alters the surface drift by only a small proporti if the water is deep. Stokes performed this operation and showed that when d $> \lambda$ the drift velocity at any depth was

$$U = a^2 \sigma k e^{-4\pi y/\lambda} - \frac{a^2\sigma}{2d}$$

hence $U_0 = a^2 \sigma k \left(1 - \frac{1}{2kd}\right)$

Stokes also obtained the drift in water of finite depth, but in view of shear at the bed in shallow water this would not be expected to apply.

Longuet-Higgins' theory applies to wave propagated in a viscous fluid of finite depth, the motion being assumed to be rotational but non· turbulent. The theory predicts a velocity at the bed of

$$U = \frac{5}{4} \frac{a^2 \sigma k}{\sinh^2 kd}$$

In this formula there is no restriction on the wave amplitude, provided the flow is laminar. However in an appendix to the present paper* he has indicated reasons why the above formula may be applicable also to the case of turbulent flow.

According to Longuet-Higgins, the flow in the interior of the fluid may be expected to depend on the ratio of the wave amplitude to the thickness of the boundary-layer δ, which is of the order of 0.5 mm. When the wave amplitude is small compared with δ (a condition very rarely satisfied) the drift velocities are given by the "conduction solution"

$$U = \frac{a^2 \sigma k}{4 \sinh^2 kd} \left[2 \cosh \left(2kd(\mu-1) \right) + 3 + kd \sinh 2kd \; (3\mu^2-4\mu+1) + 3 \left(\frac{\sinh 2kd}{2kd} + \frac{3}{2} \right) (\mu^2-1) \right]$$

But when the wave amplitude is large compared with δ, as was the case in the present experiments, the velocity profile in the interior is not predicted, but is said to depend on the conditions at the boundaries, in this case the wave-generator and the beach.

The physical reason for this as given by Longuet-Higgins is that under normal conditions ($a >> \delta$) the transport of vorticity by convection along the streamlines will be much greater than the diffusion of vorticity by viscous conduction. Initially, however, that is to say before any vorticity has had time to penetrate the interior of the fluid, the motion there should be as given by Stokes' irrotational solution.

Both in Stokes' solution and in Longuet-Higgins' conduction solution $a^2 \sigma k$, which is a velocity, gives the scale of the velocity while the remainder of the expressions show how the velocity varies with depth.

It will be seen from the experimental results presented below that many of the drift profiles happen to agree with those predicted by the conduction solution, although the waves are far too high for it to apply. Similarly other results agree quite well with the Stokes' solution and these are shown superimposed upon it. Neither solution is applicable, the Stokes' solution because it considers only an inviscid fluid and the conduction solution because it considers waves of such low height; but both have nevertheless been used where they tend to agree with the experimental results.

SUMMARY OF PREVIOUS EXPERIMENTAL WORK

In the experiments made by Caligny in 1878, the drift velocities at the bed are in agreement with Longuet-Higgins' theory. In the interior, the velocities are forwards near the bed and the surface and backwards at intermediate levels.

In some experiments made by the U.S. Beach Erosion Board drift velocities were observed under conditions that represented a wide range of kd. When kd > 3 fairly good agreement was found with Stokes' theory. The scatter of points with shallow water waves was so large as to render the results valueless.

Bagnold observed the form of the mass transport curve, and measured drift velocities at the bed. His results differ from those predicted by the Longuet-Higgins theory by a maximum of 15% and the curves are of the same form as the conduction solution. However only long waves of low amplitude were used.

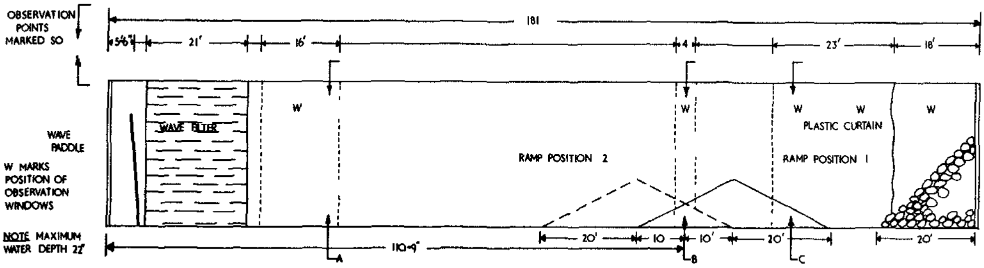

Fig. 1. Sketch of wave channel (vertical scale is ten times horizontal scale).

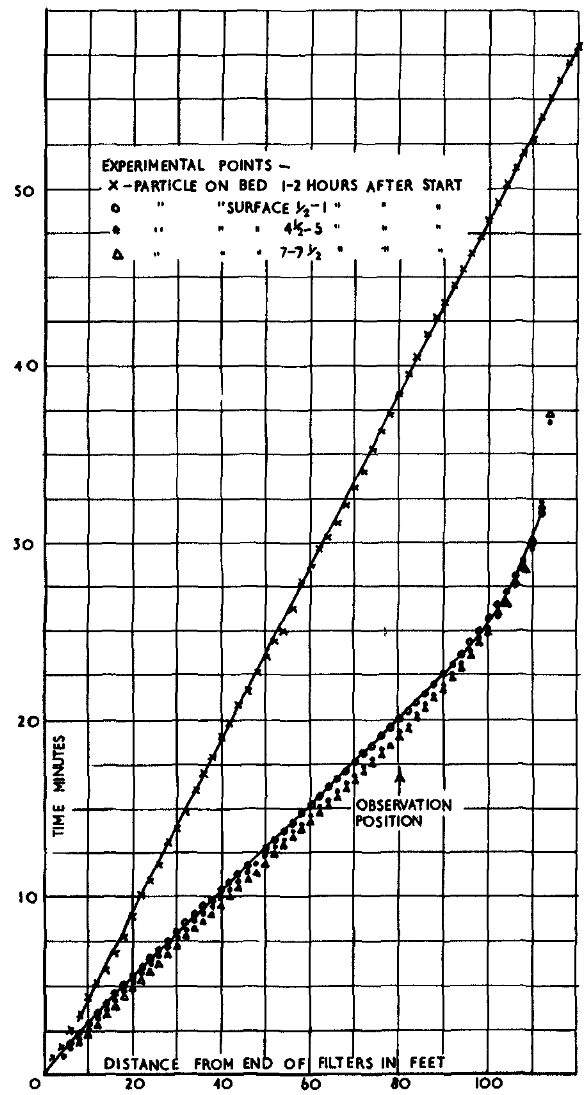

Fig. 2. The variation of drift velocity at the surface and near the bed with distance along the channel ($H = 46$ in., $T = 1.5$ sec, $d = 20$ in.).

AN EXPERIMENTAL INVESTIGATION OF DRIFT PROFILES IN A CLOSED CHANNEL

The Hydraulics Research Station carried out some earlier experiments in a 54-ft long wave channel, in which drift velocities were obtained by observing the motion of compounded particles having the density of water. A series of curves, mostly "S" shaped, were obtained with a forward drift at the bed and a backward drift at the surface. The present series of experiments indicate that the channel was too short; and that the results were affected by circulations set up by the waves when they broke.

APPARATUS

The wave channel is 185 ft long 4 ft wide and will contain water having a maximum depth of 22 in. Windows are provided at intervals along one side of the channel. The walls are vertical, plane and parallel and the bed is horizontal to within limits of 1/20 in. At the wave-generator end of the channel the width increases to 6 ft and the 21-ft long tapered part of the channel is fitted with filters made of curved perforated plates.

The wave generator is a paddle, driven by a two-speed induction motor through an infinitely variable friction gear-box and a quick-return mechanism. The linkage allows the paddle movement to simulate that of a paddle hinged at the bed or at an infinite depth below the bed, or at intermediate levels.

At the other end of the channel is a beach of $\frac{3}{8}$-in. shingle with 3-in. shingle on the surface. It has a slope of 1 in 10.

A twin-wire resistance wave recorder that is sensitive to changes in water level of 1/5000 ft is set in a hollow at the back of the shingle beach. At this point the fluctuations of water level caused by the generated waves are small but seiches in the channel arrive with little attenuation.

For taking photographs of the water motions a Rolleiflex camera is used, the shutter operated by a solenoid. The solenoid is actuated by a small pendulum that is moved to and fro by the waves some distance along the channel, and ensures that all photographs of the same waves are taken at precisely the same phase.

METHODS OF MEASUREMENT

In these experiments the principal method of recording drift velocities involved the timing of small particles that had the same density as water as they travelled known distances along the channel. The particles were normally followed for a period of between 20 and 40 secs but exceptionally the period was as short as 15 secs or as long as 80 secs. The maximum error in the measurement of this period was of the order of 1 wave period. The mean depth of the particle was estimated from horizontal grid lines drawn both on the glass and on the other wall of the channel, and could involve a maximum error of $\frac{1}{4}$ in. Particles which rose or fell more than 1 in. during an observation were ignored. The lamps illuminating the particles were shaded so that only particles near the centre of the channel were apparent. The illuminated slice of water was 6 in. wide at the surface increasing to 1 ft at the bed. The particles were scattered into the water near the observation point before an experiment began. They beca diffused throughout the fluid and it was only necessary to wait for a particle to appear at any desired level. Drift profiles were plotted f at least 25 velocity measurements of individual particles.

A second method of recording drift velocities was used when there a seiche in the channel. The seiche had a period of 51 secs and obscur the drift velocities, when they were found by observing particles. The method involved photographing a dye streak as it was deformed by the dr The method had the advantage that a seiche might carry the whole dye streak laterally; but the true drifts could still be obtained, if it wa assumed that the drift profile had to be a balanced curve involving no flow in either direction. (The velocities at the very bottom, which ar theoretically not influenced by the seiche, were not treated in this wa The dye streak was of fluorescein and was obtained by coating a neutral buoyancy particle with fluorescein powder, damping it slightly and dropping it into the water. A fine bright streak was obtained.

In order to disperse the film of dirt which invariably formed on tl surface of the water, Teepol was added until the surface drift velociti became stable and uniform across the section. Experience soon showed tl amount required.

PRELIMINARY EXPERIMENTS

These were experiments designed to ensure that the drift velocities obtained were not affected by proximity to the wave generator, to the beach or to the sides of the channel. Observations were also made on tl stability of waves as they travelled down the channel, on seiches that w sometimes set up, on the degree of turbulence in the water and on the ti taken to achieve steady conditions.

End Effects. Each time the wave length was altered by a large amou a float was timed as it drifted on the surface from the filters down the channel. Fig. 2 shows the results of one such experiment. For wave periods between $1\frac{1}{4}$ and 3 secs it was found that the waves breaking on th beach caused a very strong backward drift at the surface, which apprecia distorted the drift profiles as far back as the observation window B. A the paddle end of the channel the uniform drift persisted up to 10 ft fr the filters. Actually at the filters there was no observable drift. Th strong drift initiated near the beach was greatly reduced by installing near the beach a flexible plastic curtain in the waves, which hung from floating wooden bar at its upper edge and carried an iron bar at its low edge. The device was moored by feeble elastic threads. The device had no apparent effect on the waves but ensured that there was zero drift at the point where it was installed. The experiment to which Fig. 2 refers was made with plastic curtain in place. The figure shows that the surfa drift is uniform over a distance of 80 ft and that the observation windo is suitably placed for measuring it. The uniformity of the surface drif along the greater part of the channel with divergences only at the ends was taken by us to indicate that the drift was not in fact determined by conditions at the ends of the channel but were independent of them. Fig. contains five drift profiles all for the same wave conditions and depth of water, but measured at different places in the channel. It is clear that only close to the beach or very close to the glass window is the drift profile significantly altered.

Side Effects. Floats were dropped into the water simultaneously on a line parallel to the wave crests and it was observed that except for the floats within 2 in. from the walls they remained in a straight line. Those close to the walls moved ahead faster than the others. It can be seen that floats move away from the wall when the water is rising and towards the wall when the water is falling: they are furthest from the wall when the water is moving forward and closest to the wall when the water is moving backwards. This is sufficient to explain their additional drift velocity. The movement towards and away from the wall is thought to be caused by the phase lead of the motion near the wall on the motion elsewhere[(6)] and is associated with small vortices having their axes parallel to the axes of the channel at the intersection of the water surface with the wall.

Steady Conditions. Experiments were made to determine the time required to achieve steady conditions. If stability was to be achieved, the time varied with the waves that were produced but seldom exceeded one hour and never exceeded three hours. There was one series of experiments in which the drift never settled down, one of the experiments being continued for seven hours without the drift showing any signs of becoming steady. In these experiments which related to very long waves, kd was 0.29, the drift velocity at the bed was steady and was recorded.

Fig. 2 relates to the more normal experiments where steady conditions were achieved in the first hour. The figure shows by the parallelism of the lines at the observation point that the surface drift there was the same after 1 hour as after 5 hours and $7\frac{1}{2}$ hours.

Stability of the Waves. It was found that waves with periods shorter than $\frac{3}{4}$ sec were unstable and about half-way along the channel broke up into groups of five waves with almost undisturbed regions, a wavelength long, between them. The effect appeared with longer waves if the height was increased. That the effect was not caused by irregularities in the speed of the wave generator was verified with a stroboscope. Drift profiles with these short waves were observed at point A shown on Fig. 1, where the waves were still of constant height.

Seiches. When the wave period exceeded 3 secs it was found that the seiche that was set up when the wave-generator was started did not die down after 20 min, as was otherwise the case, but reached a steady height between .001 and .003 ft. The seiches resulted in a variation of velocity of up to $1\frac{1}{2}$ ft per minute during their 51 secs period and made it impossible to obtain systemmatic results from the observation of particles. The photographic method was therefore used and the results were adjusted where necessary for no net flow across the section.

Turbulence. Dye streaks were observed under all conditions in order to obtain the general form of the curves: most of them were photographed. The dye streaks remained intact and undiffused for very long periods -

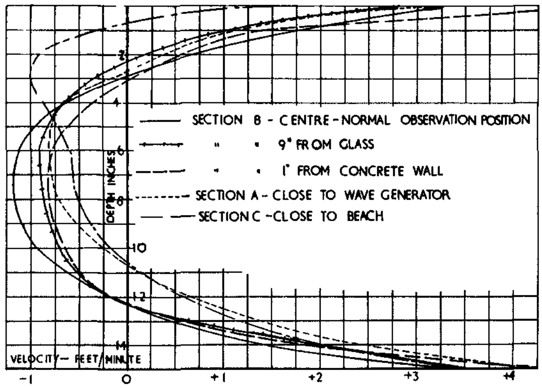

Fig. 3

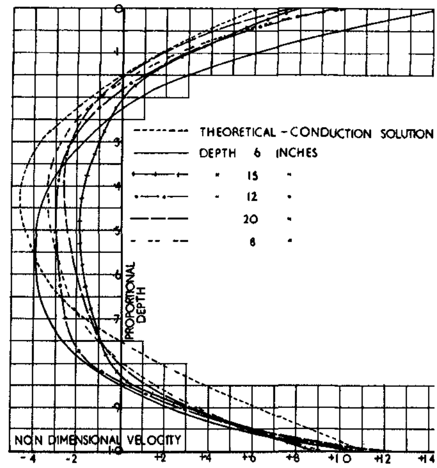

Fig. 4

Fig. 3. Drift profiles at different places in the channel (H = 3.6 in., T = 1.5 sec, d = 15 in.).

Fig. 4. The variation of non-dimensional drift profile with scale (a/d = 0.23, kd = 0.92).

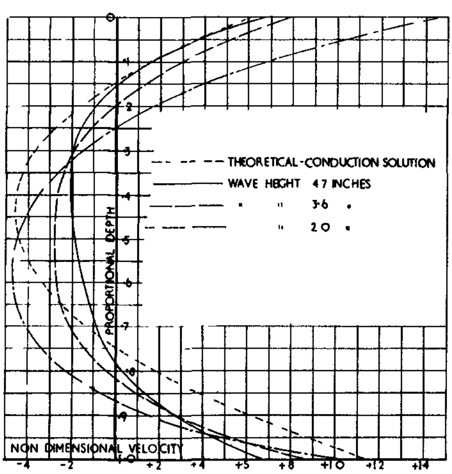

Fig. 5

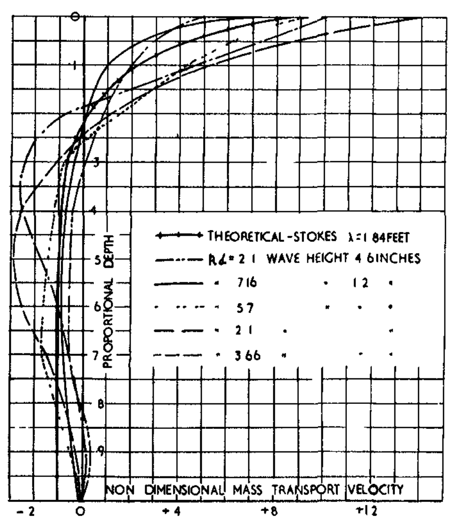

Fig. 6

Fig. 5. The variation of non-dimensional drift profile with wave-heigh T = 1.5 secs, d = 15 in.).

Fig. 6. The variation of non-dimensional drift profile with kd. Deep water waves (d = 20 in.).

indicating that the flow was laminar - except within 1 in. of the bed, where a diffuse cloud of dye was formed whenever kd < 1.5. The turbulent layer was thicker for higher velocities at the bed. It should be noted that in the turbulent layer near the bed the dye spreads both forwards and backwards; and that the advance of the front of the cloud is faster than the drift indicated by a particle.

EXPERIMENTAL RESULTS

Fig. 4 shows the results of a series of experiments using geometrically similar waves, designed to show whether the drift profiles scaled according to the ratio $a^2\sigma k$. Each experiment was done with a different depth of water but the ratios a/d and λ/d were kept constant. The drift velocities should be proportional to $a^2\sigma k$: accordingly if the measured drift velocities are divided by $a^2\sigma k$, and plotted against y, the proportional depth, identical curves should result. The figure shows that except when the depth of water was as little as 6 in. the profiles were similar and close to the profile of Longuet-Higgins' conduction solution.

An experiment with a water depth of only 4 in. failed to produce a repeatable curve. Although a dye streak indicated a profile similar to those in Fig. 4, there was a greater forward drift on the surface down one side of the channel than down the other; indicating a large scale circulation in the horizontal plane. It is possible that this will always be the case when the width of the channel is large compared with the depth of water.

Fig. 5 shows the results of experiments to determine whether drift velocities were strictly proportional to a^2. It is found that when plotted non-dimensionally the low waves result in faster drifts than the high waves; and this indicates that the velocities are proportional to rather less than the second power of a.

Fig. 6 contains drift profiles for deep water waves in a depth of 20 in., measured at Section A in the channel. The agreement with Stokes' theory, modified to produce a curve balanced about the axis, is good except near the bed. The conduction solution for these conditions gives surface velocities many times too big. It was thought possible that a different drift profile might be obtained if an experiment were run continually for the time d^2/ν that would be needed for the conduction solution to apply. To test this hypothesis an experiment was run for 24 hrs with water $11\frac{1}{2}$ in. deep, so that d^2/ν was 18 hrs. It was found that the surface drift had a slight tendency to decrease after 6 hrs running, but remained close to the Stokes value.

Figs. 7 and 8 show the observed drift velocities for kd values of 0.5 and 1.25. The scatter of points can be seen. The conduction solution is shown for comparison. The experiment to which Fig. 7 refers was the only one in which a negative drift at the surface was found.

Fig. 9 is similar to Fig. 6 but relates to shallow water. It shows the variations in the drift profiles for different values of kd. They are in qualitative agreement with Longuet-Higgins' conduction solution where

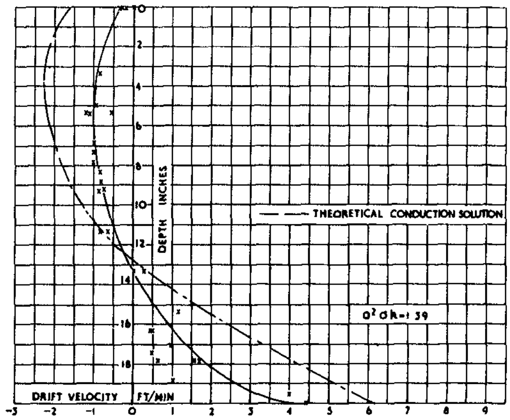

Fig. 7

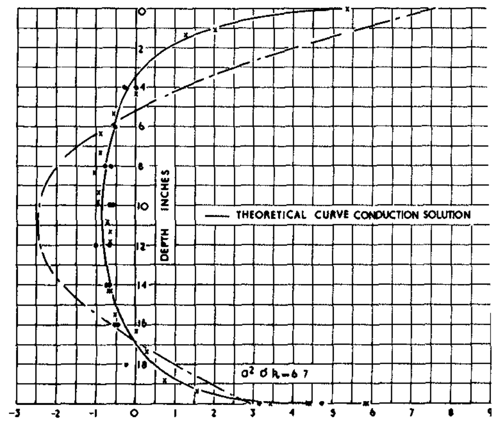

Fig. 8

Fig. 7. Comparison between the experimental and theoretical drift pr file when kd = 0.5 (H = 4.6 in., T = 3 secs, d = 20 in.).

Fig. 8. Comparison between the experimental and theoretical drift pr file when kd = 1.25 (H = 4.6 in., T = 1.5 secs, d = 20 in.).

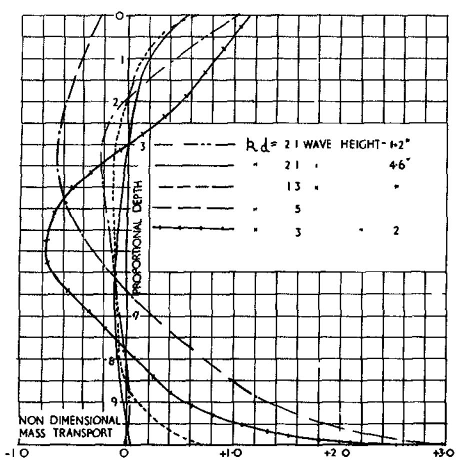

Fig. 9

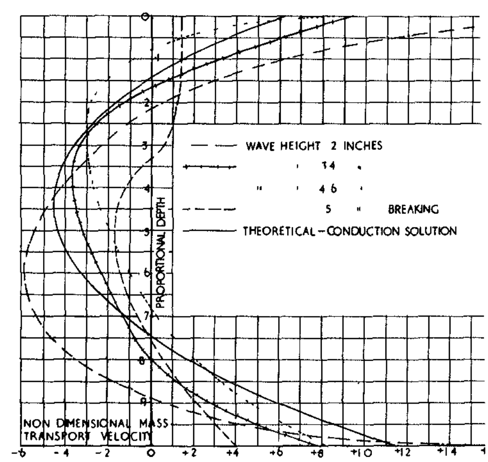

Fig. 10

Fig. 9. The variation of non-dimensional drift profile with kd. Shallov water waves (d = 20 in.).

Fig. 10. The variation of non-dimensional drift profile with wave-heig the bed having a positive slope of 1 in 20 (T = 1.5 secs, d = 1ξ kd = 0.92).

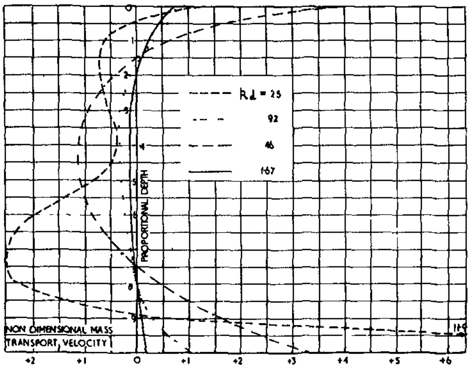

Fig. 11

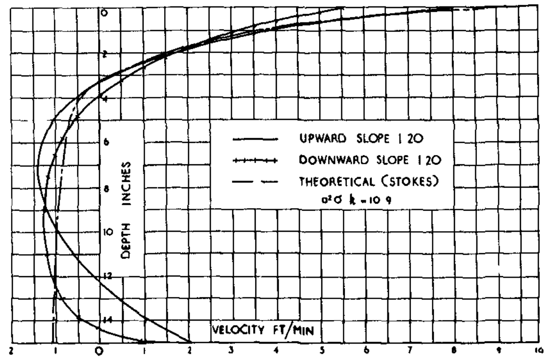

Fig. 12

Fig. 11. The variation of non-dimensional drift profile with kd. Shallow water waves on a positive slope of 1 in 20 (H = 3.6 in. d = 15 in.).

Fig. 12. The variation of non-dimensional drift profile with slope (H = 3.4 in., T = 1 sec, d = 15 in., kd = 1.67).

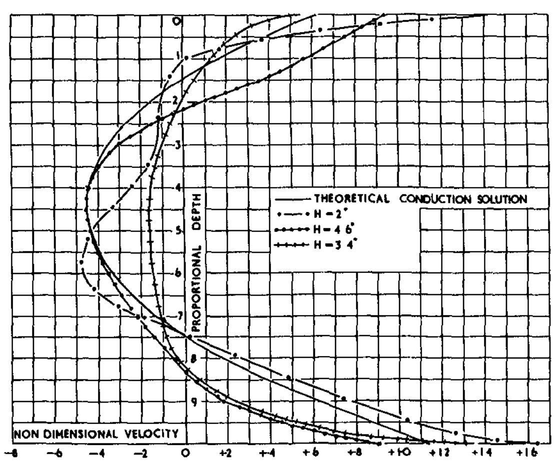

Fig. 13

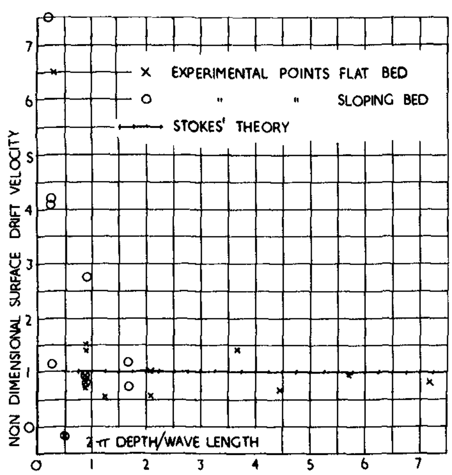

Fig. 14

Fig. 13. The variation of non-dimensional drift profile with wave-height, the bed having a negative slope of 1 in 20 (T = 1.5 sec, d = 15 in., kd = 0.92).

Fig. 14. Comparison between theoretical and experimental non-dimensional surface drifts.

kd > 0.5. When kd < .3 the profiles varied unsystemmatically with small variations in wave characteristics and usually involved flows in directions other than in the direction of wave propagation. Many of the drift profiles became stable in 15 min: others were unstable and altered continuously. The single curve that had approximately the theoretical form when kd = .3 was a very low wave, only 2 in. high. Although at the low values of kd the drift in the body of the fluid varied unsystematically, that at the bed was consistent and was recorded. The velocities are assembled with others in Fig. 15.

Fig. 10 and 11 are similar to Fig. 5 and 9 except that they relate waves running up a slope into shallower water (positive slope). The depth of water, d, is taken to be that in the middle of the slope where the observations were made. Unless the waves broke on the crest of the bar, the drift profiles were little affected by the slope.

Fig. 12 shows that the effect of a change in slope from +1/20 to -1/20 is small for short waves.

Fig. 13 is similar to Fig. 10 and 5 but relates to a downward slope of 1 in 20. Except for the wave 5 in. high that broke on the crest of the bar the profiles are not very different. The 5-in. breaking wave caused a strong forward drift at the surface and a strong negative drift at the bed. There was so much turbulence that no observations on the drift in the body of the fluid could be made. Other experiments performed on the downward slope, with kd <.9, failed to produce stable drift profiles: it was found however that when the waves broke near the crest there was invariably a negative drift on the bed as far back as the point at which the waves broke; and a forward drift at the surface. If the waves spill only very slightly the bed drift could be in the positive direction.

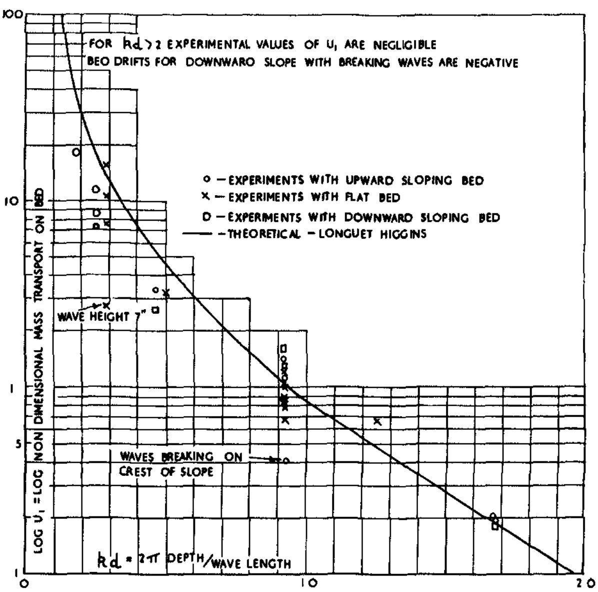

Fig. 15. Comparison between the experimental non-dimensional drift at the bed and that according to Longuet-Higgins.

AN EXPERIMENTAL INVESTIGATION OF DRIFT PROFILES IN A CLOSED CHANNEL

CONCLUSIONS

It was found possible to obtain drift velocities set up by progressive waves in a closed channel, which were independent of position and time. It is probable however that the velocities would be disordered by circulations in a horizontal plane, if the waves were not confined to a narrow channel.

Stable drift profiles were not obtained with the very longest waves, those amounting to a succession of solitary waves, except when the waves were very low. Possibly the channel, which was only long enough to contain four of these waves at a time, was too short.

Near the bottom the drift velocities are as predicted by Longuet-Higgins for all vaues of kd that were investigated. Fig. 15 shows this agreement. This is in spite of the fact that the theory applies in the first place to laminar conditions, whereas the flow was nearly always turbulent. However in an appendix Longuet-Higgins has given reasons why the formula may be generally applicable in the turbulent case also.

For the surface and interior of the fluid there is no strictly applicable theory. However in deep water the surface drifts are found to be as in Stokes' irrotational theory. Further, when $0.7 < kd < .13$ the profiles in the interior are fairly well fitted by Longuet-Higgins' conduction solution. There is only one departure from this curve which is systemmatic. It is that for a given value of kd the lower waves produce bigger non-dimensional drift velocities.

The drift profiles are disrupted in the neighbourhood of breaking waves. This provides a mechanism capable of sustaining an offshore sand bar, because opposed bed drifts are set up which meet at the top of the bar.

When the waves do not break, the slope of the bed does not alter the drift curves a great deal and the theoretical values of the drift at the bed are almost equally applicable to waves over a horizontal or a gently sloping bed.

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APPENDIX

THE MECHANICS OF THE BOUNDARY-LAYER NEAR THE BOTTOM IN A PROGRESSIVE W

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National Institute of Oceanography,
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Mr. Russell has asked me to give a brief theoretical account of th somewhat paradoxical forwards drift in the boundary-layer near the bott which he and Mr. Osorio have measured. A general treatment of such boundary-layer effects is to be found in a previous paper(2); but in th following I shall try to give a simple physical picture of one particul case, namely where the wave motion is purely progressive, and the bottc is rigid and level.

It is assumed at first that the viscosity is constant and that the motion is laminar - a condition not always satisfied in Mr. Russell's experiments. Under these circumstances it is shown that the mass-transport velocity near the bottom and just outside the boundary-layer is given by

$$U = \frac{5A^2}{4c}$$

where A is the amplitude of the horizontal oscillatory motion at the bottom and c is the wave velocity. Since, however, the observations are in agreement with this result even when the flow is turbulent, I also consider the case where the (constant) coefficient of viscosity is replaced by a coefficient of eddy viscosity depending on the distance from the bottom. I find then that the above formula is valid independently of the functional form of the viscous coefficient. This appears to be a step towards the explanation of the phenomenon in the turbulent case.

(1) <u>The boundary layer at the bottom</u> According to the first-order theory of surface waves, and from observation, a wave in water of finit depth produces near the bottom a horizontal oscillatory velocity given l

$$u_\infty = A \cos (\sigma t - kx) \tag{1}$$

approximately, where

$$A = \frac{a\sigma}{\sinh kd}.$$

However, on the bottom itself the velocity must be zero. It appears, then, that there is a region of strong shear very close to the bottom, where viscous stresses are important, and outside which they are relatively small. This region may be called the "boundary-layer".

To determine the horizontal motion within this layer, compare an element of fluid within the layer with an element just outside. (Fig.A1) The forces accelerating each element horizontally are the pressure

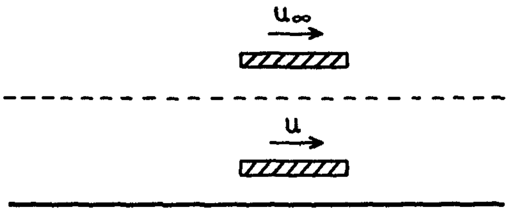

Fig. A1. Comparison of the motion of two fluid elements in and outside the boundary-layer.

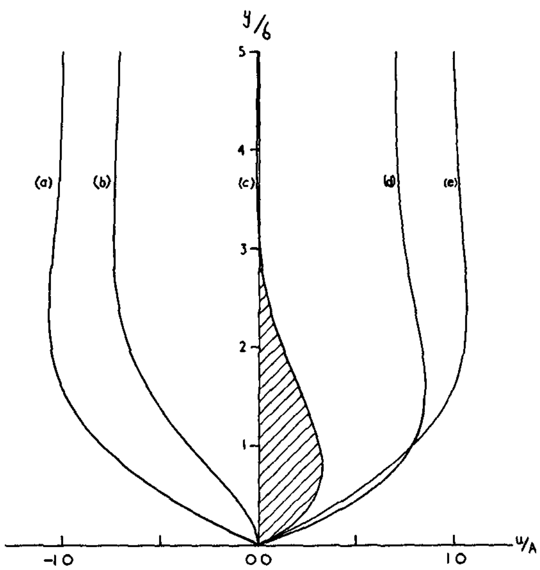

Fig. A2. The velocity-profiles (correct to first order) in the boundary-layer for five successive phases of the motion at intervals of T/8. (Vertical scale greatly exaggerated).

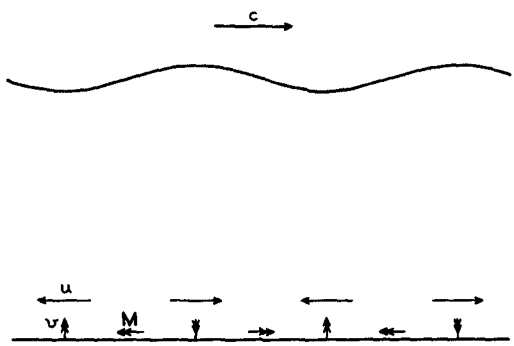

Fig. A3. Diagram showing the origin of the vertical motion in the boundary-layer.

gradient $\partial p/\partial x$ and the viscous stress $\partial/\partial y\,(\rho\nu\,\partial u/\partial y)$. Now since the layer is very thin and the vertical acceleration is not large, the pres gradient is practically the same for the two elements, while the viscou stress is appreciable only for the element within the layer. So the difference in their horizontal acceleration is due to the viscous stres only. Hence (neglecting second-order terms)

$$\frac{\partial u}{\partial t} = \frac{\partial u_\infty}{\partial t} + \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right). \tag{2}$$

When the viscosity is constant, the solution of this equation is

$$u = A\left[\cos(\sigma t - kx) - e^{-y/\delta}\cos(\sigma t - kx - y/\delta)\right] \tag{3}$$

where

$$\delta = \left(\frac{2\nu}{\sigma}\right)^{1/2}.$$

(Here y is measured vertically upwards from the bottom).

The motion is illustrated in Fig. A2, where the velocity profile i shown for various phases, at a fixed point. Effectively the velocity i the same as for a uniform fluid oscillating in the neighbourhood of a plane wall (see Lamb[5] § 347). The velocity tends very rapidly to its value u_∞ just outside the layer, and the total thickness of the layer i of the same order as δ .

An important feature of the motion is that the phase of the veloci inside the layer tends in general to be in advance of the velocity just outside. The integrated flow in the layer increases indefinitely, of co as y tends to infinity. But the component of the integrated flow which is in quadrature with u_∞ is finite, and is given by the shaded area of the velocity-profile curve (c) in Fig. A3. Denoting this by M we have

$$M = \int_0^\infty u\,dy = \frac{1}{2}A\delta\sin(\sigma t - kx). \tag{4}$$

Now if the flow were uniform horizontally, as in Lamb's solution just mentioned, there would be no vertical component of motion. But sir u varies sinusoidally with x, so also does the total flow M; this produces a piling-up of mass within the layer which gives use to a small but important vertical component of velocity. From Fig. A3 we see that just behind a crest the flow M is negative, and just in front of a crest it is positive. Hence beneath the crest itself there is "stretching" of the layer, giving a downwards velocity. Similarly beneath a trough ther is a piling-up in the layer, giving an upwards velocity. Analytically, we have

$$v = \int_0^y \frac{\partial v}{\partial y}\,dy = \int_0^y\left(-\frac{\partial u}{\partial x}\right)dy = -\frac{\partial}{\partial x}\int_0^y u\,dy.$$

Paying attention only to the part of v that is in phase with u_∞, that i the part arising from M we have

$$v_\infty = -\frac{\partial M}{\partial x} = -\frac{1}{2} A k \delta \cos(\sigma t - kx). \tag{5}$$

This shows that the mean value of the product $u_\infty v_\infty$ is negative:

$$(\overline{uv})_\infty = -\frac{1}{4} A^2 k \delta < 0. \tag{6}$$

the significance of which will soon become apparent.

(2) <u>The mean stress on the bottom</u> If first-order terms only are considered, the mean stress on the bottom is identically zero. But we shall show, by a straightforward consideration of momentum, that to second order, the mean stress must in fact be positive.

Imagine a rectangle, one wavelength long, drawn in the fluid with its upper side CD just outside the boundary-layer and its lower side $C'D'$ on the bottom. When fluid having a horizontal velocity u crosses the upper side CD with velocity v there is a transfer of momentum across the boundary at the rate ρuv per unit horizontal distance. The mean rate of transfer of momentum across CD in this way is given by $\rho\overline{uv}$, the familiar Reynolds stress. Consider then the momentum balance inside the rectangle $CDD'C'$. Along the upper side viscous stresses are negligible and there is a transfer of momentum due to the Reynolds stress $\rho(\overline{uv})_\infty$. On the lower side $C'D'$ the Reynolds stress vanishes (since $v = 0$), but there is a mean viscous stress $\rho(\overline{\nu \partial u/\partial y})$. On the two vertical sides the conditions are identical by the periodicity of the motion, and so the transfer of momentum across one side just cancels the transfer across the other side. But the total momentum within the rectangle remains unchanged; therefore the viscous stress on the bottom must just balance the Reynolds stress at the top. In other words

$$\left(\overline{\nu \frac{\partial u}{\partial y}}\right)_0 = -(\overline{uv})_\infty. \tag{7}$$

We have seen from eqn. (6) that the mean product $(\overline{uv})_\infty$ is negative. In other words there is a downwards transfer of momentum into the boundary-layer. To balance this, there must be a backwards stress on the layer at the bottom, that is to say a forwards gradient of mean velocity. In the case when the viscosity is constant we have

$$\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\nu}(\overline{uv})_\infty = \frac{A^2}{2c\delta} > 0. \tag{8}$$

The velocity gradient at other levels within the layer may be obtained by considering a rectangle $CDD''C''$ (see Fig. A4) which has its lower side $C''D''$ at an arbitrary level within the boundary layer. The same considerations of momentum apply, but now account must be taken of both the viscous stress and the Reynolds stress at the level $C''D''$. This leads us at once to the relation

$$\overline{\nu \frac{\partial u}{\partial y}} = \overline{uv} - (\overline{uv})_\infty. \tag{9}$$

If the viscosity is given, the profile of the mean velocity $\bar{u}$ may be de by direct integration. This is done in §5, and we find

$$\bar{u} = \frac{1}{c}\,\overline{u_{\infty} u} + \frac{1}{2c}\,\overline{u^2} - \overline{\frac{\partial u}{\partial y}\int v\,dt} \qquad (1$$

a result which does not depend upon the distribution of viscosity withi the layer. For constant viscosity we obtain the left-hand curve shown in Fig. A5.

(3) <u>The mass-transport velocity</u> It is essential to distinguish betwee the mean velocity $\bar{u}$ measured at a fixed point and the mass-transport velocity U, which may be defined as the mean velocity of the same particle of fluid averaged over a complete period (both $\bar{u}$ and U being assumed small compared with the orbital velocity u). For example in tl Stokes irrotational wave the mass transport velocity is always positive relative to the mean velocity. This is for two reasons: first because as a wave crest passes overhead the orbital velocity is positive, and so the particle "stays with the wave", spending slightly longer on the forwards part of its orbit than on the backwards part; secondly, the velocity of a particle is slightly greater at the top of its orbit, whe it is travelling forwards, than at the bottom, where is is travelling backwards (see Fig. A6).

The same considerations apply, in general, in the boundary-layer; although the vertical displacements are very small, the vertical gradie of velocity is correspondingly large, so that both the effects just mentioned become appreciable. However, the phase difference between horizontal and vertical components of velocity is a function of the mean position of a particle within the layer.

Analytically, if P is the point on the orbit of a particle whose me position is Q, the instantaneous velocity at P will differ from that at Q by an amount

$$\Delta u = \frac{\partial u}{\partial x}\Delta x + \frac{\partial u}{\partial y}\Delta y$$

where $\Delta x, \Delta y$ are the horizontal displacements of P from Q. These displacements are given by

$$\Delta x = \int u\,dt, \qquad \Delta y = \int v\,dt \qquad (11$$

approximately, (apart from a constant term). Hence the difference between the <u>mean</u> velocity of the particle and the velocity at Q is given to the second approximation, by

$$U = \bar{u} + \overline{\frac{\partial u}{\partial x}\int u\,dt} + \overline{\frac{\partial u}{\partial y}\int v\,dt}. \qquad (12$$

On the bottom itself, u, v, $\partial u/\partial x$ and $\partial v/\partial y$ all vanish, and so on differentiation we find

$$\frac{\partial U}{\partial y} = \frac{\partial \bar{u}}{\partial y} > 0. \qquad (13)$$

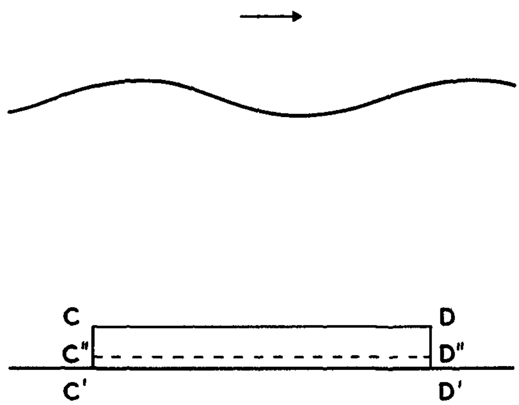

Fig. A4. Diagram for deriving the stress on the bottom.

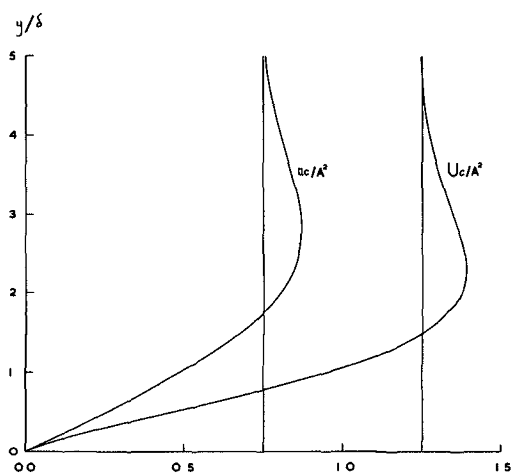

Fig. A5. The mean velocity ū and the mass-transport velocity U in the boundary-layer.

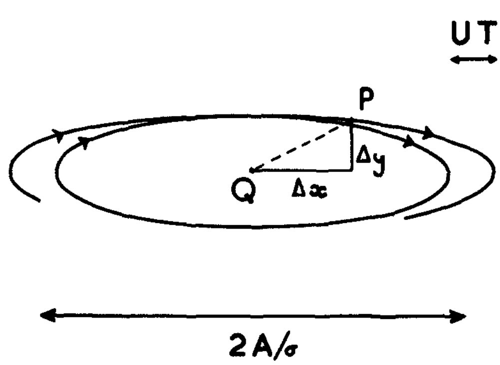

Fig. A6. How the mass-transport velocity arises, when ū = 0.

Since U vanishes on the bottom itself, this shows that U must be positi very close to the bottom; here at least there is a forwards mass-transp velocity.

At other levels within the boundary-layer, however, the last two terms in eqn. (12) are not negligible. For a progressive wave we have

$$\overline{\frac{\partial u}{\partial x}\int u\,dt} = -\frac{1}{c}\,\overline{\frac{\partial u}{\partial t}\int u\,dt} = \frac{1}{c}\,\overline{u^2}$$

by eqn. (21) below, and so from (10)

$$U = \frac{1}{c}\left(\overline{u_\infty u} + \frac{3}{2}\,\overline{u^2}\right). \qquad ($$

When $u \to u_\infty$ we have

$$U \to \frac{5}{2c}\,\overline{u_\infty^2}\,. \qquad (.$$

These remarkably simple formulae are independent of the absolute value of the viscosity and even (as will be shown) independent of the form of the distribution of viscosity within the layer. However, in the specia. case when the viscosity is constant and the motion sinusoidal we have oı substitution from (3)

$$U = \frac{A^2}{4c}\left(5 - 8e^{-y/\delta}\cos y/\delta + 3e^{-2y/\delta}\right). \qquad (:$$

This distribution is shown by the second curve in Fig. A5. U is always positive and has a maximum value

$$U = 1{\cdot}376\ldots\ A^2/c \qquad (1$$

As y/δ tends to infinity

$$U \to 1{\cdot}25\ \ A^2/c \qquad (1$$

compared with the limiting value

$$\bar{u} \to 0{\cdot}75\ \ A^2/c \qquad (1$$

for the mean velocity.

(4) <u>Discussion</u> We have remarked that the formulae (14) and (15) are independent of the distribution of viscosity within the layer, provided that the flow is laminar. Now for turbulent but steady boundary-layers it has been shown that the flow may be quite well approximated by the laminar velocity profile, provided that in the outer part of the layer the ordinary viscosity is replaced by a uniform coefficient of eddy viscosity[6]. Now if the eddy viscosity fluctuates according to the instantaneous velocity gradient, then an oscillatory boundary-layer will not be strictly comparable with a steady boundary-layer. If on the othe hand we assume that the eddy-viscosity of a small element of fluid does not fluctuate appreciably throughout a wave period it is possible to replace the ordinary kinematic viscosity ν by a coefficient which is constant for a particle, though varying with the mean distance of the

particle from the bottom. Our result then indicates that the equation

$$U_\infty = \frac{5}{2c}\overline{u_\infty^2}$$

for the velocity just outside the boundary-layer is valid also when the flow is turbulent.

Moreover this expression is valid even when the motion, though periodic is not strictly harmonic, as will happen with long waves in shallow water when the form of a solitary wave is approached. Suppose that the velocity near the bottom, instead of being simply harmonic is given by an expression of the form

$$u_\infty = A_1 \cos(\sigma t - kx) + A_2 \cos 2(\sigma t - kx) + \cdots + B_2 \sin 2(\sigma t - kx) + \cdots$$

in which the coefficients may be deduced theoretically or found from observation by Fourier analysis. Then the above expression gives

$$U_\infty = \frac{5}{4c}\left(A_1^2 + A_2^2 + \cdots + B_2^2 + B_3^2 + \cdots\right). \tag{20}$$

Although the mass-transport velocity just outside the layer has been shown to be independent of ν, eqn. (2) shows that the distribution of the velocity within the layer is certainly dependent on the form of the viscosity. For this reason the expression (17) for the maximum velocity within the layer may not be valid for a turbulent layer.

However, what is observed in practical experiments is less likely to be the maximum velocity than the velocity just outside the layer, where the velocity gradients are less steep - especially if the boundary layer is thus and observations are made with streaks of dye. For, a slender tongue of dye is less easy to observe than a diffused cloud moving forwards with a relatively uniform velocity. It is fortunate that the latter velocity appears to be more easily predictable.

(5) Proof of eqn. (10) Finally shall prove our statement that eqn. (10) is true independently of the viscosity. Although the argument can at some length be translated into physical terms, it is simpler at this stage to give an analytical proof.

We start from eqn. (2) and (9), which are both valid even when the viscosity is a function of time and position. From (2) we have by integration with respect to t,

$$u = u_\infty + \int \frac{\partial}{\partial y}\left(\nu \frac{\partial u}{\partial y}\right) dt$$

and therefore

$$\overline{uv} - (\overline{uv})_\infty = \overline{u_\infty v} + \overline{\int \frac{\partial}{\partial y}\left(\nu \frac{\partial u}{\partial y}\right) dt \,.\, v} - (\overline{uv})_\infty .$$

Now from the equation of continuity

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x} = \frac{1}{c}\frac{\partial u}{\partial t}$$

and so

$$\overline{u_\infty v} - (\overline{uv})_\infty = \int_\infty^y \overline{u_\infty \frac{\partial v}{\partial y}}\, dy = \frac{1}{c}\int_\infty^y \overline{u_\infty \frac{\partial u}{\partial t}}\, dy$$

where the limit ∞ denotes a value of y large compared with δ but small compared with the wavelength or total depth. On substituting for $\partial u/\partial t$ from eqn. (2) we have

$$\overline{u_\infty v} - (\overline{uv})_\infty = \frac{1}{c}\int_\infty^y \overline{u_\infty\left[\frac{\partial u_\infty}{\partial t} + \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)\right]}\, dy = \frac{1}{c}\overline{u_\infty \nu}$$

since $\overline{u_\infty . \partial u_\infty/\partial t}$ is identically zero. Further, if f and g denote any two periodic quantities then

$$\overline{f\frac{\partial g}{\partial t}} + \overline{\frac{\partial f}{\partial t} g} = \overline{\frac{\partial}{\partial t}(fg)} = \frac{1}{T}(fg)_{t=0}^{t=T} = 0 \qquad (21)$$

and so in any averaged product of this type the operator $\partial/\partial t$ may be transferred from one member to the other, provided the sign is reversed at the same time. Thus for example

$$\overline{\int \frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)dt \, . \, v} = -\overline{\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)\int v\, dt}\, .$$

On substituting these results in eqn. (9) we obtain

$$\overline{\nu\frac{\partial u}{\partial y}} = \frac{1}{c}\overline{u_\infty \nu \frac{\partial u}{\partial y}} - \overline{\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)\int v\, dt}\, . \qquad (22)$$

We assume that the viscosity is <u>constant following a particle</u> but is a function $N(Y)$ of the mean height Y of the particle above the bottom, then at any fixed point (x,y) the viscosity will be a slightly varying function of the time. To the first approximation

$$\nu(x,y,t) = N(Y) - \frac{dN}{dY}\Delta y = N - \frac{dN}{dY}\int v\, dt.$$

Substituting in (22) and neglecting third-order terms we have

$$\overline{\left(N - \frac{dN}{dY}\int v\,dt\right)\frac{\partial u}{\partial y}} = \frac{1}{c}\overline{N u_\infty \frac{\partial u}{\partial y}} - \overline{\left(N\frac{\partial^2 u}{\partial y^2} + \frac{dN}{dY}\frac{\partial u}{\partial y}\right)\int v\,dt}$$

The terms involving dN/dY cancel, and on dividing through by N, which is a function of Y only, we obtain

$$\frac{\partial \bar{u}}{\partial y} = \frac{1}{c}\overline{u_\infty \frac{\partial u}{\partial y}} - \overline{\frac{\partial^2 u}{\partial y^2}\int v\,dt}\, .$$

This relation is entirely free from N . The last term can be written as

$$-\frac{\partial}{\partial y}\left(\overline{\frac{\partial u}{\partial y}\int v\,dt}\right) + \overline{\frac{\partial u}{\partial y}\int \frac{\partial v}{\partial y}\, dt}\, ,$$

and since

$$\overline{\frac{\partial u}{\partial y}\int\frac{\partial v}{\partial y}\,dt} = \overline{\frac{\partial u}{\partial y}\int\frac{1}{c}\,\frac{\partial u}{\partial t}\,dt} = \frac{1}{c}\,\overline{\frac{\partial u}{\partial y}\,u} = \frac{1}{2c}\,\frac{\partial}{\partial y}\,\overline{u^2}$$

we have

$$\frac{\partial \bar{u}}{\partial y} = \frac{1}{c}\,\overline{u_\infty\,\frac{\partial u}{\partial y}} + \frac{1}{2c}\,\frac{\partial}{\partial y}\,\overline{u^2} - \frac{\partial}{\partial y}\left(\overline{\frac{\partial u}{\partial y}\int v\,dt}\right).$$

On integrating from $y = 0$, where u and v vanish, we find eqn. (10)

Since the relation (12) between U and $\bar{u}$ is purely kinematical, it follows that eqn. (14) and (15) also are independent of N.

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CHAPTER 11

SCALE EFFECTS INVOLVING THE BREAKING OF WAVES

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1. INTRODUCTION

When in the period after 1953 the projects to damm up the estuaries in the south western part of the Netherlands took shape and it turned out to be necessary to let very wide discharge sluices into at least one of them, the problem arose which waves had to be taken into account at the structure before, during and after the construction. The waves occurring at high tides and storm surges are determinant for the definite shape and dimensions of the sluices. More frequently occurring waves have to be reckoned with during the construction. Though, especially in recent years, many measurements have been done in nature by the Rijkswaterstaat, supplementary calculations and model investigations appeared necessary, to determine the design criteria at rarely occurring circumstances.

The shoals before and in the estuaries have a very capricious shape. This fact and the often very strong tidal currents practically prevent accurate calculations of refraction and diffraction. Moreover, there has to be reckoned with the influence of the local wind and with the breaking of the waves on the offshore bars.

All this considered it was decided to built a small-scale model representing the estuary and the offshore area.

For the interpretation of the results of such a model, which by the large extent of the prototype should have very small scales, it was necessary to analyse certain factors, viz. the exact influence of the internal and external friction on waves and the relation of the friction to the scale of the model, the transfer of energy from wind to waves and the relationship between the scale of the model and the loss of energy of waves at breaking.

Although the investigations of these problems are not yet all completed, the preliminary results of the latter scale effect may herewith be communicated.

2. DESCRIPTION OF THE MODEL

The loss of energy of waves passing submerged barriers has been a subject of investigations in several countries for a long time. We may cite among others the publications of Johnson on scale effects in wave models, ref. 1, and of Johnson, Fuchs and Morison on the damping action of submerged breakwaters, ref. 2.

Apparently in these and other studies the width of the crest of the submerged obstruction has been taken small, compared with the length of the waves, or the influence of the absolute wave dimensions on the results have not been taken into account.

Although rarely stated, the periods of the model waves seem to have been mostly rather long, viz. about 1 or 2 seconds.

Among the results it is mentioned that the loss of energy of steep waves, passing a certain obstruction, exceeds that of flat waves and that the transmission of the energy decreases with increasing height of the obstruction. These phenomena were also observed in Delft, where it was above all a matter of study of very small wave periods and of great width of the obstructing bar in the direction of the wave motion.

For this purpose tests have been carried out in a glass walled flume, wide 0.5 m, with models as shown in figure 14. The models consisted of a gentle slope of 1 : 10, at the end of which was a horizontal crest, representing a bar, followed by a channel. The dimensions of the model are expressed in the deep-water wave length L_0, so that at all different wave periods – hence lengths – a suited model was used. The top of the bar was placed at a half wave length above the level of the bottom of the flume, so that the incident wave always was a deep-water wave. The width of the bar in the direction of the wave motion was three wave lengths. The models were made of glass to reduce the wall and bottom friction to a minimum and to permit an accurate finishing.

The water level ranged from 0.05 L_0 to 0.3 L_0 above the level of the bar and the wave steepness from 0.02 to 0.08. The investigated wave periods were 0.31, 0.43 and 0.55 seconds; the corresponding wave lengths were 0.15 m, 0.29 m and 0.47 m respectively.

In order to determine the correct scale laws for very small wave models, it is necessary to continue these tests with greater wave periods.

3. THE TESTS

During the tests the surface of the water was repeatedly cleaned because of the experience, that even a thin and unvisible cover of dust increases the surface tension and diminishes considerably the height of waves of small periods. Separate tests have been carried out in the same flume to determine the loss of energy due to viscosity and side-wall friction. The results of these tests

Fig 1

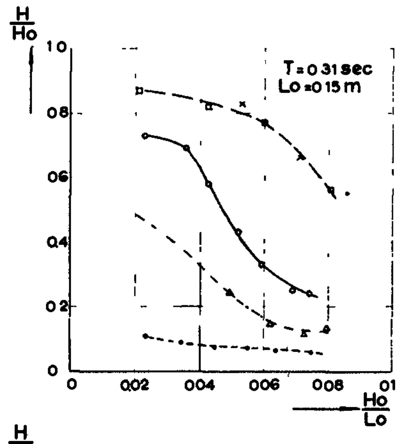


Fig 2

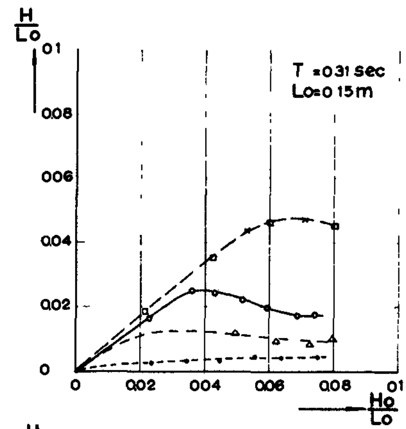


Fig 3

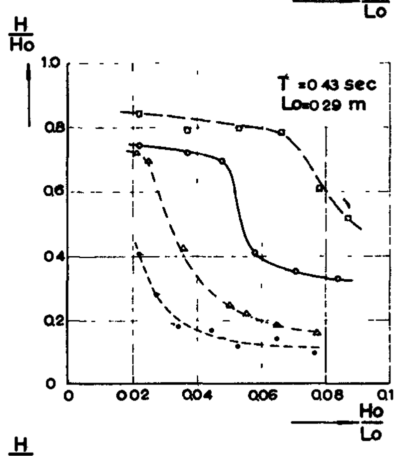


Fig 4

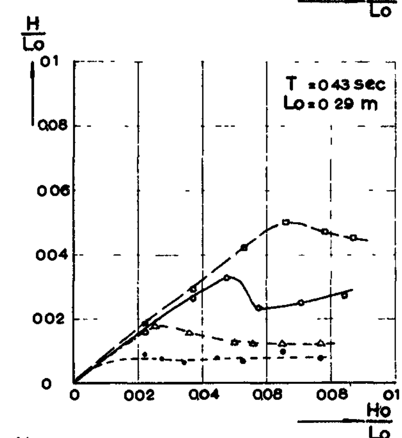


Fig 5

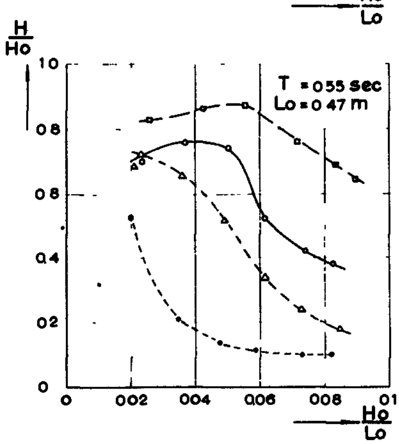


Fig 6

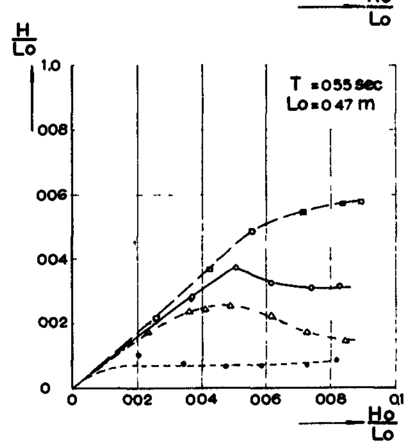


Fig 1,3,5 - Relationship between the ratio of the wave height behind the bar to the incident wave height and the wave steepness in deep water

Fig 2,4,6 - Relationship between the steepness of the waves before and behind the bar

- - - • - - - D = 005 Lo
- - ∆ - - D = 0075 Lo
—o— D = 01 Lo
- - ▫ - - D = 02 Lo
- - x - - D = 03 Lo

have been taken into account in the interpretation of the tests concerning the breaking of the waves.

The experimental data are shown in a number of diagrams. The figures 1, 3 and 5 show for different wave periods - i.e. different wave lengths - the relationship between the steepness of the incident wave $(\frac{H_o}{L_o})$ and the ratio of the wave height before the bar to the wave height behind the bar $(\frac{H}{H_o})$ the transmission coefficient. The figures 2, 4 and 6 show the relationship between the steepness of the waves before and behind the breaking zone, $\frac{H_o}{L_o}$ and $\frac{H}{L_o}$ respectively.

The tests have been performed at four different water levels. The figures 7, 8, 9 and 10 show the results for each water level separately. It appears that the water depth above the bar is of outstanding significance. For each water depth there exists a definite wave steepness for which the transmission coefficient is maximum. Of more importance is, however, that this relation varies with the wave period. With deceasing periods the transmission decreases. The ratios $\frac{D}{L_o}$ and $\frac{H_o}{L_o}$ being constant, the transmitted wave energy with a wave period of 0.55 seconds may be the three- to fourfold of that with a wave period of 0.31 seconds. With larger waves, having periods of 2 to 3 seconds, the quantity of transmitted energy will be ever greater.

For a small-scale model this means a limitation of the possibilities. The given boundary conditions may not always be met and often smaller wave heights will have to be used, to avoid breaking. For refraction models this is rather inconvenient. The influence of this effect can hardly be corrected, especially when the waves break more than once. The cause of the stated differences can be found in the properties of the liquid, notably the surface tension and the viscosity.

From the relationship between the wave height and the covered stretch in the model, see figure 14, it appears that the decrease of the wave height does not remain constant for waves having the same steepness but different periods, see figures 15 and 16. The relative depths, where a marked decrease of the wave height begins, - which for very small waves is not identical with the depth, where a visible breaking occurs - increases with decreasing wave period. In the case of the smallest waves the conversion of the energy occurs by viscous friction after the origination of a capillary wave train. It is the intention to repeat a part of the tests with

Fig 7

Fig 8

Fig 9

Fig 10

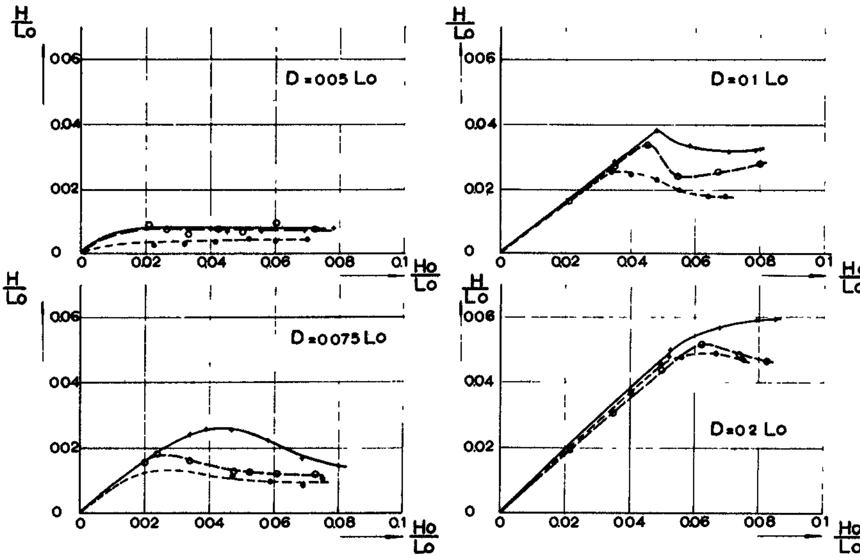


Fig 7,8,9,10 - Relationship between the steepness of the waves before and behind the bar

Fig 11

Fig 12

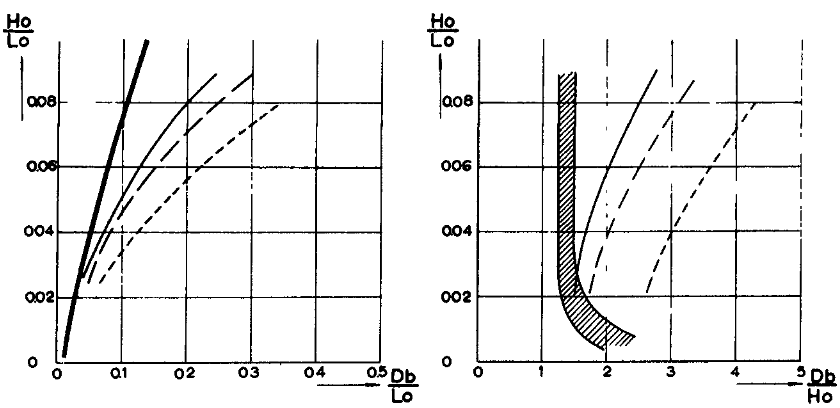


Fig 11 - Relationship between the steepness of the incident wove and the rotio of the depth ot breaking to the deep-woter wove length

Fig 12 - Relotionship between the steepness of the incident wove ond the rotio of the depth at breaking to the deep-woter wove height

Experimentol data of foreign investigotors Wove periods 1-3 s

Wave period 0 55 sec

" " 0 43 "

" " 0 31 "

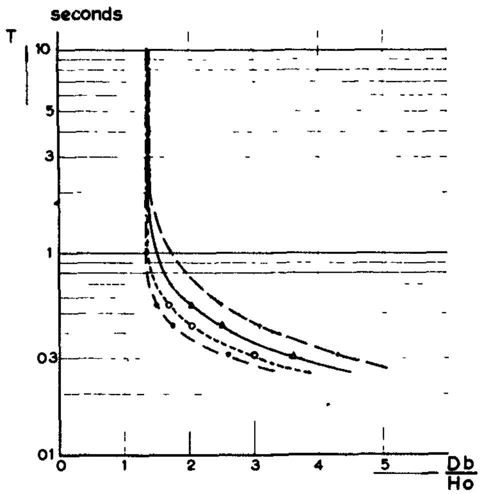
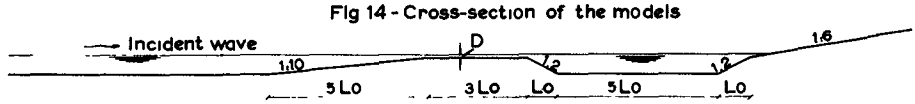


Fig 13 - Relotionship between the wove period ond the rotio of the depth ot breoking to the height of the deep-woter wove

Fig 14 - Cross-section of the models



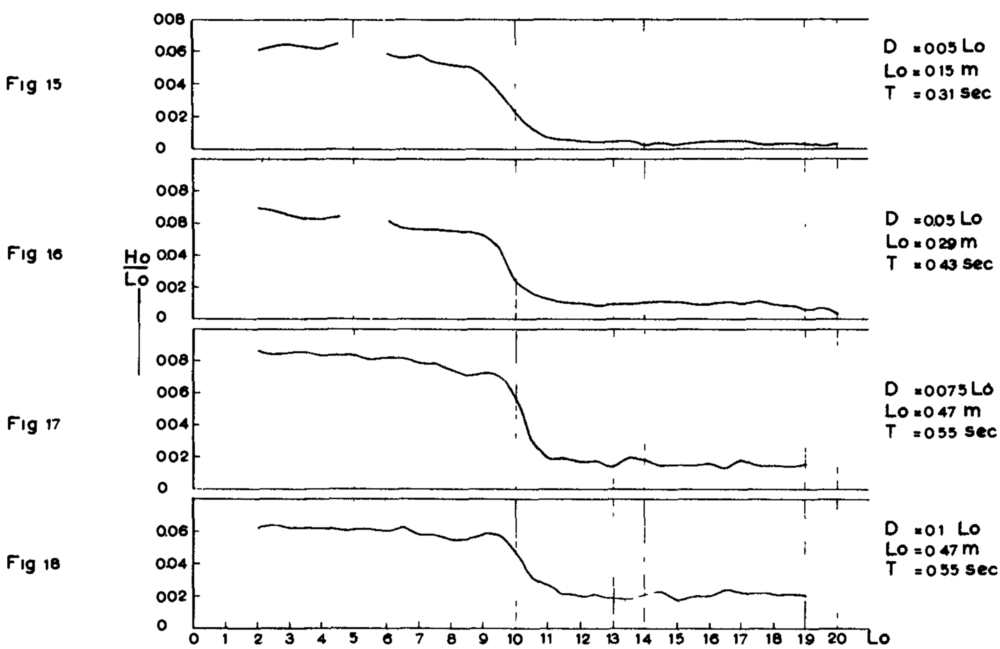


Fig 15,16,17,18 - Typical wave height diograms a liquid having another - smaller - surface tension.

On the figures 11 and 12 the relationship is shown between the steepness of the incident wave $\frac{H_o}{L_o}$ and the ratio of the depth of breaking to the length and to the height of the incident wave, $\frac{D_b}{L_o}$ and $\frac{D_b}{H_o}$ respectively. On these are also indicated the experimental data of other investigations, viz. Larras, ref. 3, Iversen, ref. 4, Hensen, ref. 5, and the University of California, ref. 1. No tests have been performed in Delft with very small steepness. The deviations are considerable in the case of very small periods. Larras mentions this phenomenon but gives no further data.

The relationship between the ratio of the depth of breaking to the wave length $\frac{D_b}{L_o}$ and the wave period is shown on figure 13 The period is of no influence if it is longer than about 2 seconds Most investigators used waves with periods longer than 1 second and by that reason they could observe little or nothing of the influence of the surface tension.

4. CONCLUSIONS

With the breaking of waves on a submerged bar there exists a considerable scale effect in the transmission of the energy - hence the height - of the waves. With small wave periods more energy is converted into heat than with long periods. This applies notably to obstructions having a width of the crest in the direction of the wave motion of more than one deep-water wave length. This is caused by the surface tension. It is impossible to obtain absolute quantitative data from small models, e.g. of large estuaries, because the effects of scale impose too many limitations.

In calculations of refraction is, behind the point of breaking, always reckoned with the maximum height of a wave that could pass without breaking. This supposition appears to be safe.

The influence of the wave height and the viscosity on the velocity of propagation can be neglected in practice. Hence, a model of an estuary, when it is inacessable for calculations, can provide very usefull indications for the diagrams of refraction, especially when the tidal currents interfere.

The ratio of the depth of breaking to the deep-water wave length $\frac{D_b}{L_o}$ decreases with inceasing wave period. Therefore, when results of model tests are published, even if this is done in dimensionless formulas and diagrams - otherwise very desirable -, it is absolutely necessary to mention the real dimensions.

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CHAPTER 12

DETERMINATION OF THE WAVE HEIGHT IN NATURE FROM MODEL TESTS SUPPLEMENTED BY CALCULATION

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1. INTRODUCTION

This paper deals with the problem of determining the wave characteristics in shallow water from those in deep water. In general this can be done by means of a refraction calculation. If the sea bottom topography is too irregular the height of the waves can be determined by means of a small-scale refraction model. In both cases, however, some additional effects have to be taken into account, viz. the influence of the bottom friction and the influence of the wind.

Since a small-scale model does not correctly reproduce the breaking of the waves, this should be avoided by using such small waves, that no breaking in the model occurs.
The influence of the breaking of the waves must then be studied in a separate model on a larger scale and the results of these separate tests must be taken into account as a correction factor by which the results of the refraction model have to be multiplied.

If a model is built in concrete, and the waves are long with respect to the water depth, the bottom friction in the model is no in accordance with that in the prototype. This can be compensated to a certain extent by a distortion of the model. In case of a very small scale of the model, however, this is not sufficient and additional measures have to be taken to compensate this scale effect.

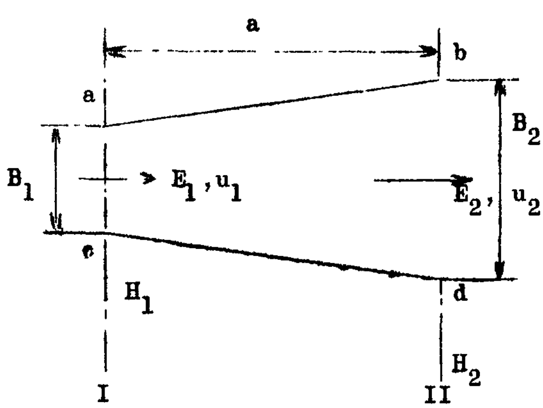
With regard to reproducing the influence of the wind on the height of the waves, it is often very difficult to generate wind in the model, in which case also this effect has to be taken into account as a correction factor.

For these reasons a small-scale refraction model cannot produce exact quantitative results and such a model will only give a correct representation of the refraction pattern.

The present paper describes a method for determining by means of a small-scale model, supplemented by calculations, the correct wave height in the prototype from the wave heights measured in the model. It is based upon the consideration of an energy balance for the prototype as well as for the model. This method may also be used if the refraction is not determined by a model but by calculations only. It can only be applied if the following conditions are fullfilled:

1) no breaking or surfing of the waves may occur
2) there must be only refraction of the waves and no reflection
3) the stretch of water, to which the method is to be applied, must be so short that the deviation of the wave height and the group velocity from the average values is not too great.

2. GENERAL PRINCIPLE



Let a b and c d be two approximately straight orthogonals and I and II two sections normal to these orthogonals, then the transport of wave energy through section II must be equal to the transport of wave energy through section I, if between the sections I and II no energy is lost nor gained.

However, neither in the model, nor in the prototype this is the case. In the prototype, as well as in the model, energy is lost due to bottom friction, while in the prototype energy is gained from the wind.

The energy balances for prototype and model can be written as follows if losses due to internal friction are left out of consideration.

Prototype

$$E_2 = E_1 - E_b + E_w \qquad \text{watt}$$

Model

$$E_2' = E_1' - E_b' \qquad \text{watt} .$$

In the above equations is:

E_w = energy supplied by the wind in the stretch I - II of the prototype

E_b = loss of energy due to bottom friction in the stretch I - II of the prototype

E_b' = loss of energy due to bottom friction in the stretch I - II of the model.

In the following it will be shown that the unknown wave height H_2 in section II of the prototype can be calculated from the known wave height H_1 in section I by means of the above energy equations, if the corresponding wave heights in the model H_1' and H_2' are measured and the losses in model and prototype, due to bottom friction are known.

3. TRANSPORT OF ENERGY IN SECTION I and II

The wave energy passing section I per second, may, with sufficient accuracy, be written as:

$$E_1 = \frac{1}{8} \rho g \, H_1^2 B_1 u_1 \qquad \text{watt}$$

and through section II:

$$E_2 = \frac{1}{8} \rho g \, H_2^2 B_2 u_2 \qquad \text{watt.}$$

Hence, without wind and without bottom friction:

$$\frac{1}{8} \rho g \, H_1^2 B_1 u_1 = \frac{1}{8} \rho g \, H_2^2 B_2 u_2 \; .$$

After substituting u_1 and u_2 by:

$$u_1 = n_1 \frac{L_1}{T} \text{ and } u_2 = n_2 \frac{L_2}{T} \quad \text{respectively,}$$

the above equation becomes:

$$\frac{1}{8} \rho g \, H_1^2 n_1 B_1 L_1 = \frac{1}{8} \rho g \, H_2^2 n_2 B_2 L_2$$

The latter equation expresses that the wave energy present in section I on an area of $n_1 B_1 L_1$ is equal to that present in section II on an area of $n_2 B_2 L_2$.

The time required by the energy on the area $n_1 B_1 L_1$, to move from I to II amounts to $\frac{a}{u}$ sec, where u = the average energy velocity between I and II.

4. LOSS OF ENERGY DUE TO BOTTOM FRICTION

In the prototype, as well as in the model, there is a loss of energy due to bottom friction. Between section I and section II these losses amount to E_b and E_b' respectively.

According to a theory on the dissipation of wave energy by bottom friction developed by Putnam and Johnson, ref. 1, the loss of energy per unit of area, due to bottom friction, can be expressed by:

$$\Delta E_b = k\rho H^3 f\ (D,T) \qquad \text{watt/m}^2$$

where: H = wave height in m

ρ = density of the water in kg/m^3

$f(D,T)$ = a function of water depth and period

k = dimensionless coefficient of friction from the formula: $v_* = \sqrt{k}\cdot v_D$, in which $v_* = \sqrt{\frac{\tau}{\rho}}$ m/sec

τ = shear stress in N/m^2

v_D = velocity at the bottom in m/sec.

The function $f(D,T)$ is:

$$\frac{4\pi^2}{3T^3}\left(\frac{1}{\sinh\frac{2\pi D}{L}}\right)^3 \text{ with the dimension of } \left[t^{-3}\right].$$

The above formula for ΔE_b is valid only for an impervious sea bottom, having a coefficient of bottom friction which is independent of the magnitude of the velocity at the bottom and of the water depth in the area under consideration, while the velocity must be sinusoidal.

From wave measurements in Lake Okeechobee, ref. 2, it appeared that k has a nearly constant value: $k = 0.01$.

From the results of separate model tests on bottom friction, carried out in the Delft Hydraulics Laboratory, it appeared that the friction coefficient k for a model on a small scale is dependent on the water depth D and the period T. With the help of the in this way obtained values of k the correction for the bottom friction in the model could be determined.

The total loss of energy, due to bottom friction, in the stretch from section I to section II is then:

$$E_b = \Delta E_b \frac{B_1 + B_2}{2} a \qquad \text{watt .}$$

5. ENERGY SUPPLIED BY THE WIND

From the diagram showing the growth of wind-generated waves in shallow water, ref. 3, it appears that, after a certain period of time, for each water depth and each wind velocity, a state of equilibrium will be reached in which the waves no longer grow and the entire quantity of energy supplied by the wind is dissipated by the bottom friction.

By studying the amount of energy lost due to bottom fricti for various water depths and wave periods, an approximate constan value ΔE_w was found for the increase by wind of the wave energy p sq.m over one metre.

For deep water, ΔE_w can directly be calculated from the dia gram showing the growth of wind-generated waves in deep water, ref. 4. For each value of the wind velocity, a value for ΔE_w is th found that is independent of the wave period and of the wave heig This value does not differ much from the above mentioned value fo shallow water. This value is per metre displacement of the wave:

$$\Delta E_w \doteq \frac{\frac{1}{8}\rho g H\ (H\Delta L + 2L\Delta H)}{\Delta a L} \qquad \text{joule/m}^3$$

The total increase in energy, due to the wind, in the stret from section I to section II is then:

$$E_w = \Delta E_w \frac{B_1 + B_2}{2} ua \qquad \text{watt .}$$

6. ENERGY EQUATIONS

Based upon the foregoing considerations, the following ener equations for prototype and model can be written:

<u>prototype</u>

$$E_2 = E_1 - E_b + E_w$$

or:

$$\frac{1}{8}\rho g H_2^2 B_2 u_2 = \frac{1}{8}\rho g H_1^2 B_1 u_1 - \Delta E_b \frac{B_1 + B_2}{2} a + \Delta E_w \frac{B_1 + B_2}{2} au \qquad (1$$

<u>model</u>

$$E_2' = E_1' - E_b'$$

or:

$$\frac{1}{8}\,\rho g H_2'^2 B_2 u_2 = \frac{1}{8}\,\rho g H_1'^2 B_1 u_1 - \Delta E_b' \,\frac{B_1 + B_2}{2}\, a \,. \tag{2}$$

In the above equations, in the term regarding the wind energy, the product of the average width of the stretch $\frac{B_1 + B_2}{2}$ and the average energy velocity u has to be written as the average product $\frac{B_1 u_1 + B_2 u_2}{2}$; for the terms regarding the bottom friction the same may be done with fair approximation.
Thereupon both equations are divided by $B_2 u_2$ and after eliminating $\frac{B_1 u_1}{B_2 u_2}$ the following equation is obtained:

$$H_2^2 = \frac{\left(\frac{1}{4}\,\rho g H_2'^2 + \Delta E_b'\, a u^{-1}\right)\left[\frac{1}{4}\,\rho g H_1^2 - \left(\Delta E_b a u^{-1} - \Delta E_w a\right)\right]}{\left(\frac{1}{4}\rho g H_1'^2 - \Delta E_b'\, a u^{-1}\right)\frac{1}{4}\,\rho g} + $$

$$- \frac{\Delta E_b a u^{-1} - \Delta E_w\, a}{\frac{1}{4}\,\rho g} \tag{3}$$

In order to simplify equation (3) the following substitutions are finally introduced:

$$\frac{\Delta E_b'\, a u^{-1}}{\frac{1}{4}\rho g} = P \tag{4}$$

$$\frac{\Delta E_b\, a u^{-1}}{\frac{1}{4}\rho g} = Q \tag{5}$$

and

$$\frac{\Delta E_w\, a}{\frac{1}{4}\rho g} = R \tag{6}$$

Equation (3) then becomes:

$$H_2^2 = \frac{(H_2'^2 + P)\,(H_1^2 - Q + R)}{H_1'^2 - P} - Q + R \tag{7}$$

The substitution (5) contains the term

$$\Delta E_b = k\rho \left(\frac{H_1 + H_2}{2}\right)^3 f\ (D,T)\ .$$

Since the prototype wave height H_1 is given and the model wa heights H_1' and H_2' can be measured in the model, the value of H_2 i the prototype can be determined by solving the equations (5) and (7 after having inserted the values for

$$\Delta\ E_b', \quad \Delta\ E_b \quad \text{and} \quad \Delta\ E_w\ .$$

As mentioned before, the above method can only be applied if no crossing energy transport occurs and if in the stretch I - II the deviation of the wave height and the group velocity from the average values is not too great, say about ten percent.

For large stretches the total length has to be subdivided into short stretches and the calculations repeated.

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CHAPTER 13

A WAVE HEIGHT AND FREQUENCY METER

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INTRODUCTION

During World War II a group of Naval officers conducted visual measurements of ocean waves simultaneously with instrumental recordings. A comparison of the visual and instrumental values indicated ".... the natural tendency for the observer .... to record not the average wave height but a wave height based on some kind of average of the highest waves. The general experience is that an observer will give a value for the wave height which represents the average of the highest 20 to 40 per cent of the waves" (SIO, 1944). The average height of the highest one-third of the waves, $H_{1/3}$, was therefore suggested as the characteristic (or significant) wave height. "Characteristic wave period" was given a corresponding definition as the average period of the highest one-third waves.

An interpretation of characteristic wave height in terms of standard statistics was made by Putz (1952) and Longuett-Higgins (1952). They related theoretically rms height, mean absolute ordinate a_o, and the average height of the highest "p" per cent. It turns out that $H_1 = 5.02a_o$. Underlying assumptions are that the ordinates are Gaussian and the spectrum moderately narrow. A summary of observations relating to statistical properties of wave heights was recently made by Wiegel and Kukk (1957).

Measurement of the average absolute ordinate can easily be accomplished. In this paper we describe two systems which lead to a direct measurement of $5a_o$ ($=H_{1/3}$). The overall situation is therefore satisfactory inasmuch as a naturally reported quantity, the characteristic wave height, can be defined in terms of fundamental statistics and objectively recorded by automatic means.

The situation is not nearly as satisfactory with regard to wave period. The original definition of characteristic wave period as the period of the highest of one-third waves could not be interpreted in terms of standard statistics, nor is it possible to design reasonably simple devices for recording this quantity. An alternate definition is based on the average time between zero crossings. This can be inte] preted in terms of standard statistics, and instruments can be devise for recording it. Moreover, a definition based on zero crossings is much in accord with the natural tendency of observers as the previou definition of characteristic period. It would seem that characteristic period is not a useful concept and should be dropped. In this paper w describe two instruments for directly measuring the average zero crossing frequency.

OCEAN WAVE RECORD ANALYSIS

Recent research on ocean wave record analysis has dealt with two problems: (1) the description of data obtained from wave meters which record the time history of surface elevation or subsurface pressure fluctuation and (2) the formulation of a theoretical model for the analysis of the data. Two points of view have often been taken in the work on these problems, resulting in the methods of "wavewise" analysis and "ordinatewise" analysis. The first considers the record as a sequence of oscillations defined by the successive points on it which correspond to relative maxima and minima. The second treats the record as a continuous function of the time variable which may be considered either on its own merits as an isolated piece of data or as a particular sample of finite length from a continuing process. Thus, from the second viewpoint, the given time history may be analyzed by taking into account all of the information which is present in it - corresponding to the information in the ordinary Fourier spectrum (specifying amplitude and phase vs. frequency), or it may be analyzed by taking into account all the information present except that which depends upon the value of the absolute time variable corresponding to the power spectrum (specifying amplitude, but not phase.) The latter method thus seeks to describe the record in terms of the statistical distributions of the various ordinates on the time-history curve. Such a description for a stationary Gaussian process may be considered complete if the correlation function is specified.

A wavewise analysis is often convenient to apply, whereas the ordinate-wise analysis furnishes a more complete and sophisticated description of the data. The existence of the two methods of description creates the problem of relating the measured parameters which result from the two types of analysis. It is thus of interest to see how to obtain information about the power spectrum (or, equivalently, about the correlation function) from a wavewise analysis

Wavewise analyses of time-history records have been reported by many observers. In these analyses measurements were made on the empirical distributions of wave height and wave period, referred, in each case, to troughs and crests. Certain regularities were found in these distributions, particularly those for wave heights. From measured inter-zero-crossing period distributions the correlation function and hence, in principle, the power spectrum, has been estimated (Putz, 1957).

Studies of the Fourier spectrum of observed wave records have been made by Barber and Ursell (1948), using analog computer methods. The results of those studies are useable spectral curves, which, however, reflect the finite length of the analyzed record and the finite resolving power of the analyzing instrument.

The introduction of the notion of a stochastic process to describe ocean wave records was pioneered by Seiwell (1951) and Rudnick (1951), among others. The usefulness of the Gaussian model was recognized by Rudnick, who also computed for a number of wave records the correlation function - the element remaining to complete the picture given by the stationary stochastic process model. The study of the applicability of this stationary Gaussian model was taken up by Pierson (1952) and Putz (1953), among others.

The stationary Gaussian model was found to be in reasonably good agreement with data and to explain a number of the previously-observed relations brought to light by wavewise analyses. These relations were found to be derivable from the mathematical model, which had, in fact, been discussed earlier in some detail by Rice (1944-5). Wave-height distributions, predicted for a narrow-band spectrum, were found to agree with observed distributions, both as to shape and relative values of parameters. The distributions of wave periods, redefined in terms of zero-axis crossings, were found to be directly related to the power spectrum of the individual wave record, and a means of estimating the spectrum from these distributions was developed. Putz (1957a, b). In addition, various relations between parameters of these zero-crossing period distributions and the power spectrum (or, equivalently, the correlation function) were shown to hold.

RELATIONS INVOLVING a_o AND $f_{z.c.}$

The present report considers techniques for the measurement of two quantities, and shows how these may be used to obtain information about the power spectrum and the wave-height distribution. The first of these is the so-called mean deviation a_o of the time-history ordinate, f(t), i.e., the value of $\overline{|f(t) - \xi_o|}$, averaged over time, where ξ_o is the mean ordinate, such that $\xi_o T = \int_0^T f(t)\,dt$, where $\underline{T}$ is the total duration of the record. The second is the so-called mean upcrossing frequency, f_o, i.e., the number of times the time-history curve crosses the mean ordinate level per unit time. It convenient to define also the corresponding angular frequency, ω_o. Thus if N_o is the total number of crossings (in both directions), then $f_o = N_o/(2T)$, and $\omega_o = 2\pi f_o = \pi N_o/T$. The notation "$f_{z.c.}$" is also used below for the quantity $N_o/(2T)$.

If ω is the general angular frequency and $\Phi(\omega)$ is the spectral density function describing the power spectrum, we define the absolut spectral moment M_k of order k by the expression $M_k = \int_{-\infty}^{+\infty} |\omega|^k \Phi(\omega)$ $k = 0, 1, 2, \ldots$. Thus the zero-order moment M_o is the total power in the spectrum. The first-order absolute moment becomes, after division by M_o, the mean angular spectral frequency, $\mu_\omega = M_1/M_o$. From μ_ω and the reduced second-order moment, $\mu_2 = M_2/M_o$, is derived the concept of the spectral bandwidth σ_ω, defined by the expression $\sigma_\omega^2 = \mu_2 - \mu^2_\omega$. Further, the relative spectral bandwidth, δ_ω, may be defined as σ_ω/μ_ω, with the result that $\sqrt{\mu_2} = \sqrt{\sigma_\omega^2 + \mu_\omega^2} = \mu_\omega \sqrt{1 + \delta_\omega^2}$.

It is well known (Rice, 1944-5) that for a Gaussian process, $a_o = (2/\pi)^{1/2}\sqrt{M_o}$, and $\omega_o = \sqrt{M_2/M_o} = \sqrt{\mu_2}$, so that $M_o = (\pi/2)a_o^2$ and $\mu_\omega = \mu_o (1 + \delta_\omega^2)^{-1/2}$. It is seen that observation of a_o and ω_o (or f_o) leads to the determination of M_o and, in the case of small relative bandwidth, of an approximation (on the high side, within 5% if δ_ω does not exceed a value of 1/3, approximately) to μ_ω.

To obtain information regarding the wave-height distributions in a wave record, additional theoretical relations may be used. Thus it follows from the theory of the stationary Gaussian stochastic process that the mean height of the envelope to the time-history graph is $(\pi/2)^{1/2}(M_o)^{1/2} = (\pi/2)\,a_o$. In the case of small relative bandwidth the height of the double envelope may be taken to be the mean trough-to-crest wave height; in this case, then, $\pi\, a_o$ is the mean wave height. More generally, it is known that the mean height of the ordinates at the relative maxima (or the mean depth of the ordinates at the relative minima) is given by $\mu_M = \alpha\,(\pi/2)\,a_o$, where $\alpha = f_o/f_1$, and f_1 is the mean up-crossing frequency for the first derivative curve. The mean trough-to-crest wave height will be given by $2\mu_M = \alpha\pi a_o$. A typical value of α for ocean swell is 0.85. It follows also from the theory of the stationary Gaussian process that the doubled mean height of the highest one-third of the ordinates on the envelope curve is $2(2.00\sqrt{M_o}) = 5.02\,a_o$.

More general relationships holding for the total spectral power and the reduced second spectral moment may be obtained by considering measurement of mean ordinate deviation and mean up-crossing frequency made about an arbitrary ordinate level. Thus, if we measure the average value, a_h, of $|f(t) - h|$, where $h = \xi_o + k\sqrt{M}$

we find that $a_h/a_o = \exp(-k^2/2) + k.\ \Psi(k)$, where $\Psi(k) = \int_o^k \exp(-t^2/2)dt$. Also, for the mean up-crossing frequency, f_h, at ordinate level $\underline{h}$, we have $f_h/f_o = \exp(-k^2/2)$, where, as before, $k = (h-\xi_o)/\sqrt{M_o}$.

It will be noted that both ratios reduce to unity when k = o. Further, the differences $a_h - a_o$ and $f_h - f_o$ are of higher order in k, so that the effect of a small error in locating the mean ordinate level vanishes rapidly with k. In fact, for values of k not in excess of about 0.4, the approximations $a_h/a_o \doteq 1 + k^2/2 \doteq f_o/f_h$ may be used. For example, an error of 5 or 10% in the determination of ξ_o should not correspond to a value of over 0.4 for k, which yields a relative error of 8% in the value of a_o.

DESCRIPTION OF THE ANALYZER

The block diagram of the wave analyzer is shown in Figure 1. The wave data enters a point A as an electrical signal which varies about a zero potential in proportion to the amplitude of the wave about the mean pressure level. This signal is simultaneously applied to two channels of the analyzer. The first channel reverses the polarity of the signal each time it becomes negative to produce a full wave rectification of the incoming signal. By averaging the rectified signal with an appropriate filter a measure of the average absolute ordinate, a_o, is obtained.

The second channel produces a unit pulse for each positive zero-axis crossing of the incoming signal. By again averaging these pulses with an appropriate low-pass filter a mean zero-axis crossing frequency can be obtained.

AN ELECTRONICALLY OPERATED ANALYZER

This analyzer (Figure 2) operates as shown in the block diagram Figure 1). All amplifiers indicated are type K2W Philbrick functional d-c amplifiers and the circuits are discussed by Philbrick (1955).

The rectifier circuit provides full-wave rectification about the zero axis with d-c coupling and a common ground between input and output. With the input connected to ground the zero adjustment potentiometer is set for zero output of the rectifier for rectification at zero input voltage level.

The pulse generator circuit consists of an overdriven amplifier and relay operated pulsing circuit. The amplifier has positive feedback with a loop gain of slighly greater than unity and therefore tends to assume one of two stable states. When the input exceeds a critical voltage, the amplifier output is positive and the relay is energized. When the input voltage returns to a value slightly below the critical voltage the amplifier output becomes negative. With negative voltage the relay is de-energized due to the action of the shunting diode.

The difference between the level at which a positive-going voltag causes the relay to energize and a negative-going voltage causes the relay to de-energize is the circuit hysteresis. The hysteresis prevent chattering of the relays when the input voltage is zero. The amount of hysteresis is controlled by the 50M feedback resistor. For zero axis adjustment the zero adjustment potentiometer is set in the center of the hysteresis range with the amplifier input connected to ground.

Relay operated pulse generators are used in preference to electronic circuits. When relay A is de-energized a 300 μfd electrolytic condenser is charged to 10 volts; when relay is energized the condenser is discharged through relay B energizing it for 3 seconds. A 1.35 volt pulse is therefore applied to the input of the filter for each positive zero-axis crossing.

Stability of the two filter circuits is the primary concern of their design. All electrical components must be of high quality with low temperature and aging coefficients. Sufficient stability was obtained only by using heavy negative feedback.

The time constant of the filter, being essentially equal to RC, can be made equal to 2 seconds for adjusting the circuit and 600 seconds for normal operation. With a 600 second time constant the high frequency cutoff occurs at $1/2\pi$ 600) = 1 cyc/hr. Recording millimeters can be driven directly from the output of the filter. Sensitivity of the circuits is adjusted by potentiometers connected in series with the recorders.

A RELAY-OPERATED ANALYZER

This wave analyzer, designed to operate with a Mark IX Pressure Head (Snodgrass, 1955), consists of a thyratron-operated relay circuit which drives two panel meters filled with 12,500 centistoke silicone fluid (Figure 3). The viscous damping of the meter movement provides a response time-constant of twenty minutes. These meters are equivalent to a single stage RC filter with a high frequency cutoff at $1/(2\pi$ 20 min) = 0.5 cyc/hr. (Snodgrass, Putz 1954)

The second meter receives as an input the rectified current of the a-c wave signal driving the Esterline-Angus recorder. The wave height meter is 10 times more sensitive than the Esterline-Angus recorder; its full scale reading corresponds to an average absolute

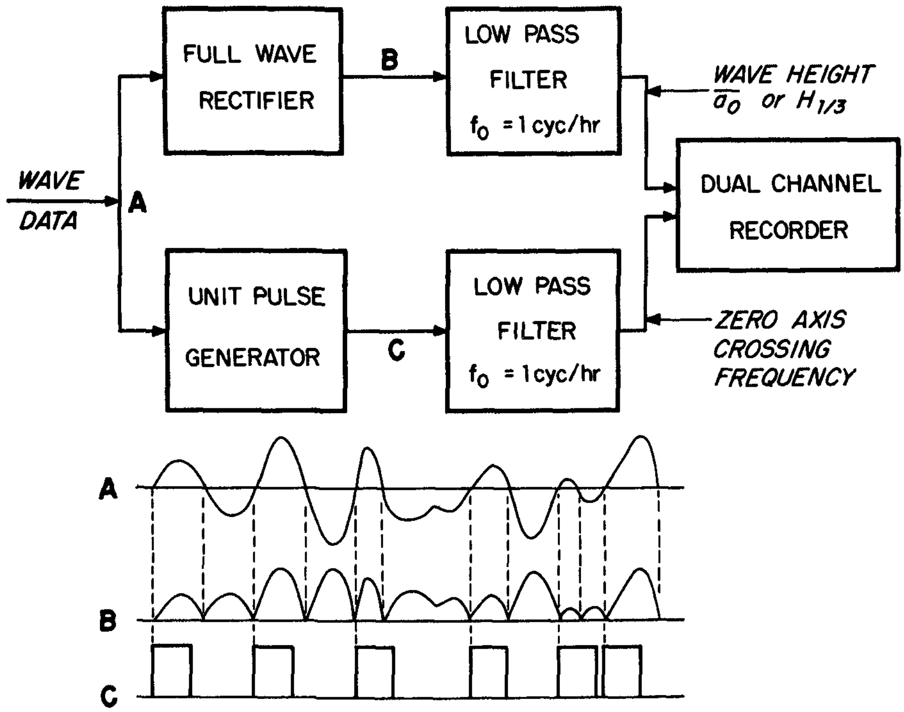

Fig. 1. Block diagram of the analyzer. Voltage signal from wave gage enters A; the average values of the rectified wave record at B and zero-axis-crossing unit pulses at C are recorded as wave height and frequency.

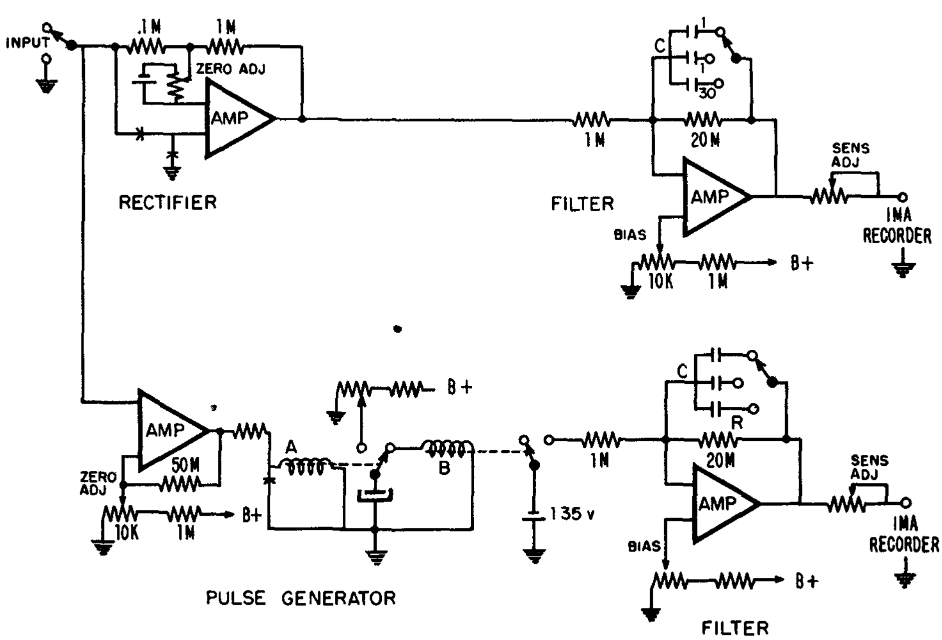

Fig. 2. Functional diagram of an electronically operated analyzer. AMP symbols are Philbrick d-c operational amplifiers.

ordinate value of 0.1 of full scale of the Esterline-Angus. If $H_{1/3}$ is assumed to be equal to approximately $5a_o$, the characteristic wave height $H_{1/3}$ is $(5CR_h)/10$ where C is the full scale calibration of the Esterline-Angus recorder and R_h is the reading of the wave height meter assuming full scale to be unity.

The first meter receives a pulse of current each time the wave record passes through the zero axis in a positive direction. By proper adjustment of the circuit the meter reads one milliampere for an input signal with a frequency of 0.1 cyc/sec. The zero crossi frequency then equals 0.1 R_f, where R_f is the reading of the frequenc meter.

Circuit operation - A combination of a d-c bias voltage and a 9 leading a-c bias voltage connected in a cathode circuit of the thyratr tube (Figure 4) prevents the tube from conducting unless the grid voltage exceeds a given value. The exact value of the grid voltage necessary to cause conduction is determined by the setting of potentiometer P_1. This voltage will be approximately 12 volts with respe to the negative side of the d-c bias supply, since the "wave-recorder potentiometer" is in its center position with no ocean-wave signal applied. Resistor R_1 and Condenser C_1 in the grid of the thyratron decouple the thyratron grid from the recording circuit of the wave re corder and prevent high-frequency noise from firing the tube.

Resistor R_2 and Condenser C_2 shift the filament voltage 90° leading with respect to the plate voltage. With this voltage connecte in the bias circuit, the tube will conduct the entire positive half of th plate voltage swing as soon as grid voltage exceeds the critical value With only the d-c bias connected in the circuit, the tube would conduc only one-half of the positive plate voltage.

Relay operation - The relay circuit operation is as follows: Assume the thyratron grid voltage is below the critical value, the tub is not conducting, and the relays are de-energized. If a signal is supplied from the wave recorder which causes the grid voltage to increase above the critical value, the tube will conduct and relay A wil be energized, closing contacts A_1 and A_2. Contact A_1 energizes Relay B, which closes contacts B_2 and B_3 and opens contact B_1. Contact B_1 disconnects relay A, but relay A will remain energized b cause of the charge in condenser C_3. The contacts of relay A remai closed for 1.4 seconds each time the circuit is actuated. Contact A_2 causes a unit pulse current of 7.1 ma for 1.4 seconds to flow through the "wave period" meter each time the grid signal exceeds the critic value. Contact B_2 prevents relay B from de-energizing when contac A_1 opens. Relay B, therefore, will remain energized as long as the

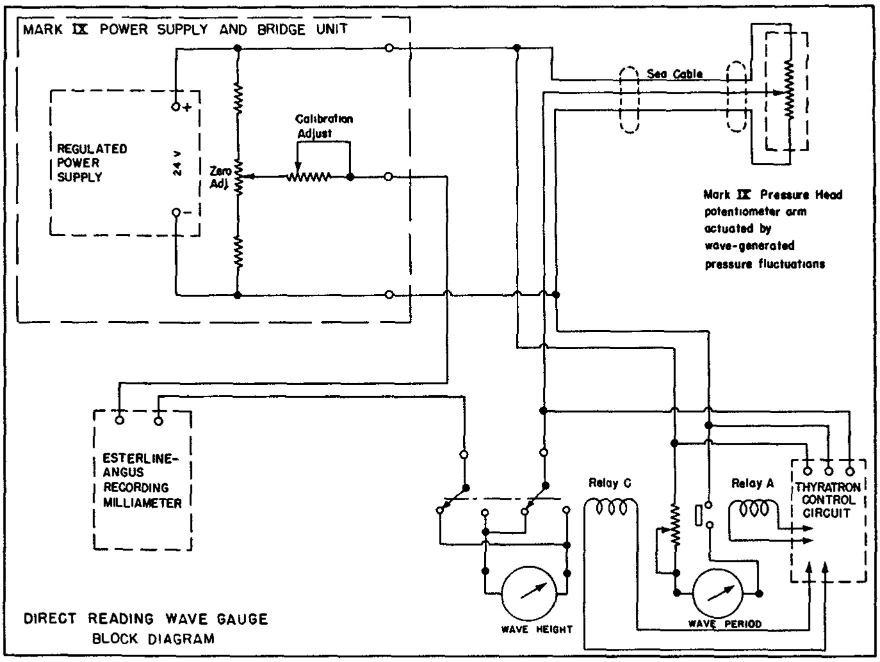

Fig. 3. Functional diagram of the relay operated analyzer for the Mark IX pressure gage.

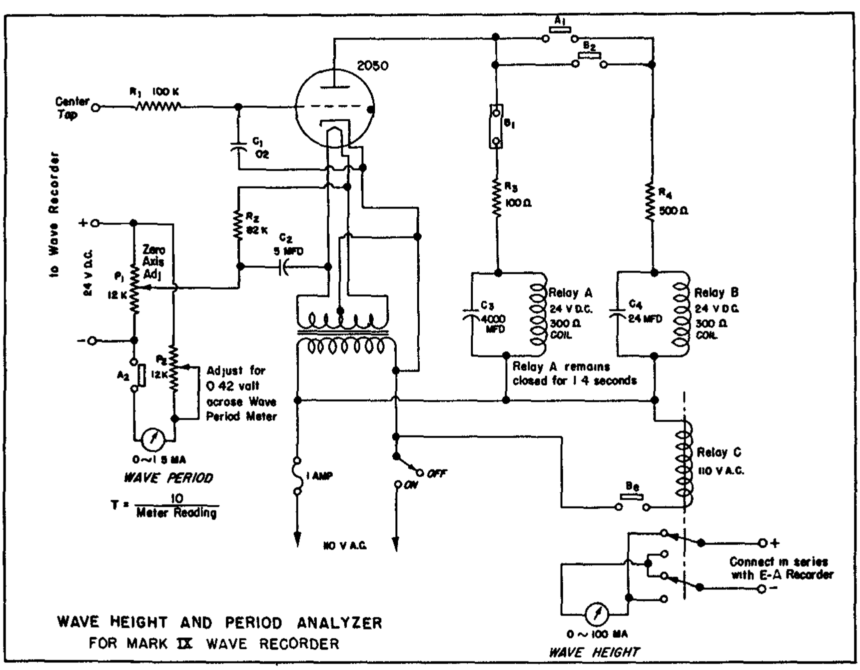

Fig. 4. Schematic diagram of the relay operated analyzer. The wave height and period meters are submerged in 20,000 centistoke oil for averaging of signals.

TABLE I*. TEST RESULTS

DATE 1954	$1/f_{z.c.}$	$T_{1/3}$	a_o	$H_{1/3}$	DATE 1954	$1/f_{z.c.}$	$T_{1/3}$	a_o	$H_{1/3}$
1/26(1)	-	10.9	-	18	2/5	-	12.5	-	11
	11.1	12.0	3.2	17		12.5	12.3	3.5	17
	11.5	12.9	3.0	15		14.7	11.9	3.8	20
	-	13.3	-	16		-	11.9	-	20
1/27	-	13.7	-	17	2/6(2)	-	11.9	-	17
	14.1	13.9	2.6	15		12.5	12.8	3.6	19
	13.2	14.6	2.9	16		14.8	11.5	2.4	13
	-	13.8	-	17		-	11.9	-	16
1/28(2)	-	13.4	-	16	2/7(2)	-	13.0	-	13
	14.1	13.9	2.7	16		14.7	13.2	3.2	17
	15.4	13.2	2.8	14		13.5	12.9	2.9	17
	-	14.6	-	15		-	12.6	-	16
1/29(2)	-	14.9	-	17	2/8	-	12.9	-	14
	15.4	15.2	2.7	15		13.3	13.5	2.7	15
	13.2	14.1	3.2	16		13.0	13.3	3.1	16
	-	15.1	-	16		-	13.6	-	16
1/30	-	16.3	-	20	2/9	-	13.2	-	16
	14.7	17.3	4.0	23		14.3	13.9	3.2	17
	12.5	16.9	4.5	19		14.5	14.9	3.6	19
	-	17.9	-	18		-	14.2	-	17
1/31	-	16.1	-	24	2/10	-	14.5	-	20
	12.7	16.2	4.2	22		12.5	13.9	3.0	18
	12.0	16.1	3.4	20		11.7	13.5	3.6	19
	-	14.6	-	18		-	13.2	-	18
2/1	-	-	-	15	2/11(1)	-	14.5	-	22
	14.3	15.7	3.0	15		12.5	13.9	6.0	30
	14.9	14.7	2.6	14		11.7	13.5	6.3	30
	-	13.7	-	16		-	13.2	-	26
2/2	-	15.1	-	17	2/12(1)	-	13.9	-	28
	13.3	14.4	4.0	22		12.7	13.6	4.8	23
	14.1	13.6	3.4	17		12.5	14.0	5.1	25
	-	14.5	-	16		-	16.0	-	24
2/3	-	14.3	-	16	2/13(1)	-	16.0	-	22
	14.1	13.3	3.2	18		12.3	15.2	11.6	58
	12.8	13.5	3.0	16		7.7	12.0	10.0	43
	-	12.7	-	15		-	14.0	-	52
2/4	-	13.2	-	13	2/14(1)	-	13.3	-	44
	13.3	12.1	3.0	16		10.7	13.9	9.2	48
	12.8	13.3	2.4	14					
	-	13.1	-	16					

*Four measurements were made at 0415, 1015, 1615, and 2215 of each day.

$f_{z.c.}$ and a_o were read from the relay-operated analyzer. $H_{1/3}$ and $T_{1/3}$ were measured from 20-minute wave records by the personnel of the Surf and Weather Station, Camp Pendleton, California.

[1]Local storm present which generated strong secondary waves.

[2]Sections of the record were nearly flat making zero axis crossings difficult to measure.

grid signal exceeds the critical value. Contact B_3 causes relay C to operate as a rectifier by reversing the polarity of the "wave height" meter. The critical grid voltage is adjusted so that the meter polarity is reversed as the current in the Esterline-Angus recorder passes through zero current.

Condenser C_4 prevents relay B from chattering due to the half-wave rectified current flowing in the 2050 thyratron tube. Resistor R_4 limits the current so that 24 volts appear across Relay B with the thyratron conducting. Resistor R_3 limits the current in the thyratron to its maximum allowable value, while Relay A is being energized.

TESTS AND RESULTS

The relay operated analyzer was tested for 23 consecutive days at the Surf and Weather Station, Camp Pendleton, California. The readings of wave height and period obtained from the oil immersed meters were compared to the values determined by the standard manual analysis. The test indicated that $H_{1/3}$ is approximately equal to $5a_o$ (Figure 5).

The frequency meter indicated a wave period that was generally less than the significant wave period particularly during local storms as indicated in Table I. At the outset of the local storm high frequencies could be seen in the wave record which would considerably affect a reading on the zero crossing meter; the manual analysis of the record would fail, however, to indicate this change in sea state until the amplitude of the high frequency waves was appreciable in comparison to the underlying sea swell.

Figure 6 indicates results obtained by comparing the value of the zero-axis crossing frequency, measured by the electronic analyzer, to the value computed from the spectrum. The spectra were obtained from data consisting of 3000 values at one-second intervals read from a 50-minute wave pressure record (gage depth 14 feet), (Munk, 1957). The value at A is the mean frequency of the record, while the value at B is the theoretical zero-axis crossing frequency. The arrow labeled C indicates the range of values obtained from the analyzer during the 50-minute period of the spectrum data. Good agreement was found between the computed and measured zero-axis crossing frequency.

The continuous recording of wave height (or energy) and frequency is perhaps more important than the determination of the average values of $H_{1/3}$ and $f_{z.c.}$ discussed above. Large variations in $H_{1/3}$ and $f_{z.c.}$ can occur in relatively short periods of time as shown in Figures 7 and 8. This record indicates that 15-minute records taken a half hour apart could differ by 30 or 40 per cent.

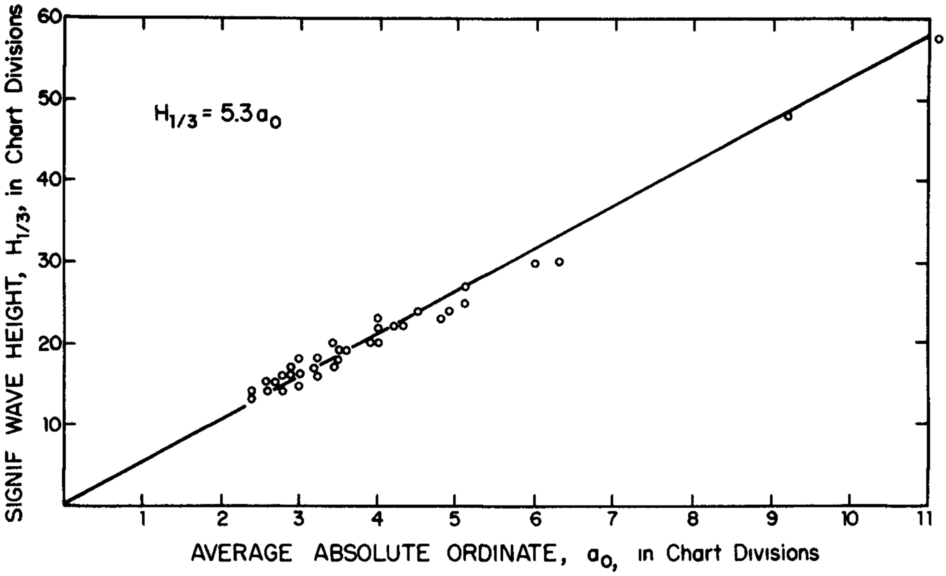

Fig. 5. Experimental data comparing the average absolute ordinate, a_o, to the characteristic wave height $H_{1/3}$.

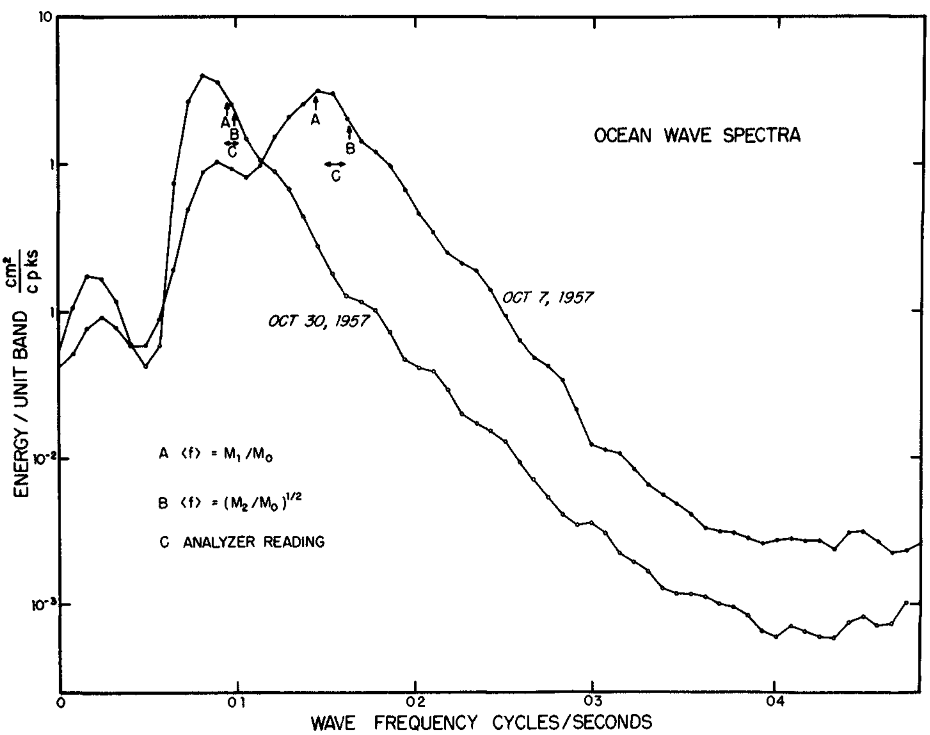

Fig. 6. Experimental data comparing wave spectra, calculated mean frequency (A) and zero crossing frequency (B), to measured zero crossing frequency (C).

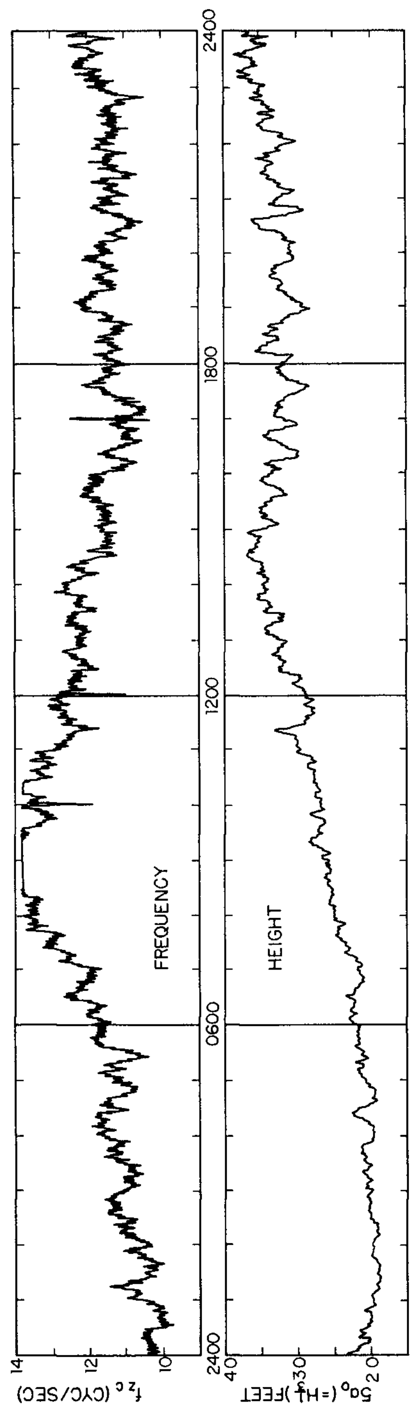

Fig. 7. Analyzer output Nov. 1, 1957. The input signal was obtained from a pressure head 12 feet below MLLW at Scripps Pier. The waves were generated by 15-knot winds that started late October 31 and lasted through November 1.

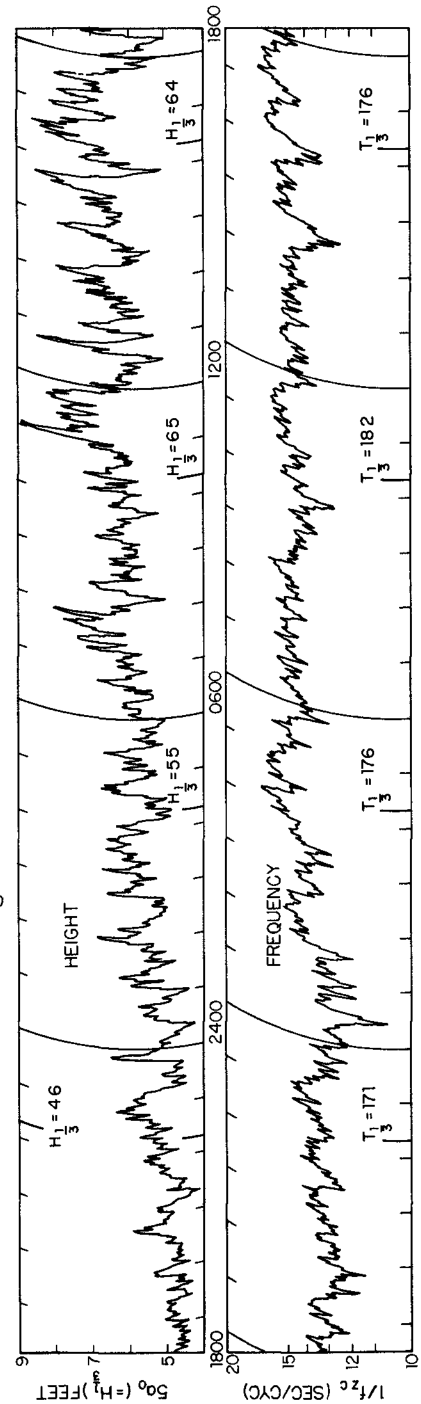

Fig. 8. Analyzer output July 4 - 5, 1956. Local winds were less than 7 knots. Waves were typical "heavy southern swell." Values of $H_{1/3}$ and $T_{1/3}$ were obtained by manual analysis of twenty-minute records at the time indicated.

The change in sea state indicated in Figure 7 was caused by a local 15-knot wind which started late October 31 and lasted through November 1. The wave record was obtained from a pressure gage 12 feet below MLLW in 20 feet of water at Scripps Pier. The zero crossing frequency increased sharply, reached a peak and then decreased before the wave energy increased appreciably.

The record shown in Figure 8 was obtained from a pressure gage 30 feet below MLLW in 35 feet of water at Camp Pendleton, California. The local winds were less than 7 knots. The waves were described as "heavy southern swell" being very low frequency and from the south. Characteristic of southern swell recordings from the analyzer, the zero crossing frequency is low and steady with large variations in wave height. Some correlation is apparent between the frequency and height recordings, with low frequency occurring at times of high energy. The values of $H_{1/3}$ and $T_{1/3}$, indicated in Figure 8, were obtained by manual analysis of twenty-minute records at the time indicated.

The fluctuations of the records pose some interesting questions. Can the fluctuations be predicted for various sea states or for an aging storm? What are the expected correlations between the height and frequency records? Can the spectrum of the analyzer output be predicted from the raw wave record?

ACKNOWLEDGMENT

The authors wish to thank Walter Munk for his many helpful suggestions, both during the study and in the preparation of the manuscript. The assistance given by L. G. Boyd, CWO Marine Corp, and the staff at the Surf and Weather Station, Camp Pendleton, in obtaining wave data was greatly appreciated.

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CHAPTER 14

IMPROVEMENTS IN THE ELECTRIC STEP GAUGE FOR MEASURING WAVE HEIGHTS

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SUMMARY

Continuous systems are compared with a step system. The influences of parasitic series and coupling resistances are examined.
Methods of decreasing these parasitic resistances are considered. Relays are being used to eliminate the errors of these effects. When no relays are used a decrease in the effect of the parasitic resistances is obtained by switching in condensers or inductors instead of resistors in an LC system as part of an oscillator circuit. Some considerations on design are given.

1.1. In fulfilment of the Dutch Rijkswaterstaat our Laboratory is engaged in the development of an electrical system for measuring wave heights.

1.2. An electrical system was preferred to a mechanical one because the latter was expected to have a much shorter life.

CONTINUOUS SYSTEM.

2.1. Some types of continuous electrical wave gauges are: a resistance type consisting of a double metal wire placed vertically in the sea, the resistance between the wires producing a straight-line decrease when plotted against the water height. A capacitive type is obtained by placing an insulated wire vertically in the sea, the capacity increasing in a straight line when plotted against water height. Another electric gauge is the inductive type in the form of a long, thin coil (delay line without screening) placed vertically in the sea. The result is changing impedance because of the "secondary" charge produced by the conducting seawater.

ERRORS DUE TO PARASITIC EFFECTS.

2.2. In the resistance type, corrosion or fouling of the wires will result in an unknown increase in the shorting resistance. The same happens if the salinity of the water changes (estuaries).
The same sort of calibration change appears in the capacitive type in which any fouling affecting the insulating material will change its capacity. Because the inductive type generates current in a "thick" water layer such

fouling is of less consequence, but changing salinity will result in large errors which are difficult to compensate for if the salinity changes at different depths.
A second type of error applying to all these continuous systems is the effect of the film of water remaining after the passage of a wave.

ADVANTAGES OF THE STEP SYSTEM.

Because of these errors a step system was chosen instead. For a continuous system the total error is the sum of all the small errors caused by the film of water remaining and changing surface conditions. A step system, however, can be furnished with a threshold high enough to suppress the effects of parasitic coupling between "wet" electrodes above the surface of the sea and low enough to take the highest electrode series resistance into account, supposing that the first effect is smaller than the second. In that case though the total error may be much larger than the threshold value no error will appear.

PARASITIC EFFECTS IN THE STEP SYSTEM.

A series resistance does arise because of the limited conductivity of seawater. For a spherical form of electrode with a diameter of 1 cm and a conductivity of 4.4 $ohm^{-1}m^{-1}$ the resistance will be 3.6 ohm; in estuaries, where the system should also work in relatively fresh water (4.4 x 10^{-2} ohm^{-1} m^{-1}) this will increase to some 400 ohms.
The series resistance will be further increased as a result of the resistance near the electrode surface caused by fouling and electrochemical action. Both effects decrease if the frequency of the electric current applied is raised. Owing to electrolytical action corrosion of the electrodes will take place except if spectrographically pure carbon is used.

Concerning the parasitic coupling between electrodes caused by the remaining film of water we have found by calculation that the water film resistance between the electrodes and earth is of much greater consequence than the resistance between successive electrodes.
Any earthed metal construction parts giving rise to a water film coupling with the electrodes should therefore be carefully avoided.

The resistance between successive electrodes will be increased if the electrodes are mounted on thin wires. In this way a water film of small cross-section is obtained, giving rise to a relatively high coupling resistance. We

have achieved this by mounting the electrodes on hard drawn copper wires insulated with black polythene. The wires are bunched together with nylon cord. Each wire with its terminal electrode is bent at right angles to the bunch. This assembly is attached to a support in the sea. This idea was first used in Indonesia by engineers of the B.P.M.
The cable surface should be smooth, so as to ensure minimum water adherence.

RESISTANCE STEP GAUGE.

Our first experience with step gauges was gained with the well-known parallel resistance type. In this gauge resistances are connected in parallel by the seawater-electrode "switches". The resistance then decreases as a straight line when plotted against water height. When an electrical potential is applied to this resistance a current will flow that increases in a straight line when plotted against water height.
If in such a system the resistances have a value R and the parasitic series resistance a value R/100 the indicated height will be 1% too low or one step if a total of 100 electrodes is used.
If for this system the electrodes above water level are wet and coupled by means of a water film resulting in a resistance between two successive electrodes of R/2 the system will indicate one step too high.

If this is the maximum allowable error the given example shows that for the resistance parallel type of gauge a ratio of 50 between coupling and series resistance should not be exceeded.
In our estuaries it was feared that this ratio would be much smaller.

THE RELAY STEP GAUGE.

For the reasons given above a threshold should be introduced. To achieve this the sea-electrode switches should operate through relays. By setting these relays sharply to operate at the maximum expected series resistance an ideal on-off system was produced by a group of Dutch Rijkswaterstaat workers.
Relays with as narrow an on-off current ratio as possible should be selected for this purpose. The use of relays however introduces a partly mechanical element resulting in a short life because of the high switching rate.
The relays should therefore be easily accessible, making an expensive multicore cable necessary.

THE CONDENSER STEP GAUGE.

To reduce the error from the parasitic resistances we have in our design made use of the 90 degrees phase

shift between the voltages across either a condenser or an inductance on the one hand and a series resistance on the other. Because of this phase shift the impedance of the condenser (inductance) will increase only by the square of the ratio between the impedance and the parasitic series resistance instead of in a straight line when resistances are used.
If a series resistance of say ten percent is introduced the impedance will only increase with 1%.

Condensers are preferable because of the ease which with they can be combined to make any value. They are also very easily obtainable.

Together with this switched condenser system where the capacity changes with the water height an electrical circuit should be found that translates this capacity change into a quantity that is easily transmitted by cable or by ratio transmitter if a cable cannot be used.
We have decided on a frequency modulated system. In this system the changing capacity is converted into a changing frequency. At the receiving point this changing frequency can easily be converted into a changing voltage. The advantage of this is that this conversion is independent of the amplitude of the voltage the frequency of which is to be measured. This is of great importance because unpredictable attenuation of the transmitted voltage easily occurs.

The translation of the capacity change into a frequency change is done by including this capacity in the tuned part of an oscillator.
For an RC oscillator two gauges are necessary; for an LC oscillator only a single gauge is needed.
The frequency of an LC oscillator is expressed as:

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

The capacity in the LC circuit (fig.1) is for example made up of 100 condensers one pole of each being connected to a common line, the other poles being connected to the electrodes. Condenser values are chosen making a capacity of:

$$C_h = \frac{C_{min}}{(1-\frac{2}{3}\frac{h}{H})^2}$$

in which h = waterlevel and H = highest electrode level

The frequency decreases as a straight line when plotted against water level. The ratio of the condenser C_{min} and the switched condensers establishes the frequency range, for which 500 c/s to 1500 c/s has been decided on.

For this system (with 100 electrodes), if all the electrodes are submerged and a parasitic series resistance with a value of $\frac{7}{\omega_{max}C_{min}}$ appears for each electrode calculated is found to be equivalent to one step too low.
If for the same system with all the electrodes out of the water, each electrode is coupled to its neighbour by resistance of $\frac{35}{\omega_{max}C_{min}}$ the resulting error is equivalent to one step too high.
This example, in which the series resistance has a value of 1/5 th of the parasitic coupling resistance, shows that a situation has been created which only relays can cope with. If we compare this with the system in which a chain of resistances is employed and in which an error of one step were to appear for a ratio of 50 between the parasitic coupling resistance and the series resistance, we see a tenfold improvement.

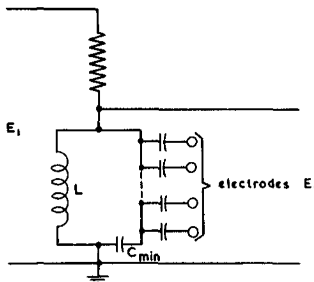

Fig. 1

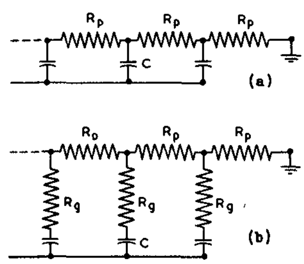

Fig. 2

THE EFFECT OF WIRE MOUNTED ELECTRODES.

In the above mentioned case in which we presume the parasitic coupling resistance of $R_p = \frac{35}{\omega_{max}C_{min}}$ only the effect of coupling along the cable between two adjacent electrodes is considered (see fig. 2a). By mounting the electrodes on projecting wires each electrode is coupled to the cable through an individual resistance (Rg) caused by the waterfilm on this wire (see fig. 2b).

In this case a lower value of Rp might appear for the same error.
If for example Rg = 10 Rp the value of Rp for an error of one step is found to be:

$$Rp = \frac{8}{\omega_{max}C_{min}}$$

The same thing applies to the parallel resistance gauge mentioned earlier, in which an error of one step arises for Rp = R/2 in the situation shown in fig. 2a.
With the electrodes on projecting wires and Rg = 10 Rp we find:

$$Rp \geqslant \frac{R}{4.3}$$

if the error is to be kept down to one step or less.

The use of frequencies between 500 and 1500 c/s makes the series resistance caused by fouling and polarisation of the electrodes negligible compared to the few hundred ohms we anticipate in estuaries.

A GAUGE WITH RADIO TRANSMISSION AS ALREADY CONSTRUCTED.

To predict wave heights near the construction pit for the sluices to be built in the mouth of the Haringvliet, waves are being measured well to seaward.
As a cable was considered unreliable wireless transmission was decided upon.

Following the design considerations given above, a system was built and is now working consisting of a double condenser step gauge together with an RC oscillator. Though we would now prefer an LC system enabling a single electrode system to be used, this idea only occurred to us after the construction of this gauge was completed. The gauges operate over a range of 7.5 meter and both gauges have 50 electrodes. The relative height of the two gauges is such that the electrodes of the one are just between the electrodes of the other, hence the frequency is changed every 7.5 cm.

Because electric coupling between the two electrode systems could not be tolerated the two electrode systems are some distance away from each other.

By designing the oscillator carefully the frequency to water height relation is a straight line within 1%. The signal is transmitted by a small crystal-controlled 170 Mc/s transmitter having an output of 0.2 Watts. The signal is received 5 km away on a Yagi 10 element aerial. The modulation signal is fed to a Hewlett Packard frequency meter model 500 B. A feature of this meter is its scale spread of three or ten times which facilitates reading the waves. The frequency meter output can also be recorded.
The modulation frequency range of 500 c/s to 1500 c/s makes transmission over telephone lines possible.

The power consumption of the transmitter is 4.6 Watts. The high tension voltage is obtained from a specially designed transistor oscillator.
The five Leclanché elements of 1.3 Volt and 2000 Ah require renewal once every two months.

The influence of the parasitic series and coupling resistances in this gauge depends greatly on the difference between these resistances in both electrode systems and is therefore difficult to estimate.
Supposing a difference of 20% we find for errors of one step or less:

$R_p \geqslant \frac{p}{15} r$ where p is the number of parasitically coupled electrodes.

CHAPTER 15

LABORATORY FACILITIES FOR STUDYING WATER GRAVITY WAVE PHENOMENA

by

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Department of Engineering, University of California
Berkeley, California

ABSTRACT

Details are given of the design of laboratory facilities at the University of California for studying water gravity wave phenomena.

INTRODUCTION

During the past two decades a series of facilities have been designed and built by the College of Engineering, University of California, Berkeley, California, for the purpose of studying the generation and characteristics of water gravity waves and their effects on coastal sediments, marine structures, amphibious craft, and ships (including ships' moorings). At the present time the following facilities are in operation:

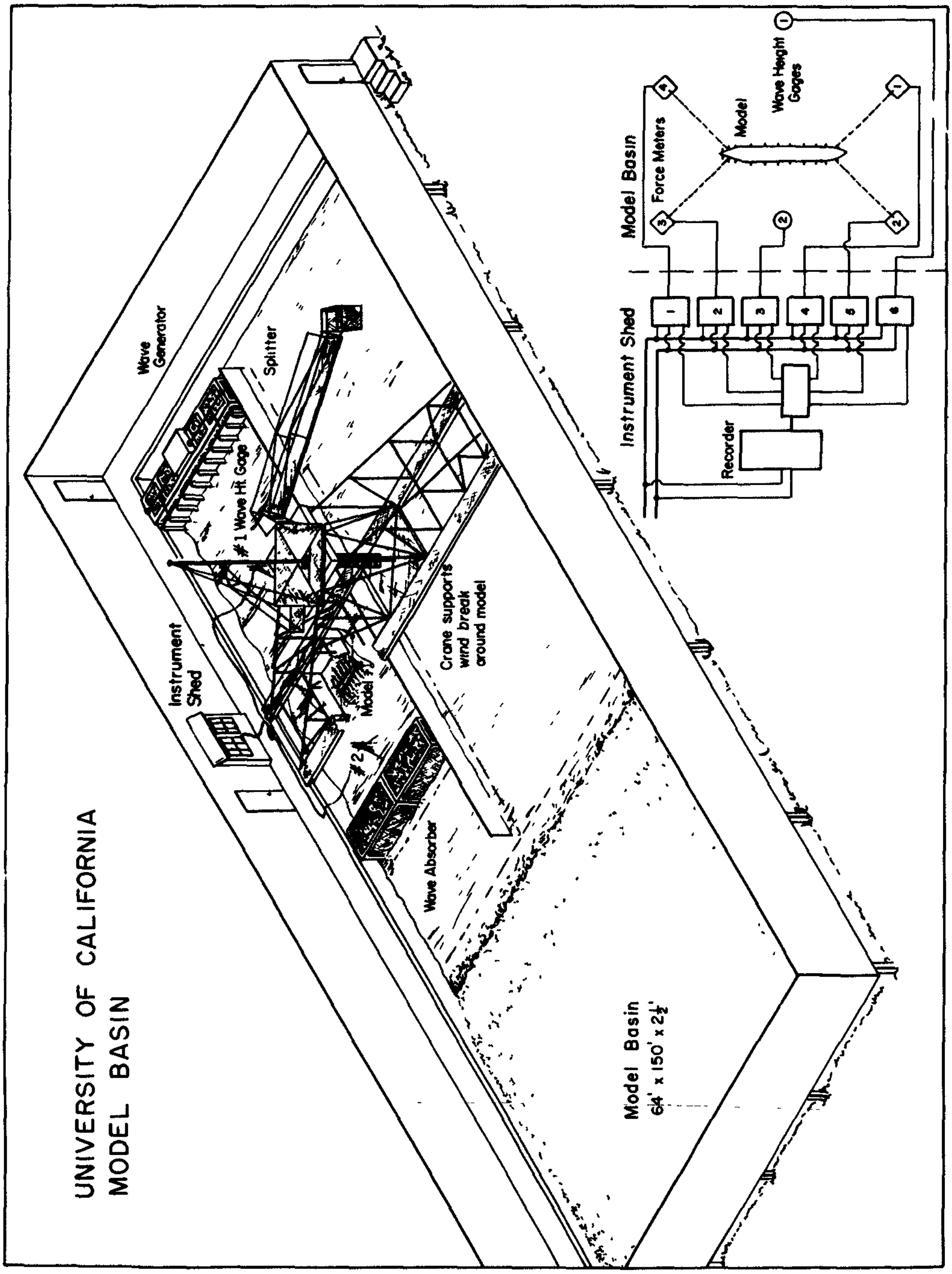
1. Model Basin, 64 ft. by 150 ft. by 2.5 ft. deep.
2. Wave-Towing Tank, 8 ft. by 200 ft. by 6 ft. deep.
3. Wave Channel, 1 ft. by 60 ft. by 3 ft. deep.
4. Wind-Wave Tunnel, 1 ft. by 60 ft. by 1.25 ft. deep.
5. Wave-Sediment Basin, 6 ft. by 12 ft. by 1 ft. deep.
6. Ripple Tank, 4 ft. by 20 ft. by 0.5 ft. deep.

The tanks and equipment have been designed over a period of years to study particular engineering problems. Experience gained in operating each facility led to improved designs for each succeeding one. Because of this, details will be given herein of only the latest designs while a minimum of information will be presented on the earlier equipment.

Several of the tanks are located at the Engineering Field Station of the University of California in Richmond, California (Bermel, 1955) while the remaining tanks are located on the Berkeley campus.

MODEL BASIN

The new model basin, located at the Engineering Field Station, is used to study the effect of groins on the deposition of sand on a beach, the refraction and diffraction of waves with respect to harbor and coastal structure design, the motion and mooring forces of a ship moored at an angle to waves, and other problems which require a large area. The basin, 64 ft. by 150 ft. by 2.5 ft. deep, is equipped with a movable wave generator, a movable photographic crane, and sand beaches and stainless steel filings for absorbing the wave energy. A corrugated aluminum fence, reaching $9\frac{1}{2}$ ft. above the top of the basin, surrounds the basin to pro-


UNIVERSITY OF CALIFORNIA
MODEL BASIN
Wave Generator
Splitter
#1 Wave Ht Gage
Instrument Shed
Crane supports wind break around model
Model
#2
Wave Absorber
Model Basin 64' x 150' x 2½'
Model Basin
Force Meters
Model
Wave Height Gages
Instrument Shed
Recorder

tect it from the wind. A schematic drawing of the basin is shown in Figure 1 which includes an instrument wiring diagram for a sample experimental set-up. Details of the basin construction, designed by K. J. Bermel and N. A. Jensen, are shown in Figure 2.

The flap-type wave generator (See Suquet, 1951, for definitions of wave generator types) used in the original model basin, located on the Berkeley campus, was used in the new model basin until a piston-type wave generator, designed by C. M. Snyder, was constructed. This piston-type wave generator is basically the same type as the one used in the wave-towing tank except for required size modifications and the addition of a remote control system for operating the amplitude mechanism. Both the wave amplitude and wave period can be varied while the wave generator is in operation.

The gear box (Figures 3 and 9) is the same in principle as the gear box in the wave generator used in the wave-towing tank, except that instead of using a hand wheel to operate the input shaft of the amplitude control mechanism a motor was substituted so that it can be controlled by power. On the input shaft there is an electric brake to keep the amplitude from drifting when the motor is not running. A double-ended miter gear box is connected to the amplitude control shaft with a sylsyn generator on one side and a limit switch on the other (See Figure 4 for the wiring diagram). A forward and reverse push-buttom station and sylsyn repeater which drives a counter were installed in the operation building.

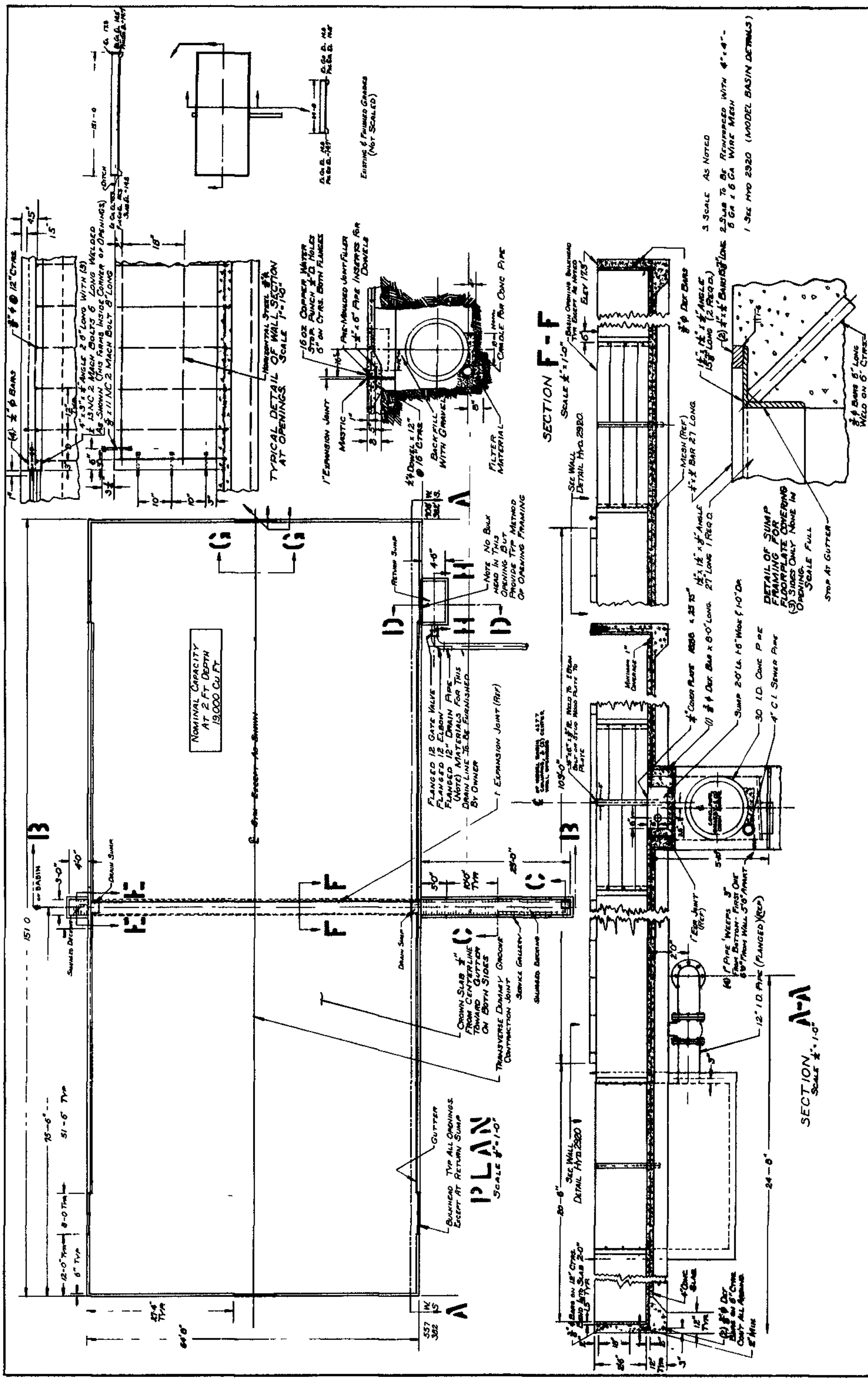
The main motor speed control is obtained by a modified Ward-Leonard system. This has a 5 hp shunt motor which has its field excited by an electronic power supply. The motor armature current is supplied by an electronic exciter motor generator set. This type of drive has an advantage in that it is quite stable and has a large speed range. Wave periods from 3/8 second to at least 10 seconds have been obtained by this drive.

The wave generator was built in three sections. Each section was made 21' 2-1/4" long. The sections (Figure 5) consist of a box-like steel girder frame, with a piston that rides in a channel track. The box frames can be connected to each other by a set of pin connectors. The pistons are linked through a rocker arm shaft. This shaft is in turn linked to the gear box on the center section crank by a connecting rod. This design allows the generator to be made up into three different lengths 21' 2-1/4", 42' 5-3/4", and 63' 7". The center section is the power section.

The flap-type wave generator has been retained so that it is possible to generate "cross seas" using the two generators simultaneously.

WAVE-TOWING TANK

As a part of the program to move large engineering facilities from the Berkeley campus to the Engineering Field Station, it was necessary to move the ship model towing tank. Because of the type and condition of the tank it was decided to design a new tank which would include a wave generator so that experiments involving water gravity waves


PLAN
SECTION A-A
SECTION F-F
TYPICAL DETAIL OF WALL SECTION AT OPENINGS
NOMINAL CAPACITY AT 2 FT DEPTH 19,000 CU FT

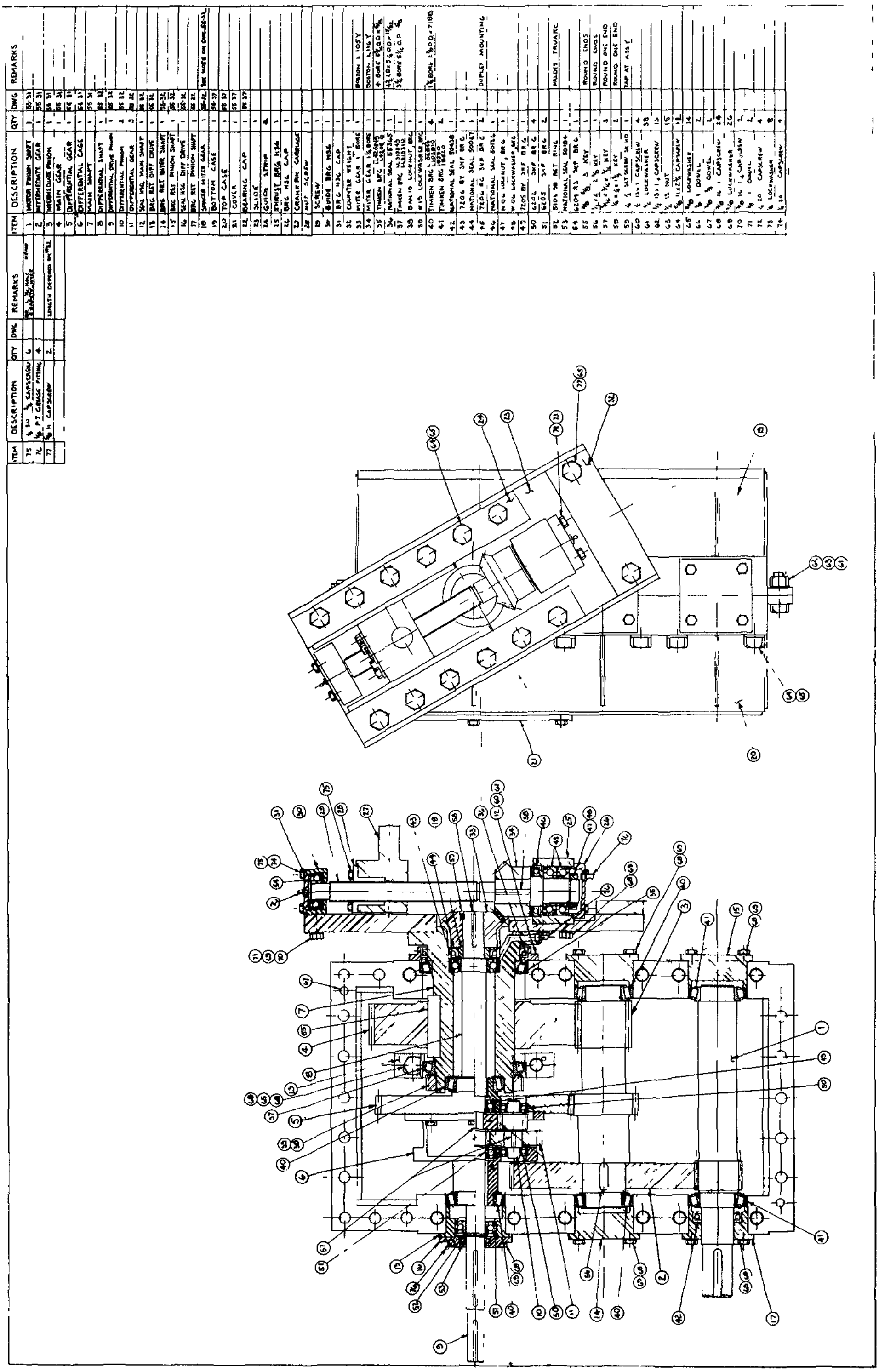


Fig. 3. Gear box assembly, model basin wave generator.

could be performed on a larger scale than was possible with the other wav tanks at the University. The overall dimensions of the tank were determined by economic and engineering considerations and were fixed at 8 ft. by 200 ft. by 6 ft. deep. A schematic drawing of the tank is shown in Figure 6, which includes an instrument wiring diagram for a sample experimental set-up. The tank is housed in an all-metal industrial building. The tank is equipped with piston-type wave generator, a stainless steel filing beach for absorbing the wave energy, two sections of observation windows along one side of the tank, a towing carriage, an overhead travelling crane system, a camera pit, and a bank of photographic lights. The general arrangements can be seen in Figure 7. The tank was designed by K. J. Bermel and N. A. Jensen and the wave generator was designed principally by C. M. Snyder.

WAVE GENERATOR SPECIFICATIONS

The dimensions of waves needed by naval architects were small; hence, the sizes of the waves to be generated were fixed by considerations of other problems. Laboratory studies of the behavior of waves in shoaling water and their effect on the motion of sand and the study of wave forces on marine structures pointed up the limitations of the wave generating facilities then in existence. Working from the given dimensions of the tank (8 ft. x 200 ft. x 6 ft.), it was decided that the maximum wave should have a period of five seconds and a height of two feet. A review of the studies by the Beach Erosion Board (1949), together with past experience at the University, led to the decision to use a piston-type wave generator. Using the portion of the work of the Beach Erosion Board (1949) for a piston-type generator, it was estimated that the wave generator stroke should be approximately one and a half feet.(total motion of three feet). The minimum wave period for design purposes was about one-half second with the wave height being the value associated with the maximum possible wave steepness, which fixed the wave height at less than one-tenth of a foot (Suquet and Wallet, 1953).

The power of the generated waves was determined from wave theory (O'Brien, 1942), where

$$\text{Wave power} = \frac{\text{Group velocity} \times \text{Wave energy}}{\text{Wave length}}$$

The electro-mechanical power necessary to generate waves of a given power was estimated from the experimental work of the Beach Erosion Board where the relationship between the actual power-time history of the generator and the wave power was determined experimentally for a model piston-type wave generator. The Beach Erosion Board studies (1949) showed that the average power needed was almost that of the wave horsepower but that the peak power needed for certain waves was approximately three times the wave power. Because of the fact that a motor can be overloaded for a small portion of the cycle the motor chosen was only a 15 horsepower, 230 V.D.C., 350-1750 RPM shunt motor.

Motor speed control is obtained at present by adjusting a field reostat, giving wave periods from 3/4 to 3-3/4 seconds. If

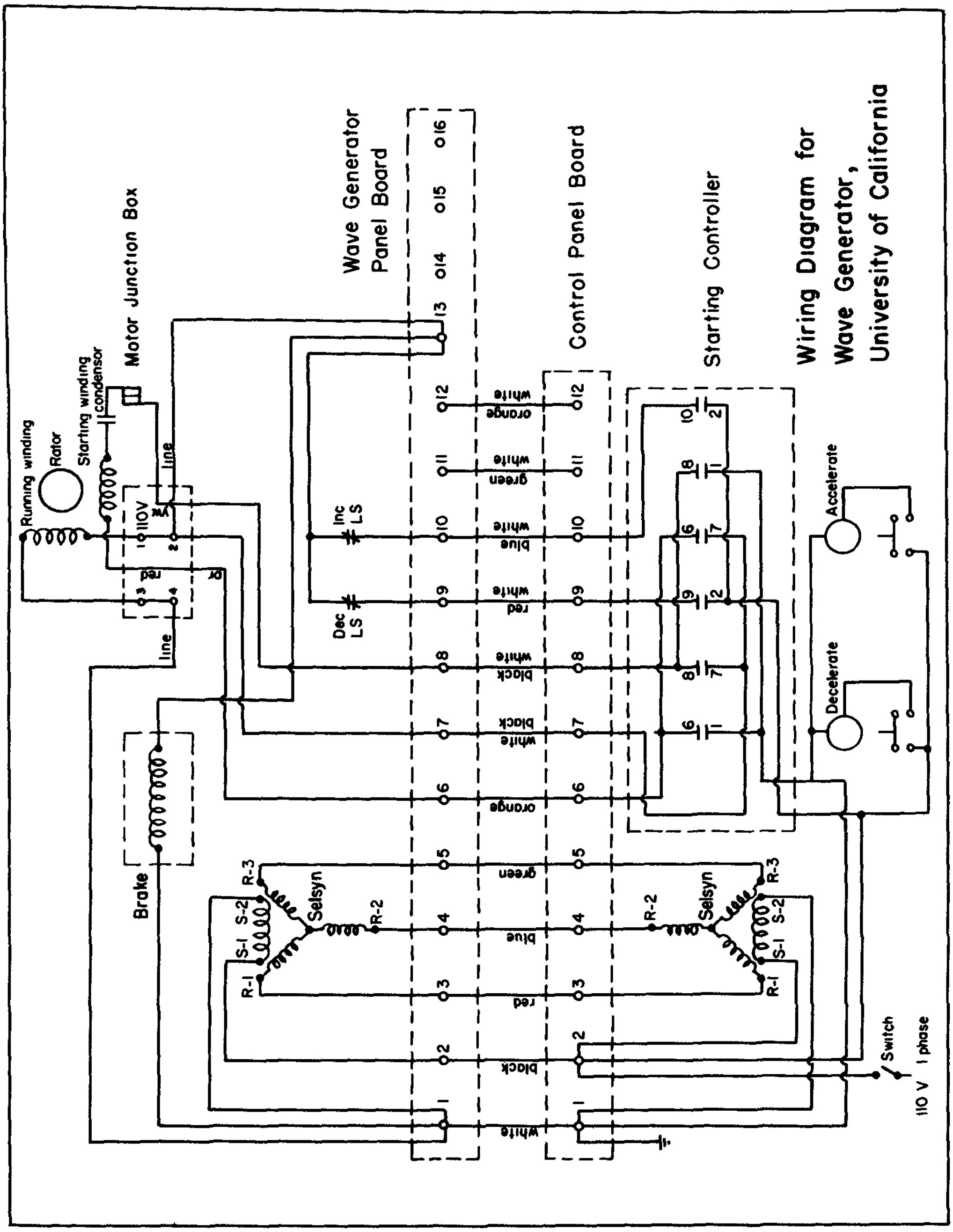
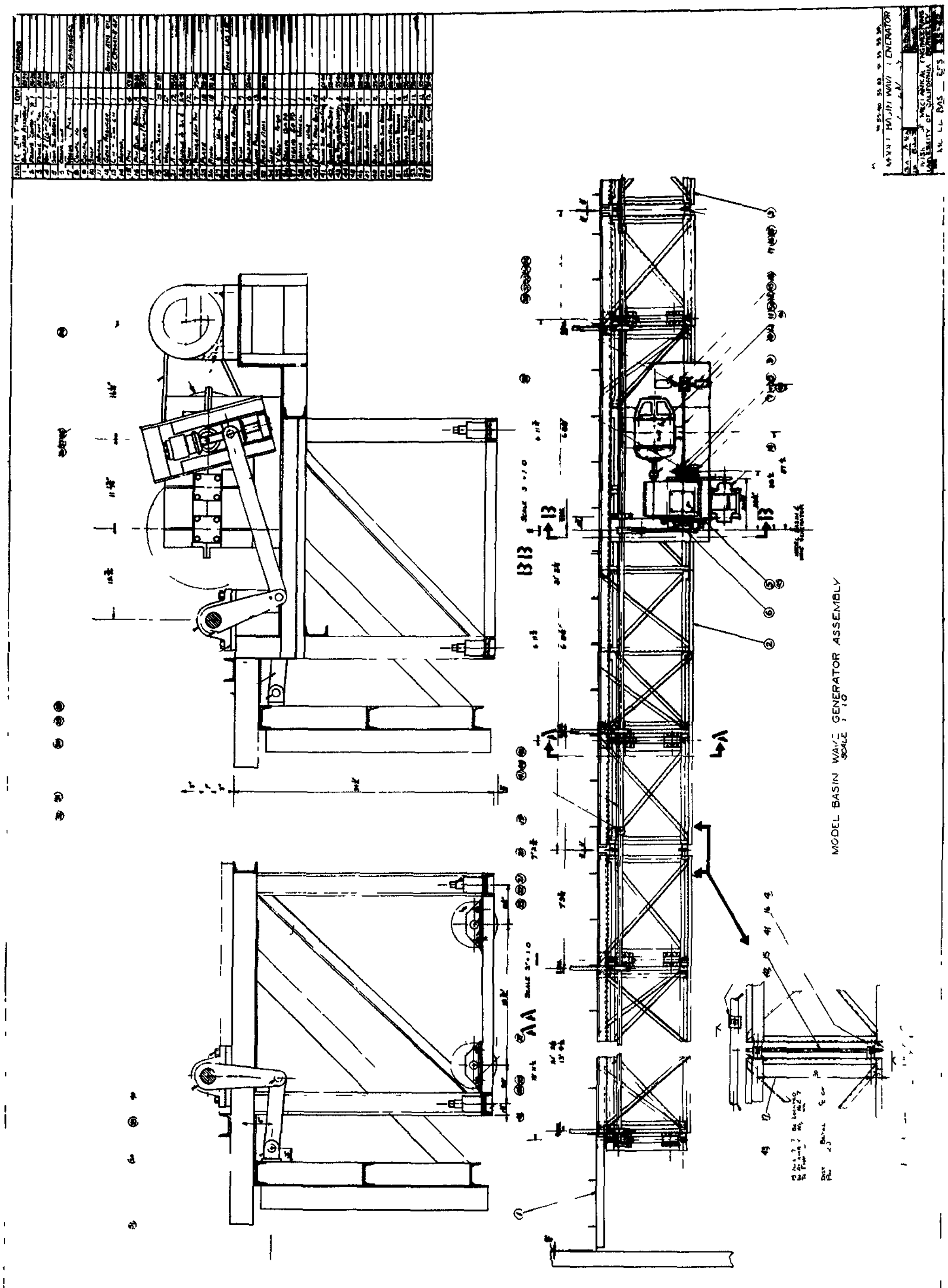

Wiring Diagram for
Wave Generator,
University of California
Wave Generator
Panel Board
Control Panel Board
Starting Controller
Motor Junction Box
Running winding
Rotor
Starting winding
condensor
line
110V
red
Inc
LS
Dec
LS
Brake
Selsyn
R-1
R-2
R-3
S-1
S-2
Accelerate
Decelerate
Switch
110 V
1 phase
white
orange
green
blue
red
black

Fig. 4


MODEL BASIN WAVE GENERATOR ASSEMBLY
HYD-7645

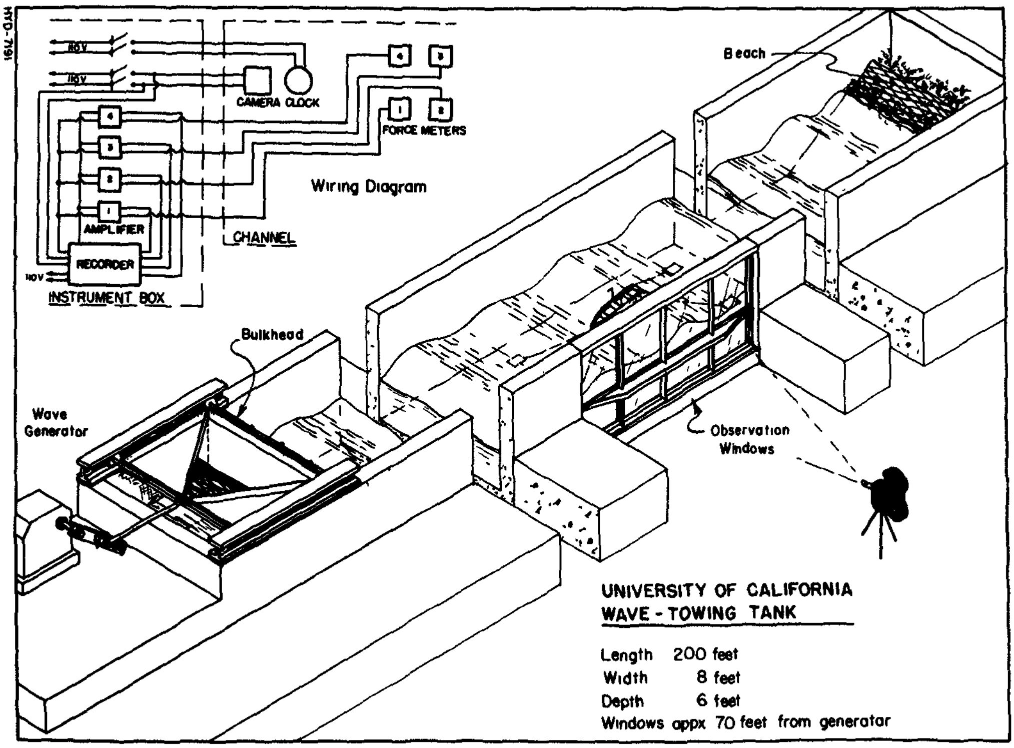


Fig. 6

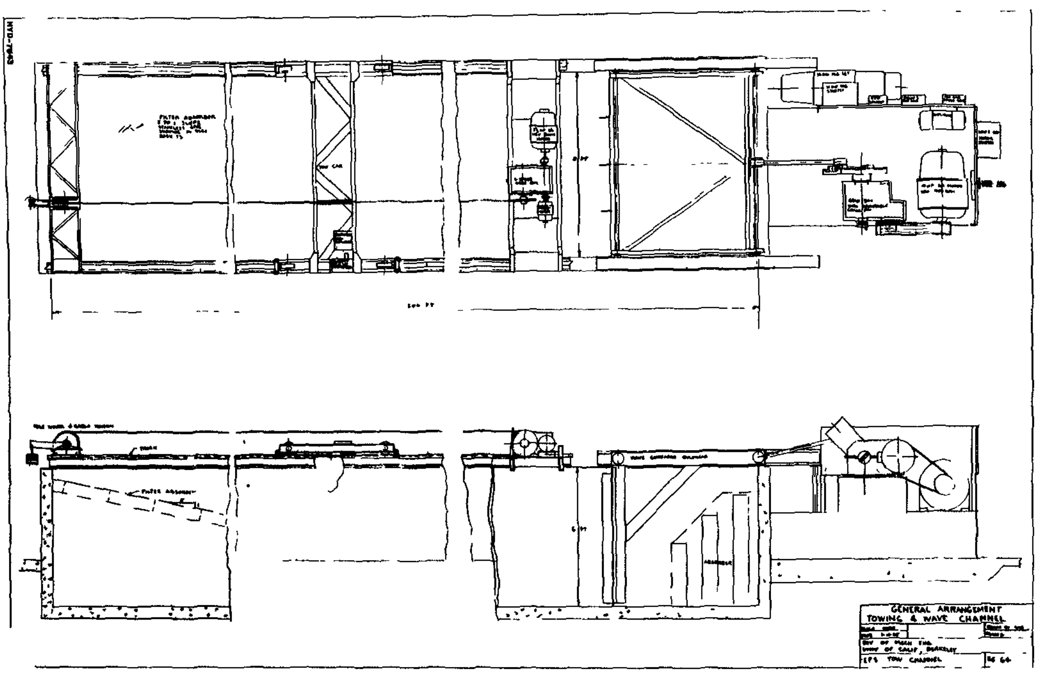
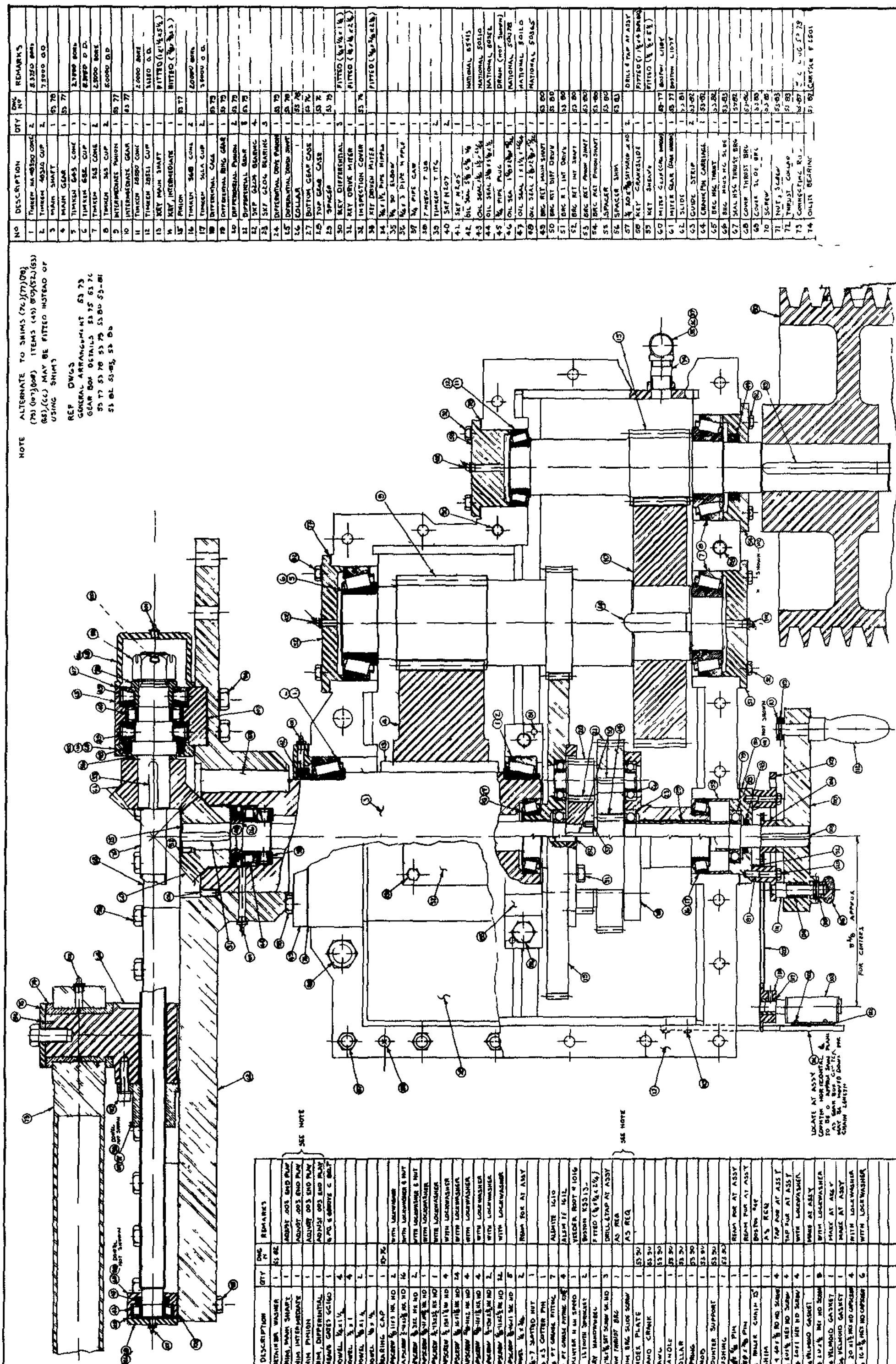


Fig. 7. General arrangement, towing and wave channel.



it becomes necessary to have a greater range, the drive can be modified by changing pulley sizes.

Power for the wave generator at present is furnished by a 230 V, 50 Kw motor generator set.

PISTON-TYPE WAVE GENERATOR

The generator consists of a welded steel piston (bulkhead) which is connected by a connecting rod to an adjustable crank mounted on a specially built gear box (Figure 7).

The gear box (Figure 8) was designed to allow for adjustment of the crank-pin position while running. This was accomplished by using an epicyclic gear train. This allows shaft Q to turn at the same rotation and angular velocity as shaft G when shaft U is held stationary (Figure 9). When shaft U is turned, it causes a change of rotation speed of Q with respect to shaft G causing screw T to move the crank-pin. The gear ratios were selected to allow one complete turn of U to increase the amplitude of the piston by 1/10 of an inch. The gear ratios which were used in the wave-towing tank generator and the model basin generator are tabulated in Figure 9.

The piston (Figure 7 and 10) was fabricated from 1/8" plate and 6" channel. The wheels on which the unit rolls were mounted above the water line; the wheels on one side of the piston were grooved and the wheels on the other side of the piston were flat. Special double track was constructed to keep the wheels in a confined path. Mounting the bearings, etc., out of the water eliminated the excessive wear and corrosion which has occurred in previous wave generator designs. The piston was connected to the crank through a connecting rod.

The stroke of a piston-type wave generator can be computed by an equation developed by Biesel (1951). The equation is stroke = wave height divided by 2K, where K is a function of the ratio of wave length to water depth, and the stroke is measured from its mean position.

MEASURED CHARACTEISTICS OF WAVE GENERATOR

Two sets of measurements were made on the wave generating equipment: (1) the wave generator stroke necessary to produce different wave heights and periods; and (2) the horsepower input to the electric motor necessary to generate waves of different wave heights and periods. The data are presented in Figure 11. It can be seen that Biesel's (1951) equation for the necessary stroke predicts correctly the required stroke. Use of the Beach Erosion Board data (U. S. Army, 1949) results in an overpowered system by about a factor of 4/3. It can be seen that except for low values of wave horsepower a factor of three will permit the prediction of peak horsepower where the horsepower is taken as the horsepower to the motor minus the tare horsepower of the motor and the gear box. It is fortunate in that the tare horsepower decreases to about one horsepower for the longer wave periods which are the one requiring the greatest net horsepower. Later tests by the Beach Erosion Board (Caldwell, 1955) also showed that the peak load for this type of wave generator was approximately three times the average wave horsepower.

TOWING CARRIAGE

The towing carriage mounted on the tank is a light-weight aluminum carriage, designed to tow cylinders with relatively high speeds compared with ship model speeds and with rapid acceleration in order to obtain data on drag and inertia forces (Laird and Johnson, 1956). The four-wheeled carriage is pulled by a cable that spans the length of the tank. It rides on special rails and is guided laterally by means of horizontal guide wheels which bear on the machined sides of the rail. The rails were machined 100-lb. rail obtained by the University as surplus property from the Bureau of Ships, U. S. Navy. The rails were leveled using the water level in the tank as the datum, and their level is checked periodically.

A $7\frac{1}{2}$ horsepower shunt-wound D. C. motor is used to drive the towing carriage through a two range gear box (See foreground of Figure 10). The two speed ranges are 0.5 to 5 and 2 to 20 feet per second. Power for the drive motor comes from a 10 KW motor generator set which is excited by a General Electric Co. Electronic Amplidyne. A tachometer is mounted on the motor pinion shaft of the carriage gear box. This furnishes a voltage that is balanced against a reference voltage in the electronic circuit of the Amplidyne. When the output voltage of the tachometer equals the reference voltage, the carriage is running at constant speed. Any change in speed will cause the Amplidyne to force the field of the generator which, in turn, adjusts the speed of the motor. Close speed regulation is obtained. It is in the order of 1/10%. Speed adjustment is attained by the setting of a 10,000 OHM, 10-turn micro-potentiometer which adjusts the reference voltage level. The car is stopped by the Amplidyne's reversing the power to the drive motor and limiting the current. When the carriage is running at maximum speed, it can be stopped within a distance of 20 feet. If the operator does not push the stop switch in time, the braking is actuated automatically by limit switches located at the proper distance from the two ends of the tank. The carriage, drive, and braking system were designed by C. M. Snyder.

The speed of the towing carriage is measured similar to the method used by the Massachusetts Institute of Technology (Abkowitz and Paulling, 1953).

The carriage has an idle wheel that runs on the rail. The diameter of the wheel is such that it turns at 1 rps. when the speed of the carriage is 1000 knots. A disk is attached to the wheel that has 1,000 equally spaced slots 0.010" wide. A light source on one side of the disk is focused on a 0.010" slot in front of the disk. When the disk slot and the other slot line up the light passes through and is picked up by a photoelectric cell. The impulse picked up by the photocells are amplified and feed into a digital electronic counter.

The counter counts the number of impulses against a crystal control time base that has an accuracy of the order of a few millionths of a second. The counter in turn is coupled to a digital recorder that prints the number of impulses on a tape for the permanent record of the speed. The counter is limited to counting the number of impulses to ± one pulse which limits the accuracy of measuring the speed of the

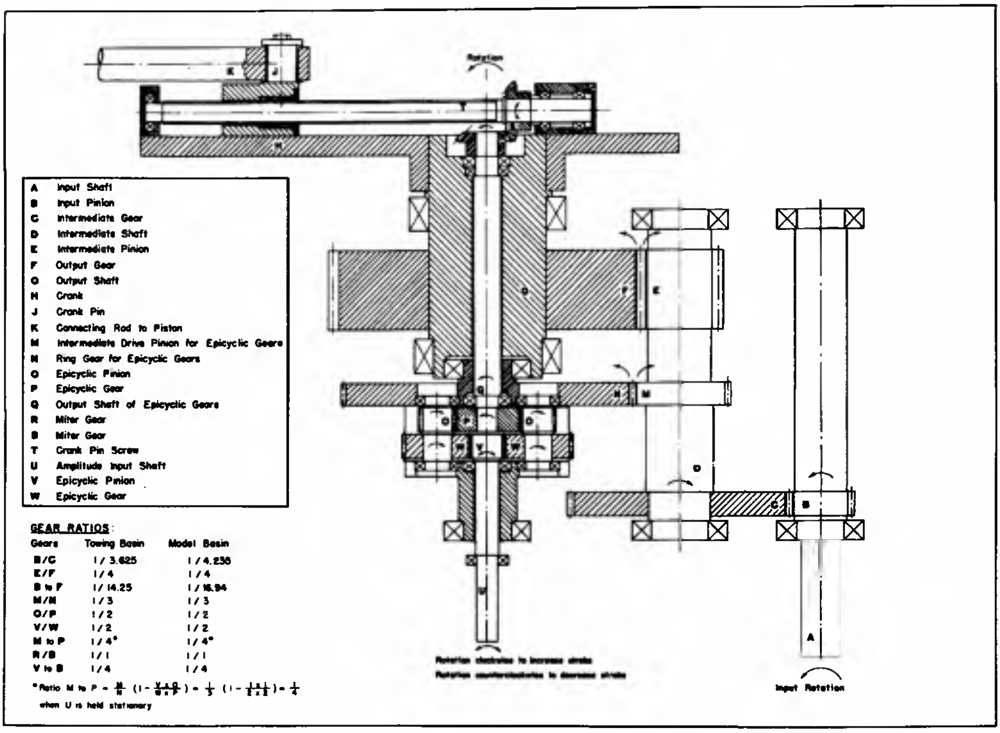


Fig. 9. Schematic drawing. Gear box for wave generator.

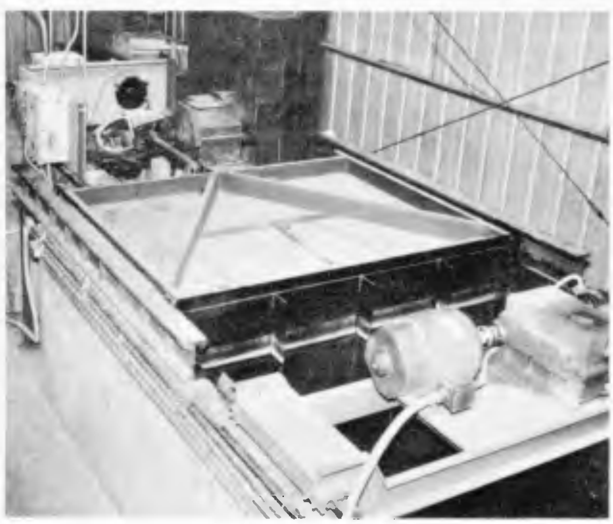


Fig. 10. Photograph of wave-towing tank wave generator.

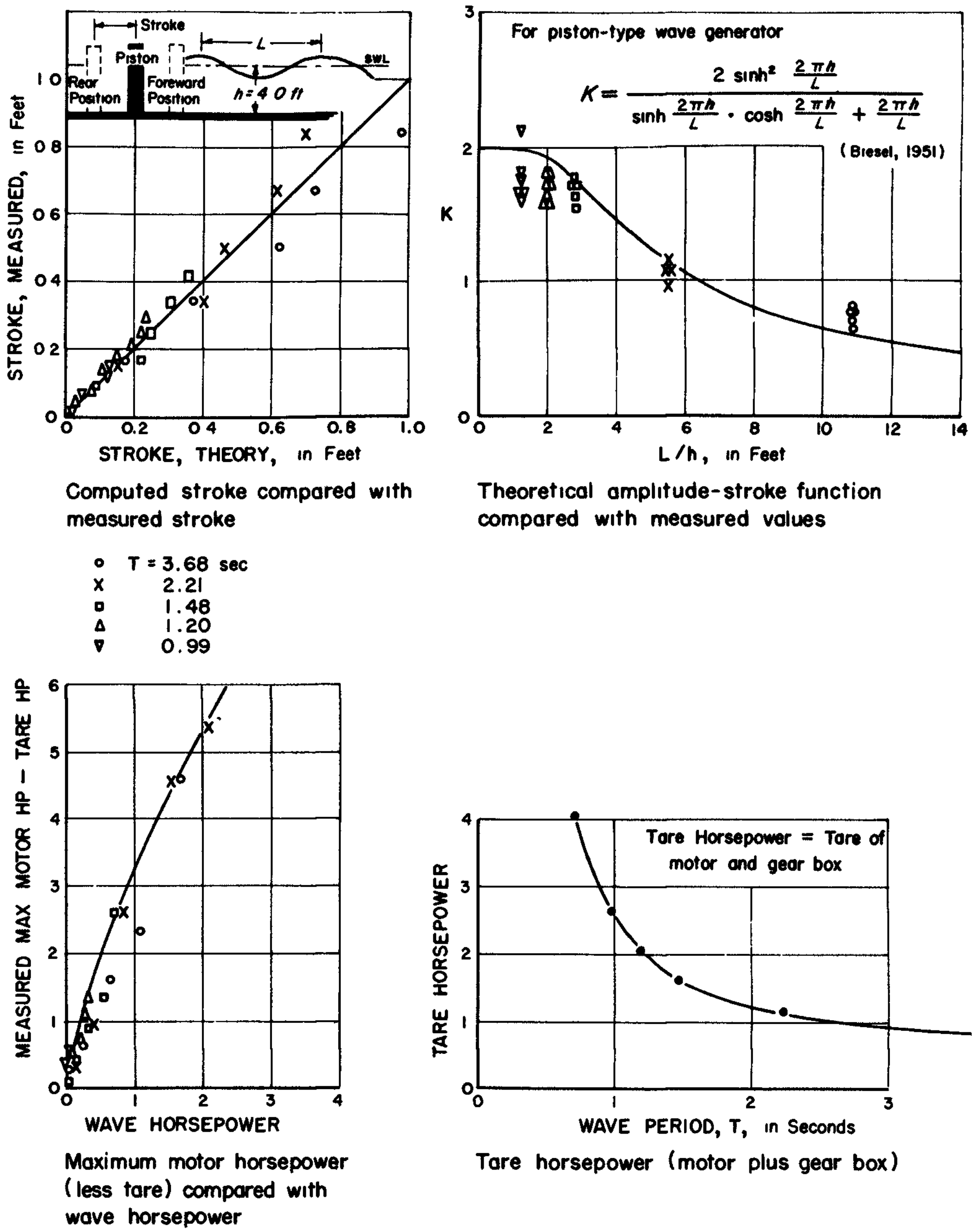


Fig. 11. Characteristics of the wave-towing tank wave generator.

carriage to within ± 0.001 knot.

The electronic counter and digital recorder were built by the Berkeley Scientific Company of Richmond, Calif. The speed measuring wheel amplified and photocell system were built by the University of California. The amplifier used was basically the same as that used by the Massachusetts Institute of Technology with the exception that the power supply was specially designed.

WAVE CHANNEL

The 1 ft. by 60 ft. by 3 ft. deep wave channel was the first wave tank built at the University (Figure 12) and was designed by M. P. O'Brien. The wave generator first used in this wave channel was of the flap-type with a motor-driven crank moving the top of the flap back and forth and with the bottom of the flap hinged in a fixed position. The wave generator (Figure 13) was redesigned by A. B. McIntyre and F. M. Sauer so that the top and bottom of the flap could be moved simultaneously by motor-driven coupled crank (Boucher, 1947). The amplitudes of the top and bottom motions of the flap can be set at the horizontal amplitudes of the water particle orbits of the wave to be used. The amplitude can be changed only while the unit is stationary and is accomplished by turning the screws on the two horizontally mounted fly-wheels. The period can be adjusted by means of a Vari-drive unit attached to the D. C. motor which drives the wave generator.

The channel is equipped with a series of observation windows through which motion pictures can be taken of the wave action.

Absorption of wave energy is usually accomplished by means of a sloping aluminum beach (with a slope of 1:10 or less) covered with sand, gravel or crushed rock.

WIND-WAVE TUNNEL

The wave channel described above was also used as a wind-wave tunnel (Johnson and Rice, 1952). However, due to a heavy work load it was necessary to build a separate unit for studying wind tides (Sibul, 1955). A schematic drawing has been given in Figure 14, together with sketches of an experimental set-up for measuring wind pressure distribution, waves, and wind tide. The tunnel and equipment was designed by J. W. Johnson, K. J. Bermel and O. J. Sibul.

The tunnel was constructed of wood, with one side made of plate glass for observation and photographic purposes. The wind was generated by a blower mounted on one end of the channel, driven by an A-C motor. The wind velocities could be varied from zero to fifty feet per second by varying the air intake area at the blower. A honeycomb section was installed between the blower and the channel to straighten the wind flow. Sloping wood planks were installed at both ends of the channel to guide the wind onto and off of the water surface. The plank at the exit end of the channel was covered with burlap to absorb the energy of the relatively small wind waves generated by the blower.

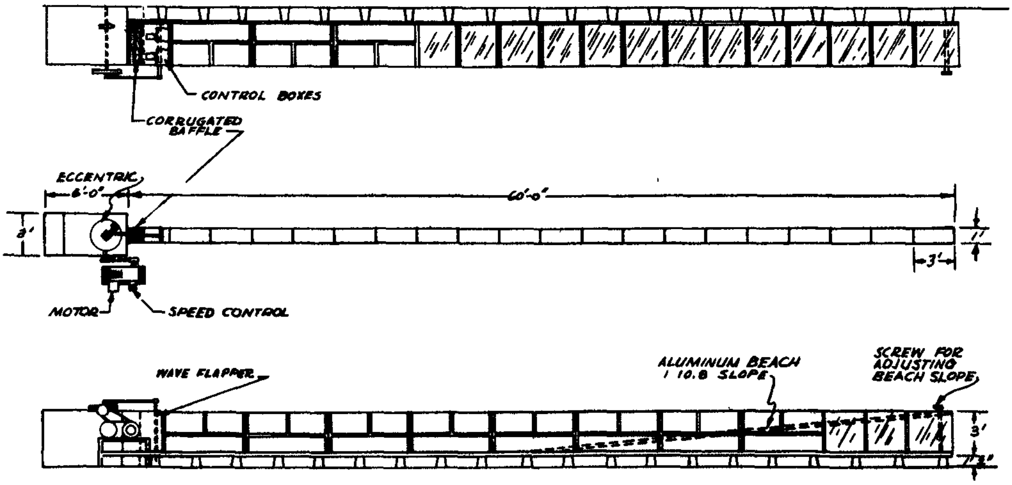


Fig. 12. One-foot wide wave channel.

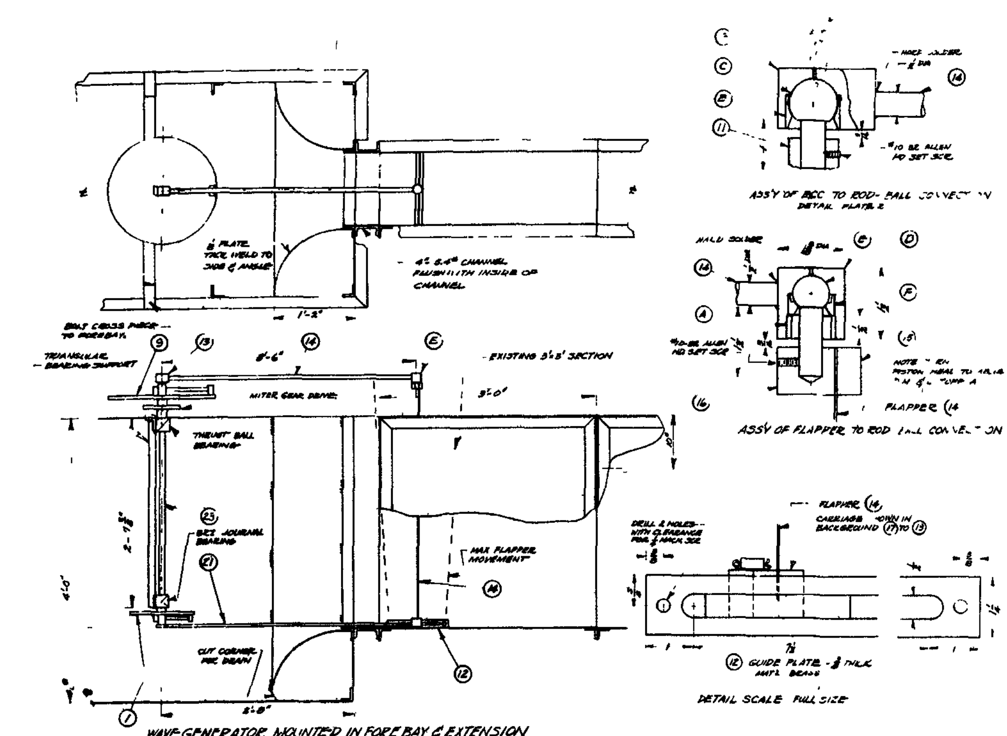


Fig. 13. Wave generator, one-foot wide wave channel.

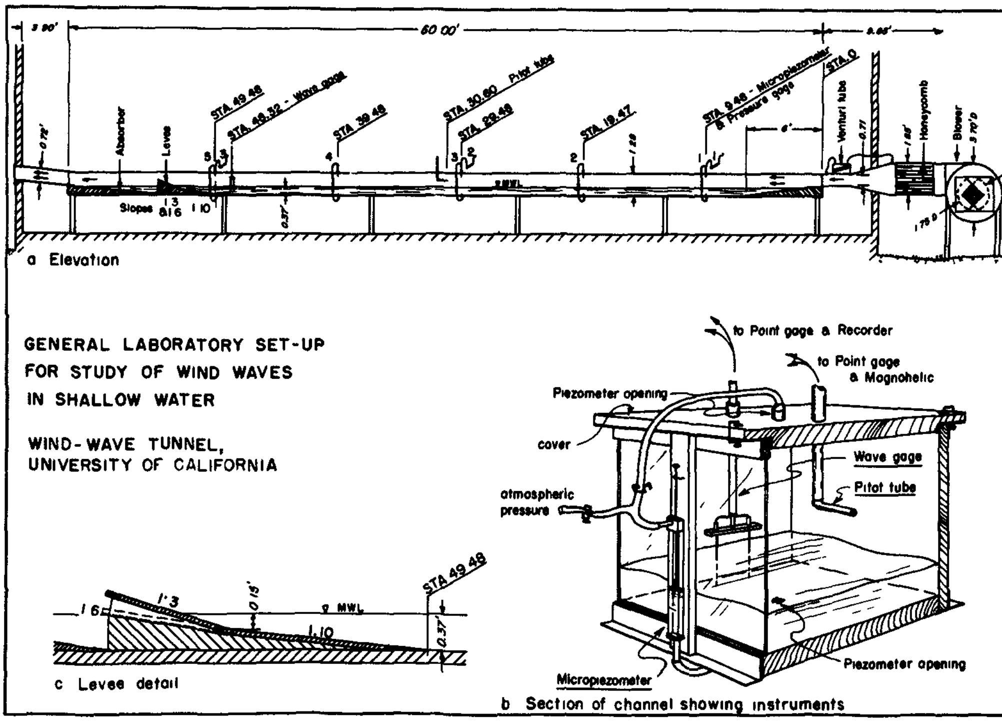


Fig. 14. Channel for studying wind waves in shallow water.

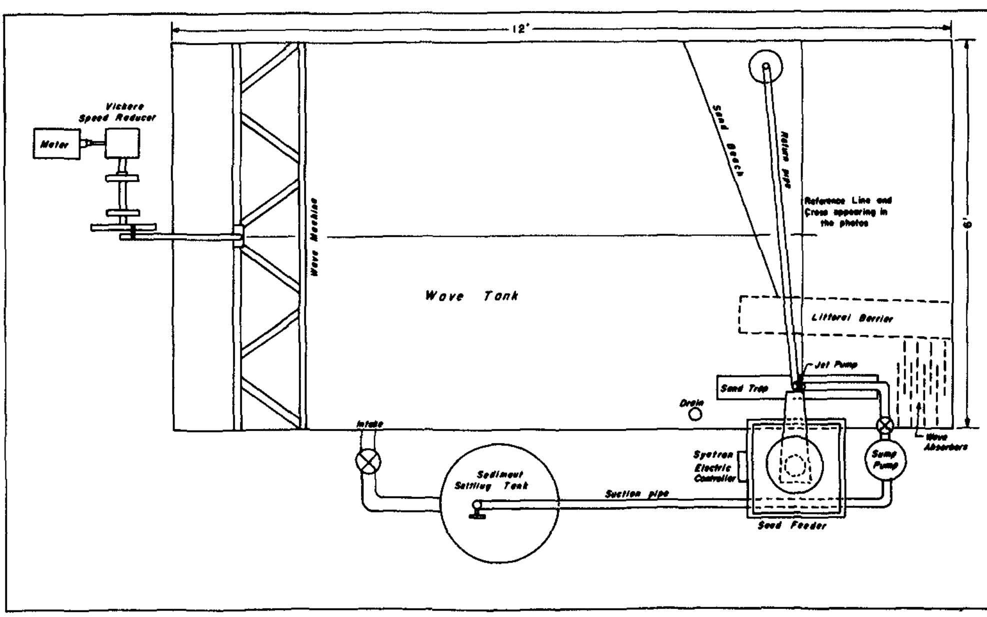


Fig. 15. Sketch of wave-sediment basin.

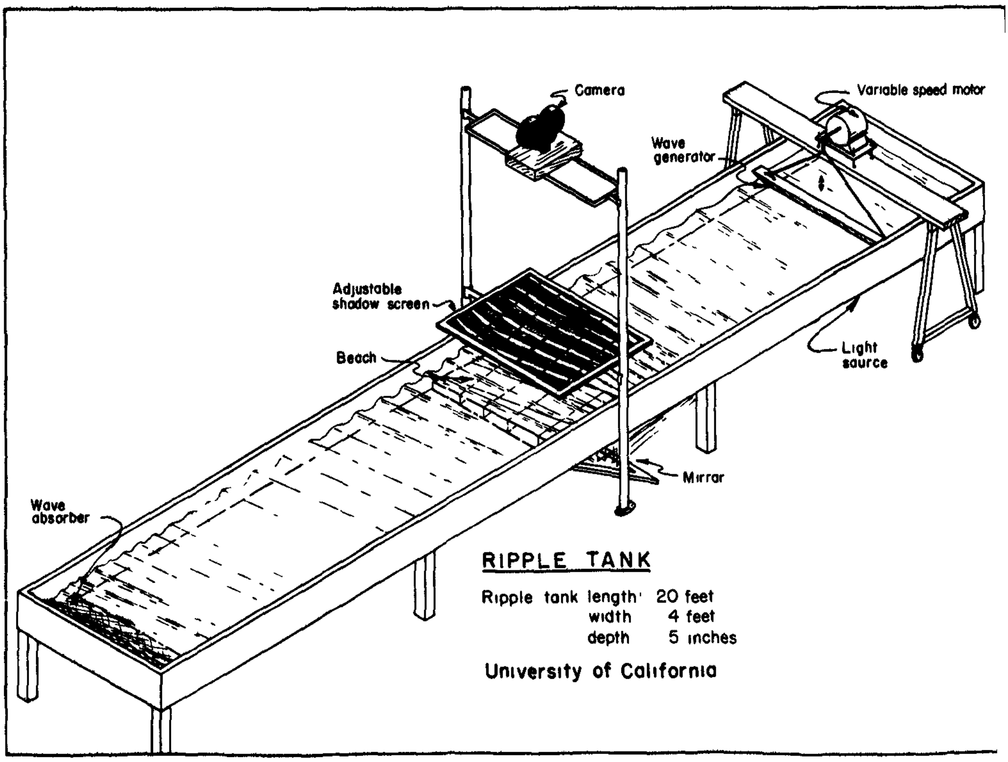


Fig. 16. Ripple Tank.

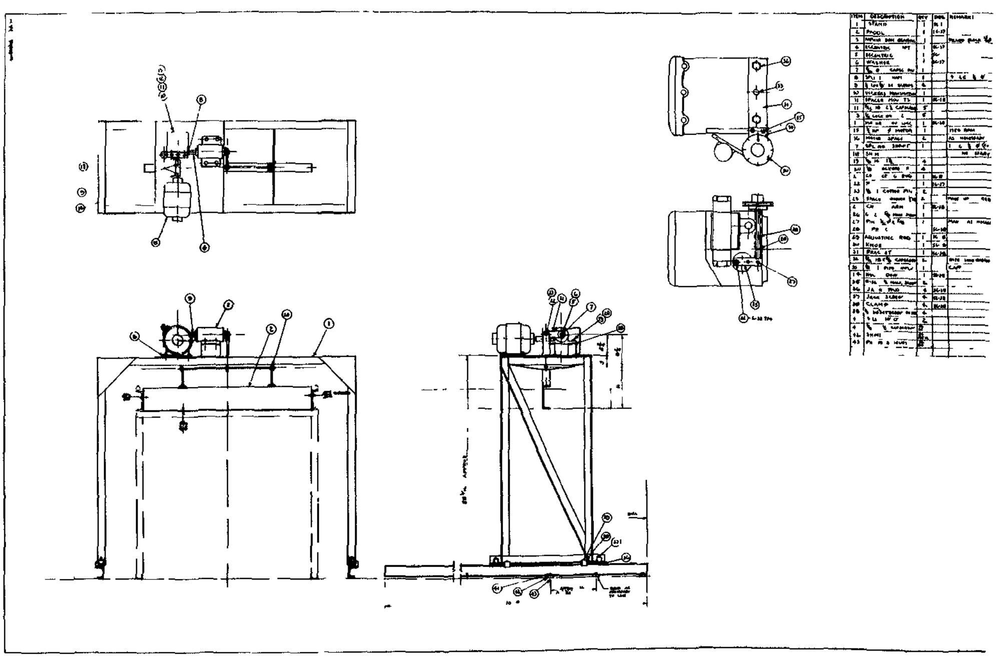


Fig. 17. Wave generator, ripple tank.

LABORATORY FACILITIES FOR STUDYING WATER GRAVITY WAVE PHENOMENA

WAVE-SEDIMENT BASIN

A small tank was designed by A. B. McIntyre for studying the effect of a littoral barrier on a sandy coast (Chien and Li, 1952). The tank is 6 ft. by 1 ft. deep. A piston-type wave generator is located at one end of the basin (Figure 15) which is driven by an A-C motor. The period of the wave generator is controlled by a Vickers speed reducer mounted between the motor and the drive crank, and the amplitude of the wave generator is controlled by an adjustable eccentric. At the opposite end of the tank a triangular-shaped sand trap was installed at the "downcoast" side of the beach which collected the littoral drift of sediment. This sediment was pumped to the "upcoast" end of the beach by a jet pump and again injected into the littoral system.

RIPPLE TANK

The ripple tank is 4 ft. by 20 ft. by 0.4 ft. deep with a 45 in. by 72 in. by 3/8 in. glass section located at the middle of the tank bottom (Figure 16). A mirror oriented at 45 degrees with respect to the vertical reflects light from a point source through the glass section, and the waves focus the light on an adjustable shadow screen. A carbon arc lamp is the source of light (Chinn, 1949). Wave energy is absorbed by means of a sloping plank covered with thick burlap. The equipment was designed by H. A. Einstein and A. J. Chinn. The wave generator has been redesigned by C. M. Snyder (Figure 17) to give a greater range of wave dimensions and more accurate control of wave direction.

The equipment was modified by Ralls and Wiegel (1956) as shown in Figure 18 to study the three-dimensional problem of the generation of short-crested waves by wind. Sections of 3/4 in. plywood, coated with a water resistant material, were placed on top of the tank to form a rectangular duct. A 44 in. by 39 in. by 3/8 in. plate of glass was placed directly above the glass section mounted in the tank bottom. The top was mounted flush with the inside of the tank to prevent generation of disturbances in the air stream, and was fabricated in sections, permitting the air intake to be placed at several positions to vary the fetch. Gaskets at the joints made the sections airtight, and the whole top was fastened to the tank with C-clamps. Modeling clay placed at the joint prevented air leakage.

Intake and exhaust units were made of 18-gauge galvanized iron, according to NACA specifications (McLellen and Bartlett, 1941). A 100 mesh strainer cloth (Dryden, 1947) was installed on top of the intake unit to help prevent irregularities in the velocity distribution. A honeycomb section was placed inside the tank just downstream from the intake unit as an air-straightening section (Prandtl, 1953) to ensure that the air entering the test section would be parellel to the walls. A suction fan was connected to the exit development piece by a piece of flexible rubber which prevented fan vibration from being transferred to the walls of the tank.

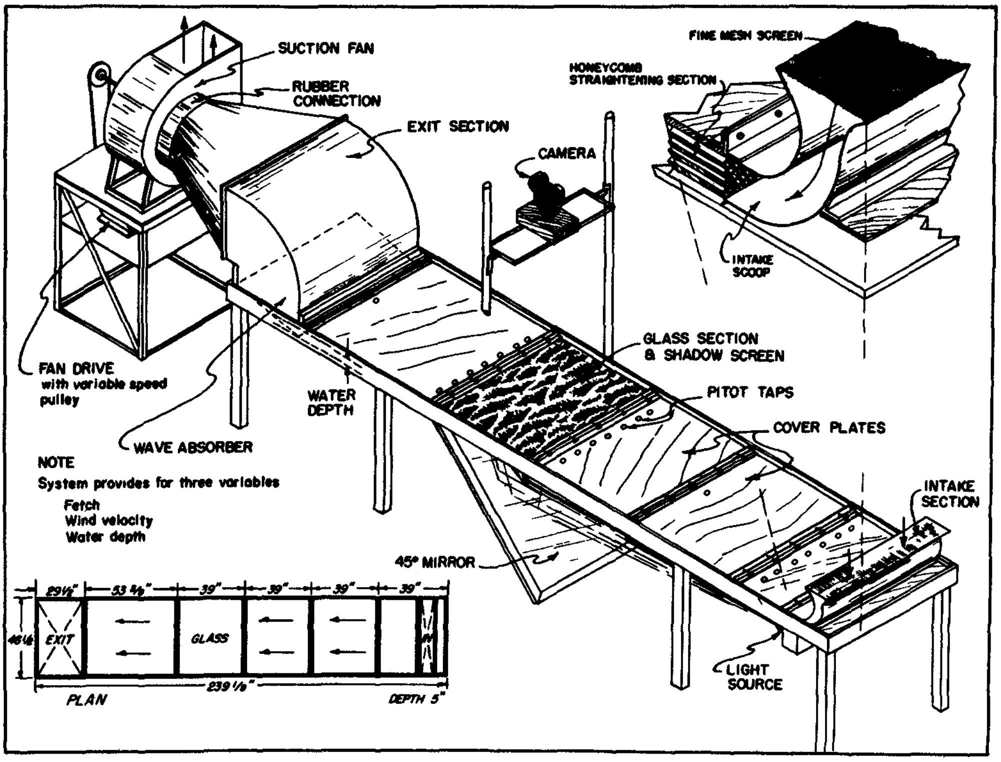


Fig. 18. Ripple tank modified to wind tunnel.

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CHAPTER 16

THE USE OF THE STOKES-STRUIK APPROXIMATION FOR WAVES OF FINITE HEIGHT

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ABSTRACT

A formal approximate solution is derived for the profile and velocity components of a wave with permanent form of finite height in moderate water depths. The approximation is carried to the third order, sufficiently far to represent all except the very high "design" waves. The relationship of the formulas to others found in the literature is discussed.

The wavelengths and the coefficients in the third-order series for the wave profile, and the water particle velocities and local accelerations are tabulated for approximately 2000 waves. The depths, heights, and periods for the listed wave conditions vary respectively from 10 to 500 feet, 5 to 40 feet, and 4 to 20 seconds. The range of applicability of the theory is discussed and approximate limits estimated.

As an aid in calculations, tables of the trigonometric and hyperbolic sines and cosines for integral multiples of the argument are included.

DERIVATION OF FORMULAS

We wish to construct potential flows satisfying Bernoulli's theorem on the free surface $\eta(x)$ whose shape is one of the unknowns. The problem is one of some difficulty mathematically because of the non-linear boundary condition. The usual general theorems of existence of solutions for linear problems are not valid and an *ad hoc* verification of existence of a solution must be made to be sure that the formal solution (Stokes, 1847), which can be rather easily obtained, exists. The existence proof was first carried out by Struik (1926) following the method of Levi-Civita (1925) for deep water waves. Struik also obtained formulas for the profile, velocity components, and the necessary auxiliary functions to relate the parameters in these formulas to the usual physically observed wave characteristics.

Unfortunately all these formulas have algebraic errors, as was first pointed out by Hunt (1953) and then independently by Tanaka (1953). However the results of Hunt and Tanaka do not, at first sight, agree. It seemed advisable to make a new and simpler derivation of these results in a slightly different way, in order to clear up these discrepancies. A more recent calculation by De (1955) has checked the results of Hunt and extended them to the fifth approximation.

We introduce the device of a moving coordinate system whose velocity, C, is the celerity of the waves. In this coordinate system, the motion is steady and two-dimensional. It is then convenient to use complex variables (Lamb, 1932) and a complex potential w, whose real and imaginary parts are the velocity potential ϕ and stream function ψ,

$$w(z) = \phi(x,y) + i\psi(x,y), \tag{1}$$

and whose derivative is related to the velocity components v_x and v_y

$$\frac{dw}{dz} = - v_x + i v_y. \tag{2}$$

The mathematical problem may be described briefly. We wish to construct a function w of complex variable z which has the properties that dw/dz has a real period L,

$$\frac{dw}{dz}(z) = \frac{dw}{dz}(z + L), \tag{3a}$$

that there is no flow through the bottom,

$$\text{Im } w = 0 \qquad \text{on} \qquad \text{Im } z = 0, \tag{3b}$$

and that Bernoulli's equation is satisfied

$$\frac{1}{2}\left|\frac{dw}{dz}\right|^2 + g \text{ Im } z = \text{Const.}, \tag{4a}$$

on the free surface

$$\text{Im } w = lC. \tag{4b}$$

We will assume that the solution to the problem has the form

$$w = C\left\{z + \frac{L}{2\pi}\sum_{n=1}^{\infty} a^n A_n \sin nkz\right\}, \tag{5}$$

where $k = 2\pi/L$. We note that equations (3a) and (3b) are satisfied if A_n are real. The success of this assumption will presently justify the form. We know that Bernoulli's theorem must hold on the line given by putting the imaginary part of w equal to a constant lC in equation (5), whose real and imaginary parts are then parametric equations for the profile. The parameter is ϕ, the real part of the potential. We see that we will need to invert equation (5) to obtain z as a function of w, in order conveniently to satisfy Bernoulli's theorem, equation (4a). It is also necessary to obtain dw/dz as a function of w. We will indicate formally how these steps can be carried out, although to justify each step mathematically we would have to investigate the convergence of the assumed solution after we have calculated the values of the coefficients, A_n.

We differentiate equation (5) with respect to z and obtain

$$\frac{dw}{dz} = C\left\{1 + \sum_{n=1}^{\infty} a^n n A_n \cos nkz\right\}. \qquad (6)$$

We assume z as a function of w is given by

$$Cz = w + C\,\frac{L}{2\pi}\sum_{n=1}^{\infty} a^n B_n \sin\frac{nkw}{C}, \qquad (7)$$

the justification being that we are able to substitute equation (7) int
equation (5) and solve successively for B_n as functions of A_n.

Then we substitute equation (7) in equation (6) and obtain

$$\frac{dw}{dz}(w) = C\left[1 + \sum_{n=0}^{\infty} C_n \sin\frac{nkw}{C}\right], \qquad (8$$

where C_n are functions of a and A_n.

We put Im $\dot{w} = lC$ in equation (8) and equation (7) and substitute in equation (4a). We find

$$\frac{1}{2}\left|1 + \Sigma\, a^n C_n \cos\frac{nk}{C}(\phi + ilC)\right|^2 + \frac{gL}{2\pi C^2}\,\mathrm{Im}\, z(\phi + ilC) = \mathrm{Const.} \qquad (9$$

We can also put

$$C^2 = \sum_{n=0}^{\infty} D_{2n} a^{2n}. \qquad (1$$

since the celerity may be a function of wave height and depth. Bernoulli's theorem, equation (9), must be valid for all ϕ and a, and we r
put the separate coefficients of $a^n \cos m\phi$ equal to zero. When this i
done, the resulting equations may be solved successively to obtain

$$A_1 = 1$$

$$A_2 = -\frac{3}{4(\cosh 2kl - 1)}$$

$$A_3 = -\frac{2(\cosh 2kl - 11)}{16(\cosh 2kl - 1)^2}\,.$$

$$D_0 = \frac{\sinh kl}{\cosh kl}\,\frac{gL}{2\pi}$$

$$D_2 = \frac{(2\cosh^2 2kl + 2\cosh 2kl + 5)}{4(\cosh 2kl - 1)}\,\frac{gL}{2\pi}\,\frac{\sinh kl}{\cosh kl}\,.$$

The profile $\eta(x)$ is found by eliminating ϕ from the real and imaginary parts of equation (7), evaluated on $w = \phi + ilC$. Then we obtain the m
water level d by integration:

$$d = \frac{1}{L}\int_0^L \eta(x)dx = l + \frac{L}{8\pi}a^2 \sinh 2kl \;, \tag{13}$$

and the wave height H by some algebra,

$$H = \eta\left(\frac{L}{2}\right) - \eta(0)$$

$$= \left[2a \sinh kl + 3a^3 \frac{4\cosh^3 2kl + 4\cosh^2 2kl + 1}{8(\cosh 2kl - 1)^2} \sinh kl\right]\frac{L}{2\pi} \,. \tag{14}$$

Finally, the celerity is given by equation (10),

$$C^2 = \frac{\sinh kl}{\cosh kl}\left[1 + \frac{2\cosh^2 2kl + 2\cosh 2kl + 5}{4(\cosh 2kl - 1)}\right]\frac{gL}{2\pi}, \tag{15}$$

and the particle velocities from minus the real part and the imaginary part of equation (6),

$$\begin{aligned} v_x = -\operatorname{Re}\frac{dw}{dz} = -C\bigg[1 &+ a\cos kx \cosh ky - \frac{3\cos 2kx}{2(\cosh 2kl - 1)}\cosh 2ky \\ &- \frac{3(2\cosh 2kl - 1)}{16(\cosh 2kl - 1)^2}\cos 3kx \cosh 3ky\bigg], \end{aligned} \tag{16}$$

and

$$\begin{aligned} v_y = \operatorname{Im}\frac{dw}{dz} = -C\bigg[a &\sin kx \sinh ky - \frac{3\sin}{2(\cosh 2kl)} 2kx \sinh 2ky \\ &- \frac{3(2\cosh 2kl - 1)}{16(\cosh 2kl - 1)^2}\sin 3kx \sinh 3ky\bigg] \,. \end{aligned} \tag{17}$$

We can transform back into a stationary coordinate system by the substitution

$$\begin{aligned} x &= x' - ct, \\ v_x &= v_x' - c, \end{aligned} \tag{18}$$

and obtain, on dropping the primes,

$$\begin{aligned} v_x = C\bigg[a &\cos k(x - ct)\cosh ky - \frac{3\cos 2k(x - ct)}{2(\cosh 2kl - 1)}\cosh 2ky \\ &- \frac{3(2\cosh 2kl - 1)}{16(\cosh 2kl - 1)^2}\cos 3k(x - ct)\cosh 3ky\bigg], \end{aligned} \tag{19}$$

and

$$\begin{aligned} v_y = -C\bigg[a &\sin k(x - ct)\sinh ky - \frac{3\sin 2k(x - ct)}{2(\cosh 2kl - 1)}\sinh 2ky \\ &- \frac{3(2\cosh 2kl - 1)}{16(\cosh 2kl - 1)^2}\sin 3k(x - ct)\sinh 3ky\bigg] \,. \end{aligned} \tag{20}$$

The set of three equations (13), (14), and (15) have as unknowns the three auxiliary parameters a, k, and l. We see that, given the wave height, the water depth, and the wave period, they may be solved for these auxiliary parameters. The parameter k has a simple interpre tation as seen from these equations since

$$\frac{2\pi}{k} = L. \qquad (2$$

Also, l and a are related to the depth and height, respectively.

The equations here given check with those of Hunt and Tanaka. T compare with Hunt and De it is necessary to obtain a as a function of μ, which may be done from the equations for the depth and height. It is then found that a is an odd function of μ,

$$a = \sum_{n=0}^{\infty} \mu^{2n+1} M_n(l). \qquad ($$

To compare with Tanaka, it is only necessary to identify his par meter d with our l. His statement that his d is the depth is mislead ing, since clearly his d is not the mean water level. Finally, it is relevant to notice that the equations pertaining to the Stokes-Struik theory in the Beach Erosion Board Technical Memoranda 1 and 2 are incorrect, as is pointed out by Hunt.

APPLICATION OF THE THEORY

After a theory is explicitly stated, normally two steps remain t be completed before the Engineer or Oceanographer can readily apply t theory to an actual problem. First, some procedure must be given to overcome the computational difficulties involved in the use of the theory, and second, the range of applicability must be indicated so that it is possible to determine when the theory should be used.

COMPUTATIONAL FORMULAS

The Stokes-Struik theory as presented can be put in a somewhat r convenient form for computations as follows. We multiply eq. (13) b eq. (15) and ignore terms containing powers of a greater than the th Then we divide both sides of the resulting equation by $L^2 g$ to obtain

$$\frac{d}{gT^2} = \frac{\tanh kl}{4\pi^2}\left\{kl + \frac{a^2}{4}\left[\sinh 2kl + \frac{kl(\cosh 4kl + 2\cosh 2kl + 6)}{\cosh 2kl - 1}\right]\right\}.$$

Similarly we multiply eq. (14) by eq. (15) and divide by $L^2 g$ ignorin powers of a greater than the third, and obtain

$$\frac{H}{gT^2} = \frac{a \tanh kl}{2\pi^2}\left\{\sinh kl + \frac{a^2}{8}\left[\frac{\sinh kl\ (2\cosh 4kl + 3\cosh 2kl + 10)}{\cosh 2kl - 1} + \frac{\sinh 3kl}{2}\,\frac{(3\cosh 4kl + 4\cosh 2kl + 2)}{(\cosh 2kl - 1)^2}\right]\right\}. \tag{24}$$

Normally at the start of the problem, the depth d, wave height H, and wave period T are known. The symbols π and g are the mathematical constant 3.1415....and the acceleration due to gravity respectively. Hence kl and a are the only two unknown quantities in equations (23) and (24), and the problem becomes one of solving the two nonlinear equations in two unknowns. While this is difficult by hand, it can be solved easily by iteration on an electronic computer. Once kl and a are known, the other properties of the wave follow immediately. An equation for the wavelength in same general form we have been using is obtained by dividing eq. (15) by gL and remembering that $C = L/T$. Hence

$$\frac{L}{gT^2} = \frac{\tanh kl}{2\pi}\left[1 + a^2\frac{(\cosh 4kl + 2\cosh 2kl + 6)}{4(\cosh 2kl - 1)}\right]. \tag{25}$$

The wavelength is thus determined. The wave profile* and the water particle velocities and local accelerations (equations 19 and 20) can be reduced to the convenient form

$$\begin{aligned}
\eta &= d + \eta_1\cos\theta + \eta_2\cos 2\theta + \eta_3\cos 3\theta\\
v_x &= v_1\cos\theta\cosh ky + v_2\cos 2\theta\cosh 2ky + v_3\cosh 3\theta\cosh 3ky\\
v_y &= v_1\sin\theta\sinh ky + v_2\sin 2\theta\sinh 2ky + v_3\sinh 3\theta\sinh 3ky\\
a_x = \frac{\partial v_x}{\partial t} &= a_1\sin\theta\cosh ky + a_2\sin 2\theta\cosh 2ky + a_3\sin 3\theta\cosh 3ky\\
a_y = \frac{\partial v_y}{\partial t} &= -a_1\cos\theta\sinh ky - a_2\cos 2\theta\sinh 2ky - a_3\cos 3\theta\sinh 3ky
\end{aligned} \tag{26}$$

where η_1, η_2, η_3, v_1, v_2, v_3, a_1, a_2, and a_3 are functions solely of L, T, kl and a, and where

$$\theta = 2\pi\left(\frac{x}{L} - \frac{t}{T}\right),\quad k = \frac{2\pi}{L}.$$

Explicitly

$$\begin{aligned}
\eta_1 &= \frac{La}{2\pi}\left[\sinh kl + \frac{a^2}{64}\,\frac{(9\sinh 5kl + 15\sinh 3kl + 6\sinh kl)}{\cosh 2kl - 1}\right]\\
\eta_2 &= \frac{La^2}{16\pi}\,\frac{(\sinh 4kl + 4\sinh 2kl)}{(\cosh 2kl - 1)}
\end{aligned} \tag{27}$$

***Your attention is drawn to the fact that the wave height computed from $H = \eta(0) - \eta(\pi)$ may occasionally be slightly different from the initial wave height chosen at the start of the problem. This discrepancy is due to the approximations made.**

$$\eta_3 = \frac{La^3}{256\pi} \frac{(3 \sinh 7kl + 15 \sinh 5kl + 27 \sinh 3kl + 39 \sinh kl)}{(\cosh 2kl - 1)^2}$$

$$v_1 = \frac{La}{T}$$

$$v_2 = \frac{3\, La^2}{2T\, (\cosh 2kl - 1)}$$

$$v_3 = - \frac{3\, La^3}{16T} \frac{(2 \cosh 2kl - 11)}{(\cosh 2kl - 1)^2} \quad (2$$

$$a_1 = \frac{2\pi v_1}{T}$$

$$a_2 = \frac{4\pi v_2}{T}$$

$$a_3 = \frac{6\pi v_3}{T} .$$

As an examination of the equations will show, η/L, v_xT/L, v_yT/L, a_xT^2/L and a_yT^2/L are all dimensionless and depend only on kl and a or in turn only on d/gT^2 and H/gT^2. The same dimensionless form extends to the coefficients.

In all of the preceding formulas, a coordinate system with its origin at the sea floor directly beneath the wave crest when $t = 0$ was used. The x axis is horizontal and positive in the direction of wave propagation while the y axis is positive upwards.

When the quantity d/L becomes very large, the hyperbolic functions of ky in the equations become unmanageably large and exceed most tables of hyperbolic functions. In order to avoid these large numbers, a different coordinate system is used whenever $d/T^2 > 2.56$. This is approximately the same as changing the coordinate system whenever $d/L > 0.5$. The new coordinate system has its origin at the sea surface directly below the wave crest when $t = 0$, and has its y axis positive downward. The x axis remains the same as before. The equations for the velocities and accelerations assume a slightly different form under this transformation. Let y' be the new vertical axis and y be the old one. Then

$$y = d - y' \quad (2$$

We substitute this into the previous formulas, dropping the prime, and obtain v_x, v_y, a_x, and a_y as:

$$v_x = v_1 e^{-ky} \cos \theta + v_2 e^{-2ky} \cos 2\theta + v_3 e^{-3ky} \cos 3\theta \quad (2$$

$$v_y = v_1 e^{-ky} \sin \theta + v_2 e^{-2ky} \sin 2\theta + v_3 e^{-3ky} \sin 3\theta$$

$$a_x = a_1 e^{-ky} \sin \theta + a_2 e^{-2ky} \sin 2\theta + a_3 e^{-3ky} \sin 3\theta$$
$$a_y = - a_1 e^{-ky} \cos \theta - a_2 e^{-2ky} \cos 2\theta - a_3 e^{-3ky} \cos 3\theta \tag{29}$$

TABLES OF COEFFICIENTS

Although all coefficients in the equations for η, v_x, v_y, a_x, and a_y can be stated in a dimensionless form as functions only of d/gT^2 and H/gT^2, a table of the coefficients versus the three variables, wave depth d, wave height H, and wave period T, is more convenient for most computations. Accordingly, the coefficients for approximately 2000 different wave conditions are tabulated in Appendix I. Water depths from 10 feet to 200 feet are listed in 10-foot intervals, and from 200 feet to 500 feet in 20-foot intervals. Wave heights from 5 to 40 feet in increments of 5 feet, and wave periods from 4 seconds to 20 seconds in increments of 2 seconds are given wherever the Stokes-Struik theory is not demonstrably false. In some cases it was convenient to measure the elevation y, from the water surface positive downward instead of from the sea floor positive upward and to use equations (29). The waves for which y should be measured from still water level are indicated in the tables by a minus sign on the wave period (T). The negative sign is used here merely as a convention to indicate the transformation of z.

A heavy preceding dot has been attached to the wave periods on certain waves. This dot indicates that we do not actually believe the Stokes-Struik theory is applicable to those particular waves. However, since a frequently used method for computing the velocities and accelerations in waves for which no theory is available consists of interpolation between the several most nearly applicable theories, the coefficients for the wave are included in the table even though the theory does not apply.

The position of the decimal point is indicated at least twice on each page of the table. The decimal point for any other number in the column can be located by comparison.

CIRCULAR AND HYPERBOLIC SINES AND COSINES FOR INTEGRAL MULTIPLES OF THE ARGUMENT

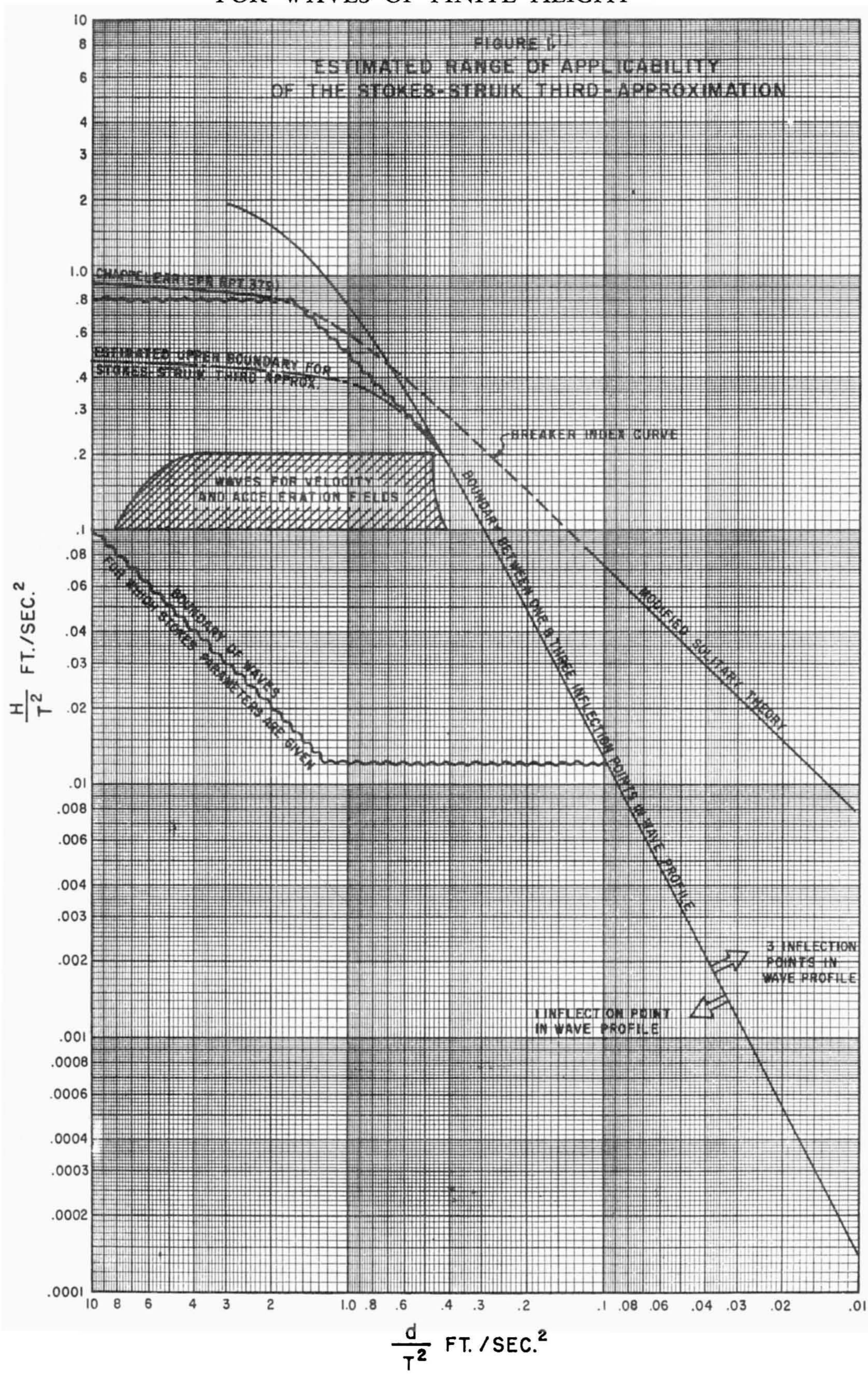
To facilitate computation of the three-term series, several auxiliary tables were prepared. Table 1 in Appendix II gives sin θ, sin 2θ, sin 3θ, cos θ, cos 2θ, and cos 3θ as a function of θ for each degree of θ between 0° and 360°. Table 2 in Appendix II gives sinh $2\pi y/L$, sinh $4\pi y/L$, sinh $6\pi y/L$, cosh $2\pi y/L$, cosh $4\pi y/L$, and cosh $6\pi y/L$ as a function of y/L in increments of 0.01 from 0 to 1.0. These two tables reduce the labor of looking up the functions for a given computation from six entries in a standard table to only two entries.

RANGE OF APPLICABILITY

Our estimate of the range of applicability of the theory is given Figure 1. For each value of d/T^2 there is a limiting value of H/T^2 suc that waves will break before H/T^2 grows larger. A line connecting thes limiting points is called the breaker index curve. In Figure 1, for th estimation of the breaker index curve, the modified solitary theory was used for $d/T^2 < 0.1$ while the results of an unpublished study made by J. E. Chappelear were used for $d/T^2 > 2.0$. The position of the line in between was estimated and is shown by dashes. This breaker index curve is an obvious upper limit to the use of any wave theory. A somewhat smaller upper boundary can be developed from the wave profile equation. From wave tank studies and from intuition, we are fairly sure that wave have only one maximum per wave cycle, i.e., the wave crest. Even more stringently, we suspect that the waves do not have any jogs or semi-steps in the profile. This is the same as saying that the wave profile should have only one inflection point between the wave crest and trough The boundary line between those waves, as computed by the Stokes-Struik third approximation, that have three inflection points between their crests and troughs and those that have only one is shown in Figure 1 as a solid line. Although this criterion strictly is intuitive, it seems reasonable to the author. The inflection point boundary and the breake index curve cross at about $d/T^2 = 0.7$. Hence the inflection point criterion is unsuitable for larger d/T^2 values, and a different criteri was developed.

The percent of the wave height above still water level for maximum waves where $d/T^2 > 2.0$ was known by a previous unpublished study. Grap of the percent of wave height above still water level were plotted vers H/T^2 holding d/T^2 at a constant value both for the Stokes-Struik third approximation and Chappelear's limiting case. The Stokes-Struik values departed from a smooth curve through the points at about $H/T^2 = 0.45$. The point of departure decreased somewhat with a decrease of d/T^2. Thi departure line is shown as a dash-dot line in Figure 1. Reid and Bretschneider (1953, p. 12) estimate the second approximation of the Stokes Struik theory to be accurate in deep water for approximately $H/T^2 < 0.3$ Hence the boundary line reaching $H/T^2 = 0.45$ for the third approximatic is reasonably consistent with their estimate.

In the preceding work, the basic formulas were derived by Chappele while the section on the application of the theory together with the various tables were prepared by Borgman. Both authors wish to express their appreciation to the Shell Oil Company and the Shell Development Company for permission to publish the paper.


ESTIMATED RANGE OF APPLICABILITY
OF THE STOKES-STRUIK THIRD-APPROXIMATION
ESTIMATED UPPER BOUNDARY FOR
STOKES-STRUIK THIRD APPROX.
BREAKER INDEX CURVE
WAVES FOR VELOCITY
AND ACCELERATION FIELDS
BOUNDARY BETWEEN ONE & THREE INFLECTION POINTS IN WAVE PROFILE
MODIFIED SOLITARY THEORY
BOUNDARY OF WAVES
FOR WHICH STOKES PARAMETERS ARE GIVEN
3 INFLECTION
POINTS IN
WAVE PROFILE
I INFLECTION POINT
IN WAVE PROFILE
H/T² FT./SEC.²
d/T² FT. / SEC.²
10
8
6
4
3
2
1.0
.8
.6
.4
.3
.2
.1
.08
.06
.04
.03
.02
.01
.008
.006
.004
.003
.002
.001
.0008
.0006
.0004
.0003
.0002
.0001

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LIST OF SYMBOLS

a	=	a mathematical parameter in the Stokes-Struik third approximation. The physical meaning is given by the formulae.
A_n	=	expansion coefficients to be calculated.
a_x, a_y	=	horizontal and vertical components of the local accelerations of water particles, i.e., $a_x = \partial v_x/\partial t$, $a_y = \partial v_y/\partial t$; (ft/sec^2).
a_1, a_2, a_3	=	Stokes-Struik third approximation coefficients for the local accelerations.
B_n	=	expansion coefficients to be calculated.
C	=	wave celerity, or speed of advance of the waveform (ft/sec).
C_n	=	expansion coefficients to be calculated.
d	=	still water depth, or depth the water would assume if th waves were calmed down and the water was still (ft).
D_{2n}	=	expansion coefficients to be calculated.
g	=	acceleration due to gravity (nominally taken as 32.16 ft/sec^2).
H	=	wave height, or vertical distance between a crest and th preceding trough (ft).
k	=	$2\pi/L$ (ft^{-1}).
l =	=	a mathematical parameter in the Stokes-Struik third approximation. The physical meaning is given by the formulae.
L	=	wavelength, or horizontal distance between two succeeding troughs or crests in the direction of wave propagation (ft).

t = time measured from the instant when the wave crest was above the origin of the coordinate system (seconds).
T = wave period, or length of time for two succeeding crests to pass a fixed point (seconds).
v_x, v_y = horizontal and vertical components of the water particle velocities (ft/sec).
v_1, v_2, v_3 = Stokes-Struik third approximation coefficients for the water particle velocities (ft/sec).
$w(z)$ = complex potential, $\phi + i\psi$
for T marked plus - x, y = horizontal and vertical coordinate axis, respectively. The origin is at the sea floor directly below the wave crest when $t = 0$. x is positive in the direction of wave propagation and y is positive upwards.
for T marked minus - x, y = horizontal and vertical coordinate axis, respectively. The origin is at still water level directly beneath the wave crest when $t = 0$. x is positive in the direction of wave travel and y is positive downwards.
z = = complex variable, $x + iy$
$\eta(x)$ = vertical elevation of the wave profile above the sea floor, (ft). η is a function of x or of θ depending on what horizontal measure of distance is being used.
η_1, η_2, η_3 = Stokes-Struik third approximation coefficients in the equation for the wave profile (ft).
θ = angular phase position or distance from crest measured so as to make
$\theta = 0$ at the crest and $\theta = 180°$ at the preceding trough. In general,
$\theta = 2\pi(x/L - t/T)$, but usually t is set to zero and the simpler formula
$\theta = 2\ x/L$ is used (dimensionless)
$\phi(x, y)$ = velocity potential
$\psi(x, y)$ = stream function

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS

Column Headings:

d = water depth (ft)
H = wave height (ft)
T = wave period (sec)
L = wavelength (ft)
η_1, η_2, η_3 = profile coefficients (ft)
v_1, v_2, v_3 = velocity coefficients (ft/sec)
a_1, a_2, a_3 = local acceleration coefficients (ft/sec^2)

(The reader is referred to the list of symbols for more detailed definitions, and to the text for the mathematical series in which the coefficients are to be used.)

The decimal point position is indicated in the first line of each depth group and on the first line of each page. The decimal positions of the other numbers in a given column are the same as those where the position is indicated.

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS

d	H	T	L	η_1	η_2	η_3	V_1	~~V_2~~ $\frac{1}{2}V_2$	V_3	O_1	~~O_2~~ $\frac{1}{2}a_2$	O_3
18	5	*4	668	2346	.6583	.2075	3100	.1953	.0188	4870	.6136	.0886
20	3	4	787	2437	.3030	.0307	1582	.0089	.0002-	2485	.0278	.0010-
20	3	6	1377	2431	4169	754	2368	838	49	2480	1796	154
20	3	8	1947	2368	6137	1441	2594	2205	443	2037	3464	1848
28	10	*4	838	4823	9915	2939	3237	460	16-	5084	1445	75-
20	10	*6	1440	4632	14044	4605	4494	3452	465	4706	7187	1460
30	5	4	830	2472	.2336	.0330	.792	0005	.0000	1244	.0016	.0000
50	5	6	1565	2475	2356	259	1686	180		1765	377	
58	5	8	2270	2459	3267	420	2049	648	58	1609	1019	90
30	5	10	2932	2428	4531	749	2195	1323	190	1379	1662	358
30	5	12	3625	2383	3990	1273	2243	2128	526	1175	2228	827
30	5	14	4294	2517	7497	2006	2233	2987	1090	1002	2681	1467
30	10	*4	880	4879	7630	1893	1706	31	1-	2680	97	5-
30	10	6	1607	4851	8506	1774	3322	756	5	3479	1584	15
30	10	8	2326	4751	11674	2869	3933	2529	317	3089	3972	746
30	10	10	3036	4573	13461	4818	4092	4841	1392	2571	6084	2624
30	15	*6	1662	7144	17118	5058	4912	1818	41	5144	3807	127
30	15	8	2401	6878	23190	8135	5646	5372	1126	4434	8752	2653
40	5	4	842	2474	.2172	.0293	.386	.0000	.0000	.606	.0000	.0000
40	5	6	1683	2485	1665	142	1224	45		1281	95	2-
40	5	8	2318	2483	2078	181	1630	238	4	1296	373	9
40	5	10	3314	2472	2808	296	1847	359	33	1161	703	66
40	5	12	4091	2450	3720	491	1941	974	119	1016	1020	187
40	5	14	4858	2424	4771	787	1983	1462	281	891	1312	379
40	5	16	5620	2383	3891	1192	1994	1994	342	783	1566	638
40	5	18	6380	2339	7056	1721	1982	2553	915	692	1782	958
40	5	20	7137	2265	8076	2307	1938	3064	1370	609	1925	1291
40	10	*4	893	4892	7039	1661	871	2		1369	6	
40	10	6	1721	4913	6161	1003	2451	196	4-	2566	410	12-
40	10	8	2556	4881	7787	1320	3241	954	36	2546	1499	85
40	10	10	3367	4806	10346	2111	3575	2167	279	2246	2724	526
40	10	12	4168	4688	13277	3357	3688	3637	883	1931	3809	1387
40	10	14	4964	4538	16300	5060	3685	5213	1942	1654	4679	2614
40	15	*6	1768	7280	12616	2922	3685	485	11-	3839	1015	35-
40	15	8	2609	7172	16126	3952	4754	2166	138	3734	5405	326
40	13	10	3441	6974	21039	6200	3132	4707	938	3237	5915	1768
40	20	*6	1820	9618	20461	3990	4935	955	22-	5168	2001	70-
40	20	*8	2673	9383	26339	8298	6200	3907	376	4870	6136	886
50	5	4-	845	2476	.2133	.0283	7630	.0023	.0005-	11985	.0079	0021-
30	5	6	1739	2490	1354	101	894	12		937	26	
50	5	8	2708	2490	1494	100	1353	99		1063	135	
58	5	10	3614	2488	1937	147	1582	270	8	994	339	15
30	5	12	4491	2480	2539	254	1702	507	34	891	531	53
50	5	14	5330	2456	3229	363	1760	789	88	790	709	119
50	5	16	6201	2430	4071	362	1803	1130	186	708	887	219
50	5	18	7047	2430	4970	828	1821	1501	356	636	1048	352
50	5	20	7888	2388	5842	1149	1812	1872	537	569	1176	306
30	10	*4-	897	4896	6886	1604	14511	140	45-	22794	440	211-
50	10	6	1793	4937	5045	717	1807	34	1-	1892	113	4-
30	10	8	2738	4930	5712	731	2684	401	2	2108	629	5
50	10	10	3651	4897	7369	1096	3108	1067	65	1953	1341	122
50	10	12	4542	4839	9503	1704	3309	1960	263	1733	2053	413
50	10	14	5422	4735	11895	2396	3391	2999	663	1522	2692	895
50	10	16	6298	4643	14377	3778	3399	4111	1314	1335	3229	1548
50	10	18	7170	4308	16786	5220	3335	5223	2219	1171	3647	2324
50	13	*6	1898	7337	10393	2099	2749	137	4-	2878	288	13-
50	15	8	2781	7301	12092	2312	3984	924	11	3129	1451	27
30	15	10	3705	7201	13310	3367	4555	2374	223	2862	2983	425
50	15	12	4616	7044	19650	5122	4790	4241	862	2308	4441	1354
30	15	14	5522	6830	23961	7529	4836	6291	2070	2170	5647	2787
30	20	*6	1889	9712	16901	4305	3725	279	10-	3900	584	32-
30	20	8	2833	9608	20102	4957	5251	1691	38	4124	2656	89
30	20	10	3771	9403	25635	7209	5924	4180	536	3722	5252	1049
50	25	*6	1942	12087	24293	7541	4733	497	21-	4959	1041	65-
30	25	*8	2890	11873	29392	8771	6495	2732	99	5101	4291	254
50	25	*10	3845	11530	37246	12733	7227	6487	1137	4541	8152	2143
60	5	4-	846	2475	.2125	.0283	7626	0006	.0001-	11979	.0018	.0005-
60	5	6	1804	2494	1201	83	652	3		683	7	
60	5	8	2853	2494	1170	66	1123	44		882	69	
60	5	10	3865	2494	1442	86	1372	142	2	862	178	4
60	3	12	4837	2489	1850	128	1508	286	11	789	300	17
60	5	14	5785	2488	2368	198	1591	471	33	714	423	44
60	5	16	6720	2477	2954	297	1637	687	73	643	539	86
60	5	18	7646	2458	3592	432	1660	926	137	580	646	143
60	10	*4-	898	4897	.6849	.1590	14510	0035	.0011-	22792	.0109	.0053-
60	10	6	1837	4947	4479	590	1327	15		1389	32	
60	10	8	2881	4951	4513	496	2237	180	1-	1757	282	3-
60	10	10	3892	4937	5569	650	2715	565	16	1706	710	29
60	10	12	4873	4907	7099	967	2966	1126	87	1553	1180	137
60	10	14	5836	4860	8924	1455	3097	1817	252	1390	1631	340
60	10	16	6789	4800	10950	2142	3160	2604	547	1241	2045	644
60	10	18	7737	4709	13006	3011	3166	3428	987	1105	2393	1034
60	10	20	8682	4613	15155	4111	3149	4296	1602	989	2699	1510
60	15	6	1882	7364	9235	1726	2038	40	1-	2134	83	3-
60	15	8	2918	7363	9671	1552	3542	418	3-	2625	657	8-
60	15	10	3935	7312	11955	2051	4018	1273	56	2525	1600	106
60	15	12	4929	7223	15095	3016	4352	2486	294	2279	2603	462
60	15	14	5911	7092	18659	4442	4499	3927	819	2019	3525	1103
60	13	16	6887	6921	22358	6322	4531	5482	1701	1779	4305	2004
60	20	*6	1932	9759	15007	3331	2790	83	3-	2922	173	9-
60	20	8	2964	9724	16242	3369	4437	774	6-	3484	1215	14-
60	20	10	3988	9609	20104	4488	5272	2268	143	3312	2850	269
60	20	12	4998	9428	25139	6532	5657	4326	699	2962	4531	1098
60	20	14	6002	9178	30579	9422	5789	6683	1864	2598	6000	2510
60	23	*8	3013	12051	23950	6017	5525	1264	6-	4339	1986	14-
60	25	10	4048	11848	29680	8083	6487	3563	302	4076	4478	569
60	23	12	3077	11543	36741	11642	6893	6630	1373	3609	6943	2157
60	30	*6	2039	14562	28658	9095	4416	257	12-	4625	538	38-
60	30	*8	3070	14331	32569	9321	6607	1906		5189	2994	2
60	30	*10	4115	14038	40402	12898	7666	5172	565	4817	6499	1065
70	5	4-	846	2475	.2123	.0283	7623	.0001	.0000	11978	.0004	.0001-
70	5	6	1829	2493	1120	74	473			495	2	
70	5	8	2968	2495	975	48	937	20		736	32	
70	3	10	4075	2496	1135	56	1202	79		755	99	
70	5	12	5139	2492	1421	78	1350	172	4	707	180	6
70	5	14	6173	2495	1800	117	1441	295	13	647	263	18
70	5	16	7188	2485	2231	172	1494	441	32	587	346	37
70	5	18	8192	2479	2730	251	1530	609	63	534	425	66
70	5	20	9187	2466	3268	354	1550	794	109	487	499	102
70	10	*4-	898	4897	6839	1586	14509	9	3-	22791	27	13-
70	10	6	1862	4955	4183	527	969	4		1014	9	
70	10	8	2991	4965	3783	366	1873	84		1471	132	2-
70	10	10	4098	4960	4430	429	2390	316	3	1501	397	6
70	10	12	5167	4942	5533	603	2673	683	31	1399	715	49
70	10	14	6210	4922	6940	889	2838	1139	105	1274	1040	141
70	10	16	7239	4875	8503	1290	2923	1708	244	1148	1342	288
70	10	18	8259	4826	10241	1837	2969	2326	471	1036	1624	494
70	10	20	9274	4737	12034	2325	2979	2977	797	936	1871	751
70	13	6	1907	7377	8618	1542	1501	11		1572	24	
70	15	8	3026	7396	8147	1153	2809	196	3-	2206	309	7-
70	13	10	4133	7376	9616	1373	3555	717	13	2234	901	24
70	15	12	5210	7321	11957	1922	3952	1524	107	2069	1396	168
70	13	14	6268	7239	14800	2785	4161	2539	347	1867	2279	467
70	15	16	7315	7134	17956	3991	4261	3700	791	1673	2906	932
70	15	18	8356	6987	21160	5512	4279	4923	1472	1494	3437	1542
70	15	20	9394	6805	24270	7317	4240	6148	2397	1332	3863	2259
70	20	*6	1959	9783	13978	3144	2076	25		2174	51	2-
70	20	8	3068	9789	13764	2522	3747	367	7-	2943	576	17-
70	20	10	4178	9731	16354	3048	4691	1286	34	2948	1616	63
70	20	12	5266	9618	20253	4256	5180	2683	259	2712	2810	406
70	20	14	6340	9455	24810	6091	5414	4398	809	2430	3947	1089
70	20	16	7408	9231	29553	8310	5487	6271	1779	2155	4926	2096
70	25	*6	2013	12188	20001	5310	2692	47	2-	2819	98	3-
70	25	*8	3116	12149	20380	4323	4688	604	13-	3682	949	31-
70	25	10	4230	12035	24372	5533	5802	2035	74	3646	2557	140
70	25	12	5330	11840	30037	7728	6358	4155	517	3329	4351	813
70	25	14	6423	11566	36400	10915	6593	6688	1555	2959	6003	2093
70	30	*6	2067	14607	26542	8015	3349	81	3-	3507	170	11-
70	30	*8	3167	14492	27827	7186	5635	920	20-	4426	1445	48-
70	30	10	4287	14300	33458	8947	6892	2974	146	4330	3737	275
70	30	12	5402	13996	41013	12404	7490	5937	918	3922	6217	1442
70	35	*8	3221	16825	35979	10522	6589	1326	28-	5175	2083	67-
70	35	*10	4348	16531	43436	13263	7961	4114	261	5002	5170	492
70	35	*12	5478	16100	52979	18330	8583	8032	1500	4494	8411	2356
80	5	4-	846	2475	.2123	0283	7626	.0000	0000	11979	.0000	.0000
80	5	6	1842	2494	1080	70	341			357		
80	3	8	3054	2498	832	38	784	10		616	15	
80	3	10	4253	2496	934	39	1060	46		666	57	
80	5	12	5404	2497	1141	52	1219	108	1	638	113	2
80	5	14	6520	2493	1417	75	1314	193	6	590	173	8
80	5	16	7614	2497	1762	109	1379	298	15	542	234	18

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_5	a_1	a_2	a_3
80	5	20	9759	2484	.2571	.0220	1445	.0534	.0036	434	.0348	.0052
80	10	•4-	898	4897	6837	1586	14509	2		22791	7	3-
80	10	6	1876	4956	4827	496	703	1		757	2	
80	10	8	3076	4969	3307	292	1369	40		1233	63	1-
80	10	10	4273	4969	3661	306	2112	183		1327	230	
80	10	12	5427	4959	4465	404	2420	431	12	1267	452	18
80	10	14	6350	4946	5537	377	2604	763	46	1169	687	62
80	10	16	7632	4919	6786	828	2711	1165	117	1065	915	138
80	10	18	8745	4894	8213	1180	2780	1626	239	971	1135	230
80	10	20	9825	4853	9667	1616	2805	2113	417	881	1328	393
80	13	•4-	936	7534	12373	3836	20836	8	4-	32760	27	17-
80	15	6	1922	7384	8287	1447	1100	3		1132	7	
80	13	8	3109	7416	7155	925	2363	94	2-	1836	148	4-
80	13	10	4303	7409	8006	988	3134	418	1	1982	523	2
80	15	12	5462	7376	9750	1308	3397	969	40	1883	1014	63
80	15	14	6595	7532	12028	1855	3832	1700	156	1729	1526	209
80	15	16	7712	7233	14388	2626	3987	2557	386	1566	2008	455
80	15	18	8820	7134	17357	3649	4031	3505	765	1414	2447	801
80	15	20	9922	7029	20196	4921	4062	4501	1312	1276	2828	1237
80	20	•6	1974	9795	13417	2942	1535	7		1607	13	
80	20	8	3150	9827	12121	2030	3164	177	4-	2485	278	10-
80	20	10	4342	9801	13719	2213	4178	753	4	2625	946	8
80	20	12	5508	9725	16676	2934	4736	1713	98	2480	1796	154
80	20	14	6654	9621	20410	4121	5041	2968	367	2262	2664	494
80	20	16	7788	9471	24530	5766	5188	4410	889	2037	3464	1048
80	20	18	8915	9280	28802	7868	5233	5957	1720	1827	4159	1801
80	25	•6	2029	12206	19152	4950	2010	14		2104	30	1-
80	25	8	3196	12212	17995	3652	3974	295	8-	3121	463	19-
80	25	10	4589	12153	20594	4073	3189	1198	12	3260	1505	23
80	25	12	5563	12025	25021	5410	5847	2678	199	3061	2805	313
80	25	14	6723	11839	30360	7521	6181	4559	716	2774	4092	963
80	25	16	7875	11582	36101	10377	6319	6677	1688	2481	5244	1989
80	30	•6	2084	14631	25346	7437	2523	26		2642	53	3-
80	30	•8	3246	14578	24613	5810	4795	453	14-	3766	711	33-
80	30	10	4440	14466	28433	6606	6185	1760	28	3886	2211	52
80	50	12	5624	14265	54492	8789	6922	3854	559	3624	4036	564
80	30	14	6799	13971	41554	12121	7271	6459	1237	3263	5797	1666
80	35	•8	3297	16936	31861	8510	5628	659	22-	4421	1035	51-
80	35	10	4495	16752	57107	9850	7171	2448	55	4505	3076	104
80	35	12	5690	16455	44919	13114	7967	5246	596	4172	5494	936
80	35	14	6882	16046	55769	17965	8319	8659	1971	3754	7773	2654
80	40	•8	3551	19293	39661	11757	6473	920	32-	5084	1445	75-
80	40	•10	4553	19016	46506	13824	8146	3271	101	5118	4110	191
80	40	•12	5760	18609	56177	18421	8988	6863	930	4706	7187	1460
90	5	4-	846	2475	2122	.0283	7625	.0000	0000	11978	.0000	.0000
90	3	6	1850	2494	1058	67	245			257		
90	5	8	3119	2498	770	32	656	5		515	7	
90	5	10	4403	2497	796	30	940	27		590	34	
90	5	12	5639	2503	948	37	1109	71		581	74	
90	5	14	6854	2498	1159	51	1210	132	3	543	118	4
90	5	16	8002	2494	1419	72	1274	207	7	500	163	9
90	5	18	9152	2495	1729	103	1321	298	16	461	208	17
90	5	20	10288	2488	2066	144	1349	398	30	424	250	28
90	10	•4-	898	4897	6837	1386	14510			22793	2	
90	10	6	1884	4957	3946	480	509			533		
90	10	8	3140	4976	2997	248	1317	19		1034	30	
90	10	10	4421	4977	3130	232	1875	109		1178	137	
90	10	12	5658	4968	3707	288	2201	281	4	1153	294	7
90	10	14	6858	4961	4543	597	2401	522	21	1077	469	29
90	10	16	8033	4945	5545	361	2524	819	59	991	643	70
90	10	18	9192	4923	6689	788	2602	1164	127	908	812	132
90	10	20	10340	4893	7938	1087	2648	1546	231	832	972	218
90	15	•4-	956	7335	12575	3835	20857	2		32762	7	5-
90	15	6	1930	7388	8110	1398	802			848	2	
90	15	8	3172	7428	6486	785	1986	46		1560	72	2-
90	15	10	4448	7429	6867	734	2805	249	1-	1762	313	2-
90	15	12	5688	7416	8171	944	3286	635	15	1720	665	23
90	15	14	6895	7375	9940	1290	3564	1165	72	1599	1046	97
90	15	16	8081	7329	12069	1808	3733	1815	198	1466	1425	253
90	15	18	9254	7263	14421	2507	3828	2551	415	1336	1781	435
90	15	20	10419	7175	16908	3401	3872	3348	744	1216	2104	701
90	20	•4-	1012	9835	18829	6829	26894	7	4-	42244	23	18-
90	20	•6	1983	9802	13111	2835	1130	2		1183	4	
90	20	8	3212	9854	11013	1728	2668	87	2-	2096	136	5-
90	20	10	4484	9845	11829	1702	3727	452	2-	2342	568	4-
90	20	12	5727	9803	14875	2157	4341	1129	37	2273	1183	58
90	20	14	6944	9729	17087	2916	4693	2055	173	2106	1844	252
90	20	16	8143	9623	20564	4040	4889	3161	461	1920	2482	543
90	20	18	9333	9488	24327	5530	4985	4393	950	1740	3067	995

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
90	20	20	10317	9313	28168	7370	5009	.5689	.1668	1574	.3573	.1572
90	25	•6	2038	12216	18682	4756	1493	4		1564	9	
90	23	8	3258	12231	16364	3112	3563	143	4-	2641	228	18-
90	25	10	4326	12226	17837	3148	4641	721	3-	2916	906	6-
90	23	12	3774	12143	21246	3970	5575	1749	76	2815	1833	119
90	23	14	7001	12003	25646	3384	5779	3174	540	2394	2849	457
90	25	16	8217	11833	30685	7408	3994	4837	889	2554	3799	1047
90	23	18	9424	11591	35832	9960	6068	6622	1784	2118	4623	1868
90	30	•6	2094	14645	24674	7123	1892	8		1981	17	
90	30	•8	3307	14634	22398	4952	4072	223	7-	3198	354	17-
90	50	10	4573	14376	24736	5154	3549	1064	3-	3486	1337	6-
90	30	12	3828	14442	29491	6505	6387	2538	159	5344	2679	219
90	30	14	7067	14235	35478	8799	6835	4333	595	3068	4869	801
90	30	16	8299	13957	42037	11957	7045	6810	1312	2766	5549	1782
90	30	18	9526	13609	48682	15902	7095	9221	2977	2477	6438	3118
90	35	•6	2150	17098	31005	9903	2524	14		2434	29	1-
90	33	•8	3358	17010	29003	7251	4797	531	12-	5768	520	28-
90	35	10	4624	16896	32387	7682	6449	1486		4852	1867	2-
90	35	12	5886	16698	38653	9783	7377	3499	235	3862	3664	369
90	35	14	7139	16392	46249	13162	7830	6110	956	3525	5484	1280
90	35	16	8387	16008	54454	17745	8050	9075	2371	3161	7126	2793
90	40	•8	3410	19384	36103	10009	5536	466	18-	4348	752	44-
90	48	•10	4678	19207	40744	10827	7549	1996	6	4618	2309	11
90	48	12	5948	18920	48614	13834	8548	4599	373	4571	4816	585
90	40	14	7215	18514	57949	18558	8840	7923	1451	5967	7112	1954
100	5	4-	846	2476	.2123	.0283	7627	.0000	.0000	11980	.0000	.0000
100	5	6-	1854	2495	1048	66	5208	13	1-	5454	32	4-
100	5	8	3167	2498	714	29	549	2		431	4	
100	3	10	4530	2498	697	24	835	16		323	21	
100	5	12	5847	2499	802	28	1010	47		529	49	
100	5	14	7119	2504	973	37	1120	92	1	503	85	2
100	5	16	8360	2506	1186	51	1192	150	4	468	118	5
100	5	18	9578	2499	1427	71	1237	218	9	452	152	9
100	5	20	10781	2480	1683	97	1261	292	17	396	183	16
100	10	•4-	898	4897	6836	1585	14509			22791		
100	10	6-	1888	4958	5903	472	10230	66	11-	10713	138	33-
100	10	8	3187	4978	2783	219	1103	9		866	15	
100	10	10	4546	4982	2747	185	1669	66		1049	83	
100	10	12	5863	4978	3163	216	2013	188	1	1054	197	2
100	10	14	7139	4981	3829	289	2228	368	10	1000	330	14
100	10	16	8384	4959	4624	396	2357	591	31	925	464	37
100	10	18	9610	4951	5577	553	2448	858	71	854	599	74
100	10	20	10823	4921	6598	755	2500	1153	133	785	725	125
100	13	•4-	956	7534	12573	3834	20836			32760	2	1-
100	15	6-	1935	7590	8016	1372	15009	167	38-	15717	349	120-
100	15	8	3219	7437	6033	697	1668	22		1310	35	
100	15	10	4572	7443	6044	604	2501	152	1-	1571	191	3-
100	15	12	5890	7432	6981	710	3006	425	5	1574	445	8
100	15	14	7170	7403	8590	938	3508	820	34	1485	736	46
100	15	16	8424	7374	10143	1292	3499	1316	105	1374	1034	123
100	15	18	9661	7328	12125	1779	3616	1892	233	1262	1321	244
100	15	20	10887	7270	14287	2417	3685	2532	436	1138	1591	411
100	20	•4-	1012	9835	18827	6828	26893	2	1-	42244	7	5-
108	20	•6-	1988	9806	12946	2778	19564	337	98-	20487	705	307-
100	20	8	3259	9870	10249	1534	2247	43		1765	67	2-
100	20	10	4605	9874	10442	1370	3329	276	3-	2091	346	6-
100	20	12	5924	9848	12102	1621	3983	759	13	2086	794	20
100	20	14	7211	9798	14540	2144	4374	1453	83	1963	1304	112
100	20	16	8476	9727	17473	2931	4607	2311	247	1809	1815	291
100	20	18	9727	9625	20713	3992	4738	3290	541	1654	2297	567
100	20	20	10970	9513	24219	5367	4807	4366	998	1510	2743	941
100	25	•6-	2043	12222	18425	4652	23938	598	206-	25068	1233	446-
100	25	8	3304	12275	15228	2761	2840	72	2-	2231	113	5-
100	25	10	4645	12276	15796	2543	4154	442	6-	2610	555	11-
100	23	12	5966	12225	18370	3033	4946	1193	27	2590	1249	42
100	25	14	7261	12133	22014	4005	5408	2259	165	2427	2028	223
100	25	16	8538	12001	26287	5433	5671	3557	480	2227	2794	566
100	25	18	9805	11828	30925	7324	5805	5014	1032	2026	3501	1081
100	25	20	11066	11615	35706	9673	5850	6562	1861	1838	4123	1754
100	30	•6-	2100	14653	24299	6950	28169	975	382-	29499	2043	1201-
100	30	•8	3353	14672	20852	4395	3451	113	4-	2711	177	8-
100	50	10	4689	14652	21973	4164	4978	654	9-	3128	822	17-
100	30	12	6014	14563	25627	5002	5892	1730	51	3085	1812	80
100	30	14	7318	14415	30632	6595	6415	3237	291	2879	2906	392
100	30	16	8609	14212	36383	8890	6698	5048	827	2630	3965	974
100	30	18	9892	13944	42437	11850	6821	7037	1741	2381	4913	1823
100	35	•6-	2136	17107	30484	9639	32288	1494	651-	33812	3130	2044-
100	35	•8	3405	17061	27000	6433	4079	167	6-	3204	262	14-
100	35	10	4737	17002	28852	6251	5800	917	13-	3644	1153	24-

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
100	35	12	6066	16864	33744	.7566	6823	.2375	.0088	3572	.2487	.0138
100	35	14	7381	16647	40225	9959	7394	4387	473	3319	3938	637
100	35	16	8686	16350	47479	13323	7684	6763	1307	3017	5312	1540
100	35	18	9987	15971	54935	17573	7786	9325	2696	2718	6510	2824
100	40	•8	3457	19445	53587	8865	4723	238	9-	5709	373	22-
100	40	•10	4788	19336	56356	8823	6622	1236	16-	4161	1553	31-
100	40	12	6122	19135	42618	10751	7739	3132	145	4052	3279	224
100	40	14	7448	18835	50660	14128	8349	5709	723	3747	5124	974
100	40	16	8769	18435	59471	18783	8637	8706	1947	3392	6838	2294
110	5	4-	846	2475	.2123	.0283	7626	.0000	0000	11978	.0000	.0000
110	5	6-	1856	2494	1041	66	5203	8		5449	16	2-
110	5	8	3202	2497	676	26	458	1		360	2	
110	5	10	4637	2498	624	20	744	10		468	13	
110	5	12	6032	2498	695	21	924	32		484	33	
110	5	14	7379	2503	829	28	1039	66		466	59	
110	5	16	8690	2495	994	37	1109	109	2	436	86	2
110	5	18	9975	2509	1209	52	1167	164	5	407	115	5
110	5	20	11243	2488	1419	70	1193	222	10	375	139	9
110	10	•4-	898	4897	6836	1585	14509			22791		
110	10	6-	1890	4958	3880	467	10226	34	5-	10708	71	17-
110	10	8	5222	4981	2638	201	923	5		725	7	
110	10	10	4652	4980	2459	154	1486	41		934	51	
110	10	12	6046	4988	2757	169	1847	128		967	134	
110	10	14	7395	4985	3275	217	2069	263	5	929	236	7
110	10	16	8710	4981	3945	293	2214	438	17	869	344	20
110	10	18	10002	4972	4730	402	2310	646	41	806	451	43
110	10	20	11277	4949	5591	545	2370	881	80	745	554	75
110	15	•4-	956	7334	12573	3834	20856			32760		
110	15	6-	1937	7391	7967	1558	15004	88	20-	15712	183	63-
110	15	8	3254	7442	5718	638	1398	11		1098	17	
110	15	10	4676	7451	5429	503	2231	94	1-	1402	118	2-
110	15	12	6070	7446	6090	556	2758	290	1	1444	503	2
110	15	14	7422	7434	7232	711	3084	590	17	1384	530	22
110	15	16	8745	7408	8670	957	3289	975	57	1292	766	68
110	15	18	10045	7375	10344	1304	3422	1431	136	1194	999	142
110	15	20	11350	7328	12181	1761	3504	1942	263	1101	1220	248
110	20	•4-	1012	9855	18827	6828	26893			42244	2	2-
110	20	•6-	1990	9807	12857	2747	19560	180	53-	20483	377	165-
110	20	8	3294	9878	9713	1405	1888	21		1483	33	
110	20	10	4708	9897	9410	1146	2976	170	2-	1870	214	5-
110	20	12	6101	9880	10596	1276	3662	519	3	1918	543	5
110	20	14	7458	9847	12567	1632	4083	1047	40	1832	940	54
110	20	16	8787	9791	15016	2188	4341	1718	136	1705	1349	160
110	20	18	10099	9719	17811	2958	4500	2502	317	1571	1747	332
110	20	20	11599	9624	20837	3958	4591	3372	607	1442	2119	572
110	25	•6-	2046	12225	18281	4594	23935	325	113-	25065	681	354-
110	25	8	3340	12294	14445	2531	2595	36		1881	56	2-
110	25	10	4746	12511	14258	2131	3720	274	5-	2337	344	9-
110	25	12	6159	12280	16143	2399	4555	817	8	2385	856	12
110	25	14	7501	12216	19124	3069	5061	1633	81	2271	1465	109
110	25	16	8841	12120	22772	4099	5365	2659	266	2107	2088	314
110	25	18	10166	12001	26890	5513	5545	3848	612	1955	2687	641
110	25	20	11482	11819	31118	7263	5623	5123	1150	1766	5219	1083
110	30	•6-	2105	14656	24086	6854	28171	539	214-	29500	1130	671-
110	50	•8	5589	14700	19771	4026	2919	56	2-	2293	88	4-
110	30	10	4789	14706	19878	3499	4467	407	8-	2806	511	15-
110	30	12	6182	14651	22617	3979	5439	1190	15	2848	1246	24
110	30	14	7552	14550	26783	5096	6022	2351	144	2703	2110	194
110	30	16	8902	14387	31718	6762	6356	3789	461	2496	2976	543
110	50	18	10242	14188	37168	9003	6538	5429	1040	2282	3790	1089
110	30	20	11575	13926	42754	11761	6605	7170	1926	2075	4505	1815
110	35	•8	3440	17093	25574	5882	3460	84	3-	2718	132	7-
110	35	10	4835	17075	26139	5261	5213	572	12-	3275	719	23-
110	35	12	6230	16985	29879	6045	6310	1637	28	3304	1714	44
110	35	14	7607	16829	35350	7739	6957	3195	236	3122	2867	317
110	35	16	8970	16594	41663	10222	7315	5101	734	2872	4006	865
110	55	18	10325	16510	48555	15508	7495	7244	1627	2616	5057	1704
110	35	20	11675	15954	55437	17494	7542	9489	2969	2369	5962	2798
110	40	•8	3493	19487	31797	8097	4019	121	4-	3157	190	11-
110	40	•10	4884	19429	32990	7439	5963	774	17-	3747	973	32-
110	40	12	6282	19292	37858	8620	7172	2164	48	3755	2266	75
110	40	14	7668	19067	44680	11035	7871	4168	363	3533	5741	488
110	40	16	9045	18754	52506	14525	8247	6596	1102	3239	5181	1298
110	40	18	10414	18370	60791	19046	8415	9281	2396	2937	6479	2509
120	5	4-	846	2475	.2122	.0283	7625	0000	.0000	11978	0000	.0000
120	5	6-	1857	2494	1039	65	5202	4		5448	8	1-
120	5	8	3227	2498	651	25	382			500		
120	5	12	6196	2502	.0617	.0017	.850	.0022	.0000	.445	.0023	0000
120	5	14	7615	2500	719	21	967	48		434	43	
120	5	16	8995	2497	856	28	1042	82	1	409	64	1
120	5	18	10346	2494	1021	38	1094	124	3	382	86	3
120	5	20	11677	2503	1223	52	1136	173	6	357	109	6
120	10	•4-	898	4897	6836	1585	14510			22792		
120	10	6-	1891	4958	3869	465	10223	18	3-	10706	57	9-
120	10	8	3248	4981	2537	189	770	2		605	3	
120	10	10	4741	4988	2252	132	1328	25		834	32	
120	10	12	6209	4982	2432	136	1693	88		887	95	
120	10	14	7630	4983	2844	168	1926	191	2	864	172	3
120	10	16	9012	4979	3592	221	2077	328	9	816	257	11
120	10	18	10368	4985	4067	301	2185	495	24	765	346	25
120	10	20	11704	4965	4796	405	2251	684	49	707	430	46
120	15	6-	1938	7391	7941	1351	15002	46	11-	15710	96	33-
120	15	8	5280	7445	5500	599	1170	5		919	8	
120	15	10	4764	7459	4969	453	1993	58		1252	73	1-
120	15	12	6231	7457	5403	450	2536	201		1328	210	
120	15	14	7653	7454	6322	556	2881	451	8	1293	587	11
120	15	16	9041	7435	7522	732	3100	735	32	1217	577	38
120	15	18	10403	7401	8918	981	3241	1098	81	1131	766	85
120	15	20	11749	7368	10505	1317	5336	1513	163	1048	951	154
120	20	•6-	1992	9808	12809	2751	19557	96	28-	20481	201	08-
120	20	8	3320	9887	9546	1520	1584	10		1244	16	
120	20	10	4795	9908	8617	987	2661	106	2-	1672	155	3-
120	20	12	6259	9901	9423	1036	3372	360		1765	377	
120	20	14	7685	9682	11018	1280	3818	766	19	1714	688	26
120	20	16	9079	9837	13066	1679	4097	1297	76	1609	1019	90
120	20	18	10451	9794	15505	2254	4282	1935	191	1495	1551	200
120	20	20	11008	9713	18122	2994	4589	2645	380	1379	1662	358
120	25	•6-	2048	12226	18203	4563	23935	177	62-	25064	570	193-
120	25	8	3366	12307	13889	2375	2015	18		1582	28	
120	25	10	4832	12335	13083	1841	3332	171	3-	2093	215	6-
120	25	12	6294	12321	14400	1956	4201	568		2200	595	
120	25	14	7724	12273	16820	2417	4740	1197	39	2127	1074	53
120	25	16	9125	12206	19934	5171	5078	2016	150	1994	1583	177
120	25	18	10508	12105	23478	4215	5283	2979	369	1844	2079	387
120	25	20	11080	11977	27323	5568	5400	4050	727	1697	2545	685
120	30	•6-	2105	14659	25970	6801	28173	298	119-	29502	624	373-
120	50	•8	3415	14714	18997	3772	2463	28		1934	44	2-
120	30	10	4875	14740	18258	3026	4006	255	6-	2517	320	11-
120	30	12	6535	14708	20218	3253	5022	828	2	2630	867	2
120	30	14	7769	14631	23627	4028	5648	1726	70	2555	1549	95
120	30	16	9180	14526	27940	5277	6054	2886	262	2370	2267	309
120	30	18	10575	14360	32717	6961	6254	4229	633	2183	2952	663
120	30	20	11961	14163	37859	9120	6370	5706	1230	2001	5585	1159
120	35	8	3467	17116	24566	5507	2929	43	1-	2300	67	3-
120	55	•10	4918	17127	24051	4560	4684	360	9-	2943	452	17-
120	35	12	6579	17070	26792	4959	5837	1141	4	3056	1195	7
120	35	14	7819	16954	31315	6156	6541	2553	116	2935	2112	157
120	35	16	9239	16782	36872	8024	6960	3897	420	2733	3061	495
120	35	18	10648	16560	43046	10551	7196	5677	998	2512	3963	1046
120	35	20	12050	16273	49437	13694	7299	7594	1910	2293	4771	1800
120	40	•8	3520	19517	50520	7571	3413	62	2-	2680	97	5-
120	40	•10	4966	19495	50374	6451	5366	489	13-	3371	614	25-
120	40	12	6428	19404	34026	7095	6644	1512	9	3479	1584	15
120	40	14	7874	19241	39781	8829	7418	3081	181	3329	2766	243
120	40	16	9505	19005	46697	11474	7866	5057	633	3089	3972	746
120	40	18	10728	18693	54220	14986	8102	7304	1479	2828	5099	1549
120	40	20	12145	18301	61843	19271	8185	9682	2784	2571	6084	2624
130	5	4-	846	2475	.2122	.0283	7625	.0000	.0000	11978	.0000	0000
130	5	6-	1857	2494	1037	65	5202	2		5448	4	
130	5	8	3245	2498	653	23	318			250		
130	5	10	4801	2499	529	15	592	4		372	5	
150	5	12	6342	2491	549	14	778	15		408	16	
130	5	14	7832	2500	635	17	902	35		405	32	
150	5	16	9278	2506	753	22	985	63		387	49	
130	5	18	10693	2509	896	29	1041	97	2	363	68	2
150	5	20	12085	2496	1051	39	1075	135	4	338	85	4
130	10	•4-	898	4897	6857	1586	14511			22793		
130	10	6-	1892	4960	3865	464	10225	9	1-	10707	19	5-
130	10	8	3266	4983	2468	180	642	1		504	2	
130	10	10	4815	4989	2089	117	1185	16		745	20	
130	10	12	6354	4990	2190	113	1560	62		817	65	
130	10	14	7845	4991	2515	134	1801	142	1	808	127	1
130	10	16	9294	4989	2973	173	1960	250	5	770	197	6
130	10	18	10712	4967	3503	228	2060	381	15	719	266	15
150	10	20	12108	4980	4168	509	2144	540	31	673	359	29

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_5	V_1	V_2	V_5	O_1	O_2	O_5
130	15	8	3298	7448	.5349	.0575	.977	.0003	.0000	.768	.0004	.0000
150	15	10	4038	7463	4613	583	1781	36		1119	46	
150	13	12	6374	7469	4069	575	2338	141		1224	147	
150	15	14	7866	7457	5584	444	2692	319	4	1268	286	5
150	15	16	9318	7446	6390	572	2924	561	18	1148	440	21
130	15	18	10742	7422	7785	737	3076	853	49	1074	597	52
130	15	20	12146	7407	9178	1012	3185	1199	104	1001	733	98
150	20	•6-	1992	9809	12704	2722	19356	51	15-	20479	107	47-
130	20	8	3338	9890	9084	1260	1326	3		1042	8	
130	20	10	4868	9919	8012	873	2380	67	1-	1495	84	2-
130	20	12	6401	9917	8497	864	5109	252	1-	1628	264	2-
130	20	14	7894	9902	9768	1028	3574	560	9	1604	309	12
130	20	16	9351	9067	11494	1320	3872	992	45	1520	779	51
130	20	18	10703	9829	13369	1745	4069	1509	117	1420	1053	122
150	20	20	12198	9790	15933	2321	4204	2106	244	1321	1323	230
150	23	•6-	2049	12227	18162	4546	23934	96	39-	23064	201	105-
130	25	8	3384	12314	13498	2268	1692	9		1329	14	
150	23	10	4904	12334	12177	1632	2984	107	2-	1073	135	4-
130	25	12	6434	12340	13009	1656	3877	399	2-	2050	417	3-
130	23	14	7929	12315	14970	1951	4443	889	18	1995	798	25
130	25	16	9393	12266	17616	2508	4809	1347	86	1888	1215	101
130	25	18	10034	12196	20730	3304	3042	2340	228	1760	1634	239
150	25	20	12260	12101	24166	4351	5187	3238	470	1630	2035	443
130	30	•6-	2106	14660	23905	6772	20174	164	66-	29503	344	206-
130	50	•8	3434	14726	18434	3599	2074	14		1629	22	
130	30	10	4945	14769	17009	2665	3392	161	4-	2237	202	7-
130	30	12	6472	14753	18309	2729	4641	382	3-	2430	609	5-
130	30	14	7970	14703	21106	3269	5507	1285	33	2382	1153	45
130	30	16	9441	14612	24765	4189	5722	2218	150	2247	1742	177
130	30	18	10892	14491	29010	5486	5981	3331	392	2088	2325	410
130	30	20	12331	14340	33649	7177	6134	4580	798	1927	2878	752
150	33	•8	3487	17133	23850	5249	2475	21		1943	34	1-
150	35	10	4989	17166	22422	4049	4206	228	6-	2643	286	11-
130	35	12	6514	17135	24313	4172	5401	804	4-	2828	842	6-
130	35	14	8016	17051	28046	5011	6153	1755	56	2762	1575	73
130	35	16	9494	16920	32861	6417	6619	3007	242	2399	2362	285
130	35	18	10957	16740	38339	8364	6897	4485	621	2407	3131	650
130	33	20	12410	16516	44218	10860	7049	6121	1247	2214	3846	1175
130	40	•8	3540	19539	29609	7206	2892	31		2272	49	2-
130	40	10	5036	19546	28333	5731	4826	310	9-	3032	390	17-
130	40	12	6560	19494	30947	5985	6157	1068	5-	3224	1118	7-
130	40	14	8066	19371	35732	7214	6989	2302	88	3137	2066	118
130	40	16	9533	19186	41783	9224	7495	3913	367	2943	3073	433
130	40	18	11028	18941	48573	11972	7788	5796	926	2718	4047	970
130	40	20	12493	18631	35711	13432	7952	7852	1833	2492	4933	1728
140	5	4-	046	2475	2122	.0283	7623	.0000	.0000	11978	.0000	.0000
140	5	6-	1858	2494	1036	65	5201	1		3447	2	
140	5	8	3258	2498	620	23	265			208		
140	5	10	4863	2499	497	13	528	2		332	3	
140	5	12	6472	2505	304	12	721	11		377	11	
140	5	14	8030	2502	567	14	845	26		379	24	
140	5	16	9542	2497	660	17	926	48		364	38	
140	5	18	11019	2498	780	23	984	76	1	343	53	1
140	5	20	12471	2507	926	31	1028	109	3	323	68	2
140	10	•4-	898	4897	6836	1385	14509			22791		
140	10	6-	1892	4958	3860	464	10221	5		10704	10	2-
140	10	8	3279	4983	2420	173	339			420		
140	10	10	4077	4990	1965	106	1058	10		665	12	
140	10	12	6483	4991	1994	96	1438	44		753	46	
140	10	14	8042	4990	2246	110	1685	106		756	95	
140	10	16	9535	4988	2626	138	1850	193	3	726	152	4
140	10	18	11055	4983	3096	180	1961	301	9	685	210	10
140	10	20	12491	4968	3628	238	2037	428	20	640	269	19
140	15	6-	1939	7392	7919	1346	14999	13	3-	13707	26	9-
140	15	8	3311	7451	5246	555	815	1		640	2	
140	13	10	4899	7468	4343	346	1591	23		1000	29	
140	15	12	6502	7463	4425	318	2153	99		1127	104	
140	15	14	8061	7466	5001	364	2522	239	2	1132	214	2
140	15	16	9577	7458	5844	458	2765	434	10	1086	341	12
140	15	18	11062	7444	6880	399	2928	676	31	1022	472	32
140	15	20	12524	7441	8106	795	3048	963	68	937	605	64
140	20	•6-	1993	9809	12770	2718	19556	27	8-	20479	57	25-
140	20	8	3352	9893	8903	1220	1109	2		871	4	
140	20	10	4929	9926	7542	790	2128	42		1337	53	1-
140	20	12	6528	9934	7764	739	2870	178	1-	1303	187	2-
140	20	14	8087	9919	8765	845	3352	425	4	1504	382	5
140	20	16	9607	9893	10224	1061	3666	769	25	1440	604	29
140	20	18	11098	9866	12019	1384	3878	1194	73	1354	833	76
140	20	20	12568	9832	14074	1823	4022	1689	159	1264	1061	150

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	O_1	O_2	O_5
140	25	•6-	2049	12229	18140	.4538	23935	.0052	.0018-	25065	.0109	.0037-
140	25	8	3390	12320	13225	2194	1418	4		1114	7	
140	25	10	4964	12367	11469	1477	2671	68	1-	1678	83	2-
140	25	12	6539	12369	11893	1399	3581	202	3-	1875	296	4-
140	25	14	8119	12347	13462	1610	4174	667	8	1873	599	11
140	23	16	9644	12307	13704	2023	4358	1200	49	1790	942	58
140	25	10	11142	12248	18399	2627	4809	1855	142	1679	1293	149
140	23	20	12622	12174	21456	3437	4973	2604	307	1562	1636	289
140	30	•6-	2106	14662	25871	6756	28173	91	36-	29505	190	114-
140	30	•8	3448	14734	18070	3479	1744	7		1370	11	
140	30	10	5005	14793	16038	2433	3220	102	2-	2023	128	4-
140	30	12	6595	14707	16765	2339	4292	413	4-	2247	432	7-
140	30	14	8157	14751	19021	2703	4988	966	15	2238	867	20
140	30	16	9687	14687	22175	3399	5435	1723	87	2134	1353	102
140	30	18	11194	14591	25905	4400	5721	2650	246	1997	1850	258
140	30	20	12685	14464	30029	5718	5899	3701	525	1833	2325	494
140	35	•8	3501	17146	23344	3070	2087	11		1639	17	
140	35	10	5049	17196	21144	3670	3773	144	4-	2372	182	7-
140	35	12	6633	17188	22311	3586	5001	572	6-	2619	399	10-
140	35	14	8199	17123	23344	4161	5791	1321	25	2599	1106	54
140	33	16	9735	17027	29519	5229	6296	2344	141	2472	1841	166
140	35	18	11252	16881	34368	6742	6609	3577	392	2307	2497	410
140	35	20	12756	16712	39752	8741	6803	4977	827	2137	3127	780
140	40	•8	3555	19352	28952	6950	2446	16		1921	25	
140	40	10	5096	19388	26736	3198	4538	198	6-	2726	240	11-
140	40	12	6679	19362	28432	5152	5706	760	9-	2988	796	13-
140	40	14	8245	19470	32370	6009	6386	1736	40	2956	1358	54
140	40	16	9789	19329	37647	7545	7141	5036	214	2884	2400	232
140	40	18	11316	19131	43724	9704	7478	4637	587	2610	3237	614
140	40	20	12832	18802	30286	12486	7670	6402	1220	2410	4022	1150
150	5	4-	846	2473	.2122	.0283	7623	.0000	.0000	11978	.0000	0000
150	5	6-	1858	2492	1034	65	5197			5442	1	
150	5	8	3267	2498	612	22	220			172		
150	5	10	4914	2498	472	12	471	2		296	2	
150	3	12	6387	2498	462	11	663	8		347	8	
150	5	14	8212	2500	511	12	792	20		355	18	
150	3	16	9788	2503	392	14	878	38		345	30	
150	5	18	11326	2510	698	19	940	61		328	42	
150	5	20	12836	2503	815	24	980	87	2	308	35	2
150	10	•4-	898	4897	6836	1383	14509			22791		
150	10	6-	1892	4938	3838	463	10221	2		10704	5	1-
130	10	8	3288	4984	2387	171	444			349		
150	10	10	4928	4989	1866	97	944	6		593	8	
150	10	12	6598	4996	1839	84	1327	31		695	33	
130	10	14	8223	4993	2031	92	1581	80		710	72	
150	10	16	9800	4998	2355	113	1733	151	2	688	119	2
150	10	18	11340	4986	2751	145	1867	240	6	652	160	6
150	10	20	12853	4998	3244	191	1955	348	13	614	219	13
150	15	6-	1939	7591	7914	1344	14997	7	2-	15705	14	3-
150	15	8	3320	7451	5172	543	679			553		
150	13	10	4950	7469	4128	318	1420	14		892	18	
150	15	12	6616	7471	4084	277	1988	71		1041	74	
150	15	14	8240	7462	4514	304	2364	180		1061	161	
150	13	16	9819	7464	5232	374	2618	339	6	1028	266	7
150	15	18	11363	7433	6123	481	2789	339	19	973	376	20
150	15	20	12882	7438	7159	628	2908	774	44	914	487	42
150	20	•6-	1993	9810	12765	2716	19557	15	4-	20480	30	13-
150	20	8	3361	9896	8777	1193	926	1		727	2	
150	20	10	4979	9935	7179	728	1903	26		1195	33	
150	20	12	6640	9938	7157	643	2650	127	1-	1387	133	2-
150	20	14	8264	9932	7943	709	3148	322	1	1413	289	2
150	20	16	9846	9917	9185	871	3477	602	14	1365	473	17
150	20	18	11395	9900	10750	1121	3702	955	46	1292	667	48
150	20	20	12920	9836	12510	1454	3850	1366	105	1209	858	99
150	25	•6-	2049	12228	18126	4532	23934	28	10-	25063	59	31-
150	25	8	3408	12324	13033	2143	1187	2		933	3	
150	25	10	5015	12377	10915	1361	2389	43		1501	34	1-
150	23	12	6670	12389	10993	1222	3311	201	2-	1734	211	4-
150	25	14	8294	12372	12225	1354	3923	503	3	1761	434	4
150	25	16	9879	12332	14109	1659	4323	939	28	1697	737	33
150	25	18	11434	12293	16474	2129	4593	1483	90	1603	1035	94
150	25	20	12969	12231	19214	2775	4780	2122	204	1502	1333	193
150	30	•6-	2107	14661	23849	6746	28174	30	20-	29504	105	63-
150	30	•8	3458	14741	17801	3396	1464	3		1150	5	
150	30	10	5055	14805	15261	2241	2884	64	1-	1812	81	3-
150	30	12	6705	14814	15508	2044	3970	295	4-	2079	308	6-
150	30	14	8329	14787	17299	2280	4692	732	5	2106	657	7
150	30	16	9918	14737	19988	2801	5163	1353	50	2028	1063	39
150	30	18	11481	14671	23296	3587	5476	2128	157	1912	1485	164

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	σ_1	σ_2	σ_3
150	30	20	13024	14566	26978	.4629	5675	3019	0350	1783	1897	0330
150	35	• 8	3512	17152	22976	4942	1757	5		1380	8	
150	35	10	5099	17214	20126	3381	3385	92	2-	2127	115	4-
150	35	12	6744	17221	20650	3136	4629	408	6-	2424	428	9-
150	35	14	8368	17179	23099	3517	5454	1003	10	2448	900	13
150	35	16	9963	17101	26679	4324	5989	1841	81	2352	1446	96
150	35	18	11533	16989	30990	5516	6334	2877	250	2211	2008	262
150	35	20	13088	16842	35794	7095	6552	4064	553	2058	2554	521
150	40	• 8	3566	19562	28486	6771	2066	8		1623	13	
150	40	10	5145	19616	25462	4792	3896	126	4-	2448	158	7-
150	40	12	6786	19611	26356	4515	5288	544	9-	2769	570	14-
150	40	14	8412	19553	29577	5096	6211	1320	16	2787	1185	21
150	40	16	10012	19435	34125	6264	6802	2405	124	2671	1889	147
150	40	18	11591	19269	39519	7962	7175	3733	375	2505	2606	392
150	40	20	13157	19071	45515	10207	7406	5249	0821	2327	3298	.0774
160	5	4-	846	2475	2123	02827	7626	.0000	.00000	11979	.0000	.00000
160	5	6-	1858	2494	1035	650	5200			5446		
160	5	8	3273	2498	606	219	182			143		
160	5	10	4956	2498	452	115	420			264	1	
160	5	12	6688	2505	432	94	614	6		321	6	
160	5	14	8378	2499	465	98	743	15		333	14	
160	5	16	10017	2498	532	117	830	30	1	326	23	1
160	5	18	11615	2512	627	151	897	49	5	313	34	5
160	5	20	13182	2501	726	194	936	71	11	294	45	10
160	10	• 4-	898	4897	6836	15853	14509			22791		
160	10	6-	1892	4958	3857	4631	10221	1	2-	10703	3	6-
160	10	8	3294	4984	2364	1683	369			290		
160	10	10	4970	4988	1789	905	841	4		529	5	
160	10	12	6699	4992	1707	739	1224	22	1-	641	23	2-
160	10	14	8388	4992	1852	780	1484	61		666	55	
160	10	16	10028	4980	2109	927	1656	118	10	650	93	11
160	10	18	11628	4992	2469	1184	1781	193	36	622	135	38
160	10	20	13197	4984	2876	1529	1865	282	88	586	177	83
160	15	6-	1939	7392	7914	13441	14999	3	8-	15707	7	25-
160	15	8	3327	7451	5120	5340	565			443		
160	15	10	4992	7471	3961	2975	1268	9		797	11	2-
160	15	12	6717	7473	3799	2453	1836	50	4-	961	53	6-
160	15	14	8405	7474	4131	2599	2223	137		998	123	
160	15	16	10046	7470	4727	3112	2483	267	33	975	210	39
160	15	18	11648	7468	5508	3947	2663	434	123	930	303	129
160	15	20	13222	7441	6391	5071	2783	631	296	874	396	279
160	20	• 6-	1993	9809	12759	27140	19555	8	23-	20478	16	72-
160	20	8	3368	9898	8690	11736	772			606		
160	20	10	5021	9936	6886	6800	1699	17	2-	1067	21	4-
160	20	12	6740	9939	6659	5686	2447	91	10-	1281	95	15-
160	20	14	8427	9941	7263	6056	2959	245		1328	220	
160	20	16	10070	9930	8312	7257	3300	475	81	1296	373	95
160	20	18	11677	9912	9659	9176	3533	769	293	1233	537	307
160	20	20	13257	9886	11234	11835	3695	1118	704	1161	703	663
160	25	• 6-	2050	12228	18119	45293	23934	15	53-	25063	32	168-
160	25	8	3415	12327	12898	21077	993	1		780	2	
160	25	10	5056	12385	10479	12724	2137	27	5-	1342	34	9-
160	25	12	6769	12398	10244	10833	3061	144	19-	1603	151	29-
160	25	14	8455	12389	11193	11582	3690	386	3	1656	346	4
160	25	16	10100	12366	12809	13892	4109	743	161	1614	583	189
160	25	18	11712	12324	14850	17513	4390	1196	575	1532	835	602
160	25	20	13299	12280	17242	22547	4585	1735	1371	1440	1090	1292
160	30	• 6-	2107	14661	23838	67416	28174	28	110-	29504	58	347-
160	30	• 8	3466	14745	17609	33376	1228	2		964	3	
160	30	10	5096	14817	14653	20962	2581	41	8-	1622	51	16-
160	30	12	6803	14836	14478	18173	3674	211	32-	1924	221	50-
160	30	14	8488	14822	15877	19561	4419	559	9	1983	502	12
160	30	16	10136	14779	18163	23473	4910	1071	285	1928	841	336
160	30	18	11754	14732	21080	29662	5244	1722	1005	1830	1202	1052
160	30	20	13350	14653	24396	38028	5465	2484	2371	1717	1561	2235
160	35	• 8	3519	17157	22720	48541	1478	3		1161	4	1-
160	35	10	5140	17235	19334	31644	3034	58	14-	1907	73	26-
160	35	12	6841	17251	19291	27902	4287	293	50-	2245	307	79-
160	35	14	8525	17223	21220	30216	5139	767	20	2306	688	26
160	35	16	10177	17166	24299	36351	5702	1459	466	2239	1146	549
160	35	18	11801	17073	28104	45739	6072	2331	1605	2119	1627	1681
160	35	20	13406	16956	32440	58453	6315	3348	3749	1984	2104	3533
160	40	• 8	3574	19570	28154	66446	1743	4		1369	6	2-
160	40	10	5187	19641	24455	44829	3496	80	22-	2197	101	41-
160	40	12	6882	19651	24643	40208	4901	391	73-	2566	410	115-
160	40	14	8566	19607	27196	43825	5855	1010	39	2628	906	53
160	40	16	10222	19523	31147	52801	6483	1909	717	2546	1499	845
160	40	18	11854	19392	35965	66336	6890	3033	2419	2405	2117	2533
170	5	4-	846	2475	2122	02827	7625	.0000	00000	11978	0000	00000
170	5	6-	1858	2494	1036	650	5201			5447		
170	5	8-	3278	2498	602	217	3924	8	4-	3082	13	9-
170	5	10	4990	2497	437	109	374			235		
170	5	12	6779	2501	404	84	566	4		296	4	
170	5	14	8531	2490	425	84	696	12		312	10	
170	5	16	10231	2500	484	99	789	24		310	19	
170	5	18	11888	2501	561	124	852	39	3	297	27	3
170	5	20	13511	2499	651	158	896	58	7	282	37	7
170	10	• 4-	898	4897	6836	15854	14510			22792		
170	10	6-	1892	4958	3857	4630	10221		1-	10703	1	3-
170	10	8-	3299	4984	2347	1665	7797	34	32-	6124	54	76-
170	10	10	5004	4993	1732	858	751	2		472	3	
170	10	12	6789	4994	1603	665	1131	16		592	17	1-
170	10	14	8540	4991	1703	674	1395	47		626	42	
170	10	16	10241	4994	1929	788	1576	94	5	619	74	6
170	10	18	11899	5000	2236	986	1703	157	23	594	110	24
170	10	20	13525	4999	2601	1264	1792	233	60	563	147	57
170	15	6-	1939	7392	7913	13438	14999	2	4-	15707	4	13-
170	15	8-	3331	7451	5085	5282	11584	81	111-	9098	127	262-
170	15	10	5026	7473	3831	2817	1131	6		710	7	1-
170	15	12	6806	7483	3574	2213	1698	36	3-	889	38	5-
170	15	14	8556	7478	3806	2251	2091	105	2-	938	95	3-
170	15	16	10257	7475	4305	2629	2358	212	18	926	167	22
170	15	18	11918	7479	4990	3286	2546	353	79	889	247	83
170	15	20	13547	7468	5789	4199	2677	522	202	841	328	191
170	20	• 6-	1993	9809	12757	27133	19555	4	12-	20478	9	38-
170	20	8-	3373	9901	8632	11610	15272	152	272-	11995	239	640-
170	20	10	5055	9939	6658	6437	1516	10	1-	953	13	3-
170	20	12	6829	9951	6265	5127	2264	65	7-	1185	68	12-
170	20	14	8577	9949	6696	5252	2784	188	5-	1249	169	6-
170	20	16	10280	9941	7579	6141	3136	378	44	1232	297	52
170	20	18	11944	9933	8764	7656	3380	626	188	1180	437	197
170	20	20	13578	9905	10143	9751	3548	922	478	1115	579	450
170	25	• 6-	2050	12228	18115	45279	23934	8	29-	25063	17	91-
170	25	8-	3420	12329	12804	20831	18846	253	548-	14802	398	1292-
170	25	10	5091	12390	10134	12046	1909	17	3-	1199	21	5-
170	25	12	6857	12407	9630	9757	2831	104	14-	1482	109	22-
170	25	14	8604	12400	10326	10053	3473	296	8-	1559	266	11-
170	25	16	10308	12390	11705	11790	3909	591	89	1535	465	105
170	25	18	11975	12350	13479	14616	4201	973	369	1466	679	387
170	25	20	13616	12305	15580	18592	4404	1430	930	1384	898	877
170	30	• 6-	2107	14662	23832	67389	28174	15	61-	29504	32	191-
170	30	• 8-	3471	14748	17475	32969	22325	390	982-	17534	612	2314-
170	30	10	5131	14826	14172	19848	2309	26	5-	1451	32	9-
170	30	12	6890	14854	13625	16392	3401	152	25-	1781	159	39-
170	30	14	8635	14840	14660	16997	4161	430	12-	1867	386	16-
170	30	16	10341	14807	16603	19927	4672	852	158	1835	669	187
170	30	18	12014	14772	19170	24818	5022	1402	647	1753	979	677
170	30	20	13661	14698	22104	31472	5255	2052	1614	1651	1289	1521
170	35	•8-	3524	17161	22539	47922	25717	568	1619-	20198	892	3815-
170	35	10	5175	17248	18700	29960	2717	37	8-	1707	47	15-
170	35	12	6927	17274	18163	25183	3970	212	39-	2079	222	61-
170	35	14	8670	17258	19635	26332	4844	590	15-	2174	529	21-
170	35	16	10379	17210	22258	30945	5430	1163	261	2132	914	307
170	35	18	12057	17139	25630	38416	5823	1902	1037	2033	1328	1086
170	35	20	13713	17049	29550	48746	6090	2778	2568	1913	1745	2420
170	40	• 8-	3580	19575	27918	65557	29030	794	2510-	22800	1247	5914-
170	40	10	5221	19659	23651	42435	3135	51	13-	1969	64	24-
170	40	12	6968	19686	23225	36338	4543	283	58-	2379	296	91-
170	40	14	8709	19654	25194	38252	5524	778	17-	2479	698	23-
170	40	16	10421	19590	28594	45083	6180	1524	404	2427	1197	476
170	40	18	12105	19485	32881	55903	6616	2479	1567	2309	1731	1641
170	40	20	13770	19346	37786	70640	6902	3601	3832	2168	2263	3612
180	5	4-	846	2475	2122	02826	7625	0000	00000	11978	0000	00000
180	5	6-	1858	2494	1035	650	5200			5446		
180	5	8-	3281	2498	599	215	3922	6	3-	3080	9	6-
180	5	10	5018	2500	426	104	333			209		
180	5	12	6858	2500	381	76	522	3		273	3	
180	5	14	8671	2500	397	75	657	9		295	8	
180	5	16	10431	2498	443	85	749	19		294	15	
180	5	18	12146	2500	510	104	814	32	2	284	22	2
180	5	20	13824	2509	594	133	863	49	5	271	31	5
180	10	• 4-	898	4897	6837	15856	14510			22793		
180	10	6-	1892	4959	3858	4631	10222			10705		2-
180	10	8-	3302	4986	2339	1655	7797	24	22-	6124	37	52-
180	10	10	5032	4994	1686	822	669	2		420	2	
180	10	12	6868	4999	1519	607	1046	12		547	12	1-

THE USE OF THE STOKES-STRUIK APPROXIMATION FOR WAVES OF FINITE HEIGHT

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
180	10	16	10441	4993	1766	00673	1497	.0075	.00003	588	.0059	.00005
180	10	18	12156	4992	2028	825	1625	128	15	567	89	16
180	10	20	13856	4990	2346	1044	1717	195	41	559	121	39
180	15	6-	1939	7392	7912	13436	14999		2-	15707	2	7-
180	15	8-	3335	7452	5061	5242	11580	56	77-	9095	88	181-
180	15	10	5054	7474	3727	2694	1008	4		633	4	
180	15	12	6885	7481	3379	2014	1568	26	2-	821	27	4-
180	15	14	8695	7484	3535	1980	1969	81	3-	884	73	4-
180	15	16	10456	7481	3952	2254	2243	170	9	881	133	11
180	15	18	12173	7461	4517	2743	2428	287	50	848	200	53
180	15	20	13896	7448	5213	3460	2562	430	137	805	270	129
180	20	• 6-	1993	9810	12756	27130	19555	2	6-	20478	5	20-
180	20	8-	3376	9899	8585	11510	15263	105	188-	11987	165	444-
180	20	10	5083	9944	6482	6162	1353	7		850	8	1-
180	20	12	6907	9956	5931	4678	2093	47	5-	1096	49	8-
180	20	14	8715	9955	6219	4618	2622	145	6-	1177	130	9-
180	20	16	10477	9957	6970	5281	2986	303	23	1173	238	27
180	20	18	12196	9927	7967	6429	3230	511	120	1128	357	126
180	20	20	13884	9933	9240	8172	3415	768	329	1073	482	310
180	25	• 6-	2050	12228	18114	45272	23934	4	16-	25063	9	49-
180	25	8-	3423	12330	12738	20659	18840	176	383-	14797	277	902-
180	25	10	5119	12396	9862	11524	1704	11	2-	1071	14	3-
180	25	12	6935	12415	9121	8906	2619	75	11-	1371	78	17-
180	25	14	8740	12413	9600	8852	3272	228	12-	1469	205	16-
180	25	16	10503	12408	10765	10138	3721	474	47	1461	372	55
180	25	18	12226	12379	12329	12385	4027	798	238	1406	557	250
180	25	20	13919	12350	14222	15623	4243	1192	642	1333	749	605
180	30	• 6-	2107	14662	23829	67376	28175	8	34-	29504	18	105-
180	30	• 8-	3474	14749	17380	32683	22320	272	690-	17530	428	1625-
180	30	10	5159	14834	13792	18988	2064	16	3-	1297	21	5-
180	30	12	6968	14864	12907	14966	3147	110	18-	1648	115	29-
180	30	14	8770	14862	13647	14995	3923	332	20-	1761	298	26-
180	30	16	10533	14830	15280	17147	4449	683	84	1747	537	99
180	30	18	12261	14808	17545	21042	4815	1150	418	1681	803	437
180	30	20	13959	14743	20163	26413	5061	1708	1110	1590	1073	1047
180	35	• 8-	3528	17165	22411	47490	25713	399	1145-	20195	627	2697-
180	35	10	5203	17259	18196	28655	2432	24	5-	1528	30	9-
180	35	12	7004	17290	17217	23011	3677	155	29-	1925	160	46-
180	35	14	8804	17284	18285	23240	4569	456	29-	2050	409	39-
180	35	16	10569	17249	20519	26689	5175	933	139	2032	733	164
180	35	18	12301	17192	23500	32649	5588	1562	671	1951	1090	703
180	35	20	14007	17129	27057	41127	5875	2320	1775	1846	1458	1673
180	40	• 8-	3584	19580	27752	64934	29028	561	1785-	22798	881	4207-
180	40	10	5250	19673	23010	40573	2808	33	8-	1764	41	14-
180	40	12	7044	19709	22026	33223	4210	205	44-	2204	214	68-
180	40	14	8842	19694	23493	33819	5213	602	40-	2340	540	54-
180	40	16	10609	19644	26399	38959	5895	1224	218	2315	961	257
190	5	4-	846	2475	2122	02826	7625	0000	.00000	11978	0000	.00000
190	5	6-	1858	2494	1035	650	5200			5445		
190	5	8-	3283	2497	597	214	3920	4	2-	3079	6	4-
190	5	10	5041	2498	416	101	296			186		
190	5	12	6928	2501	364	71	485	2		253	2	
190	5	14	8799	2494	369	66	617	7		277	6	
190	5	16	10619	2500	409	73	713	15		280	12	
190	5	18	12389	2489	462	87	776	26	1	271	18	1
190	5	20	14122	2506	540	111	829	40	4	260	25	3
190	10	• 4-	898	4897	6836	15853	14509			22791		
190	10	6-	1892	4958	3857	4629	10221			10703		
190	10	8-	3304	4983	2328	1643	7790	16	15-	6118	25	36-
190	10	10	5054	4992	1648	791	595			374	1	
190	10	12	6938	4999	1446	560	966	8		506	9	
190	10	14	8808	4997	1478	527	1237	28		555	25	1-
190	10	16	10627	4998	1632	585	1425	61	1	560	48	1
190	10	18	12399	4995	1858	703	1556	105	10	543	74	10
190	10	20	14133	4993	2140	880	1651	161	28	519	101	27
190	15	6-	1939	7392	7912	13436	14999		1-	15707	1	4-
190	15	8-	3337	7453	5046	5217	11579	38	53-	9094	60	125-
190	15	10	5077	7474	3644	2598	897	2		564	3	
190	15	12	6954	7482	3219	1858	1449	19	2-	759	20	3-
190	15	14	8822	7482	3300	1759	1854	63	3-	832	57	4-
190	15	16	10641	7480	3645	1953	2134	137	4	838	107	5
190	15	18	12415	7480	4158	2354	2331	237	32	814	166	34
190	15	20	14152	7475	4785	2942	2471	361	95	776	227	90
190	20	• 6-	1993	9810	12755	27128	19555	1	3-	20478	2	11-
190	20	8-	3378	9900	8555	11447	15259	73	130-	11984	114	307-
190	20	10	5106	9944	6337	5940	1206	4		757	5	
190	20	12	6977	9959	5651	4316	1935	34	4-	1013	36	6-
190	20	14	8841	9960	5814	4112	2470	113	7-	1109	101	9-
190	20	16	10661	9956	6430	4576	2841	244	11	1116	191	13

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
190	20	18	12436	9946	.7325	.05508	3099	.0422	.00078	1082	.0294	.00081
190	20	20	14177	9935	8425	6878	5282	640	227	1051	402	214
190	25	• 6-	2050	12229	18115	45270	23934	2	9-	25064	5	27-
190	25	8-	3426	12329	12689	20532	18834	122	267-	14792	192	428-
190	25	10	5141	12399	9644	11115	1520	7		955	8	2-
190	25	12	7004	12422	8696	8225	2422	54	8-	1268	57	12-
190	25	14	8866	12424	8985	7894	3085	177	13-	1385	159	17-
190	25	16	10685	12407	9935	8788	3541	381	22	1391	299	26
190	25	18	12463	12408	11349	10628	3865	660	154	1349	460	161
190	25	20	14208	12371	13013	13221	4086	998	444	1283	627	418
190	30	• 6-	2107	14662	23828	67370	28175	5	18-	29505	10	58-
190	30	• 8-	3477	14750	17314	32483	22315	190	484-	17527	299	1139-
190	30	10	5182	14843	13495	18327	1844	10	2-	1158	13	3-
190	30	12	7036	14872	12307	13821	2912	80	13-	1525	83	21-
190	30	14	8894	14873	12772	13369	3699	258	21-	1660	231	29-
190	30	16	10714	14861	14167	14965	4243	552	40	1666	433	48
190	30	18	12495	14825	16118	17999	4616	948	268	1611	662	281
190	30	20	14245	14775	18474	22390	4876	1430	769	1532	899	725
190	35	• 8-	5551	17168	22323	47191	25711	280	807-	20193	440	1902-
190	35	10	5226	17268	17796	27637	2174	15	3-	1366	19	5-
190	35	12	7072	17310	16436	21286	3405	111	21-	1783	116	33-
190	35	14	8927	17307	17155	20756	4311	354	32-	1935	318	44-
190	35	16	10748	17281	19025	23289	4936	753	68	1938	592	80
190	35	18	12532	17234	21653	28046	5365	1290	435	1873	900	454
190	35	20	14288	17167	24805	34880	5662	1942	1229	1779	1220	1158
190	40	• 8-	3586	19581	27630	64479	29024	596	1267-	22795	623	2984-
190	40	10	5273	19684	22499	39116	2514	21	4-	1579	26	8-
190	40	12	7112	19731	21029	30735	3901	149	32-	2043	156	50-
190	40	14	8963	19724	22032	30234	4921	468	46-	2209	420	65-
190	40	16	10785	19688	24502	34041	5625	988	109	2209	776	128
190	40	18	12574	19617	27871	40992	6104	1684	658	2131	1176	689
190	40	20	14336	19526	31896	50936	6435	2528	1845	2022	1589	1739
200	5	4-	846	2475	.2122	02826	7625	.0000	00000	11978	.0000	.00000
200	5	6-	1858	2494	1035	650	5201			5446		
200	5	8-	3284	2498	596	213	3920	3	1-	3079	4	3-
200	5	10	5059	2501	410	98	264			166		
200	5	12	6990	2501	348	66	446	1		234	2	
200	5	14	8917	2504	349	60	584	5		262	5	
200	5	16	10794	2499	379	64	679	12		266	10	
200	5	18	12620	2504	431	76	748	22		261	15	
200	5	20	14407	2497	491	94	794	34	2	250	21	2
200	10	• 4-	898	4897	6836	15853	14509			22791		
200	10	6-	1892	4958	3857	4629	10221			10703		
200	10	8-	3305	4984	2324	1639	7790	11	10-	6118	17	25-
200	10	10	5073	4997	1621	771	530			333		
200	10	12	6999	4995	1383	520	892	6		467	6	
200	10	14	8925	5004	1392	476	1167	22		524	20	1-
200	10	16	10802	4999	1515	513	1358	49		533	39	
200	10	18	12629	4994	1711	606	1491	87	6	521	61	6
200	10	20	14417	4991	1960	748	1588	135	20	499	85	18
200	15	6-	1939	7392	7911	13434	14998			15706		2-
200	15	8-	3338	7454	5035	5199	11578	26	36-	9093	41	86-
200	15	10	5095	7478	3582	2527	799	1		502	2	
200	15	12	7015	7481	3083	1730	1339	14	1-	701	14	2-
200	15	14	8939	7475	3094	1577	1745	49	3-	783	44	3-
200	15	16	10815	7483	3386	1714	2033	111	1	798	87	2
200	15	18	12644	7480	3829	2028	2233	196	21	780	137	22
200	15	20	14433	7479	4390	2508	2378	304	66	747	191	62
200	20	• 6-	1993	9810	12755	27129	19556		2-	20479	1	6-
200	20	8-	3380	9903	8539	11411	15260	50	90-	11985	79	213-
200	20	10	5124	9947	6226	5773	1074	3		675	5	
200	20	12	7037	9961	5417	4024	1789	25	3-	937	26	4-
200	20	14	8958	9973	5479	3712	2331	88	6-	1046	79	8-
200	20	16	10833	9960	5975	4018	2707	197	4	1063	155	4
200	20	18	12663	9948	6750	4748	2970	349	49	1037	244	52
200	20	20	14456	9951	7749	5886	3164	539	158	994	339	149
200	25	• 6-	2050	12229	18114	45272	23935	1	5-	25065	3	14-
200	25	8-	3427	12332	12660	20458	18834	85	186-	14792	133	437-
200	25	10	5160	12402	9473	10798	1355	4		852	5	
200	25	12	7065	12427	8340	7673	2240	39	5-	1173	41	8-
200	25	14	8981	12431	8456	7112	2909	138	12-	1306	124	16-
200	25	16	10857	12425	9252	7741	3378	309	8	1326	243	9
200	25	18	12689	12407	10458	9160	3704	545	98	1293	381	102
200	25	20	14485	12393	11976	11318	3939	840	309	1237	528	291
200	30	• 6-	2107	14663	23829	67371	28176	3	10-	29506	5	32-
200	30	• 8-	3479	14751	17267	32343	22313	133	338-	17524	209	797-
200	30	10	5200	14843	13246	17785	1645	7		1033	8	2-
200	30	12	7096	14880	11806	12898	2695	58	9-	1411	60	15-
200	30	14	9009	14891	12037	12069	3491	201	20-	1567	180	27-

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_0	V_1	V_2	V_3	σ_1	σ_2	σ_5
200	50	16	10884	14884	13199	.15109	4048	.0447	.00015	1590	.0331	.00018
200	30	18	12719	14857	14922	15619	4433	787	172	1348	549	180
200	30	20	14319	14798	17000	19160	4701	1204	535	1477	736	504
200	53	*8-	5533	17167	22256	46966	25707	197	568-	20190	309	1339-
200	35	10	5245	17275	17477	26840	1942	9	2-	1220	12	3-
200	55	12	7132	17520	15768	19865	5135	80	15-	1651	84	24-
200	55	14	9041	17527	16151	10741	4069	276	31-	1826	248	41-
200	55	16	10916	17509	17755	20338	4710	611	27	1850	480	32
200	55	18	12733	17280	20075	24586	5157	1072	279	1000	749	292
200	35	20	14339	17215	22884	29956	3463	1638	858	1717	1029	800
200	40	*8-	3569	19585	27546	64165	29022	280	897-	22794	440	2114-
200	40	10	5292	19692	22089	37970	2240	15	3-	1413	17	3-
200	40	12	7171	19746	20181	28697	3614	100	23-	1892	113	36-
200	40	14	9076	19750	20778	27310	4647	365	45-	2086	527	60-
200	40	16	10951	19721	22849	30036	3349	801	45	2108	629	52
200	40	18	12792	19669	25848	33652	5869	1400	424	2049	977	444
200	40	20	14605	19590	29475	43842	6215	2134	1290	1953	1341	1216
220	3	6-	1058	2494	.1055	.00650	5201	.0000	.00000	5446	.0000	.00000
220	5	8-	5284	2498	595	215	5920	1		5079	2	1-
220	3	10	9085	2501	599	94	208			131		
220	5	12	7090	2508	524	59	380			199		
220	5	14	9125	2504	514	50	519	5		255	3	
220	5	16	11111	2301	332	51	617	0		242	6	
220	3	18	13046	2504	371	38	688	15		240	11	
220	5	20	14937	2506	422	71	740	24	1	233	13	1
220	10	6-	1892	4950	5857	4629	10221			10703		
220	10	8-	5307	4984	2518	1632	7788	5	5-	6117	8	12-
220	10	10	5099	4990	1575	736	410			262		
220	10	12	7100	4995	1289	464	761	5		398	5	
220	10	14	9130	4996	1246	395	1056	13		463	12	
220	10	16	11118	3002	1320	407	1235	35		483	26	
220	10	18	15034	4996	1475	464	1374	61	2	479	42	2
220	10	20	14946	4977	1661	533	1470	96	9	462	60	9
220	13	6-	1939	7592	7912	13435	14999			15707		
220	15	8-	5340	7432	5019	3173	11572	12	17-	9009	20	41-
220	15	10	5121	7477	5409	2422	651			397		
220	15	12	7116	7487	2080	1546	1144	7		399	7	
228	15	14	9143	7487	2787	1320	1354	30	2-	697	27	2-
220	15	16	11130	7488	2968	1361	1849	73	1-	726	58	1-
220	13	18	13067	7403	3504	1533	2050	157	8	718	93	8
220	13	20	14960	7404	3749	1876	2210	218	32	694	137	30
220	20	*6-	1993	9809	12754	27125	19555			20478		2-
220	20	8-	3382	9901	8512	11354	15254	24	43-	11980	38	102-
220	20	10	3131	9948	6065	5334	030	1		534	1	
220	20	12	7138	9962	5053	3308	1327	13	1-	800	13	2-
220	20	14	9161	9960	4907	9001	2070	33	4-	929	48	6-
220	20	16	11147	9972	5245	3196	2463	131	2-	967	103	3-
220	20	10	15004	9964	5837	3652	2740	243	19	956	170	20
220	20	20	14979	9965	6630	4414	2942	307	77	924	243	73
220	25	*6-	2050	12228	18111	45262	23934		1-	25063		4-
220	23	8-	3429	12332	12623	20361	18031	41	90-	14790	64	211-
220	23	10	5187	12407	9231	10356	1075	2		676	2	
220	23	12	7164	12434	7787	6852	1915	21	3-	1003	22	4-
220	23	14	9183	12445	7610	5944	2390	84	8-	1162	76	11-
220	25	16	11168	12440	8125	6161	5075	205	4-	1208	161	5-
220	25	18	15106	12442	9069	7074	3422	381	38	1194	266	40
220	23	20	15004	12418	10264	8307	3666	604	151	1152	380	142
220	30	*6-	2107	14662	23825	67357	28174		3-	29304	2	10-
220	30	*8-	3481	14753	17214	32184	22311	65	165-	17523	102	390-
220	30	10	3228	14849	12902	17046	1307	3		821	3	
220	30	12	7196	14898	11033	11337	2307	30	5-	1208	32	7-
220	30	14	9210	14901	10829	10080	3108	123	14-	1393	110	19-
220	30	16	11193	14899	11592	10495	3685	297	6-	1447	233	8-
220	30	18	13133	14884	12921	12033	4093	549	67	1429	383	70
220	30	20	15033	14846	14613	14439	4381	867	262	1376	345	247
220	53	*8-	3555	17169	22180	46709	23704	97	281-	20188	152	662-
220	35	10	5272	17278	17008	25690	1547	4		972	5	
220	35	12	7231	17339	14735	17761	2700	42	8-	1414	44	12-
220	35	14	9240	17352	14538	15694	3627	169	23-	1628	132	31-
220	35	16	11222	17343	13609	16395	4293	406	9-	1686	319	11-
220	33	18	13163	17320	17410	18823	4764	748	109	1663	322	114
220	33	20	15068	17294	19741	22720	5102	1183	422	1603	743	390
220	40	*8-	3591	19583	27444	63787	29021	139	449-	22793	219	1057-
220	40	10	5320	19704	21503	36357	1795	5		1128	7	2-
220	40	12	7270	19772	18864	25664	3099	57	11-	1623	60	10-
220	40	14	9274	19795	18762	22935	4147	224	34-	1861	201	46-

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	σ_1	σ_2	σ_3
240	5	6-	1858	2494	.1056	.00650	5201	.0000	.00000	5447	.0000	.00000
240	5	8-	5287	2498	394	212	3919			3078		
240	5	10	5102	2499	392	91	144			103		
240	3	12	7167	2502	308	54	324			170		
240	5	14	9294	2498	284	43	461	2		207	2	
240	3	16	11388	2498	296	42	562	5		221	4	
240	5	18	13429	2501	325	46	635	11		222	7	
240	3	20	13422	2499	364	34	688	18		216	11	
240	10	6-	1892	4958	5856	4629	10221			10705		
240	10	8-	5508	4984	2315	1629	7787	2	2-	6116	4	5-
240	18	10	5116	4993	1552	719	329			207		
240	10	12	7177	4996	1224	426	649	2		340	2	
240	10	14	9302	4997	1140	540	922	8		414	7	
240	10	16	11595	4997	1182	553	1124	22		441	17	
240	10	18	13436	4997	1294	367	1269	45		445	50	
240	10	20	15429	4998	1434	433	1376	71	5	432	44	4
240	15	6-	1959	7392	7912	13433	14999			15707		
240	15	8-	3341	7452	5013	5162	11571	6	8-	9088	9	19-
240	13	10	5138	7479	3435	2360	498			513		
240	15	12	7193	7489	2753	1420	976	4		511	4	
240	13	14	9514	7489	2552	1138	1385	18	1-	621	17	2-
240	15	16	11406	7489	2647	1117	1685	49	1-	662	39	2-
240	15	18	13447	7494	2905	1236	1905	97	2	664	68	3
240	15	20	15442	7301	3269	1460	2064	160	16	648	100	15
240	20	*6-	1995	9810	12754	27125	19355			20478		
240	20	8-	3562	9901	0500	11320	15252	11	21-	11979	18	48-
240	20	10	5168	9949	5965	5389	672			422		
240	20	12	7214	9974	4805	3304	1305	7		683	7	
240	20	14	9331	9974	4503	2665	1045	35	3-	828	30	4-
240	20	16	11422	9975	4681	2624	2245	88	3-	882	69	4-
240	20	18	13463	9987	3146	2914	2337	173	6	886	121	6
240	20	20	13458	9975	3767	3422	2745	285	37	862	178	55
240	23	*6-	2030	12228	10111	45262	23954			25065		1-
240	25	8-	3430	12333	12605	20313	18030	20	43-	14789	31	102-
240	25	10	5204	12408	9076	10070	851			535		
240	25	12	7241	12437	7389	6289	1655	11	1-	856	11	2-
240	25	14	9353	12455	6976	5134	2308	52	5-	1036	47	7-
240	25	16	11441	12451	7262	5069	2005	138	7-	1101	100	8-
240	25	18	13483	12445	7965	5611	3162	269	12	1104	188	13
240	25	20	15480	12439	0943	6609	3422	442	73	1075	278	69
240	30	*6-	2107	14662	23025	67337	28174			29504		3-
240	30	*8-	3482	14734	17107	32103	22309	31	81-	17522	49	190-
240	30	10	5245	14854	12606	16309	1037	1		651	1	
240	30	12	7272	14901	10470	10383	1971	16	2-	1032	17	3-
240	30	14	9379	14917	9932	8713	2770	76	9-	1243	68	15-
240	50	16	11463	14908	10356	0652	3361	199	11-	1320	157	13-
240	30	10	13507	14907	11382	9506	3789	389	22	1322	271	23
240	30	20	15506	14904	12794	11313	4100	637	129	1280	400	121
240	35	*8-	5536	17170	22143	46584	25703	48	138-	20187	73	326-
240	35	10	3290	17288	16725	25005	1229	1		775	2	
240	35	12	7507	17352	13993	16318	2310	22	4-	1210	23	6-
240	33	14	9408	17371	13555	13569	3234	104	15-	1431	94	20-
240	35	16	11492	17570	13981	13520	3920	273	17-	1339	213	20-
240	33	18	13334	17354	13354	15020	4411	530	36	1540	370	37
240	35	20	13536	17329	17230	17690	4767	866	206	1497	344	194
240	40	*8-	3392	19587	27394	63602	29020	69	224-	22792	109	527-
240	40	10	5338	19712	21130	53349	1430	2		898	3	
240	40	12	7346	19790	17915	23581	2654	30	6-	1389	32	9-
240	40	14	9441	19824	17228	19854	3701	130	22-	1661	124	30-
240	40	16	11322	19803	18033	19840	4473	359	25-	1757	202	29-
240	40	18	13565	19791	19066	22106	5032	695	56	1737	403	58
240	40	20	15570	19747	22274	26015	5431	1130	311	1706	710	294
260	3	6-	1858	2495	.1036	00650	5202	.0000	.00000	5447	.0000	.00000
260	5	8-	3287	2498	394	212	3918			5077		
260	3	10-	3112	2499	308	90	3143	5	1-	1975	6	3-
260	5	12	7225	2500	296	50	276			144		
260	3	14	9437	2494	265	38	409	1		184	1	
260	3	16	11651	2497	268	33	312	4		201	3	
260	3	18	13774	2302	289	38	388	8		205	5	
260	5	20	15863	2500	321	43	643	13		202	8	
260	10	6-	1892	4960	3038	4632	10223			10706		
260	10	8-	3308	4984	2314	1627	7787	1	1-	6116	2	3-
260	10	10-	3126	4993	1537	708	6273	20	12-	3941	25	22-
260	10	12	7234	4997	1177	400	552			289		
260	10	14	9444	5000	1061	301	021	3		368	3	
260	10	16	11637	4999	1071	281	1026	13		403	12	

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2h

THE USE OF THE STOKES-STRUIK APPROXIMATION FOR WAVES OF FINITE HEIGHT

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	σ_1	σ_2	σ_3
260	15	8-	3541	7453	5011	.05159	11572	.0003	00004-	9088	.0004	00009-
260	15	10-	5149	7478	5397	2320	9365	45	41-	5884	57	76-
260	15	12	7250	7490	2628	1332	831	2		435	2	
260	15	14	9456	7490	2372	1007	1231	11		553	10	
260	15	16	11648	7494	2402	945	1538	33	1-	604	26	2-
260	15	18	13790	7491	2585	1007	1762	69		615	40	
260	15	20	15885	7490	2871	1156	1926	118	7	605	74	7
260	20	• 6-	1995	9810	12754	27125	19555			20478		
260	20	8-	3383	9901	8494	11517	15251	5	10-	11978	9	23-
260	20	10-	5179	9950	5904	5300	12415	83	97-	7801	104	184-
260	20	12	7272	9973	4617	3097	1111	4		582	4	
260	20	14	9475	9978	4188	2362	1643	20	2-	757	18	2-
260	20	16	11663	9978	4245	2219	2050	60	5-	805	47	4-
260	20	18	13805	9988	4585	2378	2350	124		820	86	
260	20	20	15898	9981	5088	2728	2567	210	18	806	132	17
260	25	• 6-	2050	12229	18112	45265	23934			25064		
260	25	8-	3431	12532	12594	20288	18827	9	21-	14787	15	49-
260	25	10-	5215	12414	8986	9918	15420	135	194-	9689	168	565-
260	25	12	7298	12443	7107	5903	1395	6		730	6	
260	25	14	9494	12461	6492	4553	2056	32	3-	923	29	5-
260	25	16	11681	12461	6592	4292	2562	94	6-	1006	74	7-
260	25	18	13823	12458	7112	4592	2931	193	1	1025	135	1
260	25	20	15917	12449	7894	5273	3201	328	55	1006	206	33
260	50	• 6-	2107	14662	23826	67359	28175			29505		
260	30	• 8-	5402	14754	17173	52062	22308	15	59-	17520	24	92-
260	50	10-	5256	14859	12555	16314	18364	199	541-	11558	250	642-
260	30	12	7330	14908	10069	9934	1682	8	1-	881	9	2-
260	30	14	9519	14923	9240	7724	2469	47	6-	1108	42	8-
260	50	16	11704	14940	9429	7359	5075	156	11-	1207	107	12-
260	30	18	13844	14909	10148	7827	3509	278	3	1225	194	3
260	30	20	15940	14912	11293	9024	3854	472	61	1205	297	57
260	35	• 8-	5557	17171	22126	46527	25703	23	68-	20187	37	161-
260	35	10-	5302	17292	16544	24572	21252	281	551-	15355	355	1059-
260	55	12	7365	17561	15456	15312	1973	12	2-	1033	12	3-
260	35	14	9548	17390	12445	12055	2885	65	9-	1295	58	13-
260	35	16	11729	17399	12717	11488	3585	186	17-	1408	146	19-
260	35	18	13870	17379	13729	12515	4092	381	5	1428	266	5
260	55	20	15967	17564	15256	14173	4465	643	98	1403	404	92
260	40	• 8-	3593	19589	27573	63521	29022	55	111-	22794	54	262-
260	40	10-	5350	19717	20894	54717	24089	581	840-	15136	479	1584-
260	40	12	7404	19804	17232	22135	2269	16	5-	1188	17	4-
260	40	14	9580	19837	16059	17616	3301	86	14-	1482	77	19-
260	40	16	11758	19845	16441	16876	4094	245	24-	1608	192	29-
260	40	18	13899	19826	17782	18151	4670	499	9	1630	548	9
260	40	20	15998	19799	19756	20891	5091	841	149	1599	528	140
280	5	6-	1858	2494	.1035	.00650	5200	.0000	.00000	5446	.0000	.00000
280	5	8-	3287	2501	595	215	3923			5081		
280	5	10-	5119	2498	585	89	3141	3		1975	4	2-
280	5	12	7268	2500	287	48	234			125		
280	5	14	9555	2504	251	54	566			164		
280	5	16	11843	2500	246	51	468	5		184	2	
280	5	18	14084	2499	260	31	545	6		190	4	
280	5	20	16272	2513	289	36	605	10		190	6	
280	10	6-	1892	4958	3857	4629	10221			10703		
280	10	8-	5309	4984	2314	1627	7787			6116		1-
280	10	10-	5133	4993	1526	700	6267	12	7-	3938	15	14-
280	10	12	7277	4997	1144	581	470			246		
280	10	14	9562	4998	998	273	730	3		328	3	
280	10	16	11849	4998	985	243	937	10		368	8	
280	10	18	14090	4998	1041	251	1090	22		381	16	
280	10	20	16278	5016	1150	284	1208	40		579	25	
280	15	6-	1959	7392	7911	13434	14998			15706		
280	15	8-	3341	7455	5011	5159	11573	1	2-	9090	2	4-
280	15	10-	5156	7480	5376	2297	9361	28	25-	5882	35	47-
280	15	12	7293	7484	2548	1267	706	1		370	1	
280	15	14	9574	7491	2234	912	1096	7		492	6	
280	15	16	11859	7492	2204	817	1405	23	1-	552	18	1-
280	15	18	14099	7500	2339	845	1656	50		571	55	
280	15	20	16288	7493	2563	944	1804	88	3	567	56	3
280	20	• 6-	1993	9810	12755	27127	19555			20478		
280	20	8-	5383	9901	8492	11311	15251	3	5-	11978	4	11-
280	20	10-	5186	9952	5867	5247	12410	51	61-	7797	64	114-
280	20	12	7315	9971	4481	2950	946	2		495	2	
280	20	14	9590	9983	3948	2142	1464	13	1-	657	11	1-
280	20	16	11873	9981	3900	1921	1873	41	2-	736	32	3-
280	20	18	14115	9981	4133	1984	2178	89	1-	760	62	1-
280	20	20	16301	9983	4541	2226	2404	157	8	755	99	7
280	25	• 6-	2050	12229	18113	45268	23955			25065		
280	25	8-	3431	12333	12591	20280	18828	5	10-	14788	7	24-
280	25	10-	5222	12411	.8921	.9803	15407	.0085	.00121-	9680	0104	.00227-
280	25	12	7341	12446	6903	5632	1188	3		622	3	
280	25	14	9611	12460	6112	4122	1831	20	2-	822	18	3-
280	25	16	11891	12459	6053	3713	2341	64	5-	919	50	6-
280	25	18	14130	12472	6435	3854	2723	140	3-	950	98	3-
280	25	20	16519	12460	7058	4313	3001	246	15	945	155	14
280	30	•6-	2107	14665	23827	67365	28176			29506		
280	30	• 8-	5483	14753	17166	32040	22307	7	19-	17520	12	45-
280	50	10-	5263	14857	12464	16127	18351	124	213-	11530	155	402-
280	50	12	7373	14912	9778	9474	1434	4		751	5	
280	30	14	9656	14934	8716	7010	2201	29	4-	988	26	5-
280	50	16	11912	14940	8662	6353	2810	93	8-	1103	73	10-
280	30	18	14150	14937	9198	6584	3262	202	4-	1139	141	5-
280	50	20	16340	14927	10102	7385	3595	554	27	1150	223	25
280	35	• 8-	3537	17171	22115	46490	25701	12	34-	20186	18	79-
280	55	10-	5309	17296	16431	24304	21245	176	347-	13348	221	654-
280	35	12	7409	17366	13067	14601	1683	6		881	6	1-
280	55	14	9664	17398	11729	10930	2572	40	6-	1154	36	8-
280	35	16	11937	17397	11683	9944	5276	127	15-	1287	100	15-
280	55	18	14175	17402	12453	10346	3803	276	7-	1327	193	7-
280	55	20	16564	17387	13660	11615	4188	483	44	1316	504	41
280	40	• 8-	3593	19587	27355	65458	29019	17	55-	22791	27	150-
280	40	10-	5358	19720	20744	34517	24079	240	531-	15129	301	1001-
280	40	12	7448	19811	16732	21100	1937	8	1-	1014	9	2-
280	40	14	9696	19852	15128	15987	2945	54	8-	1322	48	11-
280	40	16	11965	19859	15131	14645	3746	168	19-	1471	132	23-
280	40	18	14200	19845	16097	15237	4339	362	9-	1515	252	9-
280	40	20	16392	19839	17719	17160	4779	632	67	1501	597	63
500	5	6-	1858	2492	.1034	.00649	5197	.0000	00000	5443	.0000	.00000
300	5	8-	3287	2499	594	212	3920			5078		
300	5	10-	5123	2498	384	88	3140	2		1973	2	1-
500	5	12	7299	2500	281	46	199			104		
500	5	14	9651	2500	258	52	325			146		
300	5	16	12028	2511	231	27	430	2		169	1	
300	5	18	14365	2502	238	27	507	4		177	3	
300	5	20	16645	2502	258	29	565	7		178	5	
500	10	6-	1892	4958	3857	4629	10221			10703		
500	10	8-	5509	4985	2314	1626	7787			6116		
500	10	10-	5137	4994	1521	696	6265	7	5-	5937	9	9-
300	10	12	7309	4996	1119	368	399			209		
500	10	14	9658	4999	951	252	650	2		292	2	
300	10	16	12033	4998	913	215	856	7		336	5	
300	10	18	14369	4984	945	212	1010	16		353	11	
300	10	20	16651	5007	1034	255	1131	30		355	19	
300	15	6-	1939	7391	7910	13429	14997			15704		
300	15	8-	3342	7454	5009	5156	11572			9088		2-
500	15	10-	5160	7474	5358	2277	9351	17	16-	5875	22	29-
300	15	12	7325	7485	2494	1224	600			314		
300	15	14	9670	7490	2126	841	975	4		438	4	
300	15	16	12045	7492	2048	722	1285	16		504	12	
300	15	18	14378	7498	2134	721	1520	37		530	26	
300	15	20	16660	7492	2312	786	1693	67	1	532	42	1
300	20	•6-	1995	9811	12757	27152	19557			20480		
300	20	8-	3383	9901	8491	11309	15251	1	2-	11978	2	5-
500	20	10-	5190	9951	5841	5210	12403	32	38-	7793	40	71-
300	20	12	7346	9971	4384	2848	803			421		
300	20	14	9687	9983	3759	1976	1303	8		585	7	
300	20	16	12057	9992	3632	1703	1715	28	2-	673	22	2-
300	20	18	14390	9984	3776	1697	2024	65	2-	707	46	2-
300	20	20	16672	9982	4097	1854	2256	119	3	709	75	3
300	25	•6-	2050	12228	18111	45261	25934			25063		
500	25	8-	3451	12353	12590	20275	18828	2	5-	14788	3	11-
500	25	10-	5226	12412	8885	9740	15403	51	75-	9678	64	142-
300	25	12	7375	12448	6755	5438	1010	2		529	2	
300	25	14	9707	12463	5822	3805	1631	12	1-	732	11	2-
500	25	16	12074	12476	5640	3294	2145	44	4-	842	35	4-
300	25	18	14406	12477	5881	3297	2551	102	4-	883	71	4-
300	25	20	16688	12463	6372	3597	2816	186	6	885	117	5
300	30	•6-	2107	14662	23825	67556	28174			29504		
500	50	•8-	5483	14755	17167	32042	22310	4	9-	17522	6	22-
300	30	10-	5268	14858	12411	16017	18345	77	133-	11527	97	251-
300	30	12	7405	14921	9576	9159	1221	2		639	2	
500	30	14	9732	14936	8500	6469	1960	18	2-	880	16	5-
300	30	16	12095	14946	8057	5621	2571	64	6-	1010	50	7-
300	30	18	14425	14947	8411	5658	5033	148	6-	1059	103	7-
300	30	20	16708	14943	9137	6175	3377	269	10	1061	169	9
300	35	•8-	3537	17171	22110	46475	25701	6	16-	20186	9	39-
500	35	10-	5314	17294	16553	24121	21235	110	218-	13542	138	410-
300	35	12	7441	17372	12788	14100	1434	3		751	3	

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
500	35	14	9760	17404	11174	.10092	2292	0025	.00003-	1029	.0023	.00004-
300	33	16	12119	17417	10882	8817	3000	88	10-	1178	69	11-
500	35	18	14447	17416	11376	8864	3336	202	10-	1234	141	10-
500	53	20	16730	17406	12339	9714	3935	366	16	1236	230	13
300	40	10-	5362	19718	20643	34052	24069	130	335-	13123	189	631-
500	40	12	7480	19818	16371	20370	1632	4		865	5	
500	40	14	9791	19871	14430	14787	2627	33	3-	1179	30	7-
500	40	16	12145	19868	14077	12962	3428	116	14-	1346	91	17-
500	40	18	14473	19868	14741	13071	4037	263	14-	1409	185	15-
500	40	20	16736	19860	16034	14334	4491	479	23	1411	301	24
520	5	6-	1858	2492	.1034	00648	5197	0000	.00000	5442	.0000	.00000
520	5	8-	3287	2498	393	212	3919			3078		
320	5	10-	5126	2500	383	88	3141	1		1974	1	
520	5	12	7325	2500	277	43	169			88		
320	5	14	9730	2495	228	29	288			129		
520	5	16	12188	2511	217	23	393	1		134		
520	5	18	14614	2510	221	24	473	3		165	2	
320	5	20	16988	2499	233	23	330	6		167	4	
520	10	6-	1892	4938	3837	4629	10221			10703		
320	10	8-	3309	4984	2313	1626	7787			6116		
520	10	10-	5140	4994	1317	693	6264	3	3-	3936	6	3-
520	10	12	7332	4997	1101	339	338			177		
320	10	14	9757	5007	916	237	379	1		260	1	
320	10	16	12194	5003	860	194	784	3		308	4	
320	10	18	14619	5000	877	187	942	12		329	8	
320	10	20	16995	4998	958	198	1061	23		333	14	
520	15	6-	1959	7592	7912	15435	14999			13707		
520	13	8-	5342	7453	5007	5134	11570			9087		
320	13	10-	5162	7479	3554	2272	9334	11	10-	3878	13	18-
520	15	12	7348	7485	2453	1191	509			266		
520	13	14	9749	7495	2043	789	868	3		389	2	
320	15	16	12205	7501	1926	631	1176	11		462	9	
520	13	18	14628	7498	1968	627	1415	27		493	19	
320	13	20	17001	7494	2108	668	1591	31		500	32	
320	20	• 6-	1993	9809	12754	27125	19555			20478		
320	20	8-	3383	9902	8491	11310	15232		1-	11979		2-
320	20	10-	3192	9951	5826	3188	12400	20	23-	7791	25	44-
320	20	12	7370	9976	4519	2780	682			357		
320	20	14	9765	9982	3611	1830	1159	5		520	4	
520	20	16	12217	9991	3408	1531	1568	19	1-	616	15	2-
320	20	18	14640	9989	3486	1479	1883	48	2-	657	34	2-
320	20	20	17013	9985	3735	1575	2120	91		666	57	
520	25	• 6-	2050	12228	18111	45261	25934			25063		
320	23	8-	3431	12333	12589	20274	18828	1	2-	14788	2	5-
320	23	10-	5229	12412	8862	9699	15399	32	47-	9676	40	88-
320	25	12	7397	12447	6646	5297	858			449		
520	23	14	9786	12465	5596	3566	1451	8		651	7	
320	25	16	12233	12473	5291	2960	1960	31	2-	770	24	3-
320	25	18	14655	12481	5427	2872	2354	75	4-	822	53	4-
320	23	20	17028	12484	3826	3069	2631	143		833	90	
520	30	• 6-	2107	14662	23825	67356	28174			29504		
320	50	• 8-	5483	14754	17163	32032	22308	2	5-	17521	3	11-
320	30	10-	5271	14861	12382	15955	18345	48	83-	11526	60	157-
320	50	12	7429	14916	9416	8915	1037	1		543	1	
520	30	14	9810	14942	7980	6066	1745	11	1-	783	10	2-
320	30	16	12253	14953	7570	5063	2333	44	4-	924	35	5-
320	30	18	14673	14952	7765	4913	2822	109	6-	985	76	7-
320	30	20	17046	14949	8335	5250	3175	205	2	998	129	2
320	35	• 8-	3537	17171	22108	46468	25701	3	8-	20186	4	19-
520	35	10-	5316	17295	16309	24015	21232	69	136-	13340	86	257-
320	35	12	7465	17577	12586	13744	1220	2		639	2	
320	35	14	9838	17408	10741	9459	2041	16	2-	916	14	3-
320	35	16	12277	17427	10228	7946	2747	61	7-	1079	48	8-
320	35	18	14695	17432	10514	7738	3293	149	10-	1149	104	10-
320	55	20	17067	17419	11284	8269	3701	280	3	1163	176	3
320	40	• 8-	3593	19587	27346	63424	29019	4	14-	22791	7	32-
320	40	10-	5365	19723	20590	33913	24069	94	211-	15123	119	397-
320	40	12	7504	19822	16109	19847	1407	2		737	2	
320	40	14	9870	19870	13861	13846	2340	21	3-	1050	19	4-
320	40	16	12303	19877	13227	11675	3139	80	10-	1233	63	12-
320	40	18	14719	19890	13632	11419	3760	195	15-	1312	136	15-
320	40	20	17091	19876	14643	12223	4224	367	5	1327	230	5
340	5	6-	1858	2494	.1036	.00650	5201	0000	.00000	5446	0000	00000
340	5	8-	3287	2498	593	212	3919			3078		
[illegible]	5	10-	5127	2495	381	87	3135			1969		

d	H	T	L	η	η	η	V_1	V_2	V_8	a_1	a_2	a_3
340	3	18	14840	2497	.0204	.00021	.438	.0002	.00000	.153	.0002	.00000
340	5	20	17302	2487	214	21	496	4		156	3	
340	10	6-	1892	4958	3856	4629	10821			10703		
340	10	8-	3309	4984	2313	1626	7786			6113		
340	10	10-	3141	4993	1314	691	6262	3	2-	3934	4	3-
340	10	12	7349	4997	1089	332	286			130		
340	10	14	9801	4998	884	224	514			231		
340	10	16	12333	3003	813	177	717	3		282	3	
340	10	18	14843	4993	813	164	876	9		306	6	
540	10	20	17307	4982	837	169	995	17		313	11	
540	13	6-	1939	7392	7912	13433	14999			13707		
340	13	8-	3342	7432	5007	3153	11370			9087		
340	13	10-	3164	7480	3349	2267	9354	7	6-	5877	8	11-
540	13	12	7363	7490	2429	1173	432			226		
540	13	14	9813	7493	1980	749	771	2		346	2	
340	13	16	12342	7304	1826	593	1076	8		425	6	
340	13	18	14833	7308	1836	337	1517	20		460	14	
340	13	20	17313	7496	1937	376	1497	39		470	23	
340	20	• 6-	1995	9810	12734	27123	19555			20478		
340	20	8-	5383	9901	8490	11306	15251			11978		1-
340	20	10-	3194	9951	5817	3175	12599	12	14-	7790	15	27-
340	20	12	7387	9973	4268	2727	579			303		
340	20	14	9829	9983	3496	1736	1030	5		462	3	
540	20	16	12335	9987	3224	1397	1434	13		565	11	1-
340	20	18	14863	10001	3251	1312	1753	36	2-	613	25	2-
340	20	20	17326	9992	5458	1362	1996	70		627	44	
340	23	• 6-	2030	12228	18111	43262	25934			25063		
340	23	8-	3431	12353	12389	20273	18828		1-	14788		5-
340	23	10-	3231	12411	8846	9671	13396	20	29-	9674	25	54-
340	25	12	7414	12444	6366	3193	728			581		
340	23	14	9830	12467	5419	3384	1290	5		579	4	
340	25	16	12371	12478	5014	2707	1794	21	2-	705	17	2-
340	23	18	14879	12473	5046	2336	2190	56	5-	765	59	5-
340	23	20	17340	12479	5351	2645	2493	110	1-	783	69	1-
340	30	• 6-	2107	14662	23825	67356	28174			29504		
340	30	• 8-	3483	14734	17162	32029	22308		2-	17320	1	5-
340	30	10-	3272	14860	12358	13906	18340	30	52-	11523	37	97-
340	30	12	7446	14918	9307	8749	881			461		
340	30	14	9874	14943	7728	5737	1353	7		697	6	
340	30	16	12391	14933	7170	4626	2154	31	3-	846	24	3-
340	30	18	14897	14958	7231	4350	2628	80	5-	917	56	6-
340	30	20	17337	14957	7670	4538	2989	158	2-	939	99	2-
340	35	• 8-	3337	17171	22107	46465	25701	1	4-	20186	2	9-
340	35	10-	5318	17296	16281	23949	21230	43	83-	13339	54	161-
340	35	12	7482	17378	12437	13483	1037			543		
340	35	14	9902	17413	10402	8977	1817	10	1-	815	9	1-
340	35	16	12414	17430	9689	7261	2314	42	5-	987	35	6-
540	35	18	14917	17427	9781	6839	3064	110	8-	1070	77	9-
340	33	20	17377	17430	10388	7152	3485	216	3-	1093	135	3-
340	40	• 8-	3393	19388	27345	63422	29019	2	7-	22792	3	16-
340	40	10-	5367	19724	20553	33815	24067	59	152-	15122	74	249-
340	40	12	7322	19829	15924	19482	1197	1		627	1	
340	40	14	9934	19877	13425	13143	2083	13	2-	935	12	2-
340	40	16	12440	19895	12330	10693	2876	56	7-	1129	44	8-
340	40	18	14941	19900	12702	10118	3502	144	13-	1223	101	13-
340	40	20	17399	19897	13494	10587	3979	282	4-	1250	177	4-
360	5	6-	1858	2494	.1035	.00650	5200	.0000	.00000	5446	.0000	.00000
360	5	8-	3287	2498	593	212	3918			3077		
360	5	10-	5128	2499	382	88	3139			1972		
360	5	12	7352	2497	271	44	121			63		
360	5	14	9846	2505	217	27	228			102		
360	5	16	12447	2509	195	21	329			129		
360	5	18	13043	2489	190	18	406	2		142	1	
360	5	20	17591	2301	200	19	470	3		148	2	
360	10	6-	1892	4959	3857	4630	10222			10704		
360	10	8-	3309	4984	2313	1626	7787			6116		
360	10	10-	5142	4994	1513	690	6262	2	1-	3934	2	2-
360	10	12	7362	4997	1079	348	243			127		
360	10	14	9853	4997	861	214	456			205		
360	10	16	12452	4991	772	163	654	2		257	2	
360	10	18	13047	4999	765	148	816	7		285	5	
360	10	20	17596	5000	799	150	940	14		295	8	
360	15	6-	1939	7392	7912	13435	14999			15707		
360	15	8-	3342	7453	5008	5154	11571			9088		
360	15	10-	5165	7478	3344	2263	9351	4	4-	5876	5	7-
360	15	12	7378	7492	2410	1158	366			191		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1		308		

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
360	15	20	17603	7511	1802	.00508	1413	.0031	00000	.444	0019	.00000
360	20	• 6-	1993	9810	12754	27125	19555			20478		
360	20	8-	3383	9901	8489	11306	15251			11978		
360	20	10-	5195	9952	5812	5168	12398	7	9-	7790	9	17-
360	20	12	7399	9974	4231	2690	490			257		
360	20	14	9881	9984	3407	1683	915	2		411	2	
360	20	16	12475	9992	3079	1294	1313	9		515	7	
360	20	18	15066	9992	3043	1174	1633	26	1-	570	18	1-
360	20	20	17614	9990	3183	1192	1879	54	1-	590	34	1-
360	25	• 6-	2050	12228	18111	45263	23934			25063		
360	25	8-	3431	12333	12587	20270	18827			14787		1-
360	25	10-	5232	12414	8841	9662	15398	12	18-	9675	15	34-
360	[illegible]	12	7427	12448	6514	5129	617			323		
360	[illegible]	14	9902	12471	5281	3246	1147	3		515	3	
360	25	16	12491	12478	4784	2505	1641	15	1-	645	12	1-
360	25	18	15081	12478	4735	2277	2041	41	3-	712	29	3-
360	25	20	17627	12479	4957	2316	2348	85	2-	738	53	2-
360	30	• 6-	2107	14662	23825	67357	28175			29504		
360	30	• 8-	3483	14754	17161	32026	22307		1-	17520		3-
360	30	10-	5273	14862	12348	15886	18341	19	32-	11524	23	61-
360	30	12	7459	14923	9233	8637	748			391		
360	30	14	9926	14947	7530	5520	1380	4		619	4	
360	30	16	12510	14959	6845	4284	1971	21	2-	774	17	2-
360	30	18	15098	14953	6779	3899	2447	60	4-	854	42	5-
360	30	20	17643	14962	7110	3977	2816	122	4-	885	77	3-
360	35	• 8-	3537	17171	22108	46466	25702		2-	20186	1	5-
360	35	10-	5319	17298	16265	23911	21229	27	53-	13339	34	100-
360	35	12	7495	17378	12329	13296	881			461		
360	35	14	9954	17418	10138	8610	1616	6		725	5	
360	35	16	12533	17439	9255	6730	2302	29	3-	904	23	4-
360	35	18	15117	17434	9182	6144	2856	81	7-	997	57	7-
360	35	20	17662	17439	9633	6272	3284	167	6-	1032	105	5-
360	40	• 8-	3593	19589	27347	63427	29021	1	3-	22793	2	8-
360	40	10-	5368	19725	20530	33754	24066	37	83-	15121	47	156-
360	40	12	7535	19829	15783	19206	1017			533		
360	40	14	9986	19882	13082	12602	1854	8		832	7	1-
360	40	16	12559	19905	11987	9910	2634	39	5-	1034	30	6-
360	40	18	15140	19914	11933	9098	3265	107	10-	1140	75	11-
360	40	20	17683	19910	12519	9290	3751	219	8-	1178	137	8-
380	5	6-	1858	2494	1035	00650	5200	0000	00000	5446	0000	00000
380	5	8-	3287	2497	593	212	3916			3076		
380	5	10-	5129	2499	382	88	3139			1972		
380	5	12-	7361	2500	270	43	2621	3		1372	3	
380	5	14	9888	2501	212	26	202			91		
380	5	16	12550	2507	187	19	300			118		
380	5	18	15224	2501	181	17	380	1		133		
380	5	20	17856	2510	188	17	445	3		140	2	
380	10	6-	1892	4958	3856	4629	10221			10703		
380	10	8-	3309	4983	2312	1625	7784			6114		
380	10	10-	5143	4993	1512	689	6261	1		3934	1	1-
380	10	12-	7370	5000	1075	345	5239	10	4-	2743	11	7-
380	10	14	9895	5006	846	208	405			182		
380	10	16	12556	4999	744	153	599	2		235	1	
380	10	18	15229	5005	724	135	761	5		266	3	
380	10	20	17861	5000	745	133	886	11		278	7	
380	15	6-	1939	7391	7910	13432	14997			15705		
380	15	8-	3342	7454	5009	5156	11572			9089		
380	15	10-	5165	7482	3346	2264	9356	2	2-	5879	3	4-
380	15	12-	7387	7496	2398	1148	7841	24	15-	4106	25	23-
380	15	14	9907	7497	1890	695	608			273		
380	15	16	12565	7494	1669	514	899	4		353	3	
380	15	18	15237	7493	1620	451	1140	11		398	8	
380	15	20	17868	7488	1670	446	1327	24		417	15	
380	20	• 6-	1993	9810	12755	27126	19555			20478		
380	20	10-	5196	9952	5808	5163	12398	5	5-	7790	6	18-
380	20	12-	7408	9975	4206	2664	10415	42	35-	5453	44	55-
380	20	14	9923	9993	3342	1632	813	1		365	1	
380	20	16	12578	9996	2960	1213	1201	6		472	5	
380	20	18	15247	9987	2872	1065	1521	20	1-	531	14	1-
380	20	20	17878	9996	2972	1059	1772	42	1-	557	26	1-
380	25	• 6-	2050	12229	18112	45265	23934			25064		
380	25	8-	3431	12331	12583	20261	18824			14785		
380	25	10-	5232	12412	8831	9646	15394	8	11-	9672	9	21-
380	25	12-	7436	12454	6479	5085	12974	67	69-	6793	70	109-
380	25	14	9944	12470	5171	3137	1018	2		457	2	
380	25	16	12594	12487	4602	2349	1503	10		590	8	
380	25	18	15261	12482	4474	2070	1902	31	2-	664	21	
380	25	20	17891	12475	4621	2052	2212	66	2-	695	41	
380	30	• 6-	2107	14662	23826	67360	28175			29505		
380	30	• 8-	3483	14754	17160	32023	22307	.0000	.00000	17520	.0000	00001-
380	30	10-	5274	14860	12336	15862	18337	12	20-	11522	14	38-
380	30	12-	7468	14926	9179	8556	15507	99	121-	8119	103	190-
380	30	14	9968	14949	7375	5336	1226	3		550	2	
380	30	16	12613	14960	6576	4010	1803	15	1-	708	12	2-
380	30	18	15278	14965	6413	3551	2282	44	3-	797	31	4-
380	30	20	17906	14971	6640	3534	2656	95	4-	834	60	4-
380	35	• 8-	3537	17173	22110	464?4	25704			20188		2-
380	35	10-	5320	17299	16256	23890	21230	17	33-	13339	21	63-
380	35	12-	7504	17379	12252	13163	17999	137	194-	9424	144	305-
380	35	14	9996	17421	9929	8325	1436	4		644	3	
380	35	16	12636	17438	8889	6297	2107	20	2-	827	16	3-
380	35	18	15297	17449	8689	5596	2663	61	5-	930	42	6-
380	35	20	17924	17446	8994	5570	3097	130	6-	973	82	6-
380	40	• 8-	3593	19587	27343	63413	29018		2-	22791		4-
380	40	10-	5369	19725	20515	33715	24065	23	52-	15120	29	98-
380	40	12-	7544	19829	15682	19009	20467	183	294-	10716	192	461-
380	40	14	10028	19886	12812	12184	1648	5		740	5	
380	40	16	12661	19911	11521	9283	2411	27	3-	947	21	4-
380	40	18	15319	19920	11280	8274	3044	80	8-	1062	56	8-
380	40	20	17945	19926	11699	8261	3538	170	9-	1112	107	8-
400	5	6-	1858	2494	1035	00650	5200	0000	00000	5445	.0000	.00000
400	5	8-	3287	2497	593	212	3917			3076		
400	5	10-	5129	2501	382	88	3141			1973		
400	5	12-	7367	2501	269	43	2621	2		1372	2	
400	5	14	9921	2506	209	26	180			81		
400	5	16	12639	2493	179	18	273			107		
400	5	18	15387	2500	172	15	354			124		
400	5	20	18100	2506	176	15	419	2		132	1	
400	10	6-	1892	4958	3857	4629	10221			10703		
400	10	8-	3309	4984	2313	1626	7787			6116		
400	10	10-	5143	4993	1511	689	6261			3934		
400	10	12-	7377	4999	1069	342	5234	7	3-	2741	8	5-
400	10	14	9928	5001	830	202	359			161		
400	10	16	12644	5001	719	145	548	1		215		
400	10	18	15391	5001	687	123	709	4		247	3	
400	10	20	18104	4999	699	119	835	8		262	5	
400	15	6-	1939	7392	7911	13434	14998			15706		
400	15	8-	3342	7455	5010	5157	11573			9090		
400	15	10-	5166	7478	3341	2259	9350	2	1-	5875	2	3-
400	15	12-	7393	7489	2383	1138	7832	17	11-	4101	18	17-
400	15	14	9940	7494	1858	675	539			242		
400	15	16	12654	7495	1613	486	822	3		323	2	
400	15	18	15399	7509	1546	417	1065	8		372	6	
400	15	20	18111	7510	1577	404	1255	19		394	12	
400	20	• 6-	1993	9810	12755	27127	19556			20479		
400	20	8-	3383	9903	8492	11311	15253			11980		
400	20	10-	5196	9955	5810	5165	12401	3	3-	7792	4	6-
400	20	12-	7415	9980	4192	2650	10417	30	25-	5454	32	40-
400	20	14	9957	9984	3281	1584	721			323		
400	20	16	12666	9991	2857	1145	1098	4		431	4	
400	20	18	15409	9992	2732	979	1418	15		495	10	
400	20	20	18120	9991	2787	948	1671	33	1-	525	21	1-
400	25	• 6-	2050	12229	18113	45269	23935			25065		
400	25	8-	3431	12333	12588	20270	18828			14787		
400	25	10-	5233	12412	8829	9641	15394	5	7-	9673	6	13-
400	25	12-	7442	12454	6450	5049	12968	48	50-	6790	50	78-
400	25	14	9977	12470	5085	3053	904	1		406	1	
400	25	16	12682	12478	4440	2216	1373	7		539	6	
400	25	18	15423	12484	4254	1902	1773	23	1-	619	16	2-
400	25	20	18133	12481	4342	1844	2088	51	2-	656	32	2-
400	30	• 6-	2107	14663	23827	67366	28176			29506		
400	30	• 8-	3483	14753	17160	32023	22306			17519		
400	30	10-	5275	14860	12331	15852	18337	7	12-	11521	9	23-
400	30	12-	7474	14922	9132	8487	15496	71	87-	8114	74	137-
400	30	14	10002	14950	7252	5194	1088	2		488	1	
400	30	16	12702	14961	6354	3791	1649	10		648	8	1-
400	30	18	15439	14969	6099	3264	2127	33	3-	743	23	3-
400	30	20	18148	14968	6231	3169	2505	74	4-	787	47	3-
400	35	• 8-	3537	17171	22106	46461	25701			20186		1-
400	35	10-	5321	17300	16250	23875	21230	10	21-	13339	13	39-
400	35	12-	7511	17380	12198	13069	17993	98	140-	9421	103	220-
400	35	14	10030	17423	9764	8102	1275	2		572	2	
400	35	16	12724	17443	8593	5956	1927	14	1-	757	11	2-
400	35	18	15458	17450	8261	5142	2483	45	4-	867	32	4-
400	35	20	18165	17451	8449	5003	2922	101	6-	918	64	5-
400	40	• 8-	3593	19587	27343	63412	29018			22791		2-
400	40	10-	5370	19724	20504	33686	24063	15	33-	15119	18	61-
400	40	12-	7551	19830	15610	18870	20460	132	212-	10713	138	333-

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_5	a_1	a_2	a_3
400	40	14	10062	19894	12605	.11867	1464	.0003	00000	.657	.0003	.00000
400	40	16	12749	19911	11131	8773	2206	19	2-	866	15	2-
400	40	18	15480	19926	10730	7608	2838	60	6-	991	42	6-
400	40	20	18185	19929	10988	7417	3338	133	9-	1049	83	8-
4 0	5	6-	1858	2493	.1035	.00649	5199	.0000	.00000	5445	.0000	00000
420	5	8-	3287	2498	593	212	5918			3078		
420	5	10-	5129	2497	381	87	5136			1970		
420	5	12-	7372	2500	268	43	2619	1		1371	1	
420	5	14	9948	2505	206	25	159			71		
420	5	16	12715	2506	176	17	251			99		
420	5	18	15592	2511	166	15	352			116		
420	5	20	18925	2513	167	14	596	2		124	1	
420	10	6-	1892	4958	3857	4629	10221			10703		
420	10	8-	3309	4985	2313	1626	7787			6116		
420	10	10-	5143	4994	1511	689	6261			3934		
420	10	12-	7381	4997	1065	340	5231	5	2-	2739	6	3-
420	10	14	9955	4990	816	196	318			143		
420	10	16	12720	4998	698	138	501			197		
420	10	18	15536	5001	656	114	661	3		231	2	
420	10	20	18527	4996	659	108	788	6		247	4	
420	15	6-	1959	7391	7910	13431	14997			15705		
420	15	8-	3342	7454	5009	5156	11572			9089		
420	15	10-	5166	7475	3338	2295	9346			5872	1	2-
420	15	12-	7398	7487	2374	1131	7827	12	8-	4098	13	12-
420	15	14	9967	7489	1031	660	478			214		
420	15	16	12729	7493	1565	462	751	2		295	1	
420	15	18	15544	7498	1473	385	991	6		346	4	
420	15	20	18334	7499	1484	364	1105	14		572	9	
420	20	•6-	1993	9810	12756	27130	19556			20479		
420	20	8-	3383	9901	0489	11306	15251			11978		
420	20	10-	5196	9951	5804	5157	12397	2	2-	7709	2	4-
420	20	12-	7420	9973	41?2	2631	10406	22	18-	5449	23	28-
420	20	14	9983	9991	3241	1553	639			287		
420	20	16	12742	9992	2775	1091	1003	3		394	2	
420	20	18	15554	9999	2615	910	1322	11		462	8	
420	20	20	18343	9991	2631	860	1576	26		495	16	
420	25	•6-	2050	12228	18111	45261	23934			25063		
420	25	8-	3431	12333	12588	20271	18828			14787		
420	25	10-	5233	12413	8827	9639	15395	3	4-	9675	4	8-
420	25	12-	7447	12452	6427	5020	12962	34	36-	6787	36	56-
420	25	14	10004	12472	5018	2989	801			360		
420	25	16	12758	12481	4314	2114	1256	5		495	4	
420	25	18	15568	12494	4073	1769	1653	17	1-	577	12	1-
420	25	20	18355	12483	4099	1672	1970	40	2-	619	25	2-
420	30	•6-	2107	14662	23825	67356	28174			29504		
420	30	•8-	3483	14755	17162	32030	22308			17521		
428	30	10-	5275	14860	12528	15845	18337	4	8-	11521	6	15-
420	50	12-	7479	14921	9101	8441	15490	51	62-	8110	53	98-
420	30	14	10028	14945	7151	5078	965	1		433		
420	30	16	12777	14965	6174	3617	1509	7		592	6	
420	30	18	15584	14971	5832	3029	1983	25	2-	692	17	2-
420	30	20	18369	14972	5885	2875	2364	58	3-	743	37	3-
420	35	•8-	3557	17171	22106	46461	25701			20186		
420	35	10-	5321	17297	16241	23856	21227	6	13-	13337	8	24-
420	55	12-	7516	17382	12160	13004	17990	71	101-	9419	74	158-
420	35	14	10057	17424	9633	7929	1131	1		508	1	
420	35	16	12800	17456	8357	5690	1764	10		693	8	1-
420	35	18	15602	17464	7909	4781	2316	34	5-	808	24	3-
420	35	20	18386	17456	7981	4539	2758	79	5-	866	50	5-
420	40	•8-	3593	19587	27343	63412	29018			22791		
420	40	10-	5370	19721	20493	33658	24059	9	20-	15117	11	39-
420	40	12-	7556	19832	15559	18771	20455	95	153-	10710	99	240-
420	40	14	10088	19891	12431	11604	1300	2		583	2	
420	40	16	12825	19920	10818	8373	2018	13	1-	793	10	2-
420	40	18	15624	19934	10265	7066	2647	45	5-	924	31	5-
420	40	20	18405	19919	10363	6714	3149	104	7-	989	65	7-
440	5	6-	1858	2494	.1035	00650	5200	.0000	.00000	5446	.0000	.00000
440	5	8-	3287	2498	594	212	5919			3078		
440	5	10-	5129	2500	382	88	5140			1973		
440	5	12-	7375	2501	267	43	2620			1372		
440	5	14	9969	2498	203	24	141			63		
440	5	16	12779	2506	171	17	229			90		
440	5	18	15661	2502	158	13	308			107		
440	5	20	18527	2519	159	13	375	1		118		
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10221			10703		

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
440	10	14	9976	4998	.0810	.00194	.282	0000	.00000	126	.0000	.00000
440	10	16	12785	5002	602	133	458			180		
440	10	18	15666	5004	631	107	616	2		215	1	
440	10	20	18531	4999	626	99	744	5		234	3	
440	15	6-	1939	7392	7912	13435	14999			15707		
440	15	8-	3342	7452	5007	5153	11570			9087		
440	15	10-	5166	7479	3340	2258	9550			5075		
440	15	12-	7401	7488	2370	1127	7826	9	5-	4098	9	8-
440	15	14	9900	7485	1810	648	425			190		
440	15	16	12794	7495	1528	444	687	1		270		
440	15	18	15673	7497	1415	360	925	5		522	3	
440	15	20	18538	7485	1403	531	1114	11		350	7	
440	20	•6-	1993	9810	12754	27125	19555			20478		
440	20	8-	3383	9901	8490	11307	15251			11978		
440	20	10-	5196	9951	5803	5156	12396	1	1-	7789	1	2-
440	20	12-	7423	9975	4165	2623	10406	15	13-	5449	16	20-
440	20	14	10004	9985	3203	1524	566			254		
440	20	16	12807	9990	2706	1047	917	2		360	2	
440	20	18	15683	9992	2509	850	1232	8		430	6	
440	20	20	18547	9993	2497	787	1488	20		468	13	
440	25	•6-	2050	12228	10111	45261	23934			25063		
440	25	8-	3431	12333	12588	20271	18828			14787		
440	25	10-	5233	12413	8826	9637	15395	2	3-	9673	2	5-
440	25	12-	7450	12450	6411	5000	12958	25	25-	6785	26	40-
440	25	14	10025	12471	4965	2938	710			319		
440	25	16	12822	12481	4207	2029	1147	3		451	3	
440	25	18	15696	12489	3910	1653	1541	13		538	9	
440	25	20	18558	12486	3891	1551	1860	32	2-	584	20	1-
440	30	•6-	2107	14662	23025	67356	28174			29504		
440	30	•8-	3483	14754	17162	32028	22308			17521		
440	30	10-	5275	14860	12326	15841	18336	3	5-	11521	3	9-
440	30	12-	7485	14921	9080	8410	15487	36	45-	8109	38	70-
440	30	14	10050	14952	7081	4998	856			384		
440	30	16	12842	14966	6023	3473	1579	5		541	4	
440	30	18	15712	14980	5610	2839	1850	19	1-	646	13	1-
440	30	20	18572	14975	5585	2631	2232	46	3-	701	29	3-
440	35	•8-	5537	17171	22106	46461	25701			20186		
440	35	10-	5321	17297	16257	23848	21226	4	8-	13337	5	15-
440	35	12-	7519	17384	12135	12957	17987	51	72-	9418	53	114-
440	35	14	10078	17422	9529	7791	1003			450		
440	35	16	12864	17450	8144	5457	1612	7		635	5	
440	35	18	15731	17462	7597	4473	2159	25	2-	754	18	2-
440	35	20	18589	17450	7568	4149	2602	62	4-	818	39	4-
440	40	•8-	3593	19587	27543	63413	29019			22791		
440	40	10-	5370	19725	20496	33665	24063	6	13-	15119	7	24-
440	40	12-	7560	19832	15522	18699	20451	68	110-	10708	71	173-
440	40	14	10110	19892	12299	11406	1153	1		510	1	
440	40	16	12889	19923	10553	8042	1846	9		725	7	1-
440	40	18	15752	19941	9869	6619	2469	33	3-	862	23	4-
440	40	20	18608	19920	9836	6145	2973	82	6-	934	51	6-
460	5	6-	1858	2494	.1035	.00650	5200	.0000	.00000	5445	.0000	.00000
460	5	8-	3287	2498	593	212	3918			3077		
460	5	10-	5129	2502	385	88	3143			1975		
460	5	12-	7377	2500	267	43	2618			1371		
460	5	14	9986	2506	202	24	125			56		
460	5	16	12834	2507	168	16	209			82		
460	5	18	15776	2506	153	13	207			100		
460	5	20	18714	2503	150	11	352			110		
460	10	6-	1892	4958	3857	4629	10221			10703		
460	10	8-	3309	4984	2313	1626	7787			6116		
460	10	10-	5144	4993	1511	688	6260			3934		
460	10	12-	7387	4997	1061	338	5229	3	1-	2738	3	2-
460	10	14	9993	4994	802	191	249			112		
460	10	16	12840	5008	669	129	419			164		
460	10	18	15781	5014	612	102	575	2		201	1	
460	10	20	18718	4999	597	91	702	4		221	2	
460	15	6-	1939	7392	7912	13435	14999			15706		
460	15	8-	3342	7453	5008	5155	11571			9088		
460	15	10-	5166	7480	3341	2259	9352			5876		
460	15	12-	7403	7490	2367	1125	7827	6	4-	4098	6	6-
460	15	14	10005	7494	1799	641	375			168		
460	15	16	12849	7496	1496	430	627			246		
460	15	18	15788	7493	1364	339	860	3		300	2	
460	15	20	18725	7493	1340	307	1055	9		331	6	
460	20	•6-	1993	9810	12754	27125	19555			20478		
460	20	8-	3383	9902	8492	11311	15253			11980		

APPENDIX I

STOKES-STRUIK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	a_1	a_2	a_3
460	20	16	12861	9990	.2649	.01012	.837	0001	00000	.329	0001	.00000
460	20	18	15798	10000	2425	805	1149	6		401	4	
460	20	20	18754	9996	2382	727	1406	16		442	10	
460	25	• 6-	2050	12228	18111	45261	23934			25063		
460	25	8-	3431	12333	12588	20272	18828			14788		
460	25	10-	5233	12413	8825	9635	15394	1	2-	9672	1	3-
460	25	12-	7453	12447	6397	4983	12953	18	18-	6782	18	29-
460	25	14	10042	12475	4925	2900	629			282		
460	25	16	12877	12487	4123	1964	1049	2		412	2	
460	25	18	15811	12496	3778	1562	1437	10		502	7	
460	25	20	18745	12486	3710	1412	1756	25	1-	552	16	1-
460	30	• 6-	2107	14662	23825	67356	28174			29504		
460	30	• 8-	5483	14754	17161	32028	22308			17520		
460	30	10-	5275	14860	12325	15839	18336	2	3-	11521	2	6-
460	30	12-	7405	14921	9065	8387	15485	26	32-	8108	27	50-
460	30	14	10067	14954	7022	4952	758			340		
460	30	16	12897	14969	5898	3557	1260	5		495	3	
460	50	18	15827	14981	5414	2678	1724	14	1-	602	10	1-
460	50	20	18759	14990	5337	2436	2110	36	2-	663	25	2-
460	35	• 0-	3537	17171	22106	46461	25701			20186		
460	35	10-	5321	17277	16236	23844	21226	3	5-	13337	5	9-
460	35	12-	7522	17384	12112	12922	17985	56	52-	9417	38	82-
460	55	14	10095	17422	9447	7684	889			399		
460	35	16	12919	17449	7972	5271	1475	5		578	4	
460	35	18	15845	17461	7331	4217	2012	19	2-	702	13	2-
460	35	20	18775	17467	7230	3840	2460	49	3-	773	31	3-
460	40	• 8-	3593	19588	27343	63414	29019			22791		
460	40	10-	5370	19725	20494	33660	24063	4	8-	15119	4	15-
460	40	12-	7562	19832	15495	18647	20448	49	79-	10707	51	124-
460	40	14	10127	19894	12195	11252	1023			459		
460	40	16	12944	19925	10354	7772	1687	6		662	5	
460	40	18	15866	19938	9522	6239	2301	25	2-	805	17	3-
460	40	20	18793	19946	9402	5692	2811	64	5-	883	40	5-
480	5	8-	3287	2498	.0593	.00212	3919	.0000	.00000	3078	.0000	.00000
480	5	10-	5129	2495	301	87	3134			1969		
480	5	12-	7579	2502	267	43	2620			1372		
480	5	14	9999	2506	201	24	110			50		
480	5	16	12881	2503	164	16	191			75		
480	5	18	15878	2489	146	12	266			93		
480	5	20	18886	2500	143	11	332			104		
480	10	0-	5309	4984	2313	1626	7786			6115		
480	10	10-	5144	4993	1510	688	6261			3934		
480	10	12-	7389	4997	1060	338	5229	2		2738	2	1-
480	10	14	10006	5006	800	190	221			99		
480	10	16	12886	5001	655	124	582			150		
480	10	18	15885	5002	590	96	555	1		187		
480	10	20	18889	5009	574	05	665	3		209	2	
480	15	0-	3342	7452	5007	5153	11570			9087		
480	15	10-	5166	7480	5341	2259	9352			5876		
480	15	12-	7405	7496	2368	1126	7032	4	3-	4101	5	4-
480	15	14	10018	7495	1707	635	332			149		
480	15	16	12895	7497	1470	417	573			225		
480	15	10	15890	7497	1325	523	802	5		280	2	
488	20	0-	3383	9901	0489	11306	15251			11970		
480	20	10-	5197	9951	5002	5155	12596			7789		
480	20	12-	7427	9976	4154	2612	10405	8	7-	5448	8	10-
480	20	14	10035	9986	3156	1488	444			199		
480	20	16	12908	9990	2602	983	764	1		500		
480	20	10	15900	9992	2546	761	1070	5		575	5	
480	20	20	18904	9995	2200	675	1527	15		417	8	
480	55	0-	3431	12353	12588	20272	10828			14788		
480	25	10-	5253	12412	8825	9632	15393		1-	9672		2-
480	25	12-	7455	12446	6389	4973	12950	15	13-	6701	15	20-
480	25	14	10055	12475	4892	2069	557			250		
480	25	16	12924	12488	4050	1908	958	2		576	1	
480	25	18	15915	12487	3656	1480	1558	7		447	5	
480	25	20	18915	12492	3556	1315	1659	20	1-	521	12	
480	30	• 8-	3485	14754	17161	52027	22308			17520		
480	30	10-	5275	14860	12525	15838	10557	1	2-	11521	1	5-
480	30	12-	7487	14921	9054	8571	15485	19	25-	8107	19	36-
480	30	24	10080	14954	6976	4879	672			301		
480	30	16	12943	14969	5795	5261	1151	2		432	2	
480	30	16	15929	14982	5247	2544	1607	10		561	7	
480	30	20	18929	14976	5105	2259	1991	28	2-	625	16	2-
480	55	• 8-	3557	17171	22106	46441	25701			20186		
480	55	10-	5321	17297	16255	23843	21226	2	5-	13357	2	6-
480	55	12-	7524	17382	12095	12095	17981	26	57-	9415	27	58-
480	35	14	10109	17424	9385	7604	788			554		
480	55	16	12966	17451	7632	5122	1545	5		528	5	
480	35	18	15947	17461	7104	04004	1875	0014	.00001-	.655	.0010	00001-
480	55	20	18945	17465	6921	3569	2323	39	3-	730	24	3-
480	40	• 8-	3593	19588	27344	63416	29019			22792		
480	40	10-	5370	19725	20493	33656	24063	2	5-	15119	3	9-
480	40	12-	7564	19833	15475	18610	20446	35	57-	10706	37	89-
480	40	14	10141	19895	12113	11151	906			407		
480	40	16	12991	19925	10148	7547	1541	4		605	3	
480	40	18	15968	19942	9231	5928	2145	19	2-	749	13	2-
480	40	20	18963	19950	9007	5296	2656	51	4-	834	32	4-
500	5	8-	3287	2499	.0594	.00212	3920	0000	.00000	3079	0000	.00000
500	5	10-	5150	2501	382	88	3141			1974		
500	5	12-	7380	2502	267	43	2620			1372		
500	5	14	10009	2500	199	24	97			44		
500	5	16	12920	2505	162	15	174			68		
500	5	18	15969	2501	144	11	249			87		
500	5	20	19042	2498	137	10	313			98		
500	10	8-	3309	4985	2314	1626	7788			6116		
500	10	10-	5144	4995	1510	688	6260			5954		
500	10	12-	7390	4999	1060	338	5231	1		2739	1	
500	10	14	10017	5007	796	189	195			88		
500	10	16	12926	4999	645	121	348			137		
500	10	18	15973	4992	571	91	497			174		
500	10	20	19046	5011	552	80	628	2		197	2	
500	15	8-	5342	7453	5007	5153	11570			9087		
500	15	10-	5166	7479	3540	2258	9350			5875		
500	15	12-	7406	7490	2362	1122	7826	3	2-	4098	3	3-
500	15	14	10029	7502	1781	631	294			132		
500	15	16	12955	7499	1449	408	523			205		
500	15	18	15981	7500	1288	309	747	2		261	1	
500	15	20	19052	7502	1256	268	941	6		295	3	
500	20	8-	3385	9901	8489	11306	15251			11978		
500	20	10-	5197	9955	5806	5160	12401			7792		
500	20	12-	7428	9975	4149	2607	10402	6	5-	5447	6	7-
500	20	14	10045	9986	5139	1476	593			176		
500	20	16	12947	9991	2563	959	698			274		
500	20	18	15991	9994	2285	728	997	3		348	2	
500	20	20	19060	9996	2192	632	1254	10		394	6	
500	25	8-	3431	12333	12588	20271	18828			14787		
500	25	10-	5233	12413	8824	9634	15394			9673		1-
500	25	12-	7456	12449	6586	4969	12952	9	9-	6782	9	15-
500	25	14	10066	12475	4867	2846	493			221		
500	25	16	12963	12481	3905	1859	873	1		345		
500	25	18	16004	12483	3554	1413	1246	5		435	4	
500	25	20	19071	12501	3423	1233	1569	16		493	10	
500	30	• 0-	3405	14754	17161	32026	22307			17520		
500	30	10-	5275	14062	12326	15842	18338		1-	11522		2-
500	30	12-	7488	14927	9053	8369	15408	13	16-	8110	14	26-
500	30	14	10091	14954	6938	4857	595			267		
500	30	16	12983	14971	5707	3185	1050	2		412	1	
500	30	18	16019	14978	5101	2428	1497	8		523	5	
500	30	20	19085	14990	4912	2120	1802	22	1-	591	14	1-
500	35	• 8-	3537	17171	22107	46465	25702			20186		
500	35	10-	5521	17298	16236	25845	21227		2-	13357	1	4-
500	35	12-	7525	17302	12085	12875	17979	19	27-	9414	20	42-
500	55	14	10119	17427	9359	7543	698			315		
500	35	16	13005	17451	7715	4997	1228	2		482	2	
500	35	18	16037	17462	6910	3827	1740	11		610	7	
500	35	20	19100	17473	6659	5345	2195	31	2-	690	19	2-
500	40	• 0-	3594	19588	27545	65421	29020			22792		
500	40	10-	5570	19725	20492	33634	24063	1	3-	15119	2	6-
500	40	12-	7565	19834	15463	18586	20446	25	41-	10705	26	64-
500	40	14	10151	19897	12051	11059	803			560		
500	40	16	13031	19929	9999	7568	1407	5		555	2	
500	40	18	16058	19946	8982	5667	2000	14	1-	698	10	1-
500	40	20	19110	19954	8662	4961	2509	40	3-	788	25	3-

APPENDIX II

Table 1 Circular Sines and Cosines for Integral Multiples of θ

Table 2 Hyperbolic Sines and Cosines for Integral Multiples of $2\pi z/L$

The decimal point position is indicated by the vertical lines.

APPENDIX II

TABLE I CIRCULAR SINES AND COSINES FOR INTEGRAL MULTIPLES OF θ

(Degrees) θ	(Radians) θ	Sin. θ	Cos. θ	Sin 2θ	Cos. 2θ	Sin 3θ	Cos 3θ
0	0	0	1.0000	0	1.0000	0	1.0000
1	0175	0175	9998	0349	9994	0523	9986
2	0349	0349	9994	0698	9976	1045	9945
3	0524	0523	9986	1045	9945	1564	9877
4	0698	0698	9976	1392	9903	2079	9781
5	0873	0872	9962	1736	9848	2588	9659
6	1047	1045	9945	2079	9781	3090	9511
7	1222	1219	9925	2419	9703	3584	9336
8	1396	1392	9903	2756	9613	4067	9135
9	1571	1564	9877	3090	9511	4540	8910
10	1745	1736	9848	3420	9397	5000	8660
11	1920	1908	9816	3746	9272	5446	8387
12	2094	2079	9781	4067	9135	5878	8090
13	2269	2250	9744	4384	8988	6293	7771
14	2443	2419	9703	4695	8829	6691	7431
15	2618	2588	9659	5000	8660	7071	7071
16	2793	2756	9613	5299	8480	7431	6691
17	2967	2924	9563	5592	8290	7771	6293
18	3142	3090	9511	5878	8090	8090	5878
19	3316	3256	9455	6157	7880	8387	5446
20	3491	3420	9397	6428	7660	8660	5000
21	3665	3584	9336	6691	7431	8910	4540
22	3840	3746	9272	6947	7193	9135	4067
23	4014	3907	9205	7193	6947	9336	3584
24	4189	4067	9135	7431	6691	9511	3090
25	4363	4226	9063	7660	6428	9659	2588
26	4538	4384	8988	7880	6157	9781	2079
27	4712	4540	8910	8090	5878	9877	1564
28	4887	4695	8829	8290	5592	9945	1045
29	5061	4848	8746	8480	5299	9986	0523
30	5236	5000	8660	8660	5000	1.0000	0
31	5411	5150	8572	8829	4695	9986	0523-
32	5585	5299	8480	8988	4384	9945	1045-
33	5760	5446	8387	9135	4067	9877	1564-
34	5934	5592	8290	9272	3746	9781	2079-
35	6109	5736	8192	9397	3420	9659	2588-
36	6283	5878	8090	9511	3090	9511	3090-
37	6458	6018	7986	9613	2756	9336	3584-
38	6632	6157	7880	9703	2419	9135	4067-
39	6807	6293	7771	9781	2079	8910	4540-
40	6981	6428	7660	9848	1736	8660	5000-
41	7156	6561	7547	9903	1392	8387	5446-
42	7330	6691	7431	9945	1045	8090	5878-
43	7505	6820	7314	9976	0698	7771	6293-
44	7679	6947	7193	9994	0349	7431	6691-
45	7854	7071	7071	1.0000	0	7071	7071-
46	8029	7193	6947	9994	0349-	6691	7431-
47	8203	7314	6820	9976	0698-	6293	7771-
48	8378	7431	6691	9945	1045-	5878	8090-
49	8552	7547	6561	9903	1392-	5446	8387-
50	8727	7660	6428	9848	1736-	5000	8660-
51	8901	7771	6293	9781	2079-	4540	8910-
52	9076	7880	6157	9703	2419-	4067	9135-
53	9250	7986	6018	9613	2756-	3584	9336-
54	9425	8090	5878	9511	3090-	3090	9511-
55	9599	8192	5736	9397	3420-	2588	9659-
56	9774	8290	5592	9272	3746-	2079	9781-
57	9948	8387	5446	9135	4067-	1564	9877-
58	1.0123	8480	5299	8988	4384-	1045	9945-
59	1.0297	8572	5150	8829	4695-	0523	9986-
60	1.0472	8660	5000	8660	5000-	0	1.0000-

(Degrees) θ	(Radians) θ	Sin θ	Cos. θ	Sin. 2θ	Cos. 2θ	Sin 3θ	Cos. 3θ
61	1.0647	8746	4848	8480	5299-	0523-	9986-
62	1.0821	8829	4695	8290	5592-	1045-	9945-
63	1.0996	8910	4540	8090	5878-	1564-	9877-
64	1.1170	8988	4384	7880	6157-	2079-	9781-
65	1.1345	9063	4226	7660	6428-	2588-	9659-
66	1.1519	9135	4067	7431	6691-	3090-	9511-
67	1.1694	9205	3907	7193	6947-	3584-	9336-
68	1.1868	9272	3746	6947	7193-	4067-	9135-
69	1.2043	9336	3584	6691	7431-	4540-	8910-
70	1.2217	9397	3420	6428	7660-	5000-	8660-
71	1.2392	9455	3256	6157	7880-	5446-	8387-
72	1.2566	9511	3090	5878	8090-	5878-	8090-
73	1.2741	9563	2924	5592	8290-	6293-	7771-
74	1.2915	9613	2756	5299	8480-	6691-	7431-
75	1.3090	9659	2588	5000	8660-	7071-	7071-
76	1.3265	9703	2419	4695	8829-	7431-	6691-
77	1.3439	9744	2250	4384	8988-	7771-	6293-
78	1.3614	9781	2079	4067	9135-	8090-	5878-
79	1.3788	9816	1908	3746	9272-	8387-	5446-
80	1.3965	9848	1736	3420	9397-	8660-	5000-
81	1.4137	9877	1564	3090	9511-	8910-	4540-
82	1.4312	9903	1392	2756	9613-	9135-	4067-
83	1.4486	9925	1219	2419	9703-	9336-	3584-
84	1.4661	9945	1045	2079	9781-	9511-	3090-
85	1.4835	9962	0872	1736	9848-	9659-	2588-
86	1.5010	9976	0698	1392	9903-	9781-	2079-
87	1.5184	9986	0523	1045	9945-	9877-	1564-
88	1.5359	9994	0349	0698	9976-	9945-	1045-
89	1.5533	9998	0175	0349	9994-	9986-	0523-
90	1.5708	1.0000	0	0	1.0000-	1.0000-	0
91	1.5882	9998	0175-	0349-	9994-	9986-	0523
92	1.6057	9994	0349-	0698-	9976-	9945-	1045
93	1.6232	9986	0523-	1045-	9945-	9877-	1564
94	1.6406	9976	0698-	1392-	9903-	9781-	2079
95	1.6581	9962	0872-	1736-	9848-	9659-	2588
96	1.6755	9945	1045-	2079-	9781-	9511-	3090
97	1.6930	9925	1219-	2419-	9703-	9336-	3584
98	1.7104	9903	1392-	2756-	9613-	9135-	4067
99	1.7279	9877	1564-	3090-	9511-	8910-	4540
100	1.7453	9848	1736-	3420-	9397-	8660-	5000
101	1.7628	9816	1908-	3746-	9272-	8387-	5446
102	1.7802	9781	2079-	4067-	9135-	8090-	5878
103	1.7977	9744	2250-	4384-	8988-	7771-	6293
104	1.8151	9703	2419-	4695-	8829-	7431-	6691
105	1.8326	9659	2588-	5000-	8660-	7071-	7071
106	1.8500	9613	2756-	5299-	8480-	6691-	7431
107	1.8675	9563	2924-	5592-	8290-	6293-	7771
108	1.8850	9511	3090-	5878-	8090-	5878-	8090
109	1.9024	9455	3256-	6157-	7880-	5446-	8387
110	1.9199	9397	3420-	6428-	7660-	5000-	8660
111	1.9373	9336	3584-	6691-	7431-	4540-	8910
112	1.9548	9272	3746-	6947-	7193-	4067-	9135
113	1.9722	9205	3907-	7193-	6947-	3584-	9336
114	1.9897	9135	4067-	7431-	6691-	3090-	9511
115	2.0071	9063	4226-	7660-	6428-	2588-	9659
116	2.0246	8988	4384-	7880-	6157-	2079-	9781
117	2.0420	8910	4540-	8090-	5878-	1564-	9877
118	2.0595	8829	4695-	8290-	5592-	1045-	9945
119	2.0769	8746	4848-	8480-	5299-	0523-	9986
120	2.0944	8660	5000-	8660-	5000-	0	1.0000

Vertical lines indicate the decimal point position

APPENDIX II
TABLE I CONTINUED

(Degrees) θ	(Radians) θ	Sin θ	Cos. θ	Sin. 2 θ	Cos 2 θ	Sin. 3 θ	Cos. 3 θ
121	21118	8572	5150-	8829-	4695-	0523	9986
122	21293	8480	5299-	8988-	4384-	1045	9945
123	21468	8387	5446-	9135-	4067-	1564	9877
124	21642	8290	5592-	9272-	3746-	2079	9781
125	21817	8192	5736-	9397-	3420-	2588	9659
126	21991	8090	5878-	9511-	3090-	3090	9511
127	22166	7986	6018-	9613-	2756-	3584	9336
128	22340	7880	6157-	9703-	2419-	4067	9135
129	22515	7771	6293-	9781-	2079-	4540	8910
130	22689	7660	6428-	9848-	1736-	5000	8660
131	22864	7547	6561-	9903-	1392-	5446	8387
132	23038	7431	6691-	9945-	1045-	5878	8090
133	23213	7314	6820-	9976-	0698-	6293	7771
134	23387	7193	6947-	9994-	0349-	6691	7431
135	23562	7071	7071-	10000-	0	7071	7071
136	23736	6947	7193-	9994-	0349	7431	6691
137	23911	6820	7314-	9976-	0698	7771	6293
138	24086	6691	7431-	9945-	1045	8090	5878
139	24260	6561	7547-	9903-	1392	8387	5446
140	24435	6428	7660-	9848-	1736	8660	5000
141	24609	6293	7771-	9781-	2079	8910	4540
142	24784	6157	7880-	9703-	2419	9135	4067
143	24958	6018	7986-	9613-	2756	9336	3584
144	25133	5878	8090-	9511-	3090	9511	3090
145	25307	5736	8192-	9397-	3420	9659	2588
146	25482	5592	8290-	9272-	3746	9781	2079
147	25656	5446	8387-	9135-	4067	9877	1564
148	25831	5299	8480-	8988-	4384	9945	1045
149	26005	5150	8572-	8829-	4695	9986	0523
150	26180	5000	8660-	8660-	5000	10000	0
151	26354	4848	8746-	8480-	5299	9986	0523-
152	26529	4695	8829-	8290-	5592	9945	1045-
153	26704	4540	8910-	8090-	5878	9877	1564-
154	26878	4384	8988-	7880-	6157	9781	2079-
155	27053	4226	9063-	7660-	6428	9659	2588-
156	27227	4067	9135-	7431-	6691	9511	3090-
157	27402	3907	9205-	7193-	6947	9336	3584-
158	27576	3746	9272-	6947-	7193	9135	4067-
159	27751	3584	9336-	6691-	7431	8910	4540-
160	27925	3420	9397-	6428-	7660	8660	5000-
161	28100	3256	9455-	6157-	7880	8387	5446-
162	28274	3090	9511-	5878-	8090	8090	5878-
163	28449	2924	9563-	5592-	8290	7771	6293-
164	28623	2756	9613-	5299-	8480	7431	6691-
165	28798	2588	9659-	5000-	8660	7071	7071-
166	28972	2419	9703-	4695-	8829	6691	7431-
167	29147	2250	9744-	4384-	8988	6293	7771-
168	29322	2079	9781-	4067-	9135	5878	8090-
169	29496	1908	9816-	3746-	9272	5446	8387-
170	29671	1736	9848-	3420-	9397	5000	8660-
171	29845	1564	9877-	3090-	9511	4540	8910-
172	30020	1392	9903-	2756-	9613	4067	9135-
173	30194	1219	9925-	2419-	9703	3584	9336-
174	30369	1045	9945-	2079-	9781	3090	9511-
175	30543	0872	9962-	1736-	9848	2588	9659-
176	30718	0698	9976-	1392-	9903	2079	9781-
177	30892	0523	9986-	1045-	9945	1564	9877-
178	31067	0349	9994-	0698-	9976	1045	9945-
179	31241	0175	9998-	0349-	9994	0523	9986-
180	31416	0	10000-	0	10000	0	10000-

(Degrees) θ	(Radians) θ	Sin θ	Cos. θ	Sin. 2θ	Cos. 2 θ	Sin 3 θ	Cos 3 θ
181	31590	0175-	9998-	0349	9994	0523-	9986-
182	31765	0349-	9994-	0698	9976	1045-	9945-
183	31940	0523-	9986-	1045	9945	1564-	9877-
184	32114	0698-	9976-	1392	9903	2079-	9781-
185	32289	0872-	9962-	1736	9848	2588-	9659-
186	32463	1045-	9945-	2079	9781	3090-	9511-
187	32638	1219-	9925-	2419	9703	3584-	9336-
188	32812	1392-	9903-	2756	9613	4067-	9135-
189	32987	1564-	9877-	3090	9511	4540-	8910-
190	33161	1736-	9848-	3420	9397	5000-	8660-
191	33336	1908-	9816-	3746	9272	5446-	8387-
192	33510	2079-	9781-	4067	9135	5878-	8090-
193	33685	2250-	9744-	4384	8988	6293-	7771-
194	33859	2419-	9703-	4695	8829	6691-	7431-
195	34034	2588-	9659-	5000	8660	7071-	7071-
196	34208	2756-	9613-	5299	8480	7431-	6691-
197	34383	2924-	9563-	5592	8290	7771-	6293-
198	34558	3090-	9511-	5878	8090	8090-	5878-
199	34732	3256-	9455-	6157	7880	8387-	5446-
200	34907	3420-	9397-	6428	7660	8660-	5000-
201	35081	3584-	9336-	6691	7431	8910-	4540-
202	35256	3746-	9272-	6947	7193	9135-	4067-
203	35430	3907-	9205-	7193	6947	9336-	3584-
204	35605	4067-	9135-	7431	6691	9511-	3090-
205	35779	4226-	9063-	7660	6428	9659-	2588-
206	35954	4384-	8988-	7880	6157	9781-	2079-
207	36128	4540-	8910-	8090	5878	9877-	1564-
208	36303	4695-	8829-	8290	5592	9945-	1045-
209	36477	4848-	8746-	8480	5299	9986-	0523-
210	36652	5000-	8660-	8660	5000	10000-	0
211	36826	5150-	8572-	8829	4695	9986-	0523
212	37001	5299-	8480-	8988	4384	9945-	1045
213	37176	5446-	8387-	9135	4067	9877-	1564
214	37350	5592-	8290-	9272	3746	9781-	2079
215	37525	5736-	8192-	9397	3420	9659-	2588
216	37699	5878-	8090-	9511	3090	9511-	3090
217	37874	6018-	7986-	9613	2756	9336-	3584
218	38048	6157-	7880-	9703	2419	9135-	4067
219	38223	6293-	7771-	9781	2079	8910-	4540
220	38397	6428-	7660-	9848	1736	8660-	5000
221	38572	6561-	7547-	9903	1392	8387-	5446
222	38746	6691-	7431-	9945	1045	8090-	5878
223	38921	6820-	7314-	9976	0698	7771-	6293
224	39095	6947-	7193-	9994	0349	7431-	6691
225	39270	7071-	7071-	10000	0	7071-	7071
226	39444	7193-	6947-	9994	0349-	6691-	7431
227	39619	7314-	6820-	9976	0698-	6293-	7771
228	39794	7431-	6691-	9945	1045-	5878-	8090
229	39968	7547-	6561-	9903	1392-	5446-	8387
230	40143	7660-	6428-	9848	1736-	5000-	8660
231	40317	7771-	6293-	9781	2079-	4540-	8910
232	40492	7880-	6157-	9703	2419-	4067-	9135
233	40666	7986-	6018-	9613	2756-	3584-	9336
234	40841	8090-	5878-	9511	3090-	3090-	9511
235	41015	8192-	5736-	9397	3420-	2588-	9659
236	41190	8290-	5592-	9272	3746-	2079-	9781
237	41364	8387-	5446-	9135	4067-	1564-	9877
238	41539	8480-	5299-	8988	4384-	1045-	9945
239	41713	8572-	5150-	8829	4695-	0523-	9986
240	41888	8660-	5000-	8660	5000-	0	10000

APPENDIX II
TABLE I CONTINUED

θ (Degrees)	θ (Radians)	Sin θ	Cos θ	Sin 2 θ	Cos 2 θ	Sin 3 θ	Cos 3 θ
241	4.2062	.8746-	.4848-	.8480	.5299-	.0523	.9986
242	4.2237	.8829-	.4695-	.8290	.5592-	.1045	.9945
243	4.2412	.8910-	.4540-	.8090	.5878-	.1564	.9877
244	4.2586	.8988-	.4384-	.7880	.6157-	.2079	.9781
245	4.2761	.9063-	.4226-	.7660	.6428-	.2588	.9659
246	4.2935	.9135-	.4067-	.7431	.6691-	.3090	.9511
247	4.3110	.9205-	.3907-	.7193	.6947-	.3584	.9336
248	4.3284	.9272-	.3746-	.6947	.7193-	.4067	.9135
249	4.3459	.9336-	.3584-	.6691	.7431-	.4540	.8910
250	4.3633	.9397-	.3420-	.6428	.7660-	.5000	.8660
251	4.3808	.9455-	.3256-	.6157	.7880-	.5446	.8387
252	4.3982	.9511-	.3090-	.5878	.8090-	.5878	.8090
253	4.4157	.9563-	.2924-	.5592	.8290-	.6293	.7771
254	4.4331	.9613-	.2756-	.5299	.8480-	.6691	.7431
255	4.4506	.9659-	.2588-	.5000	.8660-	.7071	.7071
256	4.4680	.9703-	.2419-	.4695	.8829-	.7431	.6691
257	4.4855	.9744-	.2250-	.4384	.8988-	.7771	.6293
258	4.5029	.9781-	.2079-	.4067	.9135-	.8090	.5878
259	4.5204	.9816-	.1908-	.3746	.9272-	.8387	.5446
260	4.5379	.9848-	.1736-	.3420	.9397-	.8660	.5000
261	4.5553	.9877-	.1564-	.3090	.9511-	.8910	.4540
262	4.5728	.9903-	.1392-	.2756	.9613-	.9135	.4067
263	4.5902	.9925-	.1219-	.2419	.9703-	.9336	.3584
264	4.6077	.9945-	.1045-	.2079	.9781-	.9511	.3090
265	4.6251	.9962-	.0872-	.1736	.9848-	.9659	.2588
266	4.6426	.9976-	.0698-	.1392	.9903-	.9781	.2079
267	4.6600	.9986-	.0523-	.1045	.9945-	.9877	.1564
268	4.6775	.9994-	.0349-	.0698	.9976-	.9945	.1045
269	4.6949	.9998-	.0175-	.0349	.9994-	.9986	.0523
270	4.7124	1.0000-	0	0	1.0000-	1.0000	0
271	4.7298	.9998-	.0175	.0349-	.9994-	.9986	.0523-
272	4.7473	.9994-	.0349	.0698-	.9976-	.9945	.1045-
273	4.7647	.9986-	.0523	.1045-	.9945-	.9877	.1564-
274	4.7822	.9976-	.0698	.1392-	.9903-	.9781	.2079-
275	4.7997	.9962-	.0872	.1736-	.9848-	.9659	.2588-
276	4.8171	.9945-	.1045	.2079-	.9781-	.9511	.3090-
277	4.8346	.9925-	.1219	.2419-	.9703-	.9336	.3584-
278	4.8520	.9903-	.1392	.2756-	.9613-	.9135	.4067-
279	4.8695	.9877-	.1564	.3090-	.9511-	.8910	.4540-
280	4.8869	.9848-	.1736	.3420-	.9397-	.8660	.5000-
281	4.9044	.9816-	.1908	.3746-	.9272-	.8387	.5446-
282	4.9218	.9781-	.2079	.4067-	.9135-	.8090	.5878-
283	4.9393	.9744-	.2250	.4384-	.8988-	.7771	.6293-
284	4.9567	.9703-	.2419	.4695-	.8829-	.7431	.6691-
285	4.9742	.9659-	.2588	.5000-	.8660-	.7071	.7071-
286	4.9916	.9613-	.2756	.5299-	.8480-	.6691	.7431-
287	5.0091	.9563-	.2924	.5592-	.8290-	.6293	.7771-
288	5.0265	.9511-	.3090	.5878-	.8090-	.5878	.8090-
289	5.0440	.9455-	.3256	.6157-	.7880-	.5446	.8387-
290	5.0615	.9397-	.3420	.6428-	.7660-	.5000	.8660-
291	5.0789	.9336-	.3584	.6691-	.7431-	.4540	.8910-
292	5.0964	.9272-	.3746	.6947-	.7193-	.4067	.9135-
293	5.1139	.9205-	.3907	.7193-	.6947-	.3584	.9336-
294	5.1313	.9135-	.4067	.7431-	.6691-	.3090	.9511-
295	5.1487	.9063-	.4226	.7660-	.6428-	.2588	.9659-
296	5.1662	.8988-	.4384	.7880-	.6157-	.2079	.9781-
297	5.1836	.8910-	.4540	.8090-	.5878-	.1564	.9877-
298	5.2011	.8829-	.4695	.8290-	.5592-	.1045	.9945-
299	5.2185	.8746-	.4848	.8480-	.5299-	.0523	.9986-
300	5.2360	.8660-	.5000	.8660-	.5000-	0	1.0000-
301	5.2534	.8572-	.5150	.8829-	.4695-	.0523-	.9986-
302	5.2709	.8480-	.5299	.8988-	.4384-	.1045-	.9945-
303	5.2883	.8387-	.5446	.9135-	.4067-	.1564-	.9877-
304	5.3058	.8290-	.5592	.9272-	.3746-	.2079-	.9781-
305	5.3233	.8192-	.5736	.9397-	.3420-	.2588-	.9659-
306	5.3407	.8090-	.5878	.9511-	.3090-	.3090-	.9511-
307	5.3582	.7986-	.6018	.9613-	.2756-	.3584-	.9336-
308	5.3756	.7880-	.6157	.9703-	.2419-	.4067-	.9135-
309	5.3931	.7771-	.6293	.9781-	.2079-	.4540-	.8910-
310	5.4105	.7660-	.6428	.9848-	.1736-	.5000-	.8660-
311	5.4280	.7547-	.6561	.9903-	.1392-	.5446-	.8387-
312	5.4454	.7431-	.6691	.9945-	.1045-	.5878-	.8090-
313	5.4629	.7314-	.6820	.9976-	.0698-	.6293-	.7771-
314	5.4803	.7193-	.6947	.9994-	.0349-	.6691-	.7431-
315	5.4978	.7071-	.7071	1.0000-	0	.7071-	.7071-
316	5.5152	.6947-	.7193	.9994-	.0349	.7431-	.6691-
317	5.5327	.6820-	.7314	.9976-	.0698	.7771-	.6293-
318	5.5501	.6691-	.7431	.9945-	.1045	.8090-	.5878-
319	5.5676	.6561-	.7547	.9903-	.1392	.8387-	.5446-
320	5.5851	.6428-	.7660	.9848-	.1736	.8660-	.5000-
321	5.6025	.6293-	.7771	.9781-	.2079	.8910-	.4540-
322	5.6200	.6157-	.7880	.9703-	.2419	.9135-	.4067-
323	5.6374	.6018-	.7986	.9613-	.2756	.9336-	.3584-
324	5.6549	.5878-	.8090	.9511-	.3090	.9511-	.3090-
325	5.6723	.5736-	.8192	.9397-	.3420	.9659-	.2588-
326	5.6898	.5592-	.8290	.9272-	.3746	.9781-	.2079-
327	5.7072	.5446-	.8387	.9135-	.4067	.9877-	.1564-
328	5.7247	.5299-	.8480	.8988-	.4384	.9945-	.1045-
329	5.7421	.5150-	.8572	.8829-	.4695	.9986-	.0523-
330	5.7596	.5000-	.8660	.8660-	.5000	1.0000-	0
331	5.7770	.4848-	.8746	.8480-	.5299	.9986-	.0523
332	5.7945	.4695-	.8829	.8290-	.5592	.9945-	.1045
333	5.8119	.4540-	.8910	.8090-	.5878	.9877-	.1564
334	5.8294	.4384-	.8988	.7880-	.6157	.9781-	.2079
335	5.8469	.4226-	.9063	.7660-	.6428	.9659-	.2588
336	5.8643	.4067-	.9135	.7431-	.6691	.9511-	.3090
337	5.8818	.3907-	.9205	.7193-	.6947	.9336-	.3584
338	5.8992	.3746-	.9272	.6947-	.7193	.9135-	.4067
339	5.9167	.3584-	.9336	.6691-	.7431	.8910-	.4540
340	5.9341	.3420-	.9397	.6428-	.7660	.8660-	.5000
341	5.9516	.3256-	.9455	.6157-	.7880	.8387-	.5446
342	5.9690	.3090-	.9511	.5878-	.8090	.8090-	.5878
343	5.9865	.2924-	.9563	.5592-	.8290	.7771-	.6293
344	6.0039	.2756-	.9613	.5299-	.8480	.7431-	.6691
345	6.0214	.2588-	.9659	.5000-	.8660	.7071-	.7071
346	6.0388	.2419-	.9703	.4695-	.8829	.6691-	.7431
347	6.0563	.2250-	.9744	.4384-	.8988	.6293-	.7771
348	6.0737	.2079-	.9781	.4067-	.9135	.5878-	.8090
349	6.0912	.1908-	.9816	.3746-	.9272	.5446-	.8387
350	6.1087	.1736-	.9848	.3420-	.9397	.5000-	.8660
351	6.1261	.1564-	.9877	.3090-	.9511	.4540-	.8910
352	6.1436	.1392-	.9903	.2756-	.9613	.4067-	.9135
353	6.1610	.1219-	.9925	.2419-	.9703	.3584-	.9336
354	6.1785	.1045-	.9945	.2079-	.9781	.3090-	.9511
355	6.1959	.0872-	.9962	.1736-	.9848	.2588-	.9659
356	6.2134	.0698-	.9976	.1392-	.9903	.2079-	.9781
357	6.2308	.0523-	.9986	.1045-	.9945	.1564-	.9877
358	6.2483	.0349-	.9994	.0698-	.9976	.1045-	.9945
359	6.2657	.0175-	.9998	.0349-	.9994	.0523-	.9986

Vertical lines indicate the decimal point position

APPENDIX II

TABLE 2. HYPERBOLIC SINES & COSINES FOR INTEGRAL MULTIPLES OF $2\pi z/L$

z/L	Sinh $2\pi z/L$	Cosh $2\pi z/L$	Sinh $4\pi z/L$	Cosh $4\pi z/L$	Sinh $6\pi z/L$	Cosh $6\pi z/L$
0	0	10000000	0	10000000	0	10000000
010000000	062873200	10019746	12599470	10079061	18961375	10178180
020000000	12599470	10079061	25398165	10317494	38598460	10719068
030000000	18961375	10178180	38598460	10719068	59611035	11641940
040000000	25398165	10317494	52409080	11290134	82747900	12979682
050000000	31935255	10497553	67048400	12039721	10883357	14779968
060000000	38598460	10719068	82747905	12979683	13879762	17106952
070000000	45414095	10982915	99755825	14124881	17370783	20043555
080000000	52409075	11290134	11834111	15493424	21480830	23694431
090000000	59611035	11641940	13879762	17106952	26356363	28189677
10000000	67048400	12039721	16144881	18990976	32171131	33689489
11000000	74750550	12495049	18665286	21175290	39132348	40389858
12000000	82747900	12979682	21480830	23694431	47488076	48529551
13000000	91072030	13525574	24636031	26588231	57536090	58398640
14000000	99755820	14124880	28180782	29902449	69634450	70348820
15000000	10883357	14779968	32171134	33689492	84214295	84805945
16000000	11834110	15493423	36670179	38009236	10179521	10228521
17000000	12831599	16268065	41749059	42929989	12300367	12340949
18000000	13879762	17106952	47488091	48529556	14859548	14893158
19000000	14982736	18013394	53977990	54896480	17948262	17976098
20000000	16144981	18990976	61321410	62131440	21676580	21699634
21000000	17370783	20043555	69634455	70348825	26177362	26196456
22000000	18665286	21175290	79048570	79678580	31610996	31626809
23000000	20033500	22390648	89712615	90268225	38171119	38184216
24000000	21480830	23694431	10179521	10228521	46091501	46102348
25000000	23012989	25091785	11548741	11591955	55654400	55663380
26000000	24636031	26588231	13100571	13138682	67200590	67208030
27000000	26356363	28189677	14859550	14893160	81141535	81147695
28000000	28180782	29902449	16853490	16883131	97974035	97979135
29000000	30116488	31733398	19113920	19140061	11829792	11830215
30000000	32171131	33689489	21676582	21699636	14283747	14284097
31000000	34352819	35778711	24581997	24602329	17246716	17247006
32000000	36670176	38009233	27876107	27894038	20824285	20824525
33000000	39132343	40389854	31610999	31626812	25143949	25144148
34000000	41749055	42929985	35845728	35859674	30359638	30359803
35000000	44530639	45639652	40647261	40659560	36657222	36657358
36000000	47488076	48529551	46091510	46102357	44261121	44261234
37000000	50633055	51611105	52264565	52274135	53442300	53442390
38000000	53977985	54896475	59264035	59272475	64527945	64528025
39000000	57536090	58398640	67200600	67208040	77913105	77913165
40000000	61321405	62131435	76199750	76206310	94074765	94074815
41000000	65348890	66109590	86403790	86409570	11358887	11358891
42000000	69634450	70348820	97974045	97979145	13715081	13715085
43000000	74195005	74865875	11109349	11109799	16560025	16560028
44000000	79048560	79678570	12596956	12597353	19995099	19995102
45000000	84214295	84805945	14283749	14284099	24142718	24142720
46000000	89712605	90268215	16196398	16196707	29150685	29150687
47000000	95565205	96086985	18365148	18365420	35197460	35197461
48000000	10179520	10228520	20824290	20824530	42498532	42498533
49000000	10842720	10888736	23612708	23612920	51314075	51314075
50000000	11548740	11591954	26774495	26774682	61958240	61958240
51000000	12300367	12340949	30359644	30359809	74810350	74810350
52000000	13100570	13138681	34424844	34424989	90328385	90328385
53000000	13952508	13988298	39034376	39034504	10906536	10906536
54000000	14859548	14893158	44261125	44261238	13168894	13168894
55000000	15825269	15856833	50187745	50187845	15900548	15900548
56000000	16853488	16883129	56907930	56908020	19198815	19198815
57000000	17948262	17976098	64527960	64528040	23181268	23181268
58000000	19113918	19140059	73168315	73168385	27989783	27989783
59000000	20355055	20379604	82965615	82965675	33795766	33795766
60000000	21676580	21699634	94074785	94074835	40806098	40806098
61000000	23083706	23105356	10667148	10667153	49270549	49270549
62000000	24581995	24602327	12095488	12095492	59490850	59490850
63000000	26177359	26196453	13715083	13715087	71831100	71831100
64000000	27876105	27894036	15551544	15551547	86731190	86731190
65000000	29684934	29701773	17633910	17633913	10472193	10472193
66000000	31610996	31626809	19995105	19995108	12644464	12644464
67000000	33661894	33676744	22672465	22672467	15267333	15267333
68000000	35845725	35859671	25708326	25708328	18434252	18434252
69000000	38171119	38184216	29150691	29150693	22258110	22258110
70000000	40647253	40659552	33053991	33053993	26875133	26875133
71000000	43283912	43295462	37479946	37479947	32449902	32449902
72000000	46091501	46102348	42498540	42498541	39181020	39181020
73000000	49081116	49091302	48189128	48189129	47308427	47308427
74000000	52264555	52274125	54641690	54641690	57121665	57121665
75000000	55654400	55663380	61958260	61958260	68970540	68970540
76000000	59264020	59272460	70254520	70254520	83277255	83277255
77000000	63107700	63115620	79661655	79661655	10055154	10055154
78000000	67200585	67208025	90328410	90328410	12140918	12140918
79000000	71558865	71565855	10242346	10242346	14659320	14659320
80000000	76199735	76206295	11613806	11613806	17700137	17700137
81000000	81141535	81147695	13168907	13168907	21371695	21371695
82000000	86403760	86409550	14932224	14932224	25804873	25804873
83000000	92007225	92012655	16931665	16931665	31157605	31157605
84000000	97974035	97979135	19198834	19198834	37620697	37620697
85000000	10432775	10433254	21769557	21769557	45424443	45424443
86000000	11109348	11109798	24684525	24684525	54846880	54846880
87000000	11829792	11830215	27989810	27989810	66223895	66223895
88000000	12596955	12597352	31737646	31737646	79960785	79960785
89000000	13413863	13414236	35987351	35987351	96547230	96547230
90000000	14283747	14284097	40806098	40806098	11657413	11657413
91000000	15210037	15210366	46270080	46270080	14075536	14075536
92000000	16196395	16196704	52465645	52465645	16995256	16995256
93000000	17246714	17247004	59490850	59490850	20520600	20520600
94000000	18365144	18365416	67456740	67456740	24777234	24777234
95000000	19556099	19556355	76489195	76489195	29916802	29916802
96000000	20824285	20824525	86731190	86731190	36122511	36122511
97000000	22174708	22174933	98344590	98344590	43615442	43615442
98000000	23612704	23612916	11151304	11151304	52662690	52662690
99000000	25143947	25144146	12644464	12644464	63586570	63586570
10000000	26774489	26774676	14337571	14337571	76776470	76776470

Vertical lines indicate the decimal point position

Palm Beach

Part 2

COASTAL SEDIMENT PROBLEMS

Palm Beach

CHAPTER 17

THE ORIGIN AND STABILITY OF BEACHES

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THE ORIGIN OF BEACHES

One should not lose sight of the fact that the origin of beaches goes back into antiquity. The story begins with the origin of matter and continues through the aeons with the evolution of the solar system and the appearance of the Earth as a fiery ball gyrating in space. As one's focus narrows there is to be seen the cooling of that ball, the formation of dense clouds of water vapour in the atmosphere, the torrential rains and the beginnings of the seas. Perhaps it is at this point that the introduction is completed and the real story of the beaches begins, for with the rains came erosion of the land masses, and the transportation of the eroded material by river and rivulet towards the sea. At the brink of the ocean a brief halt is called in its journey, for here a portion of this eroded material takes position as beaches around the coast, before ultimately joining the remainder in the depths of the sea. For many thousands of years the sediment so formed and transported collected on the sea bed, consolidated and hardened and was transformed into the sedimentary rocks which, by adjustments in the Earth's crust, were later lifted above the surface of the sea to form new islands and continents. Still the rains fall, although perhaps not so heavily as before; still the processes of erosion continue upon the land masses, old and new; still a part of the products of this erosion remain for a while at the coastal fringe before they pass on to the ocean depths - the raw material of what may be, by completion of the cycle, the continents of tomorrow. Such is the sequence of events over a period of millions of years and, as the process continues during the millions of years which the future holds, the existing land masses will no doubt be eroded away and the materials of which they are composed will finally rest again on the bed of the sea. For so long as the seas have washed the shores, and the rains have fallen and reduced the mountains and high places, there have been beaches. Those beaches which are found today may have existed from time immemorial in some form or other, perhaps since before life appeared on the surface of the Earth. Due to their position in the pattern of Nature they will have changed as the coastline changed, and as the eroded ingredients of the land which formed them changed. They will have grown when the new material supply exceeded the wastage, and they will have diminished when the wastage was more rapid than the replenishment. The changes which are taking place today and which are engaging the attention of the Civil Engineer, form an infinitesimally small incident in the history of the beaches; and in the considerations of the Engineer they should be related to the whole, of which they form a part.

Quantitative Equilibrium of Beach Material.

Eroded material, from whatever source, passes inevitably into the depths of the sea in the form of silt. It is a journey of no return, for there it will remain until resurrected by some immense adjustment of the Earth's crust capable of raising the sea bed above the surface of the waters. There is no natural process which conveys these sedimentary deposits back along a return route to cast them up around the coasts, and the origin of beaches must be sought, not in the basin of the sea, but in the land masses from which they are truly derived. The accumulations of sand and shingle which exist as beaches around the perimeters of the land masses, represent a transitional stage in the conveyance of eroded material from the land to the sea. They are the coarser materials which have been rejected by the sea and imprisoned on the beaches to be submitted once more to the mills of Nature and ground down to a degree of fineness acceptable to the discriminating sea. By the constant application of the waves the coarser materials of the beach are ground against each other, graduall wearing each other away to the fine silt which as it is formed is take up into suspension in the sea water, conveyed out to sea and deposited upon the ocean bed. As previously remarked, when the material which is added to the beach from new destruction of the land exceeds the wastage, the beach will grow in size, but, when the supply is diminished or cut off and cannot therefore make good wastage, the beach becomes smaller and suffers erosion. Thus a balance, or equilibrium is maintained by Nature, for a large well-formed beach protects in some measure the lands to be found in the rear, such that the rate of destruction is thereby slowed down. Were the beach to be diminished in size the waves would more readily have access to the cliffs which would be attacked and broken down to form new beach material. In this way Nature seeks not to destroy indiscriminately but rather to preserve, for, by affording this protective screen of beaches around the softer parts of the coast, the rate of erosion of the land is reduced to a minimum and only such material is taken as is essentiall required to maintain the bulk of the beach. It might be appropriate to consider at this point what would have been the effect upon the land masses had the Laws of Nature been so arranged that material eroded from the cliffs was forthwith swept away and dissipated by the sea. In this eventuality the waves would, with every tide, have access to the land and would exact a contribution from the 'soft' cliffs. It is considered not unlikely that, in the long interval of time which has elapsed, many of the softer coastal districts would have eroded away; and the map of the world would be very different from what it is now. In this way, by gracious natural laws, Nature has preserved our country from erosion by the sea; and the inevitabl minimum of destruction which is receiving popular attention today might well be reviewed against the gigantic erosive activity which, i the absence of protective beaches, could have taken place along the coasts.

Segregation - Critical Size.

The sea segregates the eroded matter with which it has to deal in many ways. Thus there is a difference between material which may be transported into the depths of the sea and material which must remain imprisoned on the beaches. In order that a particle may pass below low water mark it must be sufficiently fine for it to be taken up into suspension, and to remain so for an adequate length of time for it to be conveyed into deeper water and there deposited on the sea bed. Particles which are greater than this Critical Size remain above low water as sand or shingle beaches, confined by forces which will be described later. Even that material which is admitted to the sea bed is assorted, so that the finest is to be found at the greatest distance from the shore. In order to establish what that Critical Size of particle might be, the Authors carried out a number of investigations into the nature of the deposits on the sea bed below low water mark at a depth below which the average wave might be presumed to have little effect, namely about one fathom. A number of pebble beaches were tested in the Torbay area and were found to change at this depth to a fine silt but it was the sand beaches which received particular attention because the information required was the size of particle above which all particles remained on the beach and below which they were taken into suspension and conveyed to the sea bed. A number of samples were obtained which were then dried and analysed by passing through British Standard Sieves. It was established that the Critical Size was that corresponding approximately to a sieve of 100 meshes to the inch, for nearly all the silt obtained from below low water mark passed through the sieve, whilst the sand above low water mark was largely retained upon it. It was in fact remarkable how little fine material was retained on the beach and how little coarse sand had found its way below sea level. The Critical Size may vary in different localities. The importance of this investigation is in its influence on one's attitude to the sea bed as a potential source of beach material. It had once been considered by the Authors that a possible way of building up a beach would be to e xcavate material from the sea bed by means of a dredger or similar machine; then to place it upon the beach where it could be stabilised by a system of groynes. In view of the nature of the bed material, however, it is clear that such a proposal would be impracticable for, after a while, the fine silt would be taken up again into suspension and returned to the bed of the sea. All material placed upon a beach to build it up should, therefore, be of a size coarser than the Critical Size described above. The experiment also points to the unlikelihood of beaches having their origin in the bed of the sea.

Sources of Origin of Beach Material.

Beaches are formed from the destruction of the coast by the waves and from the eroded matter brought down to the coast by rivers. The former are usually pebble beaches and the latter sand, although sometimes cliffs break down into sand instead of pebbles. Sometimes both sources contribute towards the material of a beach. The best beaches are those produced by the destruction of the 'soft' coastline for the gravel and sand is ideal material and has been available until recently in unlimited supplies. Many miles of this 'soft' easily eroded foreshore have recently been protected from further destruction by the construction of sea walls and is no longer available as a source of raw material for the beaches. It is feared that, as a result, those dependent beaches will soon become depleted and will ultimately vanish, and, accordingly, it becomes the duty of the Engineer to conserve what remains to the best advantage. No doubt factors which affect the discharge of rivers, such as impounding dams, reservoirs and drainage systems, will influence the supply of material to the beaches and may to some extent be responsible for 'erosion'. The foregoing remarks have dealt with some aspects of the origin of beaches in general terms, and it is hoped that they may be applied with profit to particular cases. It is now intended to deal with matters relating to the distribution of the material once it is located on the beach, for, sometimes, it is found to be occupying a situation distant from its point of deposition on the foreshore. Thus knowledge of the processes involved would be an advantage.

Waves.

Waves are essentially surface phenomena. As they move they impart to the water what has been described as a rotary movement, the direction of rotation being forward near the surface and in the reverse direction at depth. This agitation diminishes with increasing depth below the surface and, because as might be expected the larger waves are more violent in their action and have a greater field of influence, the degree is related to the size of the wave. As the effect decrease with depth, there eventually becomes a position where it may be disregarded and, subject to the discretion of the Engineer, this may be taken as one and a half times the wave height. Thus below this depth the effect of the wave upon the sea bed becomes negligible and consists only of such slight movement as would disturb the finer silts. When a wave breaks upon a beach, its character changes. The potential ener with which it is charged, and the ordered rotation of particles of water which represents its kinetic energy are lost, and the wave become simply a body of water moving directly upon the slope of the beach. With impact, a further transformation of its energy occurs. Some is absorbed by the material of the beach as it is conveyed up the slope an is ground particle against particle; some is dissipated as eddies as the violence of the impact produces the familiar froth and foam; some is absorbed by friction as the water penetrates the permeable mass of the beach; and some is transformed into potential energy as the water flows up the beach as 'swash' before eventually halting and returning to the body of the sea. As this potential energy is again converted into kinetic energy in the flow of water seawards, it is again able to pick up and transport beach material down the slope of the beach until it is met by the next oncoming wave when the cycle is repeated over again.

It would appear that at the particular slope of the beach which will be described as the 'Slope of Equilibrium', the amount of material moved up the beach by the advancing wave is balanced by the amount which is drawn down by the receding water. Were the slope at any time to be too steep, then more would be drawn down than cast up, and vice versa. Thus Nature automatically adjusts the slope of the beach. It will be clear that, the beach being arranged on the slope, the force of gravity impedes the passage of material up the beach, whilst assisting its downward journey. Thus the excessive energy of the advancing wave compared with diminished resources of the receding water is compensated in its effect by the bias introduced by the force of gravity. The energy of the advancing wave which is expended upon the beach must, of necessity, be greater than that amount which is returned to the sea by the receding water. Therefore it follows that the overall effect of the waves is to impart energy to the beach and to exert a force upon the aggregation of the beach particles; this force acts towards the shore in the direction of travel of the waves. Thus, if the average force of the waves is towards the land, there cannot be an overall movement of beach material seaward in opposition to this force. Beach material is imprisoned upon the beach and any mass movement of this material must be confined within the approximate limits of high and low water. Consider the application of this principle, namely, that the action of the waves is such that the material of a beach (provided it is coarser than a particular critical value at which it might be taken up into suspension) is compelled to remain approximately above the level of low water mark. There will be seen here justification for the claim that a physical obstruction such as an impermeable groyne or a headland across a beach would effectively prevent the passage of material in a longitudinal direction. In a similar manner, the channel of a river intersecting a foreshore, if it were deeper than about a fathom at low tide, would be equally effective in opposing the passage of material, for the action of the waves would prevent the beach material from gaining access to the bed of the river and crossing to the other side. This phenomenon is illustrated at Westward Ho, on the North Devon coast, where the River Torridge discharges into the Atlantic. Those familiar with the area will recall that the river channel almost bisects the unit of beach between Saunton Cliffs at the north-east and Hartland Cliff at the west. The famous pebble ridge was formed from the destruction of the latter cliffs coupled with the gradual distribution of the eroded material along the foreshore, until it extended along the estuary of the river, resting on the alluvial clays. It will be observed that the eastward drift has accumulated a vast bank of pebbles south of the river, but it will also be observed that none of these have succeeded in crossing the river to Saunton Sands.

Summary of Principles.

There may be those who would say that this subject matter does not warrant the choice of title in so far as the Paper has not dealt individually in detail with specific cases. Such was not the intention; rather was it the object to discuss certain general principles bearing on the subject which the Engineer must take into account when he is investigating the origin of a particular beach. Those principles are now set out in concise form as under.

1. Beach material does not originate in the sea.
2. Beaches originate firstly in the destruction of the coast and secondly from material brought down to the coast by rivers.
3. Movement of beach material is confined within the approximate limits of low and high water.
4. Longitudinal movement of beach material may be completely intercepted by barriers across the beach which may be headlands, impermeable groynes or rivers.
5. A unit of beach between any pair of barriers is independent and isolated from the adjacent foreshore.
6. In order that material may remain in position on the beach, the particles must be greater than the Critical Size (100-mesh sieve approximately). Material less than this will be taken into suspension and conveyed out to sea.

These remarks would not be complete without reference to certain views which have influenced opinion and exerted something of a strangle hold on the course of coast protection measures in the past. This Paper is based upon fundamental facts, namely, that beach material of a kind similar to that found on the beaches is not generally to be found on the sea bed, and also that natural forces do not exist which could convey material along the upward slope of the sea bed in the direction of the shore to deposit it ultimately upon the beach. It has been suggested that there are cases where some beach material is of a character dissimilar to that of the adjacent coast or countryside, and it is contended on this account that, because the beaches do not consist of material geologically similar to the cliffs or river deposit in the vicinity, then it necessarily follows that it must have come from the bed of the sea. This argument is negative in character and is submitted only in default of other explanations for the accident of these strange ingredients. For complete proof, evidence should be produced showing that such material may be found on the bed of the sea and that the necessary transportation on to the beach could be effected under natural conditions. The origins of beaches are in some cases simple to establish, in others difficult, and perhaps nobody will ever tell their full story. The reader should return to the opening sentence of this Paper and consider how the origin of a beach may be lost in the mists of geological time; for who can penetrate the tens or hundreds of thousands of years and follow the adventures of a flint now resting on a south coast beach, which was formed in the great chalk bed which once existed.

THE FOUNDATION OF BEACHES

The wave is the cutting edge of the sea. Year by year the coast perimeters of the land masses are subjected to the destructive onslaug of the sea, and with the wave as its tool the solid rocks are reduced sediment. Over a period of infinite time enormous areas of coastal lands have thus been destroyed and have left as evidence the "Continental Shelf" which is a wave cut platform marking the advance of the sea since the continents were first formed. All the erosive qualities of Nature seem to be embodied in the wave. It has corrosive properties which soften the rock, it is capable of developing immense hydraulio pressures whereby the rock may be shattered, and by taking up the fragments into its moving stream, continual abrasive actioh may be exerted upon the rock face within the limits of wave activity, i.e. in the vertical band between high and low water mark. Eventually when the cliff above these levels is undermined and can no longer sustain itself, it collapses and in time forms the aggregate of which many beaches are composed. The foundation of a beach was therefore the wave cut platform upon which the debris of destruction had been deposited. According to the geological structure of the coastline, so the foundation of the beach varied. When the coastline was hard in character, consisting of granite, limestone, sandstone, chalk etc. there was a distinct plane of demarcation between the foundation and the beach itself. This was not so evident when the coastline was soft, consisting of sand or gravel for then the beach material was very similar in character to the foundation and only careful scrutiny would reveal the difference. There were also cases where beaches consisted of material which had drifted along the coast and formed along the perimeter of low lying land, in which cases the foundation of the beach might be a clay strata. In all cases however, a beach must have a solid foundation, the surface level of which must lie above or close to low water mark. The movement of beach material by the waves was confined within the approximate limits of high and low water and beach material could not usually be conveyed to, or deposited upon a foundation situated below this level. The nature and position of the foundation of a beach intimately concerned the Engineer, for it was upon this that he supported his works in order to give them strength and stability. The value of preliminary trial boreholes thus became evident and designs for constructional works should preferably be preceded by an investigation into the foundation of the beach.

Slope of Equilibrium.

Upon the foundation so formed, the aggregate of the beach, whether it be sand or pebbles, is deposited, and is so assorted by the waves as to take up a specific configuration in which a state of equilibrium is established. During the changes which take place to suit the vagaries of the weather, the surface of the beach may be affected by numerous incidental phenomena such as berms or channels, but it is desirable to look beyond these casual corrugations to seek the important truths which lie beneath. It may be established by inspection that the surface of the beach in contact with the waves within the limits of high and low water mark takes upon itself a uniform slope. This might be expected from academic considerations for during the interval of a tide when wave conditions might be expected to be constant, it would be unreasonable to expect different slopes at different levels, with comparable material. It may be further established by observation that materials of a similar degree of coarseness adopt a similar angle of inclination to the horizontal whererever they may be found along the coast; again if the external conditions are comparable. Finally it may be observed that the angle of inclination at which the beach is maintained between high and low water marks is related to the coarseness of the beach material, being steeper where the material is large, and flatter when it is fine. When the beach material has established itself in equilibrium with respect to the waves, this angle has been termed the "Slope of Equilibrium". Appropriate values of the Slope are as follows:- Fine Sand 1 in 50; Coarse Sand 1 in 20; Small Pebbles 1 in 10; Large Pebbles 1 in 4. These values are intended as a guide and the Engineer should take measurements on the site before committing himself to the design of sea defence works. The "Slope of Equilibrium" may be susceptible in some degree to numerous extraneous factors such as the density or porosity of the material, and the special forms of wave which might encounter the beach, but these variations are not of great practical significance. It is certainly affected to a marked degree by any feature which creates excessive turbulence in the ordered breaking of the wave, such as a vertical sea wall, rocks or similar obstructions. The passage of large quantities of subsoil water through or over the beach also reduces the angle of slope. There are many instances of beaches which consist of two different materials, sand and pebbles which are segregated from each other such that the fine material occupies the lower situation and the corase is thrust to the head of the beach. In such cases each class of material takes up its appropriate Slope of Equilibrium. Similarly when one end of a beach consists of material finer in character than the other, again each material takes up its appropriate slope.

THE STABILITY OF BEACHES

The force and the power of the sea is acknowledged by all. Its effect upon man's constructive efforts is to be found in the heaps of broken masonry which may be discovered around the shores after a storm Why is it, therefore, that whilst structures are frequently destroyed, the accumulation of loose particles of material, pebbles and sand, comprising protective beaches around the coast, are able to withstand the onslaught of wind and tide? There is no resistance inherent in the individual particle. It is readily moved by the smallest wave and yet in the aggregate, these insignificant entities are able to prove a bulwark to the sea. It is the state of equilibrium which is established within such banks by the inter-action of one particle upon another related to the forces of the waves which gives them the strength to resist the violence of the sea. It is not true, however, that all beaches are capable of equally resisting the power of the sea. Some of them are afflicted by the disease of wastage imposed by either natural or artificial causes. Some sections of the coast, however, are fortunate in being relatively free from erosion, and happily their natural beauty remains unspoilt by those devices which have been found necessary to protect other parts from the ravages of the sea. Having become familiar with the varying scene presented by the coastline, the observer will become conscious of certain features or characteristics of the stable stretches, which do not appear in those which are subjected to severe wastage. He will first realise the ordered nature of the stable beach, contrasted with the irregularities and disorderliness of the unstable beach. He will note the wonderful sweep of the Chesil Bank, for instance, as his eye reaches for its farthest end, eighteen miles away, and he will realise that he is viewing a manifestation of controlling natural laws. He will then note the presence of headlands or barriers at each end which would appear to support and contain the material within prescribed limits. This phenomenon can be interpreted as a necessary condition for a stable beach; such that the absence of one or both of these barriers would automatically render the beach unstable. The most significant feature of a stable beach of the type described is the curvature. The waves and tides of perhaps thousands of years will have shaped the beach into a regular curve from end to end until a state of equilibrium has been established, in which configuration it will remain unless, due to changed circumstances, it is disturbed. It is a necessary condition of stability in such a beach that it should be capable of assuming, between its limiting barriers, such a curve. The nature of this curve of equilibrium has received special attention from the Authors.

Barriers - Groynes.

Consider first the nature and functions of the barriers which support the extremities of a stable beach. They appear naturally as headlands and artificially as groynes and jetties, provided that the latter are impermeable and do not transmit beach material through them; are substantial in construction such that they may withstand the thrust imposed upon them; and in size are able to contain the full cross-section of the beach for such distance below low water mark as satisfactorily to prevent beach material from passing around this extremity. If there exists such a barrier at each extremity of a length of beach, then the first essential condition for stability will have been complied with. For a great many years efforts have been made to combat erosion by the construction of artificial devices such as those described. In many parts of the world erosion manifests itself in a drift of beach material along the coast, and no doubt in the early days it was instinctively felt that an obstruction to such passage would have a beneficial effect. Thus timber barriers or groynes were no doubt first constructed to prevent such movement. With the passing of time, however, the custom has grown to apply groynes wherever erosion is taking place without reference to their precise functions which, in any case, have seemed very obscure. In many instances it is clear that to them have been attributed magical properties whereby they might conjure beach material out of the sea into their outstretched arms. Such a conception is, of course, nonsense, as a groyne can only control beach material which already exists in position on the foreshore. The civil engineer, in the course of his varied duties, will have become acquainted with the properties of sand. He will have encountered it dry, under which condition it is held to provide a good foundation, and he will have encountered it

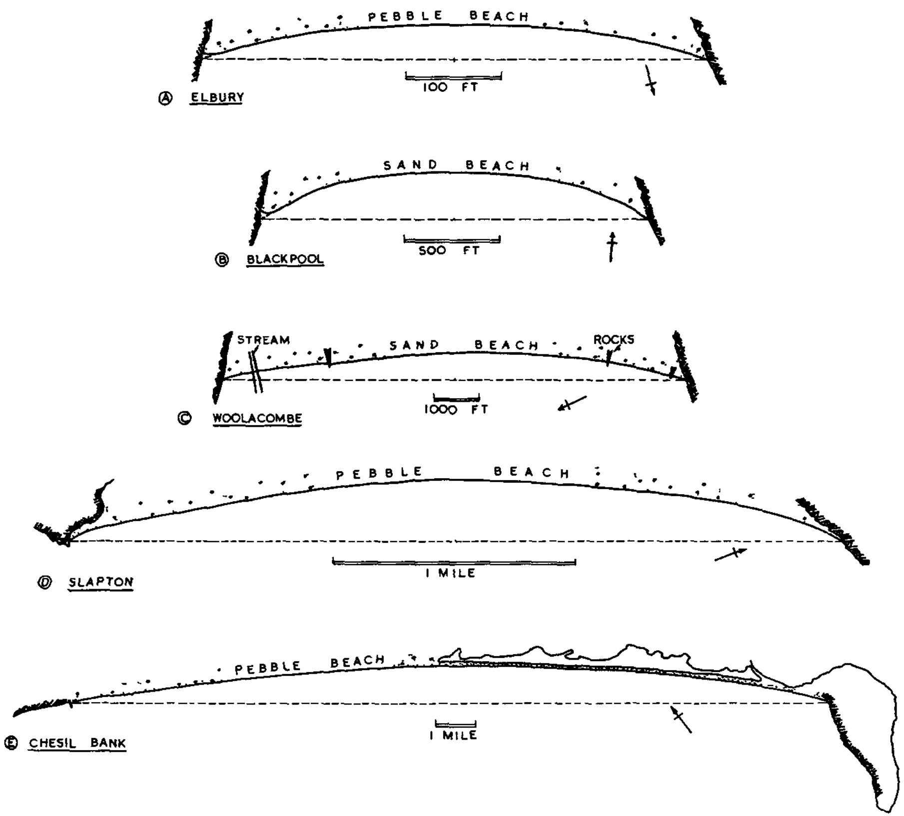

Fig. 1. The shapes in plan of a selection of beaches which are considered stable. They are between 525 feet and 18 miles long; some are pebble and some sand beaches. Note the arc formation.

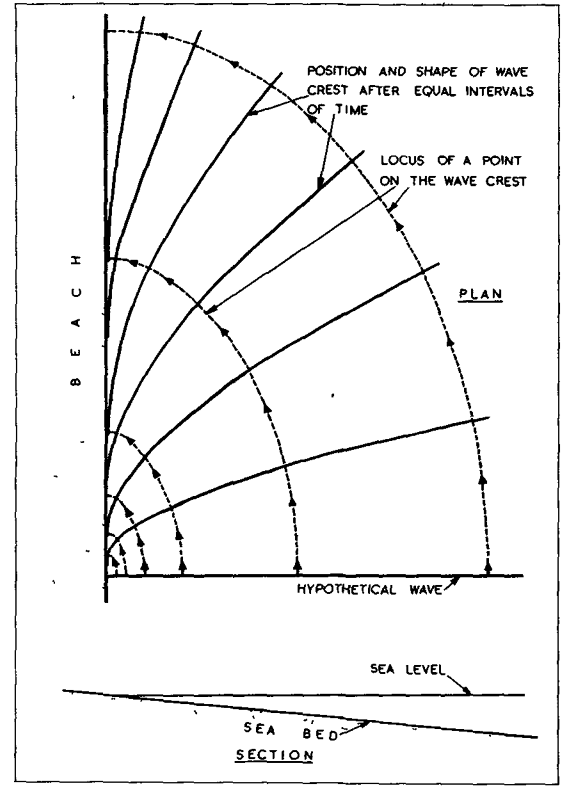

Fig. 2. Diagram showing how a hypothetical wave crest moving across the contours of the sea bed is thereby distorted. As computed from the formula $c^2 = gh$

wet as running sand which has no substance and behaves as a liquid. In the latter case it presents a major engineering problem, and it will be remembered that it is often necessary to contain it within sheet piling in order to give it the capacity to withstand load. Thus, by the application of compression to its bulk, running sand may be stabilised. The same material is found on the beaches under similar conditions. Water-logged sand below the level of the water may appear hard and firm to the foot, but a handful runs between the fingers like water, and the cavity in the beach from which it has been taken is quickly filled by other material which flows in from all sides. In the first case the sand is compressed; in the second the pressure has been released and the stability has been lost. It is an essential condition of stability in a beach that the particles of which it is composed shall be in a state of compression, and it is this compression transmitted from particle to particle throughout the length of the beach, which finally thrusts upon and is resisted by the barriers at each end. Thus the first function of a groyne must be that of a retaining wall capable of withstanding the longitudinal thrust of the beach. Moreover, although the above remarks refer to sand, the principle is equally true when the medium is coarser in character. In order that a groyne may fulfil its purpose properly, it must be impermeable to the passage of beach material and must be large enough to embrace the cross-sectional area of the beach which it is intended to stabilise. The beach must be supported beyond the highest tide and below the lowest low water, unless special circumstances apply. Of course, account will be taken of practical difficulties of construction when these rules are interpreted. Thus, by virtue of the explanation just given, it will be seen that the groyne has subtly changed in character from the mere obstruction to the passage of material to a precise instrument for imparting to the beach material an essential element of stability, i.e., compression. It will be appreciated also that the beach material can no longer be considered as able to move across the groyne barriers in its drift for, once trapped and stabilised between any pair of properly designed groynes, it is permanently established there.

Arc of Equilibrium.

The Authors investigated the nature and properties of that 'curve of equilibrium' of a beach which might properly be associated with stability. A number of beaches which fell into the stable category were considered and a selection was made of those which provided the greatest variety both in length and in class of material. In all cases these beaches were supported by a barrier at each end, and were comparatively free from external influences such as groynes and sea walls which were known to have an independent effect. With the aid of ordnance survey maps and by taking measurements on the site, the appropriate 'curves of equilibrium' were established and recorded together with other relevant information, (Fig. 1.) A close examination reveals that, when allowance has been made for local influences for which an explanation is afforded, the curve approximates to an arc of a circle. At the ends a slight deviation is sometimes exhibited which is no doubt the result of the influence which the barriers themselves exert upon the beach, and irregularities due to special physical conditions such as streams or rocks are evident. It is sufficient at this stage to indicate that the variation from a true circular arc is due to local conditions, and that in a 'perfect case' the curve of equilibrium would be an arc of a circle, which in future will be referred to as the 'Arc of Equilibrium'.

Rule of Similar Arcs.

It was further decided to examine the nature of the angle subtended at the centre of the circle by the arc of equilibrium, and it was established that these angles are nearly the same in each case, which is a remarkable fact when considered in relation to the enormous disparity in length and character of the beaches examined. The Authors crystallised this property as the 'Rule of Similar Arcs', whereby it is to be understood that 'Provided that such beaches, including the portion which extends for a material distance beneath th sea, consist of particles capable of being moved individually by the s then the "arcs of equilibrium" of all these beaches will be geometrically similar arcs, each subtending an angle of approximately 0.25 radians at the centre of its circle'. The nature of the undersea bed affects the curve of equilibrium; thus it was necessary to introduce the proviso. This principle, nevertheless, has an important and usefu application in practice.

Conditions of Beach Stability.

Before going on to examine the application of the principles expounded to practical problems of sea defence, it would be as well to assemble them in a concise form as under. It is submitted that essential conditions of stability in a beach of this class are:-

1. The beach shall be supported at each end.
2. Between the end supports the beach shall have assumed a curved shape, termed the arc of equilibrium, and the angle subtended a the centre of the arc shall be approximately 0.25 radians.
3. The slope of the beach shall have taken up its appropriate slope of equilibrium.
4. The orientation of the beach shall have been established cor sistent with prevailing climatic conditions. This will be discussed later.

The Application of Groynes to Foreshore Stabilisation.

An attempt will now be made to present the behaviour of beach material in association with the waves, as a picture of the ordered application of the foregoing principles. Let us suppose that upon a length of foreshore are tipped several million tons of beach material which are placed indiscriminately in contact with the sea. Consider what will happen to it in association with the waves. The accumulati of material will not, of course, be in stable form in accordance with the requirements just set down, and the loosely arranged material will begin to be moved freely at the dictates of wind and wave. First, throughout the length of the beach, the slope of equilibrium will be established; wherever the slope is too steep material would be dragged down; wherever it is too flat it would be filled in; so that ultimately an even slope would be established appropriate to the material used. Gradually, under the action of the waves, the material would then be extended along the foreshore until brought to a halt by a 'barrier' at each end. Finally, it would be shaped and moulded until the arc of equilibrium had been established. This activity is going on at all times around the coast; the waves are ceaselessly moving and shaping the beach material in an effort to reproduce the configuration for stability. What happens when the shape of the coastline is such that the requirements of stability cannot be met? There are examples where the barriers are miles apart and the coastline between is of such a character and such a shape that it is a physical impossibility for the curve of equilibrium to be imposed upon it. Under such conditions stability cannot be established and the loose material will be moved by the waves, sometimes to the left, sometimes to the right, in endless search for a final resting place. It would be an exceptional case, however, if the cumulative movement to the right exactly equalled that to the left; the accident of weather would introduce a bias in one direction which would reveal itself in a general movement of material longitudinally along the foreshore - hence drift. The engineer is frequently confronted with the problem of how to deal with this situation and to prevent drift of beach material from parts of the foreshore where it is required, to places where it is not. He has usually met the problem by following past example and constructing groynes across the foreshore but the results have not always been successful and have in some cases been harmful. The use of groynes in such a case is, in fact, the correct solution to the problem, provided they are scientifically designed, their object being to divide a long and unstable section of foreshore into shorter sections such that the conditions of stability can be physically applied to each separate section. Each of these sub-divisions would, in fact, constitute a separate stable beach. It will be seen that considerable control can be exercised over the beach material of a foreshore by arranging the spacing of the groynes to suit requirements. In fact, beaches can be 'designed', and the resulting configuration of the beach produced, can be anticipated in advance and designated on a plan. In other words, the effect of the groynes can be predicted and measured.

Unstable Beaches.

Natural processes operating along the foreshores are such that there is a tendency for loose material to drift and for erosion of the coast to be ordered, so that eventually a stable configuration is established. One would have thought that after many thousands of years these continued adjustments would have produced a universally stable coast in appropriate configuration. Why is it then that some sections of coastline do not conform and are subject to increased beach displacements and more severe erosion? Consider, for example, a stretch of 'soft' coastline many miles in length such as might be encountered in Norfolk and Suffolk, and let us imagine that it is entirely stable, made up of cliff barriers, bays (arcs of equilibrium), etc. Displacements of beach material and the overall rate of erosion will have been reduced to the minimum. This satisfactory state of affairs, however, is affected by other processes which lead to the destruction of the equilibrium. Most susceptible to the violence of the sea are the exposed headlands. They probably represent the more tenacious portions of the coast, but even these, when they consist of sand, gravels, and clays of varying consistency, have sometimes to succomb rapidly to the waves. There are times in the history of the coastline when those headlands under the impact of the sea are so reduced in size that they are no longer able to fulfil their functions as barriers. Thus the equilibrium of the coastline is disturbed, units of beach become merged with one another and the line of the foreshore becomes confused. This irregular line reflects the transitional stage which has been established and denotes that the shaping forces of the patient sea are once more at work in remoulding the coastline towards a new equilibrium configuration, with increased local erosion as a result. The significance and the importance of the headland in the structure of the coast will be clear. The whole line of the coast depends upon these salient features and the general rate of erosion is determined by the particular rate at these controlling points When, therefore, schemes are being considered which are directed towards the protection of long lengths of 'soft' coastline, the attention of the engineer should first be attracted towards these features, and their preservation should become a prior need.

THE ORIENTATION OF BEACHES

Those who reside by the sea will be familiar with the changing aspect of the coast as it appears in fair weather and foul, sometimes with the surface of the sea lashed to a fury by gales, with great breakers charging up the beach, and at other times like a placid lake with gentle waves idly spilling themselves on the sand and shingle. Throughout the variety of changing circumstances and different complexions of the marine panorama one thing, however, in particular remains unchanged, namely that waves travel from the horizon towards the land, never in the opposite direction. The close observer will note, in fact, that on a calm day the waves at the point of breaking on the beach will be arranged with their crests almost parallel with the shore line and that only under the influence of a severe cross wind will they be slightly deflected out of this configuration. So marked is this phenomenon that the Authors have observed along the famous Chesil Bank, Dorset, long rollers coming inwards from the Channe making almost instantaneous contact with the beach over a stretch many miles in length. What is the explanation of this phenomenon, and what natural laws are invoked to produce this manifestation of nature's controlling hand?

Conditions for Stability of Beaches.

An attempt has been made to analyse and establish those natural laws which are related to the stability of beaches. The first of those laws to be described concerns the slope at which the beach material could sustain itself in association with the sea, and to this property was given the particular name 'Slope of Equilibrium' because it was conceived that at this inclination the beach would have established itself in a state of equilibrium. It was also shown that for a beach to be in a state of equilibrium it was necessary for it to be contained or supported at each end in order that a compressive thrust might be exerted throughout its length. Finally, it was also shown that between the two end supports the surface plane of the beach would have to assume such configuration that all contours between high- and low-water mark would be arcs of circles, which were designated 'Arcs of Equilibrium'. It would be appropriate at this point to indicate what is meant by the terms stability and equilibrium. When this investigation first commenced, it was with the belief that there were two basic classes of beach, those which were stable and those which were not. The latter were naturally receiving a great deal of attention at the expense of the former, and the mass displacement of beach material, usually due to drift, was causing some concern in many localities. It was thought that an investigation into the characteristics of the two classes might reveal why one should be stable and the other not. Thus, at first, stability referred in broad general terms to the nature of the beach and its freedom from erosion. As the investigation progressed with a particular study of stable beaches it became evident that these, too, were subject to temporary displacement of beach material, according to the prevailing weather conditions. Thus it was possible for a generally stable beach to be unstable at a particular instant with reference to the conditions at the time. In the reports which follow, an effort has been made to distinguish between the general and the particular, by using the word stable to indicate the general or long-term state of the beach and the word equilibrium where the more scientific interpretation is required. Usually, in the latter case, the beach is described as being in equilibrium 'with respect to some particular circumstances'. Both words imply that there shall be no mass movement of beach material away from the beach. It will be observed that the first condition of stability is concerned with the movement of material up and down the beach between high- and low-water mark, and in nature such surface movement is continuously taking place in order to make up any irregularities which may have occurred, and to maintain the appropriate slope of equilibrium. The latter condition in a similar way is concerned with the movement of material longitudinally along the beach, in the establishment of the arc of equilibrium, but once this has been formed the beach becomes stable, and further movement ceases. What, then, is the relationship between mass movements of beach material and the circumstances of wind and wave, which are known to be the cause? These occur because changes in wind and wave produce changes in the orientation of the beach, to the accompaniment of mass transportation of material in a manner which will be described; but first it is apparent that some attention must be given to the mechanism whereby such transportations are effected. The wave is the usual medium of transportation. A careful study of the form and nature of waves is therefore a necessary preliminary to an investigation into the causes and character of beach movements.

Transportation of Beach Material by Waves.

Waves are usually produced by wind; the frictional drag of the moving air upon the surface of the sea imparts to the top layers of the water a fraction of its energy which is sufficient to create upon what would otherwise be a placid surface, numerous hillocks of water. These eventually coalesce to form larger hillocks, and also take form to become the rollers with which all are familiar, and which should not require further description here. These rollers then speed on their way obeying their own complex laws, independent of, but nevertheless still modified by, the wind which created them. Experimental investigations and mathematical analyses have been carried out into the nature of these waves, but for the present it is sufficient to understand that they are reservoirs of that energy which was originally imparted to them by the wind. This energy is capable of exerting force, and doing work on any physical obstruction which the wave may happen to strike, and is responsible for the transportation of beach material. When a sea wave or roller expends itself upon a shingle or sandy beach the kinetic energy with which it is charged imparts movement to the particles in association with it so that a portion of them are rolled up the slope of the beach and work is done upon them against gravity. With the receding flow of water, however, they are carried down the beach again, and so the slope of equilibrium is maintained, and the location of the beach material is basically unaltered. When the direction of travel of the wave crest is normal to the contour of the beach, movement of the pebbles is confined to this up-and-down movement and there is no sideways or longitudinal displacement of beach material. When, however, the wave strikes the shore obliquely, the sideways component of its kinetic energy exerts a pressure in a longitudinal movement or drift. For complete stability of a beach, therefore, which implies that there shall be no mass longitudinal drift of beach material at any time, a condition must be that the waves shall strike the beach normal to the shore line. In other words, they shall not have a sideways component in their velocity of approach. This is the basic law of stability.

The Influence of the Sea Bed on the Direction of Waves.

The controlling feature in the disposition of beach material is thus the direction of approach of the waves as they make contact with the beach. What, then, are the controlling features of the waves themselves which influence the direction of their travel? Attention is drawn to the evaluation of the velocity of an ocean wave, which has been computed mathematically. The expression which gives the velocity of propagation of an ocean wave is as follows:

$$c^2 = gh \qquad \text{(deep water)}$$

$$\text{or } c^2 = g(h + 3y) \qquad \text{(shallow water)}$$

where c = mean rate of propagation, h = mean depth of channel, g = acceleration of gravity, y = elevation of wave.

It will be apparent that the velocity of such a wave is related to the depth of water in which it travels, being greater in deep water than in shallow. Moreover, a wave travelling over a sea bed of irregular depth will suffer deformation and the wave crest will be held back in shallow water and speeded up in deep. In practice the sea bed varies in depth and, generally speaking, it gets deeper the farther away it is from the shore. It would not be far from the truth if it were assumed that the sea bed could be represented by a uniformly-inclined plane penetrating deeper into the sea from the shore line; and the deformation which would be produced on a wave crest by such a configuration would bear a close resemblance to actuality. A wave advancing in the direction of the shore directly up such an inclined plane would be travelling into shallower water and would, therefore, be slowing down; nevertheless, all points along the wave crest would at all times be over an equal depth of water and would, therefore, be travelling at equal speeds. Under these conditions, therefore, there would be no deformation and the wave crest would remain parallel to the contours of the sea bed and the shore line. It is apparent, therefore, that deformation of the wave crest would only occur when different points on the wave crest were travelling over different depths of water or, in other words, when the wave was travelling obliquely across the slope of the sea bed. In order to illustrate how the wave crest would be deformed under such conditions, an extreme case has been examined mathematically where an imaginary wave has been considered to be travelling at the moment of initiation in a direction parallel with the contours. The results of this analysis are given in Fig. 2. This diagram is intended to represent a uniformly-shelving sea bed where the depth of water is proportionate to the distance from the shore, and where on account of this the velocity of propagation of the wave crest is related to this distance by the expression $c^2 = gh$. It is then imagined that a wave is artificially produced with its crest starting at the shore and stretching an arbitrary distance between nought and infinity along a line normal to the shore such that its direction of motion at the time of origin is parallel with the shore. The object of this analysis is to examine the shape of the wave crest, at regular intervals of time after propagation, and to follow the track of a series of points on the wave crest. It is presumed that the formula for the velocity is $c^2 = gh$ for simplicity; as the more exact formula $c^2 = g(h + 3y)$ would introduce unnecessary complications. It is hoped that the figure will prove self-explanatory and will demonstrate how, under these conditions, the profile of the wave crest is altered and the direction of its travel is modified.

In practice, waves in deep water may be approaching the land from all angles, and provided information is available as to the configuration of the sea bed their paths may be computed mathematically as in the particular case described above. Without undertaking this task, it will, nevertheless, be apparent that in all cases they will finish their journey by breaking normal to the shore line. The shelving under-sea bed will have guided them inwards towards the shore, in the manner described in the first lines of this section. It is necessary to carry the matter a stage farther and consider the implications of this natural phenomenon. It would be in order, for instance, to declare that in windless weather all waves breaking upon a beach are constrained in the manner shown to advance towards the shore to strike it normally. Earlier in the Paper it has been stated that a requirement for stability of beaches shall be that very condition The implication of the two statements taken together is that beaches should always be stable, and one must reconcile this conclusion, with the undoubted facts of nature, that they very frequently are not. The qualifying and important condition upon which the truth of the above statement depends is inherent in the phrase 'windless weather' because the wind is able to modify wave motion and to impart a longitudinal or sideways component to its velocity at its point of contact with the shore.

The Effect of Wind.

The orientation of a beach is understood to mean the direction in which the beach faces, and it will at once be perceived that this feature cannot remain unconnected with the circumstances of wind and tide applicable in the area. The subject will be discussed first with reference to those localities which have a soft sea bed, and where the 'arc of equilibrium' will have taken shape. In such a case the orientation could be denoted by an axis bisecting the beach and passing through the centre of the circle of which the arc forms a part. Over centuries of time a state of overall stability will have been established in a beach such as that described above because the orientation will have adjusted itself to prevailing weather conditions and the movement of beach material by daily variations in climate to the left will have been compensated by movement to the right. The incidence, however, of a strong cross wind would temporarily upset this stable configuration and, by imparting to the waves a longitudinal or sideways component of velocity, would immediately render the whole stretch of beach unstable with respect to those particular conditions. A longitudinal movement of beach material would then take place in the direction of the wind; a denudation of the beach would occur on the windward end of the beach; and an accumulation would appear on the leeward side. The effect of this redisposition would be a tendency towards the re-orientation of the beach to suit the new conditions, and this phenomenon would involve a displacement of the axis of orientation by an angle of shift, and the establishment of the 'arc of equilibrium' in a new position. If the strong cross wind were to be sustained for sufficient time, and other factors permitted, the displacement would be sufficient to re-form the beach in its new orientation and once more to re-establish equilibrium with respect to the new conditions. The appearance of a strong cross wind would also bring about a change in the basic structural stability of the beach, which influences the process whereby the adjustment is carried out, and introduces an elaboration to the simple movement sideways of the beach material. Fig. 3. is intended to represent the arc of equilibrium of a stable beach which has been made temporarily unstable by a cross wind. The angular shift of the axis of orientation has been indicated together with the new configuration of the beach which would be stable under the new conditions. This diagram could also represent

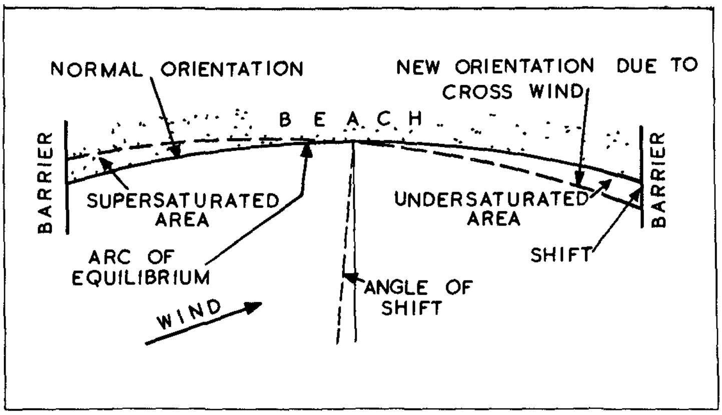

Fig. 3. Diagram showing how a strong cross wind produces re-orientation in a stable beach together with associated beach movements.

a section of beach between two groynes; thus it will be readily appreciated that this matter will have an important practical application in the engineering science of coast protection. The first curve in the diagram represents the stable configuration of the beach initially with respect to average conditions, and the second curve represents the configuration appropriate to the new conditions, involving a strong cross wind from left to right. It will be observed that during the process of re-orientation a triangular-shaped area of beach to the left of the axis has lost stability or become, as it were, supersaturated with beach material. This material cannot support itself under the new conditions, and the effect of this in nature is that it is rapidly drawn down by the waves, and slumps to form a shapeless accumulation in the vicinity of low-water mark. From this position the gradual sideways movement described, takes effect until all this superfluous material has been transferred to the other side of the axis, where there is a corresponding triangular area which is deficient in material or 'undersaturated'. The filling-in of this area restores the curve of equilibrium, but with changed orientation.

Shift of the Axis of the Arc of Equilibrium.

The long-term or average orientation of a beach is, therefore, the cumulative effect of all winds which have blown in the locality over the centuries. It could also be regarded as an average effect of winds unless major climatic changes are being experienced. The day-to-day orientation may change, producing displacements in the beach material, but these are compensatory in character, leaving the general configuration the same. Nevertheless, seasonal changes in the orientation do occur to suit the change in the direotion, intensity and duration of the winds in the area, due to the seasonal cycle of the year. It should be appreciated that re-orientation involves the transfer of beach material from one end of the beach to the other, which, of course, would be a much bigger operation in the case of a long beach than in a short one and the time taken would be proportionately larger. The strength and duration of winds blowing consistently from one direction are not always sufficient to transfer enough materia to complete the re-orientation of a beach unless it is a very short one indeed, and usually the process is interrupted and reversed by a change of wind. With long beaches a small shift or re-orientation is possible only on a seasonal basis where, for instance, the prevailin winds during the summer season are different from those in the winter. With very long beaches such as the Chesil Bank even a small trace of re-orientation could be produced only by a major climatic change in the direction of the prevailing winds. Change in orientation, however, is a common feature in short beaches, and in the case of groyne systems where each section between groynes is considered as a separate and distinct beach it is a factor which must be taken into account and provided for in the design of the groynes. Where the groynes are spaced at short intervals, a single storm may completely re-orientate the beach, and similarly a following storm from a compensatory directio may re-establish the arc in its original position. It is a matter of considerable practical importance, partioularly where the design of groyne systems is being considered, to establish the limits within which the orientation of a section of beach will be confined, under all the weather conditions which are likely to be imposed upon it. This is a factor which depends upon local conditions to some extent in so far as the nature of the prevailing winds and their direction have to be considered, and it would not be strictly correct to endeavour to apply a general formula to all cases. It is nevertheless the case that the length of the beach is the overriding factor which controls the probable degree of shift, and accordingly it is possible to establi an approximate relationship between the two which can be applied to problems involving this feature without a large error being incurred. The value of such a formula in practice more than makes up for the smal degree of inaccuracy to which it might be subject. It is oonsidered that shift may vary from an angle of 10 degrees in a short beach about 200 feet long to 2 degrees in a beach 4000 feet long.

Effect of Reflected Waves on Beach Configuration.

It may be taken as a general rule that any external agency which affects the direction of approach or configuration of the waves will b represented in the contour of the beach. The most important of these is the configuration of the sea bed to which may be added the effect o a reflected wave from some physical obstruction near the beach. When a wave encounters some structure such as a breakwater or harbour arm which juts out into the sea, it may, according to its direction of approach, be reflected in much the same way that light is reflected in a mirror. As it pursues its way after contact, in the changed direction, it then encounters the beach at a different angle to its fellow unreflected waves, and the beach material is arranged and moved until it faces the oncoming wave. The visual result is that the curve of the beach is likely to make a sharp reverse curve in the vicinity of such an obstacle. An example of the way in which a reflected wave influences the adjacent beach may be seen at Folkestone, where waves encountering the harbour breakwater are deflected on to the beach in the manner described and the curve of the beach is affected accordingly. On a smaller scale groynes have a limited local action and often produce a small reverse curve in their immediate vicinity. In some cases this may be due to reflected waves from the groyne, but it should not be forgotten that any kind of agitation or turbulence in the water produced by the waves encountering an obstacle on the beach would have the same result. Another occasion when deformation of the wave crest is mirrored in the configuration of the foreshore is when diffraction occurs due to the presence of some off-shore rock, island, or bank. When a wave advances towards the shore and encounters such a feature it naturally splits into two, and the two sections may continue towards the beach, in a very much altered form. This modified form then becomes impressed upon the material of the beach and produces appropriate configuration. Usually such a curve is opposite in character to what has previously been described as the "Arc of Equilibrium", and so it has been given the name "Reverse Curve". It may be found that long lengths of soft coastline, which may at first sight appear to be of irregular contour and therefore unstable, may in many cases be split up into a series of alternating "Arcs of Equilibrium" and "Reverse Curves" and in such configuration could be stable. In Nature it will be found that whereas the "Arc" may be of considerable length, the "Reverse Curve" is usually short and acute.

Although the natural equilibrium of the foreshore, established after many thousands of years, has in many cases been interrupted and distorted by sea walls, harbour arms and other artificial devices, such that it will never be restored to its natural shape and beauty, there are still many miles of untouched and unspoilt beaches which may still be preserved intact, providing that sufficient interest towards that end is established now. It is of great importance that effective control of the diminishing asset of the beaches should be instituted at once before the sands have all run out.

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CHAPTER 18

HOW ARE BEACHES SUPPLIED WITH SHINGLE?

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In Great Britain there are numerous types of shingle beaches, soı of which are almost wholly formed of shingle, others have a large proportion of sand intermixed with the shingle, and there are some which a virtually sand beaches on which there is some shingle.

It is now generally assumed, but by no means always proved, tha shingle on beaches is moved almost exclusively by wave action. Curreı action may sometimes be effective, but on most of the beaches in these islands the shingle is high up on the beaches and is often only touched at high water and after the waves have broken. Where shingle extends 1 lower levels it may be subjected to the action of tidal currents, but unle these run with an unusual velocity they are unlikely, by themselves, to move the stones. If, however, these stones are disturbed by waves passing above them, then it is possible that a current may move them t some extent.

As a generalisation it is true to say that along our east coast beac materials, whether coarse or fine, move southwards. There are many exceptions to this, and contrary movements are nearly always the result of a change in the direction of the wind or of the trend of the coast. On the western part of north Norfolk the movement is usually to the wes and analogous conditions exist in the Moray Firth. On our southern, Channel Coast, the movement is mainly to the east, but along the inden shores of Devon and Cornwall local conditions often predominate. The west coast is more irregular. In the seas between Wales and southwestern England beach material generally moves up-channel, in Cardig Bay the movement is towards the north-eastern corner - Tremadoc Bay In the Irish Sea there is more complexity; along North Wales and parts Lancashire material tends to move east and south. St. Bees Head, Cumberland, forms a rough divide; to the north of it the movement is tc the north-east. It is impossible to generalise for the west coast of Scotland; the very irregular nature of the coast means that local conditi must always play a major role. Some interesting local effects have been examined in the straits around the Island of Juna (Ting, 1936).

If we assume - allowing for local exceptions produced by details c topography - that there is a lateral movement of beach material in the directions just outlined, there should not be, at first sight, any great difficulty in accounting for shingle beaches and accumulations. But whe individual beaches are examined it is commonly found that there are severe complications. It is impossible in this paper to discuss details; but in the Chesil Beach, for example, the nature of the pebbles clearly implies that some come from rocks to the west of the beach, and others, especially the Portland Limestone pebbles, must come from the south-east. Moreover, there is a great likelihood that this is an over-simplification since it is possible that many of the pebbles are resorted and redistributed from former glacial or periglacial deposits. On the Sussex-Kent border there is the huge accumulation of Dungeness, and with it must be taken into consideration the old spits at Rye and Winchelsea, as well as those farther east at Hythe. The structure and history of all these have been frequently discussed, but no clear answer has yet been given about how the shingle of Dungeness crosses Rye Bay, which, presumably, it must do.

Some shingle beaches are contained between two rocky headlands, and if the beach material is the same as that of the enclosing rocks, and of those in the immediate neighbourhood, it is taken for granted that the wear and tear of these rocks have produced the shingle. If the dominant waves approach such a bay rather obliquely the shingle is usually more abundant to one side. Many of the bays on the south coast of the Lleyn Peninsula, and of Glamorganshire illustrate this point. The beaches are all in rather open bays and their nature is suggested in Figures 1 and 2.

On more indented coasts, with deep and perhaps narrow inlets, the small beaches which occur at their heads are nearly always assumed to be formed of fragments of the same rocks as those of the enclosing arms. This may well be true, but it is nevertheless an assumption, and more detailed investigations are required to prove these assumptions. It is one of the many ways in which in coastal matters, we regard as fact what is only impression.

The matter is more difficult on open coasts. Along the East Anglian coast from a few miles north of Yarmouth to the mouth of the river Orwell there is little doubt that the general travel of material is to the south. It is likely that this is the result of dominant waves set up by winds coming in from the quarter between, approximately, north and east. But anyone who knows this coast well will realize that contrary, northward, movements of shingle may prevail for days, or even weeks, (See below). Between Aldeburgh and the mouth of the Deben a great shingle foreland, Orford Ness, has grown up. If it is examined and mapped, the arrangement of the many fulls, or ridges, which compose it show that the foreland has grown from north to south. Its southern end is tapering, and just beyond the mouth of the Alde large masses of shingle, partly presumably the waste of the spit, have been thrown up at a place very appropriately called Shingle Street. The question is where did all the vast amount of shingle in Orford Ness come from? Historical records show that it has been gathering for roughly a thousand years and that its growth has been irregular. In that time we know that there has been great erosion of the cliffs of soft glacial beds and crags farther north. Cliffs, however, are not continuous along the whole length of coast between Aldeburgh and Yarmouth, and judging from their present appearance, the amount of material in them which would be used to produce the 99% flint shingle of Orford Ness is not very great. Unfortunately, however, there are no quantitative measurements available, and even if some were attempted the variables, including rate of recession of cliffs, variability of the make-up of the cliff, and other factors, are so indeterminate over a long period that they are not likely to be of much use. From what is known with some certainty of the travel of beach material along Norfolk and Suffolk, it seems unlikely that, even discounting harbour projections at Yarmouth and elsewhere, *material from beyond Winterton (six miles north of Yarmouth) travels along the coast as far south as Orford Ness.

The rivers debouching on the coast all flow through flat country and are quite incapable of bringing down any coarse material to the sea In the past, at the close of the Ice Age, conditions were probably differ and then they may well have carried large supplies of shingle to the sea Orford Ness may now be in equilibrium in so far as gain and loss is concerned, and fed sufficiently by the supply of material travelling laterally from the north along the coast to make up for the loss it suffers to Shingle Street. On the other hand the possibility of replenishment from off shore cannot be overlooked (See below).

On the north Norfolk coast there is an island, an off-shore bar, named Scolt Head Island (Steers, 1948). It has been the subject of a go deal of research, and has been well known to the writer for more than thirty years. It consists of a main sand beach, with some shingle near high water mark and a number of recurved ends formed like the modern western termination of the island, primarily of shingle. Dunes have be built on parts of the main beach and recurves, and within the island the is a magnificent series of marshes. Figure 3 shows a part of the newe western, end of the island. The main beach is on the north and the recurves and marshes on the south.

The main direction of movement of beach material is to the west fact established by a number of experiments. Occasionally westerly winds cause an eastward movement, but this is only temporary. The t

*These projections hold up sand rather than shingle, and thus render t supply to Orford more problematical.

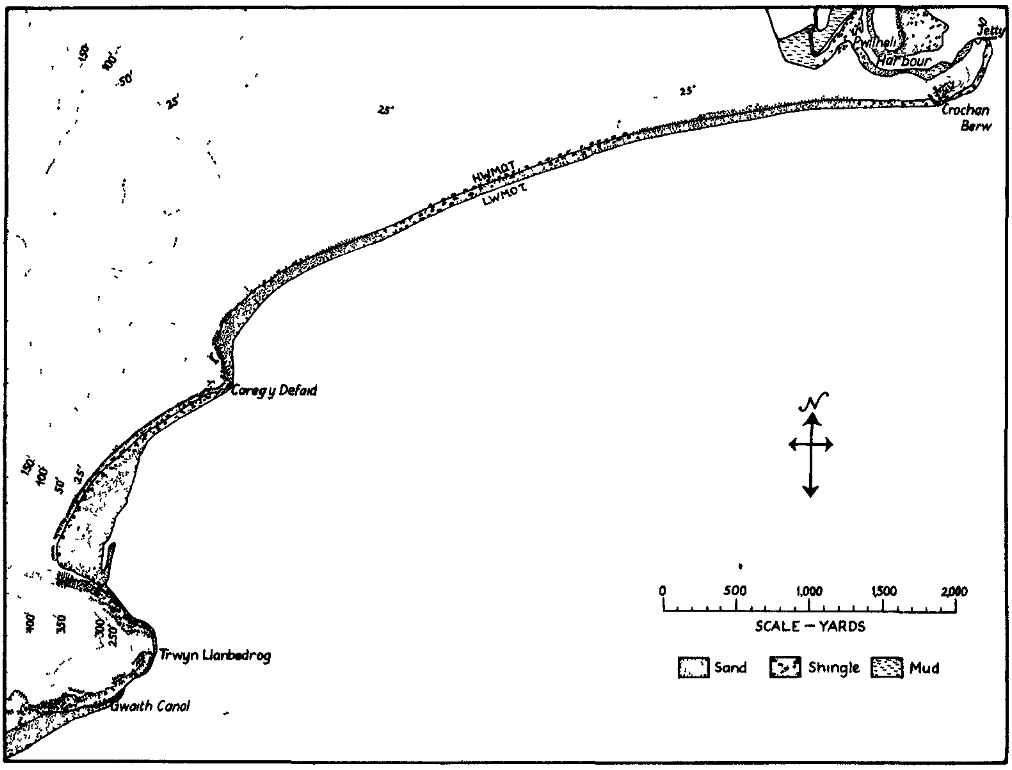

Fig. 1. Beaches near Pulwheli on the South coast of the Lleyn peninsula, North Wales.

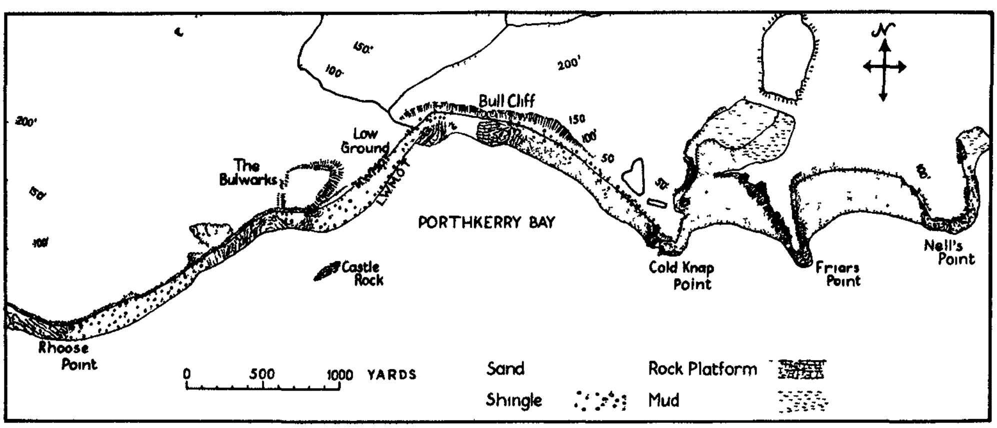

Fig. 2. Beaches on the coast of Glamorgan, South Wales.

current run parallel with the beach, and from approximately three hou before to three hours after the high water, the current runs to the eas against the growth of the beach. The shingle on the main beach, apart from a few small patches of coarse material lower down, is only touch round about the time of high water.

The east the island is separated from the adjacent shore by a ch Burnham Harbour, which is very shallow at low water, and winds abou extensive sand flats. Beyond this channel is an extremely flat sandy b a mile or more of which is exposed at low springs. There is very sca shingle on any part of this beach, and only a limited amount at its eas end, Wells Harbour, about two or three miles from Burnham Harbour Beyond Wells Harbour there is another very wide foreshore with a ver small amount of shingle, and this condition continues as far as the channel separating this part of the coast from Blakeney Point. It is or here that shingle is found in quantity, but it is more than doubtful if an it moves eastwards across Blakeney Harbour. It is most improbable that, even if any shingle crosses Blakeney Harbour, it travels even as as Wells.

To the west of Scolt Head Island the beach is remarkably free fr shingle until the mouth of the Wash is reached. There is some at Holm and more round the corner, south of Hunstanton. This emphatically does not move eastwards to feed Scolt Head Island, and the little there at Holme and Thornham seems to be stationary within certain limits, any that may move eastwards will feed the ridges (Fig. 4) forming Brancaster Golf links, and not cross the channel of Brancaster Harbou to Scolt Head Island.

How then is Scolt main beach supplied? Is it in a state of consta but slow wastage? Thirty years' knowledge of the island suggests that there is very little change in that time. This is also the view of other who know the island well. There is nothing to suggest that shingle rea the island from east or west. There is a slow landward movement of island as a result of storms over-rolling from time to time parts of th main beach and dunes, and in this way some of the beach shingle has b spread inwards. The great storm of January 31st - February 1st, 195 illustrated this very well.

Can shingle come from seaward? Since the beach is apparently fed by lateral travel, and since there is some loss by storms pushing shingle inland, it is at least a reasonable argument that some replenis ment comes from seaward. This, however, is not proof. But some considerable way towards demonstrating the possibility of such a sour of supply was made in 1956 when an experiment with radioactive shing

Fig. 3. Aerial photograph of the main beach and laterals enclosing salt marshes, Scolt Head Island, Norfolk (Photo Crown copyright; Dr. J. K. St. Joseph, Cambridge).

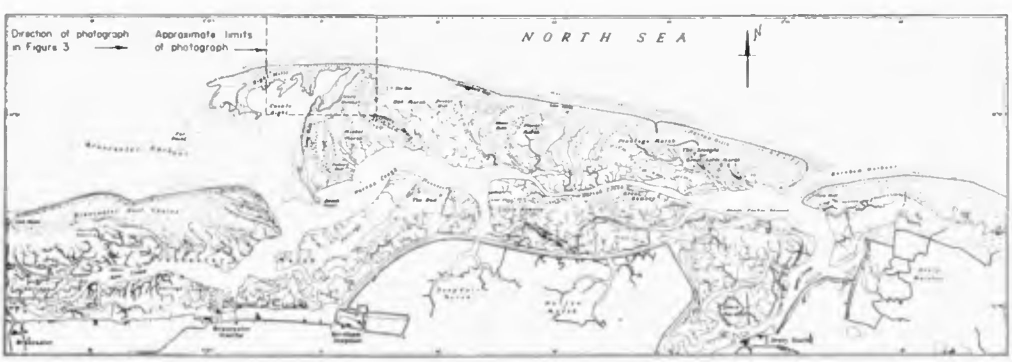

Fig. 4. Scolthead Island (Note area covered by aerial photograph in Figure 3).

was made (Steers and Smith, 1956). This was, in fact, the first time such an experiment had been made, at any rate in Britain. The beach pebbles are more than 99% flint shingle. For the purpose of the experiment softer sandstone pebbles, and artificial cement ones were use since each had to be bored with a hole half an inch deep and one-eighth in diameter. The pebbles averaged about two inches in major diameter and their specific gravity was approximately 0.2 less than that of flint.

Each pebble was then "loaded" with barium - 140, an isotope, the half life of which is twelve days. It decays by beta-emission and low energy gamma radiation into lanthanum - 140, with a half life of forty hours. The lanthanum emits gamma rays, the principal having an ener of 1.6 Me V. These were the rays used for detection under water.

About 1,200 pebbles were prepared, and they were dumped on Ap 5, 1956, from a boat at a point some 500 yards seaward from the avera high water mark off Scolt Head. The water at this point varies from 12 to 25 feet in depth according to the state of the tide. During the time the experiment ran the water was usually between 16 and 20 feet deep. The floor of the sea thereabouts is hard, consisting mainly of sand with some shingle. The place where the pebbles were dumped was marked by a buoy, and its position was fixed by triangulation from the land. When, later, observations were made from the boat, fixes were made by sextant to the surveyed points on land. Considerable care was taken in these observations, and errors arising from this source are negligib

No observations of the pebbles could be made until April 8 since the weather was squally, and on the night following the dumping of the pebbles the wind strength was 5 - 6 on the Beaufort Scale. On April 8 clear records were found that some of the pebbles had moved up to 200 feet south and west. Further observations were made on 9, 12, 20, and 23 April, and 5 and 15 May. The results were generally consistent and there is no doubt that a number of pebbles had moved inshore, in a direction a little west of south. The maximum distance measured was 260 feet. The observations made on 15 May also showed that a few pebbles had travelled about 450 feet to the north-west of the original dump, it is not certain at what stage in the experiment this movement occurred, but since the sea bed has been searched in all directions from the original dump on each date it seems that the movement took place rather late.

It is important to realize that this period - April 5 to May 15 - was not a stormy period, but one with a few squalls. Nevertheless, the pebbles were unquestionably moved over the sea-floor, and since the tidal current does not flow with a speed exceeding 2-1/2 feet per second it is most unlikely that the current itself was responsible for any movement. On the other hand, it might have helped the waves when they stirred up the bottom. Incidentally the waves during the whole of the experiment did not exceed a height of three feet.

The detection gear used for the experiment consisted of three Geiger counters enclosed in a brass cylinder one metre long. The sensitive length of the combined counter was 60 cms. It was mounted in a heavy metal sledge and had to be dragged over the sea floor. It was difficult in a choppy sea to sweep the floor carefully even over so small a radius as 200 feet from the original dump. Since the detector had to be within one foot of a pebble before it was "found" it will be appreciated that a good deal of patience was required.

This year the possibilities of using radioactive tracers have been extended. C. Kidson* has applied a new technique to shingle movement at the mouth of the River Alde near Shingle Street. The same isotope, barium - 140 was used, but a far simpler method of attaching it to the pebbles was adopted, namely by baking. Flint pebbles with a surface layer of ferric oxide were used, since it was found that the absorption of the isotope was greatest on them. Later work showed that the ferric oxide layer was unnecessary for this purpose. In all 2,600 pebbles were used. A similar method of under-water detection was used as in the Scolt experiment. As a result of the absorption of the gamma radiation from the tracer in water, the pebbles can only be detected about 10 inches away from the counter which was so mounted that when it was towed it travelled on runners which raised it two inches above the bottom.

Another part of the same experiment was the tracing of pebbles on the main beach and on off-shore shingle banks. For this purpose a scintillation counter mounted on a Dexion sledge was used. Marked pebbles could be detected at a lateral distance of fifteen feet, and buried ones at a depth of six to eight inches. Careful checks were made, as at Scolt, to record readings of pebble movement made from the boat. The movements on the main beach presented a less difficult problem.

At Orford Ness about 600 marked pebbles were deposited about 700 yards off shore in water varying, according to the tide, between nineteen and twenty-eight feet deep. Here, curiously, an entirely negative result was obtained. No movement whatever was detected, despite the fact that on several occasions wind strengths of more than twenty knots were recorded. No reason for the lack of movement can be given, but it does not follow that under more severe conditions, or even at nearby localities, movement would not take place.

*An account of this work is to appear in the Geographical Journal

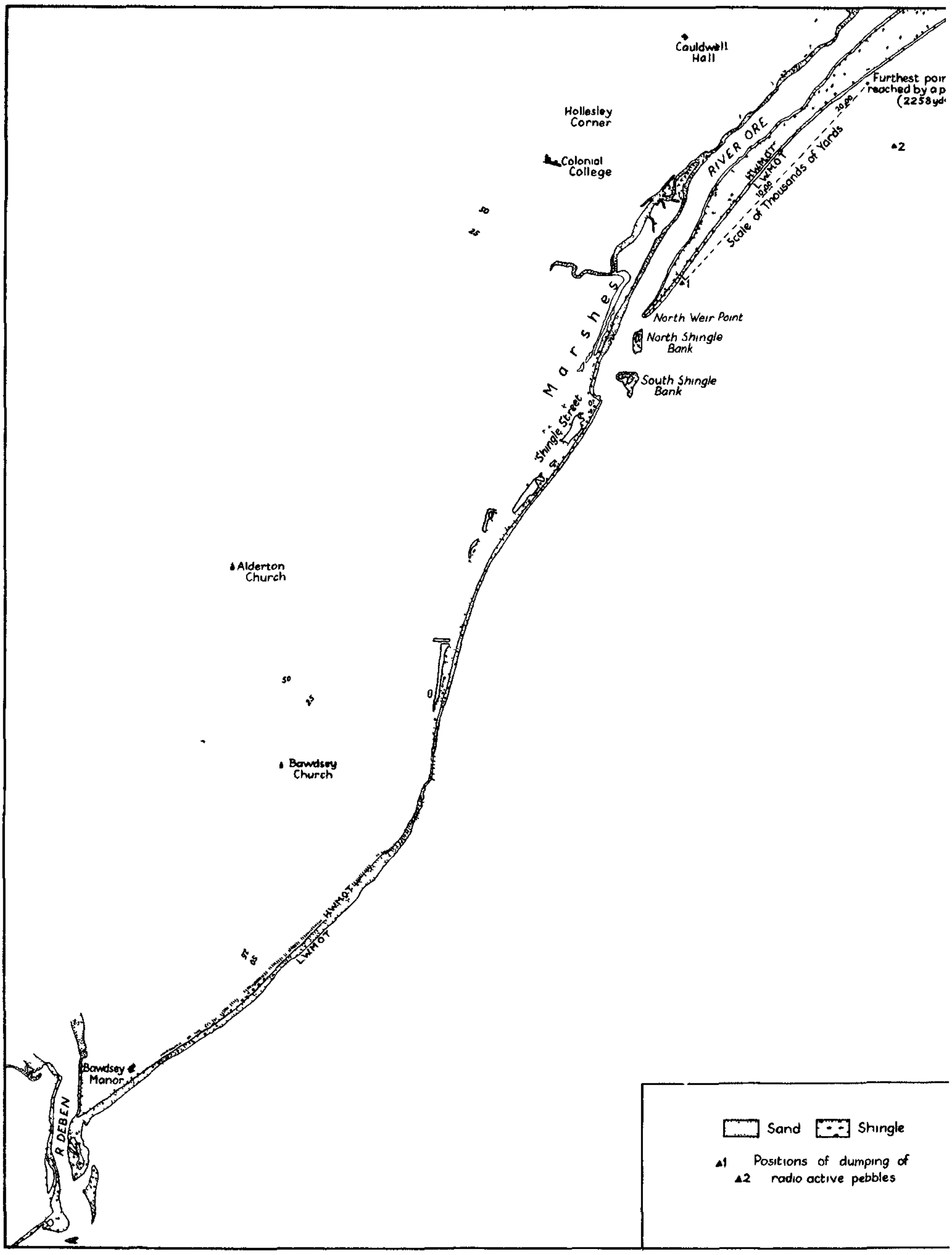

Fig. 5. The south end of Orford Ness, and Shingle Street, Suffolk, England. [To illustrate experiments made by C. Kidson (Nature Conservancy)]

The experiment of tracing pebbles along the main beach was revealing and even spectacular. About 2,000 were put down at low water on 23 January 1957 (See Figure 5). It has been said above that Orford Ness has been built from north to south, and that occasional reverse movements can take place. For the first few weeks of this experiment, south, south-west, and south-east winds prevailed. In consequence, the pebbles which were traced all moved northwards. The maximum amount moved, after four weeks, by one pebble was 2,258 yards. The mean distance of 93 was more than 600 yards northward. In the fifth week (20 - 26 February) the wind was in the north-east, and waves approached the beach with a marked northerly component. The strength of the wind did not exceed fifteen knots, and wave heights were never more than two feet. Even in this period there were winds from south-east, south, and south-west which, collectively, blew for a longer time than those from the north-east. Nevertheless, the north-east winds were the ones that mattered. The northerly drift of pebbles was arrested and reversed, and marked pebbles were found not only on the Orford beach, but on the isolated shingle banks in the haven mouth, and on the beach at Shingle Street. "From this evidence it would appear that waves approaching from a northerly quarter, combining with the tidal current (which, at springs, may reach a velocity of 7-8 knots on the ebb) in the entrance to the River Orę (=Alde) can move shingle from North Weir Point across the opposite bank. It would seem that the path of beach material which reaches North Weir Point, and continues in motion, is either up river under the influence of waves and the flood current, or, alternatively, to the shingle banks off shore, aided by the ebb tide. It is probable that the pebbles which reached the beach at Shingle Street had all arrived there by way of the off-shore banks."*

Although this experiment did not prove that the pebbles moved on shore from the point at which they were dumped, it did emphatically prove that the pebbles moved over the sea floor across a river mouth where ebb and flood currents may be very strong. In short, it showed very clearly indeed that the great masses of shingle at Shingle Street could easily have been transported from the other side of the haven mouth. Since, too, some at least of the marked shingle, travelled by way of the knolls in the haven's mouth, the experiment may be held to support the view that shingle can, under certain circumstances, travel on shore from the offing. This seems, at first sight, to contradict what has been said above about the Blakeney pebbles. Conditions are entirely different in the two places, and although I frankly admit lack of proof, I do not think any direct comparison is possible. In brief, each locality needs special study.

* C. Kidson, in a private communication.

It cannot be claimed for the two experiments - Scolt and Orford that they have proved on shore movement, but it can certainly be said that they have made it reasonable to assume that such a movement can occur. There is no doubt that with lighter material than shingle on shore movement of this type is common. Near Claigan on Loch Dunvegan (Isle of Skye) there are two unusual beaches formed almost entirely of the white skeletons of lime-secreting organisms which live just off shore. The organisms are corallines and produce great quantities of a coarse organic calcareous sand which is spread on the sea floor, and sooner or later washed up to form beaches. This is a particular instance, and may be regarded on account of the nature of the material, as a special case. But there is little doubt that ordinary sand is washed up - the common occurrence of a beach consisting of sand as well as shingle combed down in a big storm and gradually rebuilt in more normal times proves this. A striking example was the almost complete loss of beach material from parts of the Lincolnshire coast in the great flood and storm of January 31 - February 1, 1953. This coast was surveyed regularly in succeeding years and even in the early part of 1957 one or two surveyed sections showed that conditions along them had not quite returned to those prevailing before the storm.*

Undoubtedly, there is much information to be obtained by the use of radioactive tracers. For reasons of health it is not possible at present to carry on experiments in many places, but even in these islands there are tracts of shore where they could be used without danger. No other means of marking, and identifying, pebbles is at all comparable. It is expensive to carry out big experiments, but if we are to find out just how our beaches are fed and depleted we must know far more about what happens in the water immediately off shore. That movement of material, including shingle, can take place can be demonstrated by careful echo sounding. W. W. Williams, of the Department of Geography at Cambridge, has shown how comprehensive a revealing this work can be on the Suffolk coast. But whilst it shows by change of contour that movement has taken place, it cannot of itself trace the movement of any group of pebbles.

My own conviction is that many beaches are fed from off shore, especially in places, like the North Sea, where there is good reason to suppose that glaciation has left plenty of mud, stones, and sand on the floor of the sea. But conviction is not enough. We have shown that under very ordinary conditions pebbles can move landward over th sea-floor; it remains to be proved that they do so in many other places

* Private note from Dr. C. King (Nottingham University) who has carried out much work on the Lincolnshire coast.

and in large quantities.

The composition of the pebbles of the shingle beach at Gunwalloe, in Cornwall, is relevant matter for a footnote to this paper. The beach was described by Clement Reid in the Quarterly Journal of the Geological Society, 60, 1904, 113, and also in The Geology of the Land's End District, Memoirs of the Geological Society, 1907. Reid made the following analysis of the beach pebbles: Chalk Flint 86%, Greensand Chert 2%; Quartz 9%; Grit 2.5%, and Serpentine 0.5%.

There is no exposure of Cretaceous rocks within many miles of the beach. The flints are sub-angular, weathered, and in general resemble those in the Eocene gravels of Devon and Dorset. Reid's conclusion was that "both flint and chert are derived, not directly from Cretaceous rocks, but through the intermediary of some Eocene river gravel." This led him to suggest an Eocene outlier in Mount's Bay, from which the stones were thrown up by storms and buoyed up by sea-weed. They were later subjected to beach-drifting and eventually reached their present position. After a discussion of the wider implications of his view he wrote "The little evidence yet available suggests that Eocene rivers radiated from the high ground of Dartmoor............and that one of these rivers turned southward to cut through the depression" between the Land's End peninsula and the main part of Cornwall.

In discussion, the possibility that the pebbles were brought by drift ice was mentioned, but it was argued that this was most unlikely because they occur in such large quantities in one place, whilst neighbouring bays and inlets yield few flints.

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CHAPTER 19

UTILISATION DES TRACEURS RADIOACTIFS POUR L'ETUDE DES MOUVEMENTS DE SEDIMENTS MARINS

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La connaissance des mouvements de sédiments marins est un des problèmes qui conditionnent essentiellement les travaux maritimes. Des méthodes divers pour l'aborder ont été essayées avec plus ou moins de succès : levés hydrographiques - analyses granulométriques, chimiques - recherche des stocks fournisseurs de sédiments - emploi de traceurs colorés.

La possibilité d'utiliser les propriétés des radio-éléments artificiels a suscité de nombreux espoirs et dans le monde entier les ingénieurs se son penchés sur le problème. De nombreuses difficultés ont immédiatement surgi e la diversité des méthodes employées montre qu'une doctrine générale ne s'es pas encore dégagée.

On peut classer ces difficultés en trois grands groupes :
- marquage des sédiments,
- détection des mouvements,
- exploitation des résultats.

Après l'analyse des différentes méthodes employées, la présente note expose les solutions adoptées par le Laboratoire National d'Hydraulique à propos d'une campagne de mesures en précisant les difficultés résolues et celles restant à résoudre.

METHODES EN CONCURRENCE

MARQUAGE DES SEDIMENTS

Les solutions adoptées se classent en deux grandes familles :

Marquage du sédiment naturel - Les partisans de cette méthode estiment à juste titre, qu'en marquant le sédiment naturel lui-même, on est certain l'exactitude des mouvements du sédiment marqué. Trois méthodes sont en prés nécessitant toutes trois un prélèvement du sédiment.

- Activation du sédiment naturel dans un réacteur atomique [7]. Cette méthode n'est valable que lorsqu'il existe dans la composition du sédimen un élément chimique susceptible d'être activé. C'est donc une méthode qui ne présente aucun caractère de généralité. De plus, l'expérimentateur n'étant pas libre du choix de l'élément radioactif risque de se trouver en présence d'un élément à vie trop brève et par suite de n'avoir pas le temps de faire de prospections (phénomène à observer trop lent - conditions météorologiques ou maritimes ne permettant pas la prospection immédiate).

- Imprégnation du sédiment naturel par un élément radioactif. Dans ce procédé, les particules du sédiment naturel sont recouvertes par adsorption d'une couche radioactive, soit par des moyens chimiques (réduction de nitrate d'argent radioactif [4, 5, 6]) ou physiques (cuisson à 500° [14]). Cette méthode présente l'avantage sur la précédente de laisser l'opérateur libre du choix du traceur. Elle présente par contre l'inconvénient de nécessiter des manipulations longues et onéreuses d'importantes quantités de matériaux activés.

- Inclusion d'une particule radioactive dans chaque élément du sédiment. Cette méthode n'est valable que pour de très gros éléments : galets [2, 3] .

Utilisation d'un sédiment artificiel - Afin d'éviter les opérations de prélèvements, de transports et de marquage d'importantes quantités de sédiments naturels, d'autres expérimentateurs ont préparé séparément un sédiment artificiel en faible quantité mais fortement activé, appelé à être mélangé au sédiment naturel. Ce principe, le plus fréquemment adopté [2, 3, 8, 9, 10, 12, 13, 15] , n'est valable que dans la mesure où l'expérimentateur est certain d'obtenir un sédiment artificiel dont le comportement soit analogue à celui du sédiment naturel.

Pour cela, tous les expérimentateurs ont utilisé un sédiment artificiel ayant la même densité et la même distribution granulométrique que le sédiment naturel. Le Laboratoire National d'Hydraulique a pris la précaution supplémentaire de procéder à des essais comparatifs en canal dans des conditions d'essais très diverses.

IMMERSION DES SEDIMENTS

Tous les opérateurs sont d'accord sur le fait que le traceur doit être déposé sur le fond. Les différences entre les divers procédés sont essentiellement d'ordre pratique. Nous mentionnerons simplement l'avantage qu'il y a à cet égard de réduire au maximum la durée des opérations et les précautions à prendre, d'où l'intérêt de l'emploi de faibles quantités de sédiments artificiels.

DETECTION

Deux méthodes sont en présence :

- La détection par prélèvements qui est donc une détection par point. Dans certains cas [7], cette méthode seule est possible étant donnée la nature du rayonnement émis. Elle présente de plus l'avantage d'une très grande sensibilité, particulièrement avec le procédé proposé par les Portugais [4,5,6], qui consiste à récupérer sur les échantillons l'argent radioactif qui imprègne les grains, et à reconcentrer ainsi la radioactivité du prélèvement. Par contre, elle ne permet pas d'avoir une vue d'ensemble du phénomène. L'opérateur court le risque de ne pas voir une certaine partie des mouvements.

- La détection continue au moyen d'une sonde trainée sur le fond [2, 3]. Cette méthode nécessite l'emploi de traceurs émettant des rayonnements γ très pénétrants. De plus, elle est peu sensible. Par contre, elle permet d'obtenir une excellente estimation qualitative du phénomène général et d'en avoir la compréhension immédiate.

La solution idéale consisterait donc en une combinaison des deux méthodes Les prélèvements permettent l'étalonnage des détections continues. Toutefois, pour que les prélèvements soient vraiment utiles, il serait nécessaire qu'ils donnent des indications sur l'épaisseur des mouvements. Certains auteurs [4, 5, 6] l'ont estimée à 4 cm; nous pensons pour notre part que la couche intéressée est beaucoup plus épaisse (voir plus loin : expérience de l'Adour). Le problème à résoudre est donc celui du carottage, facile pour les vases, non encore résolu pour les galets.

METHODE ET APPAREILLAGES UTILISES PAR LE LABORATOIRE NATIONAL D'HYDRAULIQUE

Le Laboratoire National d'Hydraulique a effectué, en 1956 et 1957, deux séries d'expériences de traceurs radioactifs. Ces expériences avaient pour but de recueillir quelques renseignements sur le transport littoral, en vue d'étud ultérieures sur modèle réduit.

L'une de ces séries d'expériences a été consacrée à l'étude des mouvement de galets dans le cours inférieur et à l'embouchure du Var, sur la côte méditerranéenne, tandis que l'autre était destinée à l'étude des mouvements de sable au voisinage de l'embouchure de l'Adour, sur la côte atlantique.

La préparation de ces expériences a été effectuée en étroite collaboration avec les ingénieurs du Commissariat à l'Energie Atomique de Saclay (France) et a donné lieu, en particulier pour l'étude des mouvements de sable, à la mise au point d'appareillage et de méthodes de travail que nous allons décrire.

ETUDE DES MOUVEMENTS DE GALETS

Nous passerons rapidement sur l'utilisation des traceurs radioactifs pou l'étude des mouvements de galets, car les conditions locales de l'expérience que nous avons effectuée dans le Var (faibles profondeurs, zones d'études réduites) nous ont permis de travailler avec un appareillage relativement simple.

Fabrication du traceur - Les galets ont été marqués par inclusion d'une particule radioactive. Après perçage d'un trou de 8 mm de diamètre, on a introduit dans chaque galet une aiguille de tantale 182 de 5 mm de longueur. La cavité était ensuite obturée par du ciment expansif à prise rapide. Le tantal 182 est un radioisotope émettant des rayonnements gamma de 0,1 à 1,2 MeV et de 111 jours de période (demi-vie).

Immersion des galets - Etant donné la grande vitesse de chute des galets dans l'eau et les faibles profondeurs rencontrées, aucun risque de dispersion du traceur n'était à craindre pendant l'immersion. Les galets ont donc été déversés sans précautions spéciales en divers points du Var et de la côte au voisinage de l'embouchure.

Détection - Nous avons utilisé, pour la détection des galets rejetés à l'air libre, un gammamètre SRAT type GMT-14, couramment employé dans la prospection des minerais radioactifs. Il permettait de déceler la présence des galets à 2 mètres de distance. Pour la recherche des galets immergés, le gammamètre était connecté à une sonde trainée sur le fond qui pouvait détecter les galets radioactifs dans un rayon de 0,4 mètre environ.

ETUDE DES MOUVEMENTS DE SABLE

Pour l'application de la méthode des traceurs radioactifs à l'étude des mouvements de sable, nous avons été amenés à étudier et à réaliser, avec la collaboration du Commissariat à l'Energie Atomique, un appareillage spécialement conçu et à mettre au point des méthodes de détection. Certains des dispositifs imaginés ont été brevetés.

En effet, le problème posé présentait beaucoup plus de difficultés que celui des mouvements de galets dans le Var :

- Nous désirions étudier les mouvements du sable fin de la Barre de l'Adour (0,1 à 0,4 mm environ) dans des profondeurs pouvant atteindre 15 mètres.

Dans ces conditions, il était indispensable de disposer d'un appareillage d'immersion permettant le dépôt du traceur sur le fond pour éviter qu'il soit dispersé par les courants au cours de sa chute dans l'eau, et répandu sur une surface trop étendue, au début de l'expérience.

-Pendant la mauvaise saison, où se produisent les mouvements de sédiments les plus intéressants à connaître, les tempêtes se succèdent souvent à intervalles très rapprochés. Il fallait donc mettre au point un appareillage et une méthode de détection permettant de lever des zones radioactives étendues, dans le laps de temps le plus court.

Fabrication du traceur - Le traceur était constitué par un sable de verre, de densité égale à celle du sable naturel et présentant la même distribution granulométrique. L'élément marqueur était l'oxyde de Scandium $Sc_2 O_3$, fondu dans le verre au cours de sa fabrication et donnant, par irradiation dans un réacteur atomique, du Scandium 46, émetteur γ dur de 85 jours de période (demi-vie).

Immersion du traceur - Le traceur était livré par le Commissariat à l'Energie Atomique dans des tubes d'aluminium de 25 mm de diamètre et de 70 mm de longueur, contenant environ 50 grammes de sable de verre. Ces containers étaient placés, pour le transport et le stockage, dans des châteaux de plomb.

Le problème consistait à déposer au fond de l'eau le traceur contenu da ces tubes en assurant à tout moment la protection des opérateurs contre les radiations et les projections de particules radioactives. Cette opération devait, en outre, pouvoir s'effectuer à bord d'un bateau de petite taille, d des conditions d'agitation assez gênantes. (Certaines immersions ont été effectuées en mer, par les houles de plus de 2 m de creux).

L'appareil d'immersion (figure 1) est constitué d'un corps basculant, percé d'un canal central et supporté par un étrier. Une tige de blocage perm de fixer le tube contenant le traceur à l'une des extrémités du canal. A l'autre extrémité est disposée une ampoule de verre. Un dispositif, destiné à briser l'ampoule dès que l'appareil arrive au contact du fond, est fixé au dessous de celle-ci.

Cet appareil fonctionne de la façon suivante (figure 2). Après ouvertur du château de plomb et du tube contenant le traceur (2 a), l'appareil, suspe du à un mât de charge ou un bossoir, vient coiffer le tube ouvert (2 b). Aprè fixation du tube sur le corps de l'appareil, à l'aide de la tige de blocage de 2 m de longueur qui sert également à contrôler les mouvements de l'appare celui-ci est soulevé et amené au-dessus de l'eau par pivotement du mât de charge (2 c). Le retournement de l'appareil est alors effectué et le traceu contenu dans le tube est transvasé, par gravité, dans l'ampoule de verre (2 Il ne reste plus qu'à descendre l'appareil sous l'eau, où, par percussion sur le fond, le marteau vient briser l'ampoule et libérer le traceur (2 e).

Détection - L'emploi d'un traceur émettant un rayonnement gamma de haut énergie nous permettait de détecter directement le traceur sur le fond. Pou accroître encore le rendement des détections, nous avons utilisé un apparei lage enregistreur qui nous a permis de réduire les opérations de détection à celles d'un simple levé hydrographique.

Nous avons utilisé, à cet effet, un appareillage de prospection de mine radioactifs dans les forages (figure 3). Il comprenait :

- une sonde détectrice composée de 8 compteurs de Geiger-Muller pour rayons gamma, enfermés dans une enveloppe étanche,

- un intégrateur enregistreur recueillant par l'intermédiaire d'un câble, l indications des compteurs et donnant une valeur moyenne de l'activité détectée par la sonde.

Deux méthodes de détection sont utilisables avec un appareillage de ce type :

- pour lever des zones radioactives étendues, le sonde est remorquée à faib vitesse au bout d'une cinquantaine de mètres du câble, par un bateau qui peut ainsi détecter rapidement l'activité du fond le long de profils (4 a Pour permettre le report sur plan de l'activité relevée, la position du bateau est notée à intervalles réguliers et repérée sur la bande d'enregistrement;

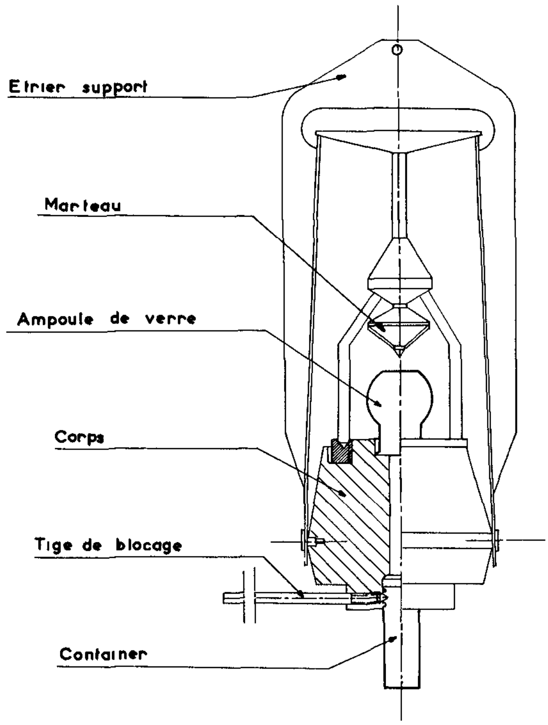

Fig. 1. Appareil d'immersion

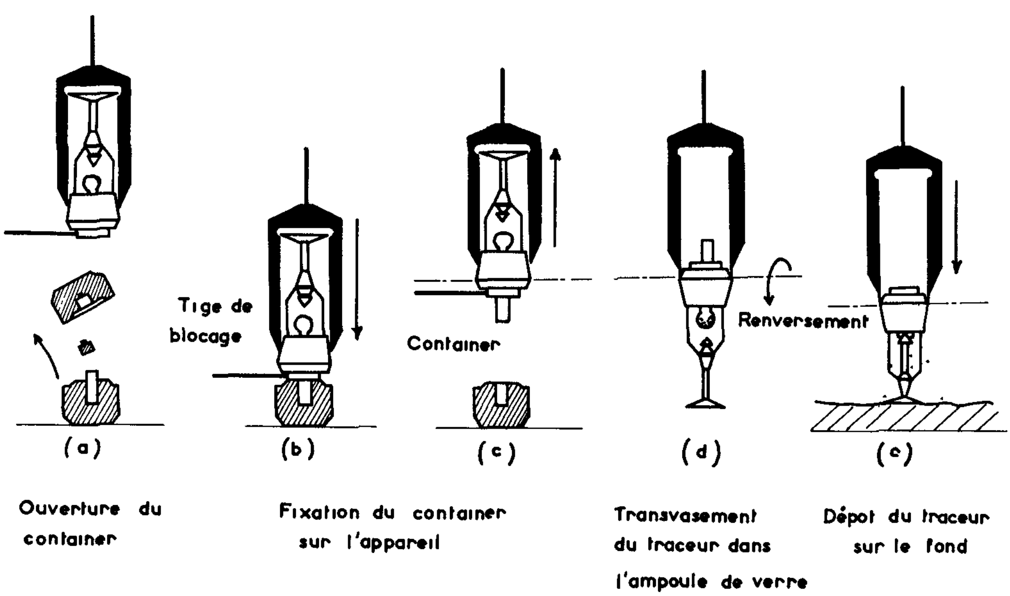

Fig. 2. Principe de fonctionnement de l'appareil d'immersion

- pour l'étude détaillée d'une zone réduite ou quand les erreurs sur les pos tions du bateau risquent d'être du même ordre de grandeur que les déplacements du traceur, le relevé de la zone active est effectué par rapport au bateau ancré. Le câble est alors déroulé sur toute sa longueur et le treuil ramène lentement la sonde détectrice à bord (3 b). On peut lever ainsi la zone active par profils rayonnant autour du point de mouillage du bateau.

A partir de cet appareillage de prospection, mis à notre disposition pa le Commissariat à l'Energie Atomique, nous avons vu définir les caractéristi d'un appareillage spécialement conçu pour les expériences de traceurs radioactifs en mer. Cet appareillage, actuellement en cours de construction, comprendra :

- une sonde détectrice de grande emprise, comportant 5 compteurs de Geiger-Muller et capable de détecter l'activité du fond sur une bande de 1,5 m de largeur environ,
- un dispositif intégrateur enregistreur de faible volume et de constructior robuste, spécialement adapté aux dures conditions du travail à la mer.
- un câble de transmission de haute résistance, de 500 m de longueur, enroulé sur un treuil.

Un soin particulier a été apporté à l'étude de la robustesse et de l'étanchéité des divers éléments de l'appareillage.

RESULTATS OBTENUS AU COURS DES EXPERIENCES

Les deux séries d'expériences effectuées par le Laboratoire National d'Hydraulique n'ont sans doute pas donné, surtout en 1956, tous les résulta espérés, mais il faut noter qu'il s'agissait, dans les deux cas, d'expérien pilotes et que nous ne disposions que d'un appareillage encore mal adapté aux conditions particulières des expériences. A l'embouchure de l'Adour, nou nous trouvions en outre sur la côte française la plus inhospitalière et bie souvent le mauvais temps persistant survenant après des immersions, nous a empêchés de suivre les déplacements du traceur aussi régulièrement que nous l'aurions désiré.

ETUDE DES MOUVEMENTS DE GALETS DANS LE VAR ET SUR LA PLAGE DE NICE

Nous n'insisterons pas sur cette campagne de mesures décrite dans une précédente communication [3]. Rappelons simplement qu'il s'agissait de mett en évidence des mouvements de galets en trois endroits différents :

- dans le cours du Var où l'on a pu déterminer les vitesses de début d'entr nement des galets et leur vitesse de déplacement en fonction de la vitess du courant,
- sur la barre de l'embouchure du Var où elle a mis en évidence le fait que les galets en place oscillaient autour d'une position moyenne, sans transports,
- au port de Cros-de-Cagnes où l'on a pu mettre en évidence la vitesse du transport Est-Ouest.

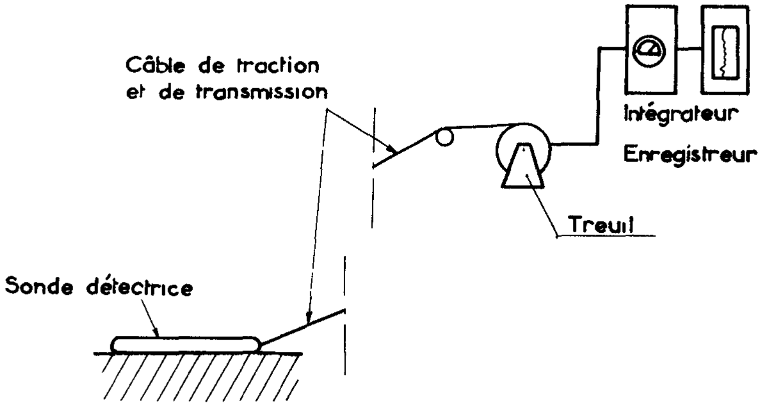

Fig. 3. Schema de principe de l'appareillage de detection

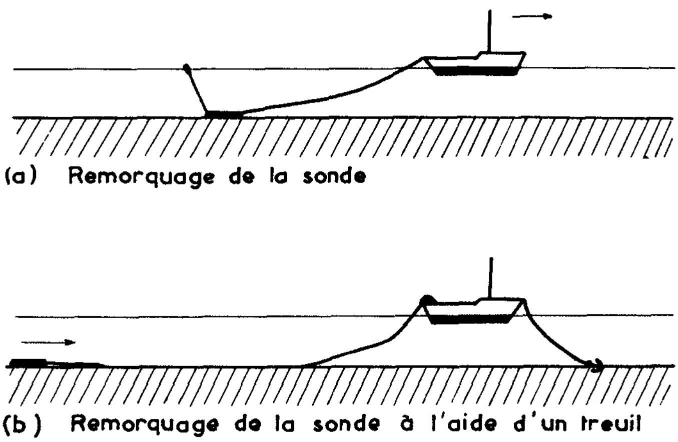

Fig. 4. Methodes de detection

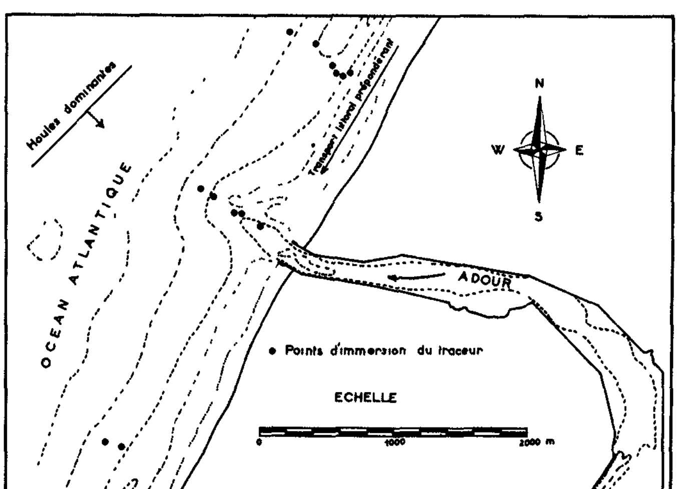

Fig. 5 CARTE DE L'EMBOUCHURE DE L'ADOUR

ETUDE DES MOUVEMENTS DE SABLE A L'EMBOUCHURE DE L'ADOUR

La campagne de mesures de l'embouchure de l'Adour a comporté deux séri d'expériences effectuées, l'une de Février à Mai 1956, l'autre en Mai 1957.

Rappelons qu'il s'agit ici du débouché en mer de l'estuaire de l'Adour. La marée à cet endroit atteint 4 m de vive-eaux, et l'action combinée de la houle et des courants de remplissage et de vidange de l'estuaire conduisent à la formation d'une barre gênante pour la navigation.

Expériences de Février-Mai 1956 - Les résultats de ces expériences ont été présentés dans une communication antérieure [3]. Elles avaient pour but d'étudier le transport littoral de part et d'autre de l'embouchure.

Deux profils, à 1400 mètres au Nord et à 1900 mètres au Sud de l'embouchure avaient été "marqués" respectivement par six et deux dépôts de traceu radioactif (figure 5). Malgré les conditions d'expérience très difficiles dues à la persistance du mauvais temps, nous avons pu mettre en évidence :

- au Nord, la composante Nord-Sud, parallèle au rivage, du déplacement des sables,
- au Sud, le déplacement des sables, perpendiculairement à la côte, vers le rivage.

Ces deux phénomènes ont été retrouvés depuis sur le modèle réduit.

Expériences de Mai 1957 - Les immersions ont été effectuées sur un profil perpendiculaire au rivage, au droit de la jetée Nord de l'embouchure. Cinq dépôts de traceur ont été immergés de la cote - 3 mètres à la cote - 7 mètres.

Les résultats des trois premières détections sont présentés sur la figure 6 où l'évolution des taches radioactives est mise en évidence par le tracé des lignes isoradioactives.

- Le lendemain de l'immersion (6 a), après quelques heures de houle de 1,40m de creux moyen, le traceur s'est répandu en nappes allongées, sensiblement parallèles aux lignes de niveau.

- Six jours après l'immersion (6 b), sous l'action combinée d'une courte tempête pendant laquelle le creux moyen de la houle a dépassé 2 mètres, et du courant de jusant à l'embouchure, le traceur provenant des trois dépôts les plus proches du rivage s'est étalé sur la Barre. Les deux dépôts du large, malgré une recherche très serrée, n'ont pas été retrouvés. La comparaison des levés hydrographiques effectués simultanément a montré par la suite que ces dépôts étaient recouverts de plus de 50 centimètres de sable.

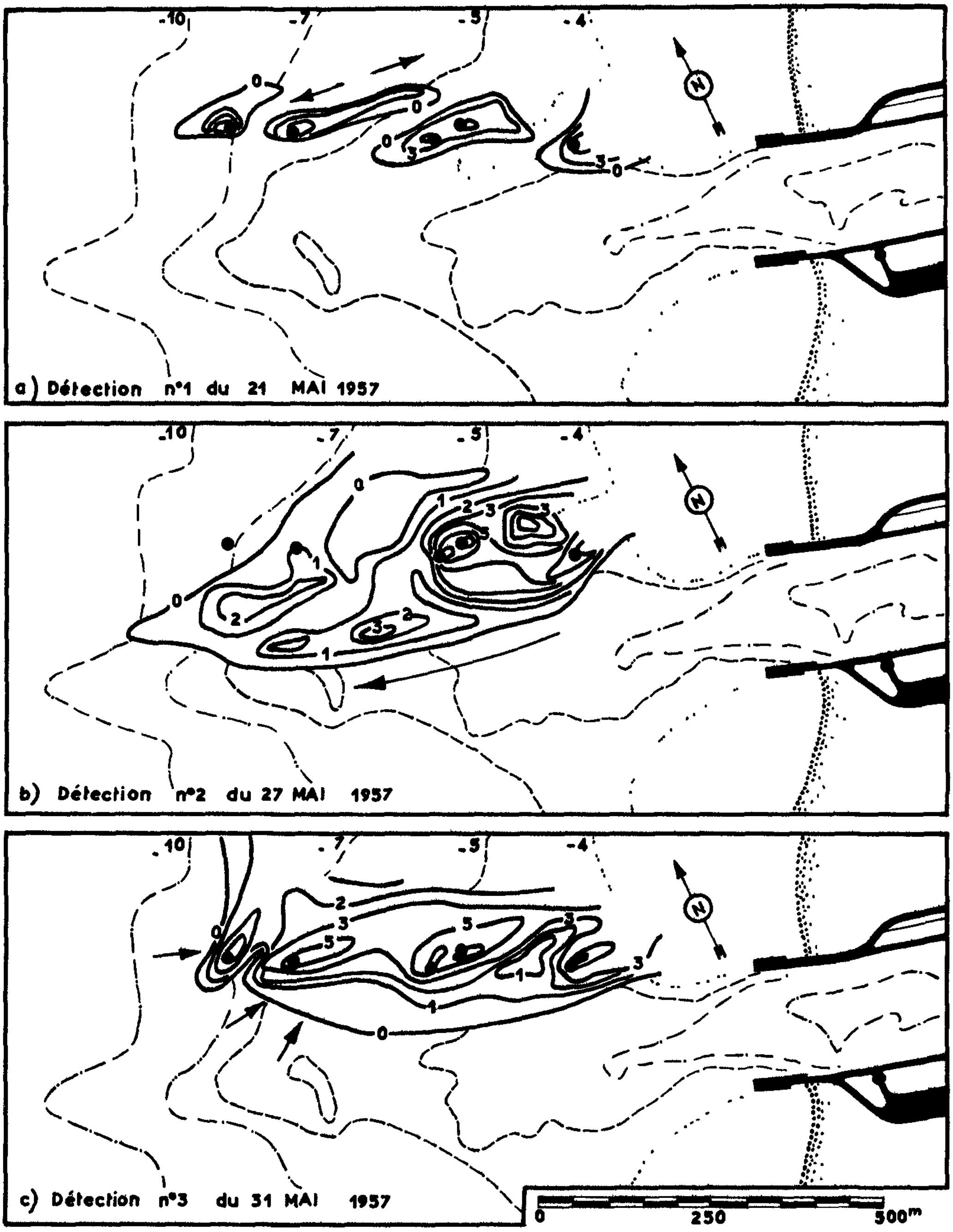

Fig. 6 MOUVEMENTS DE SABLE A L'EMBOUCHURE DE L'ADOUR
IMMERSIONS DU 20 MAI 1957
• POSITION DES POINTS D'IMMERSION DU TRACEUR

- Une détection effectuée quatre jours plus tard, après une période de beau temps,(6 c) montre d'une part la régression de la tache radioactiv et la réapparition des deux dépôts du large. On retrouve, comme dans la première détection (6 a), la tendance du sédiment à se répartir le long de lignes de niveau.

L'exploitation de ces détections et de celles qui les ont suivies, en relation avec les modifications du relief, se poursuit actuellement. Elle semble confirmer que par beau temps, le sable tend à se répartir sur le fond, sans direction bien déterminée, dans une phase de construction d'un profil d'équilibre. Par mauvais temps par contre, le longshore curren de direction Nord-Sud, développé par l'attaque oblique de la houle sur la côte Nord, se combine au courant de jusant de l'embouchure et transporte le sédiment sur la barre.

CONCLUSIONS

Les quelques résultats que nous venons de présenter sont encore fragmentaires, mais ils nous semblent, cependant, suffisamment intéressants po nous inciter à persévérer dans cette voie et à mettre la méthode définitivement au point.

Le fait que des dépôts enfouis sous une certaine épaisseur de sable aient échappé aux recherches montre que la détection en surface ne suffit pas toujours à donner une idée suffisamment précise des mouvements du traceur.

Nous pensons que la principale difficulté à résoudre réside dans la détermination de l'épaisseur de la couche active, qui pourrait peut-être, dans des cas simples où cependant on ne peut pas procéder par cubature de dépôts, permettre la détermination du débit de transport littoral.

Le problème se réduit en fait à la mise au point d'un carottier spécialement adapté aux prélèvements de sable. Pour des raisons d'échantillonnage, cette méthode nécessiterait sans doute également la mise en oeuvre d'un plus grand nombre de grains actifs et il n'est pas prouvé que, dans ces conditions, elle garde tout son intérêt économique.

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CHAPTER 20

CONTRIBUTION TO THE STUDY OF SEDIMENT TRANSPORT ON A HORIZONTAL BED DUE TO WAVE ACTION

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Summary

With a view to explaining the phenomena of sediment transport in the open sea, outside the wave breaking area, the author carried out a laboratory investigation of wave action on a horizontal bed. He puts forward a number of new results regarding :

1 - The state of turbulence near the bed and the stability of the oscillatory laminar boundary layer.

2 - The setting in motion of materials under the influence of wave alone.

3 - The entrainment current caused by wave action close to the bed.

4 - The transport of material under wave action only.

5 - The indirect action of wave on the bed.

The main conclusions reached are as follows :

1/ - The results given by Huon Li regarding the onset of turbulence within the oscillatory boundary layer overestimate the range of laminar conditions. Vo (maximum orbital velocity) and ε (roughness) are the principle factors governing the transition.

Test waves are either generally laminar, or are only slightly turbulent within the body of liquid, but they are, however, more often turbulent in the immediate neighbourhood of the bed.

2/ - The investigation of conditions for the onset of grain movement of the bed material shows that the action of wave can be appreciable, even at depths of several tens of metres. A wave of 6 metres amplitude, with a total length of 120 metres, would be capable of putting a 0.3 mm sand grain into motion at a depth of 60 metres.

3/ - The experimental investigation, as well as the viscous fluid theory, shows the existence, close to the bed, of an entrainment current of liquid particles which *always* works in the direction of wave propagation.

4/ - In test flumes, this entrainment current forms part of a mass transport within the liquid, the vertical distribution of which varies with the characteristics of the fluid motion. On a horizontal bed, it generally gives rise to an *effective sediment transport*, in the direction of wave propagation, as the preponderant part of the liquid velocity component, near the bed, is in this direction.

5/ - Owing to the existence of the mass transport current and the onset of suspensio of material above the bed, some sediment transport can exist out to sea. These results give a explanation of why, under the action of long and regular wave, material tends to be carried i the direction of the waves and build up on the beach whereas, under storm conditions, a strong resultant turbulence produces suspension and favours erosion of the beach.

6/ - On a sloping bed, transport towards the shore is counterbalanced by the effect of gravity, currents caused by winds from seaward and density currents set up in the wave breal area so that finally material eroded from land surfaces are, in part, gradually carried away towards the open sea.

CONTRIBUTION TO THE STUDY OF SEDIMENT TRANSPORT ON A HORIZONTAL BED DUE TO WAVE ACTION

GENERAL REMARKS

The problem considered here is that of solid transport along the sea bed caused by wave action, to seaward of the breaking zone i.e. outside the zone in which the chief phenomenon occurring is that usually known as littoral transport.

a) Effect of waves on the sea bed - General comments

The extent of the effect of waves on the sea bed, from the breaking zone out to fairly considerable depths, depends on the local nature of the bed, the depth of water and the characteristics of the waves. At these depths, the bed materials are usually set in motion without there necessarily being any major displacement of solid particles, and it can be said that these displacements decrease continuously as the distance from the breaking zone increases. This decrease must generally be very rapid, for turbulence has a considerable effect on solid transport, particularly near the sea bed,and should in actual fact decrease rapidly in the bottom layers of the fluid as the distance out to sea and away from the breaking zone increases. This is why suspension generally tends to fall off quickly towards the open sea and the decanting materials become correspondingly finer and finer.

Nevertheless, it should not be forgotten that waves are not the only natural factor outside the breaking zone capable of transporting materials. The solid particles set in motion by the waves may be taken up by a random sea current, even if only very slight, that will act on solid grains already dislodged from the sea bed.

It should also be remembered that wave action on the sea bed can form or produce favourable conditions for density currents, which carry the fine particles out to sea. The finer the particles and the steeper the sea bed, the farther and faster they travel.

Finally, and contrary to general belief, this bed zone can be the seat of quite considerable exchanges of solid materials. A knowledge of these movements is of great value when seeking to better understand the movements of solids in a beach profile and when tackling the problem of bar formation at the seaward end of a channel, or trying to find the best place to dump dredged or other materials at sea.

b) Experimental Tecnniques

Most of the Authors who have carried out experimental research on turbulent states or sediment transport due to wave propagation in shallow water and an horizontal sea bed (HUON LI[1]*, MANOHAR[2], BAGNOLD[3] etc have used an oscillating bed in conjunction with a still mass of water.

Although the movements near the bed produced by this method are about as extensive as in real life, the effect of a number of phenomena caused by progressive waves (such as entrainment currents, acceleration of fluid particles, pressure fluctuations in the immediate vicinity of the sea bed, development of turbulence, etc ..) on a stationary sediment layer are not completely brought into evidence.

We therefore thought it preferable to use a conventional wave flume, in which the bed materials were subjected to the effective action of progressive waves, although the results obtained would not be so directly applicable to real life conditions because of the comparatively restricted range of velocities considered.

SYMBOLS

The following symbols are used in the text :

$2a$ Total wave amplitude at free surface (vertical distance from trough to crest)

$2a_d$ Total wave amplitude corresponding to the onset of the movement of a grain of material.

d Diameter of single grain of material in the bed layer ; d is such that the diameter of 50 % of the grains is $< d$

d_1 Total travel of a fluid particle in the immediate vicinity of the bed.

$d_1 = \frac{2a}{\text{sh } 2\pi\, h/L}$ (Theoretical value, 1st order, perfect fluid)

δ Thickness of oscillatory laminar boundary layer

ε Characteristic dimension of a grain of powdery bed material (in actual fact, $\varepsilon \equiv d$ for material with a sufficiently uniform granulometry)

$\hat{g}$ Apparent gravity field $\hat{g} = g \frac{\rho_s - \rho_f}{\rho_f}$

h Height of water above bed

L Wave length (horizontal distance between two successive crests)

ν Kinematic viscosity of water

ρ_s Specific mass of grain

ρ_f Specific mass of fluid

T Wave period ($\omega = 2\pi/T$)

*. See References at end of text.

u	Maximum intensity of entrainment current near the bed at laminar conditions (mean value)
$u(z)$	Flow velocity in the oscillatory laminar boundary layer at height z above the bed level
$\tilde{u}_z$	Strength of mass transport current at height z above the bed level.
V	Velocity of fluid particle at a distance δ from the bed
V_o	Maximum value of V, being equal to the theoretical maximum orbital velocity in the immediate vicinity of the sea bed.
W	Rate of fall of a grain of diameter d in calm water

I - TURBULENT STATE IN THE VICINITY OF THE BED-STABILITY OF THE OSCILLATORY LAMINAR BOUNDARY LAYER

a) Thickness of laminar boundary layer

The pattern of the hydrodynamic forces acting on a grain of bed material depends chiefly on the flow conditions in the immediate vicinity of the bed, i.e. the actual nature of the flow around the grain.

In a viscous fluid, the overall flow pattern - even if oscillatory - causes a boundary layer - itself oscillatory - with a very steep velocity gradient to develop on the bed.

The appearance of turbulence at this oscillatory boundary layer can in general be considered as an important factor for the sediment transport in the vicinity of the bed, having a particularly marked effect on the onset of grain movement.

It is easy to imagine that the nature of the flow around a grain depends on the ratio δ/ε, i.e. on the depth of immersion of the grain into the oscillatory laminar boundary layer.

At a distance δ from the bed, the velocity of a fluid particle is a direct function of the upper flow and is given by an expression of the following form, for waves in a finite depth, and as a first order approximation :

$$V = \frac{\omega \, d_1}{2} \cos \omega t$$

Velocity V decreases very rapidly in the laminar boundary layer and becomes zero on the bed. The velocity distribution in an oscillatory laminar boundary layer has the following form (cf. LAMB, VALEMBOIS [4]) :

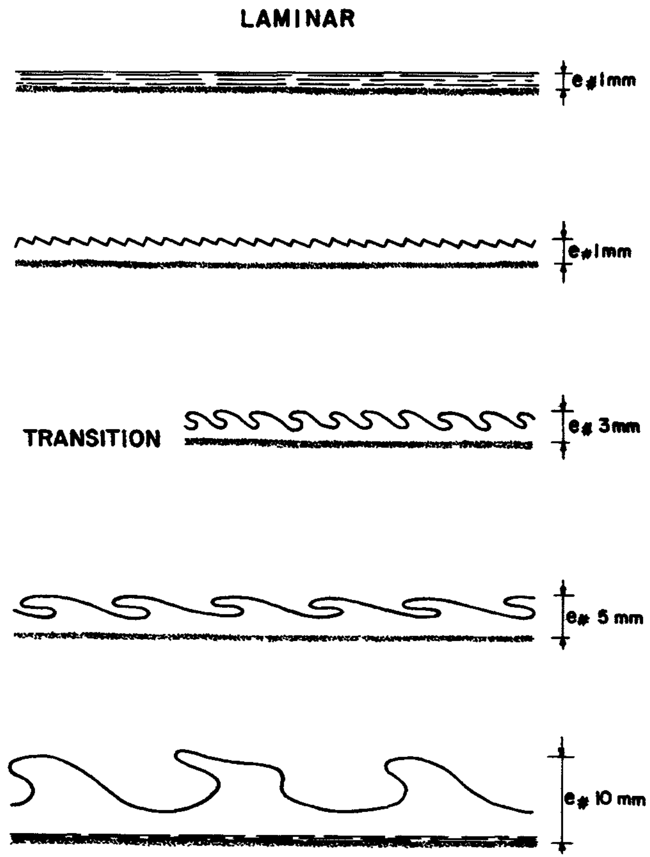

Fig. 1. Transition criterion

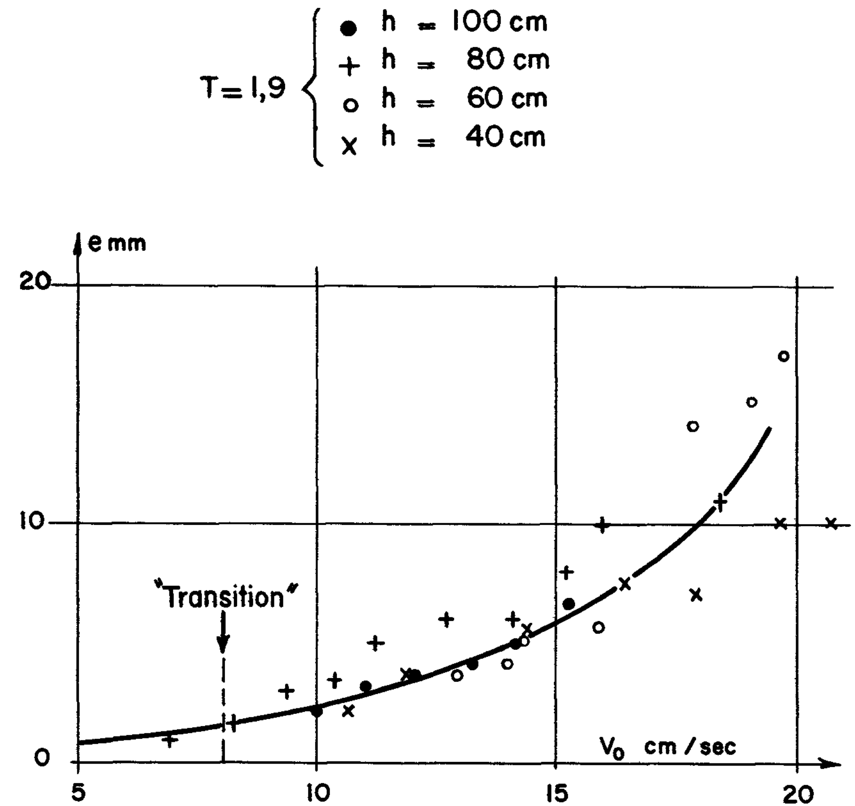

Fig. 2. Thickness of coloured "cloud" e, in mm, for varying $V_o = \frac{\pi D_1}{T}$ .sand (10) ϵ = 0.023 cm

$$u = V - \frac{\omega d_1}{2} e^{-(1/\lambda)z} \cdot \cos\left(\omega t - \frac{z}{\lambda}\right)$$

where the fictitious wave length λ has the form $(\nu/\omega)^{1/2}$ and characterises the thickness of the oscillatory laminar boundary layer ($\delta = K\lambda$) to within one coefficient. For a given fluid, δ therefore depends essentially on the oscillation period of the movement.

b) Stability of Boundary Layer - Work done by HUON LI

A very interesting experimental study of the oscillatory laminar boundary layer has recently been carried out in the United States by HUON LI [1] and completed by MANOHAR [2].

Considerable difficulties are encountered in the study of this phenomenon in progressive waves if the aim is to reproduce, even if only schematically, the boundary conditions due to the action of waves on a natural bed by means of a model.

HUON LI carried out his experimental tests in the same way as BAGNOLD, by making the bed oscillate in an horizontal plane, with the fluid mass at rest.

In this method, the periodic accelerations acting on the fluid particles * and the pressure variations occurring near the bed are not taken into account , although both very probably tend to some extent to " burst " the laminar boundary layer, i.e. to accelerate the onset of turbulence in it. HUON LI's results can therefore be expected to exaggerate the relative importance of the laminar regime compared to what it really is when the oscillatory boundary layer is formed by waves effectively progressing along a stationary bed.

The results obtained form our experimental study confirm the above comment, besides bearing out the theory that transition occurs more rapidly as the characteristic grain size, the travel of a fluid particle in the immediate vicinity of the bed and the frequency of the oscillatory movement increase. (in other words, the velocity of a fluid particle in the immediate vicinity of the bed).

c) Definition of a " transition " criterion (Fig. 1)

When the oscillatory boundary layer is *laminar*, a fine particle of colouring matter placed on the bed is seen to generate a thin coloured cloud (the thickness of this cloud varies slightly at a given point, to the rhythm of the waves) with an apparently perfectly smooth top surface. (One end of the cloud gradually travels along in the direction of wave propagation).

The mean thickness e of the coloured cloud increases slightly as the velocity of the fluid particle near the bed increases. (Fig.2)

* cf. R.MICHE in particular

At a certain oscillation rate, slight discontinuities appear on the top surface of the cloud. These form when the pressure at the bed is near its minimum value and inflect in the opposite direction to the waves, due to the momentary local flow. As the velocity increases further, they develop into small individual " flames " that become gradually more extensive.

In our view, the appearance of these " flames " is characteristic of " transition ".

All other things being equal, the rougher the bed, the lower the oscillation rate at which these " flames " appear.

There is also a detectable laminar film in immediate contact with the bed. This relatively thin film becomes still thinner as local turbulence increases.

If the boundary layer is turbulent, a grain of coloured material of approximately equal density to that of the fluid will, if released at the free surface, sink slowly, describing a very individual coloured line through the fluid down to a distance from the bed that decreases with the " turbulence ".

The above seems to show that, at regular wave conditions, [6] the turbulence produced by the bed does not propagate readily through the fluid mass.

d) Results obtained by HUON LI

According to the work carried out at BERKELEY, the bed would be smooth - in the hydraulic sense of the word - if : $\frac{\delta}{\varepsilon} > 30$ and " rough " if : $\frac{\delta}{\varepsilon} < 18.5$.

HUON LI expresses the thickness of the oscillatory laminar bounda layer in the following terms :

$$\delta = 6.5 \left(\frac{\nu}{\omega}\right)^{1/2} \quad \text{soit} \quad \delta = 2.6 \cdot \sqrt{\nu T}$$

Although this value is probably high *we have nevertheless retaine this expression for purposes of comparison.*

According to HUON LI, if the bed is " smooth ", the oscillatory boundary layer becomes turbulent at local Reynolds numbers where : $d_1 (\omega/\nu)^{1/2} = 800$, apparently even at relatively low values of : $\frac{\nu}{\omega}$

If the bed is " rough ", the transition Reynolds number $\omega d_1 \varepsilon/\nu$ is a constant for each roughness factor, or, for each characteristic bed material grain size.

e) Results obtained at SOGREAH

The values $d_1 = f(T)$ at which, in accordance with the criterion defined above, a turbulent state appears in the oscillatory laminar boundary layer, are plotted in Fig. 3. The values obtained at BERKELEY for a "smooth" bed and various materials are shown dotted and the continuous lines give the values obtained from our tests at progressive wave conditions.

These show that, with two materials with similar d (or ε), and if the oscillatory movement has a given period, turbulence appears at a lower total oscillation amplitude if the relative movement in the vicinity of the bed is caused by wave action. (For instance, if we consider two similar values of ε, as for materials (3) and (10), or (4) and (13), we find that, according to HUON LI, at constant T, transition begins to appear at a value of d_1 that is about 5 times greater than the one observed by us.)

This discrepancy in experimental results appears to be fairly general and occurs at all absolute bed " roughness " values (the roughness being due to grains of a material of characteristic dimensions ε, or, in other words, d).

We cannot state categorically that this difference also occurs with a " smooth " bed, because it is very difficult to characterise the state of such a surface. Even though the beds we considered were relatively very smooth by HUON LI's standards ($\delta/\varepsilon > 30$), consisting of a smooth metal plate (" smooth " in the physical sense) and meticulously smoothed out concrete, their behaviour from the transition point of view appeared to be that of a rough bed, inasmuch as transition occurred at a Reynolds number that still slightly depended on surface conditions.*

Our results confirm that, for a bed with some roughness, the transition Reynolds number $\omega\, d_1\, \varepsilon/\nu$ is comparatively very low, being expressed by the values given below, all of which are obtained from our various test results :

1) sand n° 4, $\varepsilon = 0.023$; $\dfrac{\omega\, d_1}{\nu}\,\varepsilon \;\#\; 36$; $\dfrac{\delta}{\varepsilon} < 19$

2) sand n° 2, $\varepsilon = 0.046$; $\dfrac{\omega\, d_1}{\nu}\,\varepsilon \;\#\; 65$; $\dfrac{\delta}{\varepsilon} < 8.5$

3) sand n° 1, $\varepsilon = 0.063$; $\dfrac{\omega\, d_1}{\nu}\,\varepsilon \;\#\; 60$; $\dfrac{\delta}{\varepsilon} < 6.2$

* If we, like HUON LI, had considered a smoother bed ("smooth" in the physical sense) such as a sheet of polished glass for instance, we might probably have reached hydraulically smooth conditions. The results obtained with a metal plate and smooth cement seem to indicate that the Reynolds number $\omega^{1/2}\, d_1\, \nu^{-1/2}$ characterising transition for a hydraulically smooth bed would not have been less than 800, from which the Reynolds number $\omega^{1/2}\, d_1\, \nu^{-1/2}$ for transition on an hydraulically smooth bed would appear to be at least 800..

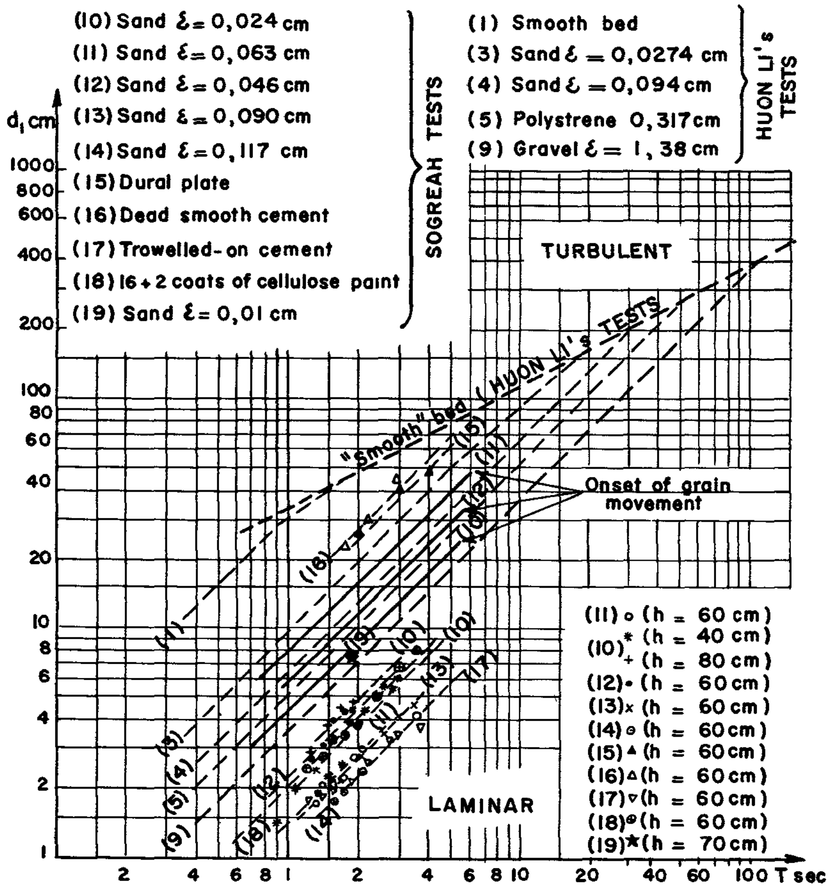

Fig. 3. Onset of turbulence in the oscillatory laminar boundary layer in progressive waves.

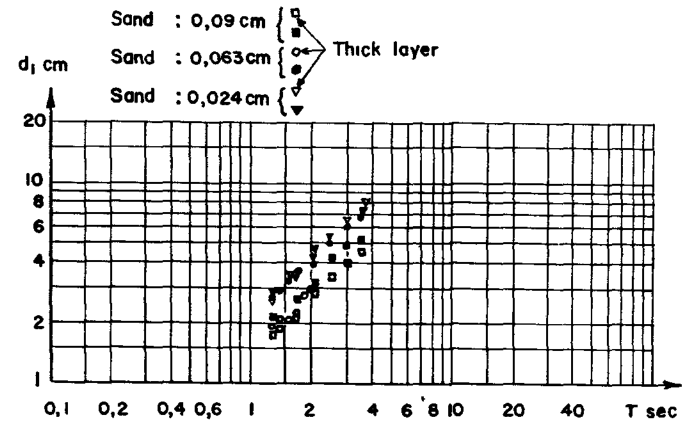

Fig. 4a. Influence of bed porosity on the onset of turbulence.

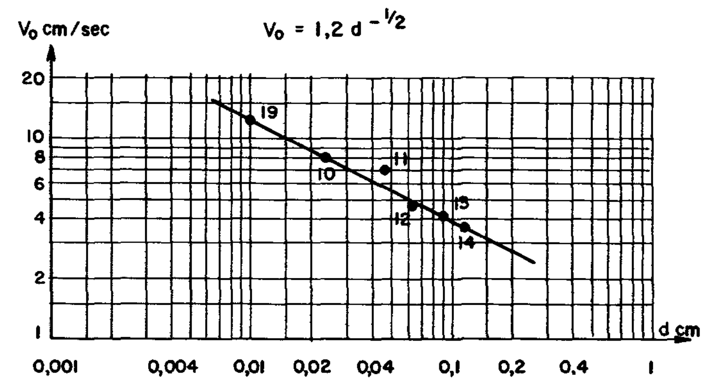

Fig. 4b. Relationship between V and d (sand

Although it is quite possible that a slight difference between the transition criteria adopted at BERKELEY and GRENOBLE could to some slight extent explain the discrepancies observed, it is more likely that they are due to the different experimental methods used. If, as an experiment, we plot the values of d_1 at which the *onset of grain movement* in the bed material occurs on the transition characteristic curve $d_1 = \psi$ (T) for the three sands tested ((10) ; (11) and (12)), we find that the value of d_1 at which the grains begin to move is itself lower than that used by HUON LI for characterising transition. And yet, with these materials, movement was already beginning to occur, although the boundary layer showed every sign of being turbulent.

Summing up therefore, we would say that the values obtained at BERKELEY appear to exaggerate the importance of the laminar conditions.

EFFECT OF BED POROSITY ON THE ONSET OF TURBULENCE

The tests carried out with sands (10), (11) and (13) were repeated, but this time with a " thick " layer (thickness of bed material layer 5 cm) and a " thin " layer (only a few grains thick, with the bottom layer grains stuck to an impervious smooth surface). The results of these tests are shown on the graph in Fig. 4a, from which it is seen that bed porosity facilitates the onset of turbulence to some extent. The influence of bed porosity however becomes very small for sands with a grain size below 0.024 cm.

EFFECT OF GRAIN SIZE

Neglecting the thickness of the bed layer, the grain diameter d, and V_0, are the main characteristic transition parameters. According to our test results, a relationship of the form $V_0 = 1.2\, d^{-1/2}$ (Fig. 4b) can be defined between these two parameters.

II - ONSET OF GRAIN MOVEMENT

Considering the evolution of movement in the immediate vicinity of the bed from the appearance of the laminar boundary layer to the instant at which well defined solid transport appears, the following phenomena can usually be observed :

1) Development of the oscillatory laminar boundary layer

2) Appearance of turbulence in the boundary layer

3) Initial setting in motion of first grains of material* (the first to be transported are often the coarsest of the bed materials)

* It may nevertheless very well occur thet the onset of grain movement precedes the appearance of turbulence if the material is very fine. According to our own experimental results, this possibility seems to arise at periods of about 2 seconds when $\varepsilon \leqslant 0.01$ cm. This comment is directly inspired by the results shown in Fig.3 and Fig.5, where it is seen that, at a given wave period, for transition, d_1 decreases as d increases, whereas in the case of nascent grain movement d_1 increases with d.

4) General grain movement

5) Appearance of riffles, which are subsequently regularised.

6) Sediment transport with characteristic conveyance and saltation in the direction of wave propagation.

7) Slow progression of turbulence towards the mass of the fluid, from the bed towards the surface.

8) First signs of transport in suspension in the lower part of the fluid mass and in the opposite direction to that of wave propagation.

9) Lengthening and gradual disappearance of riffles.

a) Earlier Results

As far as we know, R.A. BAGNOLD (3) was the first to tackle this problem by studying the setting in motion of grains of material in a layer of uniform thickness resting on a cylindrical surface oscillating in a mass of still water.

Tests in progressive waves were carried out in 1954 at the Lille Institute of Fluid Mechanics by MM. MARTINOT-LAGARDE and FAUQUET (6) (7).

In the United States, Arthur T. IPPEN and Peter S. EAGLESON (7)(8), and later Madhar MANOHAR (2), have examined the problem during very recent years. Since then, MM. LARRAS (9) and VALEMBOIS have studied the effects of waves and clapotis on sandy beds, and as a result of these tests, M. LARRAS has been able to obtain a relationship between V_0, T and W to characterise the onset of movement.

b) Criterion for the onset of movement

Experience shows that it is essential to state the criterion for the onset of movement very clearly when referring to nascent movement of a material in a moving bed ; this is all the more necessary when comparing experimental results obtained by a number of different methods.

BAGNOLD, and apparently MANOHAR too, observed the oscillation characteristics of the plate carrying the test material when the first grain started to move.

FAUQUET observed that the onset of entrainment of a few grains a smooth bed (glass) was characterised by the displacement of a least half the grains.

During our tests, the onset of movement was also characterised by displacement of the first grains. This was borne out later, when, by studying sediment transport proper along the bed, we were able to deduce at what hydraulic characteristics the solid discharge is near or at zero, with the aid of the curve defining the solid discharge Q in terms of hydraulic characteristics.

These definitions are still imprecise in some cases ; the onset of movement occurs in two distinct stages in the case of some of the very fine sands, as follows :

- Initial movement without riffling (the only case considered here), followed by spontaneous riffle formation before any appreciable sediment transport occurs.
- Initial movement with riffle formation at a fairly appreciably lower velocity at the bed. (Here, the onset of grain movement is definitely facilitated by the increased turbulence caused by the riffles).

c) Materials investigated

BAGNOLD used steel, quartz and coal grains.

FAUQUET used sand of various grain sizes as well as spherical limestone grains.

MANOHAR used various grades of sand, polystyrene and glass spheres.

We ourselves used two grades of granulated pumice and pollopas (granulated plastic) and sand in various grain sizes, their specific gravities being 1.38, 1.46 and 2.65 respectively (specific gravity of effectively transported grains).

d) Experimental results obtained at SOGREAH

The graph in Fig. 5, which is plotted on a logarithmic scale, shows the values of d_1 for various values of T at which the onset of grain movement occurs. (d_1 is the theoretical value for the total travel of a particle in the vicinity of the bed, not the observed value.)

Within the range considered, d_1 (T) can be correctly expressed by a relationship of the following form :

$$T^{-1} d_1 = \psi = \frac{V_0}{\pi}$$

where ψ is a function depending solely on the material and the fluid (kinematic viscosity)

For a given material, the relative depth h/L , i.e. the depth of water, clearly plays a certain part.

This " dispersion " can either be due to the fact that the theoretical value d_1 only provides an approximate value for the total effective travel of the fluid particle near the bed (recent tests have shc discrepancies depending on h/L) or that, in general d_1 and T cannot t themselves fully characterise the onset of grain movement in a given mater (It can been observed, for instance, that V_0 increases slightly with T for a given material and fluid). This can be explained by the fact that the thickness of the boundary layer also increases with T.

We shall see later that the maximum orbital velocity near the $V_0 = \pi d_1 T^{-1}$ (first order of approximation) is only an approximate value of the true velocity.

e) Influence of the physical characteristics of the grains

It would be misleading to try do derive a relationship between the velocity at which the grains begin to move and the physical characteri of the grains from a set of insufficiently complete results. There are ma reasons for saying this, chiefly that it is very difficult to characterise grain of material, whatever its shape, by a simple relationship, and furth because of the influence of local turbulence on grain behaviour, which is usually difficult to characterise ; these reasons hold good, however many test results may be available.

We shall therefore merely give a few comments on our results, resisting the temptation to consider certain applications and compare the results with those already published (particularly in (2,3,7 & 9)), then deducing a relationship between the physical characteristics of the grains and the depth of water and the characteristics of the waves setting them in motion.

MATERIAL	d cm	$V_0 + \bar{u}$ cm/sec	W cm/s	$\frac{V_0 + \bar{u}}{W}$ *
Sand N° 1	0.063	28.50	9.4	3.03
Sand N° 2	0.046	20.50	6.0	3.42
Sand N° 3	0.024	16.50	3.3	5.00
Sand N° 4	0.010	12.50	0.8	15.60
Pumice N° 1	0.160	11.00	8.9	1.24
Pumice N° 2	0.120	8.50	7.3	1.16
Poliopas N° 1	0.110	10.50	5.6	1.86
Poliopas N° 2	0.039	7.00	2.1	3.30

Using the results shown above, we have plotted $V_0 + \bar{u}/W$ agains d on logarithmic scales (Fig. 6).

* $\bar{u}$ being the mean velocity of an entrainment current acting on the fluid particles in the immediate vicinity of the bed.

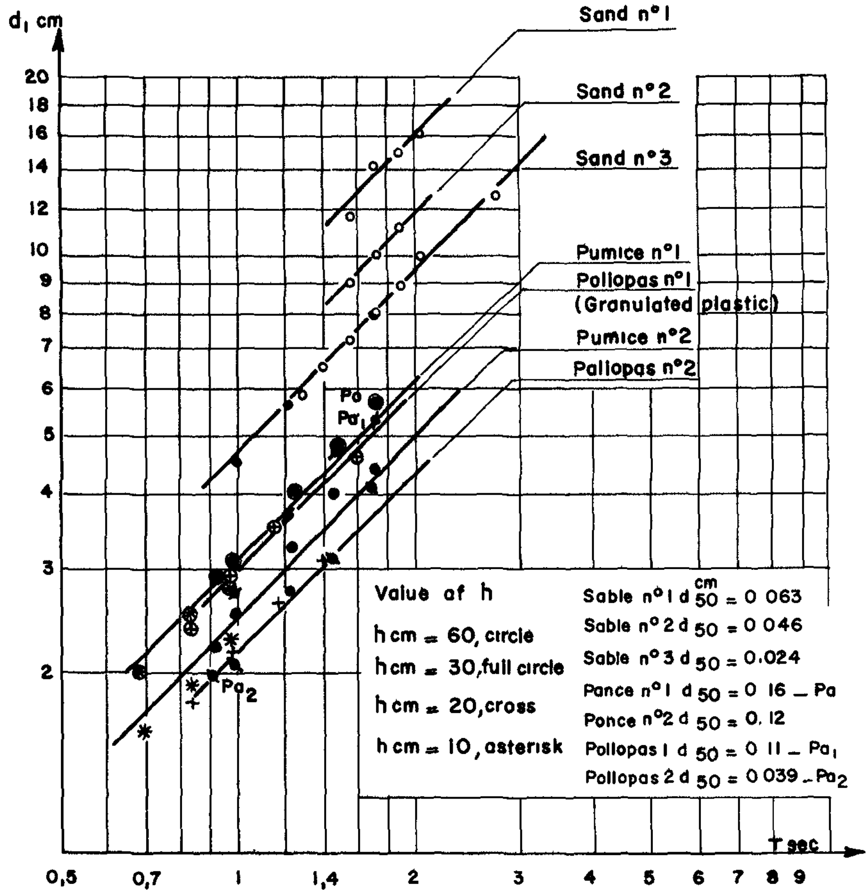

Fig. 5. Onset of grain movement.

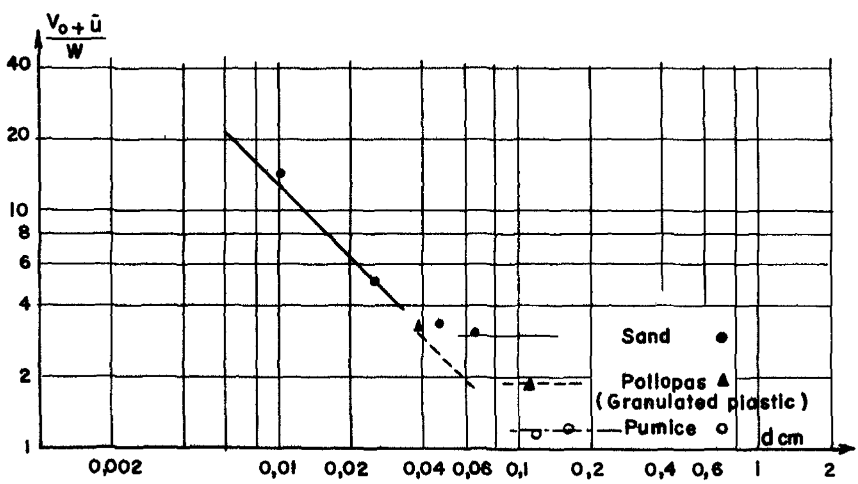

Fig. 6. Graph of $\frac{V_o + \bar{u}}{W} = f(\tilde{g}, d)$

A relationship of the form $\frac{V_0 + \bar{u}}{W} = F(d^m)$, can be found for a material of a given specific mass, with m varying as d. This result can be compared to M. VALEMBOIS' comments on the results obtained by M. LARRAS in connection with the influence of d/d_0.

At low d/d_0 i.e. when the grain of material is practically completely enclosed by the oscillatory laminar boundary layer at the onset of movement, $\frac{V_0 + \bar{u}}{W}$ appears to assume the form $\frac{V_0 + \bar{u}}{W} = \frac{k}{d}$ for a materi with a given specific mass and for a given fluid (water). However, when d/d_0 reaches values at which the grain of material is no longer protected from the upper flow by the laminar boundary layer (either because the latte is relatively too thin or partly turbulent) $\left(\frac{V_0 + \bar{u}}{W}\right)$ tends to become independent of d.

For a given fluid, the relationship between $\frac{V_0 + \bar{u}}{W}$ and d therefore generally seems to depend on the relative value d/d_0, or, final on T. With a *natural* sandy bed, where the flow in its vicinity should usually be turbulent, $\frac{V_0 + \bar{u}}{W}$ would be independent of d, depending rather to some extent on $\hat{g}$, as indicated by Fig. 6.

f) Turbulent state near the bed at the onset of grain movement

The setting in motion of the grains obviously approximatively satisfies a relationship of the form $d_1 T^{-\frac{1}{2}} = \psi$, i.e. a similar relationship to that characterising transition in the boundary layer.

By analogy with HUON LI's " transition" Reynolds number, we ca define a " setting off " Reynolds number of the form $\omega d_1 \varepsilon/\nu$, which is, on the average, a constant for each degree of roughness.

$$\frac{\omega d_1}{\nu}\varepsilon = 160 \text{ (pumice n° 2) } \varepsilon = 0.12 \; ; \; \frac{\omega d_1}{\nu}\varepsilon = 55 \text{ (pollopas n° 2) } \varepsilon = 0.039$$

$$\frac{\omega d_1}{\nu}\varepsilon = 300 \text{ (pumice n° 1) } \varepsilon = 0.16 \; ; \; \frac{\omega d_1}{\nu}\varepsilon = 165 \text{ (sand n° 2) } \varepsilon = 0.046$$

$$\frac{\omega d_1}{\nu}\varepsilon = 70 \text{ (sand n° 3) } \varepsilon = 0.024 \; ; \; \frac{\omega d_1}{\nu}\varepsilon = 305 \text{ (sand n° 1) } \varepsilon = 0.063$$

The " transition " and " setting off " Reynolds numbers are similar for the materials considered, although the former is lower than the latter. This tends to confirm the important effect of the onset of turbulence in the vicinity of the bed on the setting in motion of grains with $d > 0.01$.

The oscillatory boundary layer is usually already turbulent when the grains are set in motion (although this is not necéssarily the case if $\varepsilon < 0.01$ cm)

In real life, the thickness of the laminar boundary layer can generally be expected to be greater than that observed during our tests, since it varies as $T^{1/2}$.

For a given fluid and material, the real life and model δ/ε will not be the same ; the real bed will be " smoother ". On the other hand, since V_0 is usually greater, the boundary layer will be turbulent in the majority of cases.

III - ENTRAINMENT VELOCITY IN THE IMMEDIATE VICINITY OF THE BED

MAXIMUM FLUID PARTICLE VELOCITY IN THE IMMEDIATE VICINITY OF THE BED AT THE ONSET OF GRAIN MOVEMENT

We have seen that the expression $d\,T^{-1} = V_0$ approximately characterises the setting in motion of grains of a given material. In fact, the maximum resultant velocity of a fluid particle in the immediate vicinity of the bed differs slightly from the theoretical value V_0 characterising the velocity of a fluid particle on the bed in a perfect fluid (first order of approximation).

In a recent calculation, LONGUET-HIGGINS (10) has shown that, in a viscous fluid in a laminar state and in the immediate vicinity of the bed (or more exactly, in the boundary layer) progressive waves were accompanied by an entrainment current carrying in the direction of propagation.

The existence of such an entrainment current has been confirmed by flume tests, during which it was observed that, taken by and large, the fluid particles in the immediate vicinity of the bed moved in the direction of wave propagation.

The mean maximum value of this entrainment current has been given the symbol $\bar{u}$.

In this way, the velocity of the fluid particles acting on the grains of material in the immediate vicinity of the bed can be considered as the resultant of the two following velocities :

- The entrainment current accompanying the progressive waves
- The pulsating component, a resultant of the general oscillatory motic

Immediately near the bed, the maximum values of these two velocity components add algebraically, both being parallel to the bed ; furthermore, they are horizontal in this case.

a) Value of entrainment current in the immediate vicinity of the bed

According to LONGUET-HIGGINS's calculations, which have been confirmed by tests as we shall see, the maximum value of the entrainment current in progressive waves is given by an expression of the following form : (page 568)

$$\bar{u}_{max} = \frac{1.376\ T}{L} \left(\frac{2\pi a}{T}\right)^2 \frac{1}{sh^2 \frac{2\pi h}{L}}$$

or, in other words :

$$\bar{u}_{max} = \frac{1.376\ T}{L} V_{o\ max}^{2}$$

* As a first order approximation, the movement appears to be symmetrical ; at a given point, the velocity of a fluid particle in the direction of propagation (passing a crest) is equal to the velocity in the opposite direction (passing a trough).

As a second order, which is closer to real conditions, V(t) for a given particle is no longer symmetrical, the maximum velocity of the particle passing a crest being greater than its velocity passing a trough. We also know that a mass transport current can then occur.

In real waves and in a finite depth, tests confirm that the wave orbits are no longer symmetrical at a certain distance from the bed. If the relative depth h/L is small and the waves very steep fronted, a double asymmetry can be clearly distinguished on the orbit (vertical and horizontal asymmetry). On one orbit, the curvature is more pronounced at the upper part than at the bottom (Fig. 8a) and the orbit is not closed except at a certain distance above the bed, where the vertical disymmetry disappears.

This double dissymetry shows up on the characteristic curve for the velocity of a fluid partic along an orbit ; the shape of the curve differs quite appreciably from the sinusoidal (Fig. 8t

This difference in the velocities of the particles travelling in the direction of propagation and in the opposite direction tends to favour sediment transport towards the shore.

This difference becomes more marked as h/L decreases, i.e. as L increases at constant depth. In the limit, the waves are often comparable to solitary waves.

With shallow beds, therefore, considering the overall effect of the entrainment current and th dissymetry of the orbital velocities in the direction of propagation and against it, long shallow-fronted waves can apparently cause quite considerable shoreward sediment transport.

This would tend to explain why " fair weather waves " (long, shallow waves) that cannot cause powerful rip currents or any appreciable seaward transport of suspended solids, are particular likely to cause accumulations on beaches.

where $V_{o_{max}}$ is the maximum theoretical value of the horizontal component of the orbital velocity (first order, perfect fluid).

The entrainment current measurements carried out in the immediate vicinity of the bed during our tests confirm this expression with fair approximation. They are plotted on Fig. 7 for indicative purposes, which shows that the difference between the observed velocity and the theoretical value given by the LONGUET-HIGGINS formula is generally small at laminar boundary layer flow conditions (range of validity of the theory). This discrepancy increases as turbulence develops.

b) Maximum velocity of fluid particles in the immediate vicinity of the bed at the onset of grain movement

The resultant of the velocity of the fluid particles near the bed finally takes the form shown by Fig. 8c.

Let U_d characterise the *velocity at the onset of movement* of a given grain. The grain begins to move when the maximum value of the absolute velocity of a fluid particle in the immediate vicinity of the bed becomes at least U_d, with respect to a stationary grain of material.

As a second order approximation (perfect fluid) and in the immediate vicinity of the bed, this can be considered valid at the upper boundary of the boundary layer, the maximum velocity of a fluid particle being of the following form :

$$V_o \pm [\text{a second order term of the form } V_o^2 f(h,L)]$$

The velocity $\bar{u}$ of the LONGUET-HIGGINS entrainment current (2nd order in the boundary layer) adds to this " periodic " term for a viscous fluid.

At the onset of movement we can neglect the second order term between the square brackets and say that the grain begins to move when :

$$U_d = \bar{u} + V_o$$

We have calculated the respective values of V_o, $\bar{u}$ and $(V_o + \bar{u})$ for the various materials considered. Fig. 9 shows the values of $\bar{u} + V_o$ for various values of L for seven different materials.

$V_o + \bar{u}$ is clearly practically constant, at the *onset of movement* of grains of a given material, irrespective of the factors giving rise to these velocities (we have already seen however that V_o increases slightly with T and explained it by the increase in the thickness of the boundary layer).

The means of the resultant initial velocities are as follows, for the cases considered :

- sand n° 1 $(d_{50} = 0.063 ; \rho_s = 2.65)$; $V_o + \bar{u} = 28.5$ cm/sec
- sand n° 2 $(d_{50} = 0.046 ; \rho_s = 2.65)$; $V_o + \bar{u} = 20.5$
- sand n° 3 $(d_{50} = 0.024 ; \rho_s = 2.65)$; $V_o + \bar{u} = 16.5$
- pumice n° 1 $(d_{50} = 0.160 ; \rho_s = 1.38)$; $V_o + \bar{u} = 11.0$
- pumice n° 2 $(d_{50} = 0.120 ; \rho_s = 1.38)$; $V_o + \bar{u} = 8.5$
- pollopas n° 1 $(d_{50} = 0.110 ; \rho_s = 1.46)$; $V_o + \bar{u} = 10.5$
- pollopas n° 2 $(d_{50} = 0.039 ; \rho_s = 1.46)$; $V_o + \bar{u} = 7.0$

A quick comparison between these initial movement velocities for a given material in progressive waves and permanent flow conditions shows some variation. The initial movement velocity appears to be considerably lower at progressive wave conditions ; however it is not always easy to determine the velocity near the bed at permanent flow conditions.

Furthermore, in view of the form of the resultant velocity in the immediate vicinity of the bed, the trajectory of a grain of material in the direction of propagation of the waves will be greater than in the opposite direction. Once in motion, the grain progresses by successive forward and backward bounds, the former being greater than the latter (Fig. 8d), with the net result that, on an horizontal bed, the grains progress in the direction of wave propagation.

c) Simultaneous action of a permanent current and the waves on a bed of materials

It often occurs in practice that the action of the waves on the bed is superimposed upon that due to ordinary currents. The waves can eith run in the same direction as the current or obliquely across it, or even in the opposite direction. This is often the case for instance near a flat tidal coast or just off an estuary.

Out at sea, the ordinary currents are seldom strong and usually by themselves insufficient to dislodge the bed materials. This does not apply however if the materials have already been set in motion or suspensio by the waves ; in such cases, even the smallest ordinary current can cause quite appreciable solid transport.

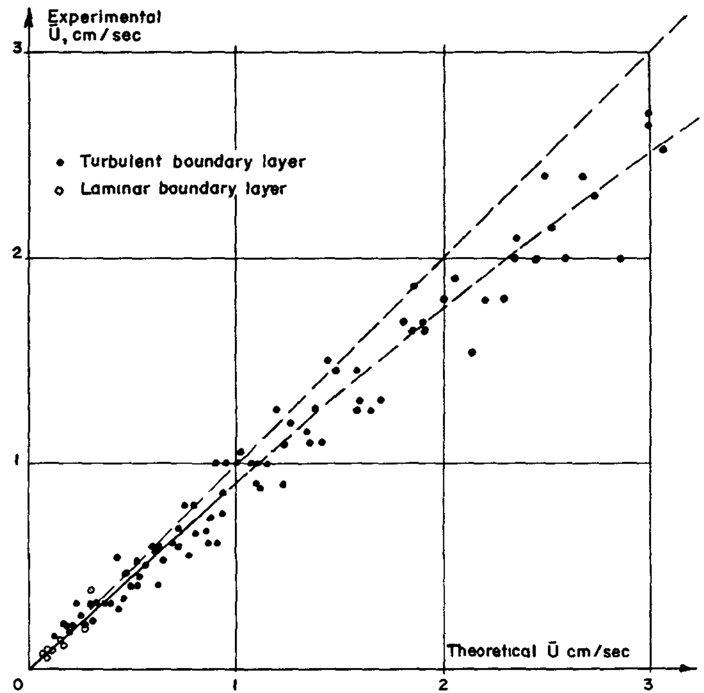

Fig. 7. Maximum velocity of entrainment current in the immediate vicinity of the bed. Comparison between experimental and theoretical values (Longuet-Higgins).

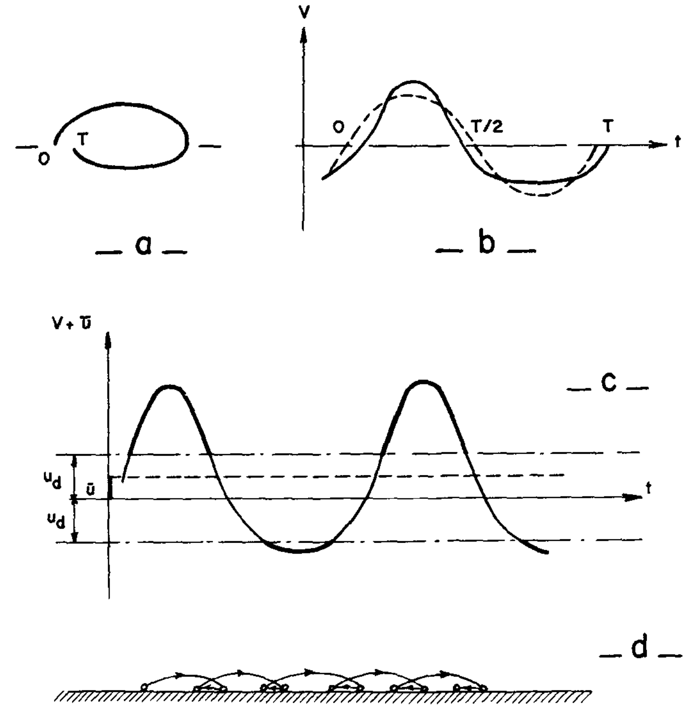

Fig. 8

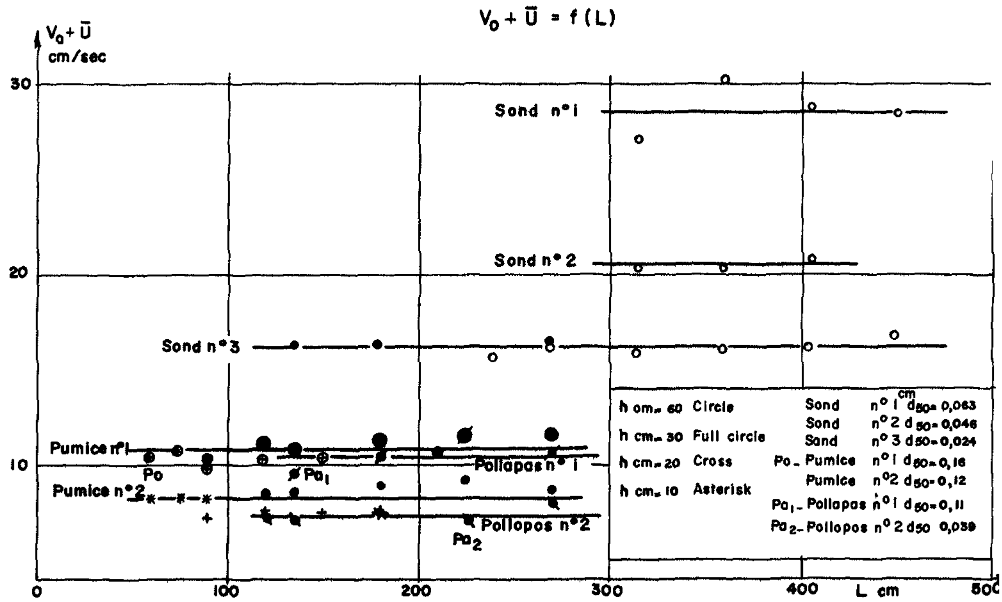

Fig. 9. Velocity at which grains begin to move.

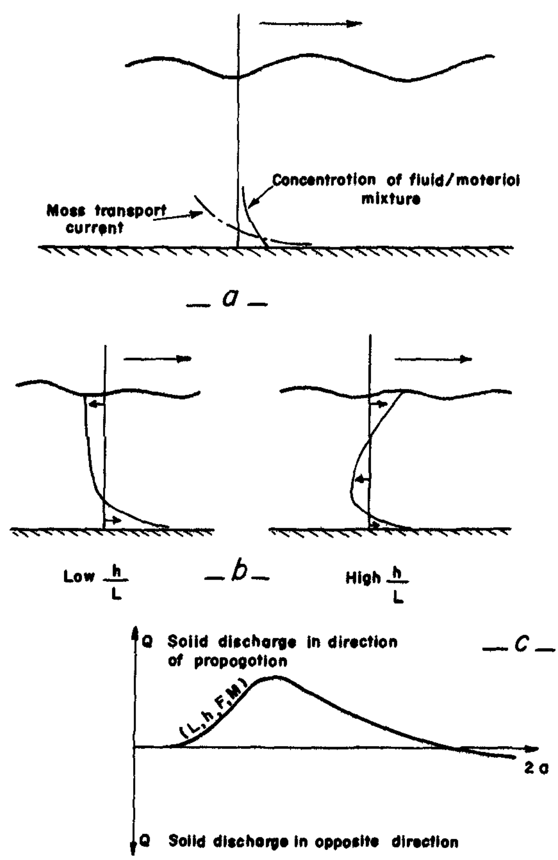

Fig. 10

The effect of the waves when superimposed on the current is either to considerably reduce the critical entrainment tractive force for the material, or, as we have already seen, to cause the grains to begin to move.

It is often most desirable to study the combined action of waves and ordinary currents ; tests have been recently carried out at SOGREAH on this subject.

IV - SEDIMENT TRANSPORT AS AFFECTED BY WAVES ALONE

Due to the fact that, particularly in the immediate vicinity of the bed, the resultant of the velocities in the direction of propagation is greater than that acting in the opposite direction, the onset of grain movement is accompanied by *real solid transport.*

a) Mass transport current in a pure single-period wave

It can be seen during flume tests that, for two-dimensional waves, the oscillatory movement of the particles in the fluid mass is accompanied by a general movement.

The vertical distribution of the velocities of this transporting current has the following characteristics :

- Near the surface, the current carries in the direction of wave propagation ; a carrier current in the opposite direction has been observed at low values of h/L during tests.

- Near the bed, the current always conveys in the direction of propagation and is particularly pronounced in a relatively thin zone in the immediate vicinity of the bed (entrainment current).

- In between these two zones the current flows in the opposite direction ; in a closed flume, where the mean discharge is zero, the discharges are equal in both directions.

Two typical distributions for this current, both of which were observed during closed flume tests, are shown diagrammatically in Fig. 10b.

* This mass transport currant appeers in the equetions for waves in a perfect fluid, from the 2nd order onwards, but the vertical velocity distribution is arbitrary.

Currents of this kind have been produced in test flumes, especielly by CALIGNY (1878), MITCHIM (1939), BAGNOLD (1947), KING (1948), and the LABORATOIRE DAUPHINOIS D'HYDRAULIQUE (1949)

Such currents may well occur at sea, providing the waves are sufficiently regular ; they may be caused by friction et tha interfeces which, as is well known, has a definite value in a real fluid.

b) Inversion of the overall solid transport on a horizontal bed

As we have seen, flume tests show that, in general, for a practically pure wave on a horizontal bed, the mass transport travels in t direction of wave propagation.

However, at certain conditions, the overall solid discharge, i.e. the solid discharge integrated across a vertical and not only that in the immediate vicinity of the bed - begins to increase with wave amplitude passes through a maximum, decreases and then, given certain particular circumstances, may invert.

This phenomenon occurs in the test flume at high turbulence, with the layer of suspended material near the bed reaching the intermediate layers of the fluid in which the transporting current is directed out to sea.

Similar effects still occur if the waves are not pure, i.e. in the case of partial clapotis ; in any case, the vertical distribution of th transporting current appears to be highly sensitive to the "purity of the waves ".

The overall solid discharge may therefore flow either with or against the direction of propagation, depending on its relative values in both directions, i.e. the distribution u_z (resultant of the velocity) and $c_m(z)$ (mean concentration of material).

Functions $\bar{u}$ and c_m usually take the form shown in Fig. 10a in the considered zone.

Lastly, the curve for the overall solid discharge Q plotted against 2a can take the diagrammatic form shown in Fig. 10c in certain cases.

It is easy to imagine the value of such results, which, togeth with density currents, provide an apparently satisfactory explanation for t existence of seaward movements of solids in a given profile.

c) Attempt at establishing an empirical relationship from the test results

In the course of our consideration of wave action on an horizontal bed in this article, we have brought a number of parameters exerting a direct influence on the solid discharge into evidence, such as the orbital velocity of the particles, entrainment currents, turbulence, et

Once sufficiently far out to sea beyond the breaking zone most of these parameters are usually insufficient in themselves to cause sedimer transport and can at most produce favourable conditions for it ; in the absence of an ordinary marine current one might think that only the entrair ing current could transport sediment effectively. The position is generall different in *shallow* zones, where the orbital velocity is appreciably dissymmetrical, or in *very shallow zones* where the oscillation wave can take the form of the translation wave that produces a considerable difference in velocity when of the wave crest or trough is passed, particularly near the bed.

This being the case, one might expect to be unable to find a simple law relating sediment transport to the hydraulic characteristics on one hand, and to the characteristics of the materials on the other, this particularly in the zones considered as shallow for the wave length considered.

Establishment of the relationship $Q_{sol} = f(V_0)$

We have seen that the expression $d_1 T^{-1}$, i.e. V_0 (maximum orbital velocity, 1st order) is a fundamental parameter characterising the onset of movement of a given material. V_0 plays an analogous part with respect to sediment transport solely caused by wave action.

In order to illustrate the principal part played by this factor, we have plotted, as an exemple, the $Q_{solid} = f(V_0)$ relationship on the graph in Fig. 11 for a medium-fineness sand, very fine sand, and pumice. We have purposely chosen two materials forming bed riffles and suspension to a varying degree (sands) and a third material not causing either, at least within the fairly wide range of hydraulic conditions considered (pumice).

These results show that :

1) For a given material and fluid, the solid discharge not only depends on V_0 , but also on the depth of water h and the period T of the wave (or the wave length L). At a given V_0, Q_{sol} increases with h and with decreasing T (this apparently contradictory result shows up the effect of V_0 very well).

2) For each material (L and h being known), Q_{sol} is expressed by a relationship of the form $Q_0 = K\, V_0^m$, with m not independent of V_0 ; at all values of L or h, m changes at a definite value of V_0 ,i.e. at a certain value of a Reynolds number $2\, V_0\, \varepsilon/\nu$ (or $\omega d_1\, \varepsilon\nu$) that can characterise the turbulence around the grain.

Since the effects are the same for both materials, one producing riffles and suspension and the other neither, one might be tempted to think that turbulence near the bed plays an important part during the first stages of sediment transport.

In actual fact, sediment transport caused by waves apparently divides into three distinct phases, as follows :

1) - Slight turbulence near the bed, with transport caused solely by bed load transport.

2) - Medium turbulence, with transport chiefly due to bed load transport and suspension ; the latter does not however extend farther than the fluid layers in which the mass current component acts in the opposite direction to the wave propagation.

3) - Strong turbulence, transport caused by bed load transport and suspension, the latter extending for quite a height above the bed We have seen that, the *overall* sediment transport is not necessarily very large in this case.

With the exception of gales, during which suspension can reach a sufficient height above the bed to show up in the zone in which the mass current runs in the opposite direction to the wave propagation, the intermediate condition is the one most frequently encountered in real life.

The essential characteristic of transport during the first and second stages is that the solid discharge proper varies very rapidly with V_0. With fine sand, for example, $Q_0 \# K\, V_0^6$ in the range observed during our tests and corresponding to the second stage referred to above (Q_0 vari very little with h and T, at constant V_0, in this particular case).

Solid discharge in a laminar or slightly turbulent medium

When the flow near the bed is laminar or slightly turbulent, the entrainment current seems to be the main driving element in the transport pattern, acting almost as a continuous current, with the superimposed oscillatory movement merely facilitating the transport. The following reasoning can be applied in this case :

The velocity gradient through the boundary layer will, if assumed constant, create the following tangential force in the boundary layer :

$$\tau = \mu \frac{\partial u}{\partial y} = \mu \frac{u\delta}{\delta} = \mu \frac{Cte\ T}{L} V_o^2 \left(\frac{\nu T}{2\pi}\right)^{1/2} \ *$$

By analogy with the expressions given for solid transport in a permanent current, we can write :

$$q_s = A\, \tau\, (\tau - \tau d)$$

or, for a given fluid and material :

$$q_s = Cte\ f\ (h,L)\ .\ a^2\ (a^2 - a^2{}_d) \quad \text{where}$$

$2a_d$ characterises the wave amplitude for the onset of motion.

Plotting :

$$q_s = f\ [a^2\ (a^2 - a^2{}_d)]$$

* $u_{\delta_1} = \frac{Cte\ T}{L} V_o^2$ (LONGUET-HIGGINS) and $\delta_1 = A\sqrt{\nu/\omega}$

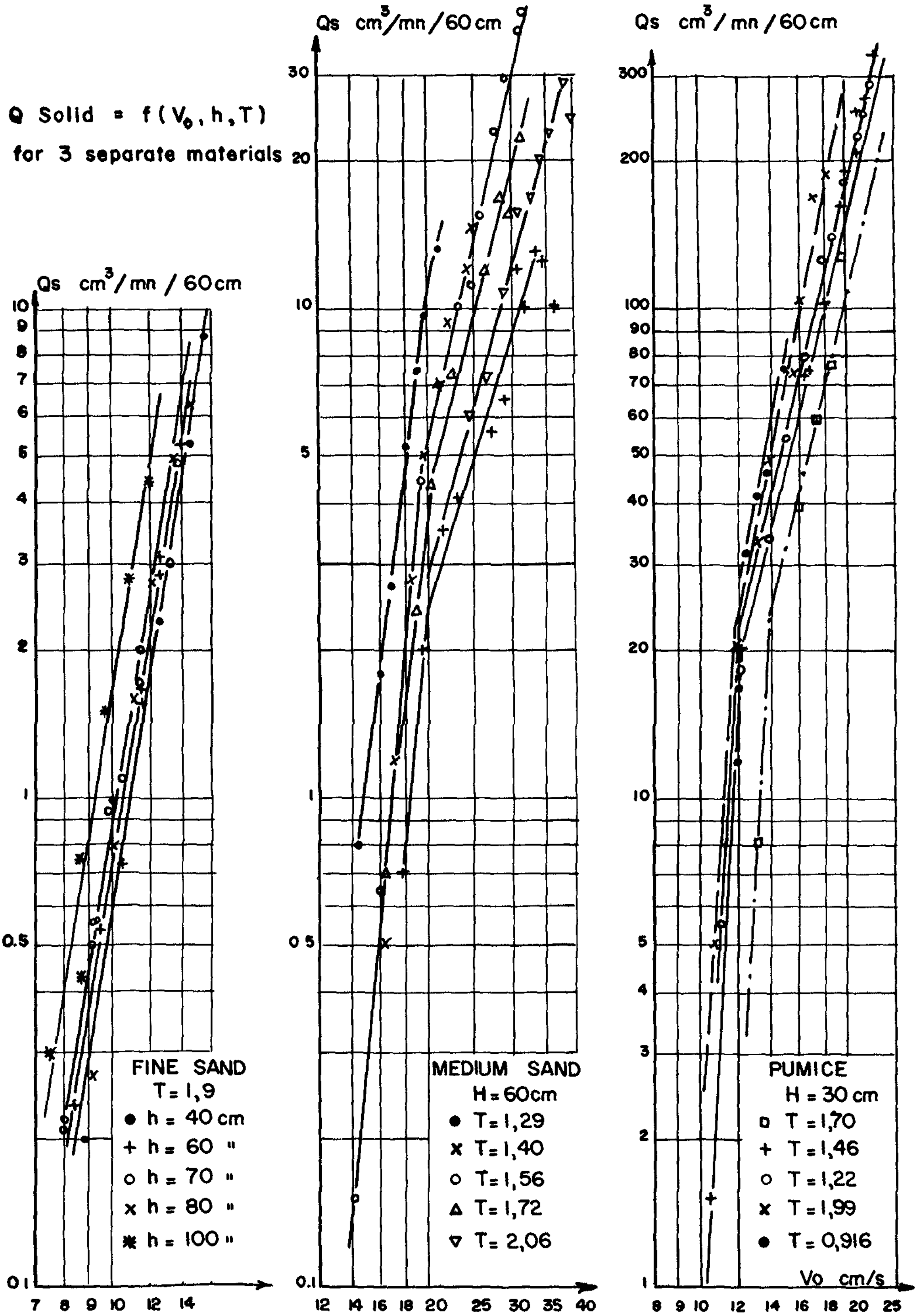

Fig. 11. Wave-generated solid transport on an horizontal bed.

we find that the form of q_s is as follows :

$$q_s = K\, a^2\ (a^2 - a^2_d)$$

For a given fluid and material, K is a constant for each value of h and L (or T), so that we can express q_s as follows :

$$q_s = a^2\ (a^2 - a^2_d) \times \varphi\ (M,\ h,\ L) \qquad (++)$$

where M is the characteristic of the material.

This expression is very well confirmed for the majority of the results we have obtained in a relatively low turbulence range.

This reasoning is strictly speaking only theoretically valid for laminar conditions, but still appears to remain applicable in cases where the flow near the bed is only slightly turbulent, an assumption that seems to be borne out by the results.

At this stage of the solid transport, q_s apparently has the following form :

$$q_s = F\ (\text{Material})\ 2\ a^2\ (2a^2 - 2a^2_d)\ L\ h^{-4}$$

where q_s is a solid discharge per unit width (in this case unit width is that of the test flume, i.e. 60 cm) its dimensional form being $L^2\ T^{-1}$. The materials function therefore has the dimensions of a velocity (e.g. falling velocity).

V - INDIRECT EFFECT OF WAVES ON THE BED

The foregoing investigation clearly shows that, near the bed, the waves transport materials in the direction of their propagation, i.e. towards the coast in real life. In this zone, the shallower the water and the finer and lighter the material, the greater generally the sediment transport in the direction of propagation, for a given wave type. One might be led to believe therefore that the materials travel indefinitely towards the coast, but in fact, the reverse very often happens, the material eroded from the coast being carried far out to sea and dispersed.

We have seen during this investigation that the overall solid discharge on a vertical line could be zero or even invert ; we have attributed this inversion of sediment transport to the leading part sometimes played by the seaward component of the transporting current.

However, we think that there are other reasons besides (not including the obvious effect of gravity, particularly near the shore where the bed slope is sometimes quite steep).

To the previously mentioned action of the waves on the bed, i.e. their capacity for developing density currents carrying the finer particles seawards (see beginning of this article) we can add the irregularity of the natural waves, particularly during gales. It was very noticeable during our tests how much the form of the transporting current for instance could be affected by a partial reflection or an irregularity in the period.

The sediment movements out to sea can also be favoured by the sometimes violent action of the surface wind, which can, particularly during a gale, set up quite an appreciable shoreward current at the surface that moves materials near the bed out to sea.

The sometimes violent currents appearing in the surf area also play a part that cannot be neglected on sediment transport out to sea, if only by creating suitable conditions for the development of density currents. There is every reason to believe that, by virtue of the principle of continuity, these " littoral currents " integrate intimately into the system of currents outside the breaking zone and which we have particularly stressed here.

Seaward sediment transport by density current

Experience shows that the waves produce density currents, similarly to flow by gravity.

When the waves are high (e.g. gale conditions), a considerable quantity of material can be dislodged from the bed and put into suspension in the breaking zone. Obviously, the finer the materials concerned and the more recent the origin of the materials on the beach subjected to the waves, the more pronounced this effect will be (Density currents chiefly consist of solid particles of approximately 50 µ or less, which are themselves able to entrain larger particles by virtue of their nature.

Because it is relatively denser than the surrounding medium, water carrying sediment sinks to the bed where it then flows towards the greater depths.

Tests (11) carried out in a wave flume show that the flow of the turbid mass towards the bed is accompanied by a counter-current of clear water in the upper fluid layers, so that the water at the surface remains clear.

In the case of waves, this phenomenon is complicated by the presence of currents linked to the very existence of the natural waves and effective right into the breaking zone.

It is probable, however, that the wave currents occurring in real li conditions only have a secondary effect upon the turbid currents. The density currents running seawards particularly tend to develop near the end of a gale, when the currents connected with the waves are in the proc of disappearing. These currents may at most delay the flow of the densit current, because, in the vicinity of the bed, they generally head towards the shore, thus directly opposing the development of the density current.

When a gale is dying out, therefore, the density current probably progressively gains an ascendancy over the mass transport current in the layers near the bed.

It is also probable that, during a gale, some of the finest particles may be able to reach the open sea, travelling in the intermediate fluid layer in which the seaward component of the mass transport current is active.

Finally, it would not be unexpected to find a fairly well-defined dividing line between cloudy and clear water a comparatively short way out from the shore. This is because the counter-current of the density curren the current due to wind action and the mass transport current caused by waves all tend to bring the clear water from the sea in towards the shore along the surface.

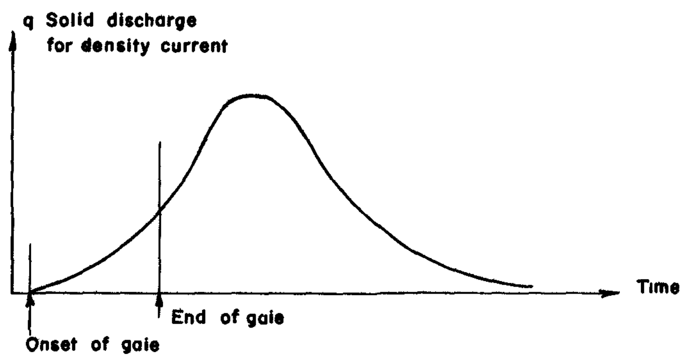

Fig. 12. Seaward movement of materials due to density currents.

Since turbulence in the breaking zone gradually decreases after a gale, the solid flood of the density current itself decreases, with a probable lag between the two events. Since the density current probably originates right at the beginning of the gale, it seems reasonable to draw the shape shown diagrammatically in Fig. 12 for the curve for the solid discharge carried out to sea by density current with respect to time.

This is why the bed materials become progressively finer further out to sea. Density currents, the progressive dying out of turbulence ... etc, contribute to this tendency for the finest materials to decant the farthest out from the shore. A correlating fact for this is that one only comes across comparatively few very fine particles on beaches, particularly after spells of bad weather.

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CHAPTER 21

A PULSATING WATER TUNNEL

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Purpose of Apparatus.

Since the basic mechanism of sand transportation in wave motion is so far unknown, there is a great need for observations of such transportation in large waves, especially, because of the possible difference of transportation in prototype wave motion from that in small model waves.

By means of the apparatus described in the present paper the water and sediment motion near the bed can be reproduced oı a prototype scale with the only modification that velocities at all points are in phase.

Principle of the Pulsating Water Tunnel.

The apparatus forms a U-channel consisting of two vertica risers (height 11 ft.) and a horizontal tunnel (length 51 ft.). The central part of the tunnel is a lucite test section, (Figs. 1 and 2)

By means of pneumatic machinery the water is made to perform oscillations in the U-channel with periods and amplitudes corresponding to the wave motion just above the bed of the sea. The water tunnel has a natural frequency (9 sec. period) close tc those of the larger motions to be produced, thus reducing the energy consumption to a minimum.

Bed material may be placed on the bottom of the test section, which has a width of 16 in. and a height of 12 in.

Range of Apparatus.

The apparatus can be applied to wave periods from 3 sec. upwards, and storm waves can be studied at full scale from 4 tc 10 sec. periods. Maximum horizontal amplitude in the test sectic is 23 ft. Thus, for instance, the motion of a storm wave of 10 period and a height of 20 ft. in deep water can be followed from deep water until a depth of 40 ft.

Problems that can be studied.

The pulsating water tunnel furnishes a means for the stud of numerous littoral drift problems, of which so far little or nothing is known, such as the maximum depth to which sediment transportation will occur, the mechanism of transportation and tl shape of the bottom (plane or rippled) at various depths for varic prototype materials. The amount of suspension may be measured

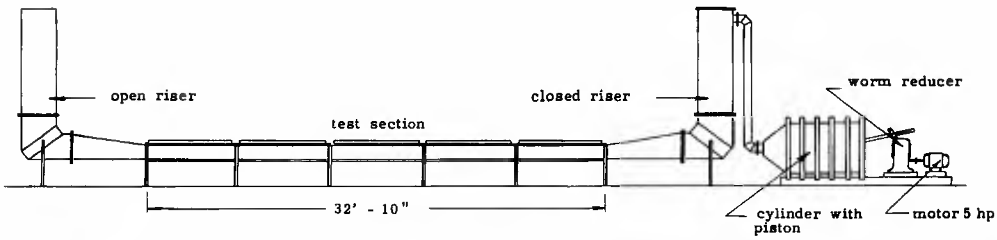

Fig. 1. U- Channel and drive system.

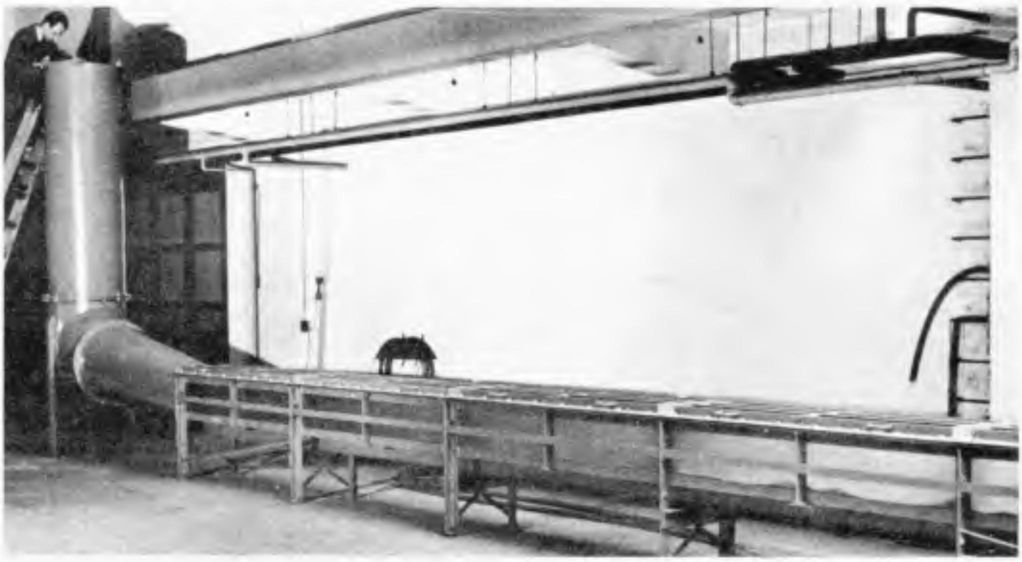

Fig. 2. View of the test section and the open riser.

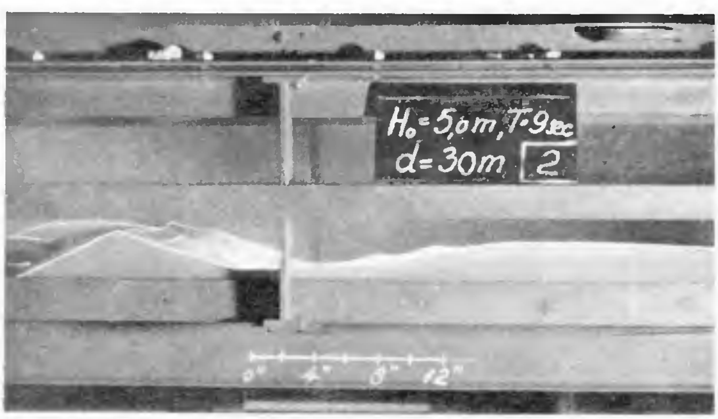

Fig. 3. Giant ripples formed 80 minutes after start of test.

and the character of the flow above the bed may be studied.

The fact that the apparatus does not reproduce the phas variation of the wave motion along the bed is probably of mino significance only because the amplitudes at the bottom are alwa small in comparison with the wave lengths.

Preliminary Results.

The apparatus has been in operation only for a few mon It has, however, been observed that under most prototype sto conditions the bed is covered by ripples of wave lengths from few inches in deep water (more than 130 ft.) to several feet in shallow water. At very large velocities the ripples are wiped (

Due to the ripples, suspended load occurs to much larg depths than usually anticipated.

The choice of a relatively low and wide cross section o the tunnel was based on the assumption that the ripples would wiped out at much smaller orbital velocities than now observec The giant ripple systems call for a much larger ratio of depth width of the cross section than applied in the present apparatu

It seems that the study of the prototype transportation pattern can be very helpful in selecting the proper material fo model investigation. Although the factors determining ripple lengths in model and in prototype are different, the ripple len in models being a function mainly of the grain size, it seems the eddy patterns are quite similar. The model material shoul therefore reproduce the prototype ripples approximately true to scale.

CHAPTER 22

SEDIMENT MOVEMENT AT SOUTH INDIAN PORTS

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INTRODUCTION

The movement of sedimentary matter along the coastal regions of the land has always been a problem in coastal and harbour engineering. Erosion and accretion of the shore and the sea bottom and the silt charge from the rivers discharging into the sea contribute the necessary sediment that moves along the coast.

Coastal sediment movement is mainly due to the action of the waves (Eaton, 1950; Johnson, 1919). The variability of the wave energy and the resistance of the sediment against transportation govern the attainment of the equilibrium profile of the shore, a condition that is only transitory. Coastal and bottom erosion and accretion are two processes which are continuous throughout all the seasons.

Though much of the sediment that moves along the coast is obtained from the surf zone, a small part of it is also derived from the shallow water and deep water zones because of the gradual shifting of the sediment at the ocean bottom especially in the shallow water zone, where the oscillatory waves from deep water transform to solitary waves (Daily and Stephen, 1951) resulting in the existence of a differential in the velocity (Munk, 1944) of the forward and backward motions of water at the bottom.

Even in deep water there is evidence of sediment movement. According to Kuenen (1950), off Land's End in Cornwall in Great Britain, stones upto one lb. in weight are moved at depths of 180'. In general, however, only very fine sediment is moved at such great depths. On reaching the surf zone, it is transported along the coast as littoral drift.

On the basis of laboratory studies (Manohar, 1955) it may be concluded that all motions of sediment beyond the surf zone occur within a boundary layer created at the bottom due to the effects of viscosity of water. Very fine sediment (less than 0.3 mm. in diameter) move in a laminar boundary layer with the movement caused by laminar shear while larger sediment move due to turbulence and lift forces in a turbulent boundary layer. Figures 1, 2, 3 and 4 are based on that study and with those nomographs and with the knowledge of the sediment sizes, depths under consideration and wave characteristics such as wave length, height and period, it will be possible to obtain an approximate estimation of sediment mov ment beyond the surf zone. Normally the sediment movement the bottom beyond the surf zone, may be in the form of rolli sliding and saltation, while under stormy conditions, it maj be in the form of suspension close to the bed. Fig. 4 may used to determine the limits of sediment motion in suspensi(

Considering the surf zone which is the chief source (sediment moving along the coast, strong local churning up oi sand due to the turbulent action of the waves occurs as the waves break in that zone at a depth approximately equal to 1 height of the waves. At the so called plung point, four t(five times as much sediment is raised as in the immediate neighbourhood. The movement of this sediment along the coast from the point from where it is disturbed depends to ε large extent on what is called 'nearshore circulation' (She· pard and Inman, 1950). Observations of nearshore circulation show that there are two inter-related current systems prevalent along the coast. The first type designated as tl coastal currents is induced due to the tides, or and winds. In general, they flow roughly parallel to the coast and con· sist of a relatively uniform drift in waters adjacent to th(surf zone.

The second type which is far more important with reference to the sediment motion along the coast is the 'near· shore system'. It is mainly due to wave action and occurs in and near the breaker zone. When waves travel shoreward there is a large transport of water shoreward rushing obliquely up the coast. This mass of water is known as swas This obliqueness in the travel and breaking of the waves ge: rate an alongshore component in the wave energy resulting i: the movement of water parallel to the coast known as alongshore or littoral current (Johnson, 1953, 1956, 1957). Whe: the swash dies away, the water that has not percolated down returns directly seaward. The seaward return flow may als generate rip currents which also move in the direction of the alongshore current. The nearshore system, therefore, consists of swash, rip currents and the alongshore currents In general, the swash provides the sediment in suspension, the longshore currents move it in the direction of their tr vel and part of it returns back seaward. The seaward move ment is, rather, very small and as such most of the sedimen churned up by wave action travels in the direction of the longshore current.

According to W.V.Lewis (Kuenen, 1950) there are two different types of waves that break on the shore and contri bute to the sediment motion along the coast. They are (1) destructive waves and (2) constructive waves. The destruc tive waves are irregular, steep, close together, have a marked orbital motion and break and plung down vertically. The power of the backwash is more than that of the swash which though large in volume mixes with the backwash of the

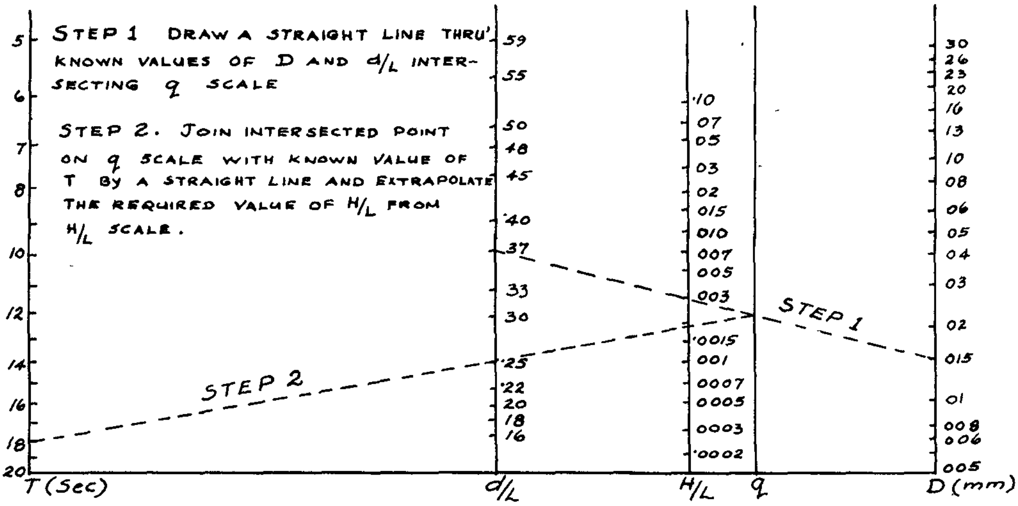


Fig. 1. Nomograph: Bottom sediment movement due to wave action in laminar boundary layer.

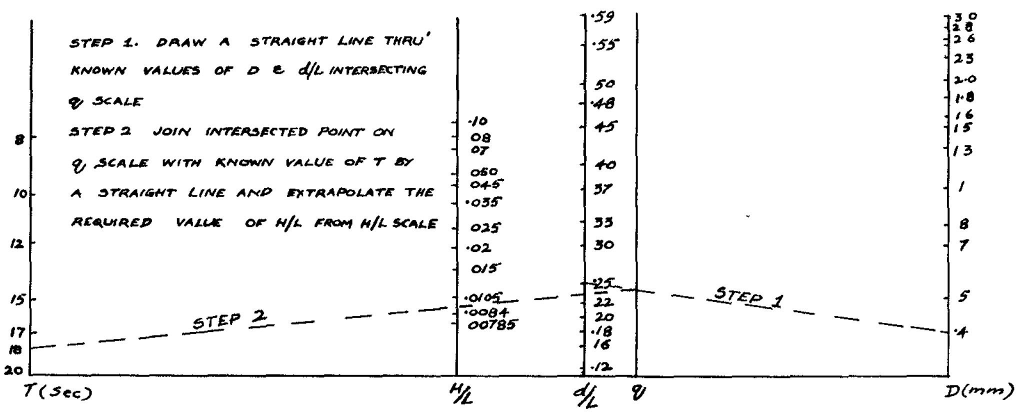


Fig. 2. Nomograph: Bottom sediment movement due to wave action in turbulent boundary layer.

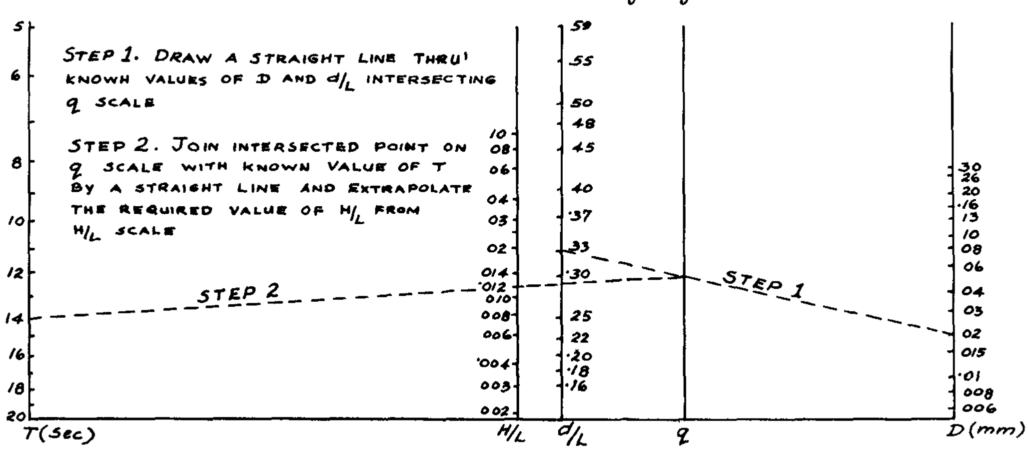


Fig. 3. Nomograph: Bottom sediment ripple formation due to wave action.

preceding wave and spreads weakly over the beach. Strong o shore winds generate this type of wave and in this, the back wash which is fairly large induces erosion of the beach and churning up of the sediment within the surf zone. On the other hand, constructive waves are long, have a regular elliptical orbital motion and break more regularly. They break less vertically and move obliquely forward. More energy is transmitted forward to the swash which though less in volume is more powerful and effective. The backwash is weaker since the swash spreads over a larger area and is los by percolation. Thus the sediment brought up by the swash is slowly added to the beach and to the alongshore movement. These waves are generated by far off winds. It is possible that a wave may act as a destructive wave or a constructive wave depending upon the nature of the wave, and profile and composition of the beach. Destructive waves may begin to work on a beach profile built up by constructive waves and vice versa. The breakers are also classified as (1) plunging (2) spilling and (3) surging waves. Usually constructive waves are assumed to be of the spilling type and destructive waves of the plunging type. All these types of breakers agitate the sediment within the surf zone and the sediment so agitated moves along the coast due to the alongshore component of wave energy.

COASTAL CURRENTS

The effect of coastal currents on sediment motion is negligible. In deep water areas, the velocity of the currents at surface seldom exceeds 3 ft. per sec. while in shallow water, it may slightly exceed that value. Similarl wind driven currents under favourable circumstances may also attain that magnitude of velocity. In general, the average velocity varies logarithmically (Kuenen, 1950) with height above the bottom with the result that the bottom velocities seldom exceed a few inches per second. These velocities ar too small to move the coarse sediment. These may cause movement of the fine sediment which is always in suspension but this type of sediment does not affect the configuration of the shoreline.

LITTORAL CURRENT, DRIFT AND TRANSPORT

As is already known (Eaton, 1950; Gilbert, 1890; Johnson, 1953, 1956, 1957) littoral currents are mainly responsible for the alongshore movement (littoral transport) of the sediment (littoral drift). These currents may act in the same direction as the coastal currents or they may act in the opposite direction. In both cases, their magnitude is far in excess of the coastal currents with the result that the littoral material moves in their direction. Though the general littoral drift may be in one direction during a particular season or period, a local drift in the reverse direction is also possible. For example at Chichester alon the coast of Great Britain, the main littoral drift is from

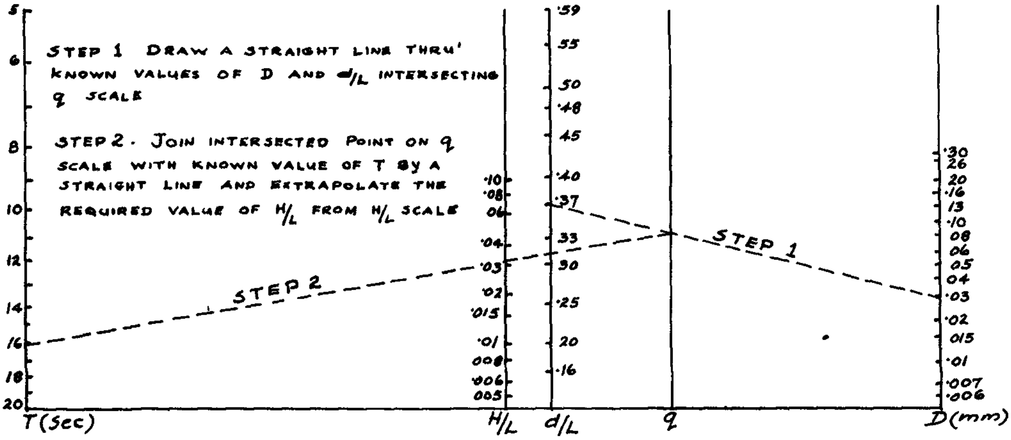
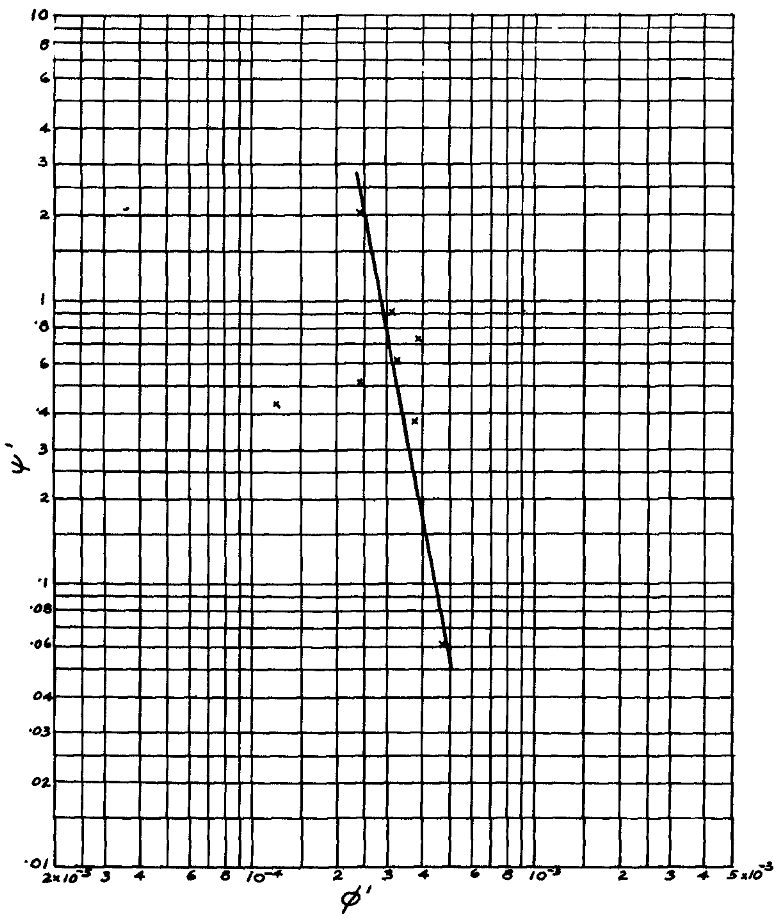


Fig. 4. Nomograph: Bottom sediment in suspension due to wave action.



No. 5. Littoral transport functions.

west to east while at the Chichester harbour itself, it is from east to west.

The general direction of littoral drift can be determined (U.S.Army, 1954) from the development of accretion and erosion near manmade structures such as jetties, groins and breakwaters, natural barriers such as headlands, sandspits and underwater bars, examination of beach and bed materials, current measurements and by refraction analysis (Dunham,195(Johnson, 1953) of wave energy at the coast in consideration. The last method loses its accuracy in zones of irregular to graphy.

Rate of Littoral Transport - The amount of sediment movement that is, the rate of littoral transport is a function of the wave characteristics, the sediment and the configuration of the shoreline. Depending upon the rate of supply and rate of transport of sediment, there can be either accretion or erosion or an equilibrium state. The only reliable method that is available at present to determine the rate of littoral transport consists in trapping and measuring the littor drift at a natural or artificial barrier. A general relationship involving the rate of transport, wave and sediment characteristics has yet to be evolved though Caldwell (1956 was able to obtain a valuable relationship between alongsho wave energy and concurrent rate of littoral transport from his studies of sand movement near Anaheim Bay in California and near South Lake Worth Inlet in Florida (Watts, 1953).

It seems to the author that the analysis of littoral transport can also be based on the concept of probability similar to the theory as evolved by Einstein (1942, 1950) for unidirectional flow. Einstein in his theory of bed lo transport in uni-directional flow, introduced two dimensionless functions, namely the ϕ function representing intensity of bed load transport and the ψ function representing the intensity of flow at the sediment level and fou that these were universally related. As a further proof that Einstein's theory was based on a correct approach to sediment transport mechanism, Tsubaki, Kawasumi and Yasutom (1953) found that Einstein's ψ function governed the dime sions of the ripples generated in uni-directional flow.

The author (1955) based his theory on bottom sedime motion due to wave action on an analysis similar to Einstein's approach and found that a dimensionless function representing intensity of flow over the sediment could be used to represent every bottom sediment motion in turbulen flow including development and disappearance of ripples. Though the flow at the bottom was oscillatory, the maximum instantaneous velocity of flow during its motion was taken to derive the dimensionless function ψ_1 . The author b lieves that a similar approach can be adopted to determine the rate of littoral transport.

When the waves break, they throw part of the sediment (finer) into suspension. The rest is in motion in the form of rolling, sliding, skips and hops. Thus the sediment in motion due to turbulence and lift forces is carried along the shore by the longshore current. The longshore current though not always strong enough to dislodge the sediment at rest acts as the transporting agent. Since the sediment and the wave characteristics govern the rate of littoral transport and since the longshore current is a function of the wave characteristics and the beach profile, it can be used along with the sediment characteristics to determine the rate of littoral transport. With this assumption many of the variables involved can be represented by a single variable namely the longshore current. On this basis and Einstein's theory of sediment transport, the author conducted a preliminary study of littoral transport from the date obtained from Anaheim Bay, (Caldwell, 1956) California. Einstein's ϕ function, namely,

$$\phi' = \frac{q_B}{\rho_s g}\left(\frac{\rho_f}{\rho_s-\rho_f}\right)^{1/2}\left(\frac{1}{gD^3}\right)^{1/2}$$

was retained as such while the ψ function was taken in the form of

$$\psi' \approx \frac{W'}{L} \approx \frac{g(\rho_s-\rho_f)A_2D^3}{C_L\rho_f\frac{V^2}{2}A_1D^2}$$

where V = longshore velocity obtained from wave characteristics and D = representative sediment diameter. The ϕ' and ψ' functions were found to be governed by a definite relationship (Fig. 5) indicating a probable approach to the determination of rate of littoral transport. However, it should be noted that the results thus obtained are based on very meagre data.

The value of the longshore current V, that is included in the expression ψ', may be obtained from the following formulae and nomograph (Fig. 6) (Inman, Quinn, 1951). The equation may be written in the following form

$$V = \frac{a}{2}\left[\sqrt{1+\frac{4c\sin\alpha}{a}}-1\right]$$

where

$$a = (2{\cdot}61\,H\,i\,\cos\alpha) \div kT$$

$$c = \sqrt{2{\cdot}28\,gH}$$

$$k = 0{\cdot}024\,V^{-1.5}$$

In the nomograph or alignment chart, the longshore current V, in ft./sec can be readily obtained when wave breaker height in ft., wave period in sec, beach slope in percent and angle of wave breaker approach in degrees are known. Though this approach is rather an approximation with uniform conditions assumed, the author believes that it should prove helpful in the determination of rate of littoral transport.

Effect on Littoral Barriers - As is well known, the movement of sediment towards and along the shore has an important bearing on the location of man-made structures and harbour sites. On the updrift side of the barrier, sediment will accumulate causing accretion and on the down drift side, deficiency in sediment supply will result in erosion. In both cases, the shore-line will tend towards re-alignment to an equilibrium profile in a direction normal to the resultant of the littoral forces which may roughly be estimated by drawing normals to orthogonals in a refraction diagram prepared for the zone under consideration. Harbour protection works such as break-waters and jetties and navigational works such as dredged channels should be aligned in such a way that they interfere as little as possible with the natural littoral transport and yet protect the harbour and the navigational channel against filling. If this is not possible, then preventive measures should be taken to prevent starvation of the down drift shore, excessive accretion of the updrift shore and keep the channel and the harbour from being put out of action. Depending upon the type of littoral barrier, the amount of littoral drift, the wind and the wave system, and the orientation of the coast, the types of protective works will vary considerably.

Types of Harbour Sites - Harbours in South India with reference to their location along the coast, may be classified differently and so it may be worthwhile to mention the different types of harbour sites and their sedimentation problems. Depending upon their location, harbours can be classified as river channel harbours, off-river harbours, fall-line harbours, tidal channel harbours, off-channel harbours and shore-line harbours (Caldwell 1950), (Mason, 1950).

River channel harbours built along the river-side wit sufficient depth for navigation can be maintained without ex cessive maintenance work. The effect of coastal sediment movement on the harbour itself is very slight. However, th formation of shoals and bars at the mouth of the river, thei frequent changes depending upon the amount of sediment brought down by the rivers, the interception of the long-shore current and therefore the littoral transport by the higher velocity of discharge from the river and the consequent settling of the coastal sediment in adjacent areas an the silting of the navigational channel due to the above causes are some of the problems involved in the upkeep of such harbour sites. In the dredged channel, since the dept is greater than normal, the waves do not break and the botto velocity is insufficient to transport material across the channel and thus the material deposits in the dredged portions. But in the case of river channel harbours, this problem is not of great magnitude since nature itself provides a channel for the discharge of river flow into the sea The maintenance of such a channel will be comparatively easy This, however, disturbs the material balance on the down-

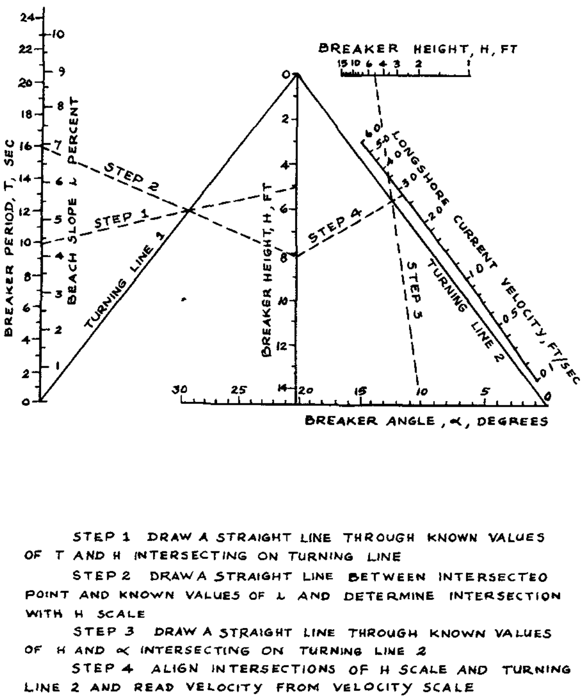


Fig. 6. Determination of longshore current (from Inman and Quinn, 1951)

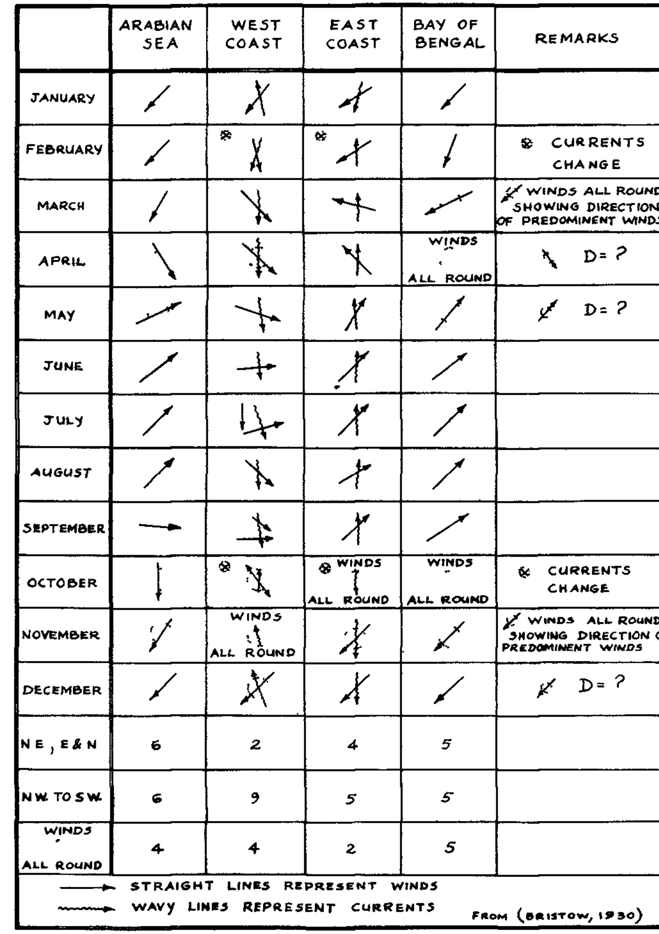


Fig. 7. Currents and winds around India.

drift side and erosion takes place in that zone.

As the name itself suggests, an off-river harbour is a stagnant water pool situated away from the river channel proper and connected to the river channel by a navigation access channel. Coastal sediment either from the sea or from the shoreline has little effect on these harbours except in the dredged navigation channel. Another trouble likely to occur with such harbours is the intermixing and the resulting flocculation and deposition of suspended material when silt and clay brought by fresh water from the river intermixes with salt water from the sea.

Tidal estuaries or bights which exist between turbulent mountain rivers and the sea sometimes offer as excellent sites for harbours. Such harbours are called fall-line harbours. In such harbours, the effects of sediment from the coast and the sea are very small except during flood time when some sediment will be carried to the harbour and into the navigational channel. The major trouble in such harbours is from the silt brought down by the river forming shoals and bars within the harbour area. Vizagapatam harbour on the east coast of South India about which reference will be made later may be classified as a fall-line harbour.

When harbours are located on tidal estuaries including tidal rivers, bays and lagoons, they may be termed as 'Channel harbours in tidal estuaries'. The effect of sediment from coasts and sea on these harbours is much more than in fall-line harbours due to great variations in tides and intermediate slack water periods. The sediment brought into the harbour area during the flood tides tends to deposit at the bottom during the slack water periods. Usually, when tidal estuaries are fed by rivers, this problem is of minor importance as compared to the formation of shoals due to the sand, silt and clay brought by the rivers as in the case of Mangalore port on the west coast of India. However, where tidal bays exist, with no major river discharging into them, sediment transport from the sea during the flood tides becomes the chief source of trouble. Upkeep of dredged navigational channels connecting the sea and the harbour provides problems similar to those mentioned earlier. In some instances, excessive flocculation of silt and clay may result in the formation of mud-banks or mud-lumps along the coast at or near the mouths of rivers discharging into the sea. Cochin on the west coast of India is an example of this type of harbour. Where excessive shoaling exists in such harbours, the harbours may be located away from the main channel in the tidal estuary. In such harbours, the effect of coastal sediment will be the same as in the previous case

When estuaries, rivers and other natural facilities do not exist for the location of a harbour, shoreline harbours are constructed directly on the open shore of oceans, bays or large lakes. Man-made structures such as breakwaters and jetties or natural barriers such as headlands projecting into the sea afford protection from waves within the harbour area. Such harbours always encounter excessive sedimentation from coastal material especially from littoral drift. These man-made structures arrest the movement of littoral drift, disturb equilibrium conditions along the coast, resulting in accretion on the updrift side and erosion on the downdrift side. In course of time, the accretion will gradually extend to the harbour entrance. It will then deposit in the lee of the breakwater depending upon the diffraction of the waves, (Johnson, 1951), and the magnitude of the velocities existing in that locality. The sediment that moves across the harbour entrance will deposit in the navigational channel causing further maintenance problems. In general, such harbours are constantly troubled by coastal sediment deposition and erosion depending upon the magnitude of littoral transport. Madras harbour on the east coast of India is an ideal example of a shore-line harbour.

With such a general analysis of coastal and bottom sediment motion, an attempt is made below to describe the conditions as they exist along the coast of South India with particular reference to four harbours namely Cochin and Mangalore on the west coast and Madras and Vizagapatam on the east coast.

WIND SYSTEM ALONG SOUTH INDIAN COAST

Sediment movement at the shore and under water is due to the action of kinetic energy on the sediment. This kinetic energy is obtained from the wind, either directly or through water waves resulting from the transfer of energy by the wind to the water-surface. Though waves may also, be generated by other sources of energy, such as earth-quakes, the principal cause is the action of wind on the water surface. In the Indian ocean, the outstanding feature of the wind system is the seasonal reversal of its direction known as the "monsoons" (India Meteorological Dept., 1941). The winds blow from a north-easterly direction during the North East Monsoon season from December to March, in which period, they are the strongest in January. From June to September, their direction is reversed and they move south-westerly and are called South West Monsoon winds. These are strongest in July. In general, the South West monsoon winds are stronger than those of the North East monsoon and as such they are the major cause of the littoral drift along the coast of India. Between these two main monsoons, there are two transition seasons so that there are altogether four seasons in a year and they may be described as follows:

a) N.E. monsoon season from December to March;

b) Hot weather period from April to May just before the S.W. Monsoons.

c) S.W. monsoon season from June to September.

d) Transition monsoon period from October to November when south westerly winds are replaced by northerly winds.

However, due to the rotation of the earth and other disturbing influences such as the mountain ranges that lie along the east and west coasts of India, the period, true di rection and force of these wind systems are different on bo coasts and also at different places on each coast (Meteorol gical office, 1940).

Wind System on West Coast - A general idea of the wind syst along the west coast may be obtained from Fig. 7 and the fo lowing table.I.

The daily variation in morning land breeze and eveni sea breeze due to the heating and cooling of the land is a marked feature of the coastal winds along the coast of Indi except during the S.W. monsoon period when the skies are generally cloudy. Since the waves that reach the coast ar generated in the centre of the Arabian Sea, these local win do not greatly affect the direction of wave approach except during the transition monsoon period and the beginning of t N.E. monsoon. The land breeze is strong from November to February though afternoon sea breeze is a regular feature throughout the season. From October to May, the winds are WNW during daytime and NE or ENE during the night. From October to January, the waves also approach the southern coast from WNW, NW or westerly direction. The maximum for of this wind system does not exceed a Beaufort scale of 2. During the S.W. monsoon period from May to September when land breeze is absent, the waves approach the southern coas from about WSW or SW with the monsoon wind blowing from W c WSW. From February to May, the wave direction at the coas is variable but generally from WSW or W especially during t latter part of the period.

COCHIN HARBOUR

Coastline - The port of Cochin (Bristow, 1930) is situated the west coast of Southern India (Fig. 4). From Cape Comc rin, the southernmost tip of India to Latitude 20° N, the west coast consists of a coastline of 800 nautical miles. Running roughly parallel to the coastline at a distance of about 20 to 50 miles inland, lies a continuous chain of mou tains, known as the Western Ghats, occasionally arising upt an elevation of 8000 feet. Most of the rivers though the rise from the Western Ghats run towards the east coast and discharge into the Bay of Bengal (Fig. 8). Only small mo

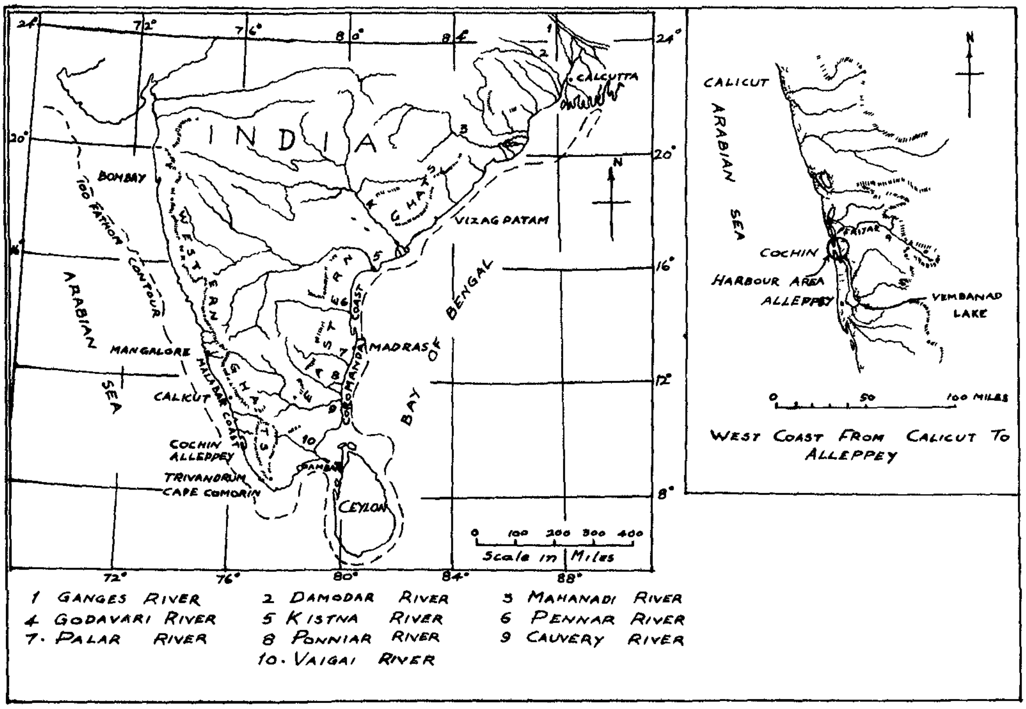


Fig. 8. Map of India with rivers and mountains.

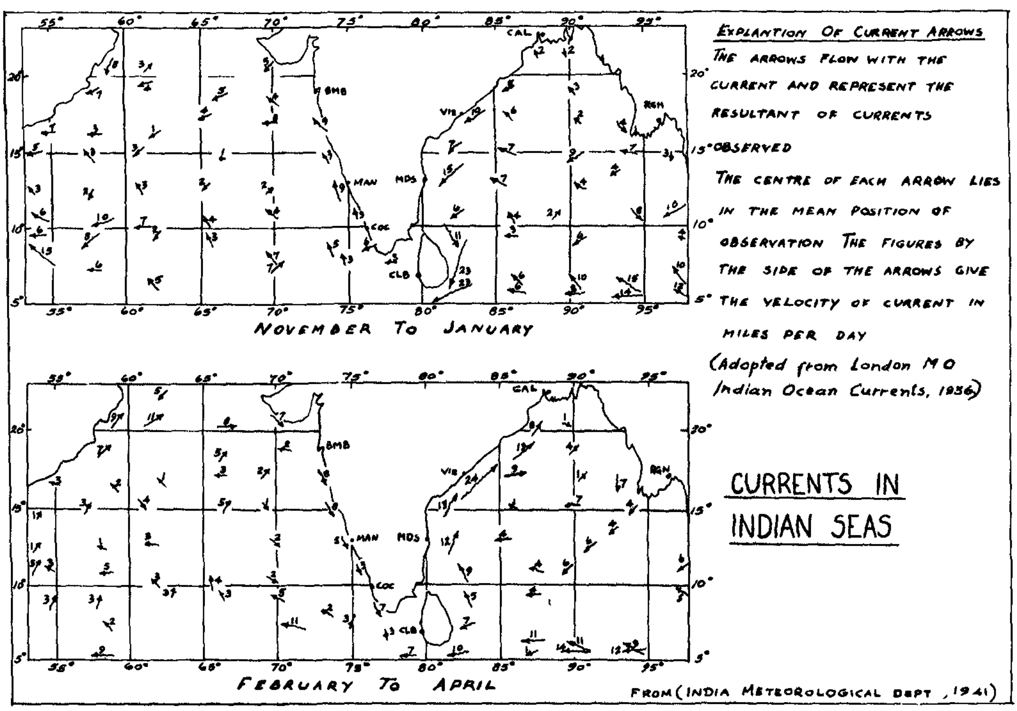


Fig. 9. Currents in Indian Seas - November to April.

Table 1.

Month	Wind direction	Wind force, Beaufort scale.	Wind direction over centre of Arabian Sea	Remarks
January	N	2 - 3	NNE, NE	
February	N, NNW	2 - 3	N, NNE	Frequent Squalls with a force of 7
March	NNW	2 - 3	N, NNE	
April	N W	2 - 3	NNW, NNE	
May	NW, W	3	NNW, to SW	Frequent Squalls of force 7
June	W, WSW	4 - 5	W, WSW	Wind force upto 8 in the centre of the sea.
July	W, SW	4 - 6	SW	Wind force up[illegible] 8 in the cent[illegible] of the sea
August	W, WSW	4 - 6	SW	Wind force up[illegible] 8-10 in the centre of the sea
September	NW	3 - 4	W, SW	Wind force up 7 in the cent of the sea
October	NW	2 - 4	NW to NE	
November	N	2 - 4	NE	
December	N, NNE	2 - 3	NE	Frequent gale of force 7

tain streams, few in number, discharge into the Arabian Sea and though the sediment brought by these rivers forms bars at their mouths, generally they bring only comparatively smaller quantity of sediment than that discharged by rivers on the east coast. On the west coast of Southern India, a strip of laterite lies outside the granitoid gneiss formation of the Western Ghats and extends roughly from 4 to 6 miles from the coast, thus indicating that the recent deposits are only a few miles wide as compared to the many miles on the eastern side. This itself is an indication of the small quantity of littoral movement along the west coast of India. For a distance of 170 miles from Calicut to Cape Comorin this stretch of recent deposit, between the laterite strip and the coast, is mainly of alluvium.

Continental Shelf - The continental shelf on the west coast of India extends outwards to an average depth of 100 fathoms. It is very wide in the north extending 120 miles seaward from the coast at latitude 20°N. It narrows towards the south and is only 30 miles wide at Cape Comorin. However, just to the north of Cochin and south of Quilon, there is a marked indentation in the 100 fathom contour such that the continental shelf is only 25 miles or less in width in these places. On the Malabar coast, in general, there is a gradual slope on the sea bottom upto 100 fathoms and then there is a sudden steep fall in the depth. But in some places there are marked deviations in the slopes. For example, along the Mangalore-Cochin section and at Cape Comorin, the continental shelf slopes gradually to about 65 fathoms and then drops rapidly to 1000 fathom line. Also at Quilon, upto 190 fathom line, the slope is gradual. Then the shelf rises seaward to 170 fathoms after which it plunges rapidly away.

Location - The harbour is located in Lat. 9° 58' N and Long. 76° 14' E at a distance of 100 miles from the southernmost tip of India. It is situated in the sheltered area of a backwater, a large expanse of water which is formed between a long narrow peninsula and the mainland, 3 miles east of Cochin. At Cochin, there is a gap, 1500 ft. in width, in the peninsula, so that inside that gap the main harbour, at once, opens 6000' wide causing all the waves entering from the sea outside the gap, to get absorbed in the backwater. Most probably, that gap was the result of a break-through during the earlier centuries from an unknown cause. On one side of the gap lies the town of Cochin and on the other side an island on which Vypeen is the most important town. The back-water is navigable for country crafts for about 125 sq. miles extending as far as 40 miles south of Cochin (Fig. 8). It drains and partly covers some 5,000 sq. miles of low country. The southern end of the Western Ghats drains into the Vembanad Lake which is a large expanse of water forming the southern portion of the backwater and it seems possible that the long peninsula upto Cochin which is of alluvium was formed by the silt brought down from the Western Ghats and drained into the Vembanad Lake. Similarly, the island of which Vypeen is the southernmost extremity was, probably, formed by the silt brought down from the Western Ghats by t Periyar river situated a few miles north (Fig. 8). The mouth of this river is at present silted up. The foreshor of Vypeen and Cochin consist of granitoid gniess sand which had its origin at the Western Ghats and which was brought t the shore by the littoral forces.

SEDIMENT PROBLEM

Cochin harbour may be classified as a channel harbou in tidal estuary. But unlike other channel harbours, it h many peculiarities. Its main features are (1) there are n rivers of importance especially near Cochin, which feed int the back-water and the only opening is at Cochin where the backwater discharges into the sea. Therefore all the smal rivers with their sediment of silt and clay drain first int the backwater. (2) Due to the largeness of the backwater and high rainfall of 120" per year with about 80" during SW monsoon period, the quantity of water flowing in and out is so great that the bottom of the backwater and the sea are covered with mud moved back and forth by the tides. The silt and mud since they require only small velocities for transportation either in suspension or at the bottom are ca ried further into the sea leaving the area near the coastli to the covered by sand brought by littoral forces. (3) Th wind and therefore the waves at the harbour are light and small creating a situation favourable for the settlement of sediment particles in the lee of the gap and at other place where the velocities are low. (4) Along the coastline t source of supply for littoral transport is the eroded mater of the coast since there are no important rivers to supply such material. Recent surveys show that as much as 40' ar eroded away at some places south of Cochin during the S. W. monsoon although 20' of the coastal strip is restored back other times. (5) In the harbour and the entrance channel, the main trouble is from the silt brought down by the tides from the backwater area. At times of flood tides, some of the silt taken out during the ebb tides finds its way back along with the littoral material and settles in the lee of the peninsula at low velocity areas. (6) Some of the littoral material finds its way into the lee of the harbour er trance due to diffraction of waves. By diffraction, the wave heights and thus the wave energy are reduced thereby allowing the sediment to settle down. (7) At the harbour, the amount of littoral drift settling down in the entrance channel is small as compared to the silt carried by the ti(from the backwater.

<u>Surface Currents</u> - The surface currents follow to a great e tent, the wind direction of the prevailing season (Figs. 7, 10). Along the west coast, at no time do they exceed 12 miles per day. This being the surface velocity, the bott(

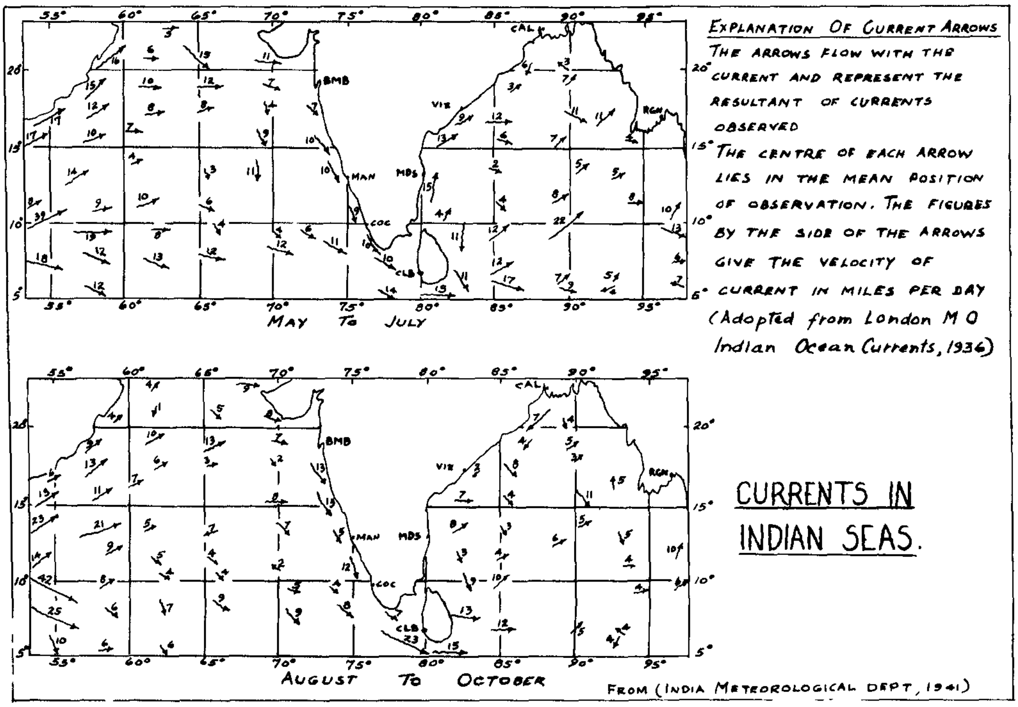


Fig. 10. Currents in Indian Seas - May to October.

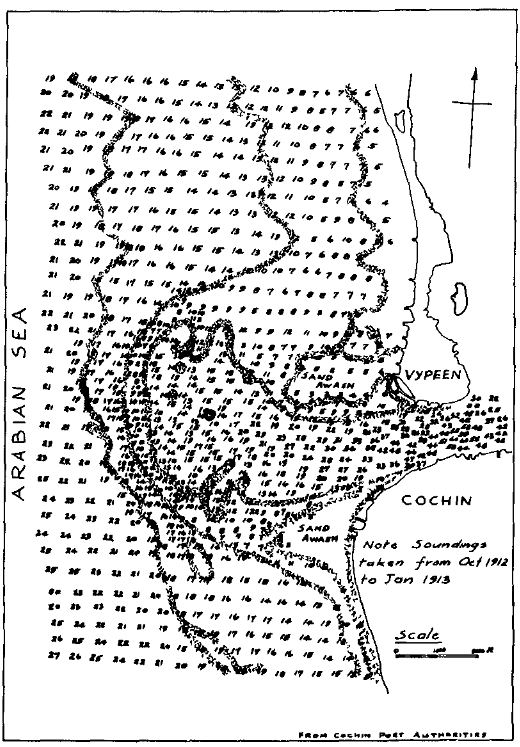


Fig. 11. Cochin harbour: Outer bar in 1913.

velocity is still less and also since the currents act at some distance away from the coastline, their effect on litt(ral drift is very slight and negligible.

<u>Tides</u> - Along the Arabian Sea Coast, as in other places, th(tides change according to the locality. From about 100 miles south of Madras on the east coast to Mangalore on the west coast, there are no tidal streams along the coast exce] just at the mouth of the rivers. Southwards from here dur- ing the flood tide, the tidal stream comes from the nortwes especially at Cochin. A peculiarity of the tides at Cochi is that they are susceptible to the influence of winds. A the Cochin harbour entrance, the ordinary spring tides rise to 3' creating currents from 1½ to 2½ knots. They extend for 40 miles southwards in the backwater area where the spring range may be as low as 8". Neap tides rise 1½ to 2 ft. creating currents of 1 to 1½ knots. However, the tide are not regular especially during the monsoons due to fresh water discharge. Depending upon the season, the ebb tide which is generally swifter than the flood tide continues fo a long time lasting 10 to 11 hours during the monsoons with current of 3½ knots and for 7 hours at other times. At ordinary times during flood tides, the ingoing current star at about 2 miles north and south of the harbour entrance wi a velocity of 1/2 to 1¼ knots which during the ebb tide mer ly reverses its direction. The effect of the tides at Cochin is to move back and forth, the backwater silt and th littoral drift.

<u>Waves and Littoral Drift</u> - The waves that approach Cochin harbour have a maximum height of 2½' at a depth of 15' and period of 10 seconds during the S. W. monsoon season. At other times, during the calm and fair weather periods, the wave heights vary from 1/2' to 1' with a period of 10 secon while the N. E. monsoon season experiences a maximum height of 2' and a minimum height of 1/2', the period remaining th same at all times.

The direction of littoral drift varies at different times of the year depending upon the direction of approach the waves. During the fair weather season and the beginni of the N. E. monsoon season since the waves are WNW, NW or westerly in direction, the littoral drift is southwards. Ho ever, during the S. W. monsoon season, with the waves ap- proaching from WSW or SW, the littoral drift moves north- wards. With the S. W. monsoon being stronger and more per sistant, the net littoral drift is northwards.

The quantity of littoral drift that moves along the coast is small due to the reasons mentioned earlier. Recer surveys show that the net northerly drift is not greater tł 42000 tons per year as against 1 million tons per year tra- velling northwards along the east coast.

An idea as to the effect of littoral drift and the backwater silt on the entrance channel, foreshore and the harbour proper can be obtained from a review of the previous and present history of the Cochin harbour.

Outer Bar and Channel - Before the outer navigational channel was dredged, there was no easy access for ships to enter into the harbour proper due to the presence of an outer bar (Fig. 11) which was formed in the form of a horse-shoe by the freshets discharging from the backwater carrying silt brought by the monsoons. The bar was formed at a maximum distance of 1½ miles from the harbour entrance between the 2 fathom contours. Probably this was the zone where the effect of ebb tide was balanced by the opposing velocity of the incoming waves resulting in low velocities ideal for sediment settlement. The bar was about 600' wide with a long flat slope on the harbour side and a steep slope on the sea side. The bar was somewhat semi-circular in shape with a radius of about 1 mile and a periphery of 3 miles but narrower on the left shoulder and wider on the right shoulder due to the predominence of the S. W. swell. At its shallowest place, the top was 10' below low water ordinary spring tide level at the worst season of the year. Dense sand, most probably brought by the littoral drift, existed at the top of the ridge while silt, mud and clay brought by the ebb tide from the backwater were found below at a depth of 20'. The depth on the bar varied very little for a long period of 89 years till it was dredged in 1922 to make way for the navigational channel. Even after the S. W. monsoon, the mean depth over the bar was never less than 9'. This may be explained from the fact that the 2' waves generally prevalent throughout that season could generate sufficient velocity at that depth to prevent the sediment from settlement. In some instances, it was noticed that the bar moved farther from the entrance during the fair weather season, as much as 600' from its original position while it was restored back to its original position after the monsoons. This gives further evidence of sediment movement under the sea towards the shore under the action of differential velocities at the bottom especially during heavy seas.

Outer Navigational Channel - A navigational channel which is made sufficiently deeper than the adjoining areas to allow for the safe passage of ships into the sheltered area is an essential requirement of a harbour. But with greater depths in the channel, sediment in motion settles down in this area. Therefore where the littoral drift is great or where the sediment brought by the rivers or bays in which the harbour is situated is great, the problem of maintenance of the channel by dredging becomes an impossible task. Some arrangement by which the sediment could be trapped and disposed off before it reaches the channel is essential in such situations. Luckily at Cochin harbour, the littoral drift is small.

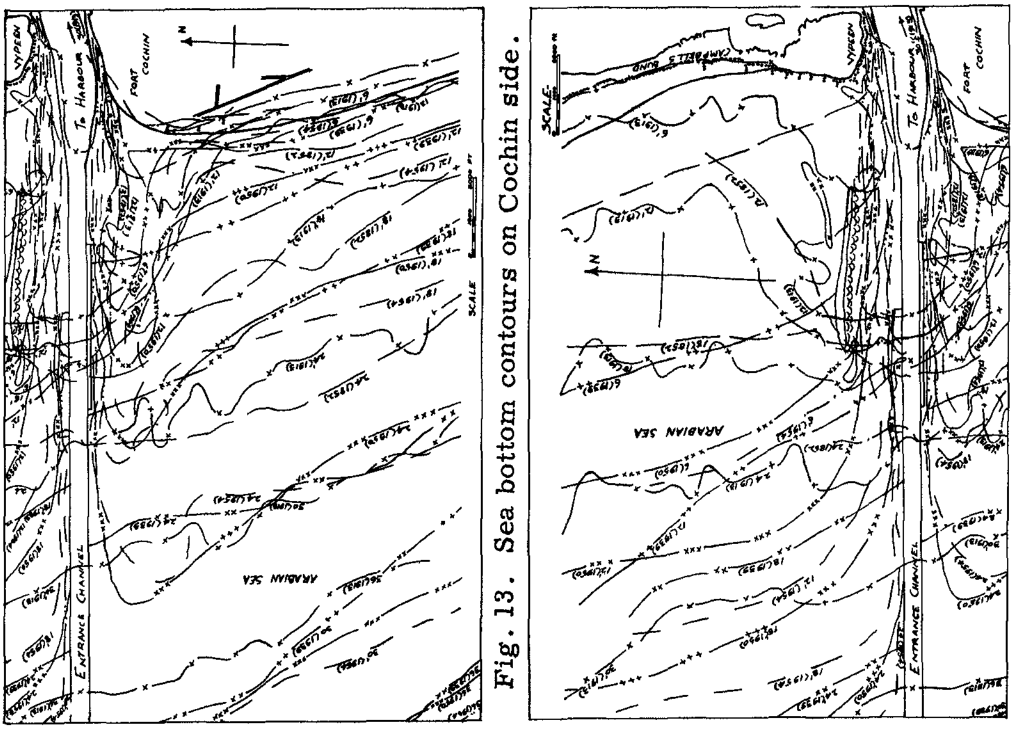


Fig. 13. Sea bottom contours on Cochin side.

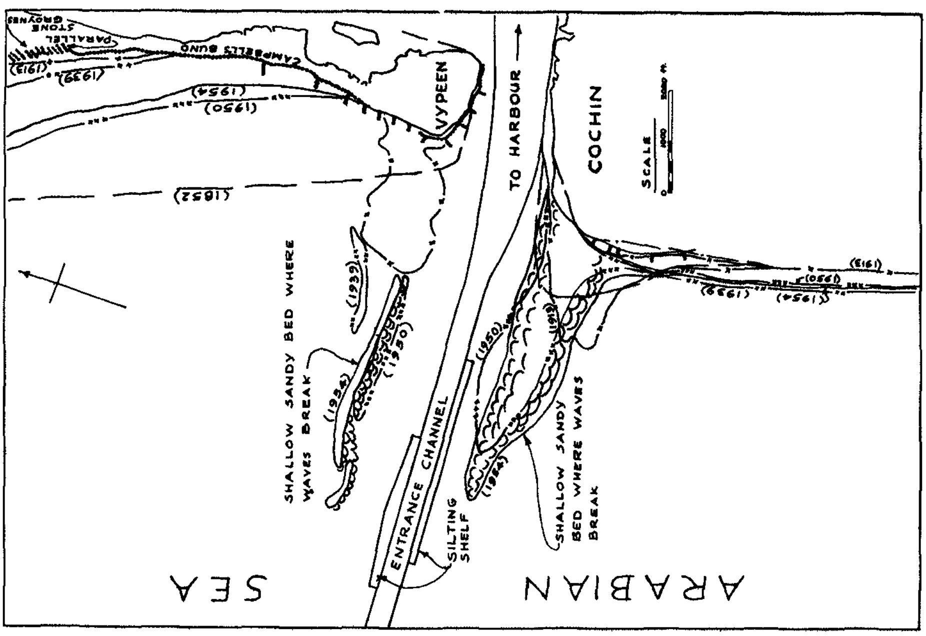


Fig. 12. Cochin harbour: Outer navigational channel

Similarly the silt brought by the backwater settling down in the channel is also not great so that maintenance by dredging for about 4 months a year is sufficient to keep a safe minimum depth of 32' at all times with the maximum depth of 38½' below L. W. O. S. T. The channel which extends upto the 33' contour is 17000' long, 450' wide and extends about 9000' beyond the old location of the bar. At that locality where the bar used to be formed, a silting shelf 100' wide and 4000' long on the south side of the channel and a similar silting shelf 100' wide and 3000' long on the north side. (Fig. 12) are provided to trap the littoral drift and silt from the backwater and also to compensate for caving in of the sides of the channel. It is worthwhile to mention that a depth of about 40' is always maintained at the entrance by the strength of the ebb tide.

The alignment of the channel is, by itself, important. At Cochin, since the prevailing sea swell and chief winds are from the WSW, fair weather winds and waves from WNW, the main littoral drift northerly along the coast and ebb current west by north, the channel is dug pointing due west so that it would have the least trouble from all the conditions existing in that region. By orienting the channel due west (1) the ebb tide is allowed to join the ocean currents with the least opposition, (2) the flood tide runs up the channel in its natural direction, (3) the dredging operations are made possible in not too rough seas and (4) the littoral drift is intercepted in as little area as possible. Upto the present time, the maintenance of this channel has not been a troublesome factor.

Erosion and Accretion of the Coast
<u>and Sea Bottom near the Harbour -</u>

With the northerly drift of the littoral material suddenly arrested by the ebb flow from the backwater and partly allowed to settle down on the Cochin side and partly deflected away towards deeper regions of the sea, the narrow spit on the Vypeen side beyond the gap is starved of the necessary littoral material resulting in considerable erosion on that side. By 1913, the narrow spit on the Vypeen side was eroding so fast that protective works in the form of stone faced bunds called Campbell's bund (Fig. 12) were constructed for the lower part of the spit but in the upper part, the spit was still eroding at 20' to 30' per year. By 1920, there was nearly a mile of this portion in a dangerously vulnerable state with only a narrow strip of land of a few feet in width lying between the backwater and the sea. In order to arrest the complete erosion of this narrow spit and thus save an important protection to the harbour it was then decided to trap the northerly drift material by the construction of a series of non-continuous stone groynes running nearly parallel to the shore and overlapping each other in an eschelon fashion for a distance of two miles (Fig. 12). These proved very effective and since then, this coastline had gradually buil up by accretion of the littoral material. They reduced th force of the waves attacking the beach, induced the waves t travel behind the groynes thus allowing the littoral materi to settle down in the calm area, and being non-continuous were susceptible to less erosion on their outer toe. Thes were found to be far more effective and stable as compared a continuous seawall at this locality. As shown in the fi gure the coast on the Vypeen side is gradually being restor to the profile as it existed in 1852. However, it looks a if it will never attain that profile. A study of the shor profiles will show that while there was gradual accretion c this coastal strip upto 1950, since then, there had been er sion at a rate of 50' per year in the upper regions. It seems therefore that the maximum accretion was reached in 1950 and that the equilibrium profile if one ever exists, lies between this profile and profile of 1913. On the otr hand, on the Cochin foreshore, there had been neither appre ciable accretion or erosion (Fig. 12) except for the formation and gradual extension of a sandy shoal parallel to the entrance channel.

Gradual silting up of the sea bottom is also one of the ways by which littoral drift manifests itself when equi librium conditions are disturbed. At the Cochin foreshore the bottom is slowly advancing towards the sea at a rate of 90' to 100' per year beyond the 24' contour and 50' to 70 per year between the 12' and 24' contours (Fig. 13). At depths less than 6', the conditions have, generally, been stable since 1937. On the Vypeen side, the rate of advan is greater with about 170' per year beyond the 24' contour (Fig. 14). At depths less than 24', the bottom advances more rapidly at a rate of 220' to 250' per year. This is contrary to what happens on the Cochin side and is most pro bably due to the effect of the stone groynes. It is interesting to note that from 1852 to 1913, there was erosion of the bottom on both Cochin and Vypeen sides at a rate of 15' per year. The considerable foreshore erosion on the Vypeen side and to lesser extent on the Cochin side also ha pened at the same time before the protective bunds and groynes were built on both sides.

Sandy Shoals - The effect of ebb flow on littoral drift is manifested in another way namely in the formation of sandy shoals parallel to the sides of the entrance channel (Fig. 12). The sandy shoal on the south side is formed by the sudden stoppage of the northerly littoral drift by the ebb tide causing it to settle in the adjacent areas while the sandy shoal on the north side is formed when the littoral drift taken into the backwater area during flood tide is taken out during the ebb tide and thrown into the low velo city region on the north side adjacent to the channel. T situation is reversed when the littoral drift changes its direction from north to south so that in both cases, the sandy shoals build up gradually. The rate of advance of these sandy shoals is on the increase from 15' per year upto 1913 to 80' per year between 1913 to 1950 and 200' per year since then. The rapid advance of the sandy shoals seems to be intimately connected to the advance of the sea bottom.

It is possible that the gradual advance of the bottom and also of the sandy shoals towards the sea, though not a threat to the maintenance of the channel upto the present time, may be a factor to be reckoned with later on. The increase in the amount of dredged material from the channel and the backwater may be due to this advance.

Sediment Samples - Bottom sediment samples taken at various places inside and outside the harbour show the presence of littoral drift material (Fig. 15, 16). Samples 1, 2 and 3 namely, those taken from the coastal region and the harbour mouth show a common origin namely, the coarse grained sand brought by the littoral drift from along the coast. Samples 4 to 8 taken at increasing distances from the harbour mouth within the backwater show a progressively finer texture in the material indicating the clay and alluvial material that have their origin in the Western Ghats and invariably brought down by the monsoons.

Effect of Wave Diffraction on Sedimentation - The sediment settlement due to wave diffraction, a phenomenon (Johnson, 1951), (Dunham, 1950) by which waves are propagated into the sheltered region of a breakwater or breakwater gap has a direct bearing on the construction and maintenance of a harbour formed in the sheltered region. By diffraction, the regular wave train is suddenly interrupted and the heights of waves entering the sheltered region are progressively reduced, thereby creating a condition by which the sediment settles down in the region of low velocity. In the case of Cochin harbour when the waves pass into the harbour through the natural gap, 1500' in width, they are diffracted. Since the waves approach the gap obliquely, the region behind the Cochin peninsula namely, the Mattanchery channel (Fig. 17) becomes an ideal place for sediment settlement due to diffraction. The sediment thus deposited is taken into the interior of the Mattanchery Channel by the flood tide resulting in shoaling of that channel while there is practically very little silting in the Ernakulam Channel. Part of the sediment deposited by the diffraction on the Vypeen side and the Ernakulam Channel finds its way out and deposits on the northern sandy shoal (Fig. 12).

MUDBANKS ALONG THE WEST COAST

The term mudbank or mudlump is used to represent islands of mud or clay that show up along the coast. They are rare in occurrence and are formed only under favourable conditions. Along the west coast of Southern India and at the

Mississippi river mouths off the Gulf of Mexico (Morgan, 1951), these form a unique phenomena and the author knows of no other locality where mudbank activity has been reported. The formation of mudbanks is independent of the littoral drift action but it is described here since it forms a part of the sediment activity along the coast. Their formation and activities along the Malabar coast are in many ways simi lar and dissimilar to those of the mudlumps of the Mississi- ppi river delta. Their activities were greatest upto 1938. Since then, many have disappeared and many have risen in other places. Before 1938, there were 4 well known mudbank off this coast, (Bristow, 1938) namely one at Alleppy (Fig. 8), one at Narakkal just north of Vypeen and two at or near Calicut. There may have been a few more at that time but they were not well known. Three of these mudbanks were either near or at the mouth of rivers while the fourth one a Alleppey though not anywhere near a river mouth was separate from the large Vambanand Lake only by a narrow alluvial stri

Before the harbour was established in the backwater, these mudbanks were a great boon to ships and country craft since they had a peculiar property of completely damping eve the roughest waves along their seaward slopes and elsewhere making it possible to unload cargo or take shelter in their vicinity. But their peculiar behaviour such as their sudde appearance above the sea and their sudden disappearance bel the sea without any previous activity, their southward and northward movements, and their occasional eruptions have be the subject of speculation and study for a long time.

Their Nature, Origin and Activity - All the mudbanks are co fined within the main body of the alluvial coastal strip on the west coast namely the coastline from Calicut to Cape Co morin (Fig. 8). The sea bed off this coast is also mainly mud stretching as far as the 20 fathom line roughly 17 mile distant from the coast. The mud on the banks and the sea bottom in these areas has the same property, indicating com mon nature and origin. It is fine grained with 70 percent of the particles being clay having an effective diameter of 0.0015 mm (Fig. 18). Borings of the sea bottom show that the alluvial material lying to a thickness of 400' to 600' above rock was most probably brought by the past and presen rivers and the backwater from the Western Ghats. The mud itself is dark green in colour when wet and being very fine it is soft and gives an oily appearance. But when it be- comes dry, it loses its oily composition, and becomes hard like ordinary mud. Though it is soft at the surface, it i compact at the bottom and forms a good holding ground.

The origin of these seem to be the rivers. In almo all cases where mudbanks appear, rivers are at moderate di tances and even at Alleppey, an opening had once existed an it seems likely that a water bearing stratum exists in that locality and elsewhere at great depths below the surface

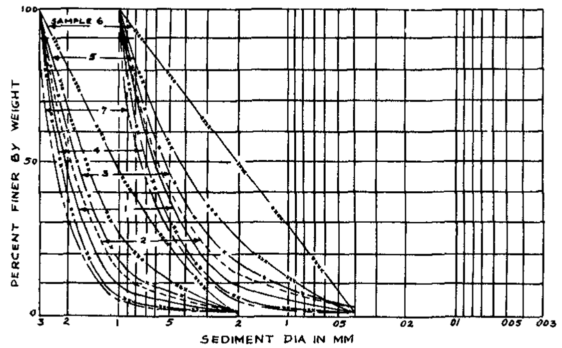


Fig. 15. Cochin harbour · Sediment analysis

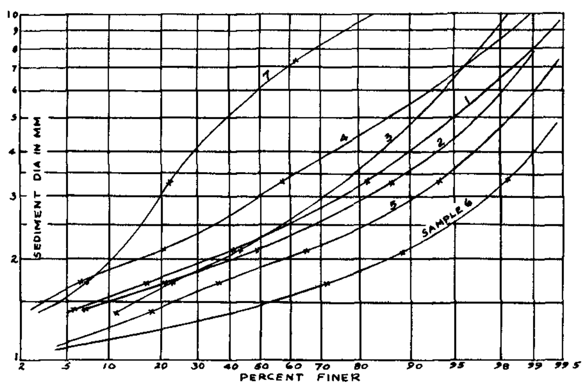


Fig. 16. Cochin harbour: Sand analysis

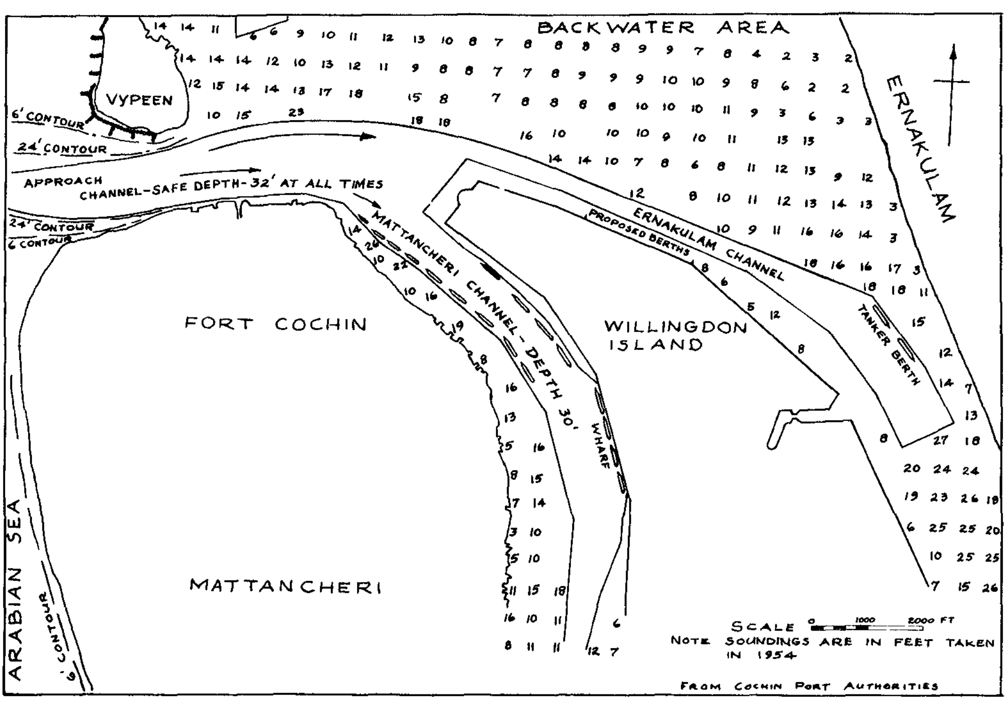


Fig. 17. Cochin harbour: Navigational channels in main harbour.

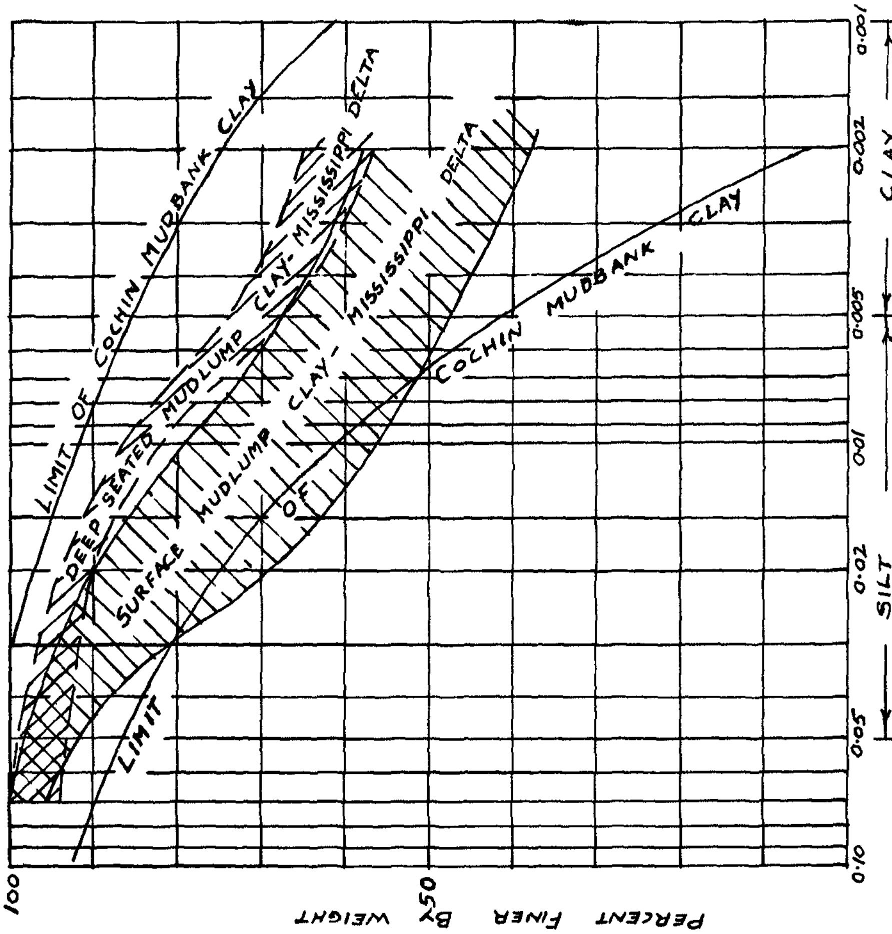


Fig. 18. West coast of South India: Hydrometer analysis of

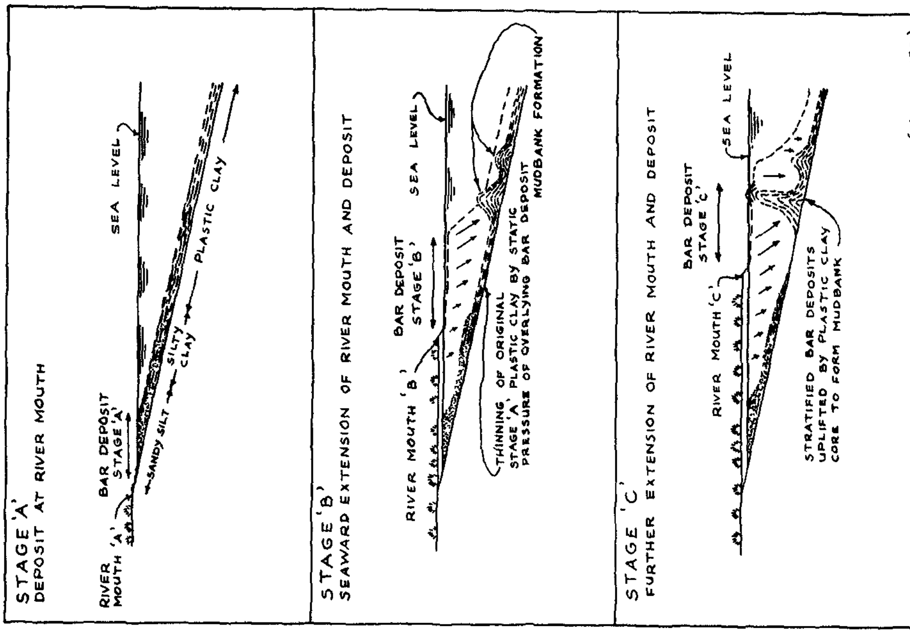


Fig. 19. Mudbank formation: Probable

through which mud from the backwater is carried out into the sea and lifted up as mudbanks. Bore holes at Cochin indicate the presence of water bearing strata which probably keep the mud in suspension at those banks. That water bearing strata exist along the coast may also be inferred from the fact that frequent eruptions and rising of mud above water and the sudden appearance of new mudbanks occur only during high water periods in backwaters. Thus it seems that the mudbanks may have been formed due to the following causes either separately or together:-

(1) by the gradual deposition of detritus mainly mud and clay brought down by the rivers;

(2) by the gradual throwing up of silt and mud through water bearing strata connecting the backwater and the sea; and

(3) by the sudden throwing up and re-deposition of mud, already in existence at the bottom, due to rough seas and seismic disturbances.

When mudbanks are formed due to category (3), they may not be stable and may disappear as fast as they appear as it had happened at Calicut and Alleppey on a few occasions.

In general, they are formed from river deposits and their processes of formation may be ideally represented (Morgan, 1951) in three stages (Fig. 19). In stage A, the sediment brought by the river which consists of plastic clay, silty clay and sandy silt is deposited at the mouth of the river with the fine plastic clay farthest from the river mouth and the coarser sandy silt nearest to the mouth. In stage B, since the quantity of sandy silt and silty clay brought by the river is greater than plastic clay, their bar deposits grow faster so that they overlie the plastic clay deposit of stage A, resulting in the squeezing ahead of plastic clay from their load pressure. During this process, the plastic clay breaks through the thin forward edge of the bar deposit starting the initial formation of the mudbank. In stage C, with the bar deposits and plastic clay increasing in quantity, the mudbank is forced to the surface and with the bar deposit surrounding it, it becomes a localised bank. A pre-requisite to the formation of the mudbanks is that the river sediment as well as the sea bottom sediment should consist of plastic clay, silty clay and sandy silt, a condition that exists along this coast.

The Alleppey and the Narakkal mudbanks when they existed used to move generally towards the south due to the stronger currents from north to south. It may be noted that the southerly direction of the currents exists for 8 months of the year. These currents are stationary for a month and for the other three months they flow from south to north. The southward movement of the mudbanks is exactly opposite to the movement of the littoral drift which is due to the fact that the coarser littoral material moves in the shallow wat zone close to the shore where it is subjected to direct wav action and moved northwards by the longshore current. Th mudbanks which are slightly farther away in the sea come under the direct influence of the southerly currents, which cause their movement because of their finer texture. Thes mudbanks gradually grow in size being nourished by the rive or by additional material thrown up from the sea bottom unt they are finally broken up by forces such as waves, swells, cyclones and seismic disturbances. Sometimes these mudban used to move suddenly to the north again due to the norther currents, or disturbances in the sea or higher flood discharges from the backwater in the case of the Narakkal mudbank. During its southerly movement, the Narakkal mudbank crossed the harbour entrance channel some years ago and either disappeared into deeper areas or dissipated away. T original Alleppey mudbank also suffered a similar fate. Ma new mudbanks have been appearing and disappearing along thi coast though at present, there are none near the harbour.

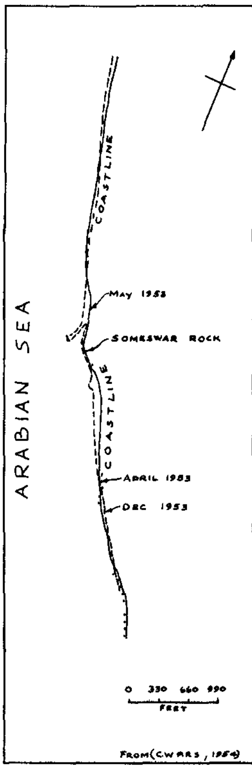
The unusually calming effect around the mudbanks due to the damping of the incoming waves may be attributed to t causes namely (1) the increase in kinematic viscosity of th suspended mud acting like a jelly and (2) the increase in t friction drag on the slopes of the mudbanks. The mud is always kept in suspension by the action of the incoming wav or by possible fresh water springs at the sea bottom.

These mudbanks are different in many ways from the mudlumps of the Mississippi river delta namely (1) they are also formed in places other than the mouths of rivers (2) they are generally in motion in the direction of coastal cu rents (3) they dampen the waves completely and no wave brea around them (4) steep slope is not a necessary criterion fc their formation and (5) their activities such as eruptions, and throwing up of mud are not frequent phenomena.

MANGALORE PORT

Mangalore port is a small port situated at Latitude 12° 52' N and Longitude 74° 51' E along the west coast, nor of Cochin (Fig. 8). Unlike the Cochin harbour which is ideally situated in a large backwater, this port is formed the junction between the Gurpur river, the Netravati river and the Arabian Sea (Fig. 20). A sand spit (Fig. 20) 3 miles long and 300' wide lies between the Gurpur river and the sea. Similarly another spit of a smaller size separat the Netravati river from the sea so that a calm area of wat exists behind the gut formed by these two sand spits.

Accretion and Erosion - This harbour may be classified a ch nel harbour in tidal estuary and it has all the problems er



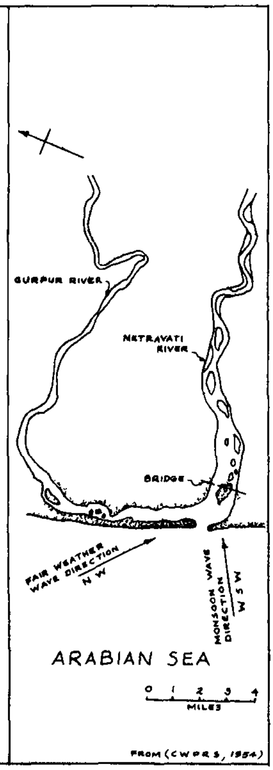


Fig. 20
Location

Fig. 21
Neighbouring coastline

Mangalore Port

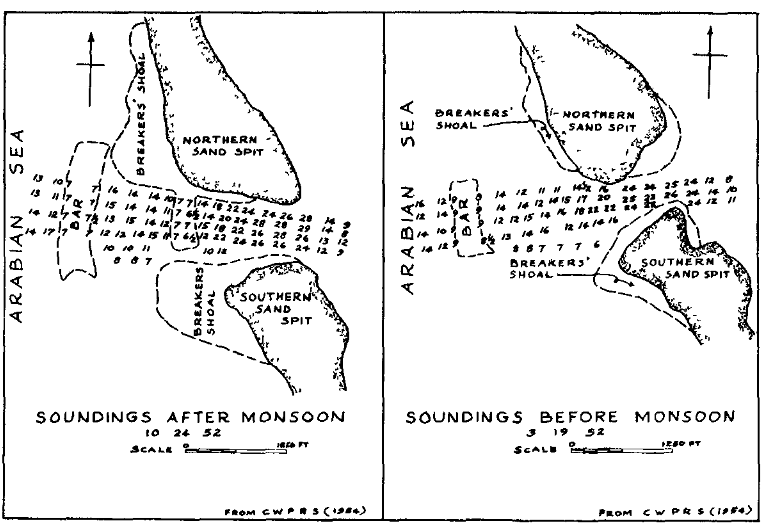


Fig. 22. Mangalore Port: Outer bar

countered in such harbours. Just outside the entrance, as was the case at the Cochin harbour, there is a large bar (Fig. 22) over which the depth varies from 7' to 9' below L. W. O. S. T. It is about 700' in width, east and west, and begins from about 2000 feet due west from the entrance. As shown in the figure, the depths over the bar decrease after the monsoons and are greater before monsoons. Unlik at Cochin where the backwater discharges only silt and clay capable of being carried further into the sea, the two rivers, Gurupur and Netravati, carry enormous quantity of coarse sand varying in dia. from 0.5 to 0.65 mm. which settles inside and just outside the entrance to the harbour resulting in heavy shoaling in these localities. The north and south sandy shoals increase in size after the monsoons due to the river deposits and are reduced in fair weather, most probably, by the littoral currents. On an average, there is change in height over the shoals of about 3 feet with quantity of accretion or erosion of 0.15 million cyds roughly 300,000 tons per year. (Central Water & Power Research Station, 1954).

Littoral Drift - The net littoral drift moving northwards along the coast is smaller when compared to this quantity c river material and is of the order of 200,000 tons per year as indicated by the erosion and accretion at Someswar, a rc outcrop 3 miles south of the gut (Fig. 21). The sources i this littoral material are the two rivers Gurpur and Netravati which have a maximum discharge of 60,000 cu.ft. per second and 120,000 cu.ft. per second respectively. This littoral drift is about five times greater than at Cochin and most probably localised in this area. The direction of tł littoral drift is the same as that at Cochin with its movement northwards during the S. W. monsoon when waves reach t shore from WSW and southerly during the fair weather seasor from December to May when waves with a period of 17 sec. a wave length of 1500' and a height of 2'4" in deep water re the shore from a northwesterly direction. The net littor drift is northwards due to the stronger and persistent S. W monsoon.

The fact that littoral drift and river deposits are large with no method of disposing off the coarse material deep into the sea or elsewhere except by continuous dredgi makes the maintenance of this harbour very expensive and a such, the improvement of this harbour into an all-weather port has been abandoned for the present. At present, it : only an open roadstead (Ministry of Transport, 1950) close during the S. W. monsoon period from May to September. I fair weather, vessels lie in the sea at a distance of two miles from the harbour entrance

As a comparison with the Cochin harbour, it may be pointed out that at Mangalore (1) littoral deposit is five times greater (2) the river deposits are so coarse and gre in quantity that their disposal by dredging is very costly (3) the formation of the large sandy bar at the entrance will be a constant trouble to be encountered with when a navigational channel is provided and (4) ideal conditions such as a large backwater with no silt bearing rivers of importance feeding into it, small littoral drift, large ebb flow and lesser maintenance work as they exist at Cochin are not found at Mangalore.

EAST COAST TO SOUTHERN INDIA

The east coast of India which extends from Cape Comorin to the mouth of Ganges has a coastline of 1300 nautical miles (Fig. 8). In the south, the stretch of coastline between Cape Comorin and Pamban (Fig. 8) is called the west coast of Gulf of Manar and this coast is shielded by the island of Ceylon. From Pamban to about Latitude 16° N, the coastline is called the Coromandal coast. The remaining coastline is divided into the Circars coast (upto Latitude 19 23' N) and the Orissa coast upto the mouth of river Hoogly.

Wind System - Table 2 below shows the average direction and force of wind system along the east coast (Meteorological office, 1940, Hydrographic Department, 1953).

Land and sea breezes are also a marked feature along this coast especially during the transition seasons. However, NE and SW monsoons dominate the surface winds of the Bay of Bengal and the east coast. SW winds other than those of the S.W. monsoon can be observed in the Bay of Bengal in March, April and May due to the heating up of the land areas to the north and east. October is the only month when the wind and weather are variable to a large extent.

MADRAS HARBOUR

The east coast of India on which Madras harbour is situated, is bounded by a chain of hills or mountains known as the Eastern Ghats running roughly parallel to the coastline (Fig. 8). Unlike the Western Ghats, they are not continuous and consist of numerous hills, some of them rising upto 5000 feet in elevation. They are situated inland with a broad strip of low lying land of alluvium between them and the Bay of Bengal. Southwards of Madras, the width of the coastline is about 80 miles as compared to a narrow stretch on the west coast. Northwards of Madras it narrows to a width of 30 miles. Thus the east coast is an eroding coast being exposed to the prevailing waves.

ORIGIN AND EFFECT OF LITTORAL DRIFT

Between the Western and the Eastern Ghats lies a plateau varying in elevation from 1000 feet to 3000 feet. Most of the rivers in south and central India which have their

Table 2.

Month	Wind direction.	Wind force-Beaufort Scale.	Wind direction over centre of Bay of Bengal	Remarks
January	NNE	2 - 4	NE	Frequent gale of force 6
February	NE, NNE	2 - 3	NE	
March	S, SW	2 - 3	SW	Frequent gale of force 4 -
April	S, SW	2 - 3	SW	
May	S, SW	2 - 4	S, SW	Frequent sto of force 8
June	SW	5	SW	Frequent gale and storms (force 6 - 9
July	SW	4 - 5	SW	Frequent sto of force 9 - and light wi of force 2 -
August	SW	4 - 5	SW	Same as in J
September	SW	4 - 5	W, SW	Frequent cal and strong winds of for
October	(NE, E (above (Lat. 15°	2 - 3	NE, E	Frequent sto of force 8 - wind variabl
	(SW below (Lat. 15°	3 - 4		
November	NE	2 - 3	NE	Frequent sto of force 9 -
December	NE	2 - 4	N to NE	Frequent gal of force 6 .

origin in the Western Ghats flow into the Bay of Bengal, between the hills comprising the Eastern Ghats, travelling through the granitoid and schistose country carrying enormous quantity of sand to the sea with some of the rivers having a course of more than 800 miles through this eroding land. This partially offsets the erosive action of the sea by providing the deficiency areas with the necessary sediment moving as littoral drift along the coast. The major rivers contributing silt to the east coast are the Mahanadi, the Godavari, the Kistna and the Cauvery (Fig. 8). With particular reference to Madras the two important rivers that contribute sediment are the Pennar, north of Madras having a maximum discharge of 620,000 cfs. from a drainage area of 20,000 square miles and Cauvery in the south with a maximum discharge of 380,000 cfs. from a drainage area of 26,000 square miles. In addition, there are many other smaller rivers that contribute to the large amount of sediment along the coast (Fig. 23).

Waves - The waves that cause the littoral transport of sediment (dia. = 0.22 mm) are northerly in direction from March to September and especially during the S. W. monsoon. They approach the coast at about 30° to the shoreline from the other direction. The surf at Madras breaks at 300' from the shore in fine, at 450' in squally, and at about 1000' in stormy weather. In fair weather, the surf wave varies from 2' to 4' in height, while in rough weather it rises upto 6' and sometimes upto 14' during gales.

The stronger S. W. monsoon and the more regular S. W. wind from March to September generate a northerly littoral transport during that period as compared to the southerly creep from October to February. Thus the sediment brought down by the rivers moves up and down the coast replacing the soil eroded by the sea maintaining the shore in rough equilibrium. The effect of currents (Figs. 9, 10) on littoral transport is very small along this coast also. They are not only feeble but act at about two miles from the shore farther away from the shallow water zone where littoral transport takes place.

Description of Harbour - As is the usual case everywhere, when the alongshore movement of sand and silt is obstructed, they tend to accumulate on one side of the obstruction and erosion continues on the other side in an aggravated form. This is the problem at Madras (Spring, 1912, 1919) where an artificial harbour classified as a 'shore-line harbour' is situated. The harbour is formed by the projection of two artificial breakwaters from the shoreline with the southern breakwater sheltering the harbour on the southern and eastern sides with an extension northward of the entrance known as the sheltering arm. The present entrance is situated between the outer end of the northern breakwater and the end of a short arm which extends northwestwards from the root of the sheltering arm (Fig. 24).

South Side Accretion - The larger northerly littoral drift a result of its stoppage by the southern arm of the harbour settles on the south shoreline resulting in accretion on th side and erosion on the north side. The effect of the weal southerly littoral drift during the North East Monsoon tryi to restore the shore back to its original profile is almost negligible. Even in the first year of its construction in 1876, because of accretion, the southern shoreline had advanced 250 feet at the harbour with the accretion extending southwards for 3/4ths of a mile. By 1910, the original entrance on the east side (Fig. 24) had to be abandoned and closed due to rapid silting and the present entrance built the northern side. Though the rate of accretion had decreased gradually in subsequent years (Table 3) by 1912, th accretion at the breakwater had extended seaward by 2540' with a consequent southward increase to 9000'.

Table 3.

Period	No. of years considered.	Seaward extension in feet.	Total Seaward extension from low water line of 1876 in feet.	Seaward extension per year	Averag seawar extensi per year i feet sin 1876.
1876-1879	3	440	440	147	147
1879-1882	3	360	800	120	133
1882-1898	16	1,020	1,820	64	83
1898-1912	14	720	2,540	51	70
1912-1919	7	230	2,770	33	64
1919-1947	28	300	3,070	11	43

Based on the amount of accretion on the south, the quantity of littoral drift was estimated to be one million tons per year. With such a high rate of littoral drift it was predicted at that time that in about 40 to 50 years, shoaling of considerable magnitude would start even at the new entrance unless some preventive measure were taken. Maintenance of the shoreline by continuous dredging and pum ing was found to be an impossible and a very expensive task due to the large amount of littoral drift in motion.

Remedial Measures - It was then decided to extend seaward t southern breakwater of the harbour by means of a masonry ar as its south-eastern corner (Fig. 24, 25) for a distance of 720 feet so that it could serve as a sand screen and deflec the littoral drift away into deeper water areas and delay t immediate extension of the shoaling process to the entrance

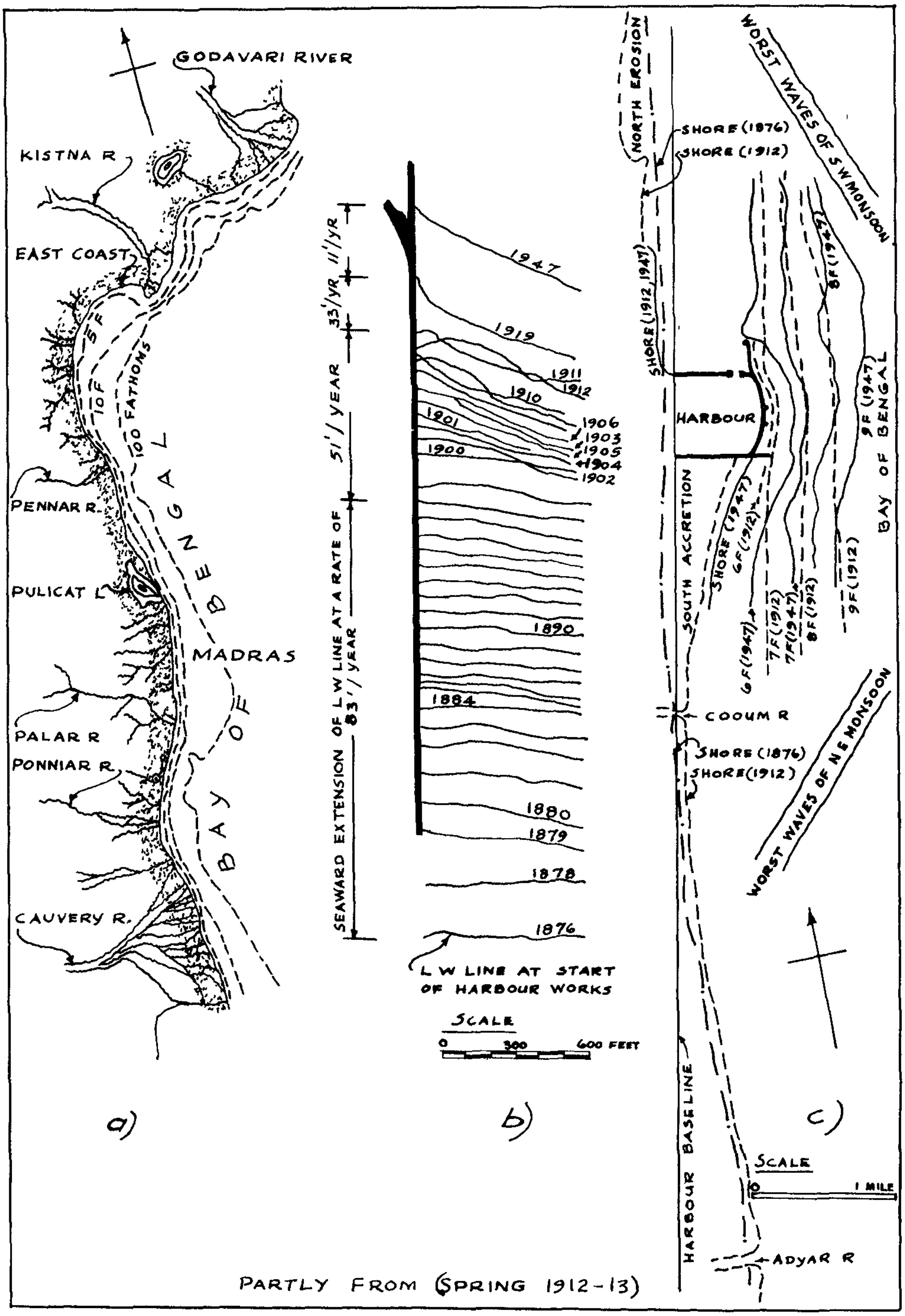


Fig. 23. Madras harbour: a) Neighbouring coast, b) Low water line at southern breakwater; c) Bottom contours in 1912 and 1947.

channel. With the provision of the sand screen, the rate c seaward extension of the shoreline has considerably decrease and it has been about 11' per year in recent years. Since the shallow coastal shelf at Madras is narrow and ocean depths are comparatively close inshore, the sand screen seem to have served the purpose in deflecting the littoral drift to deeper areas. This, together with dredging near the scre intercepting the drift passing it has lessened to a large ex tent the threat of sand drift closing the entrance.

Generally the entrance has a depth of 34' with the area inside the harbour having a least depth of 31'. The rate of shallowing of the sea has also decreased considerabl since the provision of the sand screen and the commencement of the dredging operations in that locality. For example, the 6 fathom contour which was extending towards the sea at rate of 30' per year before, is moving seaward only at a rat of 15' per year since then. Figs. 23 and 24 show the appro ximate position of the contours as they existed in 1912 and 1947.

The process of accretion and erosion when littoral movement is arrested is a manifestation by which the disturbed shoreline is trying to orientate itself normal to the direction of the waves. In the case of Madras harbour such a process will never be complete since it will be preceded l the extension of shoreline to the outer end of the harbour resulting in the unhindered movement of the littoral drift along the coast similar to what had existed before its construction. It is interesting to note that the orientation of the east coast from the mouth of the river Hooghly which discharges into the Bay of Bengal near Calcutta upto the southern end has the upper 600 miles running roughly from north-east to south-west and the lower 400 miles from nortl to south. Along this lower stretch waves generated by botl monsoons approach the shoreline at an angle of 30°, a condition which has been found by model studies to result in the maximum rate of littoral drift (Saville, 1950).

North Side Erosion - With accretion on the south side, ther was deficiency of material and the consequent erosion on th north side. In a period of 36 years an area of 450 acres land or approximately 450 million c. ft. of material was eroded along this coast for a distance of 3 miles (Fig. 23) To prevent further erosion, stone revetments were laid on this side, which has been found to be successful to this day. The N. E. monsoon which results in a southerly movement of the littoral drift at a rate of 27,000 tons per ye partly restores the eroded portion on this side but the qua tity is so small that its effect is negligible except as a source of trouble to the navigational entrance channel. In fact, a small pocket of sand fills up the northside of the harbour but a month's dredging disposes it off easily.

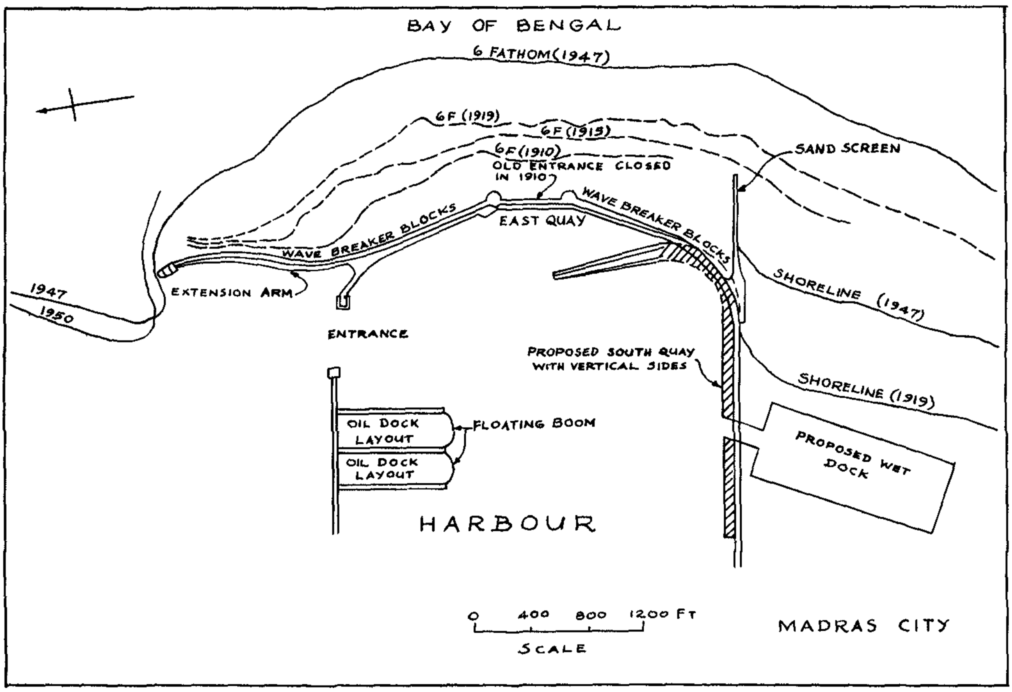


Fig. 24. - Madras harbour: Shoreline and 6 fathom contours.

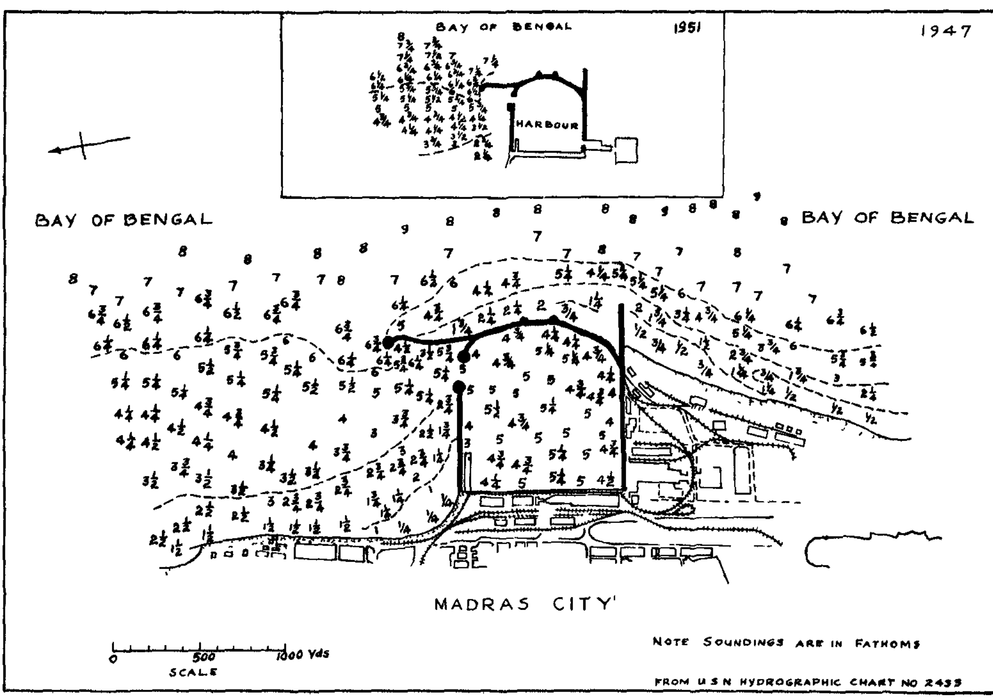
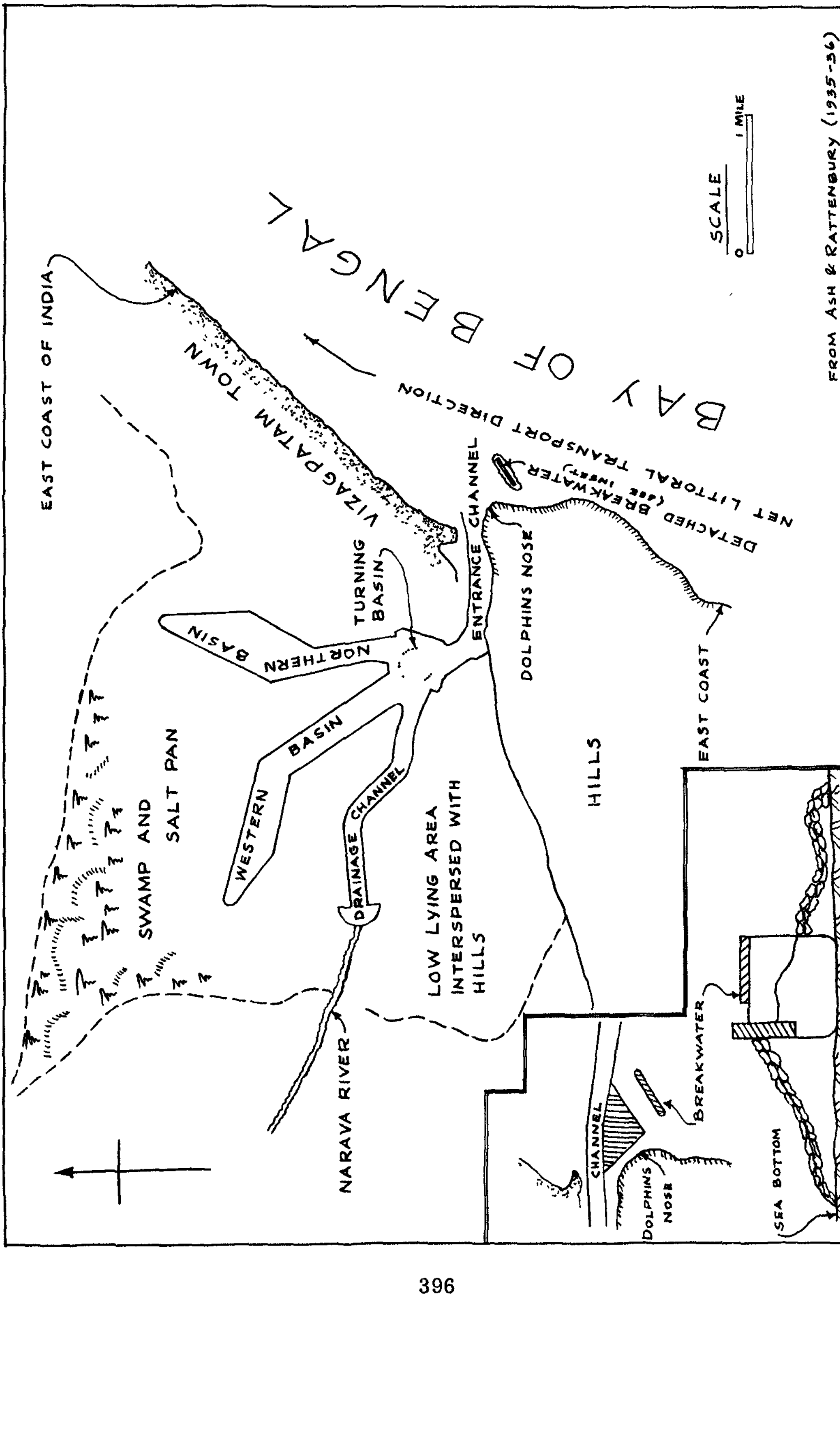


Fig. 25. Madras harbour: Bottom contours in 1947 and 1951.


BAY OF BENGAL
SCALE
0
1 MILE
FROM ASH & RATTENBURY (1935-36)
NET LITTORAL TRANSPORT DIRECTION
DETACHED BREAKWATER (see inset)
EAST COAST OF INDIA
VIZAGAPATAM TOWN
ENTRANCE CHANNEL
DOLPHINS NOSE
TURNING BASIN
NORTHERN BASIN
WESTERN BASIN
DRAINAGE CHANNEL
EAST COAST
HILLS
LOW LYING AREA INTERSPERSED WITH HILLS
SWAMP AND SALT PAN
NARAVA RIVER
CHANNEL
BREAKWATER
DOLPHINS NOSE
BREAKWATER
SEA BOTTOM

Ranging - The trouble at the harbour at the present moment is not from the littoral drift but from ranging in the harbour during the N. E. monsoon from October to December, when cyclonic storms occur in the area with the consequent danger of ships within the harbour breaking their mooring ropes. The ranging is due to short period and long period waves (Central Water & Power Research Station, 1952, 1953). At the harbour, under severe conditions, they are found to be as follows:-

Table 4

No.	Wave period in sea	Wave height at 1000' beyond shelter area	Wave direction outside harbour
1	15.9	12.5	N. E.
2	12.5	15.0	N. E.
3	10.0	15.0	N. E.
1	74	3.12	N. E.
2	59	2.5	N. E.

At the entrance, a range as high as 2'9" has been found to occur (Ministry of Transport, 1946). Such waves are sufficient to render every berth in the harbour untenable since those oscillations extend downward to a considerable depth. Unlike at other harbours where wave action is generally reduced inside the harbour, at Madras, the highest range reading of 3'6" occurs at the southern groyne and not at the entrance because of the fact that the harbour is bounded by four vertical walls which help to build up the range instead of decreasing the wave action.

VIZAGPATAM

Vizagpatam harbour is situated in Latitude 17° 41' 34', Longitude 83° 7' 45" on the east coast in the Circars coast zone. Unlike the coast at and south of Madras where there is a vast alluvial strip of 80 miles in width between the Eastern Ghats and the sea, the coast line for a considerable distance on both sides of Vizagpatam is backed by a continuous succession of rounded hills close to the sea, rocky in some places and sandy at other parts (Ash & Rattenbury, 1935). It is a port with many natural advantages situated at the mouth of Narava river which flows into a bight and then into the sea (Fig. 26). The river has a catchment area of 200 square miles with an average rainfall of 39". Sometimes severe floods are encountered in this area during the months from October to December when a rainfall of 24" may b registered in 3 days and sometimes 12" in one day. During such floods enormous quantity of silt is brought by the rive into the bight and in fact it was a large swamp before its improvement as a harbour. On the south side, it is protect ed by a rocky hill which ends in a bluff headland 4300' sout wards, rising 536' high near the sea. This headland is kno as Dolphin's Nose from its profile. On the north side, it is protected by a sandy spit on which the town of Vizagpatam with its numerous hills, is situated. Most probably the bight was once part of the sea, the sandy spit having been formed at a later stage due to littoral and silting effects in the region. Between the Dolphin's Nose and the sandy spit, a short entrance channel is provided in the gut throug which the river water is discharged into the sea. The coas line in the vicinity of the entrance channel has a north-easterly direction in the northern sandy spit while immediately south of the entrance where the coast is rocky, it takes a sudden bend in the northwesterly direction. This harbour may be classified as a "fall-line harbour". As is the case in almost all cases where a river discharges into the sea, a sand bar across the river outlet used to exist before the entrance channel was dug. The bar at the time of its existence changed its position depending upon the seasor and the year and had a depth as low as 2' below low water level.

Waves and Littoral Drift - The waves approach the harbour ir a direction of about 50° south of east from the end of February to the end of September. During the S. W. monsoon season from May to September, the waves approach the coastline, south of Vizagpatam from a southerly direction resulting in a northward littoral drift. During the roughest period of this season waves of 25' to 30' in height at depths of 25' to 30' may occur and last for 8 to 10 days resulting in northerly drift as high as 200,000 tons. During such ba weather, wind is light indicating that the huge waves are du to some disturbance far away. In October, conditions are variable but by the end of that month, the wave direction changes by about 50° and the waves approach practically due east. This being the N. E. monsoon period, cyclones are of lesser intensity and since the waves approach the shore witl a southern obliquity, there results a southerly drift of sediment of lesser magnitude. In February and October when frequent calms occur, the littoral drift is small.

Tides and Currents - The tidal range varies from 7" in the neaps to 5'9" in the springs. The tidal currents in the harbour are small though they may attain a velocity of 1 kn in the gut. The onshore currents set up by the winds foll generally the same direction as the littoral drift movement but are too feeble to affect the movement of littoral drift which mainly consists of coarse sand and very little of fine sand.

LITTORAL DRIFT AND REMEDIAL MEASURES

The problem at the harbour is, therefore, the effect of littoral drift moving up and down the coast on the entrance channel and the coast line. Just as at Madras, the S. W. monsoon being stronger, the northerly littoral drift is larger in magnitude of the order of 1 million tons per year, a fairly constant quantity that moves up and down along the east coast. At Vizagpatam, the southerly transport amounts to only 200,000 to 300,000 tons - a small quantity that is distributed over a wide area. To keep the entrance channel from getting silted up from the enormous northerly littoral drift either continuous dredging or protective breakwater or both are essential. Maintenance by dredging alone would, not only be impossible and too expensive but also hazardous due to the ordinary swells of 5' to 15' and large waves of 25' to 30' that occur during the S. W. monsoon. The situation is entirely different from that at Cochin on the west coast where the littoral drift is very small and where the ebb tide with its great flushing effect disposes off a large quantity of silt. Therefore, it is necessary to trap the littoral drift before it reaches the channel and the harbour and dispose it off to the northside to prevent erosion on that side.

At the time of harbour construction, two alternatives were possible, namely (1) a continuous breakwater from near Dolphin's Nose extending seaward for a long distance and (2) a detached breakwater serving the same purpose, a short distance away from the shore. As at Madras, a continuous breakwater from Dolphin's Nose would have caused considerable siltation on its southern side in a very short time necessitating continuous dredging on the weather side in order to keep the accretion from creeping around the breakwater and then into the channel. At Madras, there was no other choice, since the main harbour itself is situated within the breakwaters while at Vizagpatam, the harbour is situated inward from the sea in a bight. Also at Madras, low lying sandy beach and deep water areas were very near the shore on the southern side - conditions that were suitable for the construction of a shore connected breakwater with its extension arm. At Vizagpatam that was not possible in the immediate vicinity of the harbour due to the steep rocky face of the Dolphin's Nose. Southwards of Dolphin's Nose, the sea bed consisting of clean sand had a slope of 1 in 100 and majority of the littoral movement was observed to take place in localities where the waves exceeded 6' in height. These were found to be at a short distance away from the shore within a 600' zone in the shallow water areas.

Detached Breakwater - From all these considerations, a detached breakwater on the south side of the channel sufficiently far away from the shore to ensure a comparatively sheltered area and aligned in such a way that sand could be made to pass between it and the Dolphin's Nose and allowed to settle in the sheltered area in its lee, was constructed (Fig. 26). By so doing, it became possible for a dredger t work safely in the sheltered area and transport the trapped littoral material by means of a pipeline to the under-nourished sand spit and adjacent areas on the northside. The breakwater was constructed by sinking two tramp ships stern to stern with their bows forming the two ends of the breakwater. They were sited in comparatively shallow water of depths varying from 18' to 25' below low water. The ships settled only 3' on the sea bottom composed of clear coarse sand. To protect the breakwater from wave action, a rock fill was placed around the ships and in the small space provided between the ships (Fig. 26). The breakwater which wa approximately 1000' long ideally fulfilled its purpose by causing the littoral drift to deposit in the lee of the breakwater from where it was disposed off to the north side. Only 3% of the total drift passes around the outer end of th breakwater. There has been some shoaling between the Dolphin's Nose and the breakwater since its construction.

By dredging the 300' channel and the sand trap, it ha been found possible to keep the harbour in operation at all times without any difficulty. Generally a depth of 33' is maintained in the entrance channel. The harbour consists o an extensive basin eminently fitted for a large harbour and is being expanded to become one of the finest harbours in India.

SUMMARY

Littoral transport which is mainly due to the action of waves depends on rivers, sea bottom, and shore line for its sediment supply. The direction of littoral drift depends mainly upon the configuration of the coastline, the direction of the wind system and the waves. The rate of littoral transport may be obtained by methods similar to Einstein's theory of sediment transport in uni-directional flow in addition to the standard field methods. When equilibrium of movement of littoral drift is disturbed as in the case of harbours where protective breakwaters, groynes, navi gational channels and other similar facilities are provided excessive accretion and erosion take place in the neighbouring zones. To reduce and stop erosion and accretion, preventive measures have to be taken, and they depend upon the amount and direction of littoral drift and the type, locatio and size of the harbour.

The outstanding feature of the wind system in the Indian Ocean is the seasonal reversal of its wind direction known as the monsoon. During the S. W. monsoon from June t September and sometimes earlier in some regions, the wind an the waves approach the east and west coasts of S. India from the S. W. direction creating a northerly littoral drift. Fro

December to March, when the N. E. monsoon prevails, the wind and the waves approach the east coast from a N.E. direction and the west coast from a N. W. direction creating a southerly littoral drift. The stronger S. W. monsoon results in a net northerly littoral drift. Because of the fact that most of the rivers originate from the Western Ghats on the west coast and flow into the Bay of Bengal on the east traversing through alluvial country, a large amount of sediment discharged by these rivers into the sea moves up and down the east coast resulting in a net northerly littoral drift of 1 million tons per year. On the contrary very few rivers flow into the Arabian Sea on the west coast so that the net northerly littoral drift is small of the order of 42,000 tons to 200,000 tons per year with the greater quantity in motion near river outlets.

Cochin harbour on the west coast is formed in a large backwater between the mainland and two narrow peninsulas and may be classified as a channel harbour in tidal estuary. Since the littoral drift is small, erosion and accretion along the adjacent coasts are not great. A large quantity of silt and mud brought into the backwater during the monsoons from the Western Ghats are discharged into the sea by the large ebb tide from the backwater thereby resulting in less siltation in the harbour and adjacent areas. The only protective works found necessary are the groynes and bunds on the shorelines north and south of the channel. The entrance channel and the harbour are maintained by dredging for a few months. Along the west coast within short distances from Cochin there exist mudbanks similar to those found along the Mississippi river delta. They appear suddenly in different places, are not permanent and are generally moved by the stronger southerly currents.

Mangalore port situated on the west coast further north of Cochin may also be classified as a channel harbour in tidal estuary. Unlike Cochin harbour where the backwater forms the tidal estuary, at Mangalore, it is formed at the junction of two rivers and the sea. The littoral drift is about 200,000 tons per year while river deposits are still greater with the result that maintenance by dredging and protective works is impracticable. At present it is only an open roadstead.

Madras harbour on the east coast is a shore-line harbour constructed by the seaward extension of breakwaters from the shore. With the littoral drift at one million tons per year, accretion on the south side is so enormous, that the old entrance on the east side closed 34 years after its construction in 1876 is, at present, replaced by a new entrance on the northside behind a long sheltering arm. A masonry extension at the south eastern end of the harbour extending seaward to deeper areas and serving as a sand screen together with dredging in that locality has been found to reduce the rate of advance of accretion on the south side. The rate o advance of accretion is about 11 ft. per year, at present, a compared to 250 ft. per year at the time of construction of the harbour. Shoaling of entrance by accretion is no longe a threat to the harbour.

Vizagpatam situated on the east coast, north of Madra is formed in a bight fed by a small river and separated from the sea by a rock outcrop and a sandy spit and may be termed a fall-line harbour. A northerly littoral drift of 1 million tons per year exists along this coastal strip as at Madras. To keep the navigational channel from silting, a d tached breakwater about 1000 feet long in the form of two sunken ships is provided on the south side of the channel, a short distance from the shore trapping the littoral drift passing through the gap and causing settlement in its lee. Dredging of the trapped littoral drift from this calm area and its disposal by pipe line to the undernourished north si have been found to be the most effective arrangements at thi harbour.

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CHAPTER 23

LITTORAL DRIFT PROBLEMS IN PORTUGAL WITH SPECIAL REFERENCE TO THE BEHAVIOR OF INLETS ON SANDY BEACHES

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After a brief description of the littoral drift regimen in the west and south coasts of Portugal, review is made of the behavior of the works performed in three lagoon inlets located on these coasts and some general principles are inferred which are felt to be valid in the treatment of any similar problems.

LITTORAL DRIFT REGIMEN

The coast line of continental Portugal has a total length of about 480 miles, of which 380 miles form the west coast, from the mouth of River Minho to the Cape of St. Vincent, and the remaining belong to the south coast, from this Cape to the mouth of River Guadiana (see Fig. 1).

Roughly 300 miles of the shoreline are sandy beaches, sometime more than fifty miles long.

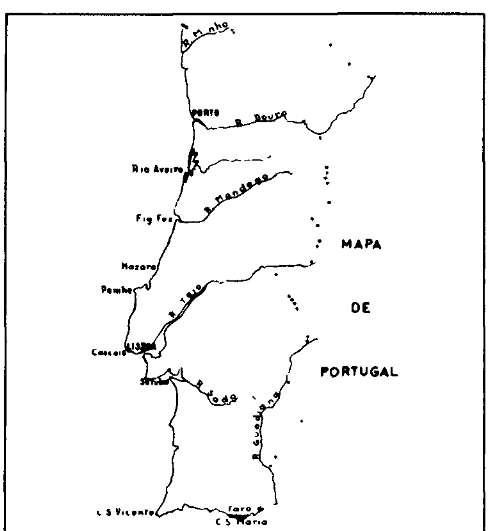

Fig. 1. The coastline of Portugal.

LITTORAL DRIFT PROBLEMS IN PORTUGAL WITH SPECIAL REFERENCE TO THE BEHAVIOR OF INLETS ON SANDY BEACHES

The west coast, which runs approximately north - southwards, is openly exposed to the winds and waves occurring in this area of the North Atlantic Ocean. In normal years, the north winds and seas are predominant, and they specially prevail in summer and the adjoining periods of spring and autumn, that is, in the dry season. South winds and seas are frquent in winter and the first half of spring, during the wet season, and the big storms usually start with their strong blowing. Meteorologically abnormal years may occur from time to time, and sometimes consecutively, in which the wet season lasts longer and the south wind and seas predominate. Similarly, it may arrive, in periods of exceptional drought, that the prevalence of the north winds and seas remains for the whole year and during consecutive years.

None of the mentioned abnormal features is very frequent, and it seems the former is the less common. While the available statistical data are not enough to allow any definite conclusion, the events in the last hundred and fifty years show that such meteorologically abnormal situations have occurred at intervals of 15 to 35 years. This means there is a really and largely predominant meteorological regimen, which therefore must model a marked littoral physiography.

As far as sediment drift is concerned - and in the Portuguese coast sediment means sand for all practical purposes - this littoral physiography is defined by an intensive alluvial movement, which proceeds alternatively southward and northward, according to the meteorological conditions at the moment, the former being predominant in normal years. Full quantitative evaluation of this littoral sand drift is unavailable, but the measures taken in some significant places where works were being carried show that in normal years the southward balance of the foreshore littoral drift amounts to approximately 200,000 c.m., which is about half of the total southward foreshore drift in those years. The offshore drift is harder to evaluate, but its volume is far in excess of this figure, and there are measures of depositions and erosions in outer bars, exclusively fed by littoral sand, at rates of one million c.m. in a few months.

This littoral drift regimen is subject to variations, according to the meteorological features of the year concerned, both as regards intensity and trend of the predominant alluvial movement.

As the major extent of this coast is straightline shaped, local regimens are very few, practically confined to the five bays of Figueira da Foz, Nazare, Peniche, Cascais and Setubal.

The southern coast of the country runs roughly in a west and east direction and its littoral drift regimen is similar to the west coast one, with the difference that modeling agents are less vigorous and local variations more pronounced. Storms are neither so frequent nor so violent. Drift is alternatively eastward and westward,

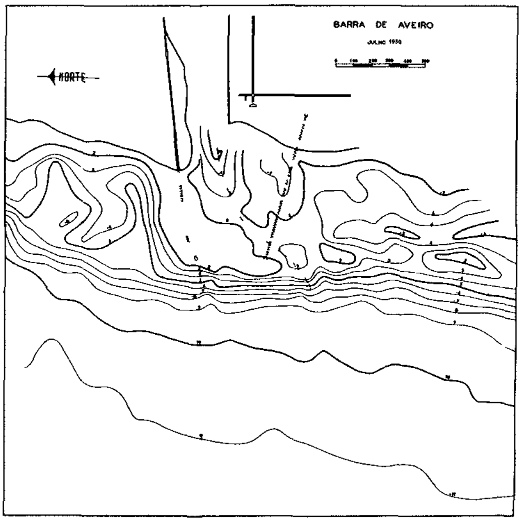

Fig. 2. July - August 1950

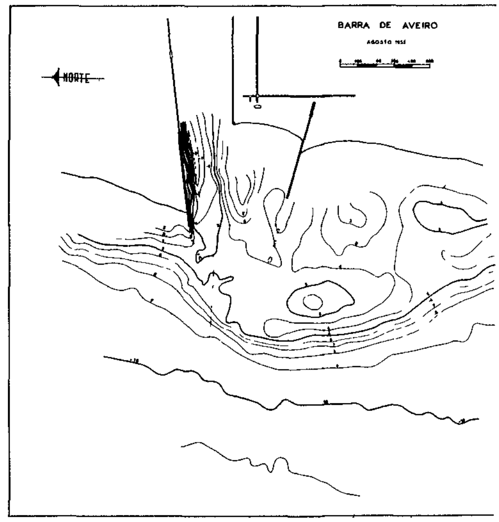

Fig. 3. August 1955

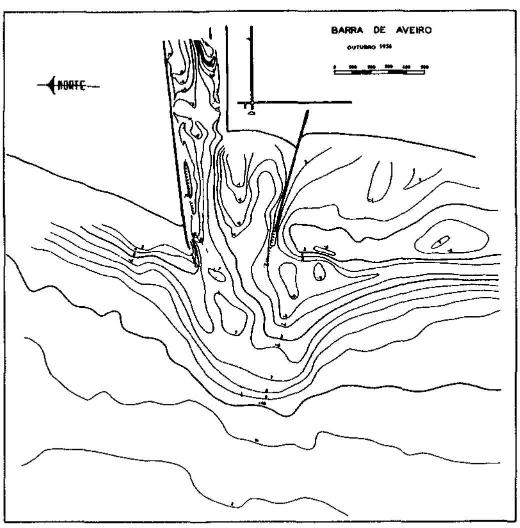

Fig. 4. April 1955

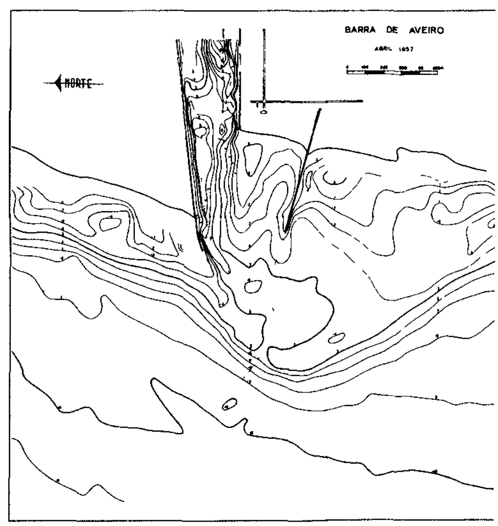

Fig. 5. April 1957

Hydrographic Surveys of the Inlet of Lagoon of Aveiro the former being more or less predominant according to the meteorological feature of the year and to the layout of the coastal stretch concerned, its prevalence being the strongest to the east of Cape Santa Maria.

While there are no systematic evaluations of the amount of sand interested in littoral drift processes, there is evidence of it being much less than the figures registered on the western coast.

The foregoing description is confined to longshore alluvial movement, which is the main process to be considered when dealing with littoral drift problems in the continental Portuguese coastline. In fact, due to the characteristics of the waves occurring and their relative frequences, as also to the nature of sediments available, transversal alluvial movements may be occasionally very intense, specially during big storms, but they are statistically much less important than the longitidinal ones. Of course, they play their part in shaping the alluvial shoreline, but the main modelling process is the longshore drift, with the possible exception of some limited stretches on the south coast.

LAGOONS AND THEIR INLETS

There are two important lagoon systems along the coast: the lagoon of Aveiro in the central west coast, and the lagoon of Faro-Tavira in the eastern south coast. As stated, we intend specifically to treat the littoral drift problems connected with the regimen of the inlet channels giving access to those large bodies of water.

THE LAGOON OF AVEIRO

In the previous paper (C. Abecasis, 1954) we gave a description of the very interesting case of the Aveiro lagoon, as known from the X^{th} century up to the results of the improvement scheme being carried on by the middle of 1954. We shall not repeat the description, but we re-insert, for the sake of confront, the hydrographic survey of the inlet in August 1950, when the scheme was started (see Fig. 2).

It is now convenient to bring up to date the analysis of the behavior of the inlet channel in its reaction to the works undertaken. For the purpose, we shall insert in the graphs and tables of the preceding study the data collected since their publication, retaining the same designations and the numbering of the tables. Some of the hydrographic surveys on which those data were based are also included (see Figs. 3 to 5).

Table 1
Changes in volume of sand on the outer bar of Aveiro

Date of surveys		Changes with reference to 1865(c.m.)	Changes with reference to the preceding survey (c.m.)	
Month	Year		Deposition	Erosion
	1865	-	-	-
XI	1914	1.661.800	1.661.800	-
I	1935	378.000	-	1.283.800
VIII	1949	3.892.700	3.514.700	-
VII	1950	2.457.400	-	1.435.300
III	1951	2.349.200	-	108.200
IV	1951	2.083.100	-	206.100
VI	1951	2.115.200	32.100	-
IX	1951	2.979.730	864.530	-
III	1952	3.104.350	124.620	-
V	1952	2.292.650	-	811.700
VIII	1952	3.180.850	888.200	-
XI	1952	4.089.320	908.470	-
I	1953	3.516.790	-	572.530
V	1953	2.813.250	-	703.540
IX	1953	2.463.100	-	350.150
I	1954	3.258.530	795.430	-
IV	1954	3.107.170	-	151.360
VII	1954	2.977.230	-	129.940
III	1955	2.650.380	-	326.850
VIII	1955	3.585.250	934.870	-
X	1956	2.964.400	-	620.850

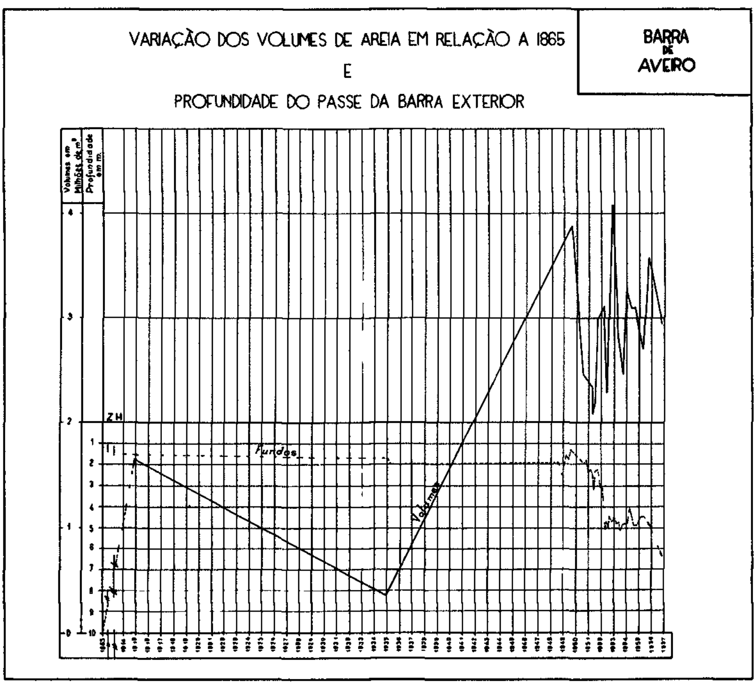

Fig. 6. Variation of the volume of sand in the outer bar of Aveiro

Table 1 and Fig. 6 show that the volume of sand in the outer bar platform remains stable, with the seasonal variations assigned before. As meteorologically anomalous years did not occur, no systematic alteration can be traced.

The projections of the longitudinal profiles of the entrance channel on a vertical plane parallel to the old southern jetty (see Fig. 7) entirely confirm what has been inferred in the 1954 analysis, i.e., as the works proceed and the lagoon's ebb current is reinforced, the outer bar is displaced seawards, widened and deepened (note the 1956 survey). Also confirmed is the then suggested envelop curve of the controlling depths at the outer bar's crest as a function of this crest's distance to a fixed base-line across the inlet canal (see Fig. 8).

Tables 3 and 4* and Figs. 9 to 12 demonstrate that the inlet channel's hydraulic characteristics favorable reaction to the works is consolidated, if not slightly improved, which, as previously stated, is of vital importance to the maintenance of depths in the whole access to the port.

In the preceding paper we pointed out the great significance of the behavior of the inner bar as an index of the soudness of the inlet's improvement scheme being executed. Fig. 13 to 16 and Table 5 show that the favorable results obtained can now be considered definitively acquired.

Moreover, additional data show that these beneficial results are not restricted to the area of the inner bar on the main lagoon channel. Thus, the access channel to the southern lagoon branch of Mira, which for many years had been starving and slowly shoaling, spontaneously deepened, specially since 1953 (see Figs. 18 and 19).

This, again, clearly means that littoral sands are not retained by the inlet in their way down-coast, as the volume of sand expelled by the ebb tide fairly exceeds that brought in by the flood. And if so, the project being carried on correctly solved the problem of assuring the depths required (indeed, more than required) in the channel with the least interference with the littoral drift regimen: in fact, a strictly localized interference, both in the time and in the space, as is also confirmed by the absence of any permanent erosion effects on the down-drift section of the sea shoreline.

* A mistake in the computations of Table 4 concerning years 1934, 1945, 1951, is now corrected. It did not affect the conclusions inferred from the Table's analysis.

Table 3

Lagoon of Aveiro
Areas of the inlet's channel cross sections

Dates Ranges	1865	1914	1934	VIII 1945	VII 1948	V 1950	V 1951	V 1952	V 1953	V 1954	V 1955	V 1956
P.1	-	410	683	-	-	913	1.062	1.220	1.497	1.595	1.679	1.654
P.2	523	403	633	-	917	909	1.032	1.191	1.640	1.640	1.594	1.634
P.3	549	499	510	804	888	802	1.068	1.237	1.743	1.511	1.722	1.724
P.4	668	647	595	769	741	789	1.216	1.234	1.677	1.635	1.720	1.669
P.5	879	520	535	721	888	866	1.186	1.102	1.730	1.528	1.626	1.626
P.6	1.064	505	591	939	918	957	1.192	1.223	1.491	1.653	1.675	1.931
P.7	736	370	623	925	1.155	1.101	1.237	1.155	1.309	1.520	1.730	1.655
P.8	599	428	621	1.229	1.345	1.138	1.254	1.154	1.390	1.471	1.607	1.712
P.9	546	390	678	1.296	1.504	1.203	1.277	1.246	1.504	1.524	1.650	1.820
P.10	505	423	659	1.342	1.825	1.266	1.417	1.308	1.674	1.691	1.955	2.010
P.11	517	643	579	1.837	2.122	1.945	1.830	1.470	-	2.139	2.000	2.050

Areas, in sq.m., under datum

Table 4 (Part A)

Lagoon of Aveiro (1934-1952)
Hydraulic elements of the inlet's cross sections

Dates	1934			VIII-1945			V-1951			VIII-1952		
Range number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m
P.1	1.123	245	4.6	-	-	-	1.522	245	6.2	1.838	308	6.0
P.2	993	200	5.0	-	-	-	1.482	238	6.2	1.974	300	6.6
P.3	850	190	4.5	1.164	216	5.4	1.526	270	5.6	2.031	308	6.6
P.4	975	210	4.6	1.039	183	5,7	1.756	294	5.9	1.877	318	5.9
P.5	775	185	4.2	1.031	160	6.4	1.746	295	5.9	1.755	324	5.4
P.6	941	160	5.9	1.289	167	7.7	1.752	307	5.7	1.727	329	5.3
P.7	1.083	255	4.2	1.295	200	6.5	1.837	311	5.9	1.956	340	5.8
P.8	1.061	238	4.5	1.689	242	6.6	1.712	305	5.6	2.024	343	5.9
P.9	1.178	255	4.6	1.856	307	6.0	1.867	308	6.0	1.988	344	5.8
P.10	1.089	228	4.8	1.902	305	6.2	2.077	342	6.0	2.100	353	5.9
P.11	1.045	213	4.9	2.467	334	7.3	2.530	359	7.0	2.354	344	6.8

Notes: a) Wetted area
p) Wetted perimeter
$R = \frac{a}{p}$ - Hydraulic radius m
Sections under mean level (+ 2,00 above datum)

Table 4 (Part B)

Lagoon of Aveiro (1953-1956)
Hydraulic elements of the inlet's cross sections

Dates	IX-1953			VII-1954			V-1955			V-1956		
Range number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m
P.1	2.070	312	6.6	2.252	303	7.4	2.280	297	7.7	2.260	300	7.5
P.2	2.051	314	6.5	2.340	308	7.6	2.200	300	7.3	2.250	301	7.5
P.3	2.127	334	6.4	2.167	300	7.2	2.305	307	7.7	2.350	308	7.6
P.4	2.414	327	7.4	2.267	305	7.4	2.356	301	7.5	2.305	300	7.7
P.5	2.349	332	7.1	2.347	330	7.1	2.347	337	6.9	2.412	339	7.0
P.6	2.291	343	6.7	2.376	334	7.1	2.334	331	7.1	2.589	335	7.7
P.7	2.190	343	6.4	2.277	337	6.7	2.382	343	6.9	2.325	341	6.8
P.8	1.989	345	5.8	2.233	343	6.5	2.312	348	6.6	2.390	349	6.7
P.9	1.941	345	5.6	2.198	343	6.4	2.308	344	6.7	2.502	345	7.2
P.10	2.167	362	6.0	2.392	352	6.8	2.712	353	7.7	2.710	354	7.6
P.11	2.405	363	6.6	2.649	354	7.5	2.713	354	7.6	2.750	355	7.7

Notes: a) Wetted area
p) Wetted perimeter
$R = \frac{a}{p}$ - Hydraulic radius m
Sections under mean level (+2,00 above datum)

Table 5 (Part A)
Lagoon of Aveiro (1948-1952)
Hydraulic elements of the inlet's cross sections
(inner channel)

Dates	VII-1948			VII-1950			V-1951			XI-1952		
Range number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m
P.12	1.639	283	5.7	1.406	281	5.0	1.774	278	6.4	1.680	280	6.0
P.13	1.818	321	5.6	1.873	323	5.8	1.984	322	6.1	2.163	320	6.7
P.14	1.870	323	5.7	1.805	327	5.5	1.792	326	5.5	1.758	324	5.4
P.15	1.895	358	5.3	1.885	360	5.2	1.967	356	5.5	2.065	358	5.8
P.16	2.190	374	5.8	2.123	379	5.6	2.188	379	5.7	2.058	375	5.5
P.17	2.320	410	5.7	2.504	409	6.1	2.350	408	5.7	2.060	407	5.0
P.18	2.742	468	5.9	2.694	468	5.7	2.392	471	5.0	2.450	469	5.2
P.19	2.625	523	5.0	2.600	522	4.8	2.438	522	4.6	2.508	522	4.8
P.20	2.665	549	4.8	2.450	549	4.5	2.330	550	4.2	2.008	549	3.7
P.21	2.506	585	4.3	2.446	583	4.2	2.294	583	3.9	1.945	582	3.3
P.22	2.311	605	3.8	1.875	603	3.1	2.208	603	3.6	1.923	603	3.2
P.23	2.302	604	3.8	1.885	603	3.1	2.027	603	3.3	1.878	603	3.1
P.24	2.225	613	3.5	2.052	610	3.3	2.158	613	3.5	2.172	613	3.5
P.25	2.347	609	3.8	2.097	605	3.4	2.321	606	3.8	2.090	605	3.4
P.26	2.440	558	4.3	1.690	552	3.1	2.105	553	3.8	1.808	550	3.3
P.27	2.290	635	3.6	2.150	632	3.4	2.367	634	3.5	1.850	632	2.9
P.28	2.345	618	3.7	2.088	617	3.3	2.174	617	3.5	1.943	615	3.1
P.29	2.195	547	4.0	2.015	546	3.7	2.135	547	3.9	1.830	543	3,4

Notes: a) Wetted area
p) Wetted perimeter
$R = \frac{a}{p}$ Hydraulic radius m
Sections under mean level (+2,00 above datum)

Table 5 (Part B)
Lagoon of Aveiro (1953-1956)
Hydraulic elements of the inlet's cross sections
(inner channel)

Dates	XI-1953			VII-1954			V-1955			V-1956		
Range number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m
P.12	2.105	281	7.4	2.025	281	7.2	1.995	284	7.0	2.232	277	8.0
P.13	2.312	322	7.1	2.430	322	7.5	2.390	318	7.5	2.513	315	7.9
P.14	2.133	324	6.6	2.245	325	6.9	2.309	329	7.0	2.542	322	7.8
P.15	2.295	359	6.3	2.288	357	6.4	2.245	357	6.2	2.492	350	7.1
P.16	2.347	372	6.6	2.650	374	7.1	2.752	375	7.3	2.580	373	6.9
P.17	2.687	408	6.5	2.589	410	6.3	2.891	411	7.0	2.444	408	6.0
P.18	2.601	469	5.5	2.592	465	5.5	2.590	471	5.5	2.673	471	5.6
P.19	2.488	523	4.7	2.650	523	5.1	2.530	523	4.8	2.825	525	5.3
P.20	2.600	551	4.7	2.425	550	4.4	2.473	550	4.5	2.795	551	5.1
P.21	2.797	586	4.8	2.579	584	4.4	2.532	585	4.3	2.755	587	4.7
P.22	2.639	606	4.3	2.495	604	4.1	2.531	604	4.2	2.717	606	4.5
P.23	2.314	605	3.8	2.425	604	4.0	2.417	605	4.0	2.532	605	4.2
P.24	2.615	615	4.2	2.381	612	3.9	2.595	612	4.2	2.554	609	4.2
P.25	2.582	607	4.2	2.967	610	4.8	2.886	595	4.8	2.812	600	4.7
P.26	2.252	558	4.0	2.448	555	4.4	2.414	550	4.4	2.448	551	4.4
P.27	2.473	637	3.9	2.527	635	4.0	2.370	634	3.7	2.394	635	3.7
P.28	2.331	617	3.6	2.404	617	3.9	2.317	616	3.7	2.485	610	4.1
P.29	2.281	549	4.1	2.341	547	4.3	2.332	542	4.3	2.280	540	4.2

Notes: a) Wetted area
p) Wetted perimeter
$R = \frac{a}{p}$ Hydraulic radius m
Sections under mean level (+2,00 above datum)

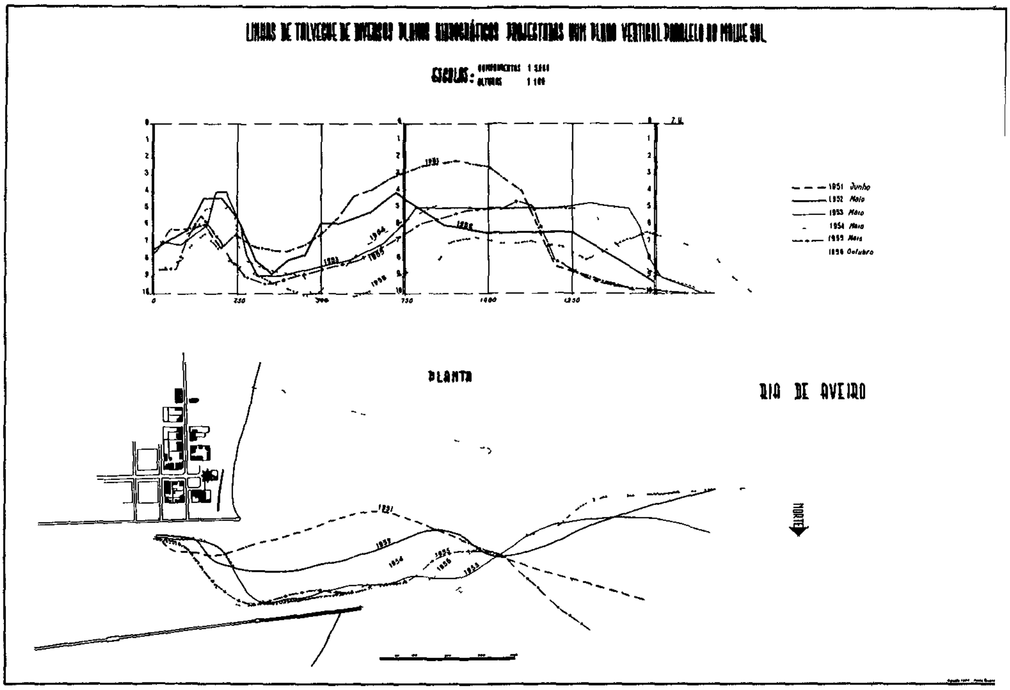

Fig. 7. Longitudinal profiles of the entrance channel projected on a vertical plane parallel to the old southern jetty

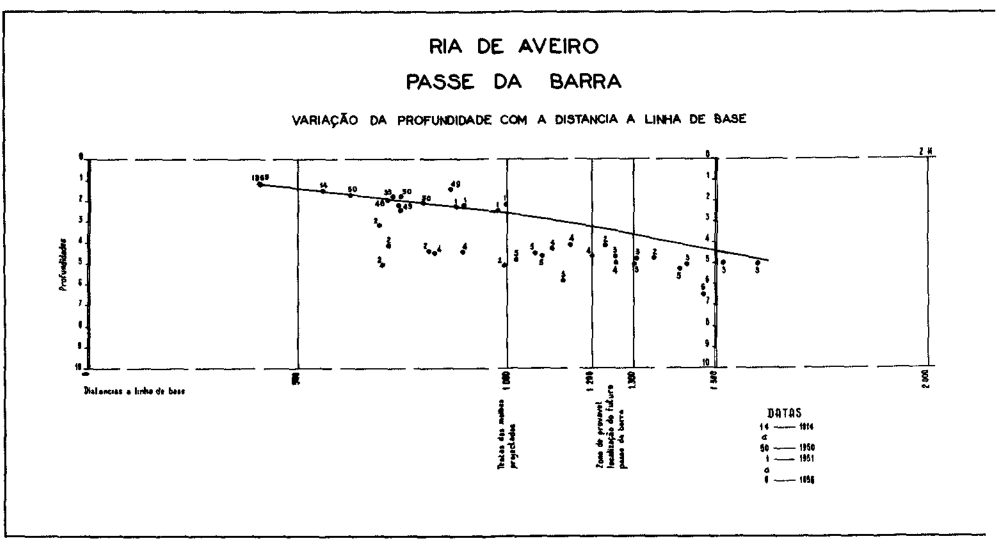

Fig. 8. Controlling depths on the outer bar (ordinates) as a function its distance to a base-line (abscissae). Envelop-curve of th minimum depths corresponding to each position of the bar-c

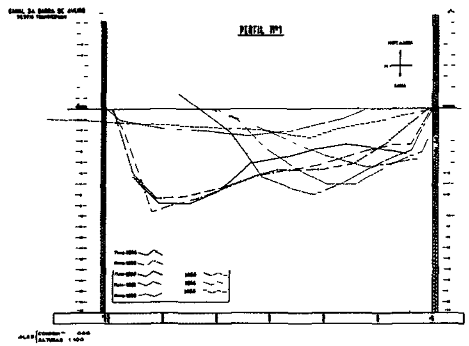

Fig. 9

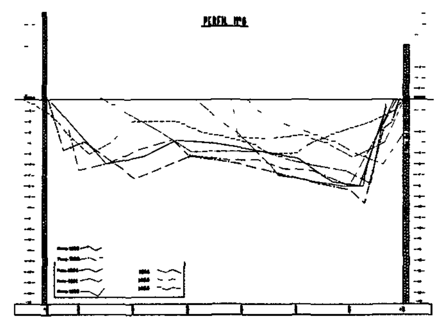

Fig. 10

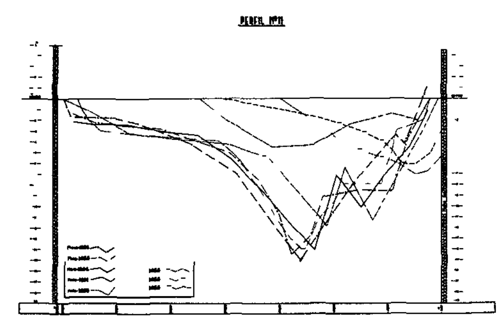

Fig. 11

Fig. 9 - Evolution of the inlet channel's cross sections due to the works (range nº.1)

Fig. 10 - Evolution of the inlet channel's cross sections due to the works (range nº.6)

Fig. 11 - Evolution of the inlet channel's cross sections due to the works (range nº.11)

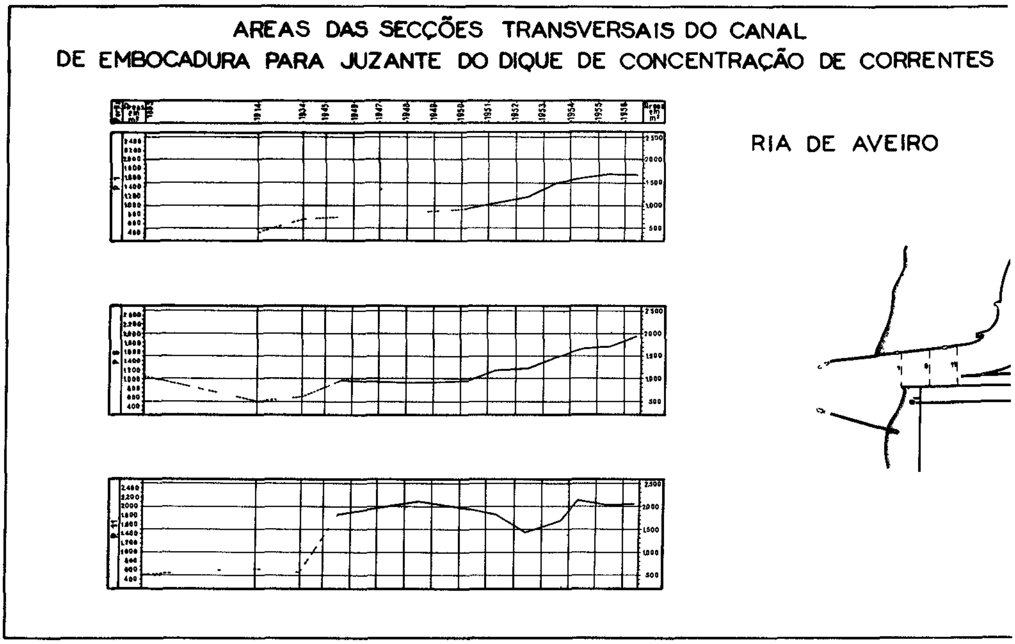

Fig. 12. Variation in the areas of cross sections at the same ranges

Fig. 13. Hydrographic survey of the inner channel in May 1956

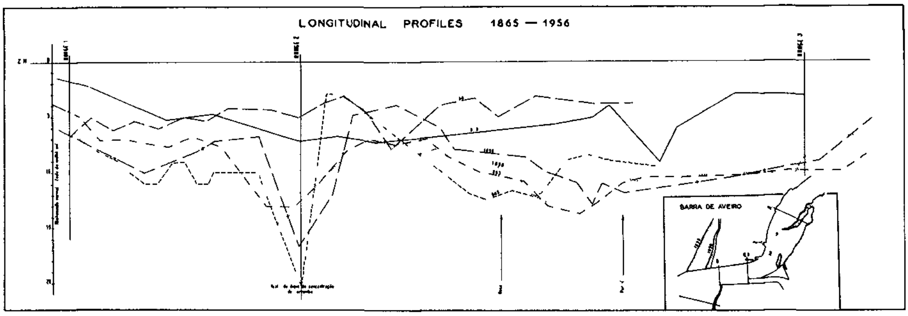

Fig. 14. Longitudinal profiles along the talweg between the inlet's mouth and S. Jacinto, from 1865 to 1956

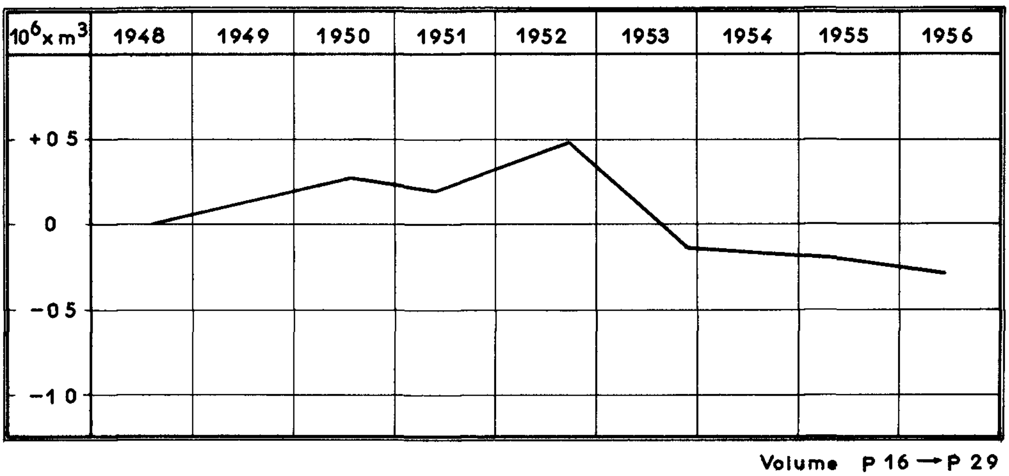

Fig. 15. Variation in the volume of sand on the inner bar from 1948 to 1956

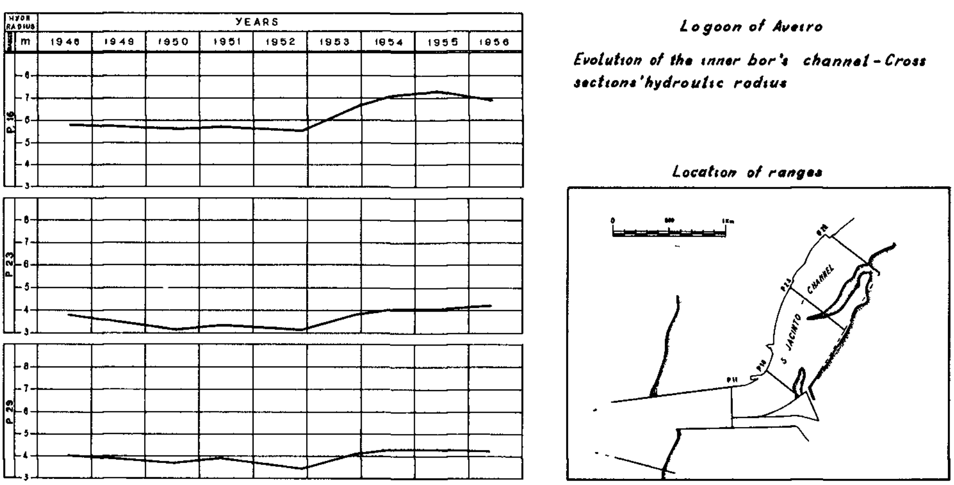

Fig. 16. Variation in the hydraulic radius of the inner channel's cross sections from 1948 to 1956

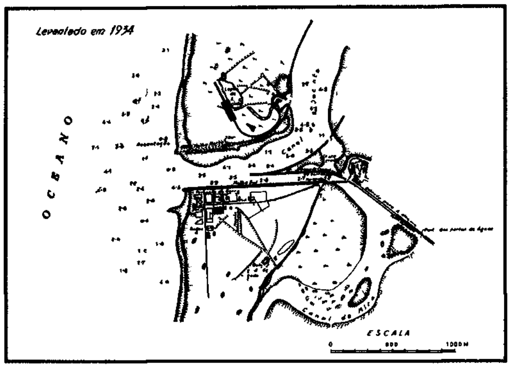

Fig. 17. The Mira inner channel lay-out

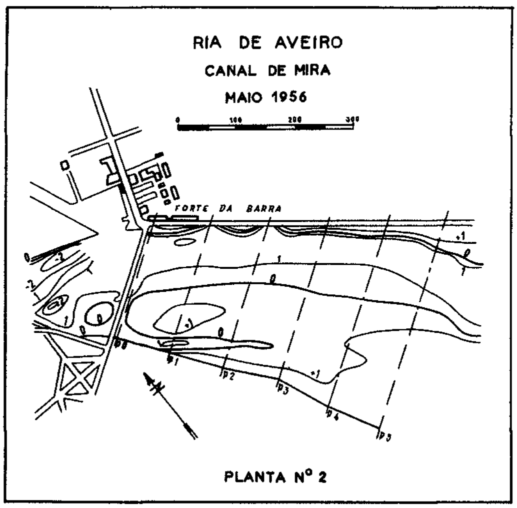

Fig. 18. The Mira channel of the Lagoon: hydrographic survey of the downstream section in 1956

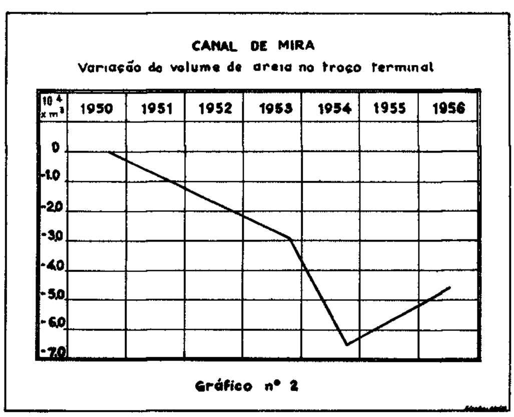

Fig. 19. Variation in the volume of sand on the same section of the Mira channel from 1950 to 1956

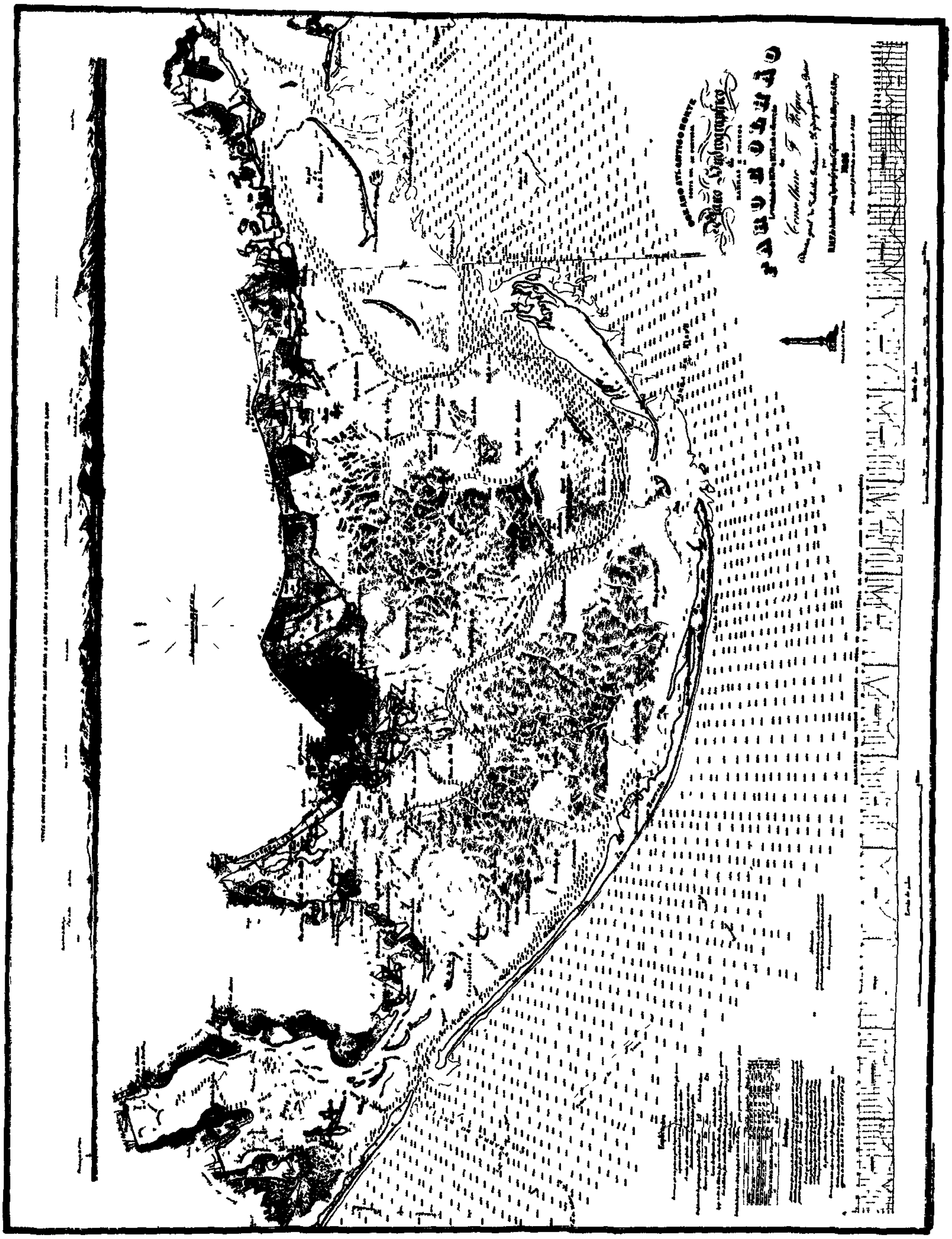

Fig. 20. The western and central part of the south coast Lagoon

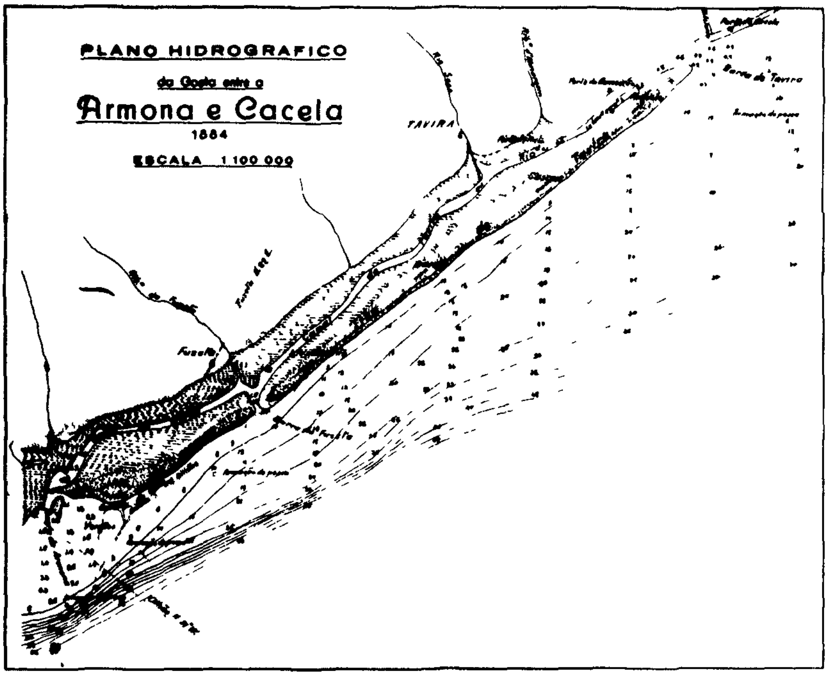

Fig. 21. The eastern part of the south coast Lagoon

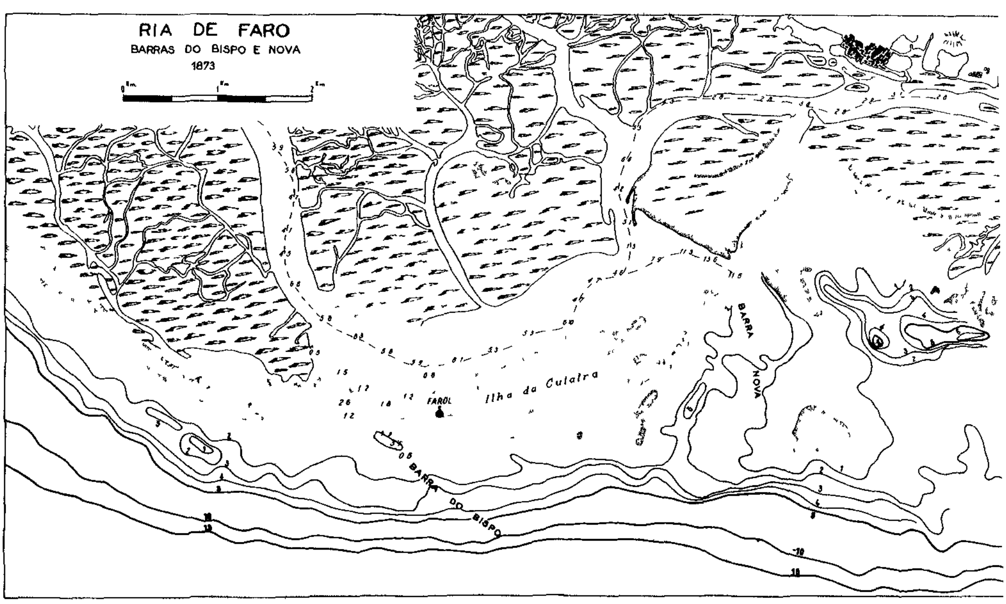

Fig. 22. The inlets of Faro - Olhao in 1873

LITTORAL DRIFT PROBLEMS IN PORTUGAL WITH SPECIAL REFERENCE TO THE BEHAVIOR OF INLETS ON SANDY BEACHES

THE LAGOON OF FARO-TAVIRA

The barrier beach limiting this lagoon extends along the whole eastern third of the country's south coast, with a length of about thirty miles. It is rather a lagoon system than a single lagoon, as there are several inlets giving access to different lagoon areas, more or less individualized, while all interconnected (see Figs. 20 and 21). These are elongated and parallel to the coast-line and in the major part relatively narrow.

Historical background of the physiography of this lagoon system has not yet been thoroughly investigated as it was in the case of the lagoon of Aveiro (C. Abecasis, 1951 and 1954). Therefore its formation is not so well understood, and is supposed to be somewhat more complex.

Nevertheless, a plausible explanation for this great accumulation of sands has been proposed and seems to be confirmed by the available data. Accordingly, the inflection of the coastline in a southeastward direction immediately to the east of Quarteira gave rise, due to the prevailing southwest winds and seas, to a massive deposition of sediments proceeding from the active erosion of shoreline to the west of it. To the east of the big projection of Cape Santa Maria, the coastline recedes and suddenly turns into a northeastward direction, with the consequence that the littoral drift, strongly pushed forward by the prevailing seas from the southwest, becomes very intense (D. Abecasis, 1926).

Behavior of the lagoon system's inlets is in agreement with this line of thought: to the west of the Cape there is only one inlet, shallow and relatively stable, while to the east, starting immediately after the Cape, there are and there have been several inlets, sometimes naturally deep but always unstable in their configuration and position, some of them fastly migrating.

We intend to deal with the cases of two inlets which in recent times were subject to improvement works: the inlet of Faro-Olhao and the inlet of Tavira, the former located near Cape Santa Maria, the latter to the far east of the lagoon.

The inlet of Faro-Olhao - The central and most important lagoon basin is situated between the towns of Faro and Olhao and the littoral sandy island or barrier beach, in the vicinity of Cape Santa Maria. Its wet area is roughly twenty square miles, including numerous marshes and the channels; its length is about eight miles and its maximum width, between the barrier beach and the mainland, approximates three miles. The Cape is just at mid-length of the basin.

This lagoon section is connected with the adjoining ones by narrow and shallow channels and with the sea by two inlets: one which was artificially opened and canalized near the Cape in 1928, and anoth one, about three miles eastward, which is called "Barra Nova", i.e. "New Inlet", a natural one (for approximate locations see Fig. 22).

It is worth to briefly review the behavior of these inlets under the acting natural agents (A. Loureiro, 1909 and D. Abecasis, 1926).

In 1861, the barrier island immediately to the east of the Cape was broken off during some storm occurring in a period of poor sediment feeding of the beach, and a narrow and deep inlet originated which was called "Barra do Bispo". Shortly afterwards, this proved to be very unstable and divided into two shoaling and moving channels, which swung across the sands of the island and migrated eastward, quickly deteriorating (see Fig. 23) and becoming completely closed by the end of the century, when the continuity of the barrier beach was re-established.

The "Barra Nova" inlet was by 1870 a wide and deep one, freely connecting with the eastern end of the main lagoon channel, where natural depths of 25 to 45 feet existed. The outer stretch of the channel points to the southwest, which is a generalized feature in this coast and is probably due to the trend of the coastal ebb-current; while the inner section is pushed to the northeast by the powerful lagoon current and the strong littoral sand drift. Those circumstances impose to the inlet channel a very defective and unstable configuration, in a long curv of small radius of curvature, across numerous shoals. Yet in the last century it was reported as a wide and fairly deep channel, but unsafe for navigation due to its instability and bent lay-out.

The subsequent free evolution of this natural inlet which for about thirty years was practically the only one connecting the main lagoon body to the ocean, as shown in Figs. 24 and 25, deserves a careful and detailed analysis by any one who may be interested in this kind of problem. For the moment, we simply want to point out that in spite of its size, of the big tidal flow circulating through and of the massive barrier of sandy islands and shoals lying down-drift, this inlet badly deteriorated and migrated eastward in the course of the years.

When, due to navigation's increased requirements and to the shoaling of the "Barra Nova" inlet, the necessity for an improved access became imperative and the impossibility of obtaining it by any natural or self-maintaining channel was evident, the argument was raised as to whether it would be advisable to artificially improve and correct the existing inlet rather than trying to establish

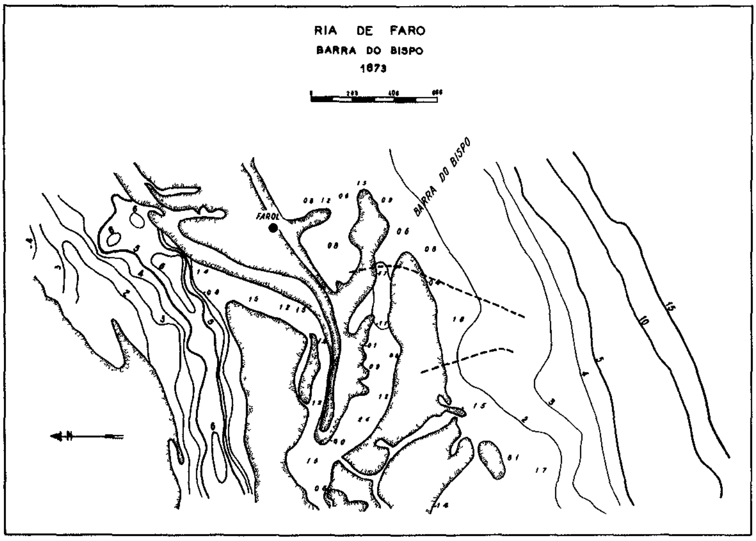

Fig. 23. The inlet of Cape Santa Maria by 1873

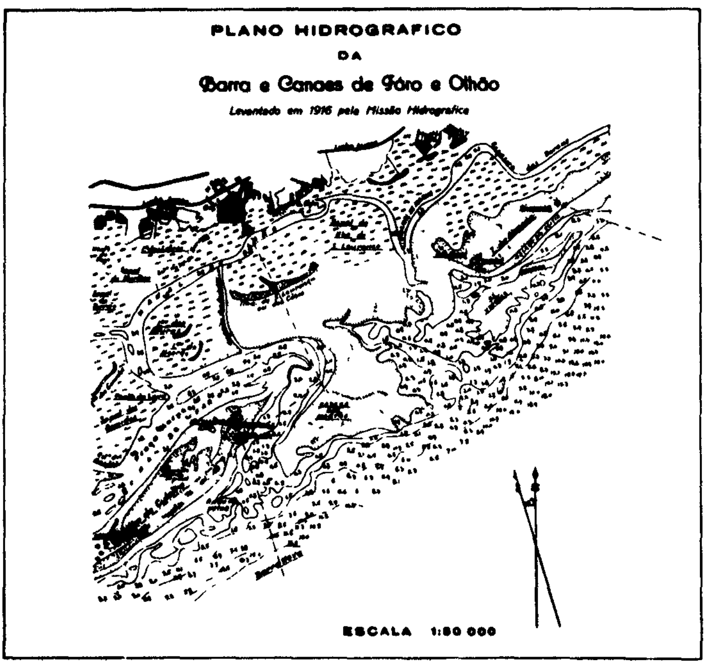

Fig. 24. The "Barra Nova" inlet in 1916

Fig. 25. The "Barra Nova" inlet in 1930

an entirely new one, and if so, as to the best location to be chosen.

The study of the local physiography which was undertaken let to the latter, based on the following reasons:

(a) It would be easy to pierce the barrier island near and to the east of Cape Santa Maria, and so to restore the ancient "Barra do Bispo" inlet, to connect it frankly with the main lagoon channel about midway between Faro and Olhao, to fix it by canalization through the barrier-beach and to direct its outer alignment in the best way concerning the circulation of the tidal flow, i.e. to SW or SSW;

(b) This site, near the projection of the Cape, would afford the occurrence of two very favorable factors as regards the self-maintenance of the new entrance channel, namely, the strength of the littoral current and the immediate proximity of the outside deep water.

(c) Should a deeper entrance channel be required, it would be easy to build a system of outer breakwaters, that wouldn't need any exaggerated length to reach the suitable depths.

Briefly, an artificial inlet so located and duly designed would be given good hydraulic conditions and the vicinity, both of the lagoon waters' center of masses and the outer deep waters.

Based on these reasonings, a scheme of works was designed and carried forward, as shown in the included hydrographic surveys. The first phase, piercing of the barrier-beach and fixing the inlet through it by means of the concave shore's revetment and the building of inner stretches of the breakwaters, was carried from 1928 to 1931. The second phase, completing of the outer stretches of breakwaters, was carried on from 1947 to 1955.

Results obtained can be seen in the hydrographic surveys (Figs. 26 to 31) and in the graphs and tables which are included and are similar to those concerning the inlet channel of Aveiro.

When comparing, it must be borne in mind that the physiographic factors' activity is much less intense in the south coast and that the lagoon tributary to the inlet of Faro-Olhao is much smaller than the lagoon of Aveiro and is connected with the ocean by another important inlet. Consequently, slower reactions and less important sediment movement are to be expected. It must also be recalled that, until now, the studies were not carried forward to the same extent as in the case of Aveiro, neither as regards the items examined nor as to the area covered.

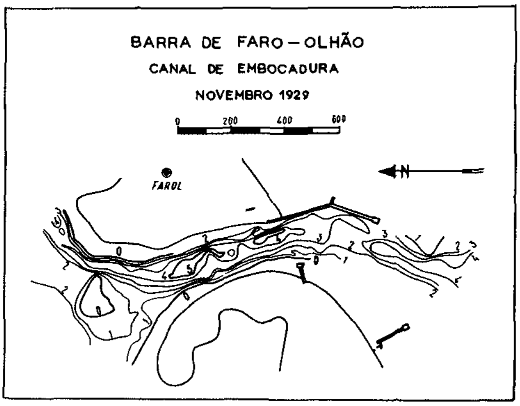

Fig. 26. November 1929

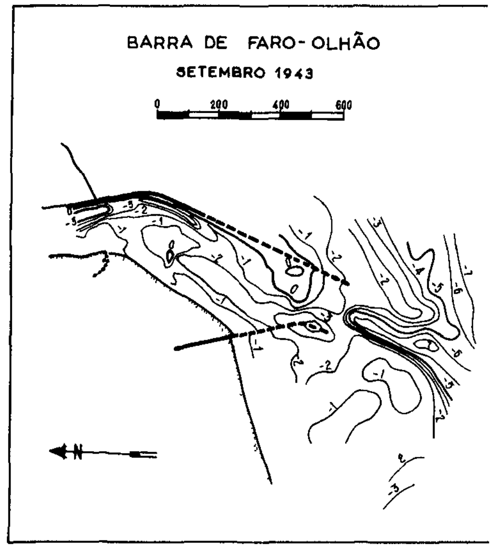

Fig. 27. September 1943

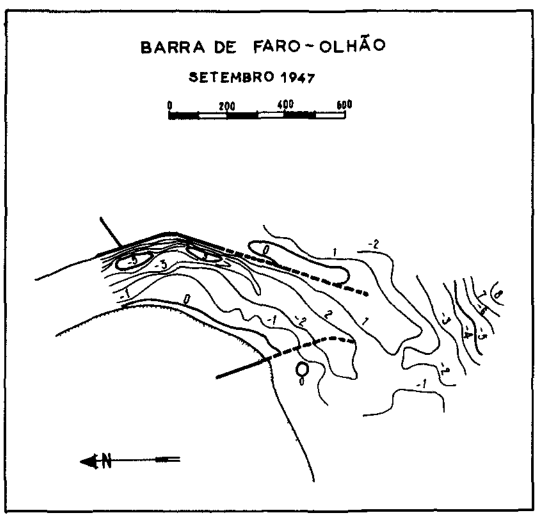

Fig. 28. September 1947

Hydrographic Survey of the Artificial Inlet near Cape South Maria

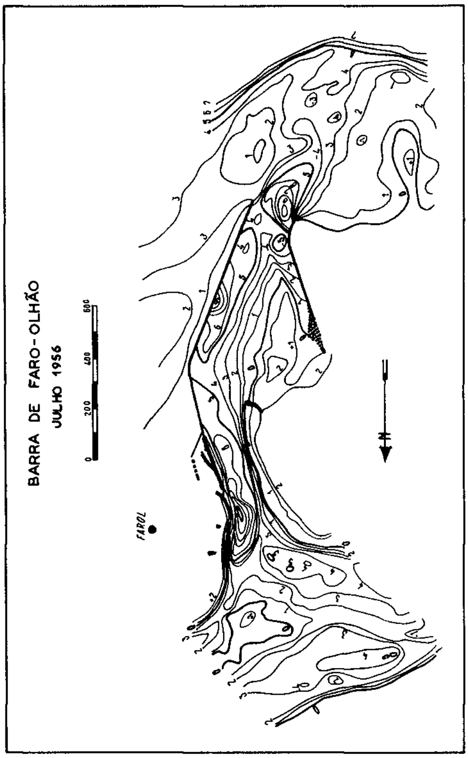

Fig. 30. August 1956

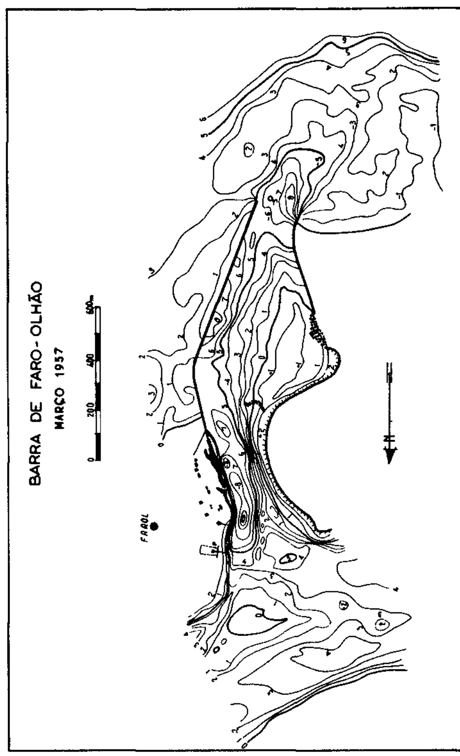

Fig. 31. March 1957

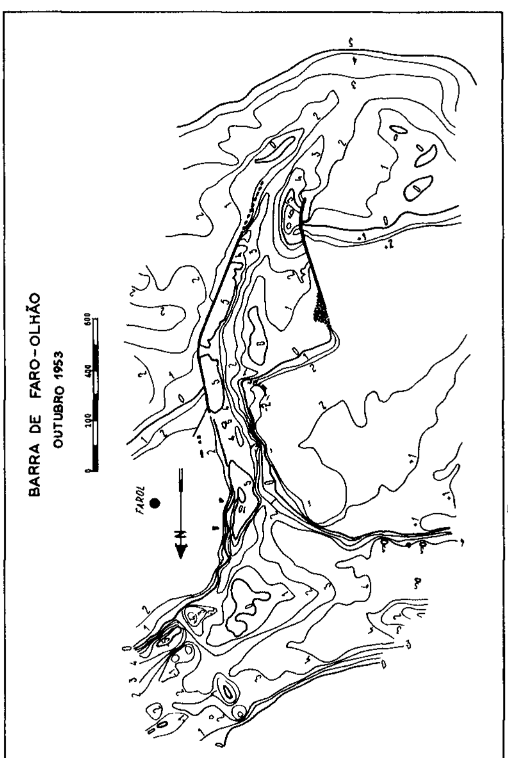

Fig. 29. October 1953

The slower reaction of the inlet to the improvement works is quite evident in the variation of the volume of sands and the controlling depths on the outer bar (see Table 6 and Fig. 32). These data also show that the meteorological influence on the volume of sands lying on the bar is of an entirely different order of magnitude than it was in the western coast's inlet. So, it is not extraordinary that the deepening of the outer bar crest was not noticed until nearing the completion of the breakwaters in 1955 (see Fig. 33). This effect was then helped by dredging in 1955-1956 some 100,000 cubic meters of sand in the outer bar and entrance channel.

But where the progressive beneficial results of the improvement works could be noticed almost step by step was in the amelioration of the hydraulic characteristics of the inlet channel as the works proceeded and in the spontaneous disappearance of the inner bar in the branch channel that leads to Faro (see Table 7 and Fig. 34 to 37). This is the best guarantee of an easier circulation of the tidal flow, whi in turn is the most efficient agent for the maintenance of depths in the entrance channel (Fig. 38).

It is also to be stressed that the absence of a sand accumulation on the updrift side of the works beyond certain limits, as well as the absence of any appreciable shoaling in the inner channel and of any systematic erosion in the down-drift beach, are good tests of the ability of the executed scheme to meet the requirements of the project. It is legitimate to infer, here again, that the littoral sands are transposing the inlet and not retained by it.

The inlet of Tavira. This inlet connects the far east section of the southern coast's lagoon system with the ocean. This section of the lagoon, which lies eastward of Olhao, is composed of a single channel with adjoining marshes and extends for about 17 miles parallel to the sea coast of the barrier-islands and to the shore of the mainland.

By 1884, the ancient inlet of Tavira, formerly situated just opposite the town and the lagoon outlet of the river crossing it, had migrated eastward for more than five miles, under the strong push of the littoral sand drift. By the end of the century, this migration had progressed for some additional 2.5 miles and the inlet reached the end of the lagoon, where it quickly shoaled. Some time later, a new inlet was opened by a storm, about one mile to the west, which took a curved configuration similar to the above-mentioned "Barra Nova" inlet and stood for years (see Fig. 39).

Later on, the deterioration of this natural inlet and the necessity of meeting the navigation requirements, specially of the important fishing activities, gave rise to the project of artificially restoring the

Table 6

Changes in the volume of sand on the outer bar of Faro-Olhao

Date of surveys		Changes with reference to 1946(c.m.)	Changes with reference to the preceding survey (c.m.)	
Month	Year		Deposition	Erosion
XII	1946	-	-	-
III	1947	101.515	101.515	-
IX	1947	152.990	51.475	-
X	1948	11.990	-	141.000
V	1949	223.525	211.535	-
XII	1949	48.530	-	174.995
IV	1950	78.490	29.960	-
VI	1950	214.970	136.480	-
VII	1950	201.810	-	13.160
IV	1951	193.710	-	8.100
VII	1951	183.875	-	9.835
III	1952	178.785	-	5.090
III	1953	35.829	-	142.956
X	1953	179.910	144.081	-
III	1954	189.950	10.040	-
V	1954	309.190	119.240	-
VII	1954	253.825	-	55.365
VIII	1954	212.885	-	40.940
IX	1954	218.200	5.315	-
IX	1954	235.100	16.900	-
XI	1954	169.255	-	65.845
II	1955	136.060	-	33.195
VI	1955	74.030	-	62.030
II	1956	4.985	-	69.045
VII	1956	41.895	36.910	-

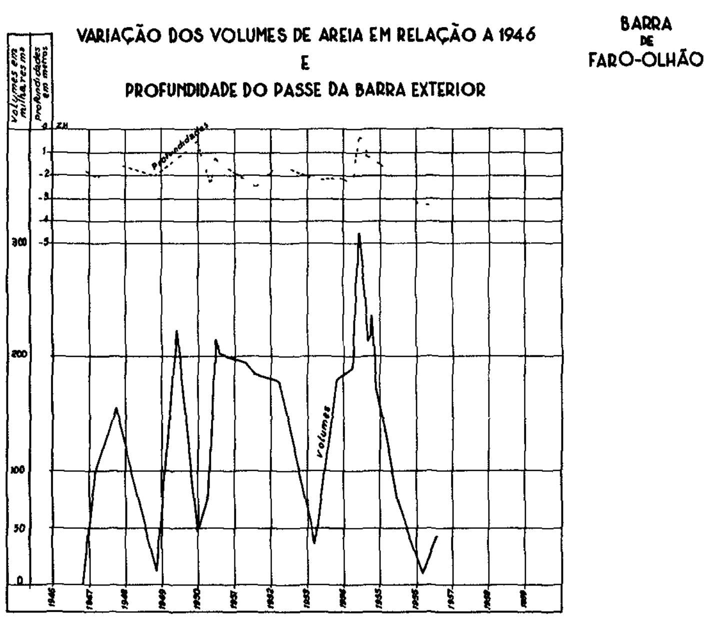

Fig. 32. Variation in the volume of sands and controlling depths on the outer-bar of the Faro-Olhao inlet

Table 7 (Part A)
Lagoon of Faro-Olhao (1929-1945)
Hydraulic elements of the inlet's cross sections

Dates	X-1929			VII-1932			VI-1933			VII-1938			VII-1942			VIII-1945		
Range number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	P m
0																		
1																		
2																		
3																		
4																		
5																404	270	1,50
6													393	290	1,35	404	278	1,45
7													424	266	1,59	346	239	1,45
8				177	140	1,26							519	248	2,09	300	202	1,49
9				215	129	1,67	344	163	2,11				503	255	1,97	309	192	1,61
10				283	120	2,36	346	175	1,98				510	225	2,27	314	167	1,88
11				217	127	1,71	350	165	2,12	446	238	1,87	348	243	1,43	366	160	2,29
12				277	178	1,56	383	166	2,31	479	222	2,16	476	235	2,03			
13				285	141	2,02	426	178	2,39	439	202	2,17	410	223	1,84			
14				255	168	1,52	418	175	3,39	400	175	2,29	416	216	1,93			
15				298	175	1,70	413	185	2,23	362	165	2,19	429	201	2,13			
16	415	170	2,44	337	152	2,22	464	167	2,78	377	140	2,69	425	186	2,28			
17	365	150	2,43	325	160	2,03	466	137	3,40	458	115	3,98	538	175	3,07			
18	377	118	3,19	409	99	4,13	483	118	4,09	417	95	4,39	383	157	2,44			
19	321	118	2,72	428	125	3,42	571	135	4,23	561	150	3,74	400	152	2,63			
20	432	125	3,46	489	145	3,47	446	124	3,60	473	135	3,50	402	152	2,64			
21	356	130	2,74	414	140	2,96	375	145	2,59	496	159	3,12	527	145	3,63			
22	488	144	3,39	461	142	3,25	383	130	2,95	418	155	2,70	544	145	3,75			
23	437	155	2,82	362	162	2,23	420	145	2,89	576	177	3,25	624	158	3,95			
24	509	140	3,64	435	155	2,81	493	149	3,31	561	160	3,51	670	150	4,47			
25	514	135	3,81	529	140	3,78	627	165	3,80	704	152	4,63	696	134	5,19			
26	516	133	3,88	468	140	3,34	539	126	4,28				649	187	3,47			
27	523	148	3,53	410	153	2,68	537	160	3,36				561	202	2,78			
28	498	240	2,08	399	220	1,81	603	240	2,51				365	155	2,35			
29	583	304	1,92	402	295	1,36	606	295	2,05				581	298	3,03			

Table 7 (Part B)
Lagoon of Faro-Olhao (1947-1956)
Hydraulic elements of the inlet's cross sections

Dates	IX-1947			VII-1950			X-1953			VII-1954			VII-1956		
Renge number	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m	a m2	p m	R m
0							530	150	3,53	803	155	5,18	895	157	5,15
1				307	156	1,97	581	150	3,87	704	152	4,53	630	150	4,20
2				388	189	2,05	717	192	3,74	810	182	4,45	831	185	4,49
3				431	224	1,92	830	235	3,53	1050	226	4,65	704	220	3,61
4				518	274	1,89	814	266	3,06	1043	265	3,93	786	264	2,97
5	393	272	1,44	474	273	1,74	780	315	2,48	964	315	3,06	745	256	2,91
6	380	247	1,54	464	263	1,76	751	355	2,11	880	355	2,48	645	196	3,29
7	405	246	1,65	470	286	1,64	655	402	1,63	889	395	2,25	670	230	2,91
8	397	260	1,53	425	280	1,52	636	390	1,63	828	394	2,10	716	257	2,78
9	425	265	1,60	356	305	1,17	725	480	1,51	806	435	1,85	699	220	3,18
10	399	240	1,66	329	245	1,34	729	470	1,55	569	330	1,72	702	328	2,14
11	459	235	1,95	305	216	1,41	617	368	1,68	708	306	2,31	680	220	3,09
12	521	248	2,10	381	205	1,86	563	312	1,80	689	263	2,62	659	245	2,69
13	554	285	1,91	471	260	1,81	615	263	2,34	734	253	2,90	693	240	2,89
14	560	225	2,49	614	255	2,41	629	317	1,98	754	232	3,25	739	214	3,45
15	521	202	2,58	604	275	2,20	648	298	2,17	720	218	3,30	721	208	3,47
16	510	199	2,56	526	253	2,08	687	256	2,68	738	245	3,01	716	201	3,56
17	575	202	2,85	469	265	1,77	569	140	4,06	875	248	3,53	709	191	3,78
18	521	170	3,06	487	175	2,78	534	152	3,51	717	180	3,98	679	180	3,77
19				521	160	3,26	431	147	2,93	711	165	4,31	740	165	4,48
20							508	166	3,06	629	150	4,19	700	154	4,54
21							412	150	2,75	636	150	4,24	714	140	5,10
22							499	172	2,90	730	172	4,24	717	165	4,35
23							543	160	3,39	727	165	4,41	770	170	4,53
24							630	168	3,75	827	183	4,52	721	180	4,00
25							728	180	4,04	887	180	4,93	911	175	5,20
26							924	200	4,62	910	205	4,44	956	200	4,28
27							880	260	3,38	915	253	3,62	1073	272	3,94
28							822	300	2,74	949	298	3,18	989	293	3,37
29							945	358	2,64	1240	355	3,49	1091	345	3,16

a) area; p) wetted perimeter; $p = \frac{a}{r}$ = Hydraulic radius Sections under datum

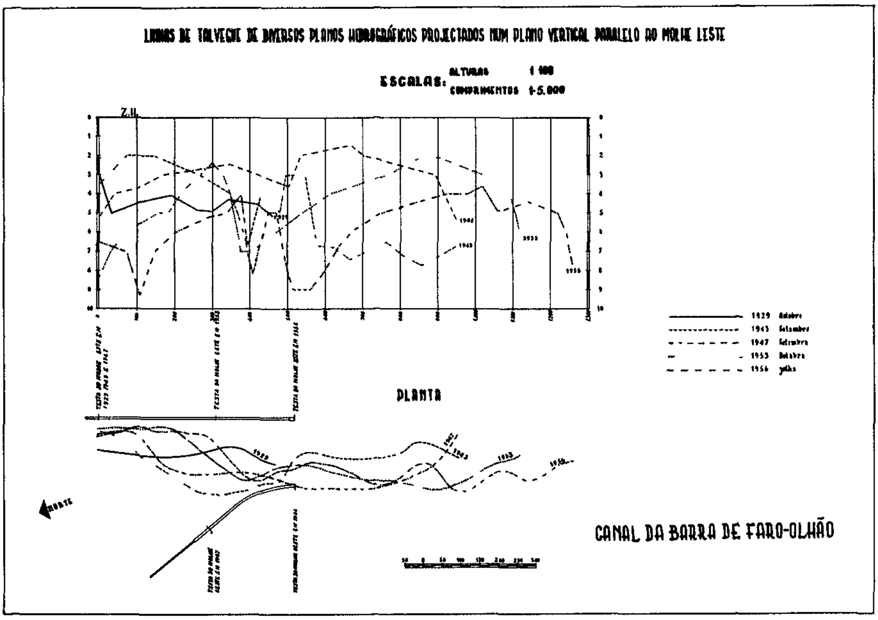

Fig. 33. Inlet of Faro-Olhao: longitudinal profiles of the entrance channel projected on a vertical plane parallel to the outer stretch of the east breakwater

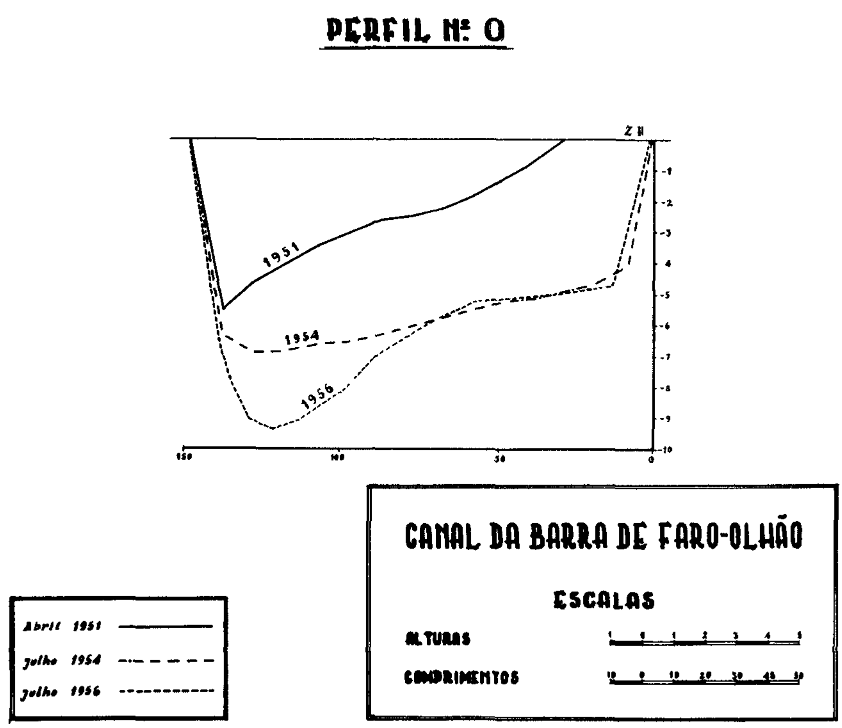

Fig. 34. Evolution of the inlet channel's cross sections due to the works (range No. 0)

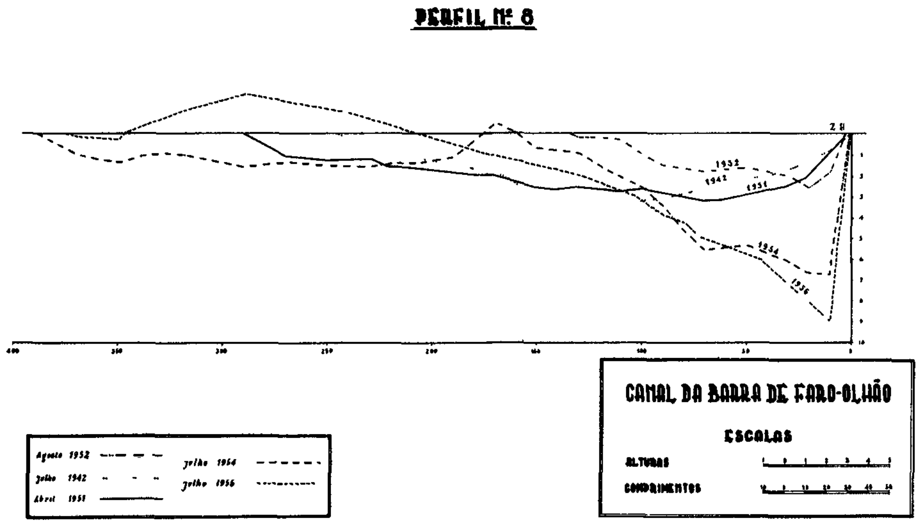

Fig. 35. Evolution of the inlet channel's cross sections due to the works (range No. 8)

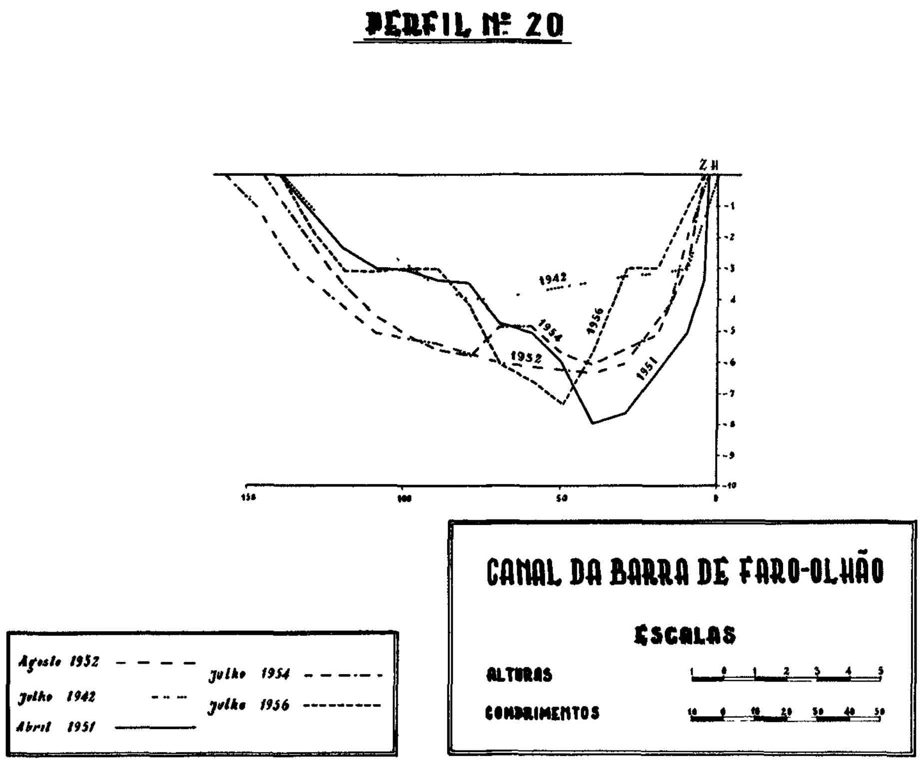

Fig. 36. Evolution of the inlet channel's cross sections due to the works (range No. 20)

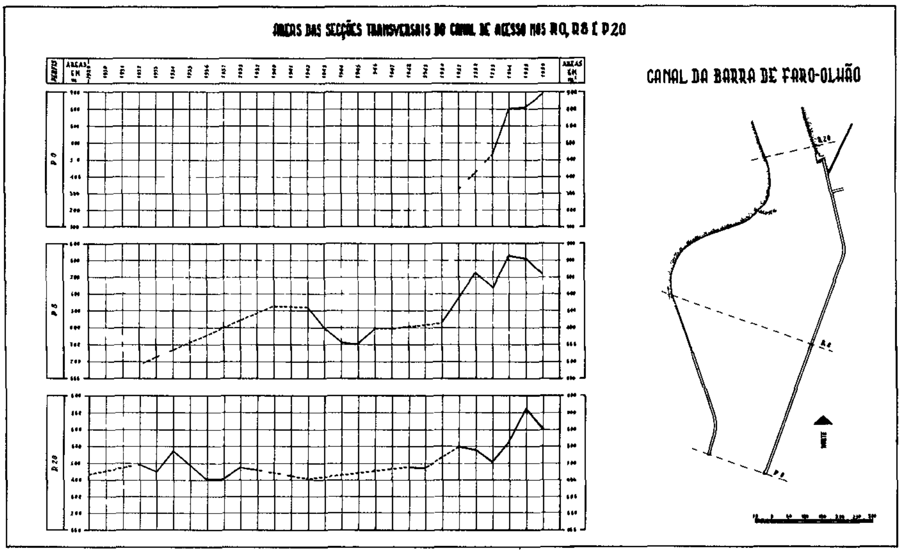

Fig. 37. Areas of inlet's cross sections

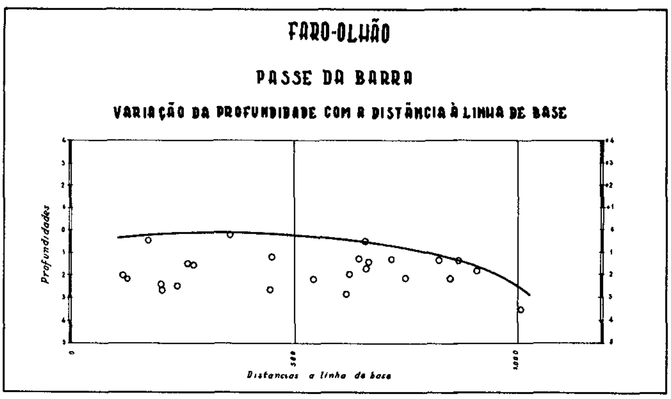

Fig. 38. Controlling depths on the outer bar of Faro-Olhao as a function of its distance to a base line

Fig. 39. Tavire inlet's hydrographic survey in 1916

ancient Tavira inlet, by piercing the barrier-beach near the town.

In fact, this was undertaken by 1927 and the artificial inlet, lyir to the SE of the Tavira's river lagoon outlet was canalized by means of two stone and concrete shore revetments across the barrier island, the eastern jetty being carried seaward for a length of about 700 feet, after wards lengthened by 250 feet more.

The 1936, 1942, 1944 and 1956 hydrographic surveys herein pre sented show what results were obtained by the initial works and the additional amendments (see Figs. 40 to 43).

Fundamentally, the works failed because they were not planned so as to obtain the transposition of the inlet by the moving littoral sands, with the less possible interference with the coastal regimen. The outer east breakwater was in fact a sand-trap in the way of the littoral drift, making a bigger amount of sand to enter the lagoon than the ebb current, although guided by it, was able to expel. Moreover, the original project contemplated a regular dredging to maintain the depths and the establishment and maintenance of an outer sand-pit, to protect the inlet against the invasion of the littoral sands. Certainly because the entrance channel itself did not shoal and the dredging was not cheap, neither item was implemented, and groynes were instead built on the west side ocean beach to prevent the littoral sands from reaching the inlet, and so to reduce the shoaling inside the lagoon. But the feeding of the barrier-beach to the east of the inlet was concurrently and substantially reduced, with the result that in 1941, during a big storm, a new inlet was opened opposite the mouth of river Almargem, due to the joint action of the river flood and the ocean waves on a weakened section of the barrier-island. In some years, the new inlet, through which the tide started to circulate, widened to hundreds of meters, due to the local destruction of the barrier-beach, and the artificial one, progressively deprived of the tidal flow and invaded by the littoral sands, completely closed.

The essential misconceptions seem to have been the obstacles raised in the way of the littoral sands, i.e., the outer eastern breakwater and specially the groynes to the west of the inlet, while the location of this one could also be a matter of argument as to the advantages of moving it a little more to the west. The elected location, based on economic grounds, does not seem to have been able to meet the modest requirements contemplated, nor its displacement to the west would have prevented the consequences of the formerly mentioned misconceptions. These, of course, could have been avoided if the port was rich enough to pay for a strong and extensive protection of the barrier beach to the east side of the inlet.

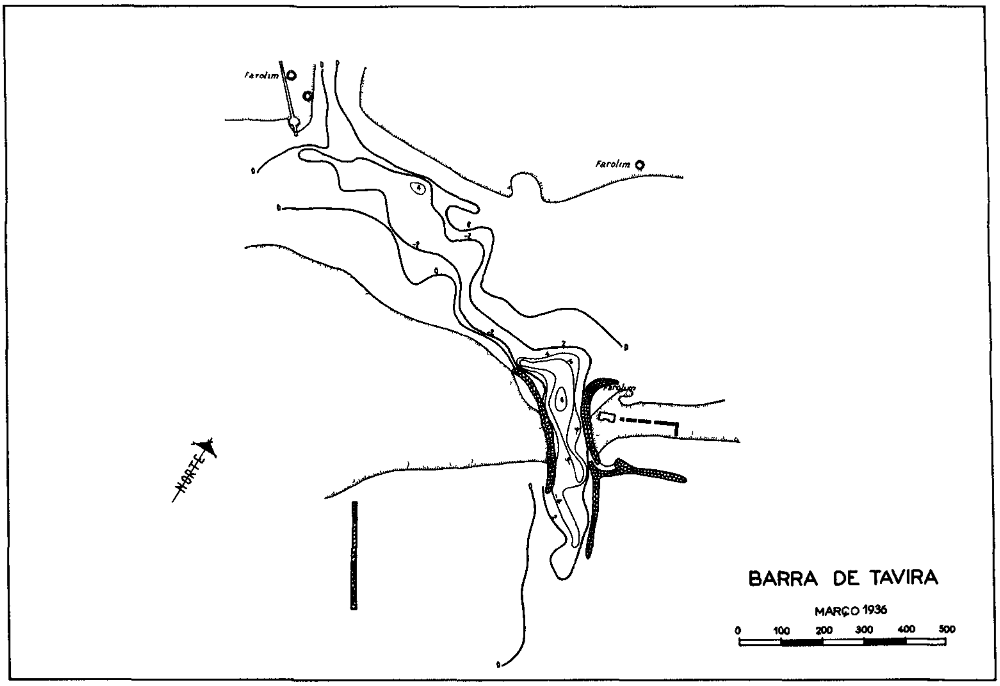

Fig. 40. Tavira inlet's hydrographic survey in March 1936

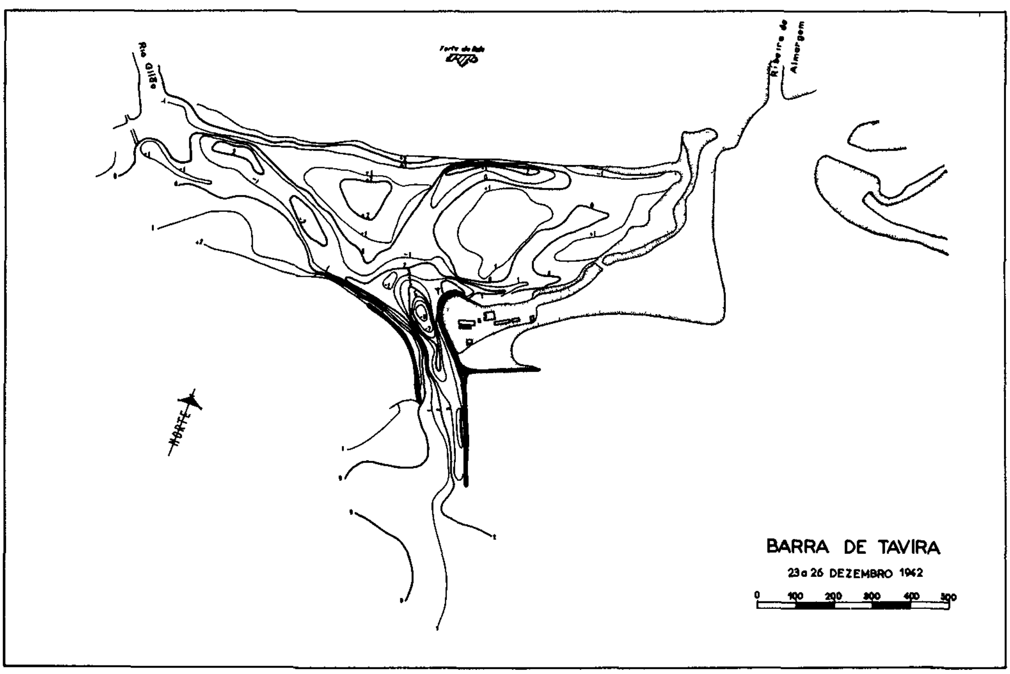

Fig. 41. Tavira inlet's hydrographic survey in December 1942

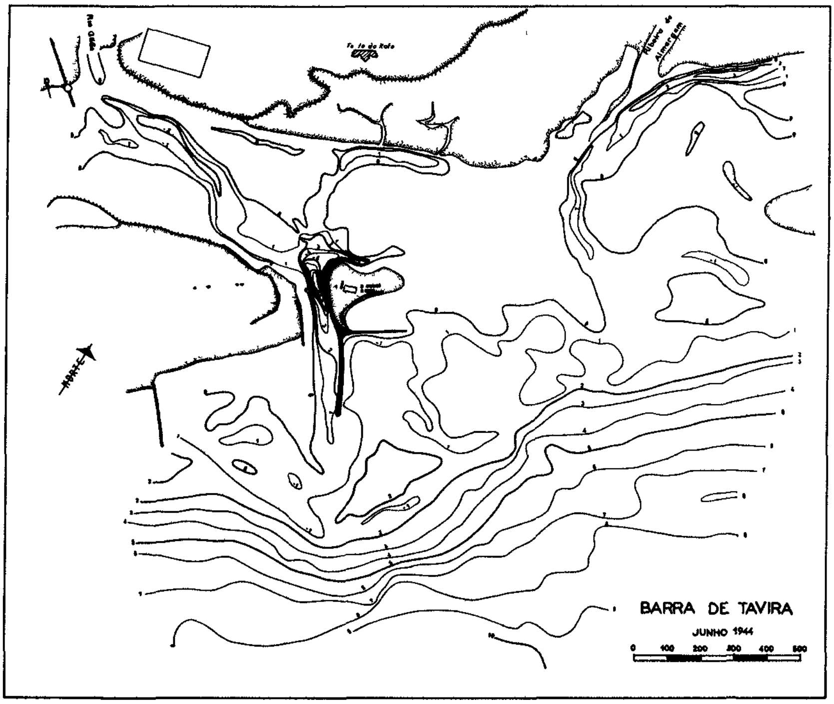

Fig. 42. Tavira inlet's hydrographic survey in July 1944

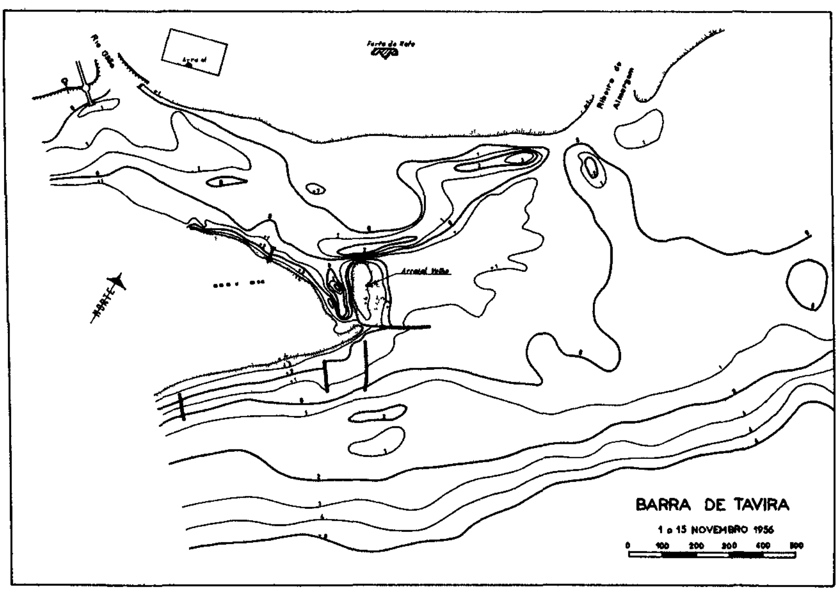

Fig. 43. Tavira inlet's hydrographic survey in November 1956

This specific case shows that the real test on the soundness of an inlet improvement scheme is not the depth on the entrance channel, but the variation of the volume of sand both on the inner bar and on the downdrift section of the coast. If the former increases and the latter decreases systematically, failure of the scheme is real, while it can be masked if there is enough money available for the purpose.

THE IMPROVEMENT OF LAGOON INLETS

In our opinion, consideration of the above-mentioned examples, together with the free evolution of a number of natural inlets on sand coasts, legitimates the inference of some principles valid in the treatment of these physiographic entities for navigation or drainage purposes. But, to prevent any misinterpretation of what has been stated, we want to stress the point that none of the herein referred successful cases is to be considered a completely solved problem, as far as taking full advantage of the lagoon-and-inlet system's potentialities for the improvement of the entrance channel is concerned: next step will probably be the correction of the hydraulic flow conditions in the inner approaches to the inlet channel.

We have had the opportunity of pointing out the fundamental differences between the suitable methods for dealing with the improvement of estuaries and those adequate to improve the lagoon entrance channels (C. Abecasis, 1954). It is enough to say that the additional data now presented solidly confirm our previous statement, so that the coastal engineer must be extremely cautious whenever he feels tempted to make use of similarity methods to solve any particular problem on lagoons' accesses by referring to the sanctioned and successful practices adhered to in estuaries' knowledge.

For the effective improvement of an inlet on a sandy coast, it seems essential, under the physiographical point of view:

(a) to increase as much as possible the relation of the volume of the tidal flow through the inlet to the volume of sands carried by the littoral drift;

(b) to interfere the less possible with the littoral drift existing along the barrier-beach, looking at that the volume of sand expelled out by the ebb be not less than that brought in by the flood tide.

Needless to say, those conditions are to a certain extent interconnected, but any of them may be more or less fulfilled in a given case. The former requires to improve as far as possible the hydraulic characteristics of the inlet channel and depends on the space available in the lagoon and the conditions prevailing along the main lagoon channels as regards the propagation of the tide; it can hardly, if ever, be performed without the canalization of the inlet channel across the barrier-beach. The latter requires a well designed and balanced canalization or protection scheme, allowing the littoral sands to reach the inlet and to follow their way downcoast, either entering the inlet or not; sometimes, dredging or canalization works may be needed in the lagoon are adjacent to the upstream section of the inlet channel to help in getting this result.

Analysis of the inner bar's and of the leeward beach's behaviors as regards accretion or erosion is the best way to check the soundness or successfulness of any inlet improvement work being or having been executed.

As practical rules for performing the above stated conditions, we advised elsewhere and now confirm that:

i. the inlet should be located as close as possible to the center of masses of the waters in the lagoon and to the biggest depths outside;

ii. the inlet should be canalized through the barrier-beach and when necessary on account of the required depths and/or on account of the volume of littoral drift, the currents from the different lagoon bodies should be harmonized and so guided to the inlet channel;

iii. the outer bar should be situated as far out in the sea as required for obtaining the depths wanted, which must be obtained by means of jetties, those being preferably slightly convergent whenever they must go beyond the previous shoreline.

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CHAPTER 24

EFFECT OF LAKE WORTH AND SOUTH LAKE WORTH INLETS ON THE MOVEMENT OF LITTORAL MATERIAL

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INTRODUCTION

DESCRIPTION

Lake Worth and South Lake Worth Inlets are artificial inlets connecting Lake Worth with the Atlantic Ocean. They are located on the east coast of Florida about 70 miles north of Miami and 300 miles south of Jacksonville. Lake Worth is a salt-water sound extending in a north-south direction, generally parallel to the ocean shore, as shown on figure 1. It is separated from the ocean by a barrier beach, 250 feet to about 3,600 feet wide and up to about 25 feet in elevation. The barrier beach is composed principally of sand, a portion of which is artificial fill over former low-lying marshy areas. There are occasional outcroppings of coquina rock on the barrier beach and in the offshore area. In this locality the offshore bottom is rather steep; the 100-fathom depth, lying closer to the shore than along other parts of the Atlantic coast, is about 5-1/2 miles offshore at Lake Worth Inlet.

Lake Worth Inlet was dredged through the barrier beach by local interests between 1918 and 1925 to develop the Port of Palm Beach. It replaced the relatively unstable natural inlet through the barrier beach several miles to the north. Two protective jetties were constructed, parallel and 800 feet apart. At the time of construction, they extended out to approximately 20-foot depth in the ocean. The north and south jetties are about 2,000 and 1,900 feet long, respectively, both measured from the 1920 shoreline. A channel 20 feet deep, 300 feet wide through the inlet and thence 200 feet wide across Lake Worth, was provided to serve the terminals of the Port of Palm Beach, located on the west shore of Lake Worth. A Federal project for maintenance of the harbor and jetties was adopted by the River and Harbor Act of August 30, 1935. In order to meet the needs of shipping in the area, the River and Harbor Act of March 2, 1945, authorized deepening of the channel and turning basin to 25 feet below mean low water. That work was completed in 1949. The location and alinement of the existing project are shown on figure 2. Palm Beach County, in cooperation with local municipalities, is constructing a sand-transfer plant on the north side of the inlet which will maintain the predominantly southerly flow of littoral drift.

South Lake Worth Inlet connects Lake Worth with the ocean near the southerly end of the lake. The inlet was dredged through the barrier beach by local interests in 1927 to provide better tidal circulation in the lake for relief of unsanitary conditions. That work provided an 8- by 125-foot channel which is used by small commercial and recreational craft. The inlet is protected by two parallel jetties about 300 feet long

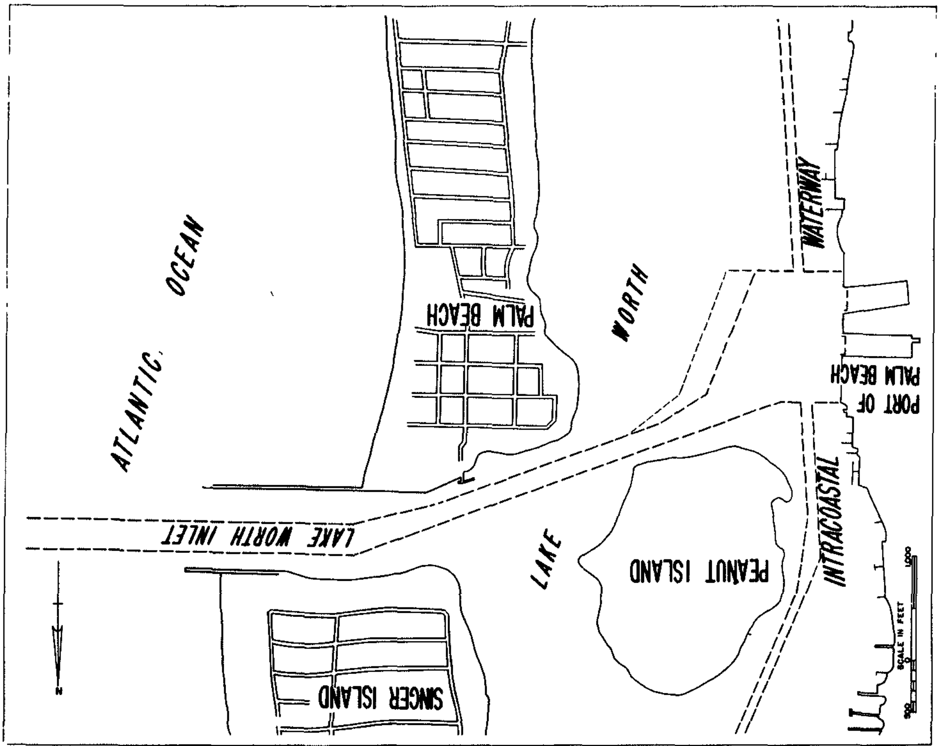

Fig. 2. Palm Beach Harbor.

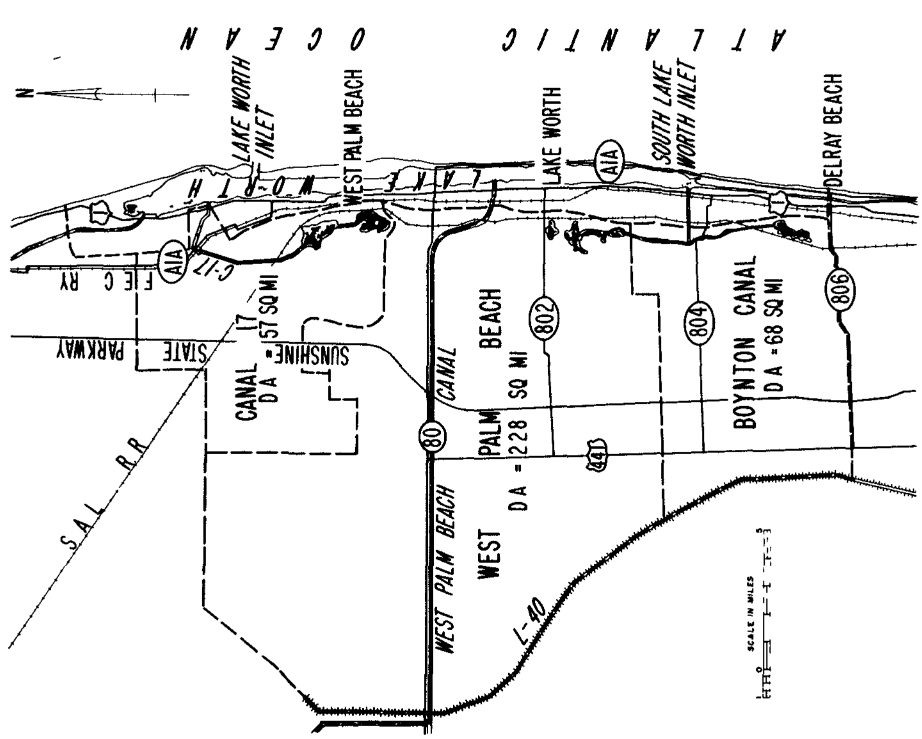

Fig. 1. Location map.

which were initially constructed to the 6-foot depth contour. The jetties were initially built to a top elevation of 5 feet above mean low water. The north jetty was raised to a height of 12 feet above mean low water in 1936 when its capacity for impounding beach material was exhausted. At that time private interests installed a sand pump near the end of the north jetty to permit transfer of impounded littoral material to the beach south of the inlet, for the primary purpose of reducing erosion of shorefront property. In restoring the natural flow of littoral material, that installation has also materially reduced erosion in the reach south of the inlet.

Lake Worth ranges from 1/4 to 1-1/2 miles in width and is about 20.6 miles long. It has a surface area of about 13 square miles. Natural depths are generally less than 8 feet. West Palm Beach Canal, Boynton Canal, and Earman River are the major tributaries. Under present conditions, they serve 228, 68, and 57 square miles respectively. Within the last 10 years, major drainage improvements by local interests and by the Federal and State governments under the Central and Southern Florida Flood Control Project have increased the rate of discharge into the lake. Additional improvements are authorized. The total drainage area of Lake Worth, including the surface of the lake and the area which drains directly into the lake, is about 400 square miles. Estimated peak discharge to the lake is given in the following tabulation.

TABLE 1

Estimated peak inflow into Lake Worth (c.f.s.)

Storm	West Palm Beach Canal	Boynton Canal	Earman River (Canal 17)	Lake surface and local area	Maximum[1] rate of inflow
		AS RECORDED			
1947--------------	5,290	2,500	700	5,000	13,000
1953--------------	4,020	1,500	500	2,500	8,000
		UNDER PRESENT CONDITIONS			
Maximum annual----	3,300	900	750	-	-
10-year storm-----	4,700	1,850	1,800	3,400	9,700
100-year storm----	6,400	2,700	2,100	8,700	19,300

[1]Considering time of concentration.

Depending on rainfall, the volume of inflow during any day will vary from practically zero to the maximum indicated in table 1. For periods of several months or more during a normal dry season, November through May, the volume of inflow will be practically zero.

Lake Worth, for its entire length, forms a link in the Intracoasta: Waterway from Jacksonville to Miami. The existing channel, 8 feet deep and 100 feet wide, was authorized by the River and Harbor Acts of Januai 21, 1927, and July 3, 1930, and completed in 1935. Enlargement of the existing project to provide a channel 12 feet deep and 125 feet wide wa authorized by the River and Harbor Act of March 2, 1945.

LITTORAL FORCES

GENERAL

The predominant flow of littoral material is from north to south, although during the summer months a reverse flow from south to north pre vails. That movement is due to the exposure of the area to winds and waves, swells, and tidal currents, including the Gulf Stream which flow northerly several miles offshore.

WINDS

The yearly average offshore winds in the Atlantic Ocean at Palm Bea summarized in table 2 are from the United States Hydrographic Office rec ords of winds reported by ships at sea during the period 1879 to 1933.

TABLE 2

Yearly average offshore winds

Direction	Percentage of time	Direction	Percentage of time
North	10	Southwest	6
Northeast	16	West	5
East	22	Northwest	8
Southeast	20	Calm	3
South	10		

The records indicate that winds from the south and southeast are experi- enced for longer periods than from any other sector--excluding those fr the east which have insignificant effect on littoral movement; however, predominant offshore winds in the area are from the north and east, witl the strongest winds from the northern sector.

The direction, duration, and velocity of the wind were observed eve 6 hours by the United States Weather Bureau at West Palm Beach during tl period July 1, 1938, to July 31, 1946. The results of those observatio are summarized in table 3 and on figure 3.

TABLE 3

Wind duration and direction--West Palm Beach

Direction	Percentage of time	Direction	Percentage of time
North------	5	South	6
Northeast--	15	Southwest	12
East-------	9	West	4
Southeast--	32	Northwest	17

The records indicate that wind velocities were greater from the northeast than from the southeast, but that the duration and wind movement were greater from the southeast sector.

SWELLS

The height and direction of movement of swells were observed at Palm Beach during the 10-year period 1932-42. A total of 40,601 observations was made. Those observations were about equally distributed over the months of the year. The results of an analysis of the observations are given in table 4.

WIND DIAGRAMS

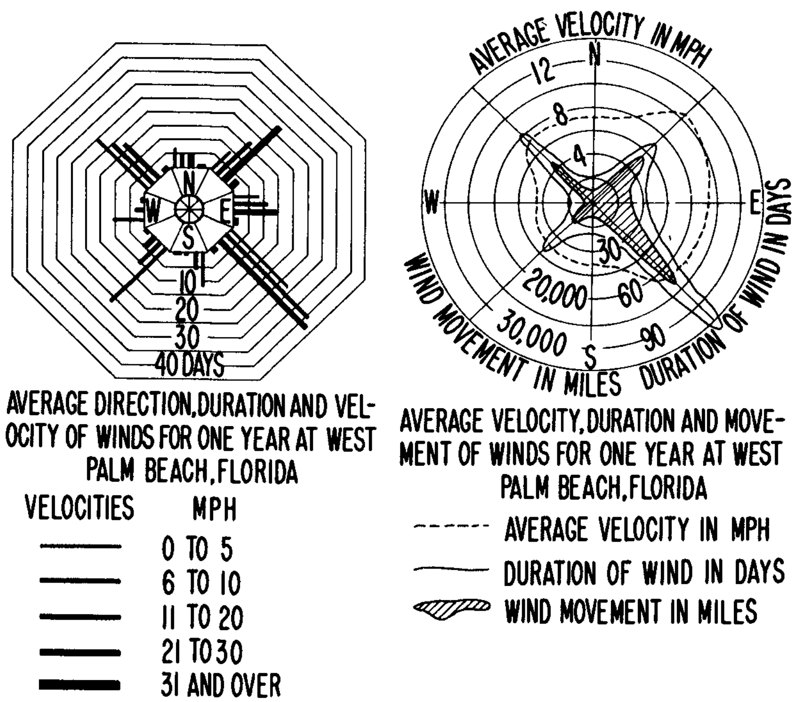

Fig. 3

TABLE 4

Swell duration and direction

Direction	Duration (pct. of time)		
	Low swells (1-6 ft.)	Medium swells (6-12 ft.)	High swells (over 12 ft.)
North------	5.0	1.6	-
Northeast--	10.2	6.3	3.8
East-------	11.8	4.4	1.8
Southeast--	9.8	1.2	0.5
South------	3.7	-	-
Southwest--	7.6	1.8	0.4
West-------	2.2	2.2	-
Northwest--	2.2	4.0	-

The records indicate that the greatest exposure is from the northeast an east. That pattern is greatly influenced by the shelter afforded this portion of the coast by the islands of the Bahama group.

WAVES

Wave observations were made at Rainbow Pier, Palm Beach, during the period April-October 1939. A record of the height and period of waves w also obtained there between March and June 1952 and since February 1954. The water depth at the observation point ranged from 12 to 15 feet. The maximum monthly waves during the period of observation varied from 7.0 t 9.5 feet in height with periods of from 4 to 13 seconds. In the period between March and June 1952, the maximum wave height was 3.0 feet and three-fourths of all waves were between 0.6 and 1.0 foot. The wave peri ods ranged up to 18.9 seconds, with 29 percent of all periods between 4. and 5.9 seconds. Observations of the direction of wave travel during th same period indicated that about 65 percent of all waves measured approached from a 40-degree sector between azimuths (true) 80° and 120°. has been reported that waves of sufficient height and force to carry san over the top of the seawalls (top elevation 16.0 ft.) were experienced i the vicinity of Rainbow Pier during the storm of September 1928.

TIDES

The mean range of tide in the Atlantic Ocean at Palm Beach is 2.8 feet; the spring range is 3.3 feet. The estimated lowest tide is 2.0 fe below mean low water. Water-level variations of as much as 7.0 feet hav occurred during hurricanes. The maximum high water of record observed a Palm Beach was 11.2 feet above mean low water during the hurricane of Se tember 1928, including the effect of waves and runup. The second highes water elevation observed was 8.7 feet during the hurricane of July 1936.

EFFECT OF LAKE WORTH AND SOUTH LAKE WORTH INLETS ON THE MOVEMENT OF LITTORAL MATERIAL

CURRENTS--LAKE WORTH AND SOUTH LAKE WORTH INLETS

OBSERVATIONS

Current observations were made in the jettied channel at Lake Worth Inlet from June to September 1939, which was in the season of northward littoral drift. The observations indicated that tidal currents in Lake Worth Inlet were high. A tidal current velocity of 3 miles or less an hour is usually found, with an occasional maximum velocity of about twice that rate. No quantitative data are available for tidal velocities at South Lake Worth.

LITTORAL CURRENTS

Littoral currents along the beach are caused by the action of swells and waves to which the beach is exposed. Because of the configuration and bearing of the shoreline under consideration--in a practically north-south alinement--swells and waves approaching from the north and northeast cause a southerly littoral current; swells from the south and southeast cause a northerly littoral current. Swells and waves from the east approach normal to the shoreline and create very little current in either direction. The wind data previously presented indicate that the area is exposed to winds producing a northerly littoral current for a greater percentage of time than winds producing a southerly littoral current. However, the winds from the north and northeast are stronger and more effective since the coast is not shielded from waves approaching from those directions by the islands of the Bahama group. Hence, it is found that the southerly littoral current predominates. Detailed examination of the records indicates that during the months of September through February the prevailing and predominant swells and waves approach from the north and northeast and set up a southerly littoral current; during March, April, and May the winds tend to be variable and low in strength, with no predominant trend discernible; and from June through August the winds and resultant predominant waves and swells approach from the south and southeast and set up a northerly littoral current. The Gulf Stream is approximately parallel and from 3 to 5 miles offshore at this point. It is possible that a secondary current flowing southerly is generated in the area by the Gulf Stream. Such a current would tend to augment the southerly littoral currents and retard northerly littoral currents. However, there are not sufficient data to determine the magnitude of the influence from that source, since it is of considerably smaller magnitude than the currents generated by swells and waves.

There have been several efforts to determine the magnitude of the littoral currents. The first, during the period June to September 1939, indicated a predominantly northward current approximately parallel to the shore. In the angles formed by the beaches and jetties, reversals in the current were noted. Current velocities observed close to shore were too low to erode material from the beach but were capable of moving beach material placed in suspension by wave action. Higher velocities were found in the deeper water offshore where the currents were capable of eroding and transporting beach material.

During the period March 6 to June 10, 1952, a series of observations were made in the reach directly north of South Lake Worth Inlet. In thos observations, fluorescein dye was used to measure currents within the breaker zone. A small paper bag containing gravel and the dye was thrown into the surf zone, and the velocity of the current was determined by tim ing the travel of the stain. The observed data are summarized in table 5

TABLE 5

Littoral current data

Location (miles north of South Lake Worth Inlet)	Accumulated duration (pct. of time) Velocity (ft./min.) 0 to 9	10 to 49	50 to 89	Estimated maximum (ft./min.)
	PERIODS OF NORTHERLY DRIFT			
0.5	8.1	63.7	92.0	130 to 139
2.0	6.1	63.2	91.7	130 to 139
5.0	9.4	62.2	92.5	160 to 169
7.0	14.4	86.6	99.0	90 to 99
	PERIODS OF SOUTHERLY DRIFT			
0.5	15.2	59.3	88.2	120 to 129
2.0	17.7	72.5	85.3	140 to 149
5.0	14.3	60.7	87.3	130 to 139
7.0	13.8	62.2	93.2	110 to 119

Examination of the data indicates that during this period (March-June) t velocity was under 89 feet per minute about 90 percent of the time and under 50 feet per minute about two-thirds of the time. The observations found no predominant current during this period and a current from eithe a northerly or southerly direction of about the same intensity could be expected, depending largely on wind conditions.

LITTORAL MATERIALS

The beach material is a mixture of silica sand and fragments of coı and shell. The silica sand is generally agreed to have been carried to the sea by the Savannah, Altamaha, and other rivers of Georgia and the Carolinas, and gradually shifted to the southward by shore currents and wave action. The underlying material is shell and coral sand which has been laid down during past periods of emergence. It varies in composit: and texture from coarse sandstone formed of consolidated coral sand (ma‹ erated shells) to a compact mass of only slightly worn shells. In seveı places along the beach, the coquina rock appears as a submerged reef th: generally parallels the shoreline. The rock reef appears at various di: tances offshore from the low-water line to 1,000 feet offshore. Sample: indicate that the composition of the beach material on the surface of tl foreshore is about 40 percent fine siliceous sand and 60 percent shell and coral fragments. The median diameter of beach material at mean sea level and at 6-, 12-, 18-, 24-, and 30-foot depths ranges from 0.15 to 1.13 millimeters, with average median diameters of about 0.64 millimeter at sea level and 0.19 millimeter at 30-foot depth. Detailed summary of the composition of the littoral material from samples obtained in 1956 is given in tables 6 and 7.

TABLE 6

Composition of littoral material along beach

Location of range	Median sand diameter in millimeters (including shell)							
	Location of sample							Average diameter
	Dune	Water's edge	Depth (ft.) 6	12	18	24	30	
8,400 ft. north of Lake Worth Inlet (profile 3N)--------	0.50	0.47	0.25	0.16	0.16	0.21	0.24	0.25
7,600 ft. south of Lake Worth Inlet (profile 3S)--------	0.38	0.35	0.23	0.24	0.28	0.21	0.18	0.25
39,600 ft. south of Lake Worth Inlet (profile 15S)------	0.47	1.00	0.20	0.19	0.20	0.19	0.17	0.33
7,700 ft. north of South Lake Worth Inlet (profile 27S)------	0.42	0.42	0.25	0.23	0.18	0.21	0.19	0.25
7,800 ft. south of South Lake Worth Inlet (profile 33S)------	0.82	1.12	0.37	0.27	0.33	0.26	0.25	0.43

TABLE 7

Composition of littoral material along alinements of Lake Worth and South Lake Worth Inlets

Location	Location of sample (depth in feet)				
	20	27	50	75	
ALONG ALINEMENT OF LAKE WORTH INLET					
North side-------	0.49	-	0.25	0.42	
Centerline-------	-	0.50	0.66	-	
South side-------	-	-	0.41	0.61	
	Location of sample (depth in feet)				
	6	12	18	24	30
ALONG ALINEMENT OF SOUTH LAKE WORTH INLET					
North side-------	0.46	-	0.26	0.24	0.24
South side-------	0.70	0.24	0.25	0.25	0.25

It can be readily seen that the tidal currents within the zone affected Lake Worth Inlet must be of considerably greater magnitude than the nor offshore littoral current. Wave action re-sorts the littoral material moving alongshore and the stronger tidal currents remove a large portio of the finer components offshore into deeper water. At South Lake Wort Inlet, the tidal currents also remove the littoral material. However, cept at very shallow depths, a smaller percentage of the finer componen is removed.

SHORELINE AND OFFSHORE DEPTH CHANGES

GENERAL

Surveys made by the United States Coast and Geodetic Survey in 188 and 1928 and by the Corps of Engineers in 1946 and 1955 form the basis determining shoreline and offshore depth changes since the dredging of Lake Worth and South Lake Worth Inlets. There is also available a surv made by the Lake Worth Inlet District in the vicinity of the inlet in 1918, immediately before the inlet was constructed. Changes in the ger eral vicinity of Lake Worth Inlet are shown on figures 4 and 5; those the vicinity of South Lake Worth Inlet are shown on figure 11. Typica bottom profiles are shown on figures 6, 7, 8, 9, 12, and 13.

FROM 1883 TO 1929

Comparison of the surveys indicates that in the period 1883 to 19 there was a general recession of from 125 to 165 feet in the shoreline

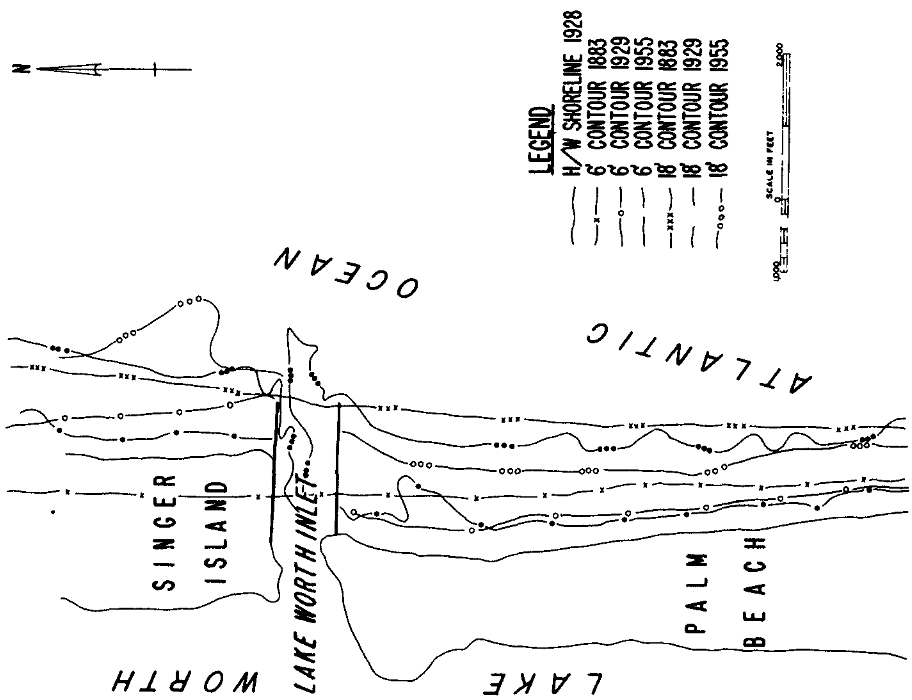
N
LEGEND
H/W SHORELINE 1928
6' CONTOUR 1883
6' CONTOUR 1929
6' CONTOUR 1955
18' CONTOUR 1883
18' CONTOUR 1929
18' CONTOUR 1955
SCALE IN FEET
1000
0
2000
ATLANTIC OCEAN
SINGER ISLAND
LAKE WORTH INLET
PALM BEACH
LAKE WORTH

Fig. 5

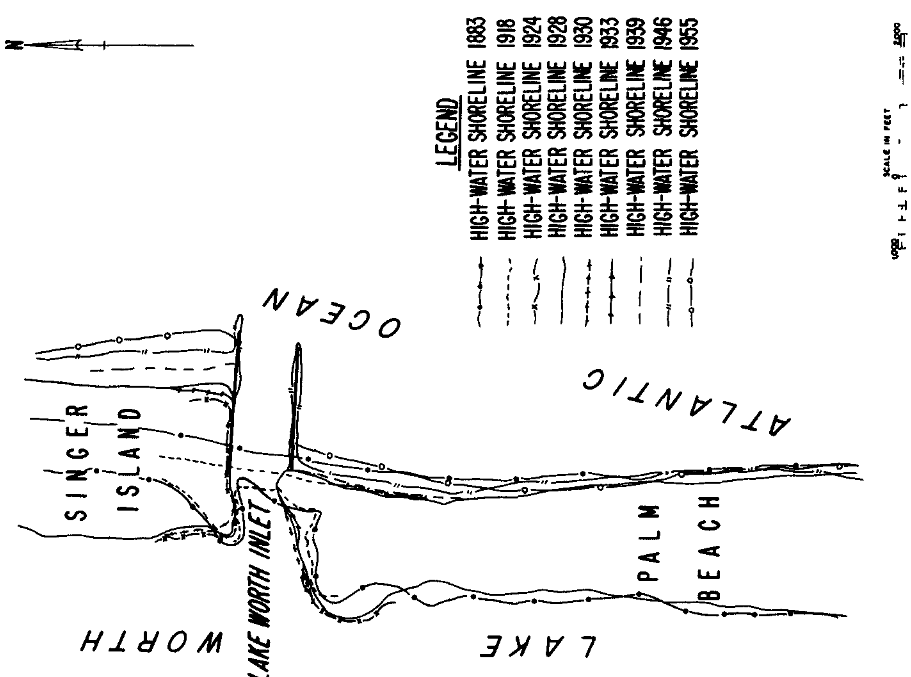
N
LEGEND
HIGH-WATER SHORELINE 1883
HIGH-WATER SHORELINE 1918
HIGH-WATER SHORELINE 1924
HIGH-WATER SHORELINE 1928
HIGH-WATER SHORELINE 1930
HIGH-WATER SHORELINE 1933
HIGH-WATER SHORELINE 1939
HIGH-WATER SHORELINE 1946
HIGH-WATER SHORELINE 1955
SCALE IN FEET
1000
0
2000
ATLANTIC OCEAN
SINGER ISLAND
LAKE WORTH INLET
PALM BEACH
LAKE WORTH

Fig. 4

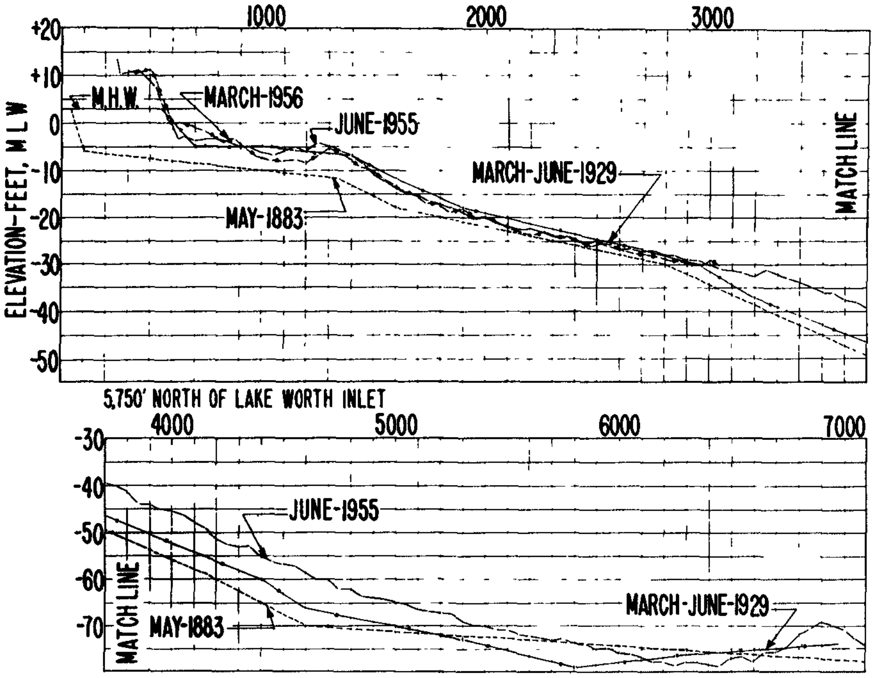

Fig. 6. Profile 2-N.

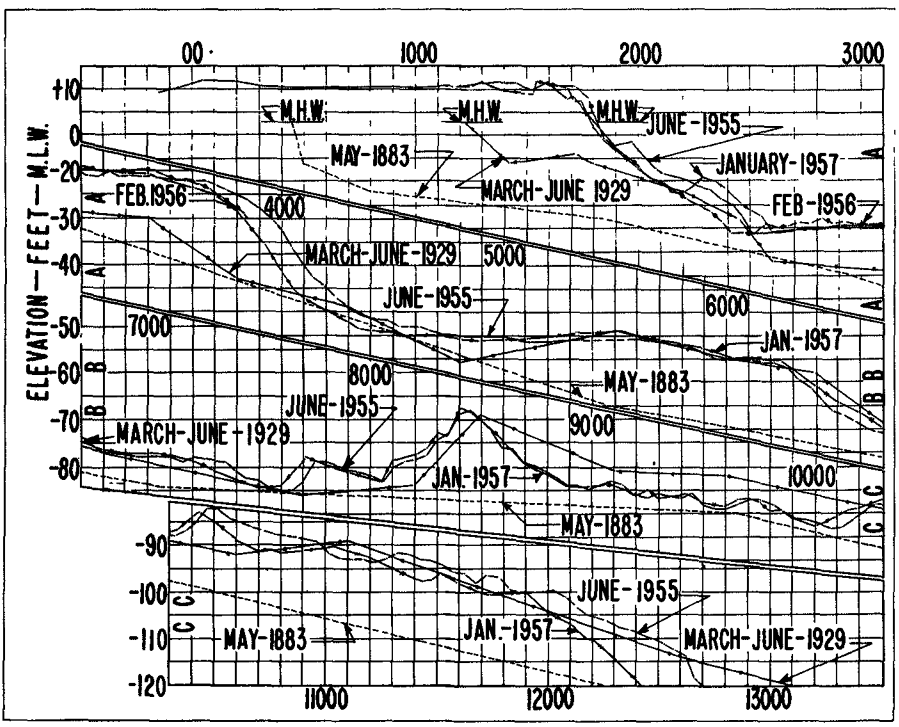

Fig. 7. Profile range 590-N (590 ft. N. of Lake Worth Inlet).

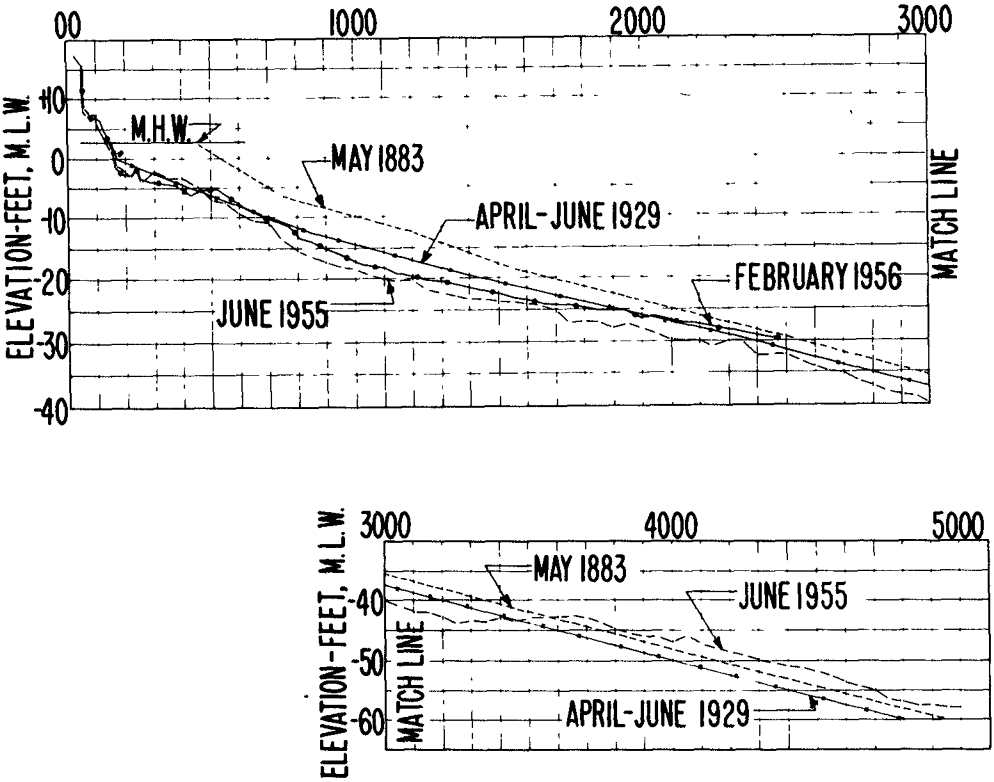

Fig. 8. Profile 2-S (5,720 ft. S. of Lake Worth Inlet).

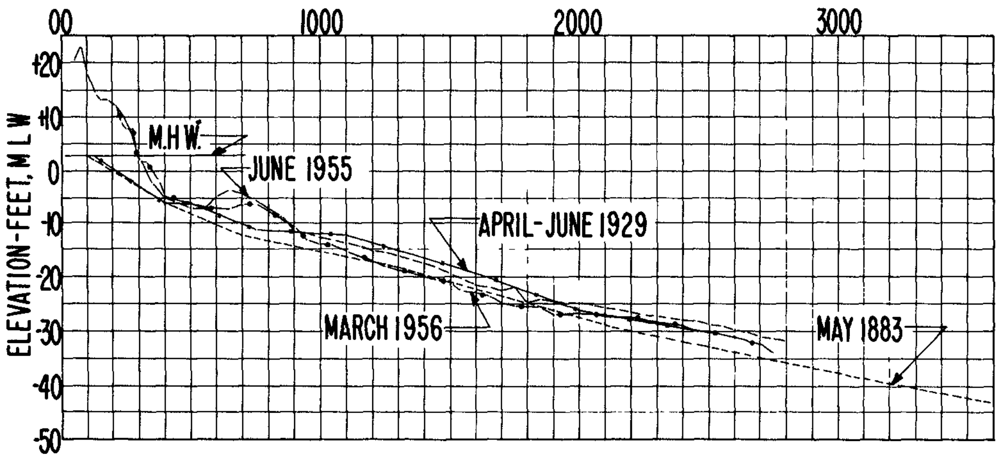

Fig. 9. Profile 14-S (36,750 ft. S. of Lake Worth Inlet).

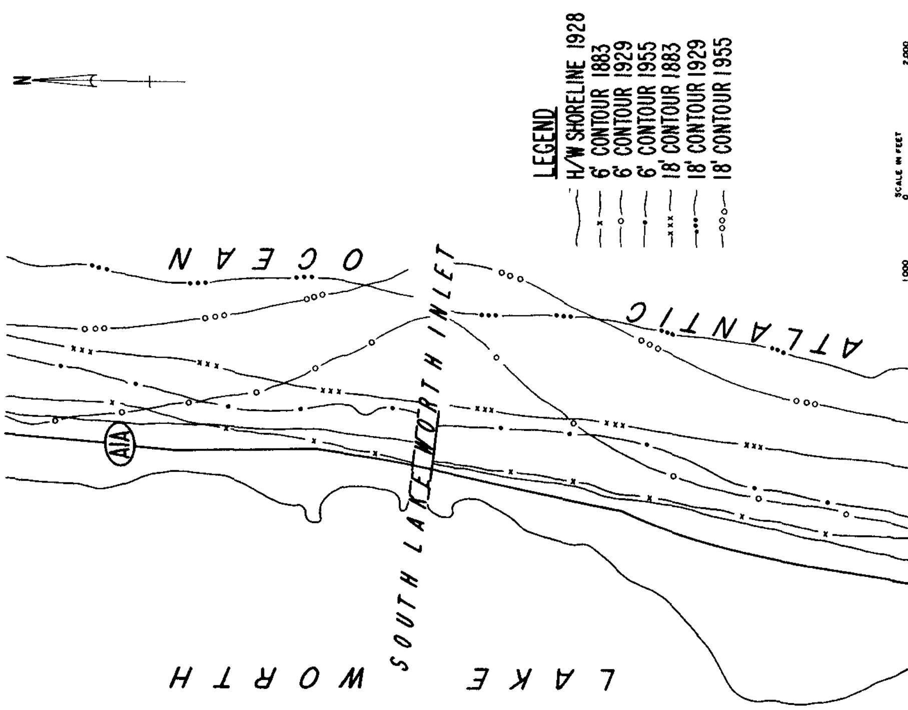
LEGEND
H/W SHORELINE 1928
6' CONTOUR 1883
6' CONTOUR 1929
6' CONTOUR 1955
18' CONTOUR 1883
18' CONTOUR 1929
18' CONTOUR 1955
SCALE IN FEET
OCEAN
ATLANTIC
SOUTH LAKE WORTH INLET
A1A
LAKE WORTH

Fig. 11

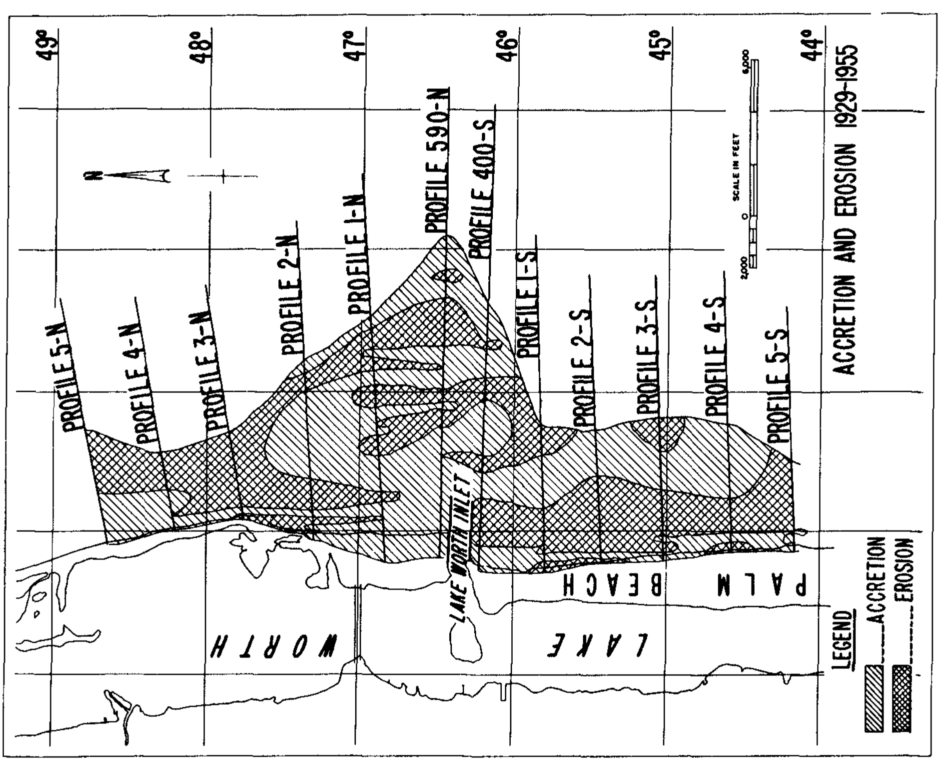
49°
48°
47°
46°
45°
44°
PROFILE 5-N
PROFILE 4-N
PROFILE 3-N
PROFILE 2-N
PROFILE 1-N
PROFILE 590-N
PROFILE 400-S
PROFILE 1-S
PROFILE 2-S
PROFILE 3-S
PROFILE 4-S
PROFILE 5-S
SCALE IN FEET
ACCRETION AND EROSION 1929-1955
LAKE WORTH INLET
PALM BEACH
LAKE WORTH
LEGEND
ACCRETION
EROSION

Fig. 10

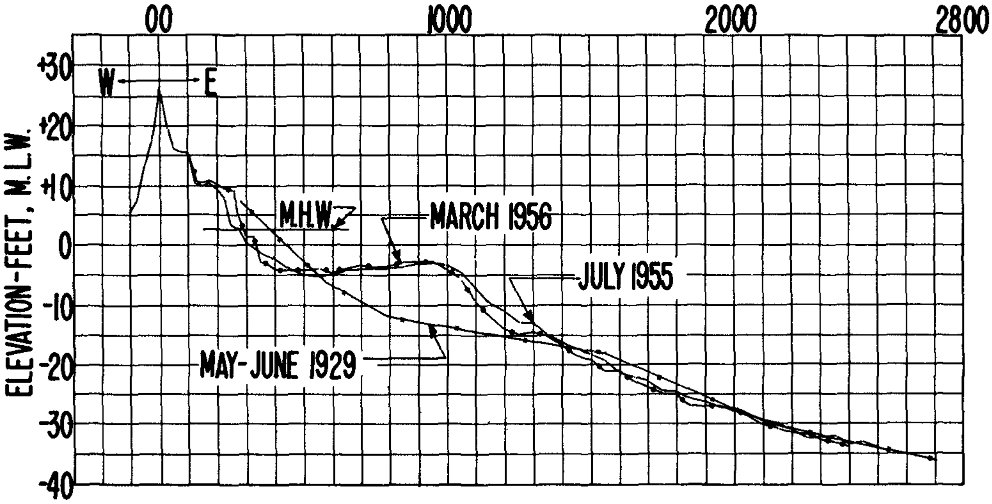

Fig. 12. Profile SLWI-1+00 (300 ft. N. of South Lake Worth Inlet).

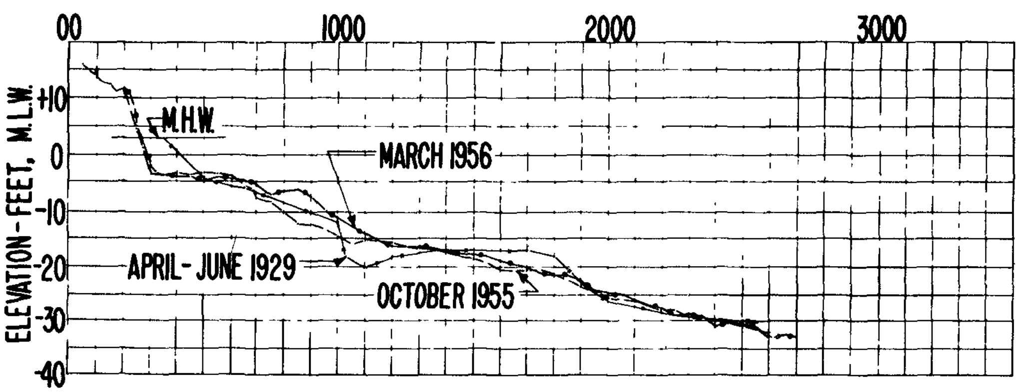

Fig. 13. Profile 31-S (2650 ft. S. of South Lake Worth Inlet).

except in the 1-1/2 miles immediately north of Lake Worth Inlet. Generally, with the exception of a 3-mile reach south of the inlet, the surveys indicate advance during that period of from 150 to 380 feet in 6-foot, 12-foot, 18-foot, and 30-foot offshore depths. It is probable tha the effects noted were a result of the 1926 and 1928 hurricanes. Those hurricanes caused widespread damage to beach protective works and expose buildings. It was reported that a lowering of the beach from 2 to 3 fee occurred in the 1928 hurricane.

The advance of the shoreline in the reach immediately north of the inlet probably is a result of the littoral barrier created by excavation of the inlet and construction of the protective jetties between 1918 and 1925. The maximum accretion occurred in depths of from 6 to 12 feet where those contours moved oceanward a maximum of 950 and 1,125 feet, respectively, with average movements of 630 and 290 feet respectively. The accretion extended to the maximum 30-foot depth covered by the surve with slightly less accretion in the greater depths.

FROM 1928 TO 1955

In the period between the 1928 and 1955 surveys, there was a genera recession of the beach except where the natural beach was affected by construction of coastal works. The 6-foot depth contour advanced in som reaches and receded in others, with a general recession south of a point about 8 miles south or downdrift of Lake Worth Inlet. At depths of 12, 18, and 30 feet, an average recession of from 130 to 200 feet was experienced along the entire reach of coast under consideration except in the general vicinity of Lake Worth Inlet. It appears that the relatively stable beach through the surf zone for the 8-mile reach immediately sout of Lake Worth Inlet was the result of the rather extensive groin system and the artificial nourishment of the beach. In August 1944, about 280,000 cubic yards of material from Lake Worth was placed on the beach about 1,500 feet south of Lake Worth Inlet. Between May and November 1948, four stockpiles of sand with an aggregate volume of about 2,400,00 cubic yards were pumped on the beach.

The accretion of the beach north of Lake Worth Inlet continued unde the influence of the jetties constructed to protect the inlet. However, analysis of the surveys indicates that the rate of accretion had been somewhat slower. The maximum advance of the beach was limited to a 1/2-mile reach immediately north of the inlet. The overall advance of the shoreline in the 27-year period (1928-55) was about 600 feet, while an advance of about 500 feet was experienced in the period of about 10 year immediately after completion of the north jetty. The 1955 survey also i dicated erosion of a portion of the material deposited immediately after the construction of the jetty in depths of about 18 feet of water. That erosion was probably caused by the tidal flow through the inlet where a 27-foot channel has been maintained. Accretion during that period continued at 30-foot depth, indicating that hydraulic forces continue to re sort and re-arrange the deposited material until it approaches the most stable position. There is a distinct tendency for the tidal currents tc erode and transport the littoral materials, redepositing them in a typical fan with a maximum elevation of about that of the maintained channel.

At South Lake Worth Inlet, there was a small accretion of material which reached a maximum at the inlet and extended from 2,000 to 3,000 feet on either side of the inlet (see figure 11). The 6- and 12-foot contours advanced a maximum of about 800 feet, while somewhat less advance was experienced at greater depths. The 30-foot contour advanced about 300 feet at the inlet. Operation of the sand-transfer plant from the time the inlet was constructed--with the exception of several years during World War II--maintained the normal flow of littoral material and tended to limit the erosion downdrift of the inlet.

Tidal currents have also conveyed a considerable volume of littoral material into Lake Worth and South Lake Worth Inlets. Examination of the dredging records indicates that about 1,450,000 cubic yards were removed from the Lake Worth Inlet channel during the period 1929-55. A middle-ground shoal known as Peanut Island was formed at the inlet. Records of maintenance dredging at South Lake Worth Inlet are not available, but periodic maintenance has been required to keep the inlet open.

VOLUME OF LITTORAL DRIFT

The continued erosion of the Palm Beach shore is an indication that the flow of littoral material is less than the ability of the littoral currents to transport material. The excess capacity has been used to erode and transport sedimentary material deposited during some earlier period when an excess of material was transported to the area. It appears from the scanty survey data available that there was a deficiency of supply with consequent erosion of the beach prior to construction of the inlet protective works. Since their construction, the inlet jetties at Lake Worth have augmented the deficiency in two ways: First, by creating conditions favorable for deposition in the reach updrift of the works, and, second, by increasing tidal currents, thus tending to move material either into the inlet channel or oceanward. From 1929 to 1955, surveys of the area to 100-foot depth showed an accretion of 3,768,000 cubic yards in the fillet immediately updrift from the north jetty. As previously stated, about 1,450,000 cubic yards were removed from the channel during that same period, including the inner bar and the outer portion of the channel between the jetties. In addition, there was an accretion of 676,000 cubic yards in the area seaward of the jetties and channels, as shown on figure 10. Assuming that the jetties were a practically complete barrier, the total volume of littoral material thus placed into a more or less permanent deposit amounted to about 6,000,000 cubic yards, an annual rate of about 230,000 cubic yards.

It is also possible to estimate the capacity of the littoral currents by analysis of the erosion which has occurred in the reach between Lake Worth and South Lake Worth Inlets. Surveys indicate that about 8,800,000 cubic yards of beach material were lost in that reach between the 1929 and 1955 surveys. Of that volume, 6,500,000 cubic yards were lost from depths of 18 feet or more and 2,300,000 cubic yards in depths of less than 18 feet. In addition, about 3,000,000 cubic yards of material placed on the beach between 1944 and 1953 were lost. The total loss was thus 11,800,000 cubic yards or an annual rate of loss or deficiency in supply of about 450,000 cubic yards.

At South Lake Worth Inlet, the sand-pumping plant has maintained a portion of the littoral drift. Since the channel depth is about 6 feet, there has also been a considerable flow of littoral material around the inlet; therefore, it is impossible to make an analysis similar to that made above for Lake Worth Inlet.

CONCLUSIONS

Lake Worth and South Lake Worth Inlets have materially affected the flow of littoral material along the reach of coastline under consideration. The initial effect at Lake Worth Inlet was primarily due to the works provided to protect the channel. In recent years, the effects of tidal circulation in the inlet have become more important as the accretion in the fillet north of the inlet has approached stability. At South Lake Worth Inlet, the effect has been much less significant because of the limited deepening of the maintained channel and operation of the sand-transfer plant since the inlet was constructed.

It is the conclusion of the writer that the deficiency in the supply of littoral material to the beach south of Lake Worth Inlet is due to a reduction in the volume of littoral material moving along that portion of the coast which has been aggravated by the circulation system established by the inlet. Analysis of the rate of accretion in the vicinity of Lake Worth Inlet indicates that the reduction of supply in recent years may have amounted to as much as 50 percent.

Tidal currents in the inlet interfere with the normal littoral currents and transfer the material to deep water where it becomes more or less permanently deposited. The Lake Worth Inlet material has been deposited in depths up to 120 feet.

Finally, the survey indicates that there is a substantial movement of the littoral materials at depths of 30 feet or more. Additional investigations should be made to determine the importance of those movements in beach stability.

ACKNOWLEDGMENTS

The data presented and the analyses thereof were accomplished in the Jacksonville District Office, Corps of Engineers, under the general supervision of Colonel Paul D. Troxler, District Engineer, and Mr. Joe J. Koperski, Chief, Engineering Division. Appreciation of the Corps of Engineers' permission to use this material is acknowledged. Personnel assisting in the study included Bryan Cornwell, Thomas E. Brannen, and William J. Bryant.

CHAPTER 25

THE SAND TRANSFER PLANT AT LAKF WORTH INLET

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It is the intention of this paper to give a description of the soon to be completed Sand Transfer Facility at the north jetty of the Lake Worth Inlet in Palm Beach, Florida.

INTRODUCTION

A technical report of this nature would not be complete if it did not bring out, however briefly, the underlying causes which led to the design and eventual construction of the subject installation. The fundamental cause would be erosion, specifically, beach erosion, one of the most destructive forces of nature and, like other such forces, one which has come under the study of scientists and professional engineers.

The beach bordering on the east coast of Palm Beach Island is composed essentially of fine sand and shell fragments which are moved by the action of littoral currents and direct attack of the shoreline by severe waves. Although contour studies made prior to 1925 indicate minor shore recession, the major problem arose with the construction of the rock jetties which protect the Lake Worth Inlet and navigation channel. The predominant southward littoral drift was intercepted by the north jetty and impounded causing an interruption in the supply of sand to the south. The very striking and visible result has been starvation of the beaches along Palm Beach Island and accretion to the north where the beach now lies approximately 1400 feet seaward of the beach line south of the inlet.

ANNUAL SAND MOVEMENT

The Beach Erosion Board of the Corps of Engineers, U. S. Army, has estimated the natural southward movement of sand at Palm Beach to be between 150,000 and 225,000 cubic yards of sand per year and has expressed the opinion that the impounding capacity of the north jetty now has nearly been reached. They further estimate that the rate of accretion at the inlet, including material removed in maintaining the channel, amounted to 230,000 cubic yards annually from 1929 to 1955.

On Palm Beach Island, landward of the 18 foot contour, annual losses of 200,000 cubic yards were experienced during the same period. This loss of 200,000 cubic yards compares favorably with the volume which was prevented from reaching the island by the inlet.

REFERENCES

Corps of Engineers, U. S. Army (1956). Beach Erosion Control Report on Cooperative Study of Palm Beach, Florida.

PREVIOUS OPERATIONS

Restoration of the beaches south of the inlet has been unsuccessfully attempted in the past by means of various type groins built normal or generally normal to the shore line. The subject is somewhat controversial but the general consensus of opinion is that while groins may be useful in intercepting sand movement and thereby impounding material, they are relatively ineffective where the primary trouble lies in a cessation of the supply of sand moving to the beach.

As an alternative to groins, stockpiles of sand amounting to 2 and 1/2 million cubic yards have been pumped from Lake Worth and deposited on the beach at various times over a period of years. Later observations indicated that this sand had noticeably benefited the beaches to both the north and south. On the basis of these operations it has been demonstrated that at least ten miles of shore line could be adequately nourished by furnishing sand at the approximate rate of 200,000 cubic yards per year.

Inasmuch as this operation appeared successful, the question naturally arises as to why it could not be continued indefinitely. Perhaps the most serious objection can be found in the inferior quality of the sand itself. The lake sand is finer grained, of a different color and intermixed with black earth, high in organic matter and gasses. These characteristics and the fact that the lake could not be considered as an unlimited source of supply forced a search for another solution to the problem.

SOUTH LAKE WORTH INLET

A situation identical in many respects to the one at Lake Worth Inlet has been successfully resolved at the South Lake Worth Inlet which lies about 15 miles to the south. In 1937, a sand transfer plant was installed at the north jetty which functioned satisfactorily until it was shut down for the duration of World War II. During its operative period it had furnished a supply of sand to the beaches south of the inlet causing them to accrete materially and considerably reduced the shoaling of the middle ground of Lake Worth. Following the termination of pumping operations in 1942 the beaches to the south eroded and shoaling was once more evident in Lake Worth. Operations were resumed in 1945 and in 1948 the 6 inch pump was replaced by the present 8 inch unit which effectively bypasses about 80,000 cubic yards of sand annually, the results of which have been indeed gratifying to local interests.

REFERENCES

Knappen-Tippetts-Abbett-McCarthy, (August 1954). Preliminary Report on SAND TRANSFER FACILITY at Lake Worth Inlet, Palm Beach, Florida - for State of Florida, County of Palm Beach, Town of Palm Beach.

THE SAND TRANSFER PLANT AT LAKE WORTH INLET

ANALYSIS OF THE PROBLEM

In October of 1954, the New York firm of Tippetts - Abbott-McCarthy - Stratton delivered an engineering report to the Florida State Board of Conservation. This report provided an appraisal of the situation and dealt with various methods of supplying sand to the depleted beaches south of the Lake Worth Inlet.

After having evaluated several methods, the consultants recommended the construction of a fixed dredge type installation modeled on the South Lake Worth Inlet plant. They concluded that the new plant should be capable of transferring sand across the inlet at the annual rate of approximately 250,000 cubic yards. The plant would bypass material of essentially the same composition as that on the shore of Palma Beach Island and would, during periods of southward littoral drift, provide an almost continual flow of such material. It is further thought that maintenance dredging in the channel will be reduced considerably by the transfer of excess sand across the inlet, a fact which may be considered as a major point in proving the economic justification of the plant.

THE SAND TRANSFER PLANT

Basically, the Lake Worth Sand Transfer Plant consists of an arrangement of pumps, piping and other equipment designed to transfer the excess sand deposited by wind and wave action on the beach north of the inlet to a point where it will serve to nourish the starved beaches to the south. The facility includes the following basic features:

(1) <u>A fixed pumping station</u> - consisting of a dredge pump installation in a reinforced concrete building designed to withstand tropical storms of hurricane intensity and to harmonize with the surrounding resort-type area.

(2) <u>A submerged discharge line</u> - made up of steel pipe and rubber hose which will carry the dredged material across the Lake Worth Inlet to the depleted areas to the south.

(3) <u>A protective groin</u> - constructed of steel sheet piling and designed to maintain the beaches to the north in their present condition.

(4) <u>Flushing Equipment</u> - which will be used to prevent plugging or stoppage in the submerged line.

PUMP HOUSE

The structure housing the pumping equipment is a heavily reinforced concrete building, elliptical in shape, and having two floors, the lower of which is at elevation - 1.0 (M.L.W.). This is the main floor of the pump house and contains the dredge pump and motor, service and gland water pumps, the sump or bilge pump and the power supplying transformer. The upper floor at elevation 11.5 houses the main operating control desk, motor controllers, ventilating equipment and the suction hose boom and mast operating equipment.

The pump house is supported on a reinforced concrete caisson, founded on sand at elevation -28.0 (M.L.W.). The lower part of the caisson has been sealed by means of a 3 foot tremie concrete slab. To increase lateral stability the caisson has been filled with about 380 cubic yards of beach sand and is closed at the top by the main floor of the pump house.

MAIN EQUIPMENT

The Dredge Pump - is a centrifugal type pump with a 12" suction inlet and a 10" discharge outlet. It has been designed for a rated clear water capacity of not less than 4000 G.P.M. at approximately 690 R.P.M. against a total dynamic head of 198 feet. The operating characteristics are based on the direct connection, through a flexible coupling, to a 400 H.P. - 720 R.P.M. wound rotor motor, all mounted on a common welded structural steel bedplate.

The pump impeller is enclosed and rotates in a volute type casing which has been designed to convert the velocity head created by the impeller to pressure head at the discharge. The casing is equipped with easily replaceable liner plates and all wearing parts are of an abrasion resisting steel alloy. The pump is capable of handling up to 15% solids at an efficiency greater than 60 percent. The rate of sand transfer, under ideal conditions, will be from 170 to 178 cubic yards of saturated sand per hour.

The Hoisting Equipment - consists of a mast and boom arrangement which carries the 12" suction hose while transporting it throughout the pumping area. The suction leg boom arrangement has been designed to control both horizontal and vertical movement over the entire pumping range. Horizontal movement is controlled by a slewing or rotating assembly which will turn the boom through an arc of 140 degrees at a periphery speed of 50 feet per minute.

The luffing or boom raising and lowering assembly is capable of raising the fully loaded boom through a vertical arc of 25 degrees in approximately one minute. Two mooring posts have been provided at the limit of westward travel to permit securing of both the boom and the hose during rough weather or inoperative periods.

POWER SUPPLY

The suppression of noise was a primary consideration in the design of the plant and it was for this reason that internal combustion engines were ruled out as a source of power. In order to insure quiet operation, all of the equipment will be powered by electric current.

The Power Supply Transformer - is a double winding, 3-phase, dry type and is rated at 112-1/2 KVA providing a primary voltage of 4160 volts. The power cables are carried within steel, zinc-coated conduits which have been encased in concrete for protection against the elements. All possible measures have been taken to prevent objectionable radio and television interference.

DISCHARGE LINE

The total length of the 10 inch discharge line is approximately 1750 feet. For a distance of 750 feet across the floor of the navigation channel, the sand and water mixture will be carried through a heavy duty wire reinforced rubber hose. To facilitate removal for future deepening of the channel, the hose is made up in 50 foot lengths with built in steel couplings for easy disconnection. The hose is of the smooth bore type and is constructed of pure gum rubber protected by multiple layers of duck. It is resistant to sand abrasion and the action of salt water and will, according to its manufacturers, prove invulnerable to the attack of teredos and other marine borers. The land portion of the discharge pipeline is constructed of extra-heavy steel pipe with 0.5 inch wall thickness and is of a type particularly suited to the transportation of dredge sand.

PROTECTIVE GROIN

The pump house with its suction hose boom lies completely within an area bounded by an "L" shaped groin to the west and north and by the easterly extension of the north jetty to the south. The primary purpose of this groin is to maintain the beaches to the north in their present or "pre-pumping" condition. In this respect, the groin acts in the manner of a bulkhead or retaining wall. Only sand which moves over the top of this groin will come within range of the suction head. The groin, including its 198 foot seaward extension, is made up of 34 foot lengths of interlocking steel sheet piling driven flush with the existing beach profile and capped. Posts have been erected along the entire length of groin which will carry a safety line marking off the area to the public.

LINE FLUSHING EQUIPMENT

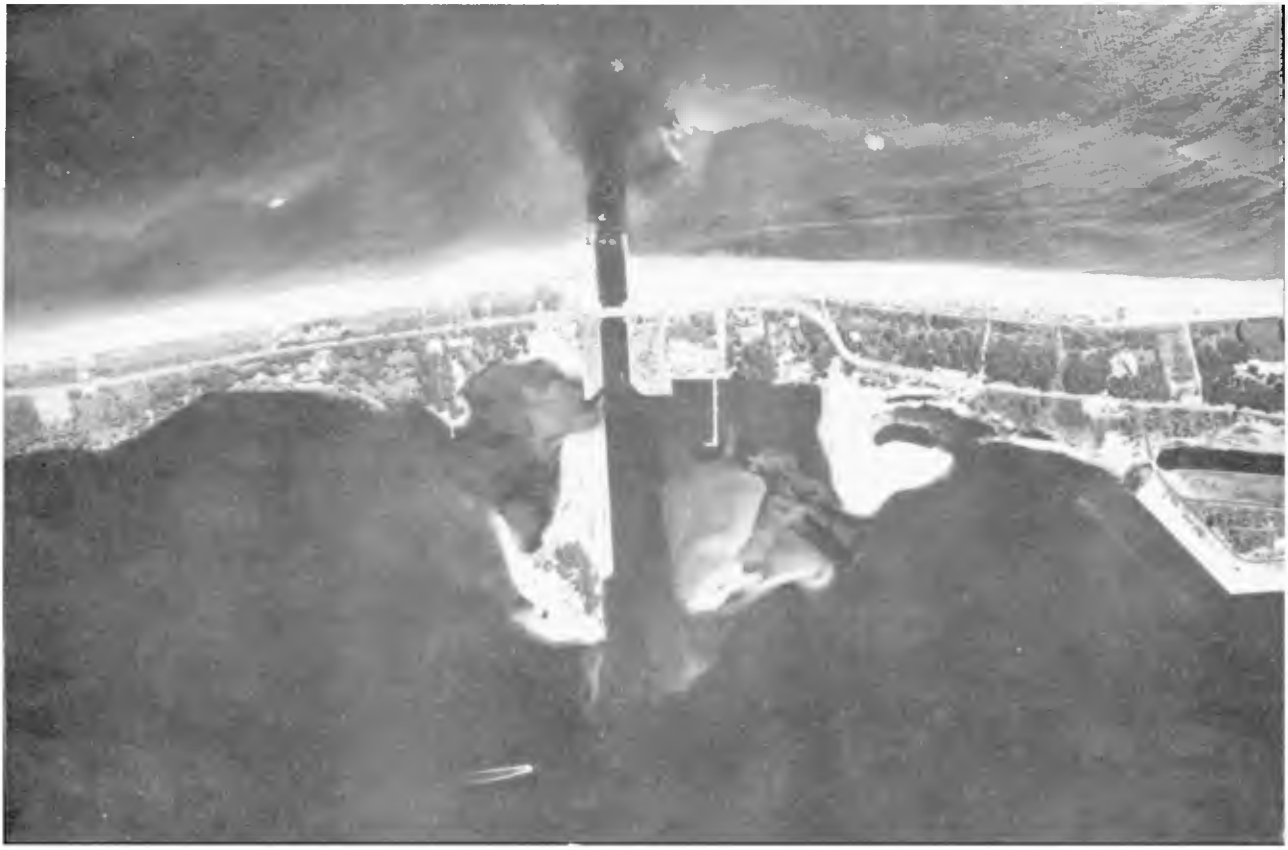
In the event that the submerged portion of the discharge line becomes clogged with sand due to a power failure or stoppage of the dredge pump, clean water, under pressure, will be automatically flushed through the line to eliminate the plug. Water for this flushing operation will be stored in a 17,000 gallon tank supported in a reinforced concrete cradle. The tank has been designed to hold sea-water under a pressure of 70 psig. A butterfly-type, air cylinder operated flush valve has been provided which under normal pumping conditions will remain closed. On failure of the power supply or other stoppage of the dredge pump, the solenoid operated valve opens an air pilot line which in turn opens the butterfly valve releasing water from the tank to the submerged line. Air required to close the valve at the end of the flushing run is supplied from a separate air receiver in which the air has been held during the pressure drop in the flush tank. A check valve will close automatically in the 10 inch line to prevent flushing water from flowing back toward the main pump while the operation is in progress. When current is restored following a power failure, the main pump will be started slowly, pumping clean water until the entire length of discharge line has been flushed out.

SUMMATION

Because the Lake Worth Sand Transfer Plant is a fixed-dredge type installation it must depend upon the action of littoral currents to bring sand within reach of the suction hose boom. Consequently the actual periods of sand by-passing will coincide with identical periods of sand movement from north to south. It would be well to point out that during these periods of southward littoral drift, the shoreline of Palm Beach Island will be experiencing sand losses due to erosion. Therefore it is assumed that during times of little or no sand movement to the pumping station only minor changes will be taking place along the beaches to the south. It is the accepted theory that southerly winds tend to build up the beaches while winds from the north cause them to erode.

The dredge pump in the new plant has been designed to handle the volume of sand as fast as it arrives within range of the suction hose. Assuming that such will be the case, it can be seen that the natural southward littoral drift will once more become available to the beaches of Palm Beach Island. The construction of this facility represents a forward step in the ever increasing program to combat beach erosion problems in the United States and throughout the world. A sufficiently wide beach or "beach berm" represents the best possible protection to shorefront and upland properties during periods of violent storms and resulting heavy seas

The future alone will determine the degree of success which will be attributed to the Lake Worth Sand Transfer Plant. For the present let its construction reflect credit on the civic-minded men responsible for its planning. Their recognition of beach sand as a valuable natural resource for the future as well as the presen should serve as an inspiration to others who are likewise dedicated to its conservation.



South Lake Worth Inlet

Part 3
COASTAL ENGINEERING PROBLEMS

Lake Worth Inlet

CHAPTER 26

FLORIDA COASTAL PROBLEMS

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INTRODUCTION

Florida is at war with coastal problems. Waves and tides threaten to wipe out its priceless heritage of sandy tropical beaches on two seas. Many communities have suffered enormous losses because of beach erosion in the last few years. The annual loss of land is approximately 300 to 500 acres.

Man-made fills in bays and waterways are detrimental to inlets and channels. Practically all of these fill operations have been carried out without any previous hydrographic or hydrological surveys.

This paper explains the nature and extent of these problems in Florida and gives recommendations for solving them.

GEOGRAPHY AND GEOLOGY

GENERAL

The State of Florida, situated in the extreme southeastern part of the United States between latitude 24.30 N and 31.00 N, and between longitude 79.52 W and 87.38 W, may be divided into two general geographic divisions: the Mainland and the Peninsula. It is bounded by two arbitrary lines, three rivers and a 1300 mile tidal shore line. The northeastern section of the state is bounded by the St. Marys River at the Georgia border, from its mouth to Ellicotts Mound (approximately 45 miles west-northwest of Jacksonville); thence the line runs arbitrarily to the confluence of the Flint and Chattahoochee Rivers (Apalachicola River); thence up the Chattahoochee River to the thirty-first parallel of north latitude. These boundaries separate the states of Florida and Georgia. The thirty-first parallel and the Perdido River separate Florida and Alabama to the northwest and west. Mainland Florida averages about 450 miles from east to west; Peninsular Florida, 400 miles from north to south, the Keys extending some 150 miles farther in a southwesterly direction.

Florida occupies only part of a much larger geographic division, a great continental plateau. This plateau is part of the old land continent of Appalachia which submerged in the Upper Cretaceous Period. Upon the metamorphic rocks of this platform marine deposits attest to several periods of uplift and subsidence. All of the territory comprising Florida today is underlain by limestone, marl and dolomite. Northwest Florida, Central Florida and the east coast areas have outcrops of "soft" limestone suitable primarily for road base material. In Hernando County on the upper west coast (north of Tampa Bay) a "hard" crystalline limestone is mined which is principally used in the production of concrete and for railroad ballast. This hard Florida limestone can be used in the construction of many types of coastal protection structures. Dolomite outcrops are interspersed along the upper west coast and as far south as Sarasota County south of Tampa Bay. Most deposits are "soft" and used for soil conditioners. A very small amount is of the hard crystalline type suitable for industrial and engineering uses.

The sand material which was built up on this footing of limestone came from the Appalachian Highland and was carried to the sea by rivers and streams. It then drifted southward along the shoreline by the action of waves and currents. Some of it was deposited in barriers, spits and recurved spits, the balance on the offshore bottom.

Although the Pleistocene ice sheets extended only as far south as the Ohio River, the change in sea level caused by their advances and recessions greatly affected the topography of Florida.

All parts of the state were subject to erosion during each interglacial stage, and increasingly large areas were above water during each successive glacial stage. Seven marine terraces are recognizable in Florida which were the bottom of the sea during seven previous high water levels:

Brandywine	(270 - 215 ft.)	Penholoway	(70 - 42 ft.)
Coharie	(215 - 170 ft.)	Talbot	(42 - 25 ft.)
Sunderland	(170 - 100 ft.)	Pamlico	(25 - 0 ft.)
Wicomico	(100 - 70 ft.)		

Directly to the east of the northern part of the Sunderland terrace in the highlands of north central Florida (longitude 82.00 W) and separating this terrace from the coastal plain to the east, stands Trail Ridge. This ridge extends from latitude 29.45 to 31.00 N. The crest of the ridge slopes from an elevation of 240 ft. at the south to 170 ft. at the point of break south of the St. Marys River (See Fig. 1). This ridge appears to be a re-curved spit created by an accumulation of sand transported by southerly littoral currents in the shallow Sunderland Sea. When the Sunderland terrace emerged the lowest part of the terrace lay directly west of Trail Ridge which obstructed drainage to the east. This low area is today known as Okefenokee Swamp, which drains tortuously into the Gulf of Mexico by way of the Suwanee River. Another interesting topographic feature in this same area, particularly from the standpoint of the coastal morphologist is the fossil recurved spit referred to locally as Baywood Promontory which lies directly east and southeast of Trail Ridge. The promontory is similar in appearance and probably in origin.

The areas of land have varied greatly in Florida during its geologic history. Although it was completely submerged for a very long period, the depth of the waters seems never to have been great. Change

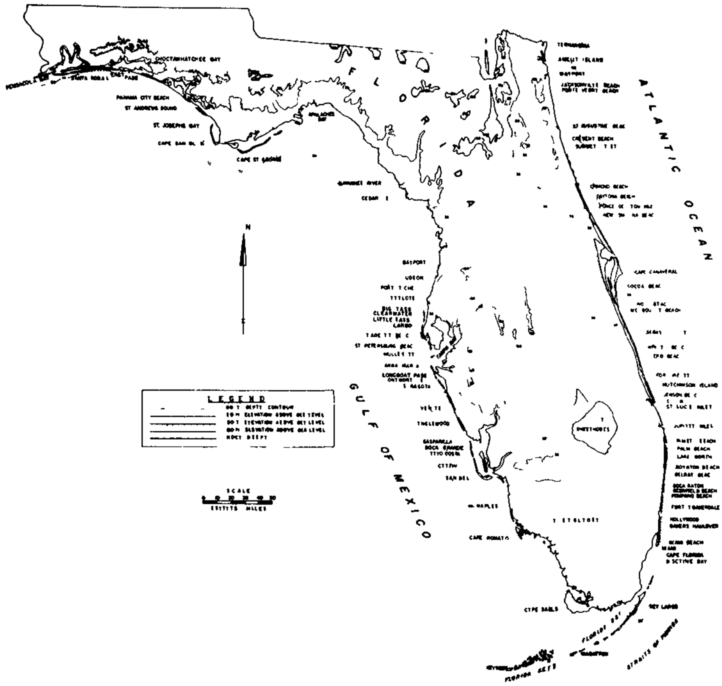


Fig. 1 State of Florida

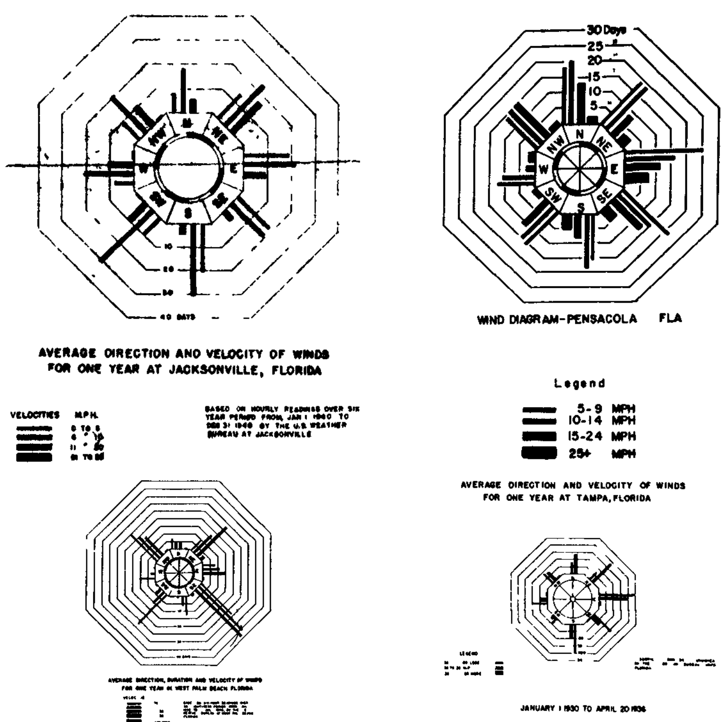


Fig. 2 Wind diagrams

of sea level produced great changes in land area due to the low relief. Present studies indicate a rise in sea level in the state of approximately one foot in 100 years with a noticeable speed-up in recent years. This is due to the melting of the polar icecaps. Between 1940 and 1950 the level at the mouth of the St. Johns River in extreme northeast Florida rose three inches; at Pensacola in the extreme western part of the state, the rise was four inches for the same period. It is not difficult to imagine the problem which may be created by the continued and increasing encroachment by the sea.

COASTAL GEOGRAPHY

Practically all of the east coast of Florida consists of a barrier island chain, broken occasionally by inlets. At Cape Canaveral the ocean beach is the greatest distance from the mainland, about 15 miles.

South of Miami Beach keys extend to Key West with small islands extending to Dry Tortugas. Mangrove swamps extend from Cape Sable to Cape Romano on the Gulf of Mexico. From Cape Romano to Anclote Keys (north of the barrier islands off Clearwater) the outer shore of the Gulf is formed of barrier islands, mostly low and without sand dunes. Off Sanibel Island in this lower west coast area is, perhaps, the only outcrop of coquina on the west coast of Florida.

In Pliocene time the coast north of St. Petersburg (Tampa Bay) was either tilted downward toward the west or lowered by other terrestial movement. This would account for the broad embayment of the shore between Anclote Key and the mouth of the Apalachicola River which is an area of salt marshes.

From Port St. Joe which is located on the lower west side of the ancient triangular delta of the Apalachicola River, the shore curves northwestward to St. Andrews Bay (about 40 miles) on the west Gulf coast. Between St. Andrews Bay and Choctawhatchee Bay the Gulf beach is on the mainland for more than 50 miles, an unusual feature in Florida coastal geography. The shore line from the mouth of Choctawhatchee Bay to Pensacola Bay, another 50 miles, is on Santa Rosa Island, a narrow barrier with extremely wide beaches.

COASTAL SANDS

All of Florida's beaches are sand beaches. Some have a large number of whole or broken shell in their content. The lack of pebble is due to the absence of hard rock material from which pebbles are formed. No firm materials are found on the Atlantic or Gulf coasts from Long Island to the Rio Grande River. The substances comprising the sand are silica (quartz) and calcite (shell). Nearly all the sand in the northern part of the state is composed of quartz. Southward along either coast the shell fragments increase.

The average grain size of sand on Florida beaches ranges from fine to medium. The coarsest siliceous sand is found on the beaches of the Gulf coast in the northwest part of the state. The wide hard beaches of the northern part of the east coast are composed of very fine sand. Some average grain sizes are the following:

Daytona Beach	.21 mm (highly siliceous)
Palm Beach	.47 mm (shell mix)
Miami Beach	.58 mm (shell mix)
Anna Maria Key	.34 mm (shell mix)
Choctawhatchee Bay	.33 mm (highly siliceous)
Santa Rosa Island (outside)	.25 - .35 mm (nearly 100% siliceous)

Most of the minerals present in Florida beach sand can be found in the northern part of the east coast, due undoubtedly to the nearness of the original source (Appalachian Highland).

It is not uncommon to find one or several of the heavy minerals (rutile, ilmenite, zircon, monazite, garnet and staurolite) on any beach where a preponderant amount of quartz sand occurs. On beaches consisting of broken coquina and coral (south Florida), these minerals rarely occur because, like quartz, they come from northern highland areas.

The brown grains of staurolite and rutile and black metallic grains of ilmenite are easily recognizable in the white quartz sand of the beaches on Santa Rosa Island in northwest Florida. The shiny diamond-like small grain of zircon are noticeable also.

Beach sands as well as inland deposits on older shorelines which are composed of more than 3% of the heavy minerals listed above are being processed commercially.

Production and exploration is continuing along the east coast as far south as Indian River County; on the Gulf coast west of Panama City (St. Andrews Bay); and along ancient shorelines in Bradford and Clay counties in north central Florida.

WIND, WAVES AND TIDES

The wind situation is shown in Fig. 2. With the exception of the wind diagram for Jacksonville, representing the northeast coast, the winds from the east are stronger and more frequent. The West Palm Beach diagram depicting the situation on the lower east coast shows that the velocities are greater from the northeast than from the southeast, but that the frequency is greater from the southeast than from the northeast.

Along the whole east coast strong northeast winds occur during the fall and winter seasons. In the spring and summer, southeast winds are predominant.

Along the west coast southwest winds are comparatively light and infrequent. Winds from the west and northwest are stronger but the velocities are not high. Offshore winds are stronger and more frequent. In regard to hurricanes see Mr. Dunn's paper in the Proceedings.

The wave action in the Atlantic Ocean is much more severe than i the Gulf of Mexico. On the east coast high waves and swells come in generally from the northeast. The frequency of waves of 12 ft. or higher is about 4% and on the west coast this frequency is less than ½%. Wave heights of 4 ft. are normal on the East coast and 1 - 2 ft. are normal on the Gulf coast. Waves of 5 to 6 ft. in height occur on every second or third year on the West coast. The mean tidal range increases from 2.4 ft. at Miami Beach to 5.7 ft. in Fernandina Beach.

The longshore component of the tidal wave is moving northward in northeast Florida and southward in southeast Florida. Along the west coast the situation is somewhat similar. On this coast the tide is a mixture of diurnal and semidiurnal; and in west Florida the tide is completely diurnal.

Along the east coast of Florida we have the Gulfstream. Near Miami Beach the average position of the axis of this stream is about 15 miles from shore and near Jacksonville Beach, 100 miles.

The velocity is about 3 miles per hour and decreases northward to about 2 miles per hour near St. Augustine. A low velocity coast current generally moves southward between the Gulfstream and the Florida shore.

LITTORAL DRIFT

PREDOMINANT DIRECTION OF DRIFT

The predominant direction of littoral drift on the east coast of Florida is from north to south. This is caused primarily by the higher velocities (30 to 50 m.p.h.) of winds from the northeast in the fall and winter seasons. The predominant wind direction as explained earlier is generally southeast, but these winds which, in particular, blow in the spring and summer are rather weak and for this reason not responsible for the quantity of littoral drift as are the northeast winds. Apart from some very limited areas at inlets where the direction of drift may be reversed, there is no place on the east coast where the predominant drift is northward.

The situation is somewhat different on the lower Gulf coast. The predominant drift is southward at most places, but less predominant, and it may be reversed at one or both sides of many passes as e.g. Boca Grande Pass, Sarasota Pass and Longboat Pass. On Sand Key south of Little Pass the drift is northward and the same is the case with the northern part of Clearwater Island at Big Pass (Fig. 1).

On the upper west coast the littoral drift is predominantly westward but it may be reversed locally at inlets. In the summer season it is westward.

QUANTITY OF DRIFT

The quantity of sand drift along the Florida shores within the littoral zone is not very well known but accumulations on the updrift side of jetties and dredging operations have given certain data which at least gives an impression of the quantity of drift material involved.

On the upper east coast from Fernandina to New Smyrna Beach, the drift seems to be 400,000 - 500,000 cu. yd. a year. At Palm Beach it is probably about 300,000 cu. yd. a year, and at Hillsboro about 150,000 cu. yd. a year.

The Gulf coast reveals much smaller figures. On the middle Gulf coast in Pinellas County (St. Petersburg) it is probably 50,000 - 100,000 cu. yd. a year; on the upper Gulf coast, about 100,000 - 200,000 cu. yd. a year.

The drift caused by currents in deeper water along the east coast (Gulf Stream) is unknown, but debris is carried northward as far as the northern part of Norway.

COASTAL MORPHOLOGY

It is not for the want of shorelines that coastal morphology was not born in Florida which, in this respect, presents a well stocked super market. Hardly any other state or country is able to make a better exhibit in all branches of coastal morphology than Florida.

The American school of geomorphology (W.M. Davis) is characterized by its main viewpoint which is that any actual form is a step in a development by which one link follows the other in a logical order consisting of standard forms.

In regard to coastal morphology one will probably have to soft-pedal standards and emphasize principles of physical nature. Visual observation and statistics give an excellent description of how this and that looks and also explain steps in development, but they are inadequate in explaining single elements in development, by which deeper understanding is obtained. Much remains to be explained, not least a better understanding of littoral drift processes, parallel as well as perpendicular to the shoreline. It is not the object of this paper to discuss such detailed phenomena.

A discussion of Florida coastal morphology must include a presen- tation of examples of coastal forms in the horizontal as well as the vertical plane.

SHORELINE CONFIGURATION

Florida's "large scale" shoreline configuration is explained unde "Geography and Geology". A description in detail will include a revie of different coastal features as barriers, spits, recurved spits, tom- bolos, angular forelands, etc., and a special mentioning of the influ- ence of inlets, unimproved as well as improved.

Barriers. - The present east coast of Florida has the appearance of a typical shoreline of emergence while most parts of the west coast in particular the southern section, has the appearance of an emerged shoreline which is now submerging again probably the result of a rise in sea level.

The barrier chain along most of the Atlantic coast and large part of the Gulf coast probably started out at some places as an offshore shell (coquina or coral) reef which was later elevated above the water surface by the lowering of the sea level and perhaps some terrestial movement. At other places an offshore bar was formed simply by the co nection of shell or coral reefs by littoral sand deposits. This bar w slowly elevated together with the reefs. If no reefs existed and the sea was shallow the bar was first built up by wave (in particular, swell) action and gradually moved on shore, widening and raising the beach, or it simply emerged gradually above the water surface after which the wind started forming dunes. Such development permits an ex- planation of the presence of successive parallel ridges consisting pri marily of silicia sand along the Florida shores. They are elevated beach ridges covered with clean sand.

There is a great variety in the width and configuration of these barriers. In some places they present themselves as barrier beaches, 50 to 150 ft. wide and so low that the wave uprush and high tide flow over them. Examples of this are to be found at many places on the eas coast e.g., at Hutchinson Island south of Fort Pierce where the bar- rier in places is only 100 ft. wide and 6 to 8 ft. above mean low wate level. At some other places on the east coast e.g., at Jupiter Island and at South Palm Beach, roads built on such narrow but higher barrier strips eroded and had to be moved to the bay side of the barrier.

On the west coast examples of such barrier beaches are numerous because the barriers generally are lower and more frequently perforate by inlets than on the east coast. Break-throughs are common. In 1957 two occurred: one at the northern tip of Longboat Key in the narrow recurved spit and one at the southern part of Anna Maria Key where the barrier was only about 50 ft. wide and 3 ft. above mean low water. Th (approximately) 150 ft. wide and 5 ft. deep break-through is now close by a new State road. At Sand Key south of Clearwater a smaller breakthrough occurred in 1957, but it closed immediately. The same storm (June 1957) overflooded parts of Madeira and Clearwater beaches where the area generally is from 4 to 6 ft. above mean low water.

Other barriers are much wider and can be characterized as barrier islands. They consist of multiple ridges of considerable height. Examples of barrier islands on the east coast are to be found at Amelia Island and Cape Canaveral on the east coast (Fig. 3). On the lower Gulf coast examples are few. Wider ridge areas exist at Sanibel Island but the height does not exceed 10 ft. above mean low water and the same is the case on some parts of LaCosta Island (Fig. 3).

The upper Gulf coast between Panama City and Pensacola presents low and narrow as well as high and wide barriers. Wide barriers exist in Walton County west of Panama City. An example of a rather narrow barrier is Santa Rosa Island which has the longest unbroken stretch of beach on the Gulf coast of Florida. Its beaches and sand dunes are snow white.

At some places barriers have, by unobstructed prolongation in the littoral drift direction, developed in such a way that big bodies of the sea have been cut off. This is the case at Apalachicola Bay, Choctawhatchee Bay and Pensacola Bay. The rising sea level is now encroaching on these bay barriers.

The development of barrier spits is mentioned under "Spits".

Keys - The word "key" is the same word as the English "cay" used as terminology for sand keys. Some barrier island in Florida are called keys whether they consist of elevated coral reefs as the big archipelago, the Florida Keys, extending about 120 miles out in the Gulf of Mexico, or they are only smaller islands mainly consisting of sand as, e.g., Key Biscayne and Sand Key in Biscayne Bay or Mullet and Egmont Keys at the entrance of Tampa Bay.

The eastern section of the Florida Keys consists of long narrow islands composed of limestone containing heads of corals. They were probably formed as coral reefs in an ancient sea. The western keys, extending with a few breaks, to Dry Tortugas, are apparently the result of shoaling action. The sand keys are only elevated shoals or smaller barrier islands disconnected at earlier times from the main barrier system because of erosion and rise in sea level.

Spits - The numerous barrier islands along the Florida coast are separated by passes, sounds, channels, and inlets, natural as well as improved. Barriers which have developed freely are usually provided with a barrier-spit at the downdrift end. On the east coast such spits are to be found e.g. at Matanzas Inlet and Ponce De Leon Inlet; on the lower Gulf coast examples are the northern end of Anna Maria Key, the southern tip of Long Key and the northern tip of Clearwater Island; on the upper west coast the Pensacola barrier tip. All of these spits are built up of material derived from the littoral drift on the sea-side combined with tidal current activity caused by the tidal exchange of water between sea and bay. Such spits often present themselves as rather wide beaches with irregular dunes and little or no formation of beach ridges. Some spits have been cut off from the barrier to which they belonged and as islands migrated across an inlet to the barrier on the downdrift side. This has been the case at St. Augustine and Matanzas Inlets. The natural cycle was that these inlets gradually moved southward until a new inlet was cut farther north after which the southern inlet closed.

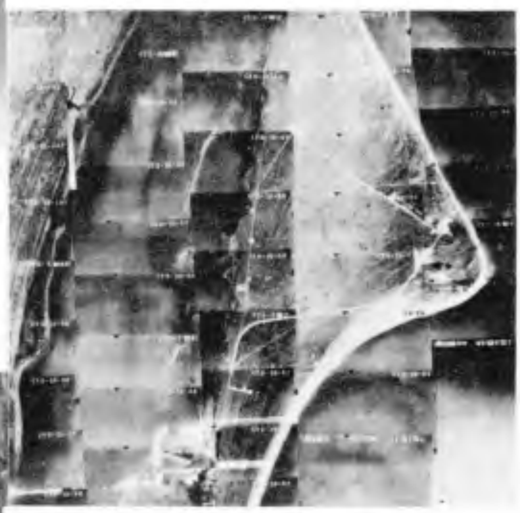
Recurved Spits - Where the open sea area on the downdrift side of a barrier is spacious and without currents of high velocity the spit eventually develops as a recurved spit. The northern part of Florida was built up as a huge recurved spit. Compared to this huge example our modern recurved spits are tiny, but they are to be found anywhere along the coast. A good example is Sanibel Island on the Gulf coast. Fig. 3 shows other (and smaller) examples at LaCosta Island and Amelia Island. Some of the recurved spits were "shaved" by inlet channels during their development so that longitudinal growth was replaced by a transversal growth. The result -- a peculiar configuration. An example is the recurved spits at Little Pass. The northernmost one, Clearwater Island, is now provided with groins on the ocean side and the southernmost, Sand Key, is seriously affected by a swash channel running close to and eroding the shore. Because of the continuous closing of inlets by natural action many recurved spits are fossilized.

Another fossil example of such "cut-off" recurved spit is at the northern end of Longboat Key. A baby spit grew out on the top of the fossil when the channel shifted northward.

Tombolos and Cuspate Forelands - Tombolos are to be found at many places in Florida because the limestone, coral reefs and sand shoals provided many opportunities for the formation of tombolos, single as well as double.

Cuspate Forelands are to be found at many inlets where shoals on the sea side caused a refraction pattern which constantly brought material toward the inlet. Examples of this are numerous and it is interesting to note that even small islands or barriers for this same reason may have two different directions of predominant littoral drift e.g. at the north end the drift is northward; at the south end, southward. At the northern part of Siesta Key at Sarasota (lower Gulf coast) a tombolo has developed by northward littoral drift caused by refraction on the shoals in front of Sarasota (Big) Pass. At the southern part a recurved spit developed at Midnite Pass because of southward drift. The big "hump" in the middle is caused by a large rock outcropping and demonstrates a northward rather than a southward drift. Similar situations are to be found at Longboat Key as well as Anna Maria Key on the lower Gulf coast.

Amelia Island



Cape Canaveral

La Costa Island

Fig. 3

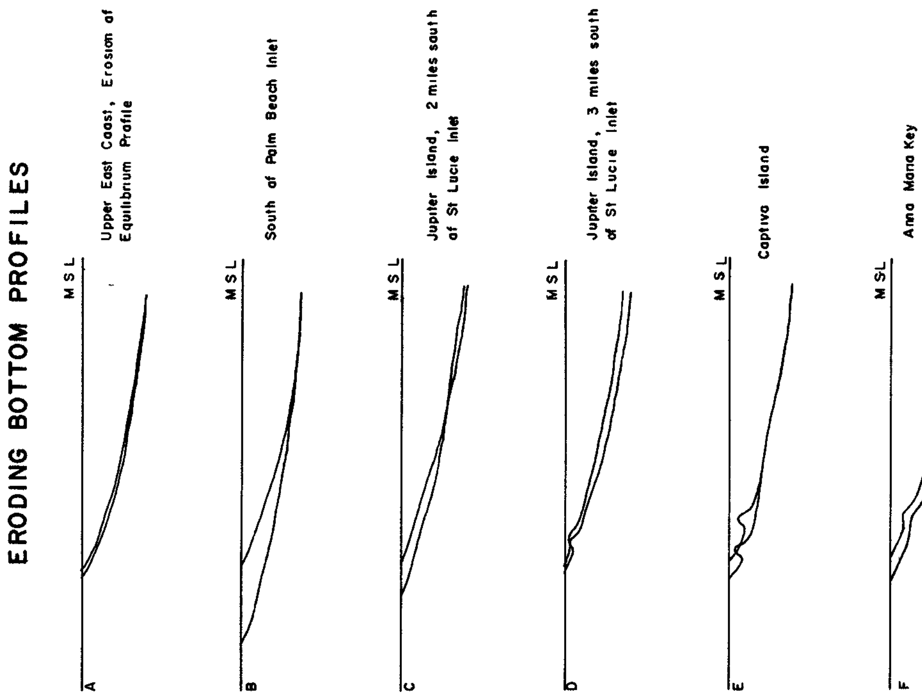


Fig. 5 Eroding bottom profiles

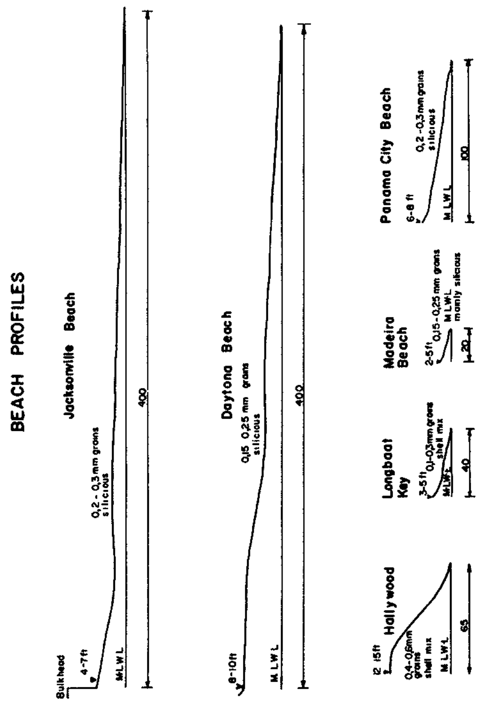


Fig. 4 Beach profiles

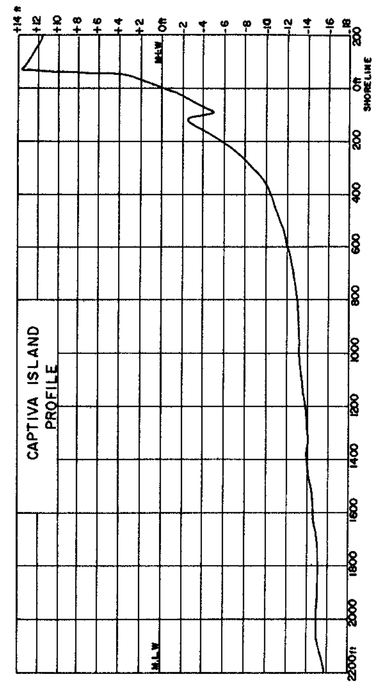


Fig. 6 Captiva Island profile

The most amazing tombolo, or perhaps it should be classified as cuspate foreland in Florida (and one of the biggest in the world) is at Cape Canaveral (Fig. 3). Its base is about 50 miles long and its tip about 15 miles east of the base line. The Canaveral tombolo must have developed for years in the shadow of rock reefs which caused one barrier system after the other to grow out based on littoral drift material (mainly sand) from the north. The tip of the tombolo gradually moved southward due to this development. For a longer period the tip may have been stationary. Comparing this development with the development at Dungeness, England as described by Steers (1954) one will find great similarities even if the foreland at Dungeness has plenty of coarse material, shingle and pebble, while Canaveral is all sand. The farther extension of the Canaveral tombolo has probably slowed down now partly because of insufficient material supply and partly because of the rise of sea level. The beach is, however, still fairly stable on the north side while erosion has started on the south (lee) side.

A very interesting example of a double tombolo is on the lower Gulf coast at LaCosta Island (Fig. 3). It developed behind a small sand key and on the top of an older system of beach ridges indicating three different shorelines of older date, probably all parts of tombolos built up behind a somewhat larger and differently located sand key which apparently moved forward and backward in front of the shoreline thereby building up different systems of beach ridges.

Angular Forelands - Angular forelands develop when barriers, spits, and recurved spits develop from different solid spots and meet, thereby creating a lagoon between them which is then gradually filled up with growth and deposits. They are not very common in Florida because they require two different directions of wave activity of fairly equal magnitude and such conditions do not exist on the exposed coast of Florida but only in the bays.

CONFIGURATION OF BEACH AND OFFSHORE PROFILES

The width and profile of the beaches depends on the character of the wave action and of the beach material. Table I shows examples of beach characteristics from different places on the Florida coast. From this table it can be seen that apart from the area around Flagler Beach the beaches on the upper east coast are rather wide and gently sloping with fine silicious sand. Beaches on the lower east coast are usually narrow and steep because of a large grain size shell mix giving high permeability.

On the lower Gulf coast the beaches are generally very narrow with varying steepness depending on grain size and shell mix. The upper west coast beaches are wider with moderate steepness in medium size silicious sand.

Fig. 4 gives an impression of the wide differences in width and profile configuration on the east and west coasts.

TABLE I

BEACH CHARACTERISTICS

East Coast

	Average Width of Beach at MLW (Ft.)	Elevation at Toe of Cliff or Bulkhead (Approx. Ft.)	Grain Size Beach Material (Approx. mm.)
Jacksonville Beach Pier	300-400	4-7	0.20-0.30 silicious
St. Augustine Beach Pier	300-400	8-10	0.20-0.30 silicious
Crescent Beach	400	8-10	0.20-0.30 silicious
Daytona Beach Pier	400	8-10	0.15-0.25 silicious
Marineland	150-300	10-12	0.30-0.40 shell mix
New Smyrna Beach 3/4 miles south of the inlet	300-350	6-8	0.20-0.30 silicious
Vero Beach	60-100	6-9	0.30-0.40 shell mix
Ft. Pierce South	60-120	3-6	0.30-0.40 silicious shell mix
Palm Beach	40-80	3-8	0.40-0.50 shell mix
Jupiter Island	50-150	3-8	0.20-0.40 silicious shell mix
Pompano Beach South	30-80	4-8	0.40-0.50 shell mix
Hollywood Beach	50-80	12-15	0.40-0.60 shell mix
Miami Beach	0-100	0-7	0.50-0.60 shell mix

Gulf Coast

Naples	60-100	4-6	0.10-0.25 silicious
Fort Myers Beach	40-120	3-6	0.15-0.20 silicious
Captiva Island	20-70	4-6	0.20-0.40 shell mix
Longboat Key	30-70	3-5	0.20-0.30 silicious shell mix
Anna Maria Key	0-70	2-5	0.30-0.40 shell mix
Madeira Beach	0-70	2-5	0.15-0.25 silicious with some shell
Clearwater Beach	0-70	3-5	0.15-0.20 silicious
Choctawhatchee Bay	70-100	6-8	0.25-0.30 silicious
Santa Rosa Island	300-500	6-8	0.25-0.35 silicious

The figures in Table I are average figures but they give an idea of the influence of wave action and grain size on the width and steepness of the beach.

The seasonal profile fluctuations are moderate apart from areas with coarse material on the upper part of the lower east coast, e.g.

at Jupiter Island where the beach may be raised and lowered by 5 ft. or more from winter to summer. The seasonal shoreline fluctuation seems to be from 30 to 60 ft. on the upper east coast and from 10 to 40 ft. on the lower east coast. On the lower Gulf coast they seem to be from 10 to 20 ft.; on the upper Gulf coast from 20 to 30 ft.

In accordance with the grain size and the shell content (permeability) beach cusps are poorly developed on the upper east coast apart from an area at Flagler Beach with much shell. They may be rather large (vertex extending 150 ft.). On the lower east coast beach cusps can be found everywhere but the dimensions are moderate (20-80 ft.) because of the moderate wave action caused by the sheltering effect of the Bahama Islands. On the upper west coast cusps extending 100 ft. can be found.

Migrating sand humps on the beach area on both sides of the average mean sea level line are common in particular on the rapidly eroding shores of Jupiter Island and South Palm Beach. The length of these migrating humps which is demonstrated by undulations in the shoreline configuration are 100 to 1000 ft. and shoreline fluctuation up to 150 ft. The direction of migration is predominantly southward and they are usually accompanied by a depression wave in the beach and shoreline on one side or both, thereby often being responsible for a considerable momentary erosion.

Rocky beaches are mentioned in a special section below.

OFFSHORE BOTTOM PROFILES

Information about offshore bottom profiles has been obtained from the U. S. Coast and Geodetic Survey, the Corps of Engineers, Coastal Engineering Laboratory surveys and private surveys. The accuracy of these surveys cannot be compared which fact should be taken into consideration in the following.

Fig. 1 shows the location of the 60 ft. depth contour along the Florida shore. It can be seen that this depth contour on the upper east coast and, in particular, on the lower Gulf coast is several miles offshore. Between Jupiter Island and Miami Beach on the lower east coast the 60 ft. depth contour is only ½ to 1 mile from the shore. It is a theory that this is caused by the erosive action of the Gulfstream. At the Choctawhatchee Bay barriers on the upper Gulf coast (west of Panama City) the profiles are of similar steepness to the 60 ft. depth contour.

At some places rock reefs have a certain influence on the steepness but the main part of the profile is still made up of sand. Fig. 1 shows where rock reefs exist. They are found between Matanzas Inlet and Ormond Beach on the upper east coast and almost continuously, but irregularly, between Ft. Pierce and Miami Beach. On the lower Gulf coast only a few rock reefs exist. On the upper Gulf coast there are none.

Even if maximum erosion as described later and maximum steepness are both concentrated on the lower east coast and the lower Gulf coast there seems to be no direct connection between erosion and steepness. The responsibility of other factors on erosion is thereby clearly demonstrated. As mentioned in the section about erosion, Florida coastal inlets have a great share of responsibility for erosion.

The influence of inlets on shoreline configuration and their specific importance for the development and shape of the beach profiles on the downdrift side of the inlet is the following: when the normal littoral drift is interrupted by a littoral barrier, e.g. a jetty protected inlet, erosion will start immediately of the nearshore beach profile because the greatest quantity of sand migrates in the area at the shoreline and this area will now suffer lack of fresh supply of sand from the updrift side. As a consequance of this the shoreline recedes and the slope of the entire profile decreases because the depth contours do not recede at the same rate. This process does not continue unlimited because the shoreline gradually turns in such a way that the rate of erosion decreases just downdrift of the littoral barrier, but at the same time the area influenced by the littoral barrier still extends downdrift. When the profile steepness has decreased to a certain extent particularly if the shore is protected with groins decreasing the recession of the offshore bottom, profiles will steepen again. This process will eventually create very unstable profiles under storm conditions.

Fig. 5 shows schematically the development of beach profiles. Profile A shows how an equilibrium profile (upper east coast) recedes as a result of erosion, e.g. caused by a slow rise in the sea level. Profile B demonstrates the development south of Lake Worth Inlet following the jetty improvement. Other examples of this are to be found at St. Lucie Inlet and Ft. Pierce Inlet as described under "Inlets". Profile C shows schematically the development of beach profiles at Jupiter Island two miles south of St. Lucie Inlet. Material eroded from the beach is here deposited offshore and not removed southward fast enough to balance the depositing. This picture will probably change in the future. Profile D shows schematically the development one mile farther south where deep water erosion takes place by which the profile steepens as a whole, because the shoreline recedes much slower than the depth contour. Profile F shows the development of beach profiles at Anna Maria Key. Profiles up to about 20 ft. depth have steepened so much that they are becoming unstable. Artificial nourishment from the bay is now undertaken on the southern part of the island where the highway is constructed and will have to be undertaken at other parts if a rapid shoreline recession during heavy storms shall be avoided.

The shoreline recession during the period 1941-1956 was at some parts of this Anna Maria Key coast 20 ft. a year. Compare a similar development in other parts of the world, e.g. on the Danish North Sea Coast (Thyboron) where an excessive steepness of the offshore bottom profiles is in places responsible for a shoreline recession of up to 20 ft. per year averaging about 10 ft. per year.

LONGSHORE BARS AND ROCK REEFS

Little information is available about longshore bars along the Florida coast. At many places on the lower east coast the offshore bar is identical with an offshore reef of coquina or coral which may appear in two or three rows. Storm waves will often break over this reef and sand will be deposited on the reef and connect holes in the reef system. Pure sand bars exist at many places on the upper east coast, e.g. at Jacksonville Beach and Fernandina Beach. The bars are comparatively wide and low.

While the northern part of the east coast has only a few rock reefs the coast south of Cape Canaveral has rock reefs about all the way through to Key West. The location of these reefs in the bottom profile is very irregular at some places, in particular to the north. For example, at Vero Beach only one row of reefs exists. At other places, in particular to the south, more than one row exists and erosion still uncovers new rock formation as on the beach at South Palm Beach. At Deerfield scattered rocks are found at 12-15 ft. depth. Farther south, at Hollywood (and partly at Miami Beach) there is some kind of an underwater "terrace" sloping gently offshore to about the 12 ft. depth contour approximately 1,000 ft. from shore. Beyond this there is a longshore channel enclosed by a reef lying roughly parallel to the shoreline. The depth on the reef varies from about 10 ft. to about 20 ft. The bottom outside the 12 ft. depth contour (approximately) is interspersed with loose rock.

In the Florida Keys numerous outcrops of limestone along the shore rise only slightly above sea level. There are places where the uneven surface of hard limestone is exposed below and in front of a narrow beach of calcareous sand.

The lower Gulf coast has only a few offshore reefs, e.g. at Englewood, Venice and Siesta Key. An offshore sand bar exists along most of the coast. The same is the case with the coast of west Florida where sand bars are well developed in the comparatively coarse silicious sand with grain size ranging between 0.25 and 0.4 mm. Lunate bar systems formed off barrier islands in the Panama City - Choctawhatchee Bay area are a modification of longshore bars probably caused by a regular rip-current system.

In regard to the cross-section of bars there is a pronounced difference between the profiles on the upper east coast which particularly in winter have a considerable amount of steep storm waves and some profiles on the lower Gulf coast where the wave activity is a mixture of small steep waves and longer swells. The swells will as in the case at Captiva Island (Fig. 6) push the bar as close to shore "as possible" but they cannot carry the bar up on the beach because the small storm waves constantly bring in water by wave-breaking and this water has to flow out again by which flow activity, an often rather deep trough is eroded between the shoreline and the bar which again results in beach erosion as already mentioned.

MANGROVE SWAMPS AND MARSHY SHORES IN FLORIDA

Florida has extensive coastal areas of low relief that have wide t narrow zones of mangrove swamps and salt marshes. These are along the coasts with little wave action and bordering many bays, sounds, and estuaries. They cover nearly 800 square miles because several zones ar broad, especially in the region from Cape Sable to Cape Romano along th southwest Gulf of Mexico coast where large mangrove swamps occur (See Fig. 1). Numerous tidal creeks and backbays dissect such areas into a labyrinth of shallow water channels. Very little erosion is taking pla in these areas because most of them are accumulating peat, muck, marl and other deposits. In fact, many of these areas are extending themselves out into adjacent shallow waters by these deposits, and the coastal problem is mostly that this process often chokes up channels an aids in the development of bars and shoals. This deposition has made necessary some dredging to keep boat channels open and to desired depths

Many mangrove swamps and herbaceous marshes on tidal flats offshor act as protective works against excessive erosion by storms because the diminish the wave and tide action. The low coasts in Florida are thus partly protected from occasional intense hurricanes by these types of vegetation. The mangrove swamps of trees and bushes are particularly effective in reducing storm damage and they should be retained where practical. Some have been removed, especially along the Florida Keys and this has led to increased erosion and other damage by storms. Many marsh and mangrove swamps have been better drained to reduce mosquitoes, make boat channels and anchorages, and improve shore property for residences. Some have been filled in, such as Miami Beach, but very few of these activities have led to any appreciable damage to the coasts. The only precaution advised is to keep the shore fringes of marsh and mangrove as intact as practical along where storm damage often occurs, suc as along parts of the Florida Keys and southern and southwestern coasts of the peninsula. Meanwhile the danger of flooding during storm and hurricane tides is considerable, because the areas are usually very low. Reference is made to the section dealing with "Development of Coastal Areas".

ROCKY SHORES

Rocky shores have only a very limited extent in Florida while rock reefs exist for considerable distances on the east coast. In no place in Florida are there high rocky cliffs, but rather small ledges reaching from low water level to a maximum of about ten feet above low water level (e.g. at the southern part of Jupiter Island).

The rocky shore at Marineland on the upper east coast represents a natural sea wall consisting of numerous small bays of tongues, which configuration will result in high energy losses and therefore good protection of the beach and shore. Similar formations exist at other places on the east coast, e.g. at Jupiter Island, South Palm Beach and Hillsboro. None of the rocks exposed are of great geological age and some of them were formed under conditions like those prevailing today in the same region.

On the Florida Keys outcrops of limestone and coral along the shore are very common but rise little above sea level. On the inner shores of the Keys, the rock is undercut by solution. At Venice on the lower Gulf coast is found outcroppings of sandy phosphatic limestone.

BEACH EROSION IN FLORIDA

GENERAL

Erosion starts when more material is carried away than deposited from a coastal area. In Florida as elsewhere one can distinguish between natural erosion and man-made erosion. Natural erosion is a result of a long-range gelolgical and climatalogical development by which land and water masses are still trying to find a balance. As mentioned under "Geography and Geology", the sea level has been rising along the Florida coast at a rate of more than one ft. per hundred years for the last few decades. Such erosion is generally rather slow, e.g. as demonstrated by a recession of the shoreline of less than one ft. per year. It is found at most places on the upper east coast, e.g. at Crescent and Daytona Beaches (See Fig. 1). At Crescent Beach, the erosion is so slight that it is difficult to recognize. In certain periods accretion takes place. On the lower Gulf coast the beaches at Naples have been stable at earlier times, but erosion has now started. On the upper Gulf coast most of the coast between Panama City and Pensacola is slowly eroding. At the passes "inlet-erosion" takes place.

Fig. 7 gives in six photographs an impression of how such erosion looks in Florida. Miles of shoreline have vanished and are vanishing in this way.

At some places as, e.g. Jupiter Island on the east coast and at Anna Maria Key on the lower Gulf coast, deepwater erosion takes place by which the offshore profiles, as mentioned earlier, are continuously steepened. This situation is unfortunate for the stability of these profiles and will, under storm conditions, eventually result in strong shoreline recessions. It is interesting to note that the strongest shoreline recessions in Florida (10-20 ft. annually) occur at these two places where the offshore profiles are steepest and where deepwater erosion at the same time takes place. But inlets play a considerable role in the development at these places. At other spots special wave and bar conditions are responsible for the erosion of the beach itself such as mentioned under "Configuration of Beach and Offshore Profiles". The amount of erosion seems to be increasing at an accelerating rate. The reason for this is partly the rising sea level, but mostly the result of man's interference with nature's regimen which gave a great number of surprises which could have been avoided if things had been evaluated properly before action was taken.

MAN-MADE EROSION

Beach erosion problems in general and the Florida man-made problems of erosion can be explained using the terms "source" and "drain" of materials.

A source of materials is a coastal area which delivers materials to other beaches. A source might be an area where erosion takes place, a shoal in the sea, for instance; the shallow area in front of an inlet which has been closed; a river which transports sand material to the sea, or sand drift from dunes to the beach. Artificial nourishment of any kind is also a source.

A drain of materials is a coastal area where materials are deposited. A drain might be a marine foreland of any kind, a spit, recurved spit, tombolo, angular foreland, etc. It might also be a bay, an inlet, or a shoal. Man-made constructions such as jetties, groins, or dredged sand traps, are also drains.

A drain has a source on its updrift side. Its downdrift side can be classified as its "littoral shadow".

In practical coastal engineering technology the following general rules are valid:

1. Coastal protection should be built in such a way that it functions as a drain. It should therefore have a source but not a drain on the updrift side. If there is a drain the coastal protection will not be very successful unless material is supplied artificially. With a sufficient supply of material further protective structures may be unnecessary.

2. A harbor or an improved inlet on a littoral drift coast should not act as a drain. It should therefore have no source but if possible a drain on the updrift side. Meanwhile it is very difficult to find a place where such ideal conditions exist, and many other factors play an important part. Most harbors are built in a sheltered area, an inlet, a bay or in a river mouth (Jacksonville, Miami, Tampa). In such areas depositions will almost always take place either from the littoral drift or as silting, which means that the harbor actually functions as a drain. Protection against the littoral drift can be effected by the improvement of the inlet by jetties. An improved inlet acts as a drain and protects the inlet, but at the same time it cuts off the supply of material to the beaches on the lee side which means that it works as a drain.

Speaking of "man-made" configuration of shorelines, this terminology is not any compliment to man's contribution to the changes of shoreline configuration in Florida. The authors of this paper do not know of any cases in Florida where man did change the shoreline intentionally to any large scale. Some bulkheads, in particular in the City of Miami Beach area, may have been built a little too far out.

The man-made changes in shoreline configuration in Florida are all unintentional and are caused by man-made littoral barriers, first of all, by the jetty-improved inlets. The reason for this erosion is well known and is only briefly mentioned under man-made changes in beach profile configurations. Table 2 is a list of some of the larger unimproved inlets in Florida. The degree of erosion they are responsible for is

a - East coast, Jacksonville

b - East coast, Jupiter Island

c - East coast, Miami

d - Gulf coast, Anna Maria Key

e - Gulf coast, St. Peterburg

f - Gulf coast, Clearwater

Fig. 7

indicated "small" as shoreline recession less than 1 ft. per year. 1-5 ft. per year is classified as being "moderate". Erosion exceeding 5 ft. per year is "severe".

TABLE 2

LARGER UNIMPROVED INLETS IN FLORIDA INFLUENCE ON EROSION

Inlet	Character of Erosion
Matanzas Inlet, East coast	Moderate to severe
Ponce De Leon Inlet	Small
Carlos Pass, Lower Gulf coast	Small
Redfish Pass	Moderate
Captiva Pass	Small
Boca Grande	Small to moderate
Gasparilla Pass	Small
Midnight Pass	Small to moderate
Sarasota Pass	Small to moderate
Longboat Pass	Small to moderate
Pass-a-Grille	Small
John's Pass	Small
Big Pass	Small
East Pass, Upper Gulf coast	Small to moderate

From Table 2 it can be seen that erosion caused by unimproved inlets in general can be classified as small to moderate. The only case where at present an unimproved inlet must probably be made responsible for a very local but rather serious erosion is Matanzas Inlet where the coastal highway is now seriously threatened south of the inlet.

The situation is different when it comes to jetty-improved inlets. Table 3 gives an overlook of such inlets with their influence on erosion classified as in Table 2 -- "severe" refers to shoreline recession of more than 5 ft. per year on the lee side.

TABLE 3

IMPROVED INLETS IN FLORIDA CAUSING LEE SIDE EROSION

Inlet	Improvement	Character of Erosion on Lee Side
	Atlantic Coast	
St. Mary's River (Fernandina)	Jetties	Moderate
St. John's River (Jacksonville)	Jetties	Small to moderate
St. Augustine Inlet	Jetties	Constructed recently
Canaveral Harbor	Jetties	Moderate
Sebastian Inlet	Jetties	Small to moderate
Ft. Pierce Inlet	Jetties	Moderate to severe
St. Lucie Inlet	Jetties	Very severe
Jupiter Inlet	Jetties	Small to moderate - Artificial nourishment from inlet undertaken
Palm Beach Inlet	Jetties	Moderate to severe - by-passing sand plant under construction. Artificial nourishment from bay undertaken.
South Lake Worth Inlet	Jetties	Sand transfer plant in operation since 1937.
Boca Raton	Jetty on updrift side	Moderate
Hillsboro Inlet	Dredging and small jetty on updrift side	Moderate
Port Everglades	Jetties	Small
Baker's Haulover	Jetties	No beach left on downdrift side.
Miami	Jetties	Small
	Lower Gulf Coast	
Gordon Pass	Dredged canal and some groins	Small
Casey Pass	Jetties	Small to moderate
Little Pass	Some dredging	Small to moderate
	Upper Gulf Coast	
St. Andrew's Bay Entrance	Jetties	Small
Pensacola Bay Entrance	Canal	Small

It is rather difficult to make a reliable judgment of what is "natural" and what is "man-made" erosion. In the case of inlets it should also be remembered that the unimproved inlets were responsible for some erosion which increased when the inlet was dredged, jetty-protected, or both, but it is obvious in most cases in Florida that man for various reasons is responsible for more than 50% and sometimes close to 100% of the erosion at improved inlets.

The erosion of the Florida sand shores has been calculated at 300 acres per year. Out of these 300 acres about 100 acres seems to be "man-made". These losses are unfortunately where property is most valuable. If the erosion of other types of shoreline is included the annual loss may be 500 acres. The seriousness of the erosion problem in Florida is thereby clearly demonstrated.

Under the assumption of erosion between levels +5 ft. and -25 ft. which are very reasonable and a replacement value of $ .50 per cu. yd., one arrives at 6 to 7 million dollars a year. The real estate value seems to be of the order of several millions of dollars but it is hard to evaluate because areas at seashores are usually sold by "front-foot".

COASTAL PROTECTION IN FLORIDA

GENERAL

There is only little natural protection along the Florida shorelines. No real solid headland exists - even if the Cape Canaveral foreland functions as such on a large scale. The coquina and shell outcroppings are too soft to form headlands but in some cases they act as sea walls or submerged or partly emerged breakwaters. In general it can be said that such rock outcroppings or reefs do good where they are. Some few of them are located in such a way that they demonstrate a predominant updrift and downdrift side with lee side erosion. This is the case at Vero Beach, and at Hillsboro Inlet on the west coast. In both cases the reefs run out at an angle from the shoreline and in both cases they are responsible for lee side erosion.

Streams and rivers bring only little material to the Florida shores and as a general rule it can therefore be said that the maintenance of the Florida shores depends on the natural equilibrium between the supply and erosion of beach material. Unfortunately unimproved and improved inlets, as already mentioned have, to an excessive degree, disturbed this equilibrium in the State.

Coastal protection in Florida heretofore has been almost entirely in the hands of individuals who, in many cases and often during an attempt to cut down real estate expenses, built their homes and other installations too close to a receding shoreline. As a result of this, very large sums of money have been spent in some localities for construction of protective works. On other beaches the owners are now facing the alternatives of complete loss of valuable property or very heavy expenditures to protect what is left of their property.

In Florida there has been a tendency to grasp at about any suggested method which seems to promise some success at the least cost. Types of constructions which have been given up elsewhere, such as permeable groins and different kinds of wave screens of piles have been built in Florida recently. The designs are often patented even if the principles they present have been well known elsewhere for a great number of years. The results, almost without exception, have been disappointing. There are a great number of examples of such work, conceived in desperation and built with insufficient or complete lack of knowledge of the complicated engineering problems involved.

There is therefore in Florida a great need for a major improvement of this situation; for guidance on all coastal protection and similar problems; for a research program which will apply to all the coastal problems and for dissemination of information to beach-front owners and communities on what to do and what to avoid.

According to 1957 legislation the Trustees of the Internal Improvement Fund, State of Florida, were appointed as the beach erosion agency of the State for the purpose of making rules and regulations necessary to carry out studies and investigations of present erosion conditions and to report the most efficient and economical method of preventing, correcting, controlling and arresting erosion along the beaches or shores of the State.

Using the terminologies "source" and "drain" as mentioned under "Beach Erosion in Florida" and the terminologies "over-nourished", "sufficiently nourished" and "undernourished" profiles as explained below it is possible to give some rules in general of how a given coastal protection will work under given curcumstances such as demonstrated in Table 4.

The overnourished beach profiles are fed with more than the waves can shape into real beach profiles. These, therefore, are irregular and often perform as irregular shoals. On the Florida shorelines we have examples of such profiles along the Florida Keys where part of the material eroded on the east coast of Florida accumulates.

There are two different types of sufficiently nourished profiles. At one of them the profiles are not fed with more material than the waves can shape into a profile, having the same "equilibrium form". At the other, the loss of material equals the supply of material and the profile still has the same equilibrium form. Some beaches north of Cape Canaveral may have profiles of the kind first mentioned, although the beach profiles at Daytona, even if eroding slightly, might be of the kind last mentioned. The beach profiles at Crescent Beach appear to be sufficiently nourished.

The undernourished beach profiles are eroded, that is, the coastline retrogrades. Profiles of that type are found at many of the beaches south of Cape Canaveral to Miami. The undernourished beach profiles will always keep an equilibrium form but the form may change from one locality to another, depending on the conditions in general.

It seems, therefore, that progradation of a coast may take place with or without equilibrium profiles while retrogradation of a shore-

TABLE 4

Coastal Protection in Relation to Source of Materials and Condition of Beach Profiles

1			2	3	4	5
Coastal Protection Actual Conditions Source of Materials and Beach Profiles			Groins or Jetties	Sea Walls	Breakwaters	Artificial Nourishment
A Plenty of source-material Over nourished profiles			Not necessary	Can never build up beach Might be necessary to avoid under extreme high water attack and storm conditions [2]	Not necessary	Not necessary
B Source material available Sufficiently nourished profiles Balance between material eroded and deposited			If a beach is wanted in a particular location[1]	Can never build up beach Might be necessary to avoid under extreme high water attack and storm conditions [2]	If a beach is wanted in a particular location Breakwaters will almost always cause erosion of the beaches on the leeside Always danger of flanking	If a beach is wanted in a particular location in short time Groins might be necessary to maintain the beach especially if the beach protrudes in proportion to the adjacent shorelines [3]
C Only limited source of materials or none at all Under-nourished beach profiles	C a Erosion only up to limited depth in the sea	C a 1 Source of material	If groins or jetties are extended beyond the depth up to which erosion takes place they should be able to maintain the beach in a future not too limited [1]	Can never build up beach They can stop erosion provided they are stable at the depth at which erosion takes place Even if groins are built a sea wall might also be necessary to avoid attack at the foot of dune or cliff under extreme high-water and storm conditions It is very important that condition[2] is fulfilled	If a beach is wanted in a particular location Must be built outside the depth up to which erosion takes place Will almost always cause erosion of beaches on the leeside Always danger of flanking	If a beach is wanted in a particular location in a short time Groins together with periodic nourishment will in most cases be necessary for maintenance of beach (See columns 2 and 3)
		C a 2 No source of material	Will not work well even if they are extended beyond the depth up to which erosion takes place unless material is supplied artificially to balance erosion totally Footnote 1 is not important	Can never build up beach They can stop erosion if they are stable at the depth up to which erosion takes place Even if groins are built and material is supplied artificially a sea wall might also be necessary to avoid attack at the foot of dune or cliff under extreme high-water and storm conditions It is very important that condition[2] is fulfilled	Will not work If material is supplied artificially they will work for a limited time Should be built outside the depth up to which erosion takes place Always danger of flanking	If a beach is wanted in a particular location in a short time Only continued supply to balance erosion totally can maintain the beach Groins will help to maintain the beach (See columns 2 and 3)
	C b Erosion up to deep water in the sea	C b 1 Source of material	Will retard retrogradation of the shoreline especially immediately after the construction But erosion will continue and continuous prolongation of land-ends will soon be necessary if materials are not supplied artificially to balance erosion [1]	Can never build up beach or stop erosion Unless material is supplied artificially to balance erosion and groins might be built, they will in any case only work for a limited time before collapsing It is very important that condition[2] is fulfilled	Will only work for a limited time After this they might do more harm than good if material is not supplied for balance of erosion Will almost always cause erosion of beaches on the leeside Always danger of flanking	If a beach is wanted in a particular location in a short time Periodic nourishment in quantities which balance the erosion can maintain the beach Groins will help to maintain the beach (See columns 2 and 3)
		C b 2 No source of material	Will not work unless material is supplied artificially and continuously to balance erosion totally Footnote 1 is not important	Can never build up beach or stop erosion Unless material is supplied artificially for total balance of erosion and groins might be built, they will in any case only work for a limited time before collapsing It is very important that condition[2] is fulfilled	Will not work If material is supplied artificially they will work for a limited time After this they might do more harm than good if material is not supplied for total balance of erosion Always danger of flanking	If a beach is wanted in a particular location in a short time Only continued supply in the same quantity as total erosion can maintain the beach Groins will help to maintain the beach (See columns 2 and 3)

1 Should be constructed in such way as to let the material pass from the updrift side to the downdrift side as soon as they are filled with material

2 Should be constructed in such a way that they contribute to erosion of the beach as little as possible

line can take place only with equilibrium profiles having a maximum steepness corresponding to the quantity of littoral drift, the waves and the material. An actual equilibrium profile therefore should be defined as a stable profile with maximum steepness. Needless to say we cannot expect that all undernourished profiles will always have the same shape, but we can expect that all undernourished and sufficiently nourished profiles will have certain standard forms, which in turn means that where one of the standard erosion forms occurs, we know that the erosion is probably not temporary. This information is very important. If on the other hand no erosion takes place we can expect that only a slight change in the littoral drift balance may start erosion.

Looking through Table 4 and taking the Florida conditions into special consideration it can be seen that groins and jetties will work well in Florida provided the problem of lee side erosion is solved adequately; that seawalls will offer the necessary protection against storm tides provided they are built to stand the accompanying wave action; that offshore breakwaters will probably not be a good solution for any beach erosion problem in Florida; and that artificial nourishment of beaches because of its prompt action, its flexibility and lack of adverse effect seems to be the best solution in most cases in Florida provided suitable material for beach nourishment is available in sufficient quantities.

EXAMPLES OF RECENT COASTAL PROTECTION IN FLORIDA

Sea Walls - Fig. 8 shows the cross-section of a revetment built on private property at South Palm Beach in connection with three adjustable groins in front of the revetment extending only to the MLW shoreline. The revetment consists of 22 in. by 22 in. by 5 in. interlocking concrete blocks, weight 180 lb. each, resting on a special type fiber-glass (Fiberglas, Inc.) which functions as a filter layer non-penetrable for sand but penetrable for water. The revetment has a toe-protection of 90 lb. "Sakrete" concrete sacks tied together with 5/8 in. deformed plastic bars. The toe-protection extending to +2.5 ft. also rests on fiberglass which extends somewhat in front of the concrete apron, by which same necessity against undercutting during extreme erosion conditions are obtained.

Groins - Groins constructed in Florida have heretofore been of the impermeable, nonadjustable type or have spot-wise been built as permeable nonadjustable or adjustable groins. The impermeable type has worked satisfactorily at some places, but as usual, it has been accompanied by lee side erosion. The excuse that a small group of groins, when kept filled with material by artificial nourishment, is not detrimental, even if they extend offshore, is not logical because it is a well known experience elsewhere that groins push the beach material out into the sea anyway, whereby it is lost wholly or at least for some distance downdrift. The best groin for conditions in Florida is either the very low type impermeable nonadjustable groin or the impermeable adjustable type (Fig. 9). The latter consists of prestressed concrete king piles with tung and groove connected with horizontal slabs of prestressed concrete or (probably better) creosoted 12 lb. pressure Florida yellow pine or imported hardwood. At exposed coasts a rock fill resting on nylon, plastic or fiberglass at the extreme end is advisable, in particular when stability difficulties arise from pile-driving through hard rock.

This type of groin is, with variations, built at Clearwater Beach, Madeira Beach and South Palm Beach. Its adjustability should be maintained with care.

Jetties and Breakwaters - The conservative type of jetty or breakwater in Florida is a rock fill built of granite (from Georgia). It is an excellent and very durable type of jetty when it is built with precautions taken against sinking, but the price of rock runs very high nowadays for which reason other types of construction are sought. Examples of such construction is shown in Fig. 10 built by the Florida State Road Department as protection jetty for the causeway at Longboat Pass. It consists of an 11 ft. wide limerock-filled crib of 12 in. by 12 in. prestressed concrete piles 40 in. apart. It has a bottom protection layer of 6 - 12 in. stones and cross-beams resting on water. Arrangements can be made for trucks to drive on the jetty for refilling with rock.

Artificial Nourishment - In 1937 the by-passing sand plant at South Lake Worth Inlet was put into operation. The plant consisted basically of an 8 in. suction line, a 6 in. 65 horse power Diesel centrifugal pump and about 1,200 ft. of 6 in. discharge line. The construction has been changed since then and consists now of an 8 in. centrifugal pump driven by a 300 horse power Diesel engine. The suction line is 10 in. in diameter, is equipped with a water jet for sand agitation and is carried by a swinging boon about 30 ft. long. The discharge line which runs across the inlet in a road bridge is 8 in. The capacity is about 80,000 cu. yd. a year.

The big new by-passing sand plant at Palm Beach Inlet (Lake Worth Inlet) is described in detail in a special paper of the Proceedings by Fred Zurmuhlen. It is supposed to by-pass about 225,000 cu. yd. of sand a year. Artificial nourishment has been undertaken earlier directly from the Lake Worth (South Palm Beach where heavy erosion makes fill operations desirable).

In 1956-1957 sand was artificially nourished from the bay to the Jupiter Island beach in the amount of about 170,000 cu. yd. Deep water erosion takes place here such as explained under "Beach Erosion in Florida" and under "Configuration of Beach and Offshore Profiles".

A similar project is planned by the Florida State Road Department at the south end of Anna Maria Key where the direction of littoral drift is somewhat uncertain. The abovementioned jetty (Fig. 10) was put in to protect the causeway and to prevent sand from being washed into the bay. From the recurved spit on the north end of Longboat Key and other observations it seems that the drift is northward for which

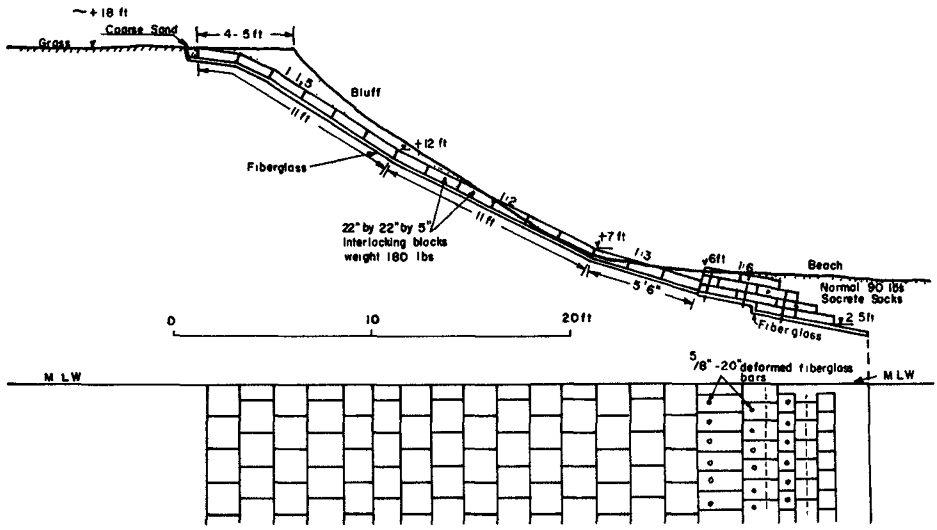


Fig. 8 South Palm Beach revetment

ADJUSTABLE GROIN

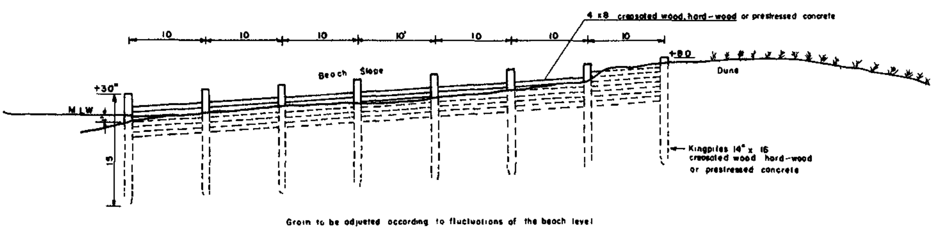


Fig. 9 Adjustable groin

JETTY AT LONGBOAT PASS

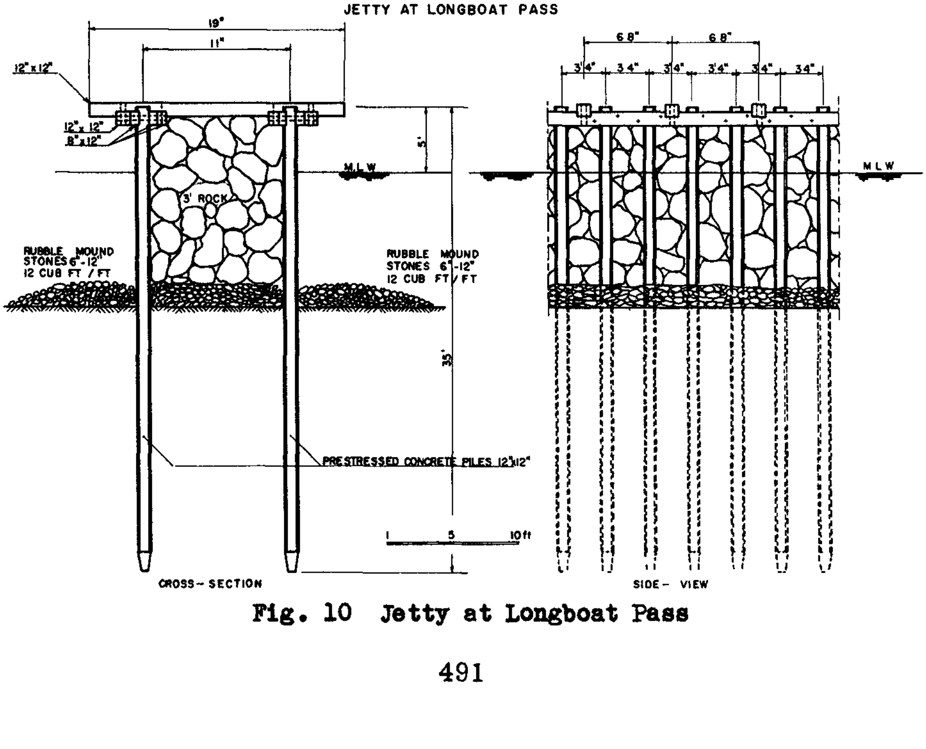


Fig. 10 Jetty at Longboat Pass

reason continuous or periodic artificial nourishment from the bay is desirable north of the jetty. Meanwhile the drift is often southward also and might even be predominantly southward outside a certain depth. At such time the jetty might do some harm to the Longboat Key beaches. For this reason a very considerable artificial nourishment (about 80,000 cu. yd.) will be undertaken by the Florida State Road Department on the one mile long beach in order to fill up the beach and the northwest corner at the jetty initially so that sand migrating southward, instead of being deposited north of the jetty, would be forced to pass the jetty southward. Surveys are planned for Anna Maria as well as Longboat Keys in order to follow the development of erosion more closely so that future maintenance operations can be planned.

FLORIDA INLETS

GENERAL

Florida's interesting shoreline has numerous tidal inlets on the east coast as well as on the west coast. Tidal currents flow through the inlets preventing them, to a great extent, from shoaling by littoral drift deposits.

Inlets can be formed in different ways. They can either have a geological, a hydrological or a littoral drift background. Each type is represented in Florida by one or more examples. The breakthrough of a barrier is the most common geological reason for inlet formation in Florida. Many smaller inlets on the east coast and west coast are examples of this type. The cause usually is erosion combined with a rise in sea level during a certain period. Tampa Bay Entrance is probably mainly of geological origin. The entrances of the St. Mary's River and the St. John's River on the upper east coast and the entrance (East Pass) to Choctawhatchee Bay on the upper Gulf coast are examples of inlets where hydrological aspects (fresh water discharge from rivers to the coast) have probably played an important part in the formation.

St. Joseph Bay Entrance (Fig. 1) on the upper west coast (West of Apalachicola) is a typical example of an inlet with littoral drift background. Predominant littoral drift is northwest in this area.

After inlets have been formed they are subject to continuous reformation and development by nature's forces including changes in shoreline configuration and migration of the channel combined with deepening or shoaling effects. The tidal currents through the inlets are responsible for the formation of outer and inner bars, the dimensions and shapes of which are related to the magnitude and direction of the littoral drift, wave and tide characteristics and other local conditions.

The ratio between the amount of littoral drift and the flow capacity of the inlet is important for the configuration and development of any inlet.

Comparing Tampa Bay entrance with Longboat Pass which have about the same littoral drift characteristics but differ greatly in tidal prism, it is found that Tampa Bay entrance has three well developed ebb channels, giving the outer bar quite a different size and shape from the Longboat Pass outer bar formation.

CONFIGURATION AND DEVELOPMENT OF UNIMPROVED INLETS

Configuration - Fig. 11 gives a few examples of inlet configurations at unimproved inlets in Florida with direction of the predominant littoral drift and flood current indicated.

From Fig. 11(1) Matanzas Inlet and 11 (2) Ponce De Leon Inlet on the east coast it can be seen that the direction of the common tangent to the shorelines on both sides of the inlet does not differ much from the general direction of the shoreline. This is because the fairly heavy wave action does not permit any point or headland to extend much into the ocean. The two examples of inlets from the east coast are "very old" (names given by the Spanish explorers in the 16th Century) and have big shoals in front establishing some natural by-passing of sand. At present lee side erosion prevails in both cases and more predominantly than earlier. Ponce De Leon Inlet (Fig. 11(2) and Fig. 13) was deepened by dredging during World War II.

On the Gulf coast wave action is much smaller for which reason the configuration of inlets holds "bigger lines". Sarasota Inlet (Fig. 11(4) is, as earlier mentioned, a good example of a cuspate bar formation because of shoals. Fig. 11(3) shows the long low barrier at Blind Pass between Sanibel and Captiva Island, now almost closed. Big Pass (Fig. 11(5) is being shoaled by littoral drift material derived from the erosion of Clearwater Island to the south and because of the breakthrough of Hurricane Pass (two miles to the north) in the October 1921 hurricane which cut down the tidal prism belonging to Big Pass. The Pensacola Bay Pass (Fig. 11(6) - which has a dredged channel - has the typical appearance of an inlet at the end of a bay-barrier where the tidal activity is small (lower velocities because of dredged channel) making it impossible to develop the spit of the barrier into the bay or out in the sea.

Shoreline configurations on either side of the inlet are predominantly determined by the local conditions regarding littoral drift and tide. Nevertheless, in many unimproved inlets in Florida, general shoreline characteristics, such as indicated in Fig. 15a are to be recognized.

Development - Inlets on littoral drift coasts are subject to continuous changes due to the action of wind, waves, currents, and littoral drift. The influence of these different elements will be discussed below and are illustrated at a number of Florida inlets.

a. Recession of the shoreline will develop after the formation of an inlet. Redfish Pass, a small inlet of restricted size on the lower Gulf coast was formed after the hurricane of 1926. Fig. 12 shows the development of the shoreline at the vicinity of the pass. The material eroded from the sea bottom and from the adjacent beaches has built up the outer and inner shoals.

Fig. 12 shows a comparison of the shoreline location before and after the breakthrough. Unfortunately no information on the location of the shoreline is available from the time before the formation of the inlet so that it is difficult to determine the actual amount of shoreline recession due to the formation of the inlet. It is most likely that erosion took place in the period between 1880 and 1926, but the greater part of the shoreline recession is probably due to the formation of the inlet.

b. Erosion and accretion on adjacent beaches. Shoals in front of an inlet can change the wave pattern as shown in Fig. 14. The building up of a cuspate bar on the downdrift side and the erosion at the updrift beach can usually be explained as a combined effect of tidal currents and wave refraction such as already mentioned under coastal morphology. The littoral drift is then reversed for a short distance on the downdrift side. In the case of Redfish Pass such building up of the downdrift beach close to the inlet created erosion problems farther down on the downdrift (south) side. The formation of Redfish Pass itself may have been caused by erosion of the beach barrier due to the influence of Captiva Pass which is located north of Redfish Pass.

c. Submerged shoal and bar formation in front of inlets. There are different types of submerged bar formations in front of tidal inlets. A few examples are given below.

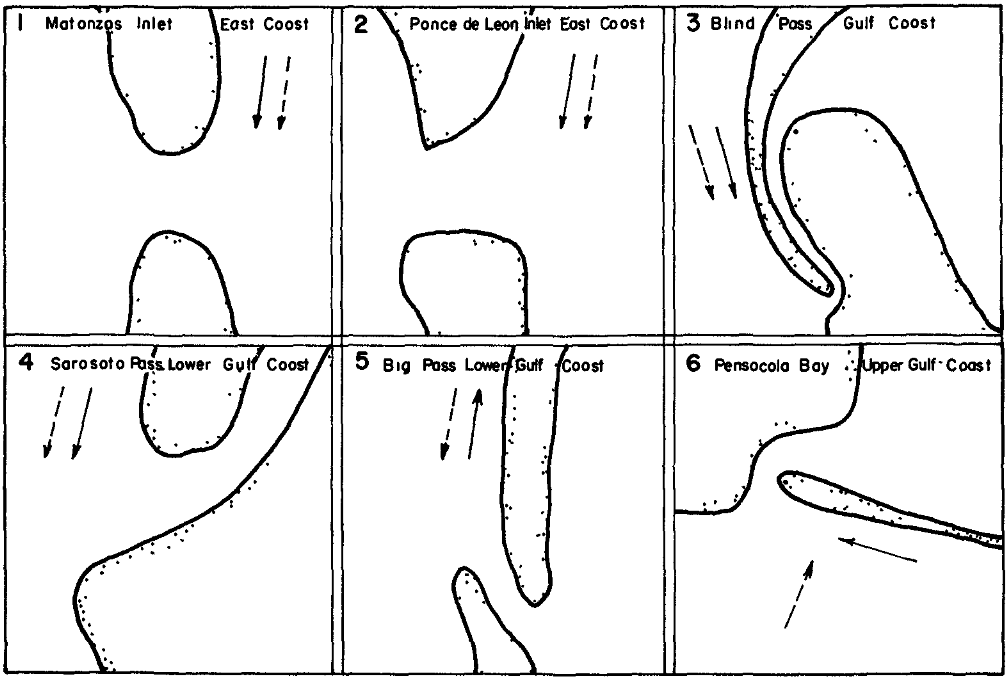
Redfish Pass (Fig. 12(1) has two wingshape bars oriented along the dominating ebb channel and a shoal at the seaward end of this channel. The channels on both sides of the inlet along the adjacent shores are minor flood channels and have a cross shoal at the end of the channels, where those flood channels meet the main ebb channel.

At the inner-bar at Redfish Pass the different flood channels are also provided with shoals at their ends. This kind of inner bar formation is characteristic for shallow bay areas.

Longboat Pass (Fig. 13) was probably created by a breakthrough. On the map of 1889 a small inlet north of Longboat Pass divided Anna Maria Key into two different islands. The closing of this inlet (which broke through again in 1956) probably has influenced the development of Longboat Pass. The outer bar formation at Longboat Pass is very similar to the one at Redfish Pass.

The formation in front of the Tampa Bay entrance is typical for inlets of greater size. Although the littoral drift situation in front of the Tampa Bay entrance is the same as in the surrounding area, a big difference in dimension and shape of the bars is to be noticed due to the large tidal prism.

Ponce de Leon Inlet (Fig. 13) on the east coast has a crescent shape bar of more symmetrical form.



Shoreline Configuration at Unimproved Inlets in Florida
⟶ Predominant Direction of Littoral Drift.
– – –→ Flood Current

Fig. 11

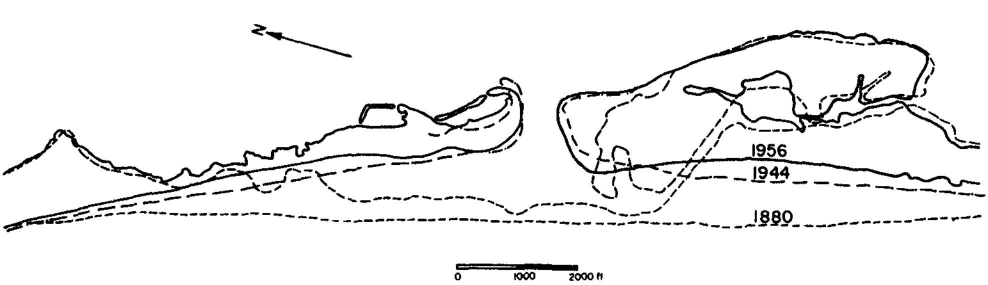


Fig. 12

d. Migration of inlets or inlet channels. Tidal inlets are subject to migration which means that the inlet itself or one or more channels outside the gorge migrate due to the action of littoral drift and tidal currents. In most cases inlets migrate in the direction of the predominant littoral drift. Under special conditions they move in the opposite direction. Examples are:

Blind Pass on the lower Gulf coast (Fig. 11(3), south of Redfish Pass, has a configuration which is quite different probably because of its location in a curved shoreline and its very small tidal prism. The inlet itself migrates to the south in the direction of the predominant littoral drift.

St. Augustine Inlet on the east coast is a very interesting example of inlet migration. Originally there was a distinct migration of the inlet to the south, in the direction of the littoral drift, which made the maintenance of a navigation channel nearly impossible. For this reason a cut was dredged through the barrier opposite Matanzas River in 1940 north of the existing inlet, after which the old inlet gradually decreased in size and was closed naturally.

CONFIGURATION AND DEVELOPMENT OF IMPROVED INLETS

At improved inlets man has interfered with nature to improve navigation conditions. Improvement is accomplished in the following ways:

a. Dredging and maintaining of a navigable channel.
b. Protection of the inlet by one jetty.
c. Protection of the inlet by two jetties.
d. Construction of a sandtrap on the updrift side of the inlet.
e. Construction of a sand transfer plant on the updrift side. This is not an improvement in itself but it will improve either one of the other improvements and first of all take care of the problem of lee side erosion.
f. Any combination of the abovementioned items.

a. Dredging and maintaining of a navigable channel through the inlet and the inner and outer shoals - In the case of Tampa Bay entrance improvement probably means little for the general regimen. For this reason the inlet is discussed with the unimproved inlets. Anothe example of an inlet improved by dredging is the Pensacola Bay entrance The stability conditions of various passes are discussed in the sectio about stability.

b. Improvement with one jetty on the updrift side - St. Lucie Inlet on the east coast developed from a man-made cut 30 ft. wide and 5 ft. deep in 1892 to a width of 1700 ft. and an average depth of 7 ft in 1898. The present width is about 1800 ft. The inlet had its maxim width, about 2000 ft. in 1922. Since 1922 it has been decreased to about 1800 ft. by deposit of spoil from dredging (1946).

Between 1926 and 1929 a stone jetty of 3325 ft. length was constructed along the north side of the inlet by the St. Lucie Inlet

Red Fish Pass

Longboat Pass

Ponce de Leon Inlet

Fig. 13

District and Port Authority, and channels were dredged from the inlet t Port Seawall.

The cut of the inlet and the construction of the jetty caused heav erosion at the beach south of the inlet creating a funnel-shaped shoreline formation (compare Fig. 15b). A submerged reef extends from the north side of the inlet in a southerly direction. This reef may play a role in beach erosion because it may concentrate ebb currents closer to the downdrift shore.

A similar influence of a reef is seen at Hillsboro Inlet farther south on the east coast.

c. Improvement with two jetties on both sides of the inlet, eithe parallel or convergent - Examples with parallel jetties are Lake Worth Inlet and Ft. Pierce Inlet on the lower east coast. Both inlets are man-made cuts through the barrier and connect the ocean with the lagoon General characteristics are accretion on the updrift side and erosion o the downdrift side. However, as shown in Fig. 15c, accretion occurs on the downdrift beach for a short distance adjacent to the jetty. Locall the littoral drift has a reversed direction due to diffraction of waves in the shadow of the jetties and refraction of the waves over the shoal in front of the inlet.

Improvements at St. John's River and St. Mary's River entrances have a long history. Both have two stone jetties, partly parallel and partly convergent. Work on the jetties of both inlets started in the early 1880's.

The shoreline developments on both inlets have been similar to wha has been described for Lake Worth Inlet. Accretion north of the north jetty and immediately (about one mile) south of the south jetty with erosion farther south. A shoal has been formed outside the jetties at the St. Mary's River entrance over which waves from the northeast refract.

d. Improvement of inlets by a sand trap - This principle has been followed at Hillsboro Inlet on the southeast coast. Sand supplied by the littoral drift can pass a low spot behind a shore jetty at the nortl side of the inlet. The sand accumulated is dredged periodically. As i the case of St. Lucie Inlet rock reefs outside the inlet influence the development. The beach immediately south of the inlet is eroding.

e. Construction of a sand transfer facility - Sand transfer facilities are built primarily for the restoration of the littoral drift along the coast at the location of the inlet to prevent erosion of the beach on the downdrift side of the inlet. Meanwhile at the same time they can improve the inlet by decreasing deposits in the inlet channel, in particular where the corner between the updrift jetty and the origial beach has been filled with sand (the shoreline has reached the end of the jetty) and where the flow capacity of the inlet is relatively small compared with the littoral drift. The inlet improving effect is probably considerably higher for the existing sand transfer plant at

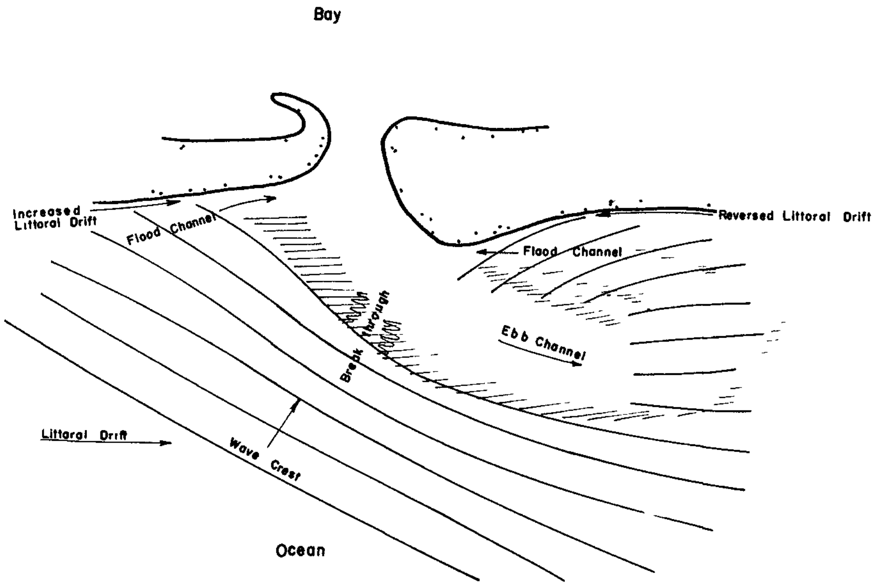


Fig. 14 Schematized hydrographic conditions, small inlet

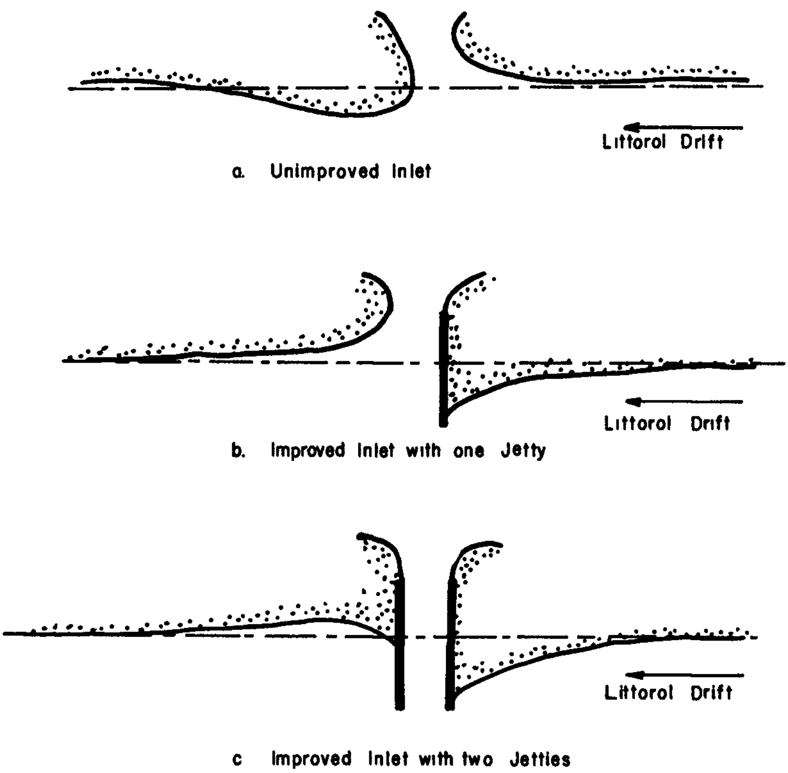


Fig. 15 Shoreline configuration, inlets in Florida

South Lake Worth Inlet than for the new plant at the much larger Lake Worth Inlet. The by-passing sand plants in the Lake Worth area are evidence of an important step toward improvement of Florida's beaches. Reference is made to other papers in the Proceedings where those plants are discussed more in detail.

STABILITY OF SOME INLETS IN FLORIDA

In a study made by O'Brien, the results of which were published in "Civil Engineering", May 1931, a relationship of empirical nature was found between the tidal prism and the cross-section of an inlet:

$$A = p\,\Omega^{0.85} \qquad (1)$$

In this formula, A is the cross-section and Ω the tidal prism. The coefficient p has a dimension and is therefore dependent on the units of A and Ω.

The Coastal Engineering Laboratory of the University of Florida in a study for the Tidal Hydraulics Committee, Corps of Engineers, of inlets in the United States as well as in other countries, has found that the relation between the tidal prism of a stable or relatively stable inlet in erodable material and the cross-section can be expressed by the formula:

$$\frac{\Omega}{A} = \frac{C\,T}{C_2 \pi} \sqrt{\frac{\tau_s}{\rho g}} \qquad (2)$$

in which Ω and A have the same meaning as in equation (1), C is the Chezy's coefficient; $C = \frac{T\,Q_m}{\pi\,\Omega}$ in which Q_m represents maximum flow rate; T is the tidal period and τ_s is the "stability shear stress". Furthermore, the specific density of sea water is ρ and the acceleration is gravity is g.

The stability shear stress (τ_s) which is the mean shear stress along the bottom of the inlet channel is probably a basic parameter for the natural stability of inlets. It is a function of the bottom material of the inlet: the amount of material in suspension both being stirred up by the tidal currents and from other sources (silt), the amount of littoral drift (from both directions), the shape of the cross-section and the rate in which the cross-section is exposed to wave action.

If an average value for the shear stress $\tau_s' = 0.4\ \mathrm{kg/m^2}$ is introduced, defined by $\tau_s' \frac{\tau_s}{K^2}$, equation (2) can be written in the form

$$\frac{\Omega}{A} : \frac{C\,T}{C_2 \pi} \sqrt{\frac{\tau_s'}{\rho g}} = K \qquad (3)$$

so that a variation in the value of the stability shear stress appears as a variation of K.

It was found that in the majority of the inlets investigated, the variation in K was smaller than 10% and predominantly determined by the relative value of the total littoral drift, that is the ratio between the total amount of littoral drift and the maximum flow rate of the inlet.

In Figs. 16a and 16b a plotting is made between Ω and A for various inlets including different inlets in Florida. These figures show that without further analysis the relation between Ω and A is already close to a straight line. The greatest deviations exist for inlets with diurnal tide characteristics (e.g. East Pass, Fig. 16a) and this is not surprising because the tidal period T occurs in the relation between Ω and A (Eq. 2). If this difference in tidal period is eliminated, East Pass shows good agreement with the theoretical relationship.

Good agreement is also found in the plottings for St. John's River entrance, St. Mary's River entrance, St. Augustine Inlet and Ponce de Leon Inlet. Gasparilla Pass and Big Pass show more deviation, in particular when the different factors of equation (2) are taken into consideration. Greater deviations are found in Lake Worth Inlet, Longboat Pass and Little Pass.

At Lake Worth Inlet either the rock bottom of the inlet, the maintenance dredging of the navigation channel, the difference between the ebb flow and flood flow pattern at the entrance (flood flow is subjected to contraction), or a combination of these factors may be responsible for the deviation.

The deviations of Longboat Pass and Little Pass are probably due to the influence of the shape of the inlet on the size of the cross-section. Longboat Pass has a regular narrow and deep cross-section almost of triangular form whereas the channels of Little Pass are relatively wide, shallow and irregular.

In Fig. 16b Pensacola Harbor seems to give good agreement but unfortunately the tide conditions at Pensacola Harbor are diurnal (such as is the case of East Pass) so that a disagreement could be expected. Pensacola Harbor probably has a cross-section which is too large compared with the average condition. This might be caused by a small value of the stability shear stress due to local conditions but more likely the maintenance of a navigable channel in the inlet of 32 ft. deep and 500 ft. wide has increased the cross-section artificially above its size of natural stability.

There are two small inlets in Florida which have not been plotted in the diagram which should be given special attention. They are South Lake Worth Inlet and Baker's Haulover on the lower east coast. Both are artificial cuts in the barrier between the ocean and a lagoon. Those inlets have small cross-sections and high velocities, compared with naturally stable inlets. The rock bottoms of those inlets prevent further scour whereby the cross-sections are kept unusually small and the maximum velocities accordingly unusually high.

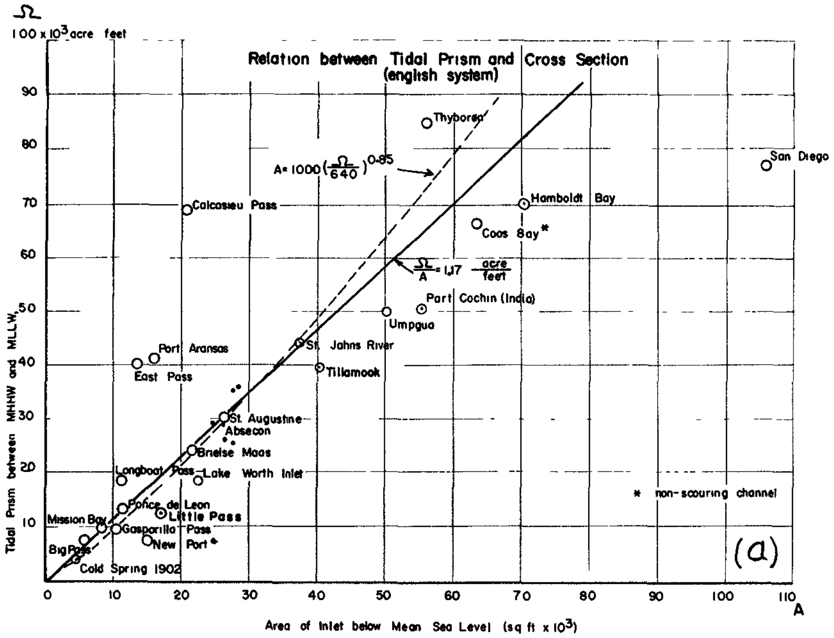


Fig. 16a Relation Between Tidal Prism and Cross-section

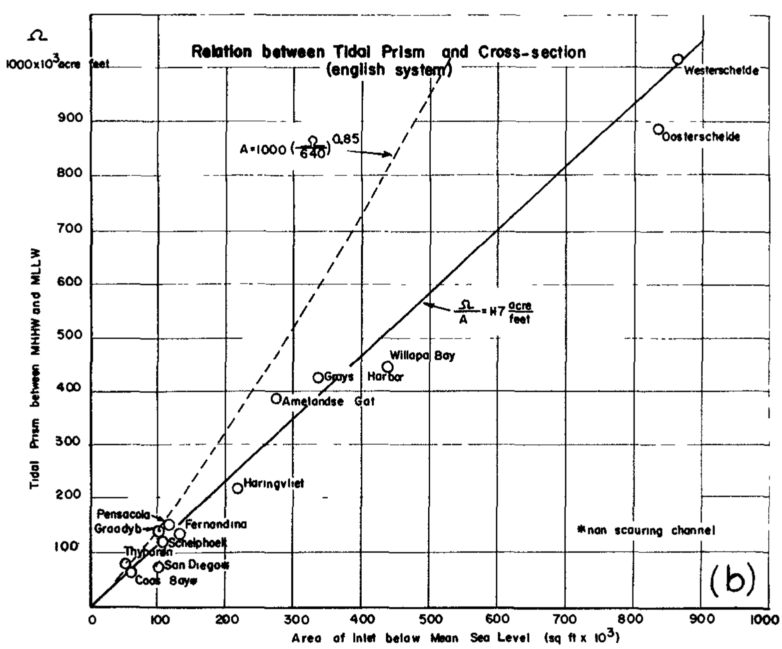


Fig. 16b Relation Between Tidal Prism and Cross-section

Fig. 16

Fig. 17 Boca Ciega Bay Fills

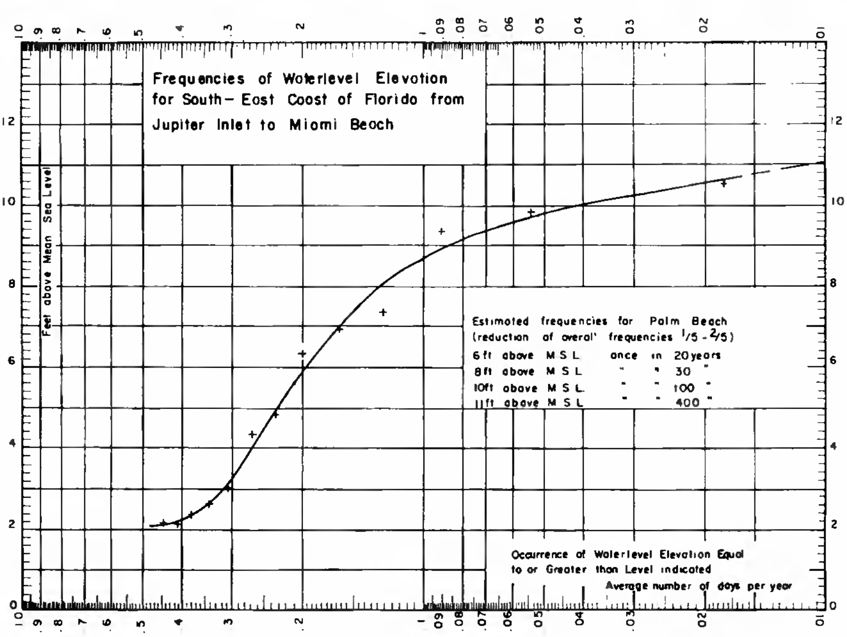


Fig. 18 Tide frequencies - Southeast Florida

COASTAL PROBLEMS IN BAYS AND WATERWAYS

Florida has an almost unbroken chain of bays, lagoons and waterways along its shoreline. Usually they are rather narrow from $\frac{1}{2}$ to 2 miles wide, yet Tampa Bay and the West Florida Bays at Apalachicola, Panama City and Pensacola are larger. These bays are usually rather shallow. Tampa Bay and St. Andrew's Bay (Panama City) are deeper.

The problem of fills in bays and waterways was briefly mentioned in the introduction. Fig. 17 shows such a fill in the Boca Ciega Bay (northern part of Tampa Bay). These bay fills which are the expression of real estate hunger for property with "bay-front" raise a lot of problems of hydrological and other nature.

Unfortunately it has been the practice through a number of years to sell submerged areas in bays (belonging to the State) to private interests without paying much attention to the effects of the fills on the abovementioned factors of hydrological nature. A permit was always required from the Corps of Engineers, taking only navigation conditions into consideration. An attempt is now being made to have this situation changed by the socalled "bulkhead law", 1957. The Board of County Commissioners shall approve bulkhead-lines defined as "the line beyond which no fill must take place". The plan is subject to final approval by the State Government through the Trustees, Internal Improvement Fund. Up to now (December 1957) a number of bulkhead lines have been approved with little or no influence on shoreline configuration and tidal flow.

The big problems still remain to be solved. At many places big fill-operations have been planned which will have a very strong influence on the hydrological and other regimen in the water area.

These problems include problems of navigation, effect on wave and current action, effect on erosion, formation of stagnant pockets, effect on hurricane and storm tides, effect on freshwater outflow and the effect on fish and wildlife. All of these questions are dealt with briefly in the paragraph dealing with coastal developments.

A mentioning of bays and waterways in Florida must include information on the socalled intracoastal waterways which on the east coast extend from the Georgia border to Miami and on the west coast along all the barrier beaches. This tremendous waterways is constructed and maintained by the Corps of Engineers.

The situation today is that the Atlantic section has been dredged to 12 ft. from the Georgia line to near Sebastian Inlet, 215 miles; thence 8 ft. to Miami, 160 miles; thence 7 ft. to Plantation Key, 63 miles. Authorizations provide for 12 ft. depth southward to Miami and 7 ft. depth thence to Key West, 150 miles.

The Gulf of Mexico section has been completed to 12 ft. from the Alabama line eastward to Carrabelle, 210 miles. 12 ft. depth has been authorized for the 44 miles from Carrabelle to St. Marks. Other reaches have been dredged southward with depths as follows:

a. 5, 7, 8 ft. through Clearwater Harbor and Boca Ciega Bay, 26 miles.
b. 7 ft. from Tampa Bay to Sarasota, 18 miles.
c. 3 ft. through Little Sarasota Bay to Venice, 20 miles.
d. 7 ft. through Pine Island Sound to Caloosahatchee River, 27 miles.

The above four sections are incorporated in the authorized (but not constructed) 9 ft. channel from Caloosahatchee River to Anclote River, 148 miles.

This big asset of the State is under continuous development furnishing still increased possibilities for commercial as well as pleasure craft.

DEVELOPMENT OF COASTAL AREAS

Coastal developments in Florida include developments on the sea as well as the bay shore.

DEVELOPMENTS ON THE SEA SHORE

Three specific problems will have to be considered for developments on the sea shore: protection from erosion by sea forces, protection against erosion from wind action, and safety against flooding.

Protection against beach erosion is dealt with thoroughly above. A paper by Dr. J. M. Davis included in the Proceedings gives a description of protection by vegetation. In regard to sand fences, refer to publication entitled "Shelter Effect" in the bibliography.

The problem of coastal safety remains. It includes measures against flooding by storm and hurricane tides with accompanying wave action.

The situation in regard to flooding is very critical at many coastal areas in Florida where developments, incredible as it may sound, have been made at 4 to 6 ft. above M.L.W. where storm tides up to 10 ft. or more can be expected.

Protection against flooding can only be obtained by having the development at a certain height above storm tide. This height is a function of the location on the sea shore and hydrographic as well as meteorological conditions.

Often the water heights experienced in the past are used as a basis for design, but because of the fact that development in coastal areas is a very recent undertaking and reliable figures are available for a short period only, this is not a reliable way of approach.

Statistical analysis of available data is one means of predicting the frequency and height of storm tides.

THE FREQUENCY DISTRIBUTION OF EXTREME HIGH WATERS

In 1939 Wemelsfelder published a statistical analysis of high tid data along the Dutch coast.

Although the amount of information on extreme high tide data is limited and of relatively low accuracy in Florida, an attempt has been made here to analyze some Florida data in a similar way.

Detailed information on extreme high tides is available from the Corps of Engineers, U. S. Army, Jacksonville: "Appraisal Report - Hurricanes Affecting the Florida Coast". In some cases piling up of wate by winds and waves might be involved in the figures of this report.

Fig. 18 is a semilogarithmic diagram of extreme high tides on the southeast coast of Florida between Jupiter Inlet and Miami Beach. The height of high water is plotted along the linear axis and the frequenc of occurrence along the logarithmic axis. The frequency is indicated by the average number of days per year that the corresponding level is equalled or exceeded. Diagrams like this are useful in particular whe an extrapolation of the frequency curve beyond the highest recorded level can be justified.

In order to have a homogeneous number of high water data, only on figure, the highest, is used for every storm during the period under consideration (1926 to 1953) so that the obtained curve refers to the conditions for this part of the coast as a whole. Further analysis of the local distribution is required to determine the frequency for a special location.

If we assume a hurricane to be effective along 50 to 100 miles of the coastline, an average figure for a special location might be found by reducing the frequencies on the diagram to 1/5 or 2/5.

On the west coast the physical factors creating extreme high tide are different from those on the east coast. However, this does not change the statistical treatment of the data available.

Records of Cedar Key on the middle Gulf coast since 1936 show no higher level than 6.5 ft. above M.S.L. Because of the restricted amount of data and limited period of which data are available, extrapolation of the frequency curve is difficult. Extrapolation such as e.g. "a", "b" and "c" are possible, giving a significant difference in height already at the frequency of 1/100.

The following figures can be read from the diagram:

6 ft. above mean sea level, occurrence once in 30 years.
8 ft. above mean sea level, occurrence once in 60 years.

The Cedar Key data show that the frequencies of occurrence for th same water level are lower for a location on the middle Gulf coast tha for one on the lower east coast, probably 1.5 to 2 times as low.

Frequency considerations like those above can be of great importance in the determination of the insurance value of real estate in coastal areas. Despite the innacuracy involved, the available information shows clearly that the possibilities for flooding is rather high and, unfortunately, entirely underestimated in Florida. At many coastal developments the most elementary considerations with respect to safety of life and damage to property have been disregarded.

A great number of causeways now connect the barrier-chain with the mainland and these causeways will make it easier to flee from the area in danger providing warnings are received, understood and followed, but the damage to property will be immense. At the same time those causeways often increase the danger of flooding in some areas, while decreasing it in other areas.

DEVELOPMENTS ALONG THE SHORES OF BAYS AND WATERWAYS

Such developments mostly take place in low marshy, swampy, or mangrove areas. They may be entirely residential or include some commercial developments as yacht and fishing harbors.

In the case of the development of low marshlands into residential areas, the danger to life by flooding ought to be taken into consideration either by having zoning regulations by which the proper use of certain areas is determined by their elevation or other precautions taken against flooding, e.g. in the form of levees.

Along tidal rivers the possibility of a water level of extreme height is increased because a storm tide from the ocean may coincide with a run-off wave from upriver. The latter is caused by heavy rainfall which usually accompanies storms and hurricanes in coastal areas. In this way such floods are of a combined hydraulic and hydrological nature. The phase difference between the storm tide phenomena in the ocean and the run-off phenomena behind the barrier is important in such cases, and can be evaluated by observations and computations. The height of the water level depends on the way the tidal wave propagates through the inlet or estuary, and the propagation of the run-off wave down river.

In the case of harbor developments on the coast, either in the creation of a new inlet or in the enlargement of an existing one, the increase in tidal height in the bay or river area should always be considered, in particular for extreme conditions. This can be done partly by computations and partly by model experiments.

FILLS IN BAYS AND WATERWAYS

Fig. 17 shows a "finger-fill" in Boca Ciega Bay. Such fills, which are real estate developments, do not make any contribution to the natural beauty of the Florida bays and, as already mentioned under "Coastal Problems in Bays and Waterways", raise many problems including:

The influence on navigation - This does not only include such obvious problems as obstructions to navigation, but also the effect on the entire adjoining bay and sea territory.

Any reduction of the amount of tidal flow caused by the filling of any area above mean low water level will account for changes in the hydrological conditions in a certain area. As explained under "Inlets" there is always a fixed relationship between the flow through any channel, pass or inlet and the cross-section of same. Any diminishing of the flow will influence the natural balance to an extent depending on the relative dimension of the change and on the age of the waterway under consideration.

The effect on wave action - Fills will almost always be bulkheaded. A normally sloping beach reflects practically no wave energy for which reason the area in front of such beach is not affected by reflected waves. A vertical bulkhead will reflect from 80% to 90% of the wave height and the area in front of such a wall will, for a considerable distance, be affected by reflected waves which are unpleasant and dangerous to navigation and often create erosion problems.

The effect on currents - Any fill will change the normal current pattern to a smaller or greater extent and may, in this way, create problems of navigation and/or erosion.

The effect on shores in regard to erosion and accretion - Because it changes the existing current and wave conditions, any fill will normally create problems of erosion as well as accretion on adjacent shores.

The effect on formation of stagnant pockets - At some places of the bay the current velocity will increase; at other places it will decrease. At certain places the possibility of the formation of stagnant pockets will arise, in particular in the case of finger-shaped fills. This is not only a question of hydrology but also a question of marine biology.

The effect on hurricane and storm tides - The height of hurricane and storm tides depends upon, among other factors, the free fetch of the wind action over the water surface, the depth of water and the configuration of the shoreline. Any fill or bulkhead will change these factors, but only an unfortunate configuration of the fill will adversely affect the height of hurricane and storm tides. The height of the fill should be determined from computed tide data. In most cases in Florida it will be inadequate to make a fill in an open bay on a level lower than 7 to 8 ft. above M.L.W. and even then it may be flooded occasionally. Bay fills may increase the water level in a bay or other waterway in open connection with the sea and these possibilities should always be investigated very carefully.

The effect on freshwater outflow - Any fill or bulkhead operation changes the existing condition of freshwater flow to the water area in question. New drainage and sewer problems arise.

The effect on fish and wildlife - This problem is of biological and zoological nature and of great importance in Florida.

TABLE 5

SOLUTION TO PROBLEMS - FILLS IN BAYS AND WATERWAYS

Problem	How To Investigate and Solve The Problem			
	Survey Information Necessary		By Calculation and/or Experience	By Model Experiments
	Existing Data	New Data		
1. Navigation	X	X	X	X
2. Wave Action	X	X		X
3. Currents	X	X	X	X
4. Erosion and Accretion	X	X	X	(X)
5. Stagnant Pockets	X	X	X (marine biology)	(X)
6. Hurricane and Storm Tides	X	X	X	(X)
7. Freshwater Outflow	X	X	X	(X)
8. Fish and Wildlife	Special investigation of biological and zoological nature indispensable			

Table 5 indicates how the different problems mentioned above can be solved by proper engineering and scientific methods.

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CHAPTER 27

SOME COASTAL ENGINEERING PROBLEMS IN INDIA

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I COASTAL EROSION IN THE STATE OF KERALA

Near the southern tip of the Indian peninsula, the State of Kerala has a 400 miles long shore line running north from the Cape Camorin. Almost all along its length i one reach or the other, the coast has fine beaches which ar continually subjected to erosion due to wave action. This process has been going on, no doubt, since the existence of the sub-continent, but it is only in recent years has there been an awakening of interest when property and plantation are being threatened, as the man-land ratio is getting dangerously high.

DESCRIPTION OF COAST

The coastal strip is narrow and lagoon-fringed at the foot of the Western range of mountains. As a result rivers are short and do not have extensive deltas as on the easter coast. A peculiar feature of the coast is the chain of backwaters or lagoons separated from the sea by sand strips from seven miles to half a mile in width. These with their connecting canals form a water communication all along the north-south length of the State by means of flat bottomed boats. The shore line is fairly regular and rock outcrops are few. Beaches are narrow and plantations of cocoanut and rice fields grow often within 100 ft of the water edge. Beach slope is steep (1:12) between the high and low water lines varying between +3.7 ft and -0.2 ft, the Indian Low Water Ordinary Spring Tide (L.W.O.S.T.) being taken as the datum. Ocean bed slope is, however, not steep, the 10 fathom depth contour being, on an average, 25000 ft from the shore. Fig.1 shows the cross-section of the coast extending up to the 60 ft contour.

WAVES

Wave action, which is the cause of erosion and of consequent littoral drift, is predominant during the S.W. monsoons, that is, during the period June-September. During this period the direction of wave approach varies between 10 to 40 degrees South of West which is the direction of the prevailing winds. During the fair weather period, October-May, the direction changes gradually northwards. As a result the direction of littoral drift is from South to North during the rough weather period June-September and in the opposite direction during October-May.

Fig.2 gives the percentage in each month of the year of waves of height exceeding four feet, which we might call a storm of low intensity. Storms, when the wave height exceeds 8 ft, are not unknown; waves of this height, when they occur concurrently with a spring tide, eat away a beach of width 20 to 30 ft, part of which is rebuilt later during the fair weather. However, as a result of the imbalance of wave action during the periods June-September and October-May, littoral drift is predominantly from south to north.

LITTORAL DRIFT

The available supply of drift is derived almost exclusively from coastal erosion, since there are no streams of consequence flowing into the sea along the Kerala coast. A convenient measure of the volume of littoral drift is possible because of the extension of a sandspit on the south of the entrance channel of the Cochin harbour (Fig.3). The presence of mud banks and the periodical dredging of the entrance channel has prevented, to a considerable extent, the northward drift of sand eroded from the coast south of the Cochin harbour resulting in the formation of the sandspit. Bearing in mind that not all quantity of littoral drift is arrested thereby, an extension of the accretion leads one to expect a sand drift varying from 0.1 to 0.2 million tons per annum, as estimated from the growth of the sandspit from year to year.

EARLY PROTECTION WORKS

The effects of erosion have made themselves felt all along the coast and valuable land has been eroded every year. At places, coastal land two furlongs wide has been lost to the sea within living memory. Measures were taken in the past when navigation canals running close by the coast or in backwaters separated from the sea by a narrow

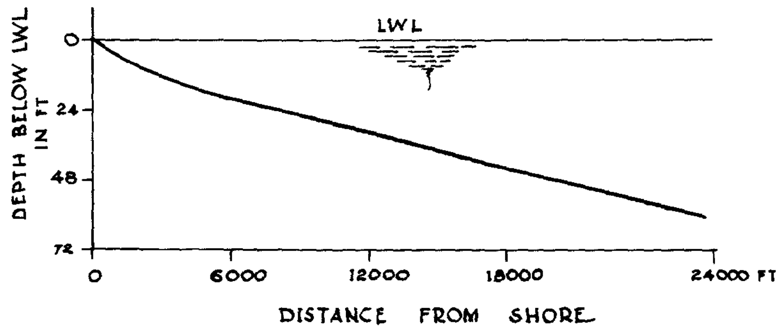

Fig. 1. Average cross-section of coast near Cochin Harbour.

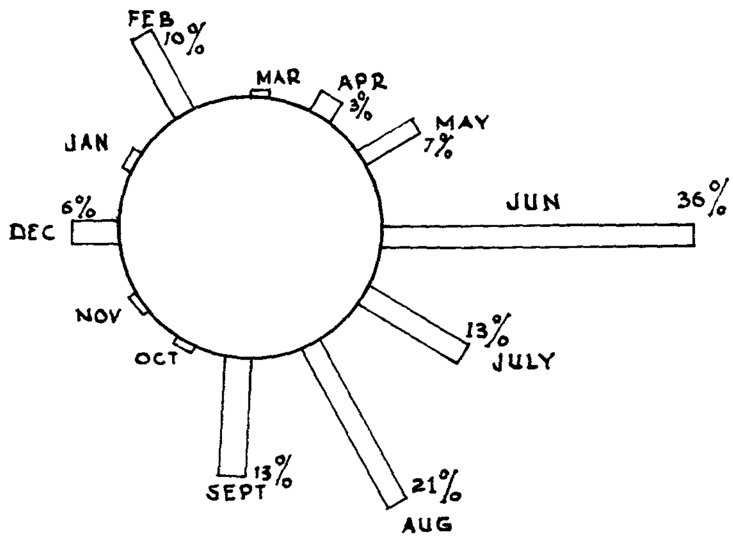

Fig. 2. Percentage of wave heights exceeding 4 ft.

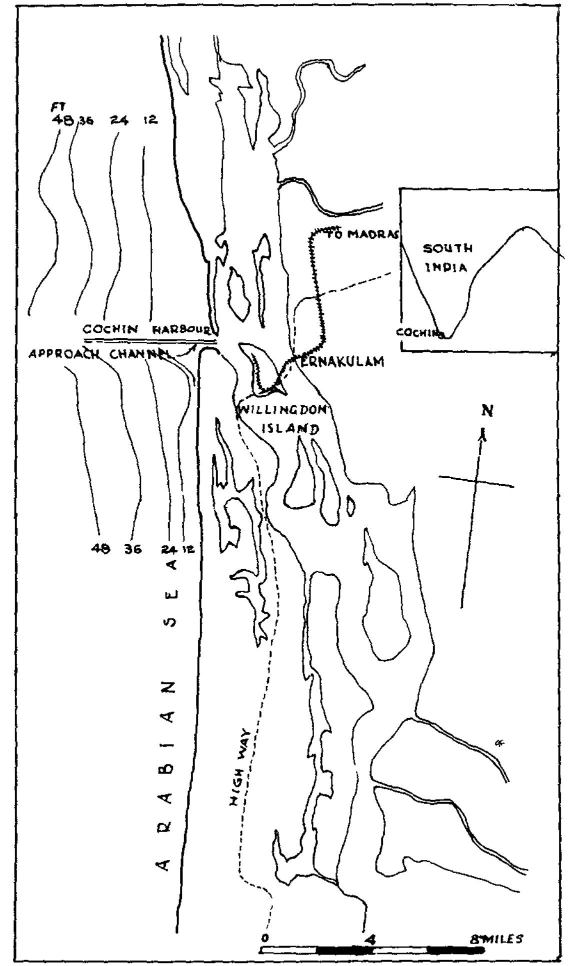

Fig. 3. Coast line near Cochin with en-

strip of land were threatened with breaches. At Varkala, about 40 miles north of Trivandrum, the State capital, only a cliff separates the sea from an inland navigation canal. The cliff was under severe attack of the waves and would have been washed away but for the construction of a series of rubble mound groynes, about 100 ft long, placed at various intervals at the foot of the cliff. Since the construction of the groynes 20 years ago, a protective beach has been formed and waves break no longer at the toe of the cliff, but spend themselves on the accreted beach.

Another major attempt was made during 1932-48 to protect the coast immediately south of the Cochin harbour by the construction of a sea wall. Rubble mound sea walls of various lengths were constructed at a number of locations over a total reach of 20 miles. Done in discontinuous portions, the protection works did not yield satisfactory results as the stones used for the sea wall were light, 100 to 200 lbs in weight, laid in a slope of 1:1 and without fascine mattresses on the bed. Moreover, maintenance of these walls was not adequate. Sea wall tops were also low, at +9.0 ft, i.e. only 6 ft above the high water level, which caused overtopping of the sea walls by waves and consequent damage. Over the last decade, most portions of this old sea wall have been disintegrated, while the sea has continued to cut into the land. Storms in recent years have been particularly severe in their damage to land and property, and the problem has demanded immediate attention.

While certain badly affected portions are given temporary protection, the question of adequate protection to about 20 miles of coastal reach is being studied both in the field and in the laboratory.

EXPERIMENTAL PROTECTIVE MEASURES

While the problem of evolving suitable and economical measures is being studied on a reduced scale model at the Central Water & Power Research Station, Poona, a one mile long experimental sea wall supplemented with groynes has been constructed in a reach most affected by erosion during the storm of 1953. Fig.4 shows a plan of the sea walls and groynes. The portion of the sea wall between groynes 1 and 5 consists of loose stones while that between groynes 5 and 10 is of rubble masonry over a core of sand. The groynes are also of loose stones and are of length 200 ft spaced 660 ft apart. Fig.4 shows cross-sections of sea walls and groynes. The weight of the sea wall stones on the seaward face does not exceed 200 lbs and those used

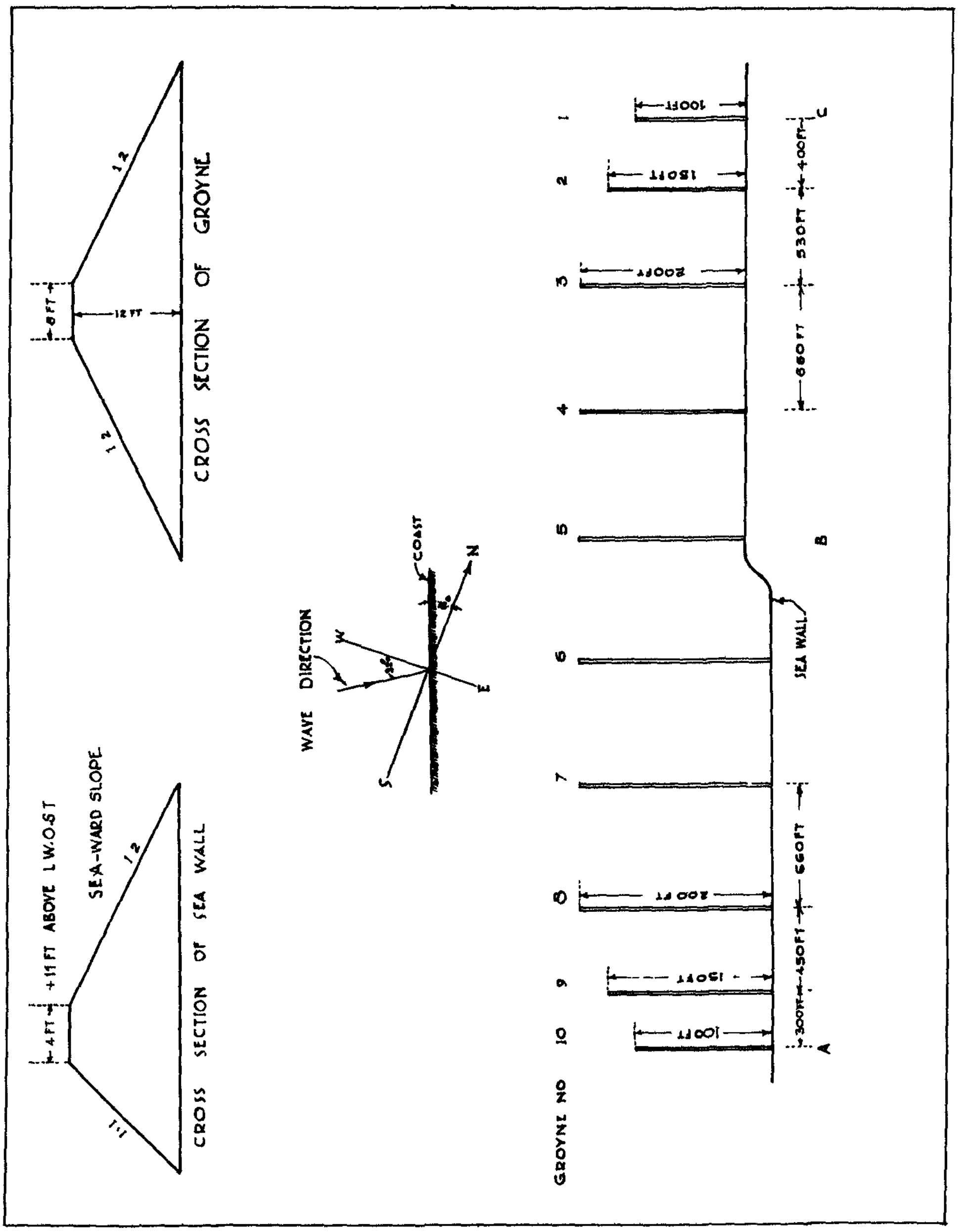

Fig. 4. Plan of sea wall and groynes in a one mile experimental reach; portion AB is ma-

for the groynes are, on an average, less than 600 lbs in weight. The construction was completed by February 1956 and during the following two monsoons the sea wall has provided effective protection to the land behind, requiring a little maintenance. Both the sea wall and the groynes have been founded on fascine mattresses and have not sunk appreciably. Compared to the damages to the sea wall in the monsoon of 1956, the damages in the 1957 season are insignificant, the loss of stone being 19% and 2% respectively of the total volume of stone used during construction. It should be added, however, that only the northern portion of the sea wall, that is between groynes 1 and 5, has suffered damage, while the southern portion has been all along separated from the water edge by a fairly wide beach. The damage suffered by the groynes is, however, considerable. During the 1956 monsoon, an average of 30 ft of each groyne near the nose was dispersed. The damages to the sea wall and the groynes have not yet been repaired, and experience of the 1957 monsoon shows that, probably, there will be no further damage and that the groynes have become stable.

It has been mentioned above that the southern half of the sea wall is untouched by water edge, while it was hugging the sea wall between groynes 1 and 5 during the monsoon. The accretion of sand between groynes in the southern half of the experimental sea wall indicates a drift from south to north, although it has to be added that wave action in the southern half has been less due to the fact that this portion has been retired by 40 ft behind the northern half. To the north of the field of groynes there has been erosion over a length of 2000 ft due, obviously, to the fact that this reach has been deprived of its supply of littoral drift, although care was taken to effect a smooth transition from the protected portion to the unprotected by means of a progressive reduction in the length of groynes.

Experience of the behaviour of the experimental sea wall after two rough weather seasons shows that the wall has provided effective protection and that the maintenance cost of the sea wall with groynes has only been 4% compared to the total cost of construction. For this experimental project a sum of Rs.1.2 million (240,000 U.S. dollars) has been spent, whereas the maintenance cost of the sea wall constructed previously and already referred to above was about 25% of initial cost.

MODEL TESTS

The problem has been studied in a 1/40 scale geometrically similar model using coal powder (0.7 mm) on bed. Experiments have shown that a sea wall alone requires considerable maintenance. Various tests were further made to ascertain the most effective length of and spacing between groynes consistent with economy. It was realised also that along a coast where the quantity of littoral drift was not considerable, groynes have to be sufficiently high so that bed material intercepted between two groynes is not lost by passing over groyne tops. The experiments have further shown that the sea wall top should be kept +11 ft above L.W.O.S. and that the groynes may have a gentle slope from +9 ft at the stem to +7 ft at the nose. Groynes of length 100 ft, 150 ft and 200 ft were tested with spacings varyin from 2 to 4 times their lengths. It has been seen that short groynes do not give adequate protection and that the minimum length of groynes is 200 ft. Spacings equal to 2 and 3 times the length of groynes gave, both, equally good results; however it has been recommended that the spacing of groynes at Cochin should be two times the length of the groynes to meet conditions arising out of a very heavy sto

NEW PROTECTION WORKS

Following recommendations made by the Central Water and Power Research Station, coastal protection works are being undertaken over a length of 20 miles. In certain portions only a sea wall has been constructed to give immediate protection and to which groynes will be added later on.

II COASTAL EROSION AT VERSOVA, NEAR BOMBAY

PROBLEM

A pleasure resort for the inhabitants of Bombay, the Versova beach is being eroded over the last 50 years. Wi land development going all around metropolitan Bombay, marshy tidal areas have been reclaimed by putting bunds across creeks nearby. Sand required for this reclamation has been removed from the foreshore of the beach. Due to this gradual depletion of sand, which served as a natural protection against the inroads of the sea, waves of amplitudes higher than before approach the coast during the monsoon. The erosion is worse when high waves synchronise with high water during spring tides.

Littoral drift along the coast is small, being due only to the material eroded from the beaches. Waves 8 to 10 ft high and of period 7 to 9 seconds strike the coast during the monsoon period, i.e. from June to September, during which the direction of waves varies from SSW to W. The waves remove beach material which appears to be travelling north where it has formed a sand bank at the mouth of the creek (Fig.5). As a result of this erosion, foundations of buildings are exposed and undermined with many collapsing. During the last 50 years, a beach of width 300 ft has been gradually eroded.

PROPOSED REMEDY

To prevent this erosion a sea wall with a seaward slope of 1 in 3, founded on fascine mattresses, is proposed. The top of the sea wall will be higher than the high water level by 7 ft. As the slope of the coast is steep (1 in 40), even waves as high as 5 ft are expected to break near the sea wall. As a result sand at the sea wall toe is likely to get washed away endangering the sea wall. In addition to the sea wall it is, therefore, proposed to construct groynes 250 ft long. The top of the groynes at the shore end would be +5 ft above H.W.L. and will be horizontal for a length of 150 ft from the sea wall; the groyne top will thereafter be sloping (Fig.5).

The groyne nose will be provided with a 4 ft stone cover, the weight of stones varying from $\frac{3}{4}$ to 1 ton, as calculated from wave energy, since the brunt of the attack due to the waves will have to be taken by the nose. The seaward slope of the wall will have a cover 3 ft thick of stones of weight 280 lbs, i.e. 2 cft in size. The hearting and landward slope of the sea wall will be of stones $\frac{3}{4}$ cft to $\frac{1}{2}$ cft.

The groynes are spaced at 500 ft interval. Figure 5 shows the layout of these protective measures: the lengths of groynes 1, 2, 10 and 11 are reduced in order to provide a smooth transition and avoid damage to the downdrift side. With the length of groynes as proposed, 6 to 8 ft high waves will break at high water between the sea wall and the end of the groynes. Since maximum movement of sand takes place in the surf zone, eroded sand, which would otherwise move towards north, will be trapped in the groyne field to form a protective beach.

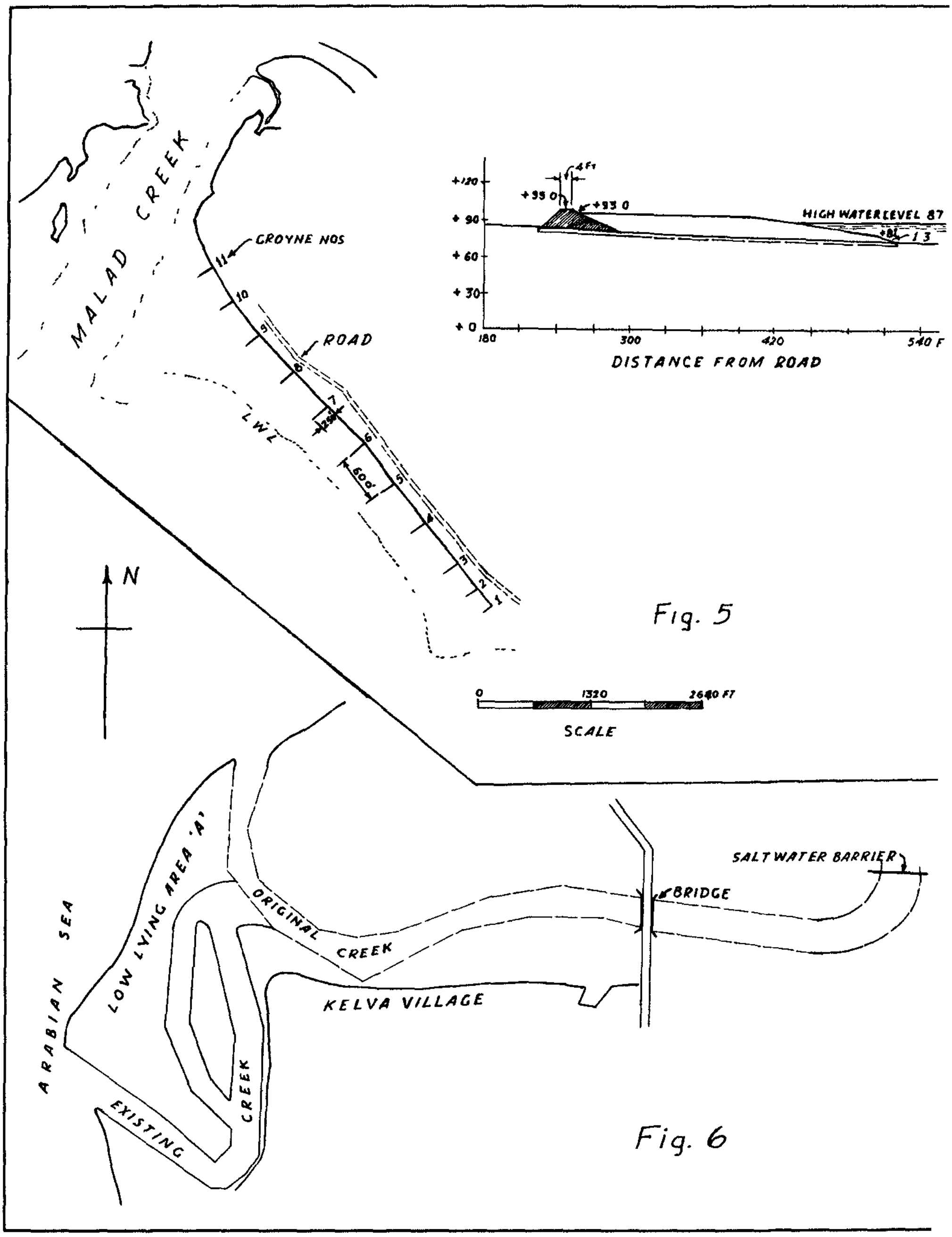

Fig. 5. Coast line at Versova with proposed protection works.
Fig. 6. Creek at Kelva.

SOME COASTAL ENGINEERING PROBLEMS IN INDIA

III COASTAL EROSION AND SHIFTING OF CHANNEL AT KELVA NEAR BOMBAY

PROBLEM

About 55 miles north of Bombay, the fishing port of Kelva is situated at the mouth of a creek. Till 1920 it was a flourishing port where vessels of draft 6 to 8 ft could enter even at low stages of the tide. Since then, however, the port is getting gradually out of use due to the silting and shifting of the mouth of the creek. The waves during a storm are now spending their energy on the coast causing severe erosion of agricultural land at a rapid rate. Moreover, the navigation channel is being silted, with the result that even fishing boats cannot enter the port and have to wait for high water.

CAUSES OF SILTING AND SHIFTING

With a view to reclaim marshy land, a salt water barrier, 2 miles upstream of the mouth, was constructed about thirty years ago across the tidal creek which was 6 miles long before the construction. Moreover, a bridge was constructed 2500 ft below the barrier constricting the waterway from an initial width of 150 ft to 45 ft (The bridge has three spans of 15 ft width each.). Due to the constriction caused by the bridge and the obstruction to the free propagation of the tidal wave by the barrier, the mouth of the creek silted gradually, as both these obstructions caused a reduction in the velocities and tidal influx into the creek. Consequently, the creek has been changing its course rapidly, the mouth is shifting from year to year and agricultural land along the creek is being eroded. Besides, the shifting channel has created a low lying area (marked A in Fig.6) along the coast over which waves, at high tide, travel and accentuate the erosion caused by the channel. Fig.6 shows the original and present course of the creek.

The tidal range at Kelva is 14 ft, same as at Versova described earlier. Conditions of wave approach are also similar. Waves as high as 10 ft are experienced in this area during the monsoon.

PROPOSED REMEDY

It is considered that the trouble can be eliminated if original conditions are restored as far as possible. It is, therefore, proposed:

a) to provide an additional waterway to the bridge;

b) to remove the salt water barrier and thus to restore the full tidal compartment; and

c) to make a suitable cut by dredging and by providing necessary training measures to stabilise the mouth.

CHAPTER 28

COASTAL DUNES: A STUDY OF THE DUNES AT VERA CRUZ

A Study of the Dunes at Vera Cruz

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Coastal dunes are formed when winds gather the dry sands from beaches and carry it inland. The repeated scouring of surface sand and its deposit in other areas does more to shape the diverse hillocks and depressions characteristic of certain coastal areas than any other physical factor.

For fifty years we have been battling the sand dunes at Vera Cruz. Some observations made from the experience gained over these years may be of interest.

NATURAL DUNE FORMATIONS

In the coastal areas near Vera Cruz the sandy dune strip (or "Faja") reaches a length that varies from 8 to 10 kilometres considering only the distance in which the wind is the predominant factor. Further inland old eolitic formations are still found, but being far from the beaches they are no longer active and the rains and vegetation govern their configuration.

Most dune strips lie on beaches exposed to the strongest winds. In the area of Vera Cruz the prevailing winds are from north to north-north-west and the gusts of these "nortes" at times exceed 40 mts. per second velocity. An aerial view of the faja which stretches from Punta Gorda (north-west of Vera Cruz) shows clearly how the process of sand bank formation follows the directi of the strongest winds (fig. 1). From the beach, banks of sand penetrate the vegetal platforms; in the photograph these sandy inlets appear like purls oriented lengthwise in the same direction as the north and north-west mean.

Each new beach penetration becomes a "fountain of sand", helping to fo successive dunes. The wind carries these sands from the beach and deposits th in small hillocks nearby. However farther on at a distance of about 2 kilometres there occurs a series of large dunes totally arid and in continuous movement reaching heights of up to 35 mts. This cordon is followed by one or two more with separations of intermediate level stretches about 200 mts. wide. Th is the most active dune area; the sands are activated by the minutest wind movement. When the winds are moderate the inside surface of the dunes undulat giving a scaly appearance; strong winds make them larger and form high inclin planes and furrows.

The front face of the dunes has the most gradual slope. The sands are impelled by the wind up the slope to the crests where strong eddies and vorti of air whirl about and the particles in suspension are in part thrown on the slope, and in part carried further afield in the general direction of the win The crests give the appearance of interlaced half-moons.

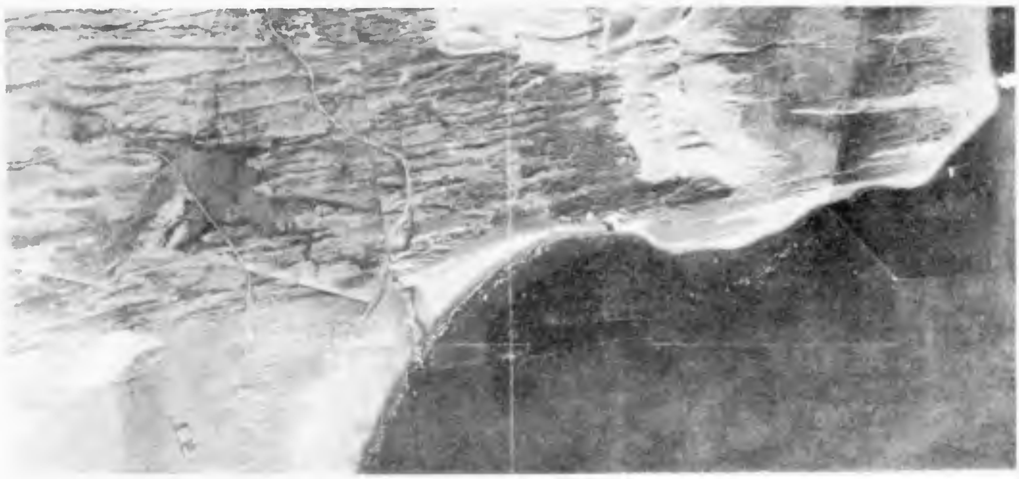

Fig. 1 - Aerial view of the strip of dunes which extends from Punta Gorda showing the process of sand bank formation.

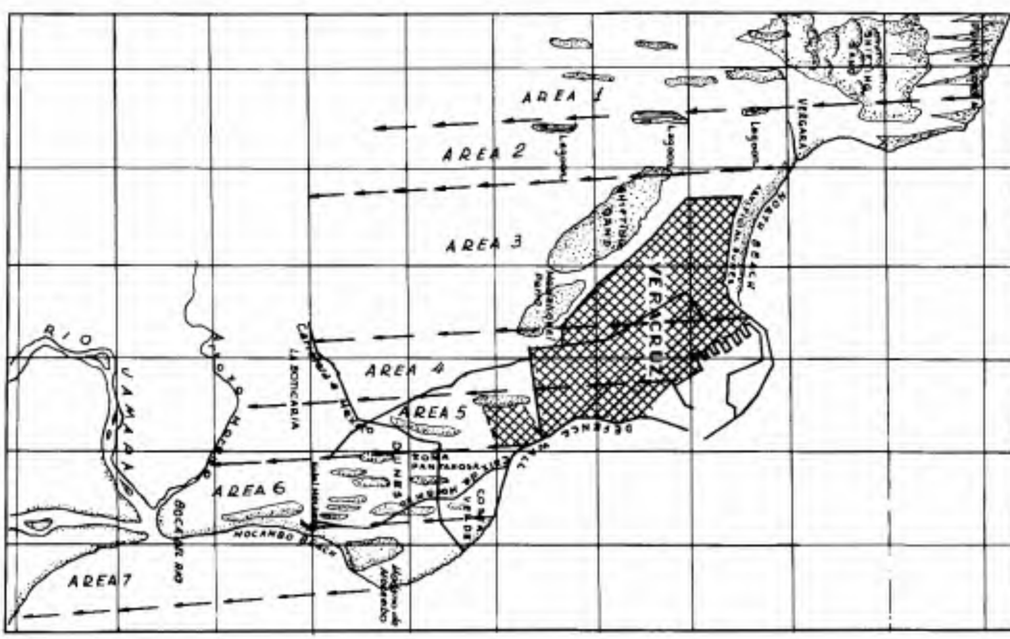

Fig. 2 - Influence of the orientation of beaches in the leeward zones.

The sands driven over the crest slide down the back slope forming an angle o approximately 33 degrees (the angle of repose) and they determine the landwa displacement of the dunes. Before the first stabilization projects were init iated Engineer Quevedo calculated the advance of the dunes at 10 to 14 mts. per annum, depending on the intensity of the winds and the relative humidity or aridness of the beach. These moving dunes have their high crests almost normal to the wind thus establishing calm leeward zones immediately to the rear. Sandfall on these zones is scarce and vegetation may flourish.

The terrain behind the active zone is composed of wooded area lying in the direction of the strongest wind in parallel lanes with low,level strips dividing it where sand deposit has been negligible. Strong winds do blow in a direction normal to them, but their force is channeled into ravine and the topography is not noticeably altered. Land behind active dunes is always a combination of woods and ravines combed in the direction of the strongest wind.

Climate and the Value of Stabilization Works - The coast of Vera Cru is the scene of a struggle of opposing forces - the wind on one hand and th rain and heat on the other. The wind hinders the growth of vegetation by smothering plants with damaging grains of sand or tearing off the topsoil a reducing seed supply. The abundant rainfall (1300 mm. annually) and intense heat (26º C. mean temperature) act in a contrary manner to encourage plants to take root. These opposing forces are pretty well equilibrated and with a little effort man can alter things in favor of vegetal growth and halt the advance of arid dunes. This has been done in Vera Cruz and could be equally well accomplished in any other area with similar characteristics. Good resu can be obtained by a close study of the behaviour of dunes in each area and the most practical methods to halt their movement and stabilize them.

Influence of the Orientation of the Beaches in the Eolitic Formation to the Leeward - It is curious to observe how the fringes of the fajas whic commence at the beaches similarly oriented hold almost identical characteri (fig. 2). In the zig-zag which forms the outline of the Vera Cruz coast, ea beach oriented in the direction east to west, almost normal to the winds of maximum velocity creates a sandhill zone very similar to that described abo The moving arid dunes appear at distances which vary between 2 and 3.5 kilo metres from the coast and this phenomenon is repeated in sudh a manner that arid dunes reproduce the form of the zig-zag coast line.

In the stretches of the coast oriented more or less north to south, appearance of the strips of ground bordering them are very distinct, the or inal arid zones being covered with vegetation even up to the beaches. From air one can observe the lots developed to pasture nearly up to the sea in t east direction, whereas to the north, the limit is 2 or 3 kilometres from t beaches.

The Mocambo beach, the favorite in Vera Cruz, has approximately the direotion north-south and is, moreover, protected from the northerly winds nearby heights. With such orientation, plantation of trees, etc., for the embellishment of the resort is possible close up to the dry upper beach. Th behaviour of the north-south beach is very different from that of the east-west front across the winds of maximum velocity, these winds supply hardly sand. In effect the northerly winds are already weakened by the immediate hillocks at the beach, and instead of picking up the sands of the sea they drift them in a parallel direction to the limiting zone of the water, and after a while increase the water-level and the wave height and thus the send of the breakers up the beach. The action wets the dry sand and impedes its removal. The "whip" of the sands is therefore weak and does not prevent the growth of vegetation.

It will be appreciated that these beaches do not release sand for the zones to the leeward; further proof is provided by the abundance of low marshy areas and lagoons all elongated to the leeward of these stretches of beach. Thus behind the beach of Vergara (fig.2) the depressions and lakes are oriented very like those behind the beach of Mocambo, although the latter is not protected by nearby high ground. The largest dunes are formed along fajas 1, 3 and 6 which originate on coasts normal to the winds; however faja 3 shows areas which have been stabilized by artifical dunes and faja 6 has a defence sea-wall. Faja 1, behind Punta Gorda, has begun stabilization thanks to the recent construotion of an artificial dune.

Faja 2 is preserved more or less in its natural state with a series of small lagoons. In faja 4 the northern end is now fully ocoupied by modern city buildings, and the quays and harbour wall of the Port of Vera Cruz; until recently this area was swampy and at a low level, whilst the original coast line ran approximately north-south.

The Beach to the North of Vera Cruz - To the north of the city the beach holds an orientation north-north-west, without any large variation for some thirty miles north of Tuxpam. As this orientation is that of the winds of maximum valocity, the formation of the sand hills have less importance than those in the fajas of the coast about Vera Cruz and in those which follow towards the south of this port.

Observed from an aeroplane one can see large stretohes in which the vegetation reaches almost to the coast. On occasions, one notes a single cordon of sand of appreciable height immediately alongside the beach which, not having received the full force of the wind charged with sand, has been able to support vegetation. On the coastal stretches forming angles of from 20º - 30º to the wind - such as that between Nautla and Tecolutla - two or three cordons of sandhills are formed without much danger from windborne sand invasion. A highway may be safely constructed right behind these cordons.

On this same coast north of Vera Cruz, when the orientation of the coast tends more towards the west, as is the case north of Punta Delgada, the same features occur as on the coast of Vera Cruz. The salient in the sea terminates in a mountainous cordon and limits beaches which face relatively to the north. The currents of air drift and lift the sands that form several sandhills and in flight, once transported over the edge of the cape, they fall into the sea. To the south of these irregularities one does not oome across the same aridity, the land remains protected from the strong winds and with the vegetation produces an agreeable countryside. However it is also subjeot to the same topographical movements as those already described at Mocambo.

PROCEEDINGS TO ESTABLISH AND ORGANIZE THE VERA CRUZ DUNE AREA

ARTIFICIAL DUNES

Anteoedents.- Engineer Quevedo, builder of the first artificial dune at Vera Cruz, attributed the calamity of moving dunes to the destruction of the "woods contigent to the founding site of the oity" by the early residents.

But the dunes were active long before the city was founded. So at least we are assured by Bernal de Castillo, soldier of Cortes and ohronioler of the Conquest: "there before that island (Sacrifioios) we all jumped to land, onto some large sand beaches where we built farmhouses and cabins with beams and sails from the ships"; and further on: "disembarked on some beaches, we made cabins on top of the sand banks which were large in that place."

Clearly then since its founding Vera Cruz has had the problem of its dunes, but an organized effort to dominate them dates from only fifty years ago with the works initiated by Miguel Angel Quevedo. This noted engineer directed the port works from 1890 to 1893 and understood the main difficulty "stemmed from the creeping sands of the beach exposed to the north winds, to the extent that the blocks for the jetties under constr-uction have been buried by high hillocks of sand." Thence dates his pro-posal to do away with the scourge of the dunes, to which he attributed, among other evils "The unhealthfulness for which the port of Vera Cruz is famed". He also recognized that to drain the lagoons formed between the sandbanks, some means had to be found to prevent the sand from piling up and sealing the natural outlets.

In 1908 Quevedo began the formation of the first artificial dune between the levee and the Vergara arroyo on the north beach. He decided to use the artificial dunes of the French Landes as his model, although he introduced some modifioations. One change he made was the use of saw-mill refuse in forming the wind-barriers thrust into the sand, in place of the usual fagots and arbours.

Sheet piling has various advantages. The logs of 2.5 to 3 mts. length are easy to transport and they may be recuperated with little effort when the movements of the dune exige a change in the location of the wall. In just two winters (1908-9-10) an artificial dune five mts. high was raised and in the ensuing year this height was raised to eight mts. Four years suffioed to produce such improvement in the area under the lee that the plugs of sand disappeared allowing drainage.

With the Revolution the works were ceased, ruining the artifioial dune. It was reconstructed from 1917-20 - abandoned again - only to be renewed in 1929 and constantly improved up to the present day.

Behaviour of an Artificial Dune - The artificial dune in Vera Cruz has functioned very well, helping with time to improve the standards of life in the port and its suburbs. The land leeward of this artificial dune is no longer subjeot to a rain of sand, vegetation abounds and the lagoons have disappeared.

The system implanted by Quevedo is still being continued, with only some alterations in the placing of the first barricades.

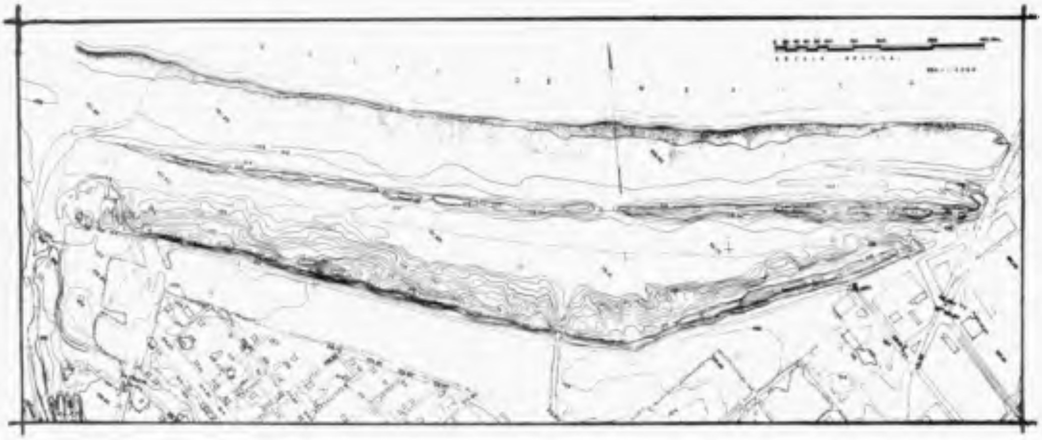

Fig. 3 - Artificial dunes in "Playa Norte" of Vera Cruz.

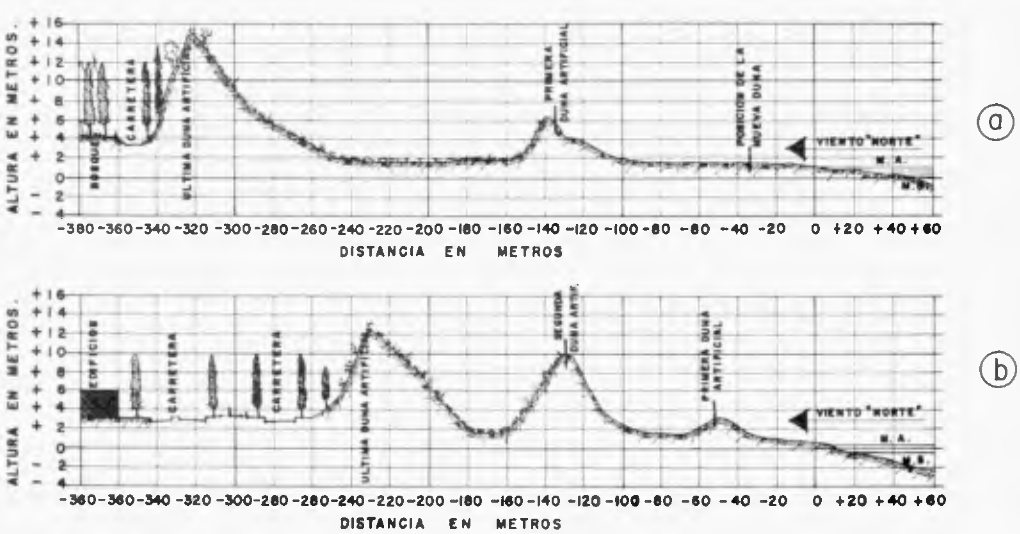

Fig. 4 - Transverse profiles of "artificial dune":

a) First dune is 135 meters from the sea, the second dune is vegetated, a new dune near the sea is necessary.

b) Two dunes with sheet piles are developing, the last dune is fixed by vegetation.

The first dune is established on the beach at about 20 mts. from the line of high tide in a direction normal to the wind. Originally Quevedo mad his first dune at 70 mts. distance from the sea (the same as that of the Gulf of Gascuña) and he recommended it to be built even farther back to avoid its destruction in seasonal gales. Sinoe then we have learnt that it is best to place the first dune as close as posiible to the shore to prevent much windblown sand from carrying over the dune(Figs.3 & 4).

The wind on encountering the dune deposits sand against the fences. A ridge or hump grows up relatively quickly and the crest displaces itself as if pushed forward by the wind. This is because the sand is precipitated in large quantity after passing the barricade rather than falling in front of it. When the dune has advanced 100 mts. from the sea, a large dry beaoh remains. The wind acts over this broad surface to raise and disperse the sand. The dune is now too far from the sea to have any effect in retaining particles in movement, so a new dune is formed by erecting another palisade close to the sea. The new dune forms and increases more rapidly than the first until it overtakes and joins it. Again it becomes necessary to form another dune close to the water, and plant the other.

Behaviour of the Artificial Dune in a Northerly Gale. - A season of "Nortes" from October, 1952 to May, 1953 allowed us to observe the behaviour of the dunes under winds of hurricane force. On the 7th of October a fierce gale from the north-north-west drifted the sands of the 20 mts. wide beach and lifted them up the slope of the first dune despite its steep inolination (fig.5). It may be seen how the talus picked up the surface sand also. The squalls of ascending air on reaching the fence enoountered other squalls and caused the dense vortices of suspended sand which was deposited about the obstruction.

The following day it was recorded that the beach from which the san was blown had entirely disappeared and the water line had encroached to th foot of the seaward dune. On the second day the wind abated and it was found that the slopes on the land side were 29-33 degrees and on the seaward side 32-34 degrees. On the seaward slope several areas showed humps o sand well above the critical profile. The humps little by little resumed a normal inclination. The crest of the sandhill had moved considerably landward and heightened, whilst the old fence and its oresting had disappeared

After two days of the north wind the slope on the seaward side had lost on the average 60 cms. thickness. The vertical fences running transversally up both faces with the aim of reducing the force of the winds of oblique incidence were all destroyed after the third day of the "norte".

One curious effect worth noting was that a single ~~sheet~~ fence driver into the beach at a point above where it was 150 mts. wide had, in three days, created a small hillock about it no less than 1.5 mts. high. The wind dropped after 4 days and continued moderate until the 3rd of Novembe: it left the first dune entirely flattened and only a few fences standing i a half-ruined condition on the seaward slope.

In these 23 days the beach soon reestablished itself, in fact it w broader than before and the first sandhill was now 4-5 mts. farther away from the sea-line. The inland dune was replaced also, but to a lesser

Fig. 5 - The first artificial dune near the sea, a) before the hurricane, b) during the hurricane.

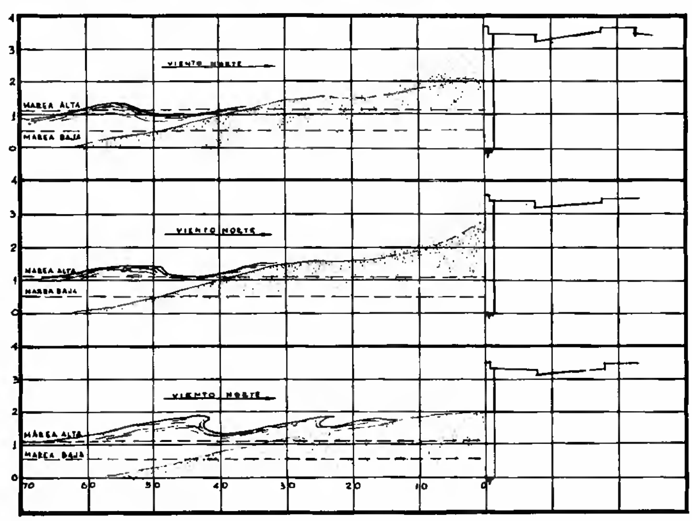

Fig. 6 - Drift and flight of sand from the beach is reduced by the sea wall. The last diagram shows the waves invading the beach and wetting the particles before being moved by the wind.

amount, although it was covered with firmly rooted vegetation. These phenomena demonstrate the wisdom of siting roads and structures well out of the range of sea action in areas which require the protection of artificial dunes.

Area Required for the Artificial Dune - In Vera Cruz the artificial dune has a normal width of 400 mts. from the beach, although it is reduced to 250 at places near the port where there is less space available. In this widht a pair of dunes are moving towards the third and last, which adjoin the highway called Circunvalación. On the other side of the highway there is a fence of loose brush ranging from 100-200 mts. width which serves to complete the functioning of the artificial dune.

In fact, then, artificial dunes act in the same way as natural dunes, only in a much more confined area. In the case of Vera Cruz artificial dunes retain sand but do not require, as do the natural dunes, a depth of borderland 8 kilometres long. When properly fenced and maintaine with adequate fencing, an efficient length of land need extend no more th 400 mts. from the beach.

EFFECTS OF PROTECTION WORKS ON THE DUNES

Sea Walls - The dykes or sea defence walls constructed in Vera Cru have proved very effective in reducing the drift and flight of sand by wind action. This, of course, was not the reason they were built, their purpose was to protect the city from the violence of the sea and at the s time provide added amenities in the form of a marine promenade, roadway a tram route.

In effect these seawalls protect even more efficiently than artificial dunes the length of the strip in which they are constructed. When t sands tend to move and encounter a solid wall 2 or 3 mts. high in their path they pile up against the wall face. The particles do not pass over because of the contained moisture and because the tide which follows the wind invades the narrow strip of beach in front of the wall, wetting it and impeding the flight of the particles. This excellent result would not have been achieved were it not for the small range of tides on this coast The maximum oscillation is 1.2 mts. On coasts where the tide movements are much larger, low tide may uncover beaches hundreds of metres wide, a various barriers must be emplanted before the seawall to diminish the sa movements caused by wind. On the Vera Cruzian coast where the sea walls a well sited, the low water does not leave a great extent of beach: 40 to 50 mts. at the most, from which the wind removes little sand over the wall. It would be an error to place the sea wall too far from the water edge in those areas fronting the north for the purpose of obtaining a wide beach - the desire of the general holiday public - because, in such a case, the dry sand would be blown towards the wall, and spilling over, would travel inland.(fig.6).

It is a proven fact that the zones of Vera Cruz situated on or near the Marine Promenade are better protected from sand invasion than those in the lee of artificial dunes. The prolongation of the sea highway with its defence wall towards the south east has permitted the construction of two new districts - "Reforma" and Costa Verde" - which would not have been possible with artificial dune areas. In the lee of the portion where the wall is constructed no sand is in movement.

Stone Barricades and Piles at Sea's Edge - Defence walls for their high cost can only be justified as a means of protecting coastal roads and urban areas. They are not the fit solution, however, to the problem of moving dunes.

But a palisade of stones, piles and horizontally placed logs can be formed which will function as well as a defence wall and at a much lesser expense. In Vera Cruz we experimented with such a palisade and got satisfactory results. The prolongation of the last stretch of the Costa Verde highway and wall was initiated in December, 1955, with the construction of the outer protective palisade shown in fig. 7 . At this point work was suspended and we have had a good opportunity to observe its behaviour with respect to the moving sands.

Fig. 7. (a) Palisade of stones, piles and logs protecting the beach in regretion S. E. of Vera Cruz. (b) Construction of the palisade.

The logs of the palisade were tied to the piles forming small compartments which were filled with stones ranging up to 500 kgs. These stones were packed as closely as possible to prevent the waves from displacing the piles and logs, while the latter in their turn helped to keep the stones fixed. The palisade is 1000 mts. long and was built rapidly and economically. The 3.5 to 4 mt. piles were sunk 2.5 mts. in the sand by means of an injection pump and a team of six men. Progress was made at the rate of 20-25 mts. daily. The rock fill was applied directly to the sand (with no excavation deemed necessary). A period of 75 days sufficed to see the work completed.

After the works of protection had been executed the waves, instead of beating against the wall, dissipated their energy over the stone rubble, and in front deposited sand. In a period of three months the stones had been almost completely covered by a depth of sand of 50 cms. The seasonal damage to the bordering vegetation ceased at once. The sand particles which had formerly penetrated this area were stopped up and plant growth allowed to flourish.

Since the completion of the palisade the beach has been renewed ar stabilized, and the dunes and sandy surfaces to the lee have diminished t the point of disappearing.

PLANTING AND OTHER METHODS OF STABILIZING THE DUNES

Without going into much detail on this subject, the type of plants which have been successfully utilised in the dunes at Vera Cruz should be mentioned.

In 1908 and 1909, when the initial phase of the construction of the artificial dunes was complete, grasses and shrubbery of the followin species were successfully planted in the surrounding coast, mostly at the Island of Sacrificios: rootstock of common reed grass (arundo donax), "riñonina" or "frijolillo", cornizuelo or coastal acacia (acacia cornig- uera). Similarly varied species of the Opuntia class were planted. In order to complete the vegetation and coverage "privilegio", "buffet" and German hay have been added.

The "frijolillo" plant covers the sand rapidly and reproduces frequently, proportioning a large quantity of dry residue and humus to the arid upper strata of sand. In the second and third years of the formation of the dune, according to Quevedo, an intermediate seeding with "carrizo" was prescribed and after 1912 the planting of shrubs, in this case the Casuar Sunningham, which today has provided the best, most verdant results.

Various Works Contributing to Stabilization - In addition to the work previously cited, viz., artificial dunes, dykes, sea walls, barriers of vegetation, and forestation, other methods have been used to stabilize the Vera Cruz area, notably, paved roads, buildings, abundant provision o water, and the cultivation of small farms and gardens. Also mounds have been leveled, swamps filled in to distribute humidity more evenly, and this has greatly favored vegetation in areas where priorly "sand fountain from the beach had impeded it.

Such results indicate that sand dunes with similar characteristics to those in the Gulf of Mexico may be kept under definite control with reasonable economy, and that, moreover, they may be transformed into verdant pastures or become the sites of human habitats.

CHAPTER 29

DUNE FORMATION AND STABILIZATION BY VEGETATION AND PLANTINGS

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ABSTRACT

Plantings of appropriate native and introduced plants and the management of extant vegetation are often very effective in promoting the development and maintenance of dunes that serve to protect shores against some types of erosion.

The importance of dunes as barriers, the formation of dunes, and the role of plants and vegetation are considered. The types of vegetation of the three zones usually present over dune fields are described, and the general management to maintain such vegetation is discussed.

Methods of making plantings on natural and artificial dunes by selecting, seeding, and transplanting the species that are most effective in building and stabilizing dunes are given.

Editor's note - This paper was a summary of the publication "Dune Formation and Stabilization by Vegetation and Plantings", by John H. Davis, Beach Erosion Board Technical Memorandum No. 101, Oct. 1957, 47 pp. The reader is referred to this publication for the complete paper.

CHAPTER 30

LEGAL ASPECTS IN COASTAL PROTECTION ENGINEERING

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A LEGAL BASIS FOR COASTAL PROTECTION

In modern systems of democratic government the basic recourse for the necessary authority and power for solving problems is in the enactment of laws. These laws may vary extensively in character and content from the comprehensive acts of a congressional body to the simplest ordinances of a town council, but each provides a means acceptable to the people for implementing a desired program. Such is the case in the preservation of beaches and the protection of coastlines.

It is safe to assert that without the benefit of some legal basis, nothing would be undertaken beyond individual attempts to control the forces, natural and cultural, which threaten the shoreline. Coastal laws are necessary. They acknowledge the problem and the need to combat it. Yet, with continued efforts to refine the law and detail the most minute principles of implementation, it loses its value as the servant of the people and becomes the master - a coastal protection program no longer flexes to a contemporary situation, but is governed by the limitations of the law.

A sound coastal protection program, then, should be based on a law more general and adaptable than specific and rigid. It should recognize the preservation of shores and beaches as a public responsibility, and should provide autho. ity and means for the discharge of this responsibility. Sucl a simple statement of the issue is misleading, however. If the legal prerequisites are so elementary, there must be som reason why every political entity with a coastal problem doe not have a basic law conducive to a successful protective an remedial program. The explanation for this is not in the law itself.

Florida is a prime illustration. Since 1931 - for more than a quarter century - this state has had in its statutes a provision intended to authorize, if not direct, a program of almost unlimited scope for the administration of state lands by the Trustees of the Internal Improvement Fund - a board comprised by five cabinet members acting ex officio. Section 253.03, Florida Statutes, reads in part:

> The Trustees of the Internal Improvement Fund of the state are vested and charged with the administration, management, control, supervision, conservation and protection of all lands and products on, under, or growing out of, or connected with, lands owned by, or which may hereafter inure to, the state, not vested in some other state agency. Such lands shall be deemed to be . . . . all lands owned by the state by right of its sovereignty . . . . all tidal lands . . . . all lands covered by shallow waters of the ocean, gulf, or bays or lagoons thereof, and all lands owned by the state covered by fresh water . . . . all lands which have accrued, or which may hereafter accrue, to the state from any source whatsoever, unless or until vested in some other state agency.

General though it may be, sufficient authority is contained in this section to have enabled long ago the initiation of coastal protection work. The aforementioned Trustees, who have the power to approve disbursements from the Internal Improvement Fund for a "liberal system of internal improvements", might legally have instituted a program of coastal improvements as well. In its permissive aspects, this law is entirely adequate. Yet, today, Florida suffers coastal problems as critical as any in the world, and lags far behind in application of modern coastal engineering techniques.

Cursory analysis is sufficient to note that coastal problems in Florida have not gone unattended through absence of legal authority to cope with the situation. Progress within the state in recent years, stimulated largely by the efforts of the Coastal Engineering Laboratory at the University of Florida, provides encouraging evidence that such activities can be conducted in harmony with, if not as a product of, the existing law. To be sure, a law with more specific references to beach erosion and coastal protection might have been utilized more extensively; but basically, the relative inactivity in this field in Florida has been a consequence of widespread ignorance and apathy on the part of the people. Despite acute natural problems and Florida's economic interest in shores and beaches of the state, a strong protective program has not developed primarily because the people have been unaware of the situation and have not been inclined to support the much needed program. This problem is not confined to Florida.

To correct this situation, no amount of legal reform will suffice. Instead, public support must be obtained through a concerted education and information program by responsible governmental agencies or interested citizens' groups. In Florida, for example, a common problem united a number of local groups and agencies into the Florida Shore and Beach Preservation Association. This organization, which received part of its stimulus from the success of similar groups in other states, has been instrumental in specifying by legislative act the responsibility of the Trustees of Internal Improvement Fund for erosion control and beach preservation. In addition it has acquired support for the program of the Coastal Engineering Laboratory, and has the promise of providing an indispensable service in educating the public.

Active participation by both the government - determine by law - and the public - determined by sentiment - is necessary for the consummate success of a coastal program. Discussion thus far has pointed up the fundamental deficiency of each. First, over-refinement of laws encourages undue reliance on the provisions of the law, limiting its application and curbing initiative. Second, without public interes and support, no law, whether infinitely detailed or broadly permissive, can provide a remedy for coastal problems. The conclusion reached is simple, and yet profound: coastal law are necessary for a remedial program, but until such time as these laws become mandatory directives, they must be drawn to enlist the fullest public cooperation, down to the last individual beach property owner.

Although coastal laws have their purposes in common, their application must vary to fit particular circumstances. If the law is not general and flexible, numerous difficulti are likely to be encountered. Procedures and policies shou evolve through interpretation of the law rather than writte into it, and local acceptance must be insured through adapt tions. If coastal conditions become so critical that hazar are created, human life is endangered and the public intere in private property is jeopardized, the law should provide for positive governmental action. Otherwise, the initiativ should be fostered at the lowest practical level.

PROVISIONS OF PRACTICAL COASTAL PROTECTION LAW

To be practical, coastal protection laws should provid for what may be done, the scope within which and the means which it may be done, and who may do it. If coastal conditions are critical enough, these provisions should require mandatory execution; otherwise permissive powers should be granted for use at whatever level the initiative is taken. As previously emphasized, the law must not be over-refined, but must authorize a liberal approach, adaptable to particu situations.

BEACH PRESERVATION LAW

In considering what the law should provide, the need for coastal protection can be logically divided according to source: human or natural. Human activities such as mining, dredging, filling and construction are often detrimental to beaches and coastlines, and may be regulated through a beach preservation law. The law may simply forbid such activities or may prescribe desirable limitations or restrictions. Offshore activities, which may be as harmful as those on the beach itself, should also be controlled. Responsibility for enactment and administration of this law should be in the political entity which legally holds title to coastal areas beyond the line of private ownership.

COASTAL PROTECTION LAW

Protection of shores and beaches from natural factors involves measures of greater complexity, and the law enacted for this purpose should provide correspondingly broader authority without attempting to prescribe superfluous procedural details. There are five basic provisions which should be incorporated into the coastal protection law:

(a) a provision creating an agency in the central government, or placing the responsibility for coastal protection in an existing agency of the central government

(b) a provision requiring certain measures to be undertaken to protect life and property, prevent hazards from products of storm and flood, and uphold the general public interest in private as well as public property

(c) a provision authorizing measures to be undertaken at lower levels to prevent and remedy damage and loss of property through natural processes such as erosion

(d) a provision authorizing the establishment of cooperative organizations for the purpose of shore and beach preservation and coastal protection at lower levels

(e) a provision authorizing participation by the central government, through financial and technical assistance, in coastal protection activities at lower levels and establishing formulae for determining the extent of governmental participation

These provisions are more or less comprehensive, and it may be desirable to delegate them to one or more lower governmental levels within the central government, depending largely on the size of the political entity assuming the responsibility and the political subdivision system in use. A small state with a relatively short coastline might easily assume each of these functions, whereas a large country with a more heterogeneous coastline might prefer to place these functions within local governments. In any case, an unequivocal line of responsibility should be maintained and the advantages of some overlap of duties at each level should be considered.

The coastal protection agency - Whenever a coastal protection law is enacted, there should properly be an agenc of the government to represent the public interest in the discharge of the provisions of the law. This agency may interest itself to some extent directly in coastal protectic and remedial programs, but primarily it serves as supervisoi and coordinator of subordinate activities, and as adviser tc the governmental executive. Liberal powers toward the execu tion of a comprehensive coastal protection program should be vested in this agency.

Mandatory requirements for protection of life and public property - Many consequences are likely to result from the action of natural forces on unregulated human development and use of coastal areas. Some of these consequences are confined in their effects, and cause no immedia public concern. There are others, however, caused or aggra vated by individual or local activity, which have far reaching effects and are of vital concern to the public as a whole. Among these consequences are the loss or jeopardy of human life through action of storms and floods on inadequately protected coastal areas, the development of public health hazards, and the destruction of public property. The coastal protection law should serve to preve or eliminate such conditions before they become consequenti requiring mandatory adherence to prescribed standards of public safety and coastal development.

Authority for individual coastal protection measures - Any individual or several coastal property owners should enjoy the right to undertake measures for the protection of their property from natural forces. To insure an orderly approach to this problem, the coastal protection law shoulc authorize private activities subject to approval and supervision by the government of the techniques and structures to be used. Since such measures frequently entail construc tion on public property below the line of private ownershi usually the mean or ordinary high water line - the law should authorize such invasion for legitimate purposes.

Local cooperative organizations.- In many cases, coastal protection or beach preservation needs do not involve the entire limits of a particular local government, yet exceed the scope of individual property owners. The desirable recourse is the establishment of a district, covering the entire problem area. This district could take the place of a local government to effectuate a cooperative program. The basic coastal protection law should provide blanket authority for creation of beach erosion districts or similar organizations, and provide a framework within which they might function.

Governmental participation in coastal protection programs - Some public benefit accrues from almost all properly planned and executed coastal protection programs. For this reason the central government may desire to participate to some extent in protective and remedial programs for private property, as well as conducting programs for entirely public property. The assistance and incentive to be gained locally from governmental participation is extremely valuable, since coastal technology is not a common science and the planning and construction of coastal projects is costly. Coastal protection laws should make some provision to enable participation by the central government, and should set forth terms on which to base the amount of assistance to local governments, beach erosion districts and possibly individual property owners where the public interest is sufficiently great.

Provision for beach preservation and coastal protection needs as outlined above, liberally drafted in a law compatible with a particular constitution or charter, will afford a general and comprehensive basis to undertake or foster the actual protective and remedial programs. It would serve little purpose to elaborate on the numerous ways by which these provisions could be represented in the law, or on the even more numerous ways by which the legal provisions could be implemented. These are considerations which must be influenced by the needs and desires of a particular government. It will probably be of value, however, to examine selected provisions of existing law to gain the benefit of experience by other governments with perhaps similar coastal problems.

PROVISIONS OF EXISTING COASTAL LAW

Coastal laws currently in use by various governments of the world have evolved - or are evolving - in a manner responsive to the needs occasioned by conditions in the area. These conditions represent a complex of physical, legal, cultural and related factors which determine differences in the laws, or whether or not there is a law at all. The result has been a wide range of experience in producing coastal protection laws among various countries, and among the various states of the United States. Provisions of some of the representative existing coastal laws are summarized below.

DENMARK

Denmark has two different kinds of beach laws:

(a) a beach preservation law, and

(b) a coastal protection law.

The beach preservation law which is now in use was issu in 1906. It provides that when it is necessary for the protection of the coast, all removal of sand, clay, gravel and stones can be forbidden. Exemptions are sometimes made for such purposes as the removal of material for coastal protection work.

Executive power is in the hands of the Ministry of Pub. Works, which, when such a question arises, establishes a "coastal commission" for each county involved. Out of the three members on each commission the chairman is selected b, the Ministry of Public Works (usually a district engineer from the ministry), and the two other members are appointed by the county commissioners, although they do not need to b county commissioners. The commission works out a proposal and holds a hearing before reaching its decision. The decision may be protested, but if the ministry sustains it, the decision is valid for five years. At the end of five years the matter can be re-considered if requested.

Coastal protection law now in use requires the approva of the Ministry of Public Works for any coastal structure built outside the mean high water line. The ministry can refuse to allow constructions which are inadequately desigr and will have a detrimental effect on the adjacent coastal property. If support from government funds is applied for, the legislators act on each individual request through the "financial committee", There are no general rules regardir the financial participation by the government. On the Nor Sea coast a contribution of 100% may be made where it is considered important for the country as a whole to counter erosion. The total amount of government funds usually is based on the percentage of public interest in the area to l protected. In the "inner seas", the Baltic and the Sounds, the government usually will contribute one-third, the coun one-third, and private interest one-third. If a city is involved, it may take over the whole cost, or share it with private interests. These are divided into two or three classes according to their interest in the matter. Class one has coastal property and pays twice as much per linear meter of protection as class two, which has its property inland from class one. Class three, if any, is inland from class two, and pays half as much as class two. The power is in the hands of a "property commission", similar to the coast commission described above. The chairman is usually a judge and is appointed by the Ministry of Public Works.

An important provision of the coastal protection law prescribes a means by which neighboring property owners may be assessed for a proportional share of the cost when benefits are derived from a project undertaken by another owner. The party initiating the project may request the property commission to determine the extent of the benefits to neighboring property and assign expenses for construction and maintenance accordingly. If the property commission deems it necessary that a coastal protection structure extend beyond the property of the builder to achieve proper results, it may grant such permission, even over the adjoining property owner's objection.

HOLLAND

Holland has no special coastal laws. Its program functions under a number of laws of more general tenor. The most important of these is the "Waterstaat" act, issued in 1900, which gives general rules for government, dealing with the regulation of the water and the defense against the sea in any situation.

Among the provisions of the Dutch law is that establishing water-divisions, or "waterschappen", which are in some ways similar to Florida beach erosion prevention districts. These waterschappen are arranged at different levels of authority and jurisdiction, and have the power to pass local legislation regarding defense against the sea. In many cases a lower level waterschappen must yield to the superior authority of a higher division, but otherwise it is responsible for coastal protection activities within its own province. In emergencies such as that which occurred in 1953, it is possible for most of the divisions to mobilize every able-bodied male between the ages of 16 and 65 for work at dikes and sea defenses.

Waterschappen are governed by a committee elected by the owners of the property within the limits of the division. The number of votes any owner has is dependent upon the size and use of his property. The chairman and members of the commission responsible for the over-all management of water divisions are appointed by the Crown. Final authority over all divisions is vested in the Crown. When the situation warrants, the official of the Ministry of Waterstaat may, in the name of the Crown, take command of any local situatic

All the beaches and the connecting dunes are part of Holland's defense against the sea, and for this reason are under control of the Ministry of Waterstaat. All land on tl seaward side of the high water line is always owned by the state. In most cases the state or division also owns a narrow strip on the shoreward side of the nigh water line, but in a very few cases this strip may be private property.

A waterschap may be established whenever a certain percentage of the property owners in a district requests it, or if the Crown deems it necessary. The Waterstaat act states that public or private property may be used for digg: surveying or erecting of certain signs necessary for the design and execution of coastal protection works, provided written notice is sent to owners or users of the property, at least 48 hours in advance. The act also provides that n coastal defense works in an area under management of a waterschap are to be approved by the county authorities, or the Ministry of Waterstaat. Every county has its own hydraulic engineering division. The waterschappen, the county authorities and the Ministry of Waterstaat are joint. responsible for the management of the coastal protection works.

Activities of possible detriment to the foreshore, dun or coastal waters are tightly regulated. For example, it i forbidden to dredge on the foreshore within a distance of 1500 feet from the toe of the dunes or the existing coastal protection works. Destruction of vegetation in these areas is especially prohibited.

If coastal protection structures built under the autho ity of a waterschap prove to be beneficial outside the division's limits, a contribution to the cost of these work may be paid by the county authorities and the government. The ratio of these contributions depends on the circumstanc but in case of emergency the state may pay the whole cost.

UNITED STATES

Provisions of United States law concerning beach prese vation and coastal protection are contained in a number of acts dating back to the important Rivers and Harbors Act of 1930. The most significant of the separately enacted laws are described here.

Public Law 520, Seventy-first Congress, 1930 - This law created a seven man beach erosion board under the Chief of Engineers, U. S. Army, to furnish technical assistance and otherwise supervise and participate in investigations and studies made in cooperation with the states to determine beach erosion needs and remedies. The Corps of Engineers was assigned the primary responsibility for conducting the cooperative studies.

Public Law 166, Seventy-ninth Congress, 1945 - Additional responsibility was placed on the Corps of Engineers, through the Beach Erosion Board, which was directed to conduct general investigations at federal expense to protect, restore and develop the beaches. The responsibility of the Board under P. L. 520 was increased to include an opinion on (a) the advisability of adopting the project, (b) what public interest, if any, is involved in the proposed improvement, and (c) what share of the expense, if any, should be borne by the United States.

Public Law 727, Seventy-ninth Congress, 1946 - For the purpose of "preventing damage to public property and promoting and encouraging the healthful recreation of the people", this law authorized federal financial assistance for the construction, but not the maintenance, of coastal protection works. The project must be for public property, and must be recommended by the Beach Erosion Board and specifically authorized by Congress. Federal funds are limited to a maximum of one-third of the total cost of the project.

Public Law 826, Eighty-fourth Congress, 1956 - Important amendments to P. L. 727 were made by this law. Shores of territories and possessions were specifically mentioned for the first time. Also, provision was made to interpret "artificial nourishment" as a limited form of construction previously provided for. Probably most important is the broadening of the provisions of the act to include all shores, whether public or private, where public interests are involved.

INDIVIDUAL STATES

Of the forty-eight united states, twenty-two have a marine shoreline and six others have a shoreline on the fresh water Great Lakes. A vast difference is manifested among the coastal and beach laws that have been developed throughout the country. Georgia, Louisiana, Maine, New Hampshire and Oregon have no beach laws to speak of, while some states operate under highly effective statutory provisions.

Florida - The basic statute under which Florida has authorized a beach preservation and coastal protection program is Section 253.03, Florida Statutes, described in the initial part of this paper. The Trustees of the Internal Improvement Fune were so empowered as early as 1931, but little has been accomplished under that authority.

In 1941, a law was enacted which became Chapter 158, Florida Statutes, authorizing the establishment of beach erosion prevention districts. This law provides that any election precinct in the state may by majority vote organize as an erosion prevention district, with all the powers and functions necessary for undertaking a program of its own or cooperating with the federal and state governments. Only a limited number of such districts has been created, but these are in some of the areas of most critic need.

It has always been the responsibility of the Trustees of the Internal Improvement Fund to administer sovereign tidal lands and regulate their alteration and development. Inconsistent legislation in past years has produced such legal confusion that the Trustees' task has been extremely difficult. The 1957 Legislature enacted Chapter 57-362, Laws of Florida, vesting an unequivocal authority in the Trustees and repealing several conflicting statutory provisions. Of greatest significance in this law is the establishment of a procedure by which local governments under the over-all supervision of the Trustees shall fix bulkhead lines in tidal waters to control dredging, filli and similar alterations. Broad application of Chapter 57-362 is currently being made.

Another important act passed by the 1957 Florida Legislature, Chapter 57-791, designates the Trustees of the Internal Improvement Fund as the erosion agency of th state, and authorizes the expenditure of surplus funds fo assistance to localities in combating beach erosion. The Trustees' responsibility in the beach preservation field is further confirmed, and a department of beach erosion may be created as a part of the Trustees' staff if it proves desirable.

Massachusetts - In Massachusetts, activities in shc protection, river and harbor development and stream imprc ment all are authorized by Chapter 91 of the statutes. Section 11 of this law, which pertains more specifically to beach erosion and harbor and channel protection, provi the Department of Public Works with broad authority to undertake such activities for improvement, development, maintenance and protection as it deems reasonable and proper. It has been the policy of the state to require a fifty per cent contribution from local sources toward the cost of beach protection works. A twenty-five per cent local contribution is required for dredging project if the general public interest is served. Local partici must petition the Department of Public Works for assistance and hold a public hearing on the proposed cooperative project.

New York - Erosion prevention and beach protection works in New York State are carried out under the Superintendent of Public Works by authority contained in Chapter 535, Laws of New York. The initiative rests with local or municipal governments, who must enter into an agreement with the state to contribute fifty per cent of the total project cost. Necessary lands or easements are provided by local interests, and plans for the project are drawn by the state, subject to local approval. The state may contract actual construction work or may undertake all or part of it with its own forces. After completion, the municipality assumes all responsibility for maintenance and repair. Provision is made to utilize assistance from the federal government in any project, but the local obligation remains at fifty per cent of the total cost. Municipalities are authorized to levy a general tax on all taxable real property therein or a special assessment on real property actually benefited by the project.

Ohio - The state of Ohio, which has no marine coastline, has a very detailed shore protection law, pertaining primarily to the shores of Lake Erie. Chapter 1507, Ohio statutes, vests the responsibility in the Division of Shore Erosion, with authority to cooperate with the federal government and to call upon other state agencies and departments for needed assistance. The Division regulates all activity, either for shore improvement and protection or for mining and removing materials from the beach or lake bottom, through the issuance of permits. The state may enter into agreements with local governments for undertaking shore protection projects. If the property to be protected is wholly public, the state assumes two-thirds of the project cost and local interests one-third; if private property is to be protected, the ratio is reversed, with the state paying only one-third. In emergencies, the state may act without an agreement for local contribution, and regardless of the ownership of the property involved. The maintenance of completed works also is shared by state and local interests. Responsibility for the preparation and continued modification of a comprehensive plan for erosion prevention is placed in the Division of Shore Erosion.

Michigan - Neither does Michigan have a marine coastline, but its fresh water shore line on the Great Lakes Superior, Michigan, Huron and Erie is extensive. The state itself, however, has not been particularly active in shore protection programs. In 1952, two measures were enacted by the Legislature to authorize initiative at the local township or municipal level. Act No. 44, 1952, authorizes any political subdivision of the state to make expenditures from its general or contingent funds for beach protection work, Act No. 278, 1952, further authorizes local governments to enter into agreements and cooperate with the federal government in any of its natural resource or conservation programs, including beach erosion control.

California - Control over beach erosion in California formerly was vested in the State Park Commission, under the immediate supervision of a beach erosion control engineer. In 1953, the office of the engineer was abolished and the powers and duties relating to control of beach erosion were transferred to the Department of Public Works. In 1956, these functions were transferred to the newly created Department of Water Resources.

Sections 330-334 of the State Water Code outline the existing authority pertaining to the control of beach erosion. Provision is made for the conduction of studies independently or in conjunction with other local, state or federal agencies. Within certain financial limitations, the Department of Water Resources may plan and construct whatever works the studies indicate to be necessary. Specific authority is provided for cooperation with the federal government or other agencies in constructing beach protection works.

CHAPTER 31

TIDAL CURRENTS IN CONSTRICTED INLETS

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INTRODUCTION

Many inlets along the coastline of North America are deep, wide bays which are connected to the ocean by a short channel of much smaller cross-section. Figure 1 is a schematic sketch of such an inlet. It is usual in these inlets that the tide curve (water surface elevation vs. time) in the bay does not coincide with that in the ocean. The range of variation is discussed by Caldwell (1) in a review of inlets in the United States. In addition, Caldwell classifies this type of inlet as one with an inadequate entrance. This term describes well the engineering problem encountered in most of them. There are high velocities in the entrance channel, usually near periods of slack water, which are inconvenient to navigation. In some instances these velocities combined with local geography constitute a serious hazard to shipping.

Typical of this class of inlet, but not so simple as the example outlined on Fig. 1, is Burrard Inlet situated on the southern mainland coast of British Columbia. Figure 2 is an outline map of this area, from which it can be seen that Burrard Inlet is three deep basins in series connected by two narrow, shallow channels. The harbour for Vancouver, Canada's major west coast port, lies along the shores of the second basin. All shipping must pass through the First Narrows in which the currents range up to 6 knots on both ebb and flood flows. This channel is too narrow and shallow for the present day volume of shipping so plans were advanced for its enlargement. At the same time the fear was expressed that the enlargement of the First Narrows would increase the currents in the Second Narrows and thus make navigation here much more difficult than in natural conditions. As a means of studying the interaction between the two Narrows a hydraulic model was proposed. However, this would need to be very large and expensive for a reasonable accuracy to be obtained. The National Research Council was requested to study this problem and recommend a means of solution. It was found that the mathematical model outlined below did provide a good approximation to the flow and the model proposal was dropped. In the following sections of this paper the accuracy of the mathematical model is demonstrated and the effects of the enlargement of the First Narrows are briefly outlined.

The analysis of the flow in Burrard Inlet is general and should be applicable to other inlets. The author has n had an opportunity to make other applications and hence is not able to state the limitations. It is hoped that throug publication of this paper that other investigations in this field can show the overall usefulness of this type of analy

MATHEMATICAL MODEL OF THE FLOW

Consider the simple situation shown on Figure 1 wher a bay of surface area A exists of sufficient depth that t time required for elementary waves to travel from end to en is smaller than the period of the tide. This is tantamount to assuming that the length of the bay be greater than one-quarter of the tidal wave length. The discharge from the ocean into the bay will then be approximately the rate of rise in the bay water level multiplied by the surface area. Connecting the bay to the ocean there is a uniform straight channel of length L and cross-section a, thus the mean velocity in the channel is:

$$V = \frac{A}{a}\frac{dh_1}{dt} \qquad (1$$

where h_1 = water level in the bay
t = time.

Equation (1) is the equation of continuity for the system which upon integration gives

$$h_1 = \frac{a}{A}\int_0^t V\,dt. \qquad (2$$

The only other equation required to define the flow a dynamic equation which is a simplified version of that pr sented by Einstein (2). The channel is assumed to be short enough that the convective acceleration is negligible, i.e. constant discharge exists along its length at any instant. Equation (7) of Einstein (2) thus reduces to:

$$\frac{dV}{dt} = g\,\frac{h_o - h_1}{L} - f\,\frac{V\,|V|}{8\;\;R} \qquad (3$$

where g = acceleration of gravity
h_o = tide level in the ocean
f = friction factor
R = hydraulic radius of the channel at mean tide.

As a boundary condition on Eqs. (2) and (3) the tide curve in the ocean must be specified. A cosine term is the simplest expression and has been found to be convenient to use.

$$h_o = H\cos\frac{2\pi t}{T} \qquad (4$$

where H = half range of tide
T = period of tide.

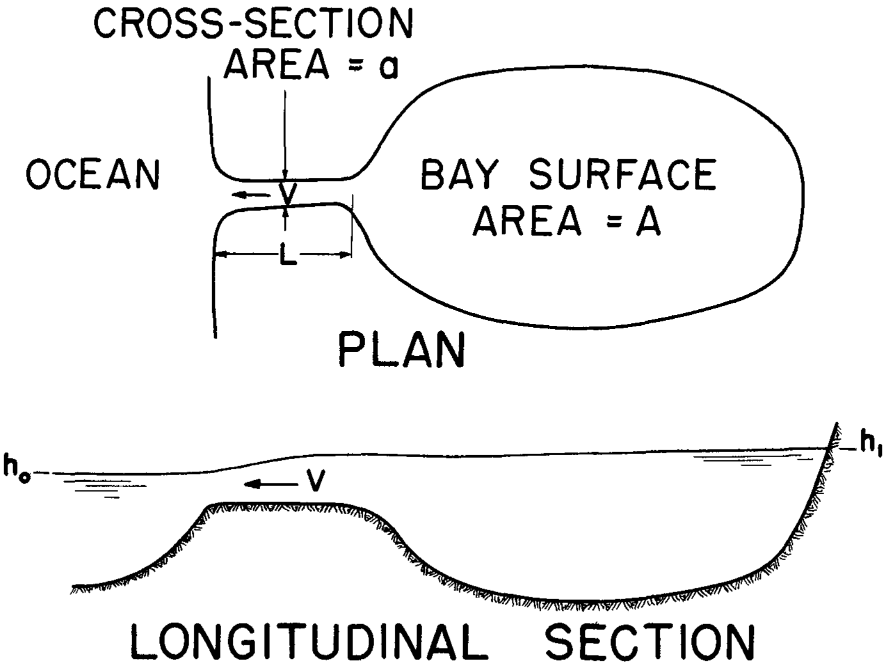

Fig. 1. Sketch of a bay with a constricted entrance channel.

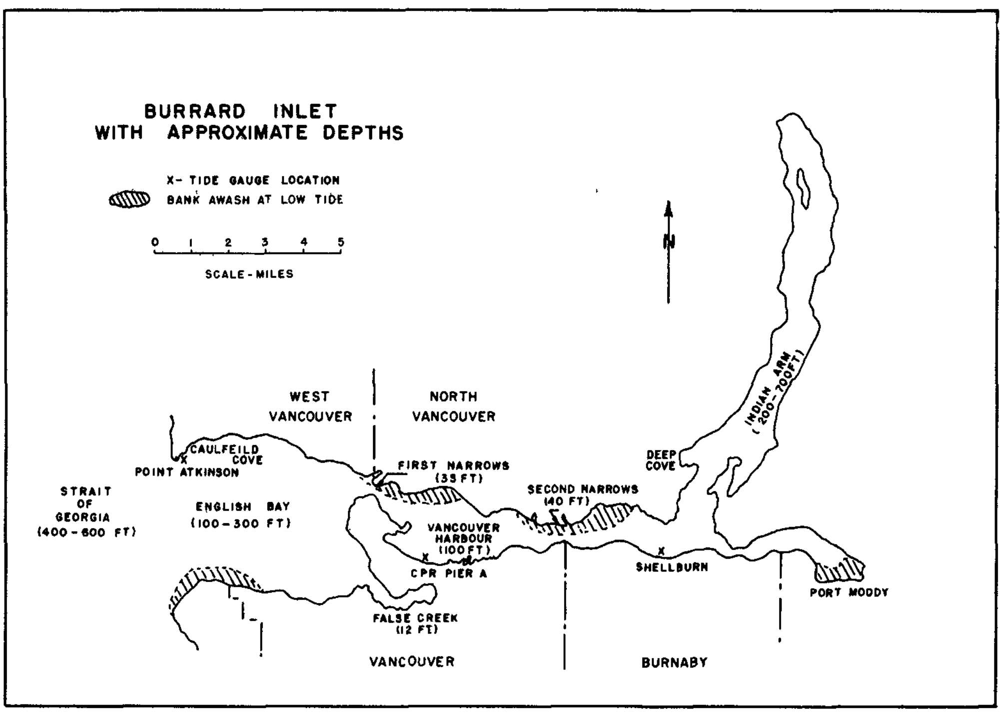

Fig. 2. Burrard inlet with approximate depths.

SIMPLIFIED SOLUTION

It is a help in the understanding of the flow to consider first the simplest approximation than can be made to the above equation. If it is assumed that friction in the channel is negligible and that the change in elevation along the length of the channel is inconsequential then Eq. (3) loses significance and Eq. (2) can be solved directly by substituting h_o for h_1 . The following simplified solution results.

$$V = -\frac{2\pi H}{T}\frac{A}{a}\sin\frac{2\pi t}{T} \qquad (5)$$

Comparing Eqs. (5) and (4) shows that the current is 90 degrees out of phase with the tide. That is the maximum flood and ebb flows occur exactly at slack tide. Of course, one of the assumptions was that $h_o = h_1$ and hence the tide curve in the bay has the same range and occurs at the same time as the tide in the ocean.

LINEAR SOLUTION

A combination of Eqs. (3), (4) and (5) gives the following equation for the velocity in the entrance channel:

$$\frac{dV}{dt} = \frac{gH}{L}\cos\frac{2\pi t}{T} - f\frac{V}{8}\frac{|V|}{R} - \frac{ga}{LA}\int_o^t V\,dt \qquad (6)$$

There is not available a method which will give an explicit solution to Eq. (6). However, if the friction term can be linearized a solution is easily obtained. This process is being widely used and Einstein discusses the consequences of such an assumption. The absolute value of V , i.e. $|V|$ is assumed constant throughout the tide cycle, the size depending upon either the best overall accuracy desired or the part of the cycle where best accuracy is desired. Thus verification of the results is required if the solution is to be of great use. For the type of engineering problem under consideration, that of navigation, the maximum currents are of primary interest so the value of $|V|$ is chosen such that the theory will match observed values at half tide.

The solution of equation (6) is easily obtained by the Laplace transform method(3) with the condition imposed that it must have a definite period. Were there a transient effect then the mean level in the bay would increase (or decrease) steadily. This is difficult to envision physically. It is found that the period of the current in the channel and the tide in the bay is identical to that in the ocean. Following are the exact expressions for these quantities:

$$V = - \frac{C_1}{C_3 - 1} \cos\alpha \quad \sin\left(\frac{2\pi t}{T} - \alpha\right) \tag{7}$$

$$\frac{h_1}{H} = \frac{C_3 \cos\alpha}{C_3 - 1} \cos\left(\frac{2\pi t}{T} - \alpha\right) \tag{8}$$

in which the phase shift angle α is defined by

$$\tan\alpha = \frac{C_2}{C_3 - 1} \tag{9}$$

and the following constants are used for convenience

$$C_1 = \frac{T\,g\,H}{2\pi\,L} \qquad C_2 = \frac{f\,|V|\,T}{2\pi \times 8\,R} \qquad C_3 = \frac{g\,T^2\,a}{4\pi^2\,L\,A}$$

The biggest difference from the simplified solution is the phase shift defined by equation (9). It is the direct result of the inclusion of friction in the equation of motion. The tangent of the phase shift angle is proportional to the constant describing the frictional coefficients of the channel. It can be seen from equations (7) and (8) that the tide lags that of the ocean by α and V lags tide of the ocean by $\frac{\pi}{2} + \alpha$. In other words, the friction delays the tidal action by a certain amount and the current is normally $\pi/2$ out of phase with the impressed tide.

Another unusual property of the above solution is the attenuation factor shown by equation (8). This term, which is commonly used in electrical engineering, is here used to define the ratio of the tide range in the basin to that in the ocean. Two factors influence it, the first and most readily understood is friction. In equation (8) the term $\cos\alpha$ represents the effect of friction in reducing the tide range in the basin. The greater the friction the smaller will be the basin tide. The other factor is related to the response to the basin to the impressed tide wave. The term $C_3/(C_3 - 1)$ will always be greater than 1. In many cases C_3 is much larger than 1 and the term reduces to unity. If $C_3 = 1$ then resonance results and equations (7) and (8) produce infinite values. In this case the solution is no longer valid; the terms having been neglected in the basic equations would put physical limits on the current and tide level. However, for C_3 not much larger than 1 the term can have values significantly greater than unity and the equations still describe the flow. In such a case the tide range in the basin will be larger than that in the ocean. Several examples of this phenomena have been given by Caldwell (1) and it appears for each case that C_3 would have a value commensurate with this analysis.

APPLICATION TO BURRARD INLET

GENERAL DESCRIPTION OF THE INLET

As mentioned previously, the Inlet consists of three wide, deep basins the outer one of which, English Bay, is connected to the Strait of Georgia without an entrance channel. Between English Bay and Vancouver Harbour the channel is called the First Narrows. The Second Narrows co nects the Harbour to the Upper Inlet which is forked into tw arms - Indian Arm and Port Moody. The depths of these basi and channels are indicated on Figure 2. The Narrows are the result of material being deposited in the inlet in recent geological time. This has resulted in a broad bench, dry a low tide, being formed on the north shore of each channel. The Capilano River which deposited the debris in the First Narrows has been diverted to empty directly into English Ba Lynn Creek and the Seymour River which empty into the Secon Narrows do not carry sediment loads under ordinary conditio However, after a heavy rainfall they rise rapidly and can move everything from clay to small boulders, which is deposited in the Narrows. The shorelines between the Narrows and along Indian Arm are for a large part deposits of unconsolidated glacial tree and alluvial deposits that form a dee cover over bedrock. About half the length of the south sho consists of outcrops of sandstones, shales and conglomerate which contribute steep slopes that are ideal for the construction of docks. Where not exposed the rock is covered by glacial till which also forms steep banks but is more subject to erosion.

Inspection of the tide curves in the three basins shows certain peculiarities not found in estuaries such as river mouths. The characteristics of these curves suggest immediately the usefulness of the simple derivation above. It is known for a start that the effect of the fresh water runoff is negligible. Rarely do the total discharges of al the rivers exceed one percent of the maximum flood discharge through the First Narrows. Fresh water discharge into the bay could be included in the above analysis and still retai an explicit solution. The only inconvenience is that of increased algebra and the difficulty of seeing the significan of the extra terms in the equation.

In the Indian Arm there is little mixing of the wate from the surface to the bottom. As a result the Upper Inle is stratified, i.e. the fresh water remains in the upper layers. Flow through the Second Narrows being highly turbulent mixes the water entering from either side and consequently Vancouver Harbour does not have such a stratification.

Within any of the basins the tide curves at all stations are virtually identical (Ref. 4). However, there are appreciable differences across each of the Narrows. Note, for example, Figure 3 which presents data pertaining to the First Narrows. The tide curve at either end has the same amplitude, that is the attenuation factor is one. (Attenuation factor for a channel is defined as the ratio of curve amplitude at the landward end to that at the seaward end.) At Pier A the shape of the curve is identical to that at Caulfeild Cove except for a displacement on the time scale. In mathematical terminology this displacement is called a phase shift. Similar relationships hold for both attenuation and phase shift for the tide curves on either side of the Second Narrows (note Fig. 4). For both Narrows the phase shift appears to be nearly constant for the periods of rapid rise and fall of the water surface. During these periods the maximum flows occur. The discharge curves are roughly sinusoidal in shape but are neither the same shape nor the same phase as the tide curves. In fact the discharge curves appear to be about 90 degrees out of phase, that is, the maximum flow occurs at half tide and slack water coincides with high and low tide.

The pattern of surface currents is typical of a long, narrow inlet. This has been measured by the Canadian Hydrographic Service (Ref. 5) for English Bay, First Narrows and Vancouver Harbour. The currents are all parallel to the axis of the inlet except in the widening portions where the flow is unstable. The instability combined with the geostrophic acceleration (a consequence of the earth's rotation) is probably the reason that the bulk of the flow is along the right bank in each instance. For example, Vancouver Harbour has expansions at both ends and it can be reasoned that this should cause a continuous counter-clockwise rotation to the water in the harbour throughout the tide cycle. This rotation has actually been measured (note Ref. 5). The overall current pattern thus does not appear to be influenced by the local form of the shoreline and any changes to it should be predictable from changes in the mean flow. This assumption was made in evaluating the proposed improvements to the Narrows.

APPLICATION OF EQUATIONS TO BURRARD INLET

Basic Equations - For each basin an equation of continuity similar to Eq. (1) can be written. It can be briefly stated: The rate of rise of a basin multiplied by the surface area is equal to the sum of the discharges into that basin. It is convenient to use in the nomenclature subscripts referring specifically to a given channel or bay. The subscripts 0, 1 and 2 are used for English Bay, Vancouver Harbour and the Upper Inlet respectively, a and b are used for reference to the First and Second Narrows respectively. With this nomenclature the continuity equations are:

$$V_b\ a_b = A_3 \frac{dh_3}{dt} \tag{10}$$

and

$$V_a\ a_a - V_b\ a_b = A_2 \frac{dh_2}{dt}$$

Upon combining these equations with dynamic equations for each channel identical to Eq. (3) there results the followi basic equations for the flow.

$$\frac{d^2 V_a}{dt^2} + \frac{f_a}{8R_a} \frac{d}{dt} (V_a|V_a|) + \frac{g\ a_a}{L_a\ A_2} V_a - \frac{g\ a_b}{L_a\ A_2} V_b = \frac{g}{L_a} \frac{dh_o}{dt} \tag{11}$$

and

$$\frac{d^2 V_b}{dt^2} + \frac{f_b}{8R_b} \frac{d}{dt}(V_b|V_b|) + \frac{g\ a_b}{L_b} \left(\frac{1}{A_2} + \frac{1}{A_3}\right) V_b - \frac{g\ a_a}{L_b A_2} V_a = 0 \tag{12}$$

Tide Curve for English Bay - One essential boundary condition on Eqs. (11) and (12) which must be specified before proceeding further is the expression for h_o which is given the apt title of the forcing function in applied mathematics. The simple cosine expression, Eq. (4) will be used to derive the linear solution of Eqs. (11) and (12) although this is not a good representation of the natural tide curve (see Fig. 3). This curve, which is assumed to be the level variation measured at Caulfeild Cove, is typical of those encountered on the Pacific Coast. There are two high and two low tides every day, the relative heights of which vary from day to day. Maximum currents are found wher the greatest difference between high-high and low-low occurs The curve for Caulfeild Cove can be described by a series of cosine terms by either a Fourier representation in terms of a basic period and its harmonics or by a series of standard tidal periods. Each of these methods is a linear combination of cosine terms and hence the solution for such a forcing function can be obtained by the same linear combination of solutions for Eq. (4). In other words, solutions obtaine by using the simple cosine expression combine by superposition. This procedure greatly simplifies working with any observed curve.

Solution of Basic Equations - The simplified solution for two basins in series is identical to Eq. (5). In terms of the above nomenclature the velocities in the Narrows are:

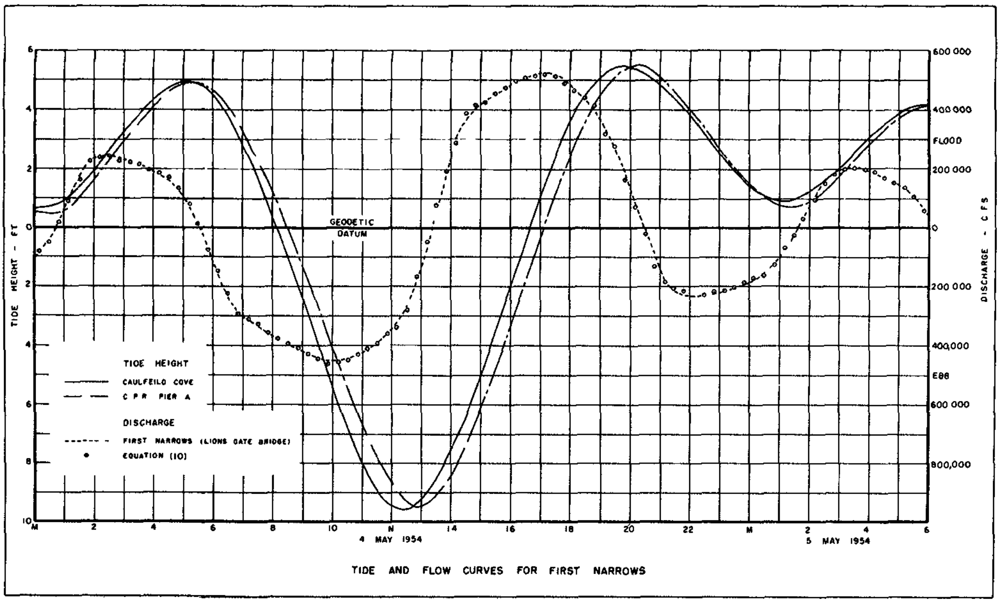

Fig. 3. Tide and flow curves for First Narrows.

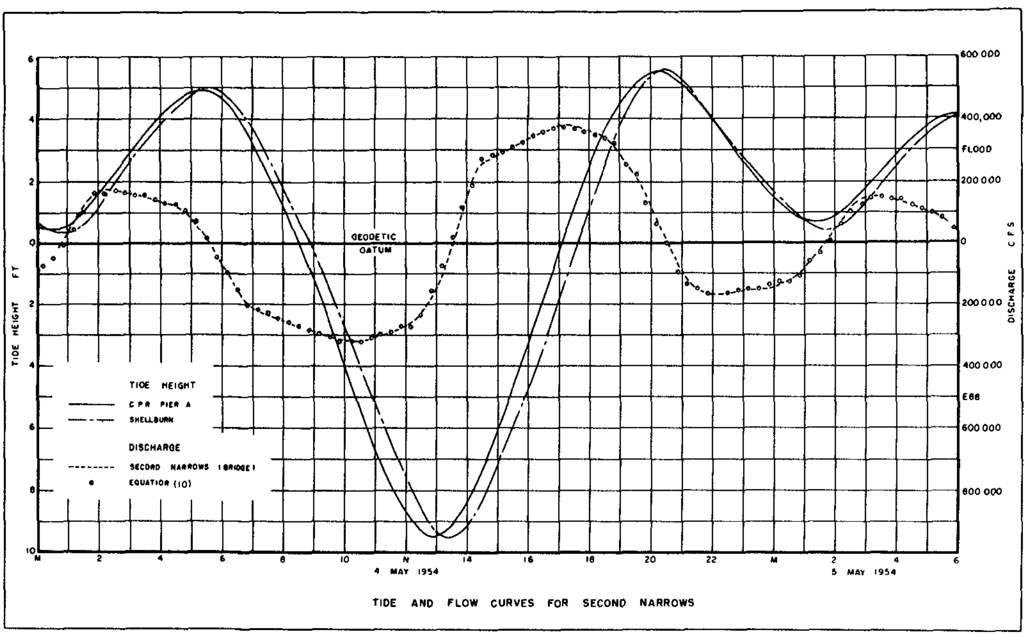

Fig. 4. Tide and flow curves for Second Narrows.

$$V_a = -\frac{2\pi H}{T}\,\frac{A_1 + A_2}{a_a}\,\sin\frac{2\pi t}{T} \qquad \text{(1}$$

$$V_b = -\frac{2\pi H}{T}\,\frac{A_2}{a_b}\,\sin\frac{2\pi t}{T} \qquad \text{(1}$$

The linear solution is, however, much more complex than for the single basin. Using Laplace Transform methoc (ref. 3) the following expressions are obtained for velocities and tide curves:

$$V_a = -\frac{C_1\,(C_7 - 1)}{(C_3 - 1)\,(C_7 - 1)\,[1 - \tan\alpha_3\,\tan\alpha_5] - C_4\;C_5}\;x$$

$$\frac{\cos\alpha_2}{\cos\alpha_3}\;\sin\left(\frac{2\pi t}{T} - (\alpha_2 - \alpha_3)\right) \qquad \text{(]}$$

$$V_b = -\frac{C_1\;C_5}{(C_3 - 1)\,(C_7 - 1)\,[1 - \tan\alpha_3\,\tan\alpha_5] - C_4\;C_5}\;x$$

$$\cos\alpha_2\;\sin\left(\frac{2\pi t}{T} - \alpha_2\right) \qquad \text{(:}$$

$$\frac{h_1}{H} = \frac{C_3\,(C_8 - 1)}{(C_3 - 1)\,(C_7 - 1)\,[1 - \tan\alpha_3\,\tan\alpha_5] - C_4\,C_5}\;x$$

$$\frac{\cos\alpha_2}{\cos\alpha_4}\;x\;\cos\left(\frac{2\pi t}{T} - (\alpha_2 - \alpha_4)\right) \qquad \text{(}$$

$$\frac{h_2}{H} = \frac{C_3\;C_8}{(C_3 - 1)\,(C_7 - 1)\,[1 - \tan\alpha_3\,\tan\alpha_5] - C_4\,C_5}\;x$$

$$\cos\alpha_2\;x\;\cos\left(\frac{2\pi t}{T} - \alpha_2\right) \qquad \text{(}$$

in which all of the constants C are coefficients of equations (11) and (12), namely:

$$C_1 = \frac{Tg\ H}{2\pi L_a} \quad ; \quad C_2 = \frac{f_b\ T\ |V_a|}{2\pi \times 8\ R_a} \quad ; \quad C_3 = \frac{g\ T^2\ a_a}{4\pi^2\ L_a\ A_2} \quad ;$$

$$C_4 = \frac{g\ T^2\ a_b}{4\pi^2\ L_a\ A_2} \quad ; \quad C_5 = \frac{g\ T^2\ a_a}{4\pi^2\ L_b\ A_2} \quad ; \quad C_6 = \frac{f_b\ T\ |V_b|}{2\pi \times 8\ R_b}$$

$$C_7 = \frac{g\ T^2\ a_b}{4\pi^2\ L_b}\left(\frac{1}{A_2} + \frac{1}{A_3}\right); \quad C_8 = \frac{g\ T^2\ a_b}{4\pi^2\ L_b\ A_3} \quad ;$$

and the angles defining phase shifts are defined by:

$$\tan\alpha_2 = (\tan\alpha_5 + \tan\alpha_3)\Big/\left[1 - \tan\alpha_5 \tan\alpha_3 - \frac{C_4\ C_5}{(C_3 - 1)\ (C_7 - 1)}\right] \qquad (19)$$

$$\tan\alpha_3 = \frac{C_6}{C_7 - 1}\ (20); \qquad \tan\alpha_4 = \frac{C_6}{C_8 - 1}\ (21);$$

$$\tan\alpha_5 = \frac{C_2}{C_3 - 1} \qquad (22)$$

Examination of these equations shows a rather unexpected relationship between the phases of the several tides and currents. There is a basic delay α_2 between the tide in English Bay and that in the Upper Inlet. The tide in Vancouver Harbour precedes that in the Upper Inlet by α_4 but there is not a simple expression for the phase difference between the tide in the Harbour and in English Bay. For the currents, that in the Second Narrows is exactly 90 degrees out of phase with the tide in the Upper Inlet, while that in the First Narrows has a different relationship to the tide in the Upper Inlet than all other of the variables. The current in the First Narrows is neither in phase with the tide in English Bay nor with that in Vancouver Harbour.

<u>Verification of Mathematical Model</u> - The only data of high enough quality for this verification are those obtained by the University of British Columbia staff in their tidal

Table I

Constants for Burrard Inlet

Symbol	Quantity	Value
	FUNDAMENTAL	
T	Lunar day	39428 sec
$\frac{T}{2\pi}$		14210 sec.
g	Acceleration of gravity	32.16 ft./sec.^{2}
A_b	Water surface area - Lions Gate bridge to Second Narrows bridge	179 4 x 10^6 ft.^{2}
A_c	Water surface area - Upper Inlet	452.7 x 10^6 ft.^{2}
L_1	Length of First Narrows	7000 ft.
L_2	Length of Second Narrows	6000 ft.
a_1	Cross-section area of First Narrows below mean tide	70,400 ft.^{2}
a_2	Cross-section area of Second Narrows below mean tide	48,500 ft.^{2}
R_1	Hydraulic radius of First Narrows at mean tide	44.4 ft.
R_2	Hydraulic radius of Second Narrows at mean tide	40.4 ft.

DERIVED

SYMBOL	VALUE	SYMBOL	VALUE
C_3	364	C_4	251
C_5	425	C_7	409
C_8	116		

Table II

Constant associated with friction terms in linear solution.

Date 1954	Current	AVERAGE PHASE SHIFT $\frac{T\alpha_2}{2\pi}$ hr.	$\frac{T\alpha_3}{2\pi}$ hr.	$\frac{T\alpha_4}{2\pi}$ hr.	$\frac{T\alpha_5}{2\pi}$ hr.	FRICTION FACTOR f_a	f_b
4 May	Slow-Flood	0.50	0.062	0.22	0.079	0.058	0.049
	Fast-Ebb	1.17	0.155	0.55	0.189	0.049	0.046
	Fast-Flood	1.00	0.147	0.52	0.141	0.045	0.044
	Slow-Ebb	0.40	0.056	0.2	0.056	0.045	0.046
5 May	Slow-Flood	0.45	0.065	0.23	0.062	0.052	0.053
	Fast-Ebb	1.18	0.155	0.55	0.192	0.050	0.046
	Fast-Flood	1.02	0.155	0.55	0.141	0.044	0.051
	Slow-Ebb	0.40	0.059	0.21	0.054	0.045	0.049
6 May	Slow-Flood	0.40	0.056	0.2	0.056	0.047	0.053
					AVERAGE	0.049	0.049

NOTE: α_2 and α_4 measured from tide curves. All other quantities computed from α_2, α_4 and constants in Table I.

survey (Ref. 4). Over the past 50 years there have been scattered measurements of tides and currents in the Inlet but these were not directly comparable because of the wide variation in the tide occurring at the seaward end. An accurate description of the tidal flow requires simultaneous measurement of the tide at a large number of locations covering all parts of the inlet together with check metering of the discharge. Such a survey was made from May 4 through May 6, 1954, and the information obtained therefrom has been accurately checked, correlated, and issued as Ref. (4). For the complete period which included two tide cycles the discharge in both Narrows was computed by the cubature method. Because the fresh water inflow is negligible the resulting figures are exact. Throughout this report this information will be used to evaluate the mathematical formulation.

The first step is to check the simplified continuity equation. With the surface area given in Table I and the graphical solution for the slope of the tide curve the Second Narrows discharge Q_b was evaluated and is plotted on Figure 4. Considering the accuracy of a graphical solution the agreement is excellent. A similar computation was made for Q_a, the First Narrows discharge. The representative tide for Vancouver Harbour was chosen as that at Pier A. The resulting values are plotted on Figure 3 whereon it can be seen that the agreement is very good. The largest discrepancies between the approximate and exact values are of the order of two percent. Therefore it is shown that the continuity assumption is valid.

The complete solution is more difficult to verify because the constant, describing the friction effect, cannot be directly measured in the inlet. The best procedure appears to be the evaluation of the friction factor from the tide survey assuming that all of the geometrical factors are accurately determined. This too is difficult to perform directly. The reason is obvious when the equations for phase shift (which are expressions of the friction effect) are compared to the measured tide curves. As an example, consider the phase shift across the Second Narrows (see Fig. 4). The time difference between the two curves is expressed in the linear solution by the angle α_4. It is readily shown that this angle is small enough that its tangent is equal to the angle itself. Furthermore, in equation (21) the constant $C_8 = 364$, hence an error of less than one percent is introduced by neglecting the factor one in the denominator. With these assumptions and writing the phase shift time $\frac{T}{2\pi}\alpha_4$ in place of the phase shift angle α_4 the following equation results:

$$\frac{T}{2\pi}\alpha_4 = \frac{f_b \; |V_b| \; L_b \; A_3}{8g \; R_b \; a_b} \qquad (23)$$

The phase shift time is the quantity found directly by measuring the time difference between the curves on Figure 4. Throughout the analysis it has been assumed that all terms the right-hand side of equation (23) were invariable with t tide conditions. This means that the phase shift time is constant and the Shellburn curve should be displaced a constant amount from the Pier A curve. This, however, is not the case as is clearly shown on Table II wherein measured values of $\frac{T}{2\pi}\alpha_4$ are entered. There is actually a variati from 0.2 hr. to 0.55 hr. Such a result does not completely negate a linear solution if a slightly different interprete tion is now put on it.

A more realistic approach is to divide the tide curv into different periods between the times of slack water. F definition these have been given the names of slow or fast, flood or ebb, to give the same sort of terminology applied the tide curve peaks, e.g., high-low tide. For each perioc all terms in equation (23) are assumed constant but the val of V_b is taken as the maximum occurring in this particul period and not that for the day. P_b is assumed as the me depth at the time V_b is a maximum. Following this procec the friction factor f_b has been evaluated and is entered the last column in Table II. There is a spread of roughly percent in the values compared to 150 percent in the phase shift time. This result is extremely good when it is reali that a phase shift is difficult to determine from the curve to an accuracy better than 10 percent. It would not be su prising if f_b were different for ebb and flood flows. Th different geometry as the flow reversed should give differe entrance and exit losses. No consistent variation shows i the results so it is assumed that the same value holds for each case. Probably a difference does exist but it is sma than the scatter of the results.

For the First Narrows a simplified equation similar to equation (23) cannot be written. However, an analysis similar to that described above can be made starting with phase shift between Caulfeild and Shellburn $\frac{T}{2\pi}\alpha_2$. This quantity has the same extreme variations as $\frac{T}{2\pi}\alpha_4$. Next equations (20) and (19) are solved in succession to yield results for $\frac{T}{2\pi}\alpha_3$ and $\frac{T}{2\pi}\alpha_5$ other phase shift times of significance. Finally, f_a is derived from equation (22) α_5 (see Table II). Again it is noted that the spread of values is much less than might be expected. A chance resu is the average value of f for each narrows being 0.049. This is not significant. Because of the very close agreem of friction factor throughout the tidal cycles it appears that the above procedure is the best method of interpreting the linear solution.

A further confirmation of this procedure is the prediction of phase for the flow curves for the First and Second Narrows. Figures 5 and 6 are plots of the linear and simplified solutions for Q_a and Q_b along with curves resulting from the tide survey. From this it is seen that the linear solution (using the phase shift times found in Table II) is in very close agreement on the time base with the exact curve. Such is taken as further proof of the accuracy of the adopted procedure because the curves on Figures 5 and 6 depend entirely on values measured from the tide curves. The simplified solution, on the other hand, is seriously in error because it does not contain an expression for phase shift but assumes that events throughtout the Inlet occur at the same instant.

EFFECTS OF CHANNEL IMPROVEMENTS

The primary reason for developing the above mathematical model and performing the lengthy observations and analysis on Burrard Inlet was the need to evaluate proposals for improving the navigation channels through the Narrows. The existing channels are too narrow and shallow for the vessels anticipated in the very near future. The major improvement is a proposed removal of about 8 million cubic yards of material from the north side of the First Narrows. This would double the width of the navigation channel at 40 feet below low water. In addition the sharp turn required of vessels entering the Narrows would be considerably reduced. The design of this enlargement was dictated by navigation requirements and not hydraulic considerations. Because of the resulting change in channel configuration it was concluded that the mathematical formulation would not accurately predict the maximum current or direction. Consequently it was decided to construct a hydraulic model of the First Narrows which could be run in a steady state at maximum flood and ebb flows. This model was built and operated by the Department of Civil Engineering of the University of British Columbia, under the supervision of Prof. E. S. Pretious. The horizontal scale was 1:400 and the vertical scale 1:100. Data from the tidal survey of the Inlet was used in the verification and the above mathematical development used to predict the discharge required after the dredging programme was completed. Model results were entirely satisfactory and showed that the enlargement would improve the hydraulic characteristics much more than the mathematical solution indicated. In evaluating the influence of this enlargement on currents in the Second Narrows the model results were used to define a new friction factor for the First Narrows. This

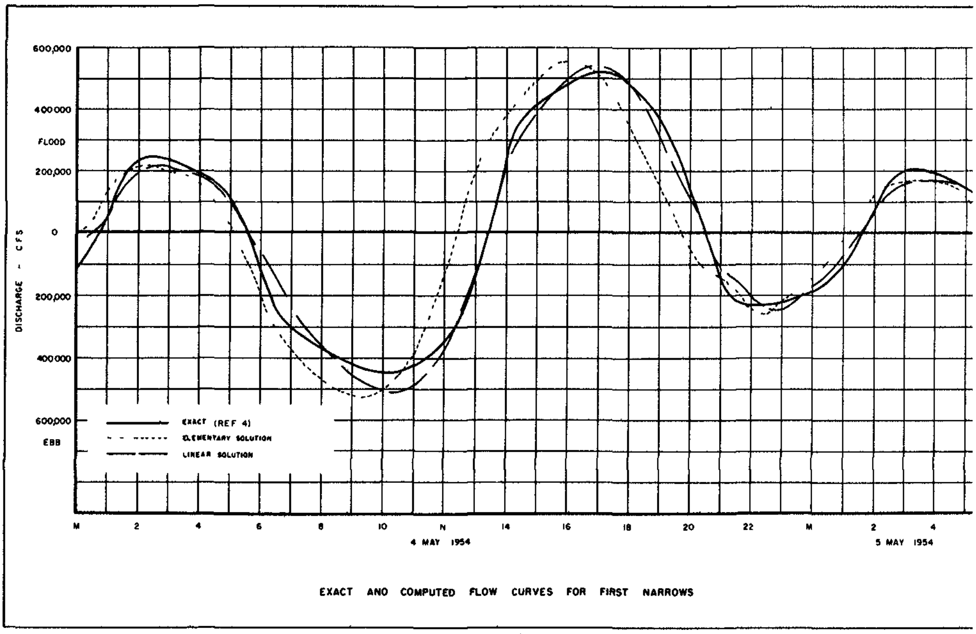

Fig. 5. Exact and computed flow curves for First Narrows.

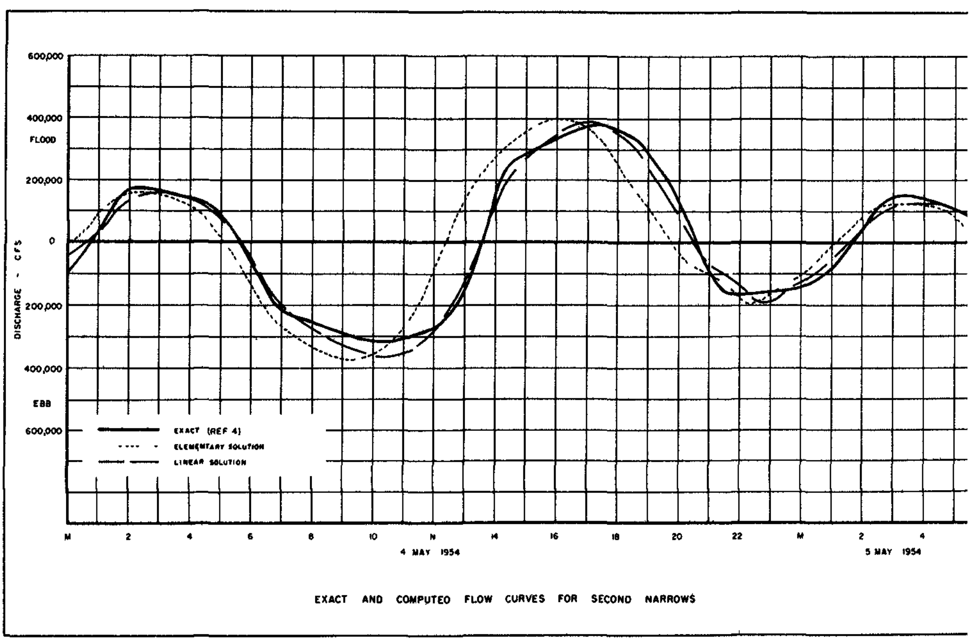

Fig. 6. Exact and computed flow curves for Second Narrows.

was inserted in equations (16) and (18) and it was found that the discharge and velocity in the Second Narrows would increase by about 1% . Other predictions were made regarding the phase differences in the tide curves but these have no practical significance. It is hoped to check this analysis by further measurements in the Inlet when the enlargement is completed.

As part of this study other possible improvements to the harbour were also evaluated. The mathematical solution was used exclusively but it was recognized that the results would not be so accurate without hydraulic model studies. Various schemes for widening and deepening the Second Narrows were considered. It was found that in every case the velocities in this channel were reduced with the accompanying increase of velocities in the remainder of the Inlet being less than 1% .

The most spectacular reduction in currents would be obtained by closing the Second Narrows. Effectively this removes over 70% of the tidal volume of the Inlet and so the discharge and velocities in the First Narrows would be reduced to about 30% of the values now existing.

CONCLUSION

A mathematical formulation for flow in a tidal basin with a constricted inlet has been developed. When applied to Burrard Inlet it has been found that the formulation gives an accurate description of the mean velocity and tide levels in all locations.

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CHAPTER 32

HYDRAULIC STUDIES IN ESTUARIES

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There is a region in estuaries where water velocities are far below critical, and where sea level variations greatly affect the hydraulic conditions, but where still distinct channels exist.

In this region the water levels are usually as much influenced by tides and meteorological conditions as by river discharges. Floods may arise from high discharges as well as from storm surges. In this study relationships are presented where hydrological and meteorological factors are included.

The affected area is treated as a system of channels with more or less unidirectional flow in each. Frequently the flow conditions vary considerably over the length or width of a sea or river arm. The determination of hydraulic parameters is, therefore quite difficult. In this paper, methods for a rational estimate of parameters have been shown.

Using these parameters, the influences of channel topography, river flow and meteorology are considered in a system of equations. These equations are transposed to an applicable form for integration by finite differences, which in dynamic cases could be carried out along characteristics.

FACTORS AFFECTING WATER LEVELS

The presented methods were deduced to study the effects of extreme hydrological conditions and also to estimate the influence of various river training works and other changes in the hydraulic properties of river channels.

It is suggested, that the river discharge is considered by inserting the discharge as a boundary condition at a point sufficiently high upstream, so that the effect of upstream water surface elevations can be eliminated. By a similar process, the effect of the sea can be reduced to a consideration of water surface elevations as a boundary condition. In the actual area under study, meteorological conditions and hydraulic parameters are inserted which determine water surface elevations and discharges in this reach as a function of the described upstream discharge and sea levels.

These known and sought quantities are illustrated on figure 1.

Statistical methods can be used to establish the design conditions, as the upstream discharge, the sea levels and the meteorological and topographical conditions in the studied area. From this statistical information the remaining data can be calculated.

MATHEMATICAL MODELS

In many cases, floods in estuaries can be studied on a mathematical model where the affected area is treated as a system of channels. The first step towards this approximation is shown on figure 2 where the same area is represented as on figure 1.

Some parameters might be required at sea to describe different levels at the mouths of various channels of the delta of the river. In many cases, however, these discharges and variations of water levels can be neglected and the whole system of channels can be treated as a linear system as shown on figure 3.

In each of the linearized channels flood routing procedures will be applied which are expanded to include the influence of flow, barometric pressure and winds. Tidal affects may be added if necessary. In the following treatment, ice is neglected since it poses a different kind of problem.

PLANNING OF MATHEMATICAL TREATMENT

The established flood routing procedures seem to require some amendments for this application.

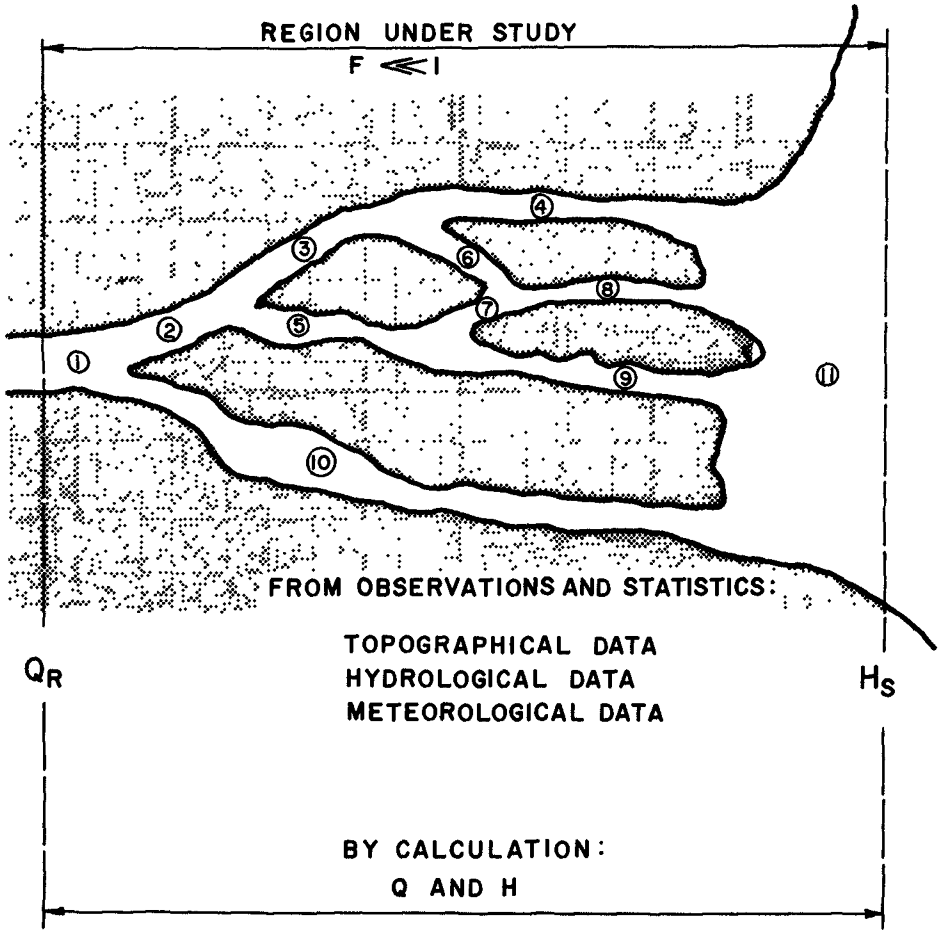
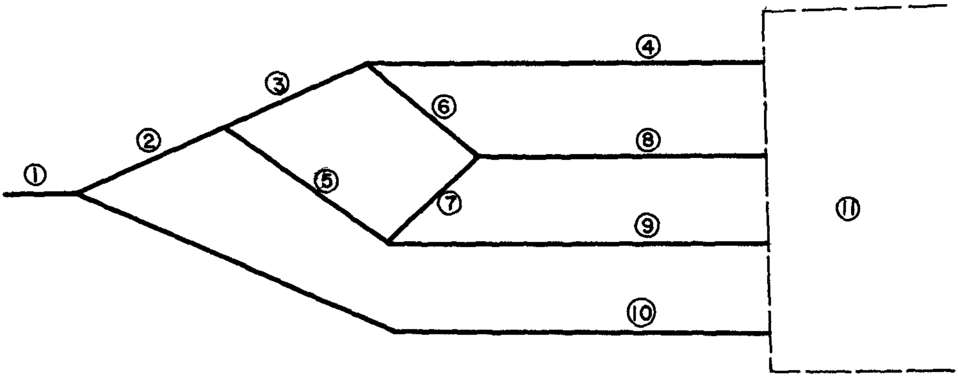


FIGURE I.
FACTORS AFFECTING WATER LEVELS.



NUMBERS INDICATE CHANNELS WHICH REPRESENT RIVER BRANCHES WITH THE SAME NUMBER AS IN FIGURE 1.

FIGURE 2.
STRAIGHT CHANNELS REPRESENTING RIVER BRANCHES.

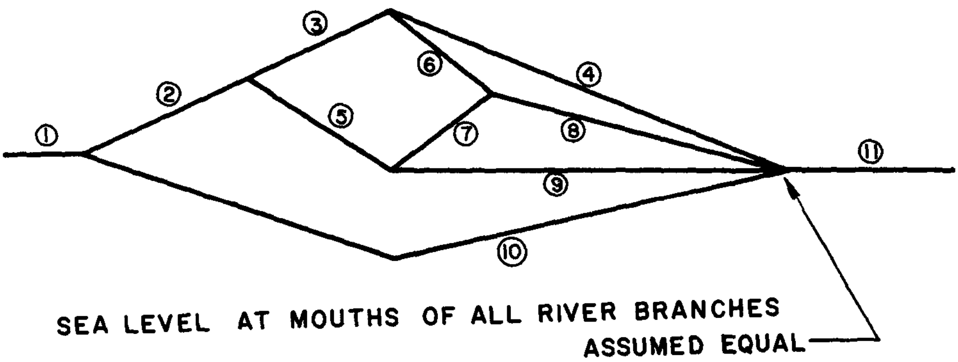


FIGURE 3.
APPROXIMATION BY A SYSTEM OF CHANNELS.

In systems which have been generally used, hydraulic parameters are usually determined by fitting data which will reproduce actual conditions. In order to speed the verification procedure and to permit a better understanding of the causes of floods, it was felt that rules for a direct computation of the parameters would be helpful. These would then need a fine adjustment only and both speed and accuracy would be gained. Such a computation procedure has been established by an integration of general flow formulae over the width of a water course.

A special application of flood routing and storm surge investigations is the study of the influence of regulating and training works. A reliable procedure of calculating parameters should add considerably to various empirical methods for the estimate of parameters in changed conditions.

Another side of the problem is the integration of the differential equations, once proper parameters are established. Peaks of floods are of special interest and peaks are probably influenced by change in river regime. It is considered, therefore, necessary to establish integration methods where truncation errors would be kept at a minimum.

NOTATIONS AND DEFINITIONS

s = length along coordinate flow line,
ds = line element along flow line which is chosen as coordinate,
$\alpha\, ds$ = line element along any arbitrary flow line,
r = transverse coordinate,
o & b = shores measured along r,
t = time,
H = water surface elevation,
D = depth of water,
f = shear at an interface,
f_f = total shear on water from flow, *
f_w = total shear on water from wind, *
g = gravity acceleration,
γ = specific weight of water,
ρ = density of water,
q = flow per unit width,
Q = water course discharge,
Ω = rotation of the earth,
φ = geographical latitude.

* Total shear on water can be expressed approximately as follows:

$$f_{tot} = f_{surface} + f_{bed} = (f_s + f_b)_w + (f_s + f_b)_f \qquad (1)$$

$$f_{tot} = f_w + f_f \, , \qquad (2)$$

where f_W is a function of wind along and f_f is a function of flow alone.

CHOICE OF COORDINATES

In the following treatment a curvilinear system of coordinates is used. Longitudinal coordinates are taken along lines where $\Delta Q/Q$ = constant. These coordinates represent some kind of flow lines. Transverse coordinates are taken at right angles to the longitudinal coordinates at all points, as shown on figure 4.

The suggested coordinate system may be found by successive approximations. First, a certain distribution of flow is assumed and coordinates are drawn. Once the coordinate system is established, the formulae below may be applied, and a more accurate flow distribution may be established, which could form the basis of a second approximation.

Flow equations are written for components along the flow lines and normal to them. From these equations, laws relating water surface elevations to discharge and meteorological phenomena are established by integration over the width of a water course. As shown in notations, the functions are given in reference to lengths along a chosen coordinate flow line.

FLOW DISTRIBUTION

Flow on a vertical is assumed to be practically unidirectional in determining flow lines and friction formulae. In one-dimensional cases the barometric effects are negligible and are, therefore, omitted below. It would cause no difficulties to include the barometric terms to the system. For a study of transverse water surface and flow variations, differential flow equations are written as given below.

$$\left.\begin{aligned}
&\frac{\partial H}{\partial s} + \frac{1}{2g}\frac{\partial}{\partial s}\frac{q^2}{D^2} + \frac{1}{g}\frac{q^2}{D^2}\frac{\partial Q/\partial s}{Q} + \frac{\alpha}{Dg}\frac{\partial q}{\partial t} - \frac{f_f\alpha}{D\gamma} - \frac{f_w\alpha\sin(w,r)}{D\gamma} = 0\\
&\frac{\partial H}{\partial r} - \frac{q^2}{D^2 g}\frac{1}{\alpha}\frac{\partial \alpha}{\partial r} + \frac{q}{D}\Omega\sin\varphi - \frac{f_w\cos(w,r)}{D\gamma} = 0\\
&\frac{\partial H}{\partial t} + \frac{1}{\alpha}\frac{\partial q}{\partial s} = 0
\end{aligned}\right\} \quad (3)$$

For the determination of flow distribution, it is assumed that bottom friction and gravity terms dominate. In most cases, these assumptions lead to a good estimate of parameters. This gives

$$\left.\begin{aligned} \frac{\partial H}{\partial s} &= \frac{f_f \alpha}{D\gamma} \\ \partial H / \partial r &= 0 \end{aligned}\right\} \tag{4}$$

By inserting friction formulae for unidirectional flow, the following expressions are obtained

$$\frac{\partial H}{\partial s} = \frac{f_f \alpha}{D\gamma} = -\frac{C_f\, q\,|q|\alpha}{D^3 g}\,, \tag{5}$$

where C_f is a slow function of depth, as

$$C_f = \frac{0.16}{\left(\ln \frac{D}{en}\right)^2}\,. \tag{6}$$

Total discharge may be expressed as

$$Q = \int q\, dr = -\left(\frac{\partial H}{\partial s}\right)^{1/2} \int \frac{D^{3/2} g^{1/2}}{C_f^{1/2}\, \alpha^{1/2}}\, dr\,, \tag{7}$$

and the relation between unit flow and total discharge is

$$\frac{q}{Q} = \frac{\dfrac{D^{3/2}}{C_f^{1/2}\alpha^{1/2}}}{\displaystyle\int \frac{D^{3/2}}{C_f^{1/2}\alpha^{1/2}}\, dr}\,. \tag{8}$$

EQUATIONS FOR TOTAL FLOW

For a study of longitudinal variations of water surface elevations as a function of total discharge, the following form of equations is written

$$\left.\begin{aligned} &\int_0^{\delta s}\int_0^{b} \frac{\partial q}{\partial t}\, dr\, ds + \int_0^{\delta s}\int_0^{b} \frac{q}{\alpha Q}\frac{\partial}{\partial s}\frac{Qq}{D}\, dr\, ds + \\ &\quad + \int_0^{\delta s}\int_0^{b} \frac{Dg}{\alpha}\frac{\partial H}{\partial s}\, dr\, ds - \int_0^{\delta s}\int_0^{b} \frac{f_f}{\rho}\, dr\, ds - \int_0^{\delta s}\int_0^{b} \frac{f_w \sin(w,r)}{\rho}\, dr\, ds = 0 \\ &\int_0^{\delta s}\int_0^{b} \frac{\partial q}{\partial t}\, dr\, ds + \int_0^{\delta s}\int_0^{b} \frac{\partial H}{\partial t}\alpha\, dr\, ds = 0 \end{aligned}\right. \tag{9}$$

By summing up the effects over the width of a water course, relationships between total discharge and certain average values of water surface elevations, wind forces and other phenomena are obtained

$$\left.\begin{aligned} &\frac{\partial Q}{\partial t} + \frac{\partial}{\partial s}\frac{Q^2}{A_e} + A_g \cdot g\frac{\partial H}{\partial s} + \frac{C_m}{A_e D_m} Q|Q| - \frac{f_w}{\rho}\cos(w,s)\, B_w = 0 \\ &\frac{\partial Q}{\partial s} + B_s\frac{\partial H}{\partial s} = 0 \end{aligned}\right\} \tag{10}$$

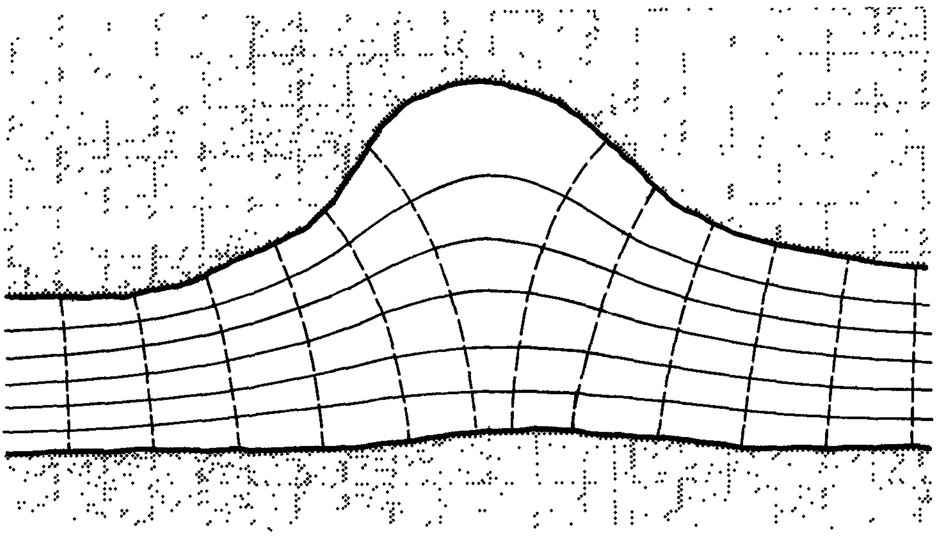
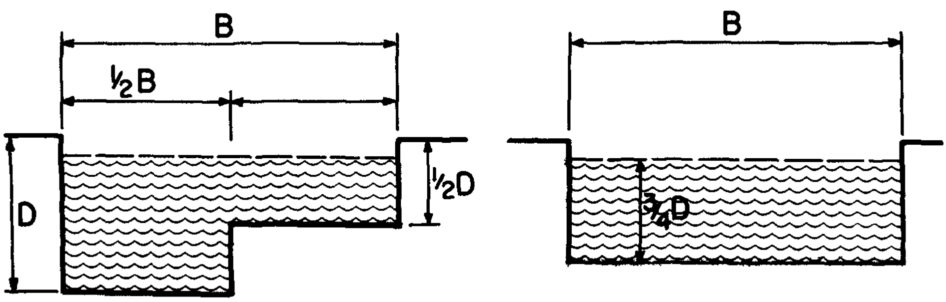


FIGURE 4.
COORDINATE SYSTEM.



$$A_1 = A_2 = \tfrac{3}{4} BD$$

$$\frac{i_1}{i_2} = 0.92$$

$$Q_1 = Q_2$$

FIGURE 5.
FLOW PROPERTIES OF VARIOUS CROSS SECTIONS.

Notations are given at the end of this section.

Above the parameters are expressed as functions of time and distance. It is more appropriate to consider them functions of time, distance and water surface elevation. This will change some of the derivatives.

The expressions may be written in the following form

$$\left.\begin{aligned} \frac{2Q}{A_e}\frac{\partial Q}{\partial s} + \left(A_g \cdot g - \frac{Q^2}{A_e^2} B_e\right)\frac{\partial H}{\partial s} + \frac{\partial Q}{\partial t} &= k \\ \frac{\partial Q}{\partial s} + B_s \frac{\partial H}{\partial t} &= 0 \end{aligned}\right\} \qquad (11)$$

where

$$k = -\frac{c_m}{A_e\, D_m} Q|Q| + \frac{f_w}{\rho}\cos(w,s)\, B_w + \frac{Q^2}{A_e^2}\frac{\partial A_e}{\partial s} \qquad (12)$$

and where the parameters are

$$\begin{aligned} A_g &= \int \frac{D}{\alpha}\, dr \\ A_e &= \frac{\left(\int \frac{D^{3/2}}{C^{1/2}\alpha^{1/2}}\, dr\right)^2}{\int \frac{D^2}{C\alpha^2}\, dr} \\ \frac{D_m}{c_m} &= \frac{\int \frac{D^2}{C\alpha^2}\, dr}{\int \frac{D}{\alpha}\, dr} \\ B_s &= \int \alpha\, dr \\ B_w &= \int \sin(s,r)\, dr \\ B_e &= \partial A_e / \partial H \end{aligned} \qquad (13)$$

It should be noted, that the above expressions were deduced under some simplifying assumptions. So are the vertical flow variations, side slopes of water surface and transverse wind force components neglected.

The effects of tidal forces and eddy losses are not shown in above formulae. Terms showing their influence may be added, however, without changing the general system.

STEADY FLOW

The following abbreviated expression is applicable to steady flow with constant discharge. Length coordinates are increasing in flow direction.

$$\frac{\partial H}{\partial s} = \frac{\frac{Q^2}{A_e}\left(-\frac{C_m}{D_m} + \frac{\partial A_e/\partial s}{A_e}\right) + \frac{f_w}{\rho}\cos(W,s)\ B_w}{A_g g - \frac{Q^2}{A_e^2} B_e} \qquad (14)$$

As above, turbulence losses are not included and require an additional term, if large enough.

The above equation permits rapid estimates of the factors influencing hydraulic parameters.

Obviously, bays at the channel shores and side channels contribute little to the heavy area, or to flow parameters, since tortuous flow paths in these areas give a high value for α.

In straight channels, the formulae for the effective area show that shallow portions of the channel contribute considerably to the effective area. Still there is a noticeable difference between the deduced parameters and formulae based on hydraulic radius. As an example, a channel is shown on figure 5 where half of the channel has double depth compared to the shallow portion.

Assuming C=constant and $\alpha = 1$, the same discharge gives an 8% smaller slope of the water surface in the left hand nonuniform section. This is reflected in the estimated parameters.

The made approximations were not chosen for direct application on wind tides in equilibrium. The assumption appears to be, however, as justified as any alternatives. For equilibrium the following equation is obtained

$$\frac{\partial H}{\partial s} = \frac{\frac{f_w}{\rho}\cos(W,s)\ B_w}{A_g \cdot g} = \frac{f_w \cos(W,s)}{\frac{A_g}{B_w}\cdot \gamma} \qquad (15)$$

DYNAMIC FORMULAE

The system of equations is well suited to a solution by final differences, integrating along characteristics. The following new system is establishe

$$\left.\begin{aligned} ds &= \beta\, dt \\ dQ - B_s \sigma\, dH &= K\, dt \end{aligned}\right\}$$
$$\left.\begin{aligned} ds &= \sigma\, dt \\ dQ - B_s \beta\, dH &= K\, dt \end{aligned}\right\} \qquad (16)$$

where K is defined above, and

$$\beta = \frac{Q}{A_e} + \sqrt{\frac{A_g g}{B_s} + \frac{Q^2}{A_e^2}\,\frac{B_s - B_e}{B_s}}$$

$$\sigma = \frac{Q}{A_e} - \sqrt{\frac{A_g \cdot g}{B_s} + \frac{Q^2}{A_e^2}\,\frac{B_s - B_e}{B_s}}$$

From these equations obviously the formulae for long waves may be obtained. The long wave celerity is

$$\beta = -\sigma = \sqrt{\frac{A_g \cdot g}{B_s}} \qquad (17)$$

which shows the slowing down caused by bays and side channels. They contribute namely substantially to B_s but hardly to A_g.

Obviously other methods are called for the treatment of other regions. Towards the sea, a two-dimensional method may be necessary, and higher upstrear friction waves may indicate different methods.

Full solutions on characteristics have been studied and adapted for electronic computers by B. Hellström, E. Asplund and the author.

CHAPTER 33

RESULTATS D'ETUDES SUR MODELE DE LA DIMENSION DES ENROCHEMENTS A UTILISER POUR LA COUPURE D'UNE RIVIERE OU D'UN ESTUAIRE A MAREE

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L'usine marémotrice projetée par Electricité de France sera située à l'embouchure de l'estuaire de la Rance, dans le golfe de Saint Malo. Ce site est particulièrement favorable, en raison de la grande amplitude des marées (13,50 m à 14 m en vives eaux exceptionnelles) et de la grande capacité de l'estuaire par rapport à sa largeur ; le volume d'eau emmagasiné par une marée de vives eaux à l'amont de l'emplacement du barrage, est de 180 millions de m^3. Le barrage aura une longueur de 800 mètres environ, correspondant à la largeur actuelle de l'estuaire, et sera établi sur des fonds rocheux situés à 10 mètres environ au-dessous des plus basses-mers. La cote supérieure de l'ouvrage sera à + 15 mètres, de sorte qu'en vives eaux le barrage n'émergera que de 1,50 m. La puissance installée sera de 350 000 kilowatts environ. La situation de l'usine marémotrice est indiquée sur la figure 1.

LES PROBLEMES POSES PAR LA COUPURE DE L'ESTUAIRE

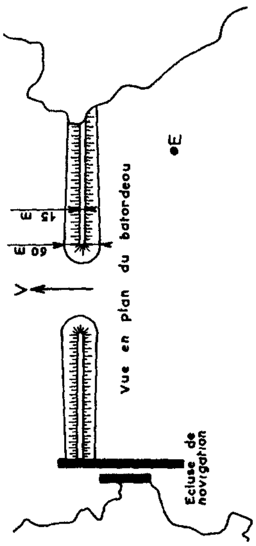
Il a été décidé de construire ce barrage à sec, à l'abri d'une enceinte étanche, dont la réalisation nécessitera un ouvrage de coupure. L'un des procédés envisagés pour cette dernière consiste à fermer progressivement l'estuaire au moyen d'un batardeau en enrochements déversés à l'avancement, simultanément à partir des deux rives. Dans ces conditions, les derniers enrochements déversés auront à supporter des courants de plus en plus forts, le moment critique se présentant quand le batardeau sera près de la phase de fermeture. Ces courants seront d'ailleurs fonction également de l'amplitude de la marée en mer, et l'on pourra profiter d'une période de mortes eaux pour réaliser la fermeture. Le problème concret posé par ce type de coupure, est :

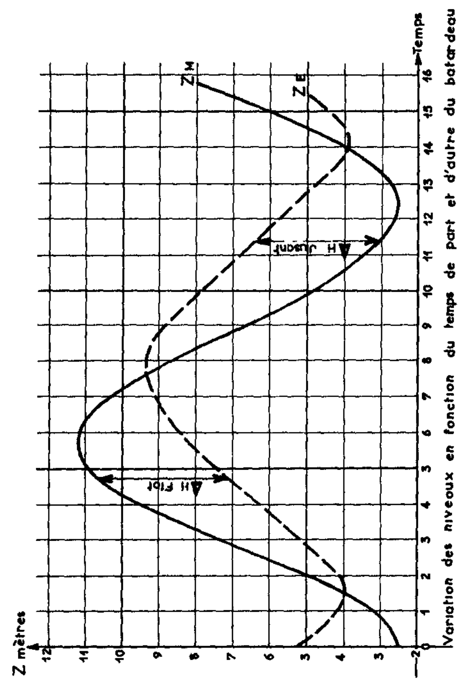
- d'une part de connaître les courants qu'on aura à vaincre au droit de la coupure, en fonction de l'avancement des travaux, et de la marée au large,

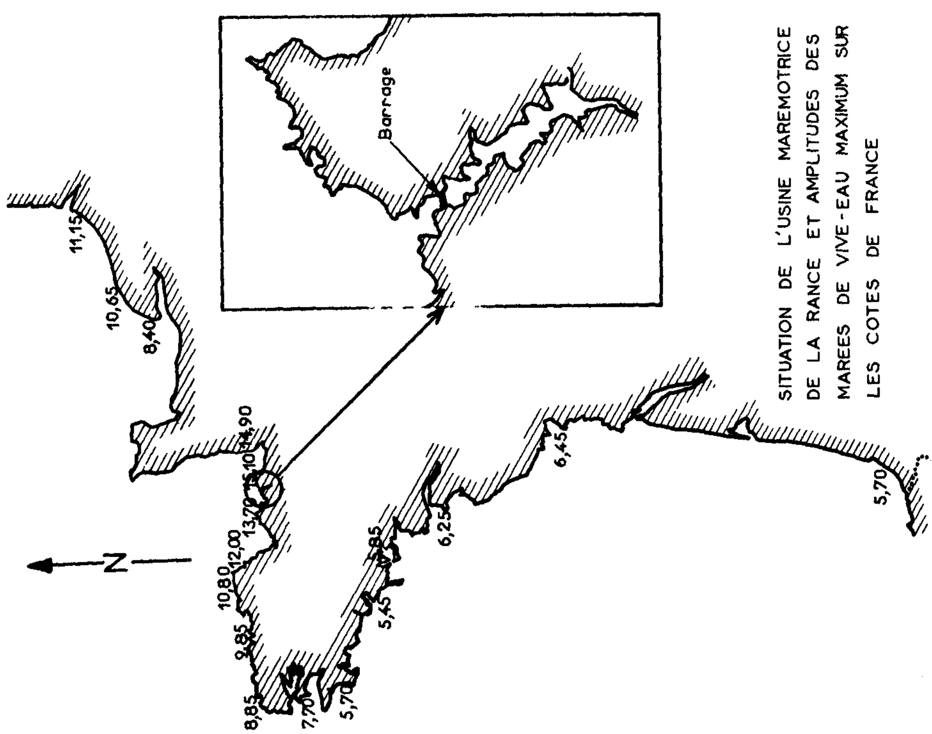
- d'autre part de prévoir les dimensions des enrochements qu'on devra utiliser pour résister à ces courants.

Ce problème a été étudié sur un modèle à l'échelle 1/150, établi sur place par le Laboratoire National d'Hydraulique. Le modèle, sans distorsion, représente l'ensemble de l'estuaire et permet de réaliser toutes les suites de marées naturelles que l'on désire.

Le problème, limité d'abord à des cas un peu particuliers, où l'on se fixait à l'avance l'étendue granulométrique des enrochements à utiliser, la cadence de déversement de ces derniers et la succession des marées correspondant à la durée de construction du batardeau, a été ensuite étendu par quelques essais complémen-







SITUATION DE L'USINE MAREMOTRICE DE LA RANCE ET AMPLITUDES DES MAREES DE VIVE-EAU MAXIMUM SUR LES COTES DE FRANCE taires. En définitive, les essais ont permis de déterminer le poids maximum des enrochements, ayant une étendue granulométrique déterminée, qui sont à la limite de stabilité dans un courant de vitesse moyenne donnée, c'est-à-dire pour une marée donnée et un avancement des travaux bien défini.

EXPOSE DES ESSAIS ET RESULTATS

Les essais ont été effectués en deux temps :

- dans une première série, le batardeau était représenté en ciment sur le modèle, de façon à éliminer le débit de percolation à travers les enrochements, et à rendre plus facile la détermination de la vitesse moyenne dans la brèche,

- dans une deuxième série d'essais, le batardeau était réalisé en enrochements représentés à l'échelle du modèle, ce qui permettait d'étudier la stabilité de ces derniers.

ETUDE DES COURANTS ET DES DENIVELLATIONS AU COURS DE L'AVANCEMENT DES TRAVAUX

La figure 2 indique l'implantation des ouvrages. Les paramètres retenus pour l'étude de la coupure sont les suivants :

s mètres carrés : section en mètres carrés de la brèche résiduelle entre les deux tronçons du batardeau de coupure, limitée par les fonds naturels et par l'horizontale de cote + 6,50 (niveau de mi-marée),

V mètres/seconde : vitesse moyenne dans la brèche, et maximum dans le temps, au cours d'une marée d'amplitude A mètres,

$\Delta H = |z_M - z_E|$ mètres : dénivellation maximum dans le temps, au cours de la même marée, existant entre 2 points M et E situés de part et d'autre des ouvrages de coupure et assez loin de ceux-ci pour que les courants y soient faibles.

La figure 3 donne les courbes de variation de V et de ΔH en fonction de s, pour $A = A_0$, $A_0 = 8{,}45$ m étant l'amplitude de la marée moyenne. On a distingué les valeurs de ΔH et V correspondant respectivement au flot (courant de remplissage) et au jusant (courant de vidange). Des courbes semblables ont été obtenues pour différentes valeurs de l'amplitude A.

On voit que la dénivellation, inexistante pour les grandes valeurs de s (correspondant au début des travaux), croît progressivement pour atteindre approximativement en fin de coupure la valeur $A_0/2$. Les dénivellations sont du même ordre au flot et au jusant.

Par ailleurs, les vitesses moyennes dans la brèche qui, pour les grandes valeurs de s, sont relativement faibles, atteignent des valeurs importantes en fin de coupure. A ce moment, on peut écrire pratiquement :

$$V = 0{,}66 \sqrt{2\, g\, \Delta H}$$

Cette relation devant n'être considérée comme valable que dans le cas particulier étudié, c'est-à-dire pour une configuration géométrique déterminée.

ETUDE DE LA STABILITE DES ENROCHEMENTS

Les enrochements utilisés ont été définis par deux paramètres :

P kg : poids maximum des enrochements du mélange utilisé,

$x = P/p$: étendue granulométrique ou rapport du poids maximum au poids minimum des enrochements du mélange utilisé, en supposant la courbe granulométrique linéaire en coordonnées semi-logarithmiques.

On a effectué des essais pour un certain nombre de valeurs de P et de x. Dans chaque cas, on diminuait progressivement la largeur de la brèche jusqu'à obtenir, pour une marée d'amplitude donnée, des conditions correspondant à la limite de stabilité.

Cette limite de stabilité, caractérisée par le poids maximum des enroche ments nécessaires, s'est révélée au cours des essais dépendre presque uniqueme de la vitesse moyenne atteinte dans la brèche, qui semble être le paramètre in portant, beaucoup plus que la dénivellation entre les deux biefs.

Les granulométries des différents mélanges d'enrochements utilisés sont indiquées sur la figure 4.

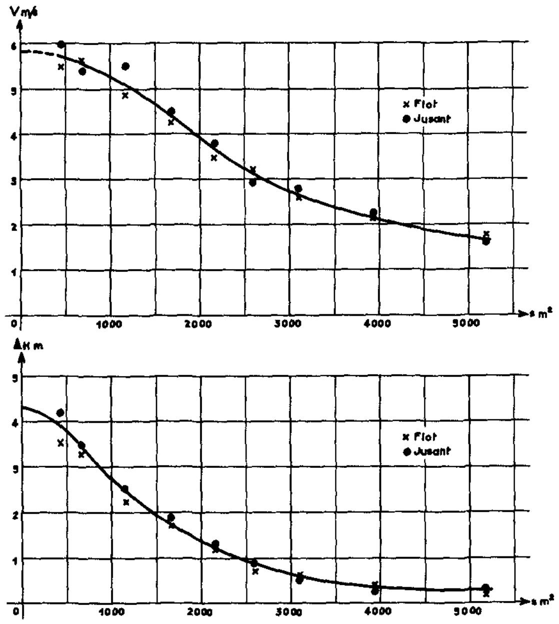
Les résultats de l'étude sont portés sur la figure 5. On voit que pour une valeur donnée de l'étendue granulométrique, x = constante, le poids maxim des enrochements à utiliser varie comme la puissance 6 de la vitesse du coura

Ce résultat confirme les lois établies pour d'autres types de coupures, notamment les coupures par tranches horizontales, lois qui sont de la forme :

$$d = k\, v^2 \qquad (1)$$

où d est le diamètre des enrochements. Toutefois, il met en évidence l'influe notable de l'étendue granulométrique, ce qui peut expliquer la grande dispers existant entre les différentes valeurs de k, dans la relation (1), suivant le auteurs.

A titre indicatif, les résultats obtenus pour le cas x = 2,5, correspon dant à un mélange d'enrochements relativement homogène, ont été portés sous



VARIATION DE LA VITESSE MOYENNE DANS LA BRECHE ET DE LA DENIVELLATION EN FONCTION DE LA SECTION DE PASSAGE A MI-MAREE POUR UNE MAREE MOYENNE (AMPLITUDE 8,45 m)

Fig. 3

COURBES GRANULOMETRIQUES DES MELANGES D'ENROCHEMENTS UTILISES

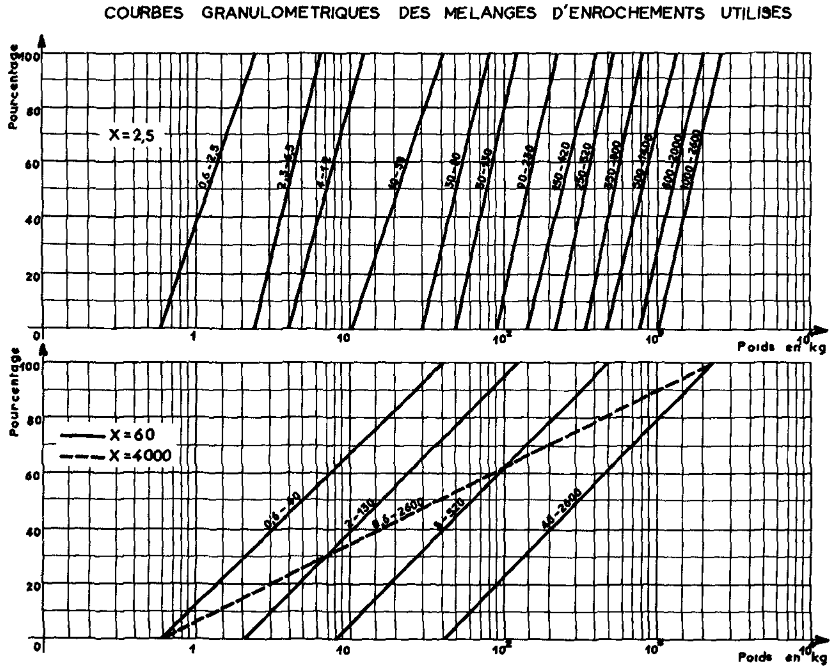
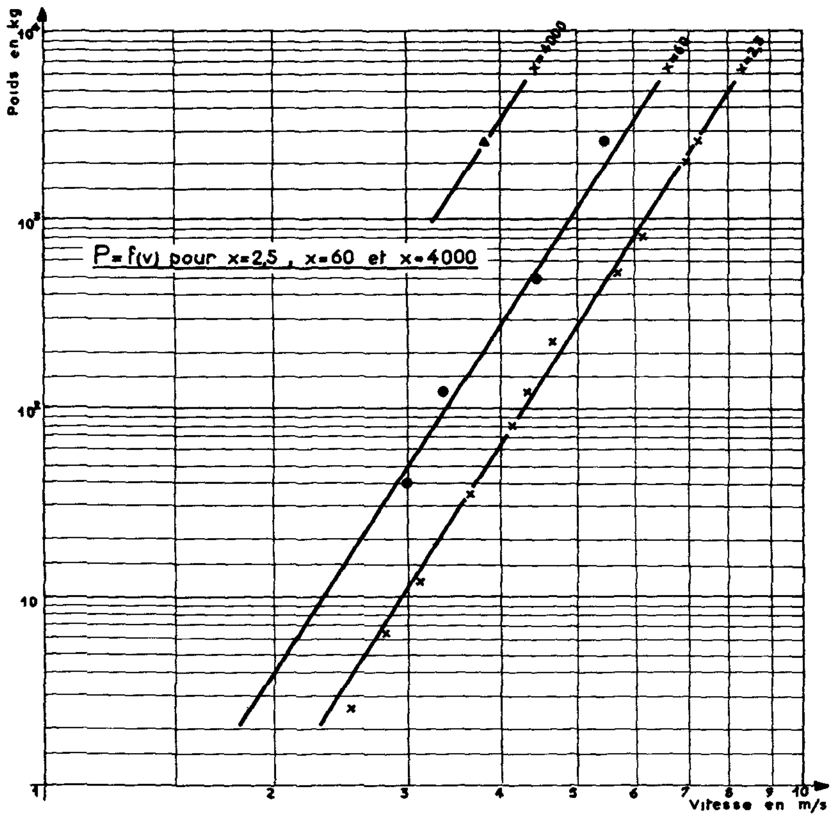


Fig. 4

COURBES DONNANT LE POIDS P DES ENROCHEMENTS ET L'ETENDUE GRANULOMETRIQUE x QUI SONT A LA LIMITE DE STABILITE DANS UN COURANT DE VITESSE MOYENNE V



VITESSE NECESSAIRE POUR ENTRAINER DES PIERRES DE DIFFERENTS POIDS

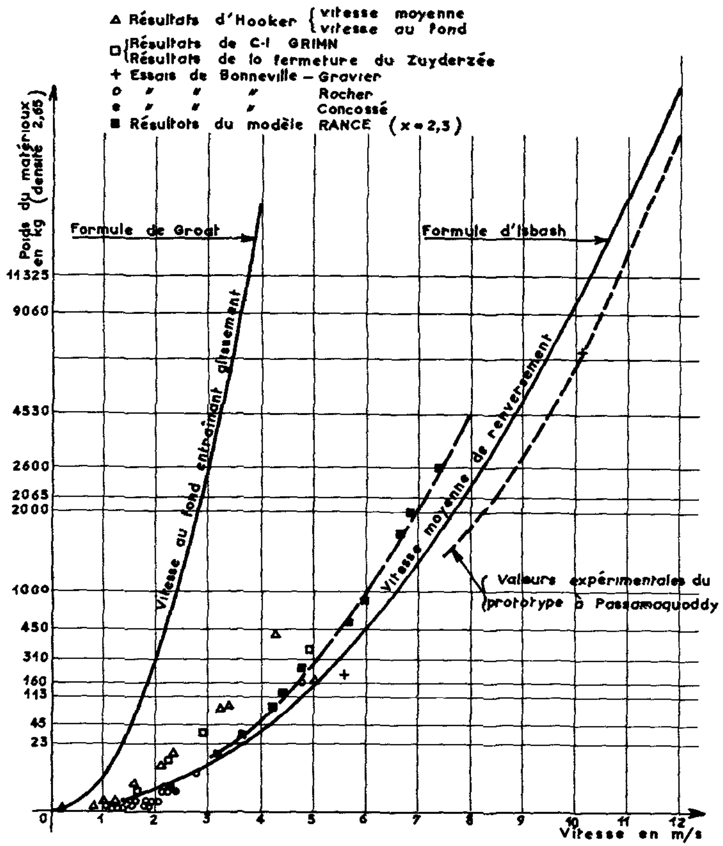


Fig. 6

la forme P = f (v), figure 6, sur un diagramme comportant différents résultats déjà connus, relatifs à des projets réalisés en Amérique ou à des études théoriques connues.

POSSIBILITE D'EXTENSION DES RESULTATS CI-DESSUS

La validité des résultats concernant les variations de la vitesse moyenne et de la dénivellation en fonction de l'avancement des travaux est évidemment limitée au cas particulier de la Rance, tout au moins en ce qui concerne les valeurs numériques.

Par contre, les résultats concernant le dimensionnement des enrochements semblent susceptibles d'une certaine généralisation. En effet, les essais ont mis en évidence l'influence prédominante, sur la stabilité des enrochements de caractéristiques déterminées, de la vitesse moyenne dans la brèche. La configuration géométrique de la brèche, variable avec l'avancement des travaux, semble donc n'avoir pas d'influence sensible sur la stabilité. De plus, les essais consistant en une recherche de limite de stabilité, le facteur temps ne semble pas devoir jouer, de sorte que les résultats, valables pour des courants de marée alternatifs, doivent pouvoir etre étendus au cas d'une coupure en rivière sans modifications notables.

CHAPTER 34

ETUDE SUR MODELE DU CALIBRAGE DU CHENAL NAVIGABLE DE L'ESTUAIRE DE LA GIRONDE

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BUT DES ESSAIS

Le Service Maritime de la Gironde effectue chaque année des dragages d'entretien du chenal de navigation de la Gironde pour son maintien à une cot fixée actuellement à 5,50 m sous basses mers, plafond qu'il serait opportun d'améliorer pour suivre l'accroissement des tirants d'eau des navires.

La répartition des volumes de dragages annuels moyens aux emplacements l plus critiques est donnée par le tableau suivant : (voir figure 1)

Emplacement	Volume annuel moyen
Passe de Cussac	450 000 m^3
Passes d'Ambès	275 000 m^3
Passe de Beychevelle	200 000 m^3
Passes chenal de la Garonne	650 000 m^3

Une campagne de mesures effectuée sur place en 1950 avait permis de déterminer les concentrations de matériaux en suspension et conduit à penser que la suspension jouait un rôle prépondérant pour les modifications du chen de navigation.

Par contre des prélèvements effectués sur les produits de dragages dans la passe de Cussac avaient montré que les apports dans cette passe étaient dı essentiellement au charriage sur le fond.

Le but de l'étude sur modèle réduit était de rechercher les emplacement et les formes des ouvrages de correction permettant de réaliser un auto-entretien des profondeurs du chenal à une cote améliorée, notamment sur les passes de Cussac et d'Ambès, en tenant compte des deux causes de transport d matériaux : charriage et suspension.

L'étude des passes de Beychevelle et du chenal de la Garonne, en amont c passes du Bec d'Ambès, n'a pas été demandée.

CARACTERISTIQUE DU MODELE

Les caractéristiques principales du modèle utilisé sont les suivantes :

- échelle en plan : 3/2000
- échelle en hauteur : 1/100
- échelle des temps hydrauliques : 3/200

La durée d'une marée est légèrement supérieure à 11 mn.

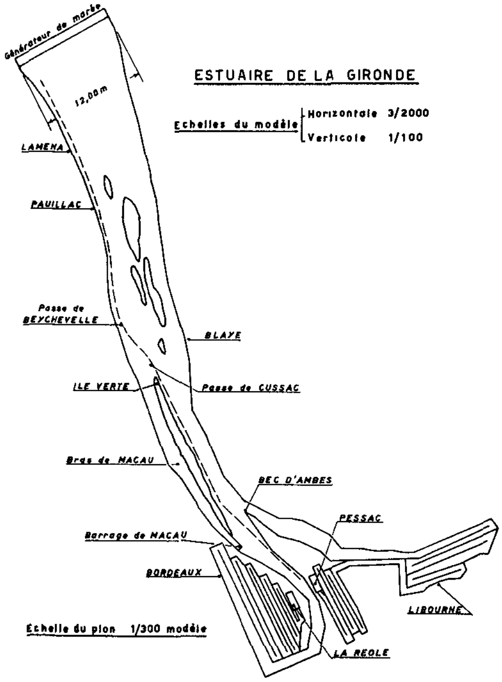

Fig. 1

L'estuaire est représenté en similitude géométrique entre le bec d'Ambès et l'aval de Pauillac (figure 1).

En amont du bec d'Ambès, la Garonne et la Dordogne sont représentées repliées en forme de labyrinthe respectivement jusqu'à la Réole et Pessac.

Dans les sections limites amont des deux cours d'eau, des déversoirs permettent d'introduire des débits variables.

Un générateur de marée produit la loi de marée par réglage du niveau à l'aval du modèle [1, 2].

METHODE SUIVIE

Deux sortes d'essais ont été effectués :
- essais préliminaires sur un modèle à fond fixe,
- essais sur un modèle à fond mobile.

ESSAIS A FOND FIXE

Ces essais ont eu un double but :

- étalonner le modèle au point de vue hydraulique, c'est-à-dire régler la rugosité des fonds de façon à reproduire une onde marée semblable à celle de la nature, rechercher l'influence des ouvrages de calibrage envisagés à la suite des études préliminaires théoriques effectuées par le Service Maritime et le Laboratoire, sur les vitesses dans le chenal et sur les lignes d'eau dans l'estuaire.

Cette recherche avait pour but d'éliminer rapidement certaines modificat envisagées a priori mais qui se révèleraient inefficaces ou même nuisibles. L critères retenus pour cette amélioration furent les suivants : les modificati à rejeter sont d'une part, celles qui diminuent les vitesses de jusant (et pa suite réduisent la puissance hydraulique d'entraînement des matériaux vers le large), d'autre part, celles qui abaissent le niveau d'eau (et par suite gêne la navigation).

Un certain nombre d'ouvrages de calibrage furent retenus à la suite de essais.

ESSAIS A FOND MOBILE

Ces essais ont comporté trois phases :
- étude du charriage seul,
- étude de la suspension seule,
- étude simultanée du charriage et de la suspension.

Ces trois phases sont exposées ci-après :

Etude du charriage seul - Elle a comporté trois stades :

1) Choix du matériau de fond.

Le matériau de fond devait remplir les conditions suivantes :
- créer des pertes de charge équivalentes à celles réalisées sur le modèle à fond fixe,
- réaliser des évolutions de fond semblables à celles de la nature,
- permettre de réaliser une échelle des temps de transport par charriage aussi faible que possible pour réduire la durée des essais.

Les essais ont montré que deux matériaux pouvaient être adoptés :
- le plexiglas de granulométrie comprise entre 0,4 et 1 mm,
- la sciure de bois de granulométrie comprise entre 0,4 et 1 mm.

Pour des raisons de commodité, le plexiglas a été utilisé pour les essai de charriage seul et la sciure de bois pour les essais de charriage et de suspension.

2) Etalonnage du modèle

L'étalonnage du modèle a consisté à reproduire des apports dans la passe de Cussac semblables à ceux de la nature pour deux situations différentes connues dans la nature (avant et après la surélévation du barrage de Macau cette passe a été choisie parce que les phénomènes de transport solide qui s'y produisent peuvent être considérés comme produits par le charriage seul.

ETUDE SUR MODELE DU CALIBRAGE DU CHENAL NAVIGABLE DE L'ESTUAIRE DE LA GIRONDE

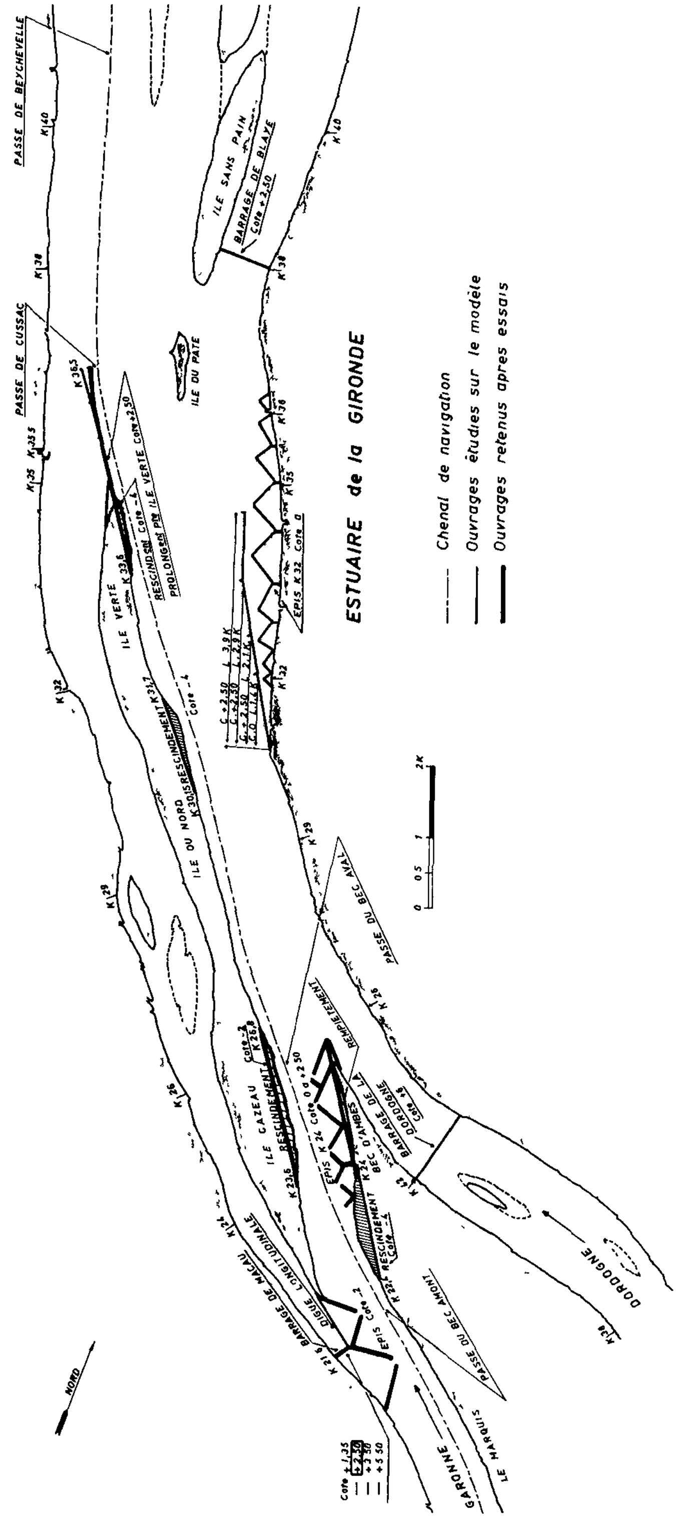

Fig. 2

L'échelle des temps de transport par charriage déterminée par cet étalonnage fut de 3/1000 pour le plexiglas (au lieu de 15/1000 pour l'échelle des temps hydrauliques).

3) Etude de l'influence des ouvrages projetés

Les essais ont conduit à rejeter un certain nombre d'ouvrages de correction retenus après les essais à fond fixe.

Etude de la suspension seule - Le but de l'étude était de rechercher :

- la nature et la granulométrie du matériau apte à représenter les phénomènes de transport en suspension en régime non permanent, dans le Bras de Macau;
- la loi de variation de l'échelle des temps de transport en suspension en fonction de l'échelle de la concentration du matériau,
- le Bras de Macau a été choisi car les phénomènes de transport solide qui s' produisent sont dus essentiellement au transport par suspension (sédimentation et remise en suspension à chaque marée).

Dans un but de simplification, cette étude a été effectuée dans un canal rectangulaire représentant schématiquement le bras de Macau [3 et 4].

Les essais ont donné les résultats suivants :
- la sciure de bois dur de granulométrie comprise entre 0 et 0,1 mm représen' bien le matériau de la nature,
- la correspondance entre l'échelle des temps de sédimentation et l'échelle (la concentration en matériau de suspension a été déterminée.

Etude simultanée du charriage et de la suspension [4]

1) Etalonnage

Au moyen des deux matériaux déterminés par les essais précédents un nouv étalonnage a été réalisé de façon à obtenir :
- la reproduction des phénomènes naturels : apports par charriage dans la passe de Cussac, dépôts par sédimentation dans le Bras de Macau lorsqu'(surélève le barrage de Macau,
- l'égalité des échelles des temps pour ces deux phénomènes de transport solide (en jouant sur l'échelle des concentrations).

Cette dernière condition a pu être réalisée, l'échelle des temps commune aux deux modes de transports solides étant de 2/1000.

2) Essais d'ouvrages

Les essais d'ouvrages de correction ont confirmé, dans l'ensemble, les résultats obtenus à la suite des essais de charriage seul.

CONCLUSION

Les résultats obtenus sont les suivants : (figure 2)

OUVRAGES INUTILES OU NUISIBLES

Certains ouvrages envisagés à la suite des études théoriques préliminair pour améliorer le chenal sont soit inutiles, soit nuisibles; en particulier :

- la digue de 4 km implantée au km 30,6 dont la construction paraissait a priori utile aurait amené dans le chenal un volume de matériau de 3 millions de m^3 (occasionnant des dépenses de dragages importantes).
- la surélévation du barrage de Macau au-delà de la cote (+ 2,50) de façon à le rendre insubmersible (arasement à la cote + 5,50) aurait entraîné un supplément de dépense et n'aurait pas apporté d'amélioration.

OPERATIONS EFFICACES

Pour la passe de Cussac :

- rescindement de l'île Verte,
- prolongement de l'île Verte par une digue de 1500 m.

Pour la passe d'Ambès :

- surélévation du barrage de Macau de (+ 1,3) à (+ 2,5),
- rescindement de l'île Cazeau,
- surélévation des épis de l'éperon du Bec d'Ambès (km 24),
- construction d'un épi au Bec d'Ambès,
- construction de deux épis sur la rive gauche (km 22).

La réalisation de ces ouvrages permet d'espérer la stabilisation des fonds de la région d'Ambès et de Cussac à une profondeur accrue. L'avantage majeur résultant de cette évolution résidera dans l'amélioration des conditions générales d'accès au Port de Bordeaux et de la sécurité de la navigation.

Par ailleurs, une économie annuelle doit être réalisée sur les dragages dont les volumes passent approximativement de :

450 000 m^3 à 90 000 m^3 à Cussac, et de
275 000 m^3 à zéro au Bec d'Ambès

soit une diminution globale annuelle de 635 000 m^3.

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CHAPTER 35

SUBMARINE WASTE DISPOSAL INSTALLATIONS

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The underlying philosophy of submarine waste disposal is economic disposal of waste without any significant adverse effect on the receiving water that would impair its beneficial use.

INTRODUCTION

A submarine outfall dispersal system is an integral part of any waste treatment facility discharging into the marine environment. Th design of the treatment facility as well as the submarine outfall installation is dependent upon the beneficial uses of the receiving water, the corresponding water quality criteria deemed necessary to protect the water use, and the waste assimilating or dissipational characteristics of the receiving waters.

The economic and technical factors related to the design and performance of waste treatment installations are familiar to most sanitary engineers. However, the quantitative resolution of the waste assimilating or dispersal characteristics of receiving waters is not well understood generally, and the problem is even more complex when dealing with coastal or nearshore marine waters. The principa reason for the complexity in the marine environment is that the waste assimilating or dispersal characteristics of coastal waters depends upon numerous physical oceanographic factors such as wind wave, sw littoral currents, variable water mass circulation systems, density gradients, upwelling, etc. in addition to the conventional physical, chemical and biological characteristics common to the aquatic environment.

FUNCTION

The obvious function of a submarine waste disposal installation is to convey a waste, treated to a suitable degree, to a point of final disposal where the effect of the waste on the receiving water is minimal even at the point of initial mixing as well as in the general area. While it may be desirable on a theoretical basis to treat a waste to a degree that even in the outfall pipe or in the area of initial mixing at the diffuser the waste has no significant effect on the receiving water; this may be unattainable from a technical viewpoint and is generally economically prohibitive. The point of final discharge must be selected on the basis of overall suitability with respect to the problem of rapid and thorough initial mixing of the waste with the receiving water and to prevent the occurrence of excessive concentrations of waste in the critical areas as a result of the subsequent transport and dispersion of the waste-sea water mixture.

RATIONAL DESIGN CONSIDERATIONS

The rational design of a submarine waste dispersion system entails consideration of a multiplicity of factors. Table 1 presents an outline-summary of the principal factors that should be evaluated.

TABLE 1

FACTORS TO BE EVALUATED IN DESIGN OF SUBMARINE WASTE DISPERSION SYSTEMS

I. Beneficial Uses of Receiving Water
   1. Bathing
   2. Marine recreation and/or working environment
   3. Fishery - propagation, migration, food organisms, etc.
   4. Shellfishery - propagation, harvesting, etc.
   5. Other marine plants, animals, i.e. kelp, etc.
   6. Industrial or commercial uses - cooling water, etc.
   7. Waste disposal
   8. Other

II. Water Quality Considerations to Protect Beneficial Uses
   1. Public Health
      a. coliform
      b. other
   2. Fishery and Shellfishery
      a. toxic substances
      b. antagonistic substances
      c. stimulants, fertilizers
      d. oxygen depressants
      e. settleable debris
      f. turbidity - suspended solids
   3. Nuisance
      a. grease and oil films
      b. floating debris
      c. settleable debris
      d. odors

TABLE 1 (cont)

4. Aesthetic
    a. sleek areas
    b. floating debris
    c. turbidity - suspended debris
    d. plankton blooms
    e. colors
    f. other

III. Oceanographic Characteristics of Site
1. General nearshore circulation system
2. Current structure
    a. surface and subsurface currents
    b. strength and direction as a function of time (ie current rose)
    c. effect of wind
        wave
        tide
        littoral drift
3. Eddy diffusivity or dispersion characteristics
4. Density structure, salinity-temperature-depth relationshi
5. Submarine topography
6. Submarine geology

IV. Waste Dispersion Considerations
1. Initial mixing process - diffuser
    a. jet mixing
    b. buoyancy forces and induced mixing
    c. interference between jets
    d. possible effect of thermoclines or density gradien to throttle rise of waste plume
    e. diffuser orientation
2. Dispersion plume and trajectory
    a. current rose
    b. eddy diffusion relationships
    c. rational dispersion equations for waste concentra
        1. no decay, ie dilution only
        2. decay or dieaway operative
            a. bacteria - coliform
            b. radioisotope
            c. other (BOD) etc.

V. Economic Analyses
1. Types of treatment, effluents, and cost in varying combinations with
2. Length, depth, cost of outfall systems and associated wa dispersal and assimilating characteristics

BENEFICIAL USES - WATER QUALITY CRITERIA

One of the fundamental requirements in the development of a marine waste disposal system is determination of the beneficial uses of the waters which are to be protected. Once this is done, suitable water quality criteria may be established to protect these uses and also provide a basis for evaluating adherence or compliance to the criteria. In some areas, there may be limited information on precise water quality standards for a respective use; however, for the most part reasonable criteria or standards can be established.

OCEANOGRAPHIC FACTORS

Oceanographic investigations of potential outfall sites are necessary to select the site having the most favorable characteristics with respect to outfall and diffuser location. It is necessary to know the general overall water mass circulation characteristics with respect to each potential site.

Current resolution - The current structure with respect to both depth and time must be studied with development of a statistically significant current rose as the ultimate objective. Sufficient studies must be conducted to provide an adequate sample of the variable currents that may exist at a particular location. Moreover, it is generally desirable to develop sufficient data so that at least a rough resolution of the effect of wind, wave, swell and tides can be made.

Current studies have been conducted using drogues of free floats, current meters, and drift cards consisting of small weighted plastic envelopes(4,14,18). Work is currently in progress on the adaptation of existing continuous recording current meters as well as the development of recording or transmitting current monitoring systems (2)(6).

If it is possible to show a fair degree of correlation between wind and current strength and direction, it is possible to employ this relationship to construct a "synthetic" current rose based on extended wind observations. This is of considerable practical importance because of the general availability of wind data and the relative ease and economy of collecting wind data as compared to current data. It should be noted that if such correlations exist, they generally apply only to the surface water layers.

Figure 1 presents a typical current rose - a plot of current direction, strength and percent (of time) occurrence for a given location. From the current rose, one can estimate the time of travel of waste material to the critical location. If data are available on

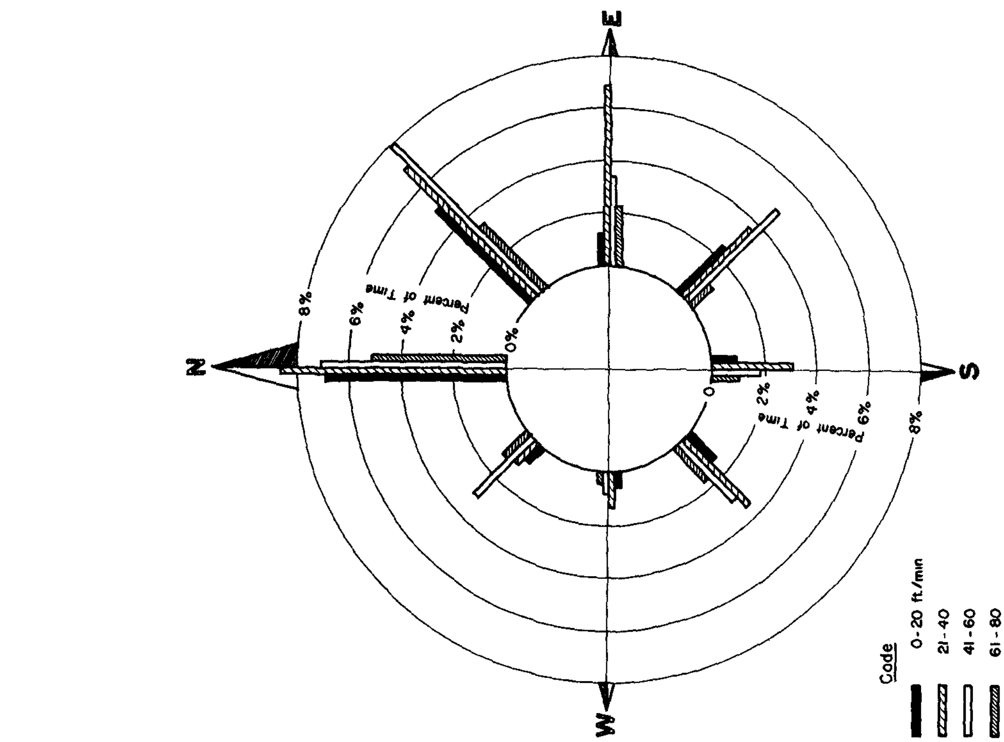

Fig. 1. Typical current rose

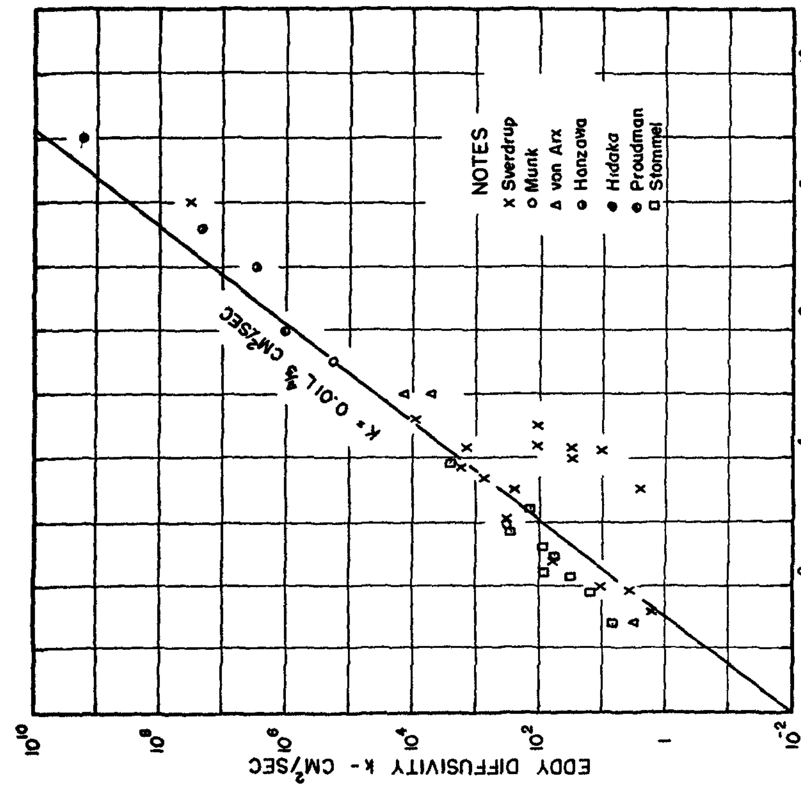

Fig. 2. Variation in eddy diffusivity, k, and scale of diffusion phenomena

the decay or dieaway reaction kinetics of the waste, quantitative estimates can be made of the effect of decay during the time of travel to the critical location. Also, it permits prediction of the probability of occurrence of a given concentration of waste at any point.

Eddy diffusivity - Evaluation of the magnitude of the coefficient of eddy diffusivity for the receiving water is necessary if quantitative consideration is to be given the effect of eddy diffusion or dispersion in reducing the concentration of waste in the waste-sea water plume. Measurement of eddy diffusivity can be accomplished most easily by the use of dye tracers such as sodium fluoroscein. Chemical tracers such as iron salts (9) and radioisotopes (7) have been used, but are generally more complex in handling and analysis.

Various investigators have observed that the magnitude of the eddy diffusivity coefficient in the ocean has been dependent upon the scale of observation employed (8)(11). Richardson and others (17)(19) have postulated theoretically that the magnitude of eddy diffisuvity is proportional to the four thirds power of the scale of the phenomena or $k = \epsilon \ell^{4/3}$. Other investigators have confirmed this by measurement of the phenomena in the ocean. Figure 2 presents field values reported by many investigators plotted to show the relationship between eddy diffusivity, k, and the scale of the diffusion phenomena observed (8)(11).

Density-temperature structure - The density or temperature-depth character of an outfall site is an important characteristics of its suitability for waste dispersion. If there is a marked density or temperature gradient at some depth below the surface, this density gradient will aid in preventing the waste-sea water mixture from rising to the surface of the sea. In fact, the density gradient acts similar to an inversion layer in the atmosphere and tends to "throttle" he waste-sea water mixture below it. This is true if sufficient jet nixing and gravitational diffusion is effected before the waste-sea water mixture reaches the density gradient region: or, in other words, he buoyancy forces due to differences in density have been reduced o a negligible degree. If possible, it is desirable to keep the waste-sea water mixture from reaching the surface of the sea because it is n the surface layers where the most rapid transport occurs. Similarly, material that might accumulate in the surface film is transported t very high velocities induced by the wind compared to the water mass mmediately beneath the surface.

In determining the density-depth characteristics of a given ite, the bathythermograph (BT) is used most frequently. BT traces re made at each station and the temperature depth relationships as

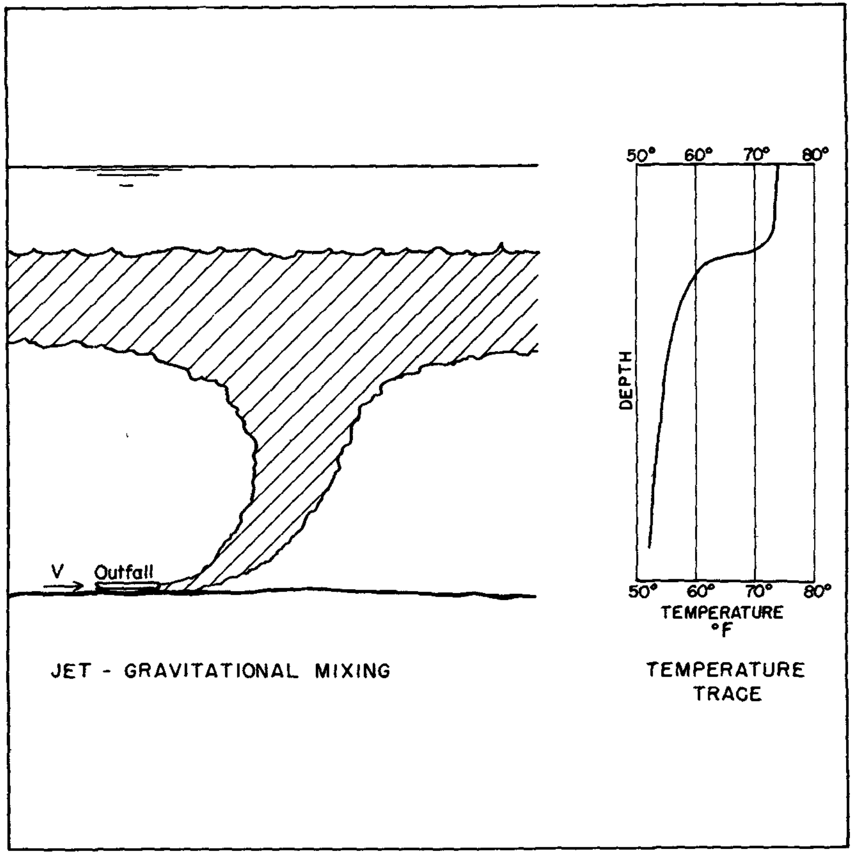

Fig. 3. Initial mixing - density considerations

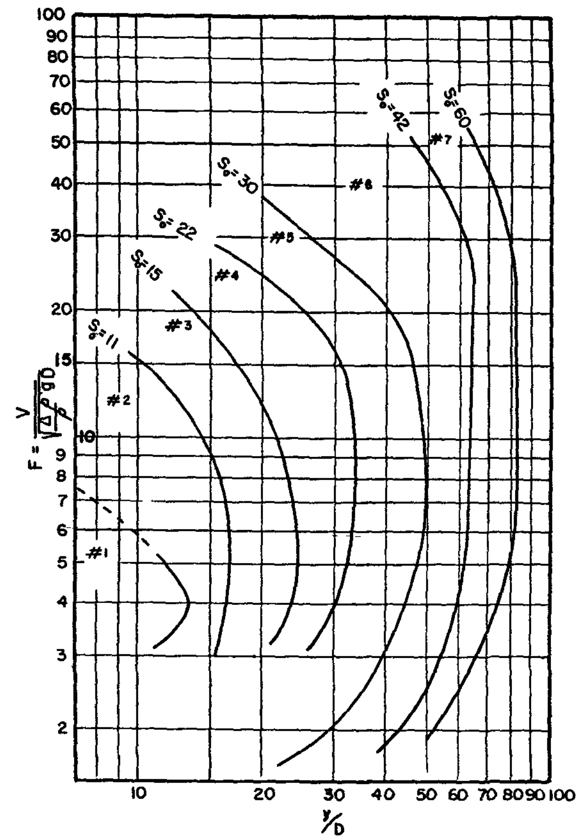

Fig. 4. Dimensionless plot of variables affecting dilution, So. based on Rawn and Palmer data for horizontal discharge

measured by a rapid temperature-depth response circuit are traced on a suitable slide. The presence of a marked density gradient is easily detected and recorded at the depth encountered. BT traces are adequate for indicating density gradients in water where no significant salinity gradient exists. However, if the outfall site is subject to considerable upwelling of deep ocean waters so that both a temperature and salinity gradient are present, it is necessary to quantitatively measure the salinity as well as temperature to properly determine the density gradient.

Figure 3 presents a sketch of a typical temperature-depth trace with a bathythermograph as well as the effect such a density gradient may have on the gravitational diffusion of waste rising from a submerged jet.

Submarine topography and geology - Submarine topography may vary widely for submarine disposal installations. Ideally what is desired is a rather uniform sloping bottom to a considerable depth within reasonable distance from shore. While there is no minimum acceptable length or depth of outfall, it would appear that for most effective initial mixing and diffusion, a depth of 200 feet or greater is desirable for large outfall installations. Also, it is desirable that the bottom topography be relatively flat in the vicinity of the diffuser installation to minimize hydraulic flow distribution problems with a multi-port diffuser.

Bottom geology is an important consideration in the selection of the outfall site. Obviously fault zones and relatively unstable bottoms are to be avoided.

WASTE DISPERSION ANALYSIS

There are essentially two major fundamental aspects of the dispersion problem. The first is concerned with the mixing and dilution of the waste in the immediate proximity of the discharge point. The second is associated with the ultimate disposition of the waste-sea water mass or the direction of movement and concentration of waste in the waste-sea water dispersion plume.

Jet mixing - gravitational diffusion - The fluid mechanics of a single port discharging into a body of water with a density different than that of the jet is complex. The mixing phenomenon is a combination of mixing resulting from jet action, ie the kinetic energy of the jet, and mixing or diffusion resulting from the gravitational or buoyancy forces due to differences in density between the waste and sea water.

Rawn and Palmer (15) have studied the problem of dilution in rising jet in large scale experiments and reported the following expression:

$$S_o - 1 = \frac{0.5(L + 3)^{2.35}}{Q^{0.61}}$$

where S_o = dilution at the head of rising column

L = centerline length of rising column in ft

Q = flow per jet in gallons per minute

More recently, Rawn, Bowerman, and Brooks (16) have re-examine the original Rawn-Palmer data based on dimensional analysis and Froude law relationships. Figure 4 presents their dimensionless pl of dilution, S_o, in terms of Froude number $F = \frac{V}{\sqrt{\frac{\Delta p}{\rho} gD}}$ and y/D, t ratio of depth to diameter of the jet. The use of the figure is obviou For a given Froude number, F, and the ratio of depth over outlet to outlet diameter, the intersection of the appropriate corrodinates is noted and the dilution, S_o, may be estimated. Rawn, Bowerman, a Brooks cite the need for caution in arriving at precise values of S_o from Figure 4 because the curves represent group averages and we not precisely defined.

Albertson (1) and Cooley and Harris (5) have developed ex perimentally similar equations for dilution in a jet. Cooley and Harris concluded that the average dilution in a jet could be express as follows:

$$S_o = \frac{L}{3\,D_o}$$

where

S_o = average dilution in jet ($Q_{jet\text{-}total}/Q_{jet}$)

L = length of axis of jet trajectory in feet

D_o = diameter of outlet, ft

The preceding equations and others (11) permit estimation o dilution of the waste in the jet mixing-gravitational diffusion plume Analysis of these expressions points up the obvious desirability of viding up a given flow and single jet into a number of smaller flow and jets such as in a multi-port diffuser.

In dispersing a given flow into the water mass passing over a multi-port diffuser, the preceding expressions imply a "new" mixing-water supply and no interference between jets. However, the latter may not be attainable practically, and the amount of "new water" passing over the diffuser may limit the maximum dilution possible. Obviously, the maximum dilution of waste immediately over a diffuser, assuming perfect mixing, is the total "new" water supply (Q_n = Vld) divided by the waste flow, Q_w. Therefore:

$$S_o = \frac{Vld}{Q_w}$$

where

S_o = average dilution of waste

V = average velocity of "new" water flow past diffuser, ft/sec

l = length of diffuser, ft

d = effective water depth over diffuser, ft.

Q_w = waste flow, ft^3/sec

The above is simply a statement of continuity and represents the maximum dilution attainable at the source with perfect mixing - the obvious objective of a multi-port diffuser.

Eddy diffusion - Once the waste discharged through a submarine outfall is effectively mixed with sea water in the immediate proximity of the outlet, what then happens to the waste-sea water mass. Because the initial mixing and dilution achieved by the diffuser section generally does not dilute the waste to a harmless level at the outlet, it is necessary that subsequent dilution processes must be considered.

Most outfall location and design has been based on judgment influenced by past experience and float studies with the objective of keeping the fresh waste away from the shore for a minimum period of time. It is hoped that during this time, sufficient dilution and bacterial (waste) decay will occur so that the water reaching the shoreline will not exceed permissible waste concentration or bacterial standards for the beneficial uses of the area.

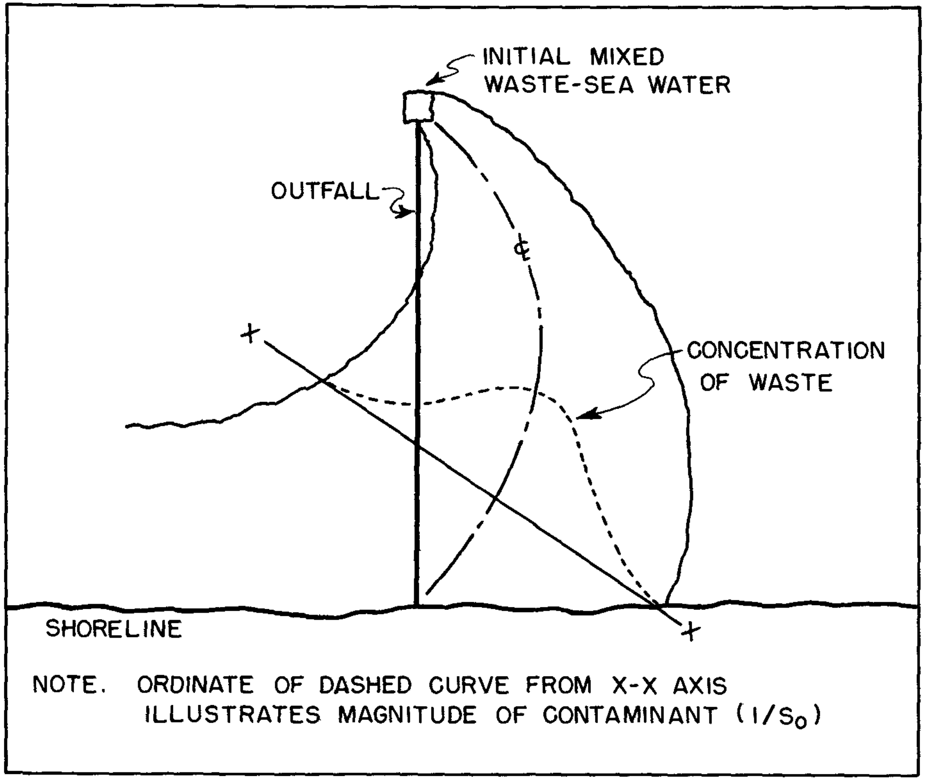

Fig. 5. Idealized trajectory of waste-sea water mass showing effect of current and horizontal diffusion

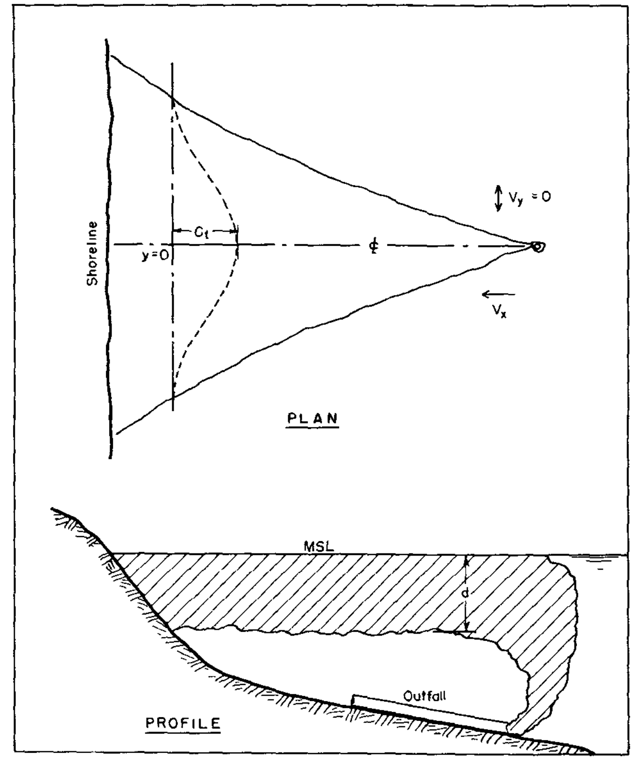

Fig. 6. Definition sketch for dispersion equations

There are numerous factors affecting the transport and dispersion of a waste-sea water mixture. General oceanographic factors such as periodic and non-periodic currents, wind currents, mass transport by waves, etc. have been reviewed by Pearson (11).

Lateral dispersion or horizontal diffusion of the waste-sea water mass occurs whether the current is such that it carries the waste away in a single direction or whether it is transported in an irregular pattern. By definition, lateral dispersion is intended to mean the dispersion of the waste-sea water mixture in a direction normal to the principal movement (advection axis) of the water mass.

Classically the current systems in nearshore waters have been assumed primarily to be rotational in character due to tidal currents (14). Figure 5 depicts an idealized trajectory of a waste-sea water mass may be likened to a plume of smoke emitted from a stack or point source. The plume follows the general direction of the current and the lateral width of the plume as it develops is a function of time and the turbulent characteristics of the receiving water (ie coefficient of eddy diffusivity).

Eddy diffusion in the ocean can be mathematically described by the Fickian diffusion formulation as follows:

$$\frac{\partial c}{\partial t} = k \frac{\partial^2 c}{\partial x^2}$$

In eddy diffusion vernacular, the diffusion constant, k, has been called the coefficient of eddy diffusivity. However, as previously pointed out, k, or the coefficient of eddy diffusivity appears to be a function of the scale of the phenomena.

The principal problem in ocean disposal of wastes is the prediction of the waste concentration at any fixed point with respect to the source. It is possible to compute this concentration if the appropriate differential equation considering continuity and boundary conditions is solved. Of particular interest in sewage dispersion is the effect of time on bacteria or coliform content of the sewage-sea water mixture. Inasmuch as this may be expressed as a decay function, it also can be included in the basic differential equation. Numerous investigators such as Ketchum and Ford (9), Munk, Ewing and Revelle (10), Pearson (11)(12) Pearson and Gram (13), Brooks (3) and others have reported solutions to the diffusion equation. Most have employed an assumed constant eddy diffusion coefficient; however, Brooks has proposed an approximate solution to the variable (function of scale, ie time) coefficient of eddy diffusivity.

Figure 6 reports an idealized definition sketch for the solution of the diffusion equation. A point source is assumed, steady unidirectional current in the direction of the shore, uniform mixing of the waste over a depth, d, and continuous uniform flow from the source. Obviously, no provision is made for return of water seaward which mus and does occur, hence the definition sketch is idealized.

A solution to the diffusion equation (point source) in terms of the minimum dilution of the waste along the axis of the waste-sea water is as follows:

$$S_o = \frac{2.35d\sqrt{kV_x \chi}}{Q}$$

where S_o = minimum dilution along axis of waste plume at distance χ from source

d = assumed vertical mixing depth, feet

k = eddy diffusivity, ft^2/sec

χ = distance from source, feet

V_x = average velocity of water mass, ft/sec

Q = waste discharge, MGD

Including the decay function for bacterial dieaway, and expressi the waste concentration in terms of coliform concentration, the above expression becomes:

$$MPN = \frac{0.438\ Q\ C_o}{d\sqrt{kV_x \chi}\ e^{(ax/V_x)}}$$

where MPN = most probable number of organisms per ml
C_o = concentration of organisms in waste, MPN/ml
a = bacterial dieaway (decay) constant, 1/sec

Similar expressions have been developed for a line source.

Brooks (3) has reported the following solution to the diffusion equation for a line source:

$$\frac{C_o}{C_{max}} = \frac{10^{\frac{t}{T}}}{\mathrm{erf}\left(\dfrac{1.23}{\sqrt{\left(\dfrac{8C_1 t}{b^{2/3}} + 1\right)^3 - 1}}\right)}$$

where C_o = initial coliform concentration

C_m = maximum coliform concentration at time, t

t = time of travel

T = time required for 90% coliform dieaway (T-90)

C_1 = constant of proportionality ($k = C_1 l^{4/3}$) based on eddy diffusivity a function of scale ($C_1 \approx 1.84\ ft^{2/3}/hr$

b = initial width of sewage field

The significant feature of Brook's equation is that it attempts to include the effect of a variable eddy diffusion coefficient. The previously cited expressions assume a constant coefficient of eddy diffusivity with selection of the appropriate value based on a representative scale of the overall diffusion phenomenon.

Diffuser orientation - As cited previously, the magnitude of the coefficient of eddy diffusivity varies as the four-thirds power of the scale of the phenomena (ie approximately as the four-thirds power of the neighbor or particle separation in the waste plume). The advantage of dispersing the sewage over as wide an area as possible, normal to the major set of the current, rather than parallel to it, is obvious. Not only is the initial dilution of the waste increased but for a given initial dilution of the waste, the waste-sea water mass having the greatest scale normal to the major current will disperse laterally at the greatest rate; hence, effecting maximum dilution of the waste. Figure 7 shows an idealized trajectory of a waste-sea water mass with respect to the orientation of the diffuser section and the relative con-

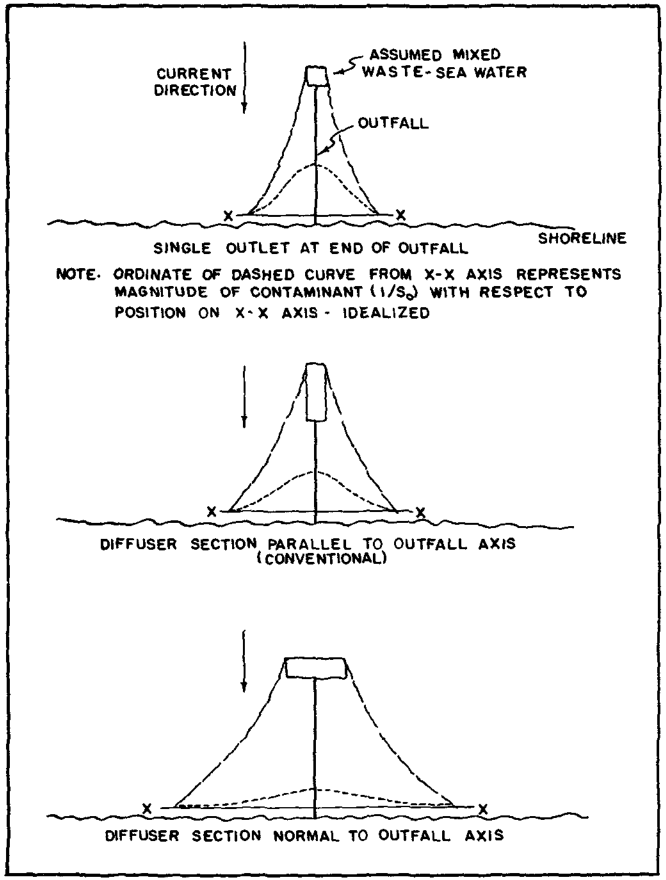

Fig. 7. Idealized representation of effect of horizontal diffusion and diffuser ori-

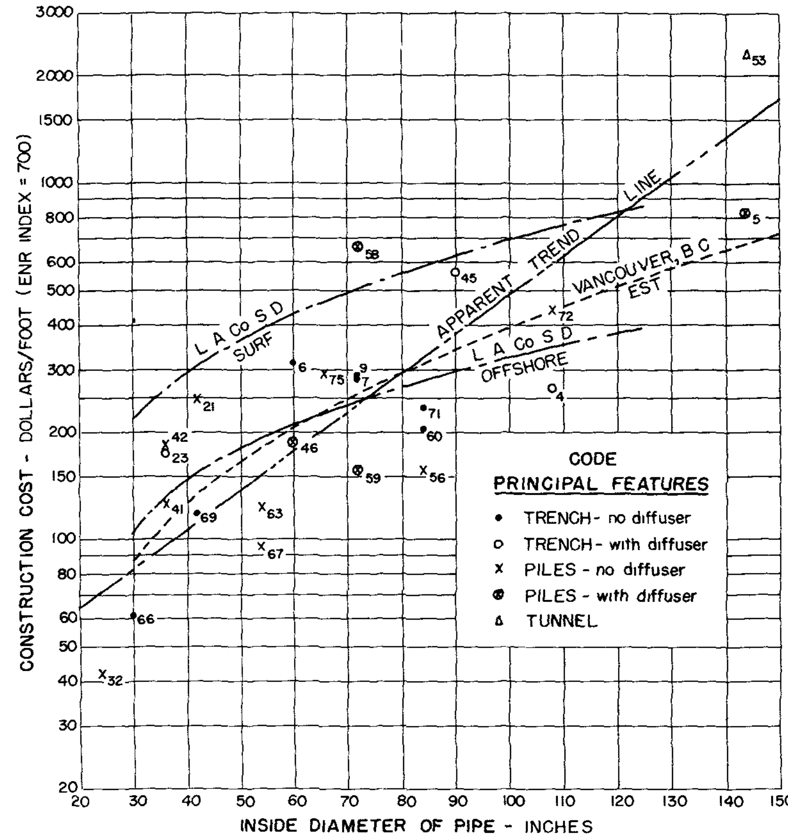

Fig. 8. Construction costs of reinforced concrete submarine outfalls

centrations of waste in the waste-sea water plume. It is apparent that the most effective orientation of a diffuser system in outfall design would be essentially normal to the axis of the so-called "critical" onshore current. In many cases this would result in the diffuser being parallel or nearly parallel to shore.

ECONOMIC CONSIDERATIONS

The design of a waste treatment plant and submarine outfall installation should be based on an analysis of all factors including economic, for all possible treatment - outfall systems. Basically, the best solution is the most economic combination of degree of waste treatment and outfall length that will produce the desired receiving water conditions. It is obvious that the difference in annual cost between varying degrees of waste treatment can be used to construct a longer outfall for the plant with the lesser degree of treatment. If the lesser degree of treatment and longer outfall is more economic than a higher degree of treatment and a short outfall; providing both alternates produce equivalent receiving water conditions at the critical point, obviously the former combination is preferred. All economic comparisons must be based on equivalent receiving water conditions or effects in the areas to be protected. Nomographic bases for economic analyses of degree of treatment and outfall length have been reported by Pearson (11). The cost of submarine outfalls is variable because of gross differences in surf and bottom condition, type of construction, anchorage, and method of construction. Figure 8 reports the unit construction costs of several large reinforced concrete submarine outfalls adjusted to an ENR Index of 700 and the relationship between unit cost and diameter of the pipe. Figure 9 presents a similar plot of unit cost versus pipe diameter for cast iron submarine outfalls.

EXISTING INSTALLATIONS

There are over 125 California coastal communities, including eleven of the thirteen largest cities, that dispose of their sewage effluent (and in some cases sewage sludge) through submarine outfalls. In addition there are a large number of industrial submarine outfall installations. The characteristics of these installations vary from relatively small (12 inch diameter) conventional pipelines to large (12 foot diameter, 5 miles long) specially designed submarine outfall installations (11).

The largest submarine outfall installation in the United States is currently under construction at the Hyperion plant for the City of Los Angeles. The effluent outfall includes a 12 foot I.D. reinforced con-

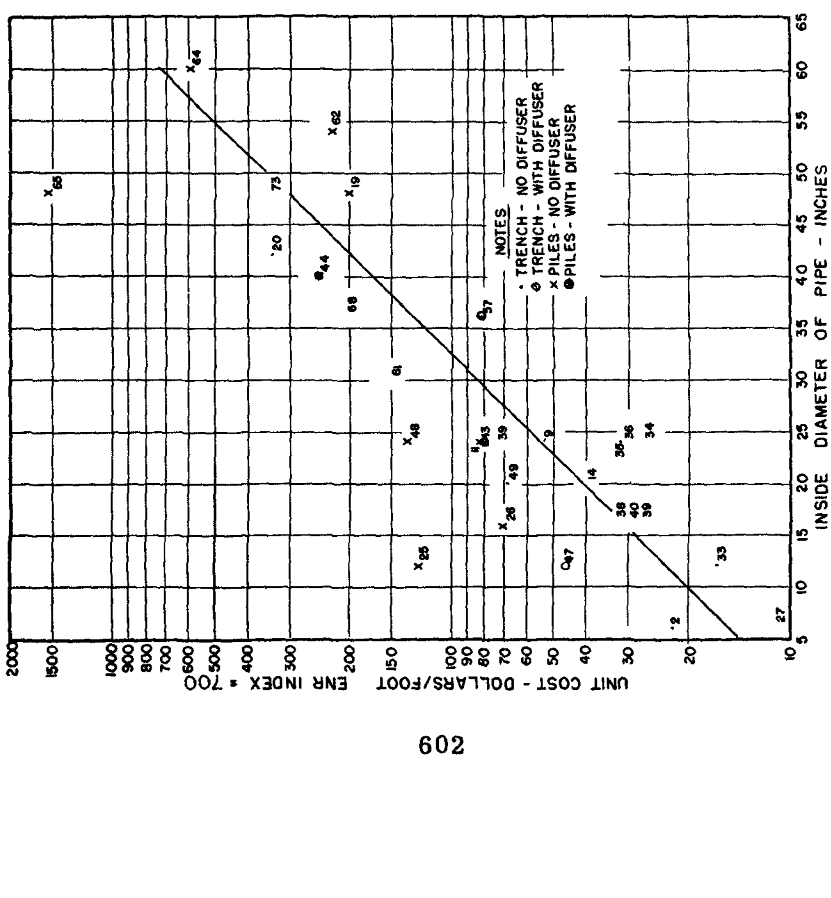

Fig. 9. Construction cost of cast iron submarine outfalls

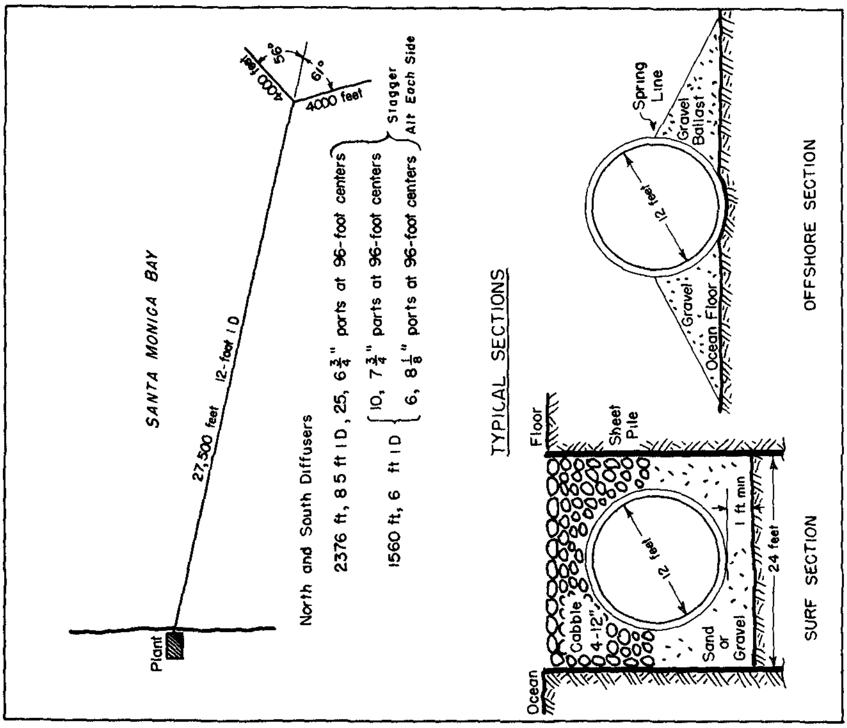

Fig. 10. City of Los Angeles effluent line

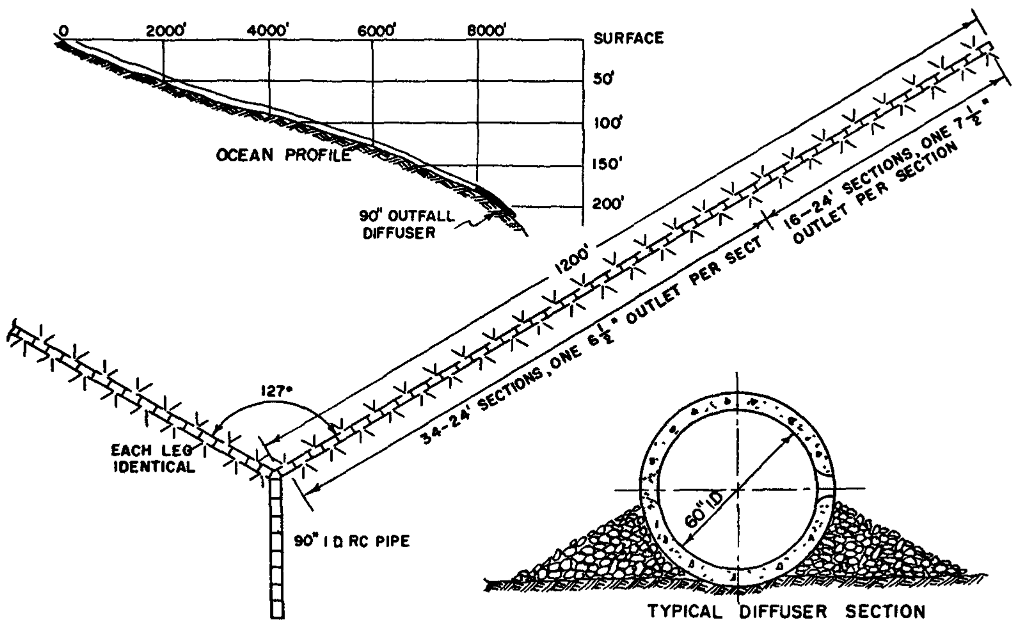

Fig. 11. Diffuser for 90 inch diameter outfall Los Angeles County Sanitation Districts No. 3

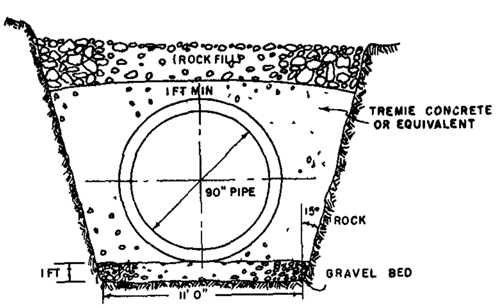

Fig. 12. Surf section, rock trench anchorage
Los Angeles County Sanitation Districts No. 3

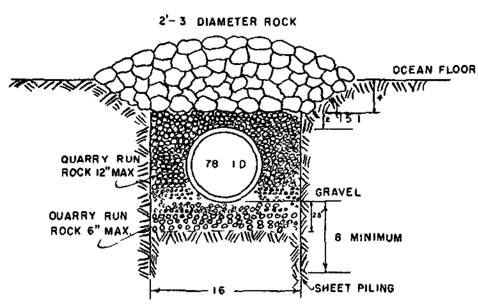

Fig. 13. Surf section anchorage Orange County Sanitation District, Calif. (1953)

crete pipe extending 5 miles offshore and submerged at the diffuser in 192 feet of water. The diffuser consists of two branches forming a 120° wye, each leg approximately 4000 ft. long and containing approximately 82 ports varying from 6-3/4 to 8-1/8 inches in diameter. Figu 10 presents a schematic view of the Hyperion, City of Los Angeles effluent line currently under construction as well as typical cross-sectic of the surf and offshore sections showing the method of construction ar type of anchorage employed.

Figure 11 presents a schematic view of the recently completed 90 inch diameter outfall line (No. 3) for the Los Angeles County Sanitation Districts.

Figure 12 and 13 show the type of construction employed in the surf zone for Whites Point Outfall No. 3 and the Orange County Sanitation Districts outfall, respectively. This type of construction is typical of that employed to protect the pipe in the surf zone.

Details of other outfalls in California, as well as in other sections of the United States are presented in a report by Pearson (11) to the California State Water Pollution Control Board.

SUMMARY AND CONCLUSIONS

The role and function of a submarine waste disposal installatior as a component of overall waste treatment and disposal systems is re viewed in detail. It is necessary to resolve the beneficial uses of the receiving water, the areas involved, and the appropriate water qualit criteria to protect the uses. Moreover, water quality criteria are necessary so that it is possible to quantitatively assay the performan of waste treatment and facilities in protecting the uses.

Consideration is given the significant oceanographic characteristics affecting the design of a submarine outfall dispersal system. The practical evaluation and importance of nearshore circulation systems, current structure, density-temperature structure, and the coefficient of eddy diffusivity in the design of submarine dispersion systems are discussed and quantitated.

Quantitative estimation of the concentration of waste in waste-sea water dispersion plumes is presented on a rational basis. The problem of initial mixing in the proximity of the diffuser is reviewed and a basis for estimating initial dilution is outlined. A definition sketch of the waste-sea water dispersion plume and solutions to the diffusion equation are presented. In this manner, a rational estimati of the concentration of waste (dilution or bacterial concentration) can be made if reasonable estimates of water mass velocities, mixing depths, eddy diffusivity, bacterial concentrations and dieaway or decay reaction kinetics are available.

Cost data on submarine outfalls are presented as well as details of the physical characteristics of some of the larger submarine waste disposal installations in California.

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CHAPTER 36

WASTE DISPOSAL IN MARINE WATERS

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Waste disposal in marine waters is of importance to coastal engineers because of the ever-increasing requirements for an effective means of dilution of both municipal and industrial wastes. There are many considerations that enter into a satisfactory waste disposal method by dilution, but to the engineer two major considerations must be made. These are, first, the rate of diffusion and, second, the level of dilution necessary to effect disposal. These considerations become quite involved because of the varying conditions which influence dilution or diffusion processes and the uses of the water, which determines the criteria for a satisfactory dilution level. Thus, the dilution levels necessary will be dependent on the nature of the material and the area of dispersal.

Generally, however, it might be considered that adequate dilution has occurred when the disposal of waste at the site of interest does not interfere with any recreational or commercial use of the area in question. Usually in the marine area the standard is set at such a level that the biological organisms endemic to the area are not harmed by the waste material.

Methods of measuring rate of diffusion have been under study for a number of years. Munk, Ewing and Revelle (1949) investigated diffusion in the Bikini Lagoon in which the radioactivity caused by the atomic explosion was used as a tracer of the water mass. Recently Seligman (1955) reported results obtained by the use of fluorescein in estimating diffusion in the Irish Sea. This work was carried out in a littoral region in order to evaluate a disposal method for dispersion of radioactive wastes from the Harwell Atomic Energy Plant. Ketchum and Ford (1952) studied the diffusion behind a barge at sea in which the ferrous ion was used as an indicator. In these experiments the peak concentration of the waste was difficult to determine because only discrete samples were obtained.

Recently radioactive isotopes, bacteria and dyes, have been used as tracers to measure diffusion rates for various applications. Ely (1957) reports on the use of Sc^{46} in following dilution of a sewage field in Santa Monica Bay. Cochrane (1956) applied P^{32} to trace sewage

TABLE I

TRACERS USED FOR DIFFUSION STUDIES

Tracer	Means of Detection	Limits of Detection	Hazard	Adaptability for Continued Analyses	Quantity/$10^6 M^3$	Expendables Cost/$10^6 M^3$
Sc^{46}	Scintillation	1.2×10^{-4} C/L*	Hazardous	Good	0.12 C	$360.00
Cs^{137}	Scintillation	2.4×10^{-4} C/L	Hazardous	Good	0.24 C	24.00
Se^{75}	Scintillation	1.2×10^{-4} C/L	Hazardous	Good	0.12 C	120.00
H^3	Scintillation	5.0×10^{-2} C/L	Low Hazard	Poor	13.5 C	1,350.00
Fe^{++}	Orthophenanthroline	20μg/L	No Hazard	Poor	44 lbs.	44.00
NH_3	Nesslerization	20μg/L	Little Hazard	Poor	44 lbs.	8.80
Lithium	Flame Photometric	3 mg/L	No Hazard	Fair	6,600 lbs.	53,000.00
Fluorescein	Fluorimeter	1μg/L	No Hazard	Good	2.2 lbs.	5.50
Serratia indica	Plate Count	10^3 C/L	No Hazard	Poor	10^3 L	Ca 350.00

*Based on requirement of 0.25 DPM/ml

movement. He also used an alternate method in which Serratia indica was added to the sewage and subsequently sampled and cultured to determine the rate of dilution. Serratia indica was also used by Robson (1956) to trace sewage pollution. Moon, Bretschneider, and Hood (1957) have reported on a method for measuring eddy diffusion in coastal embayments in which fluorescein dye was used as the tracer, and a continuous reading fluorescent meter was used as a detector. Hood, Stevenson, and Jeffrey (1957) used similar methods for detection of diffusion behind a barge at sea. Stommel (1947) has treated the topic theoretically.

If horizontal diffusion is to be known under the conditions of the environment of interest an experiment must be conducted which will allow diffusion to be directly observed. The most direct method appears to be one of addition of an easily observable tracer to the water mass, followed by detection of the tracer as an indicator of dilution.

SELECTION OF A TRACER

The selection of a tracer for use in diffusion studies is based on several criteria. These are (1) the tracer must be one which is, in a chemical state, compatible with the conditions existing in the study area; (2) the tracer must be easily detectable to a high degree of precision, preferably by a method adaptable to continuous measurement; (3) the tracer must not be hazardous to the environment or people working with the material while conducting the experiments; (4) the cost of the tracer must be reasonable; (5) the cost of capital equipment necessary for the dispersal measurement must also be reasonable.

A number of tracers have been contemplated for use and/or used, in diffusion studies. Table I gives a comparison of certain important considerations applicable to tracers for use in diffusion measurements. The radioactive tracers, which are gamma emitters which are found most useful for diffusion studies are Sc^{46}, Cs^{137}, and Se^{75}. These are all easily detected by scintillation counting with a water proof detector that can be towed to any desired depth in the sea. The limit of detection of all three is good, and the quantity of material necessary to label an area one square kilometer by a meter deep is not excessive. All of these elements are present only in trace quantities in the sea. The amount added is significant to the total amount present and, therefore, some consideration must be given to insure that absorption does not remove them from the water mass. Ely (1957) found that the addition of sodium versenate to Sc^{46} tracer minimized adsorption. The maximum level of a radioactive tracer that may be added to the water is the maximum tolerance level permitted by

AEC regulations. Assuming a detection level when diluted 1 part in 10,000 a top practical limit is then placed on the amount of water that can be labeled from a single discharge site. These isotopes are gamma emitters and are, therefore, hazardous to handle, particularly before addition to the water mass.

Tritium, which can also be detected by liquid scintillation methods, would be an excellent isotope for use in the form of tritiated water. Since total water present is infinitely greater than the amount of the water added, one would expect this tracer to follow the water mass without special removal by any other material and/or chemical process existent in the water. The chief disadvantage of this isotope, besides being somewhat more expensive, is that the technique for detection on a continuous basis has not been worked out. Other isotopes such as Na^{24} or Cl^{37} would be suitable tracers except for the difficulty of obtaining short half-life isotopes.

A number of chemical indicators are also feasible. Ferrous ion has been used as an indicator by Ketchum and Ford (1952). It may be detectec rather satisfactorily with either $\alpha\alpha$ diphyridyl or orthophenanthroline with a sensitivity which is adequate for the purpose. The cost of the test ion is low, but its use is difficult, since it is not easily adaptable to continuous measurement. Ammonium ion is very easily detected by means of Nessler reagent and is also relatively inexpensive, but is of questionable value because of the very short half-life of this material in a biologically active community. Lithium represents an ion that coul possibly be used in sea water, but the amount required for significant analytical evaluation is excessive, and the method of determination is not particularly good for continuous measurement.

Fluorescein, uranin, has been used on many occasions to trace water masses. It can be detected in very low concentrations, is adaptable to continuous analysis, and the cost of the chemical is very low. Recently Robson (1956), and Cochrane (1956) employed the use of the bacteria, *Serratia indica*, in tracing sewage effluents in the open sea. The orgai is then plated out, cultured on peptone medium, and the number of fluor cent pink colonies developing indicates the number of organisms present the water. The cost of adding sufficient quantities of these organisms represent a reasonably good dilution factor is not excessive; however, assay method will not lend itself to continuous analysis. Each of the above tracers may have specific uses in particular situations. In gene however, fluorescein seems to be as suitable as the other tracers and h considerable advantage in that its use entails no hazard, and the cost the tracer is considerably less.

TABLE II

CAPITAL EQUIPMENT

Tracer	Equipment	Approximate Cost
Sc^{46}, Co^{137}, Se^{75}	Scintillation crystal, scaler, recorder, and handling equipment	$4,000.00
H^3	Liquid scintillator, scaler, and sampler	6,000.00
Fe^{++}	Colorimeter and sampler	500.00
NH_3	Colorimeter and sampler	500.00
Lithium	Flame photometer and sampler	2,000.00
Fluorescein	Fluorescent meter, recorder and sampler	1,800.00
Serratia indica	Chemicals, sampler, and glassware	300.00

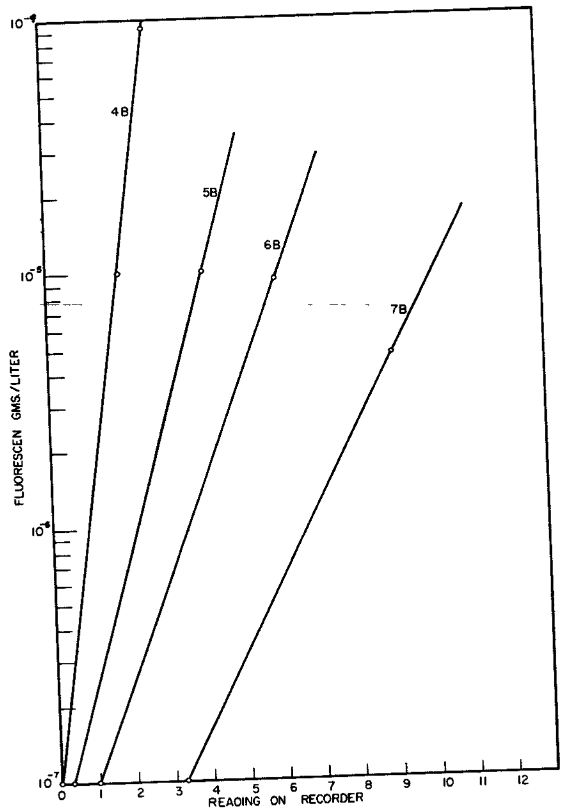

Fig. 1

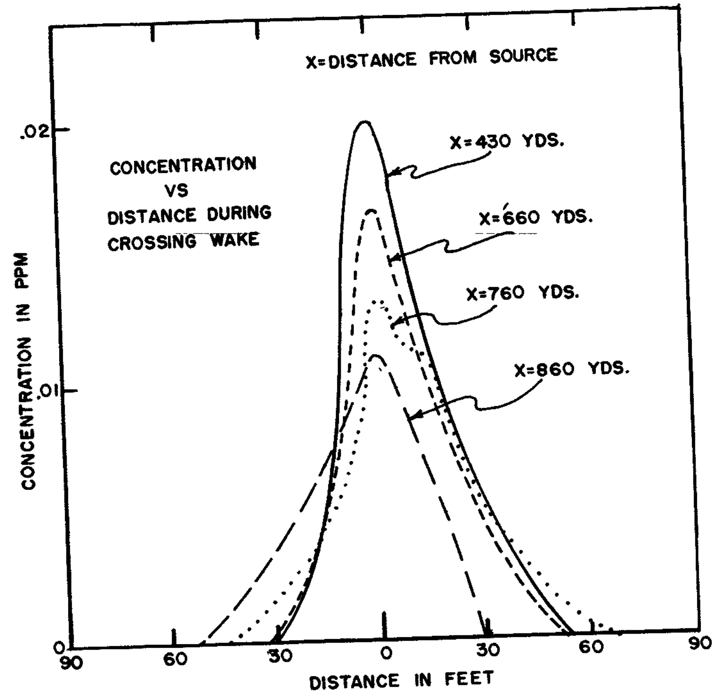

Fig. 2

Fig. 1. Concentration of fluorescein vs. scale reading at different instrument ranges. Solutions made from dilution of concentrate of fluorescein to proper value. Gulf of Mexico sea water obtained from surface at 400 fathom curve.

... concentration at various distances from point of introduction. Moon, et. al.

Fluorescein is known to decay in sunlight but for short period measurements, which are often adequate for diffusion studies, this decay would not be significant. At night, the compound is quite stable. Whether the difficulties encountered through the decay of fluorescein are greater than those incurred in some metallic ions through absorption or precipitation in sea water systems is at the present time indeterminant.

A comparison of costs for capital equipment to detect the various tracers discussed above are presented in Table II. In general, the methods which are applicable to continuous recordings require a greater capital investment. However, this difference in cost is usually offset by the decrease in personnel time required to obtain the data. The data obtained are also much more readily used in calculations than are those which are derived from discrete sampling. In view of the above considerations, studies which we have undertaken, to the present time, have all been conducted with the use of fluorescein.

DIFFUSION MEASUREMENTS

Instrumentation

A continuous recording fluorescent meter was designed and constructed for these studies. A detailed description is given elsewhere(Huebner and Hood, 1957, and Moon, 1955). In general, it consisted of a filtered ultraviolet light source which was directed to a low, fluorescent glass cell into which the sample was continuously pumped. The fluorescent light, caused by the ultraviolet absorption of the sodium salt of fluorescein, passed through glass filters (600-625 $m\mu$) and activated a photomultiplier circuit. The signal was amplified and fed to an Esterline-Angus Recorder. The instrument was standardized against sea water solutions of the sodium salt of fluorescein. The standard curves obtained in one environment are shown in Figure 1.

Field Work

A number of experiments have been conducted in different locations and under different environmental conditions to test the method. The first of these was a typical marine coastal region near Port Aransas, Texas; second, in the Gulf of Mexico in about 100 fathoms of water; and third, in the wake of a barge pumping black liquor wastes at the 400 fathom curve in the Gulf of Mexico.

In the first study, fluorescein was dispersed from a point source and the concentration of fluorescein downstream was determined in right angle crossing of the envelope at varying distances from the point of

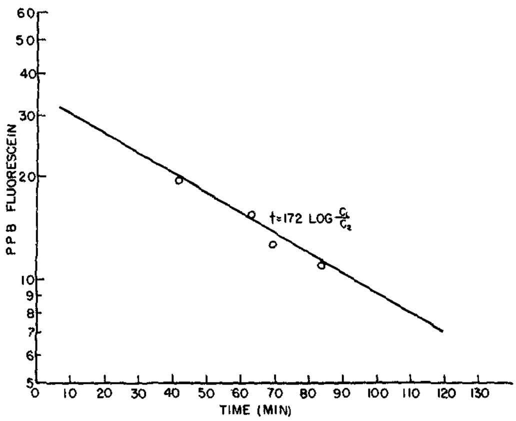

Fig. 3

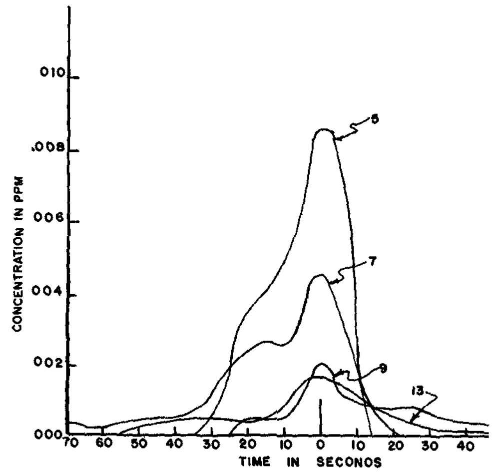

Fig. 4

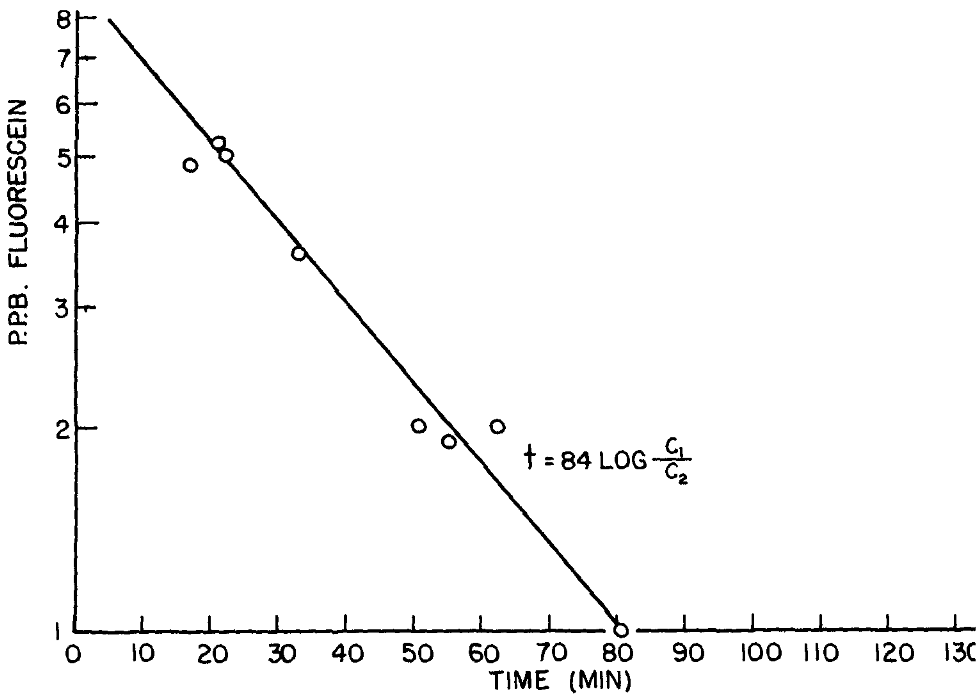

Fig. 5

Fig. 3. Rate of diffusion in tidal channel. Moon, et. al. (1957).
Fig. 4. Fluorescein concentration at various crossings of establis[hed] wake. Sea Temperature, 82°F; thermocline at depth of 175 feet; win[d] speed greater than 2 knots; sea surface, slick; location 27° 50' N, 30' W.
Fig. 5. Rate of diffusion in an established wake. Same as Fig. 4.

introduction. Figure 2 shows the distribution of fluorescein across the envelope at various distances. It will be seen that the curves are fairly symmetrical. The log of the concentration plotted against time from dispersal gives the relationship indicated in Figure 3. The details of these experiments are reported by Moon, et al. (1957).

The second experiments were conducted in 100 fathoms of water in the Gulf of Mexico. Three pounds of fluorescein dissolved in 20 gallons of sea water were dispersed over a distance of 1250 feet behind the R/V A. A. Jakkula in a manner so as to minimize the effect of ship motion on the tracer. Floating buoys were placed on either side of the labeled water, and the concentration of fluorescein in different cross sections was obtained at various times. During these studies, a current of approximately 1 knot was observed, but the sea surface was almost completely devoid of waves or swell. A portion of concentration profiles observed during this study are indicated in Figure 4. The peak concentration shown in this figure, plus other peak concentrations observed in the same study, are plotted against time in Figure 5.

The third study was conducted in connection with the deep sea disposal of a barge load of black liquor waste originating from the Champion Paper and Fiber Company of Pasadena, Texas. A barge containing 265,000 gallons of waste was labeled with 200 pounds of the sodium salt of fluorescein. Black liquor waste was loaded into the barge at a temperature 165°F and the fluorescein was added through the manholes in such a fashion as to assure uniform mixing with the entire content of the barge. At the time of addition of fluorescein to the barge, a sample of the black liquor was heated to 165° in the laboratory and suitable concentration of fluorescein dye was added. This, then, was retained as a standard for subsequent measurements at sea. The waste was pumped at the rate of 1.56×10^3 gm/cm on a course due south of the 400 fathom curve. Using the fluorescein as a guide, the A. A. Jakkula cruised in the center of the wake at various distances behind the barge while continuous fluorescein analysis was conducted. A copy of the traces obtained on the recorder of the fluorescent meter at 300 and 1,000 feet are shown in Figures 6 and 7, respectively. Reference to Figure 1 and Figure 6 indicates a range of concentration between about 10 and 92 ppb within a distance of approximately ten meters of boat travel (speed of vessel was 200 cm sec^{-1}). Because of a time lag in the sampling system of about 5 seconds and hold time in the sample cell of about 2 seconds, the extremes have probably been modified to some undetermined degree.

The above data demonstrate the stirring and mixing processes in incompressible fluids described by Eckhart (1948). The rapid fluctuations observed when samples were taken along the axis of the wake at constant time with respect to the introduction of the tracer are evidence of sharp gradients between the interfaces of the waste and the water. The energy dissipated by the tug and also by the barge and by pumping the waste is probably a major factor in creating the distorted masses of the two fluids. The 1,000 foot data shown in Figure 9 shows a decrease in the extreme of concentration gradients in the liquids. A difference of about 100 seconds in time of mixing is represented by the two traces. Continuous traces were also taken at other distances behind the barge. From these, the average values of concentration in the center of the wake were computed by graphical integration of several minutes of continuous record at each of the distances. A plot of the log of concentra tion against time for these data is presented in Figure 8.

There is some indication that the straight line plot of the data would tend to increase in slope as zero time is approached. This would be expected because of the greater energy being dissipated immediately at the site of pumping. However, more data will be necessary to determine the validity of this observation.

After observing concentration in the middle of the wake for some period of time, a floating buoy was placed immediately behind the barge in the center of the wake, and a study was initiated on the diffusion within a given water mass. These studies were conducted after dark. T data were obtained while crossing the wake of the barge at right angles the vicinity of the floating buoy. Two representative traces observed when crossing the wake are shown in Figures 9 and 10. The dispersion across the wake was such that peak concentrations, as used in previous experiments, did not yield satisfactory data for predicting the maximum concentration in the wake at any given time. Inadvertently, the analys equipment ceased to function before a sufficient number of crossings cc be made to complete the study of the diffusion pattern which occurred.

A total of fourteen crossings were made which extended over a period of two hours and twenty-four minutes. The area beneath each of the curves was estimated by a planimeter and the values obtained are reported in Table III and Figure 11. The data show that of the useful information obtained in these crossings the area beneath the trace on seven approached the value of 1.8 square inches. Three values were qu high and one very low. The indications are, however, that the fluores concentration in the wake at any time was constant, indicating little horizontal diffusion or decay of the fluorescein molecules after the initiation of the experiment.

It was not possible in these experiments to get accurate data on

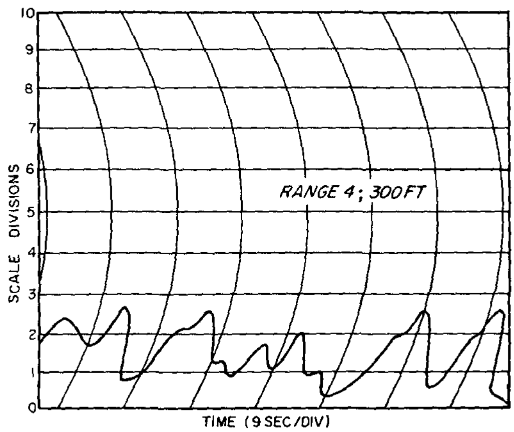

Fig. 6

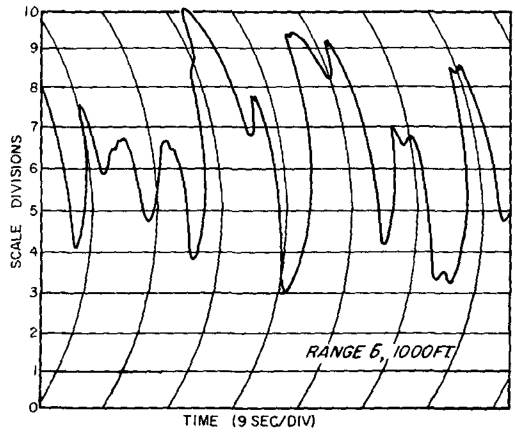

Fig. 7

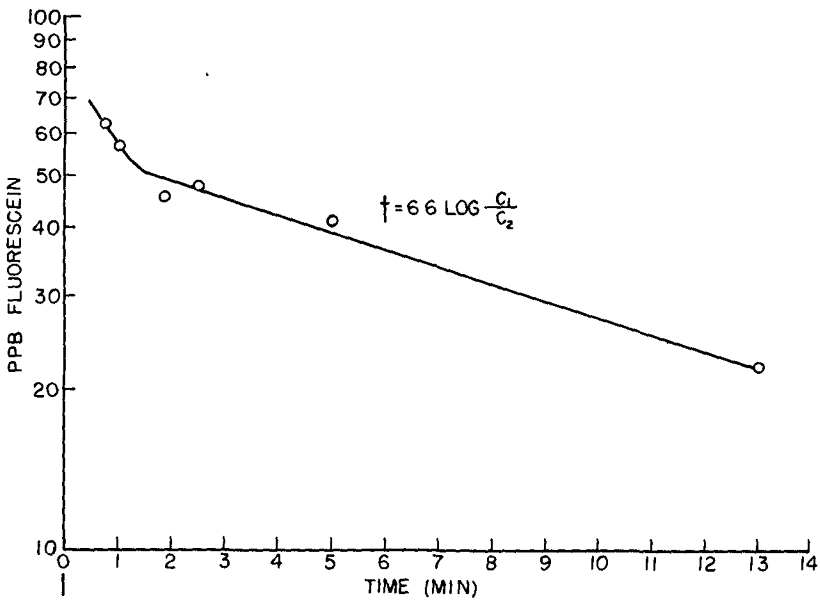

Fig. 8

Fig. 6. Concentration of fluorescein in center of wake. Distance, 300 feet aft of barge; pumping rate: of waste, 1.56×10^3 gm/cm; of fluorescein, 1.23×10^{-1} gm/cm; sea surface temperature, 78°F; strong thermocline at depth of 150 feet; wind speed, 10 knots; wave height, 4 feet; location, 27° 30' N, 94° 40' W.

Fig. 7. Concentration of fluorescein in center of wake. Distance, 1000 feet aft of barge. Other data the same as Fig. 6.

Fig. 8. Rate of diffusion in wake of barge. Conditions the same as Fig. 6.

TABLE III

AREA BENEATH TRACING AT EACH INTERCEPT OF WAKE

No.	Clock Time	Area, Square Inches	Elapsed Time, Minutes
2	1823 CST	2.01	9
3	1826 CST	1.85	12
4	1837 CST	1.80	23
5	1908 CST	0.49	54
6	1917 CST	1.78	63
7	1920 CST	1.00	66
8	1925 CST	1.64	71
9	1928 CST	3.60	74
11	2030 CST	3.60	136
12	2033 CST	6.78	139
13	2037 CST	1.80	143
14	2040 CST	1.74	146

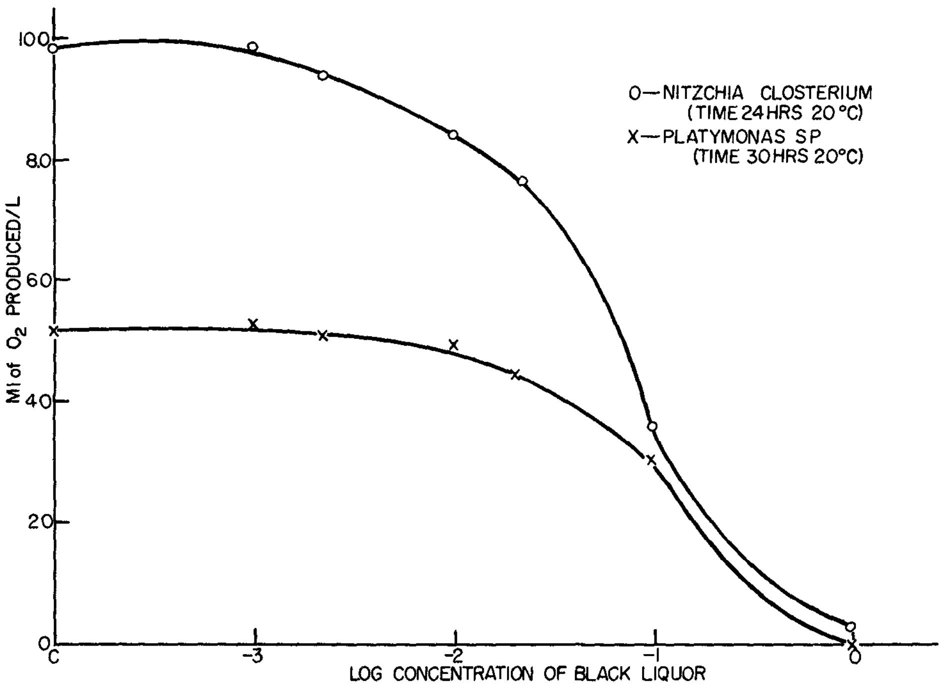

Fig. 9. Copy of pen trace of recorder (directly related to fluorescein concentration) for 6th crossing of wake of barge (1917 CST). Other data the same as Fig. 6.

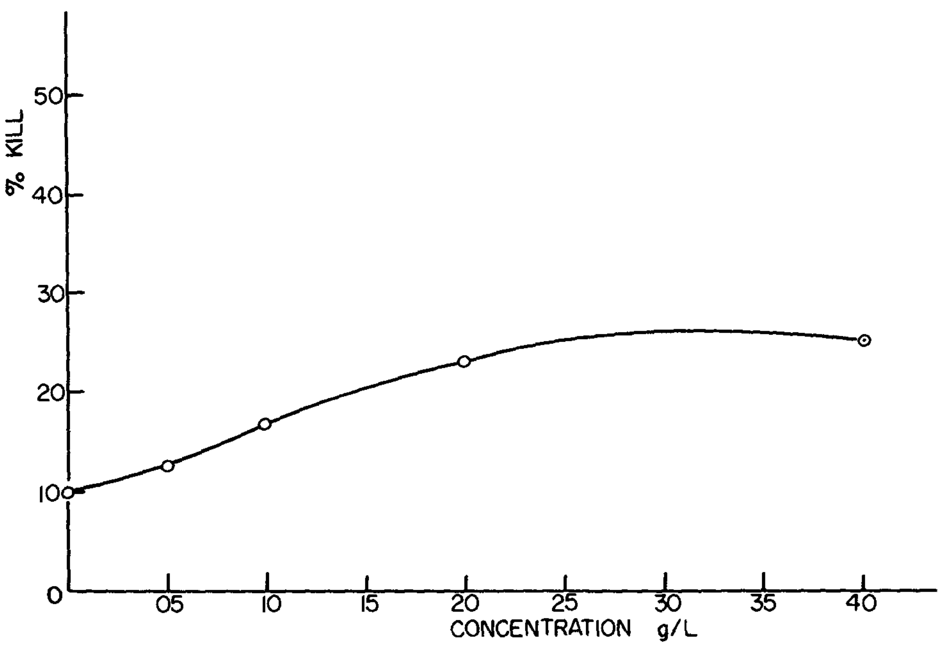

Fig. 10. Copy of pen trace of recorder for 13th crossing of wake (2037 CST). Other data the same as Fig. 6.

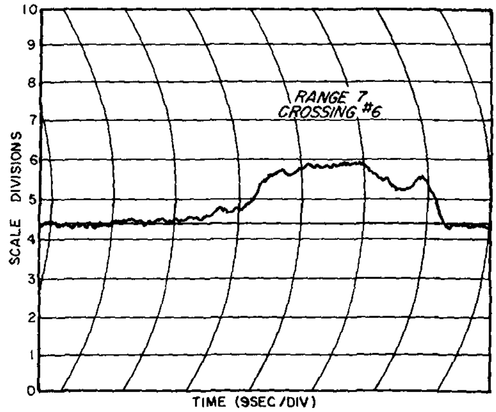

Fig. 11

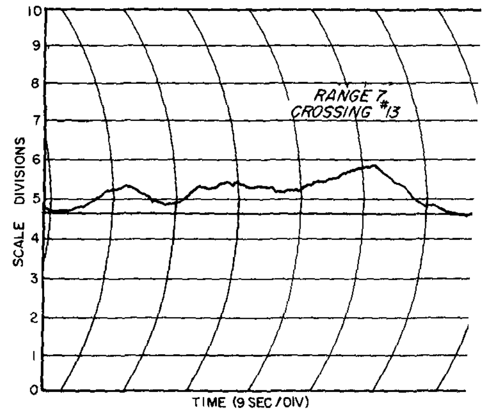

Fig. 12

Fig. 11. Area beneath chart trace at crossing vs. time after pumping barge (1816 to 2040 CST).

Fig. 12. Effect of black liquor wastes on photosynthesis of two Marin Phytoplankton. Hood, et al., (1955).

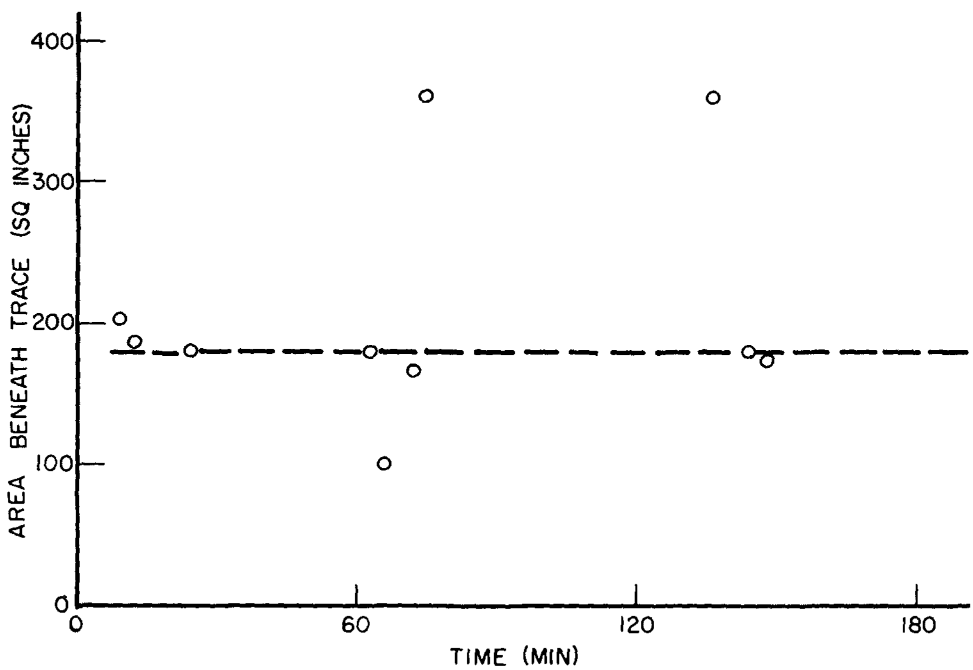

Fig. 13. Toxicity of black liquor waste to mixed zooplankton. Orga collected from Lydia Ann Channel, Port Aransas, Texas.

the concentration of the fluorescein with depth in the wake. However, attempts were made to make vertical drops of the sampling device in order to determine the depth of peak concentration. Results indicated that maximum concentration appeared near the surface, or at about the depth of sampling (8 feet), but remained fairly uniform to about 150 feet or the limit of the maximum depth of sampling.

Criteria of Dilution

The second important consideration that faces the waste disposal engineer is that of determining the dilution which is necessary in order to avoid contamination or deterioration of the water mass beyond limits that will interfere with its present use. A number of techniques, and methods, have been devised for this purpose. Among these are fish toxicities, biochemical oxygen demand, chemical oxygen demand, direct chemical analysis, and other biological and/or physical means of estimating the level of pollution. It is out of the scope of this discussion to go into details of the various methods employed and the value of each. However, it seems pertinent to describe briefly the procedure that was recently developed by Hood, Stevenson, and Jeffrey (1957) for detection of low level effects to the basic members of the biological community.

It is generally accepted that the most fundamental organisms in the marine environment or any aquatic environment are phytoplankton. Upon these all other forms of life depend for food. Since these organisms are important to the economy of any marine area, it follows that adequate dilution or waste disposal treatment must of necessity take into consideration these basic organisms. To evaluate the effects of waste on phytoplankton, a technique has been devised which determines the effect of a given waste on the photosynthetic production of a number of pure cultures of phytoplankton normally found endemic to coastal regions.

The method consists of placing a uniform number of algal cells of proper age in a medium to which has been added gradient concentrations of waste material. The solution is diluted to volume in standard oxygen bottles placed at constant temperature in a lighted water bath. After the proper duration of time (about 24 hours), the amount of oxygen produced during this period is determined. The difference between the oxygen produced by organisms in solutions not containing waste and those to which wastes have been added indicate the effect of waste material on the photosynthetic process of the organisms. The results of an experiment of this type are shown in Figure 12. From these data, it may be seen that the concentration of waste which affects the photosynthetic process of Nitzchia closterium is about 5 ppm and a similar value for Platymonas sp. is observed. Nitzchia closterium is a diatom having wide distribution in marine areas. Platymonas sp. is a green algae which is also found in marine coastal waters.

A second community which is very important to the general ecology and food supply of organisms of any region are the small animals called zooplankton. These consist of large groups of small crustaceans known as copepods and euphausids; also larval stages of invertebrates, fish eggs, and larvae. In addition, small forms of many other groups occur. The toxicity of the waste to these organisms is shown in Figure 13. The data obtained show only little toxicity in concentration of black liquor waste as high as 400 ppm. Greater concentrations cannot be used with the method employed for indicating the per cent kill. Attention is called to the difference between the concentration levels which affect phytoplankton photosynthesis and those which are lethal to the zooplankton. The time of exposure of the waste to zooplankton was only about one hour, and, this may account, in part, for the differences observed.

Based on the above data, a marine disposal operation which involves black liquor wastes would require a dilution of something below 5 ppm so as to avoid the impairment of the photosynthetic production of phytoplank It must be realized that this is only one datum point and that other considerations must be made. For example, the biochemical oxygen demand for black liquor wastes is about 50,000 ppm, and in many cases may be a limit factor in dilution.

Discussion

The above data may be used to demonstrate the effect of the environment on rate of dilution of waste dispersed in marine waters. If the diffusion curve shown in Figure 8 is extrapolated to zero time, the concentration of fluorescein in the wake immediately upon pumping would average about 56 ppb. If the assumption is made that the waste distributes in the wake the same as the fluorescein, a direct ratio may be used to compute the concentration of the black liquor in the water upon pumping. The concentration of black liquor in the sea at zero time would be 710 ppm. By substituting this value, and the 5 ppm level estimated to be non-effective to phytoplankton photosynthesis into the equation of the diffusion line, a time of about 14 minutes would be required for the black liquor waste to diffuse to a non-effective level under the conditions investigated. The same data used in estimating the time required for dilution in an established wake under the conditions studied would require about three hours. Similarly, under conditions existent in the tidal channel near Port Aransas, a time of about 62 hours would be required assuming adequate water for dilution was available.

These data indicate the value of dispersal of wastes under situations in which dilution occurs rapidly and under conditions where maximum mixing occurs. It is also evident that dispersal of waste in deep sea situations is much to be preferred over that in littoral or in-shore zones not only because of the greater rate of diffusion observed, but because the number of animals and plants in this area are minimal and, at the present time, are not exploited either industrially or recreationally.

By the use of the techniques described here, it becomes practical to study the problem of waste disposal in marine waters in a quantitative manner. If an estimate can be obtained with the quantity of effluent to be discharged from any point, a more or less complete evaluation of the effect of this effluent in the immediate vicinity can be determined. A rather detailed evaluation of the environmental conditions along with studies of the diffusion rate existent associated with these conditions will allow one to determine the extent of pollution which would arise from the discharge site and to estimate the total area influenced by the disposal operation.

Proper biological controls by approved methods, or by newer special methods, such as are suggested in this paper, allows a limit to be set on the quantity of effluent which can be discharged from single site under a given set of conditions. It is important that the combination of these two factors be taken into consideration in order to provide adequate waste disposal.

Acknowledgments

This is a contribution from the Department of Oceanography and Meteorology of the Agricultural and Mechanical College of Texas, Oceanography and Meteorology Series No. 99, based on investigations conducted for the Shell Chemical Corporation, and Champion Paper and Fibre Company of Houston, Texas.

The assistance of Mr. Charles Bretschneider and Dr. George Huebner in the collection of the physical data and instrumentation, and Mrs. Bernadette Stevenson in collecting biological data is acknowledged.

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CHAPTER 37

GENERAL ASPECTS OF A STUDY ON THE REGIMEN OF LAKE MARACAIBO

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Maracaibo Basin in Western Venezuela, Figure 1, has an area of 90,000 square kilometers. It is isolated from the rest of Venezuela on the East and South, and from Columbia on the West, by mountain ranges which reach a height of 5,000 meters at the southern boundary of the Basin. Lake Maracaibo and its marginal swamps cover 17,700 square kilometers of the Basin. The Lake proper, approximately 150 kilometers long by 110 kilometers wide, is connected to the Gulf of Venezuela by the Straits of Maracaibo and broad shallow Tablazo Bay. It is about 50 kilometers from the lake proper to the gulf. Tablazo Bay is separated from the Gulf of Venezuela by a series of shifting sand islands and bars.

The Lake Maracaibo Basin contains one of the world's large oil fields. Creole Petroleum Corporation as the largest producer of oil from this area, has a deep interest in all the factors which affect the region. From time to time we have investigated the phenomenon of nature. The principal characteristics of the region which have been studied so far are: the climate, rain fall, lake currents, lake salinity, tides, and the action of the channels and bars between the lake and the Gulf of Venezuela.

Since the entrance conditions to the lake vitally affected the transport of oil from the fields to the world's markets they were the first to be studied by the oil companies. When oil started moving out of Lake Maracaibo the draft of the vessels was limited to nine feet by an outer and inner bar, Figure 2. The channel across the outer bar was not fixed because of the shifting sand, generally the channel moved from east to west until it reached an extreme position and re-opened a new channel to the east.

The inner bar covers the whole of Tablazo Bay and it is approximately 21 kilometers across. Weather on the outer bar is generally rough as waves are built up by the north-easterly wind across the Gulf of Venezuela. On the inner bar it is comparatively calm since it is protected by the chain of islands which form the outer bar.

The first step in the study of the Bar was to form a permanent survey body which was organized by the joint action of the oil companies in 1923. Surveys made by this party determined that the natural channel migrated westward and accelerated its movements during certain periods to as much as three feet per day. In 1935, the channel had reached the extreme western position of its cycle of migration and had deteriorated to the extent that the operations of the shallow draft tankers were seriously handicapped. Records showed that a new channel would break through the bar somewhere to the east of the deteriorating channel. This cycle of migration consumed a period of about 20 years.

As oil production increased, the limitation of the bar became more serious. The oil companies found that they must open a deeper channel. A model was prepared by the Waterways Experiment Station at Vicksburg, Mississippi, to determine the most practical route for a dredged channel. The tanker Invercaibo was converted to a seagoing hopper dredge and commenced operations in 1939. Up to 1947, she completed and maintained a 20 foot high water channel. At the same time, starting in 1940, a pipe line dredge deepened the 21 kilometer long channel across the inner bar.

Additional hopper dredges assisted the Invercaibo from 1947 to 1949 but it was evident that this channel did not fulfill the needs of navigation into Lake Maracaibo. Several studies of a deep draft channel indicated tha it was possible but the studies differed in their recommendations and in the estimated cost of the project.

In April 1953, the Venezuela Government initiated a project to provide 35 foot channel through which ocean going vessels could enter and leave the lake unhampered by the former restricted draft. The southern section of the project is a channel 600 feet wide from deep water of the lake to the northe limit of the inner bar, a length of 22.5 kilometers. In the northern section the channel widens to 1000 feet for 12 kilometers across the outer bar. The southern end of the outer channel is protected by a rock breakwater 3.2 kilometers long constructed approximately 1 kilometer east of the channel.

This channel has now been in operation for approximately a year. The channel across the inner bar has been stable and requires only minor dredgi by a pipe line suction dredge to maintain it. The maintenance dredge startc widening and deepening the channel at the southern end this year as the ne: step in providing an outlet from Maracaibo for the still larger tankers ent(ing the world petroleum trade. As was expected, the outer channel require: constant dredging to maintain its position and depth. The rock breakwater protects the inner part of the channel from the westward drifting sand but there is constant sand encroachment from the east for some three kilometers north of the outer end of the breakwater. The Venezuelan Governmen. expect to maintain this channel and gradually deepens it by an ocean going hopper dredge which is now under design.

The bars have protected Lake Maracaibo from the encroachment of salt water. The opening of a deep channel from the Gulf of Venezuela into the l naturally raised questions as to its effect upon the salinity and other characteristics of the lake. Creole is interested in any change in salinit Although our oil field organizations use well water for drinking they depen upon lake water for irrigation, sanitation and industrial uses. The lake furnishes fish and shrimp for food. Any change of salinity would directly affect the animal life and would change the incidence of attack from marine borers. One of our major oil field problems is corrosion and it would be aggravated by an increase in the salt content.

In order to be prepared for future changes in the natural conditions, Creole undertook a study of the lake to determine the factors which affect: its salinity, currents, wave action and any other factor which might be di covered during the work. We requested the Woodshole Oceanographic Institu to make the first studies, set up a program of data gathering, interpret t data which they obtained and help us form an organization to carry on the work. The study started in April 1953, under the direction of Dr. Alfred Redfield.

It was thought desirable not only to secure a description of the distribution of salt in the lake as it existed in 1953, but to attempt to dra a consistent picture of the processes, motions and influences which determ this distribution as it exists and as it may vary from time to time. Dr. Redfield's group set up a program to measure water temperature, oxygen,

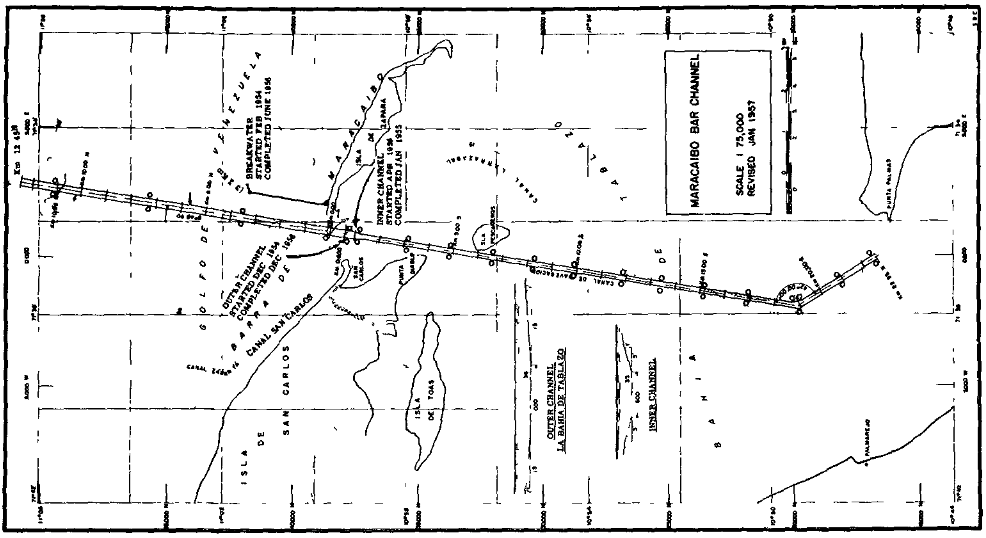


Fig. 2. Maracaibo Bar Channel.

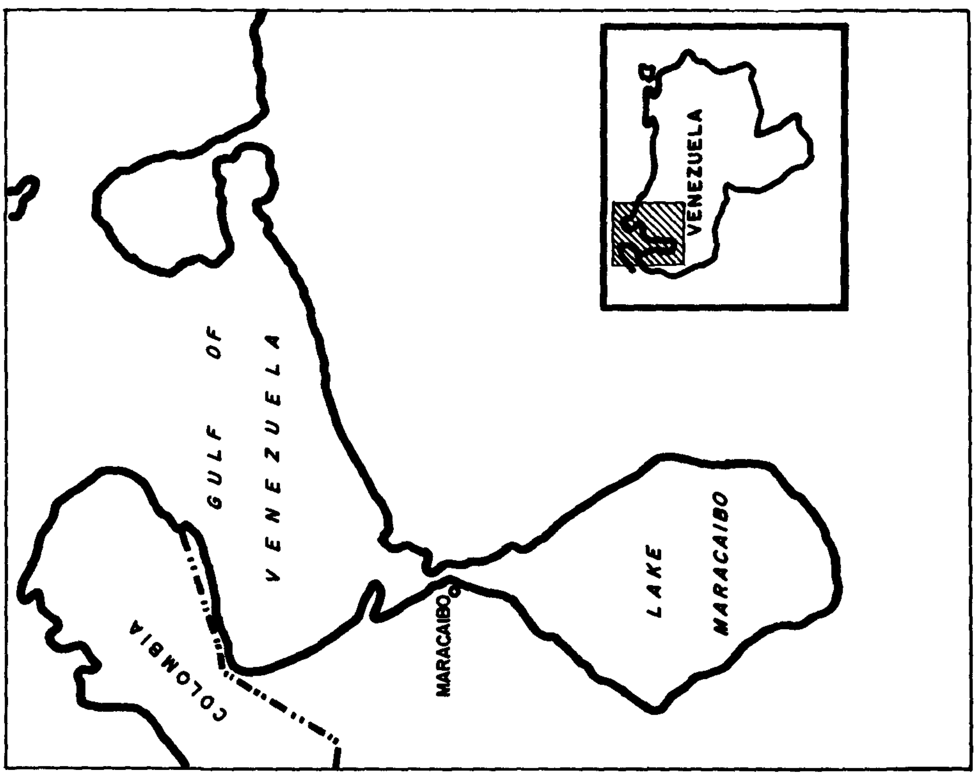


Fig. 1. Vicinity map

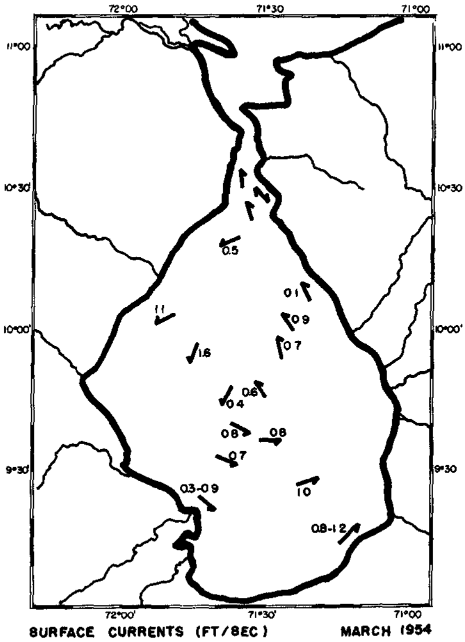
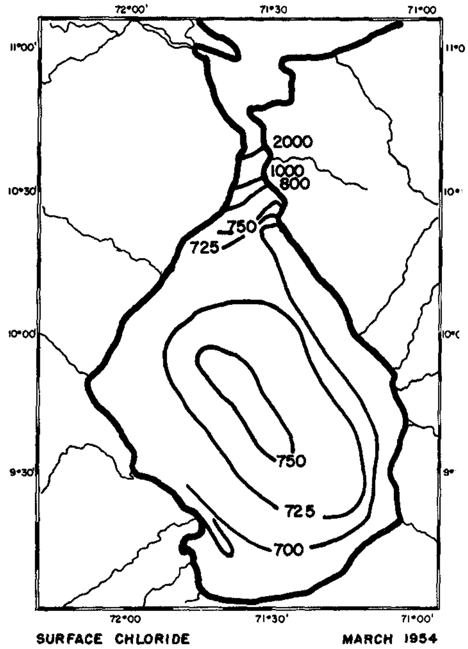


Fig. 3. Surface currents - ft. per sec. (March 1954).



Surface chloride (March 1954)

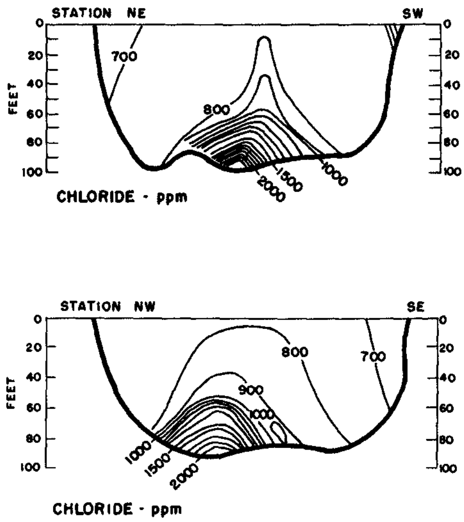


Fig. 5. Chloride concentration with depth

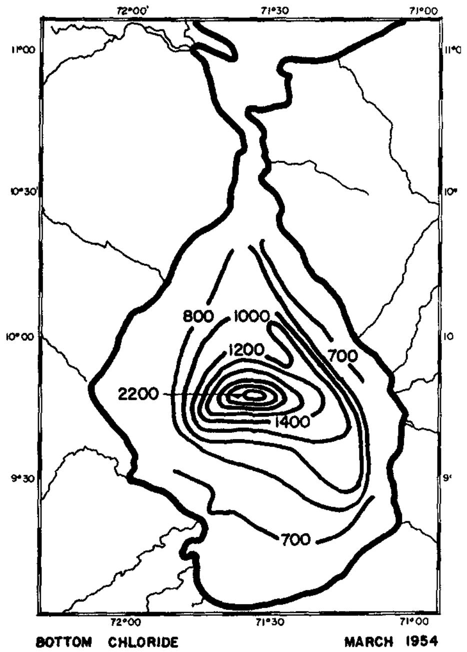


Fig. 6. Bottom chloride (March 1954).

phosphorus and chlorides. They also established stations in the Strait of Mara to observe tide cycle and the currents. They examined the available data of the salt content of the lake at previous times, pertinent information on tides, sea level, rain fall and winds. From this information they set up a program of observations to be made at intervals to provide critical information on the sea sonal and long term change in lake water. After the Spring observations by the representatives of the Woodshole Oceanographic Institution Creole personnel con data gathering until March 1954 when the representatives of Woodshole returned for additional observations. Since that time Creole staff has continued to gat data to follow the changes in the lake as they occur.

As might be expected, the study brought forth facts which were a surprise us. It was commonly thought that the chlorides varied from a maximum at the nor of the lake near the entrance to a minimum at the south. Field observations di not bear this out. The surface chlorides were found to be uniform around the edge of the lake and to increase towards the center. The cause of this distribution was the counter-clock wise circulating current in the lake.

This current is probably caused by the wind. The velocity of the current diminishes from the surface toward the bottom and has a mean value at the surface of the order of 0.7 foot per second. At this velocity a particle of water mid-way between the center of the lake and the shore would require about 10 days to complete the circuit of the lake. The current appears to fluctuate slightly as the result of semidiurnal tidal components. The velocity declines as the season advances and may become immeasurable in mid-summer. Figure 3 show the surface currents, in feet per second, which were measured in March 1954.

The lake contains two distinct classes of water which can be distinguished by the chloride content:

A. The Epilimnion, or upper layer, in which the chlorides vary only slightly from place to place, and with deptah.

B. The Hypolimnion, or deep layer, in which the concentration of chloride is distinctly higher and increase with depth.

The Epilimnion contains the large majority of the lake's water. In 1954, it constituted 90% of the volume. Its uniform chloride content indicates that the circulation and the turbulence due to wind-waves are very effective in mixing the fresh and salt water.

Chemical analyses of the chloride and other salts of the lake water show that the salinity is derived from the waters of the Caribbean Sea.

In 1953/54 the average chloride of the epilimnion was 660 PPM. The mixed water of the epilimnion consequently was a mixture of about one part Caribbean sea-water and 30 parts fresh water.

The concentration of chloride in the surface water of the lake, as observed in March 1954, is shown in Figure 4. The chloride is highest at the center of the lake and lowest in a band extending along the shore from the southwest side of the lake to the northeast. Over a greater portion of the lake's surface the chlorides range from 700 to 750 PPM in 1954, a variation of 7%. This general distribution of surface chlorides agrees with data collected by early observers and by the Woodshole Survey of 1953. During recent years chlorides in the epilimnion have ranged from 400 to 1400 PPM.

The more saline water of the hypolimnion occupies a cone with its apex at the center of the lake. The chloride concentration over the bottom of the lake in 1954 is shown in Figure 6 and should be compared with the surface chloride in Figure 4.

The salt of the hypolimnion originates from sea water which finds it way periodically into the lake from the Gulf of Venezuela. The mixturé of sea water and lake water which is produced by tidal mixing along the approaches to the lak has a greater density than the lake water. On entering the lake this water sinks the deeper parts to form the hypolimnion.

The concentration of chloride with depth is shown in Figure 5 representing two sections, made in March 1954, which crossed the lake obliquely at an angle of 60° from one another. Above the 800 PPM isochlor, the chloride concentration varies very little with depth. Below the point of 800 PPM the chloride increases gradually but at accelerating rate until it exceeds 2000 PPM at the bottom near the center of the lake.

Dr. Redfield suggests the rotary circulation of thè lake causes the dense water of the hypolimnion to be spun up into a cone with its apex at the vortex of the eddy. As a result the hypolimnion is withdrawn from the bottom around the margines and the epilimnion occupies the entire water column even to a depth of 100 feet. The epilimnion and hypolimnion mix most intensely in the vortex and consequently this area is the pricipal source of the salt in the epilimnion and the chlorides in the lake surface are highest in the center. He concludes that the velocity of the lake's circulation is adequate to cause the observed accumulation of denser water of the hypolimnion at the center of the lake.

Between 1954 and 1957 the hypolimnion disappeared completely from Lake Maracaibo. This unexpected phenomenon ocurred during the time that the outer and inner bars were being dredged. Sometime early in 1957, the hypolimnion started to reform as water of higher salt content entered the lake.

The concentration of salt depends upon a balance between the inflow of fresh water from the run off of rain fall, losses to evaporation, and the introduction of salt water from the Gulf of Venezuela by tidal exchange. The salt concentration varies with the seasonal variation in these factors. The stability of the system depends upon the relation between the volume of the lake and the rate of exchange of its water.

A study of the water balance of the Lake Maracaibo Basin by Mr. Douglas B. Carter indicates that the mean run off from the land area of the basin nearly equals the evaporation from the lake, approximately 32 x 10^9 cubic meters per year. The precipitation on the lake proper approximate 22 x 10^9 cubic meters per year and is of the same order of magnitude as the net volume of fresh water supplied to the lake.

The mean monthly values of the fresh water added to the lake is shown in Figure 7. The mean monthly values of the evaporation from the lake surface are relatively constant throughout the year, but precipitation and run-off vary in such a way that during February, March and April less water is gained than is lost by evaporation.

Changes in tide levels affect the inflow and outflow from the lake but in general terms we can expect that during nine months of the year there will be a net outflow from the lake whereas for three months there will be

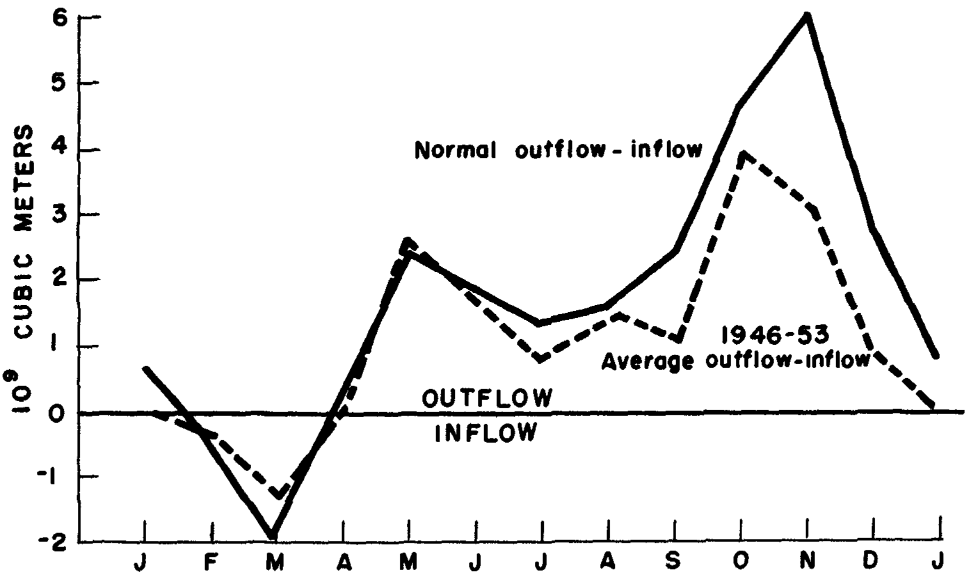


Fig. 7. Outflow and inflow of the Maracaibo basin.

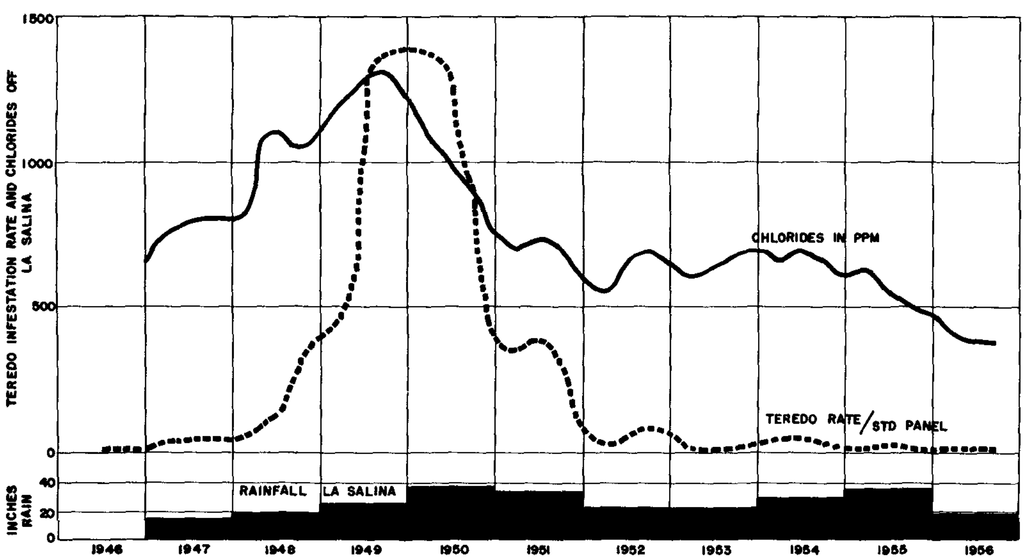


Fig. 8. Relationship between rainfall, salt content and Teredo infestation in Lake Maracaibo.

a net inflow. Our studies have not covered enough years to determine if this cycle is an average or how much it varies from the mean during years of high rain fall and years of low rain fall.

Carter estimates that the exchanged volumes of water are as follows:

Net Accession fresh water	21.34 10^9	cubic meters per year
Inflow sea water	3.3 10^9	cubic meters per year
Outflow lake water	24.37 10^9	cubic meters per year

With the knowledge of the mechanics of Lake Maracaibo gained from this excellent study by Dr. Redfield and his associates, we have prepared Figure 8 which shows the relationship between the rain fall, the salt content of the lake, and the attack of teredo Navalis, the marine borer which has caused so much destruction among our marine installations. We are now able to explain some of the anomalies of observed marine borer attack and can understand why materials tested at different times in the lake show varying resistance to mari borers.

It is too early to determine if the dredging of the deep channel through the Maracaibo bar will influence the inflow of the heavier salt water. Any effect of the channel which may have been produced to-date has been masked by a series of rainy years with high run-off. We expect to continue observations to see if a long time trend can be determined.

CHAPTER 38

PROBLEMS WITH SMALL CRAFT HARBORS

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Administration of a small craft harbor includes, among its problems, those of finance, law, public relations, policing, and engineering. This paper will relate experiences in all of these categories, but as it is written for Coastal Engineers, it will cover the engineering category most completely.

PROBLEMS OF FINANCE

Problems of this type vary considerably from harbor to harbor, as there are at least four different types of harbors which can be classified by the main service rendered.

HARBORS OF REFUGE

This class of harbor is very likely to be located in a remote section of the coastline and therefore the tax base and revenue from which finances may be derived is nill, or very small. Consequently, finances are very limited and these badly needed harbors are not constructed as much as needed. Harbors not used primarily for refuge are not included in this classification. Harbors of refuge serve the transient sailor. It is unlikely that any local government will accept the financing responsibility. This is more suited for State supported funds.

MARINAS

This type of harbor is suited to the use of harbor revenues for means of construction and operating finances. Most of the protected water area is devoted to boat slips and other revenue producing business. The main problem is to insure that harbor space demand is great enough to justify the necessary expenditures.

RESIDENTIAL HARBORS

Here is a type of harbor where people, other than boat owners or operators, are benefited. Homes along the waterfront and immediate harbor area are high in value and owners profit without using the harbor. People living within easy driving distance will come to the harbor, providing they have access to view it, to swim, or to enjoy it in any manner possible. All this benefits the property owner within the district of the harbor. Here is a case where a tax on property within this district is a logical means of raising necessary finances.

RECREATIONAL HARBORS

Every small craft harbor could be classified as recreational. Here the term is used to designate the type of harbor where private property is not a part of the development and where the public has full use of al facilities. Financing here again would best be gained by revenue from harbor use and would probably need to be supplemented by public funds. Recreation is necessary to keep the nation healthy, and cannot be expect to be entirely self supporting. Every attempt should be made to lighter the public economic burden of operation by developing sections of the harbor which will be revenue producing.

Actually, in the practical sense, most small craft harbors are included in more than one of these classifications. Each classification presents a basic financing problem which must be solved. The people whc are served by a harbor, should contribute a definite share towards its financial operations.

PROBLEMS OF LAW

In California the State "HARBORS AND NAVIGATION" Code delineates th jurisdictional powers under which the harbor administration operates. The Code distinguishes between Harbor Districts, Harbor Improvement Districts, Joint Harbor Improvement Districts, Port Districts, River Por Districts, and Recreational Harbor Districts. Therefore, each harbor ha a particular set of laws depending upon how it was organized.

At times, the law is not clear to those who operate the harbor and recourse must be made to seeking an opinion from legal counsel. Under the Code, powers are given to local jurisdictional bodies to pass ordinances to govern their harbor. Local ordinances may be <u>more</u>, but <u>never less</u> restrictive than the State Code. In the case of the Corps of Engineers, U. S. Army, who are the <u>authority</u> as to navigational aspects of a harbor, this also holds true. More restrictive harbor lines such as Pierhead and Bulkhead lines may be established by local authorities, but the <u>bayward extent</u> is set by the Corps of Engineers.

PUBLIC RELATIONS

As is the usual case with any worthwhile enterprise, good public relations are most important in the successful operation of a harbor. The harbor administrators must work harmoniously with Federal, State, County, City, and District officials in all levels of government. Permits for harbor structures and dredging, which are not adequately covered by <u>local</u> law, must be forwarded by harbor authorities to the Corps of Engineers for final approval. In problems of water pollution and highways, harbor authorities are required to work such problems out with State, County or District officials. They may also have to act between the public and other officials concerning flood control and right of way problems.

SWIMMING VERSUS BOATING

One difficult public relations problem is to settle disputes between the public over rights of using harbor waters. Swimmers demand beach space where other harbor users request piers. Where waterfront owners have property to the waters edge they usual expect and receive pier permits. However, where there is public land between property owners and the water, the problem is not so easy to settle.

A fairly successful formula has been used in Newport Harbor, California, to satisfy inland and waterfront owners on an island where a public walk surrounds the island, separating the beach from property owner's land. First, two or three beach locations are reserved on each side of the island for swimmers. No piers or moorings are permitted in these areas. Applications for piers at other locations are considered on their own merits. Each applicant must submit ten or more signatures of non-pier owners living within a 500' radius of the proposed pier location. At least 50% of these signatures agreeing to the pier installation must be those of inland lot owners. If the proposed pier location is used as a bathing beach, the applicant probably would not be able to secure the required signatures and therefore is not eligible to make application.

OUTBOARD MOTORBOATING

Another harbor use problem becoming more serious each year, is caused by the tremendous growth of the number of outboard motorboats. Nearly 600,000 such units were sold last year in the United States. The people using these boats want to waterski and race in protected waterways. However, a crowded harbor must maintain a speed limit too slow for these activities. One answer to this problem is to set aside areas limited only to these uses or to set specific time periods when such use will be permitted. Ordinances and policing must be revised to cover the added hazards to public safety caused by these activities. It is difficult for the outboard operator to judge his speed, as there is no shaft revolution counter to indicate speed. This is no reason to permit outboards to exceed the lawful speed limit. However, it takes a good job of public relations to convince these operators that they are not being cited unfairly.

SPECIAL EVENTS

During the course of a year, several groups may wish to stage events which will interfere with the normal harbor operation. Some will have a beneficial effect on the community. Crew races, speed boat races and water carnivals are of this nature. Schedules can be set so that the special event will interfere little with normal navigation and yet be effective in its intended purposes. Intermissions to permit normal traffic, also make the events less of a nuisance to regular harbor operations.

HARBOR POLICING

Harbor operation covers many phases of policing. Protection of life and property; enforcement of laws and navigation; prevention of pollution of bay waters and improper mooring of vessels, and many other harbor operations, are the everyday duties of the harbormaster and his patrol.

PROTECTION OF LIFE AND PROPERTY

To do this job, the harbor patrol must be equipped properly. Spe is of the essence. To police the 700 acres of Newport Harbor, the Ora County Harbor Department operates three speed boats approximately 20' long and able to travel faster than 30 miles per hour. Each boat is equipped with a two way radio and powdered chemical fire extinguishers During the summer, life guards accompany the boats, as they can arrive much sooner at the beach accident scene in this manner than they could through crowded streets.

A fast, radio equipped fireboat is a necessity. The first few minutes fighting a boat fire, or explosion, can mean the saving of man; lives and much property. This harbor department has a 27' fireboat capable of over 30 miles per hour. It is equipped with a separate engine driving a 300 gallon per minute fire fighting pump which is connected with a reel mounted, high pressure hose. Fog applicators and foam attachments add to its effectiveness. A sturdy tow bitt mounted amidships, permits towing a flaming vessel clear of other moored craft.

In addition, the patrol has two heavy duty work boats, each with radios and tow bitts. For patroling the shallow Upper Bay, where speeding of outboards and waterskiing is permitted, the harbor department operates two fast outboards - also equipped with radios.

The nerve center, coordinating the rescue calls to and from all patrol boats, is located at a central office. Here constant radio watc are maintained, except at night, when calls originate from the City Pol Department direct to the patrol boats.

ENFORCEMENT OF NAVIGATION LAWS

In California, State laws sets the speed limit of 5 nautical miles per hour for all boats operated within 100' of any swimmer, or within 200' of a landing float where passengers are using the facilities, or within 200' of a beach frequented by swimmers. In Newport Harbor, wher over 4,000 small craft are moored at bay moorings, or slips, along the waterfront, this enforcement is a real problem. In August of 1957, a boat count showed that 59,856 boats crossed back and forth across the harbor entrance. With this great activity, it has been necessary to establish a 5 MPH speed limit throughout the entire developed harbor. Fifteen patrolmen, divided into three shifts, man the patrol boats to enforce the laws and protect life and property. The number on duty, at any time, varys in the shifts which change in importance with the seaso Three extra men are hired in the summer months, when vacation crowds an early morning albacore fishermen place heavy burdens on both daylight and night shifts.

'REVENTION OF POLLUTION OF BAY WATERS

This becomes most important where swimming is one of the harbor uses ınd is a problem that is difficult to handle in a popularly used harbor. ?ollution can be from sewage, debris, or dead fish.

Sewage Pollution - Laws forbid the using of boat toilets inside bay waters. However, the practical way to prevent this is by attempting to place proper shore facilities within available reach of moored craft. In Newport Harbor, in all the bay mooring areas but two, any boat owner using a mooring, must be a resident in the neighborhood, directly shoreward from his mooring area. This is to ensure that anyone on his boat has shore facilities closely available. In the two areas excepted, public rest rooms are located shoreward of them. Owners of slips along the waterfront must have access to adjacent private residential facilities, and slip operators must make rest rooms available to their patrons. To ensure that these requirements are followed, no permits for boat slips are approved until rough plumbing for facilities is completed. Another way to combat the problem would be to install chemical toilets on slips installed in units in the bay, in place of individual moorings. A shore connection could be made to this arrangement to eliminate the chemical toilet feature.

To guard against pollution of harbors, water samples are tested from many stations to determine the bacteria content. If any station sample is above a safe bacteria count, the reason for this is determined and eliminated.

Pollution from debris - This is a very difficult problem to control. Articles are pushed into bay waters at any opportunity or merely left to drift away with high tide. Identified property can be traced to the owner, but most debris is not marked that clearly. An appeal to the public pride through periodic notices is helpful in preventing debris from entering harbor waters.

In the operation of a harbor, thoughtful consideration should be given to reserving space for disposal of debris. As facilities grow, such space is often overlooked, and a time will come when disposal of debris will become a noticeable operational expense. This is usually due to the necessity for hauling the debris away in order to dispose of it.

Another debris problem is caused by the careless "over the side" disposal of cans and bottles by boat passengers. Mainly, this debris comes from rented boats or fishing party boats. These passengers do not want to bother taking home empty containers. A close watch and the issuance of citations to guilty parties, soon gets the word around that such conduct will not be tolerated. If these acts are left unnoticed, the debris will soon collect.

Pollution from dead fish - Waters around fish canneries are like: to contain dead fish, due to the unloading proceedures. This problem be controlled by employing men in skiffs to pick up the dead fish bef(they are dispersed all over the bay waters, and by more careful unload

PREVENTION OF IMPROPER MOORING OF VESSELS

This can be a difficult problem in harbor operation, if definite mooring areas are not maintained, and if rigid installation specificat are not followed.

Boundaries of mooring areas are dictated by channel widths and us and should be defined on harbor maps. Moorings within the areas shoul assigned and placed under the direction of the Harbor Master to insure correct spacing for use without taking up excessive area. Commercial boats should be in areas separate from pleasure craft moorings, as the uses and construction are not similar. If moorings are vacated over a period of time, the owners should be notified that they have forfeited their space and must make way for owners who will use this space. Thi: is especially important in a harbor with limited mooring facilities.

Following good specifications for installations will prevent many mooring problems from occurring. The specifications shown below have proved themselves in Newport Harbor over a long period of time.

SPECIFICATIONS FOR SHORE MOORINGS, NEWPORT HARBOR, CALIFORNIA

Rowboats or Sailboats, Without Power, 17' or Less Overall

Maximum Length of boat	Minimum Weight of mooring	Minimum Size and Length of Chain	
12'	200 lbs	25'	3/8"
15'	250 lbs	25'	3/8"
17'	300 lbs	25'	3/8"

A 4x4 Redwood post, painted white, and with numbers assigned, painted thereon AT ALL TIMES, shall be placed against the sea wall, and project not more than 12" above the sand.

Mooring buoy shall be of an approved type constructed of metal, and painted white above the water line, with mooring numbers assigned, painted thereon AT ALL TIMES.

Buoy and post shall each have a pulley attached with a line of not less than 3/8" diameter at all times. Boat to be moored securely thereto, bow and stern, and must not be left on beach.

NOTE: Boats over 17' in overall length, and ALL power boats, regardless of size, must be moored on Off-Shore Moorings.

SPECIFICATIONS FOR OFF-SHORE MOORINGS, NEWPORT HARBOR, CALIFORNIA

All mooring weights must be metal.

All mooring buoys must be metal, or such type as approved by the Harbor Master, painted white, or aluminum, above the water-line, with numbers assigned by the Harbor Master, painted thereon AT ALL TIMES.

All vessels must be moored fore and aft.

All locations must be allocated by the Harbor Master, and moorings inspected before installation.

Top and bottom chain to be shackled together to form one continuous length.

IMPORTANT NOTICE

MOORING PENNANTS are an important part of your mooring and must be properly made up with thimble and shackled to each buoy, FOR INSPECTION AT THE TIME OF INSTALLATION OF THE MOORING. Mooring pennants must be kept in good condition AT ALL TIMES.

MINIMUM REQUIREMENTS:

Length of Boat	Minimum Dia. of Line	Length of Line
18'-20'	5/8"	Not over 10'
20'-25'	3/4"	Not over 10'
25'-30'	7/8"	Not over 10'
30'-40'	1"	Not over 12'
40'-50'	1¼"	Not over 15"
50'-70'	1½"	Not over 15'

Length of Boat	Weight of Mooring	Size of Chain Bottom	Top
20'	500 lbs.	1/2"	3/8"
25'	650 lbs.	1/2"	3/8"
30'	750 lbs.	1/2"	3/8"
35'	1000 lbs.	5/8"	3/8"
40'	1500 lbs.	5/8"	3/8"
45'	2000 lbs.	3/4"	1/2"
50'	2000 lbs.	3/4"	1/2"
55'	2500 lbs.	3/4"	1/2"
60'	3000 lbs.	3/4"	1/2"
65'	3000 lbs.	1"	1/2"
70'	3500 lbs.	1"	5/8"
75'	3500 lbs.	1"	5/8"
80'	4000 lbs.	1"	3/4"
85'	4500 lbs.	1"	3/4"
90'	5000 lbs.	1"	3/4"
95'	5000 lbs.	1"	3/4"

Length of mooring lines to be determined by the Harbor Master, he being governed by the depth.

Above specifications cover mooring for one end only.

To insure that the moorings are maintained, each mooring must be inspected every two years and all worn parts must be replaced.

EDUCATION NOT REGIMENTATION

In concluding the policing category, the importance of educating t[illegible] public in observance of laws and regulations can not be too highly stressed. Many times, laws are broken, due to ignorance along. It is not required that a boat operator of a pleasure vessel be licensed, as is the case of an automobile operator. In the past few years, the increasingly popular boating activity has resulted in vast numbers of ope[illegible] tors who know little about boat handling, laws of navigation, or what boat equipment is required. All this has lead to a move to license boa[illegible] operators. The majority of men, with boating experience, resist any regimentation along this line. They feel that education is the answer, not licensing.

In Newport Harbor, Harbor Master Albert Oberg has organized a class to train youngsters in the correct way to operate and maintain a vessel. Yacht clubs have also organized youth training in this program. Nationally, the US Power Squadron and the Coast Guard Auxiliary, have extended their training classes to reach vast numbers of these new boating enthusiasts. All these moves have appealed to the new boating public as attested by the huge response. At the end of the fiscal year June 30, 1957, a total of 139,890 people had been registered in US Power Squadron classes. The US Coast Guard Auxiliary's three pronged program of (1) courtesy pleasure boat examinations, (2) public boating education and patrol, and (3) assists to recreational boatmen, experienced considerable expansion throughout the nation during 1957. To date this year, a total of 48,781 pleasure craft inspections have been made by Auxiliarist in the Courtesy Safety Examination program, and by the end of the year this figure should exceed 50,000. Boating enthusiasts enrolling in the free public instruction courses totaled 30,216. The US Coast Guard sent 13 roving inspection teams throughout the nation last year to inspect and instruct boat operators on the spot in their harbors. It is hoped that these moves will solve the problem and result in proper boat operation by the public.

ENGINEERING PROBLEMS

Many new products made of plastics, fiberglass, styrofoam, light weight concrete, and coatings of anti-rust coverings have been developed to supplement older methods of combating the problem of sea water and other attacks on water-front structures. Here are examples of uses of these materials with a discussion of their advantages and limitations.

PONTOONS OR FLOATS

Older type pontoons were constructed of timber logs, wood planked cribs covered with paper and tar, or of steel barrels. The timber logs were subject to wood borers and to becoming waterlogged in time. The wood cribs would shrink and swell, thus allowing water to enter them, usually from wave action, or the paper would be subject to damage, thus allowing worms to bore into the timbers or water to fill the wood cells and reduce the buoyancy. Steel tanks or drums, though coated with preservatives, are subject to corrosion of their thin walls which ends in loss by sinking. Newer types of pontoons have increased the useful life of floats, but sea water still takes its toll.

Plastic Pontoons - This type is not subject to corrosion, worm attack, or waterlogging, but there are disadvantages. The thin skin is subject to vibrations which after a time cause cracks to develop and results in the pontoon sinking. Compared to older type pontoons, their first cost is higher and their useful life no longer.

Fiberglass covered pontoons - These pontoons are claimed by many to be the answer to fighting sea water attack - no corrosion, no waterlogging no borer attack. Yet they are subject to dry-rot from inside the pontoon. If any moisture finds its way inside, the lack of ventilation and presence of moisture and heat will lead to dry-rot.

Styrofoam pontoons - This light weight material usually comes in logs approximately 2'x3'x6' or 2'x3'x9'. It is an expanded material with a very irregular surface. It can easily be sawed into the required shape by passing a hot wire through the log. It is not expensive and is easy to install. However, it has disadvantages. Diesel fuel or petroleum and fish oil will dissolve the styrofoam. Also, it must be shielded from the sun, birds, and inquisitive people, as it is easily picked apart. Otherwise it has proved to be a suitable pontoon in sea water.

Light weight concrete pontoons - These have proved very serviceable providing their manufacture is correct. They are usually cast in two pieces, one piece consists of bottom, sides, and a center bulkhead. The other piece is the top which is later cemented to the sides. Here is where trouble can be found. If the top wire reinforcing is not joined to the side reinforcing wire and the top and side joint properly veed out or grooved to form a good bond for the joint, this area will later crack and leak. As usual, water content should be carefully controlled to secure strength without loss of workability. Proper curing after assembling the pontoon is most important. The advantages of this construction are its resistance to corrosion, waterlogging and dry-rot. Due to its light weight it does not sink readily if damaged. This pontoon must be shielded against impact, as it is not as strong as ordinary reinforced concrete. In certain localities, a cement borer has damaged it. It is a most successful answer to the search for a good pontoon in areas where the cement borer is not evident and provided that proper construction is used and a facer of wood is placed above its waterline to shield the pontoon.

CONCRETE AND STEEL IN SEA WATER

Concrete, if properly manufactured and installed, is a most durable product in sea water if not stressed so that cracking developes. Many maintenance problems can be avoided or reduced to a minor nature, if a few fundamental practices are followed.

1. Obtain as dense a concrete as possible by keeping the water (tent low. To avoid an unworkable mix, use admixtures as directed. Vibration to the proper degree will help.

2. Obtain sufficient concrete coverage over reinforcing steel. Where possible, have 3" cover where the structure is exposed to sea water.

3. Cure concrete immediately after initial set, either by membra coating, or application of moisture continuously for seven days.

4. Use proper handling of concrete structures to prevent crackin Prestressing concrete will help to avoid those cracks caused by handli and loading.

If these practices are not followed and concrete deterioration is occuring, do not prolong corrective repairs. Once the reinforcing is exposed, the destructive process proceeds rapidly. Clean the damaged area of all loose concrete and rust, usually best done by sandblasting. Build up the area by welding new reinforcing steel to undamaged origin steel, and gunnite the structure to build up to required size and concrete cover.

There are times when steel must be used rather than the more durat concrete. This is the case where hard pile driving conditions would crack concrete, or where long spans or lengths dictate the more easily handled steel product. Then the problem of corrosion can be reduced by applying protective coatings to the exposed steel surfaces. Properly cleaning the surface, prior to coating application, is imperative. Coatings of bitumastic, vinyl plastic, and metalized zinc have proven value, if properly applied. Some coatings stand up better on areas exposed to the sun and should be used above water level. Such is the case of bituplastic above the water, where bitumastic would alligator and peel. Pittsburg Chemical Company manufactures a protective coating that stands up well in sea water.

It has proved advisable to install cathodic protection systems on steel structures in contact with sea water, where electrolytic currents are known to exist. This is usually the case where steel ships with generators and welding equipment are located. These cathodic protection systems are most important where the steel installation costs are high and where any replacement would entail considerable expense, as in the case of sheet piling under wharves.

TIMBER IN SEA WATER

Timber structures are not as durable in sea water as sound concrete is - but there are conditions under which harbor operations dictate their use. Groin installations have not always proved to be the answer to maintaining an eroding shoreline. If the installation is of timber design, it is less costly to install and remove, if necessary, than one of steel or concrete. If a structure is liable to sudden impact forces - such as a fender system attached to a wharf, a timber designed system is recommended over a more rigid type. In other cases, timber structures, properly treated with preservatives, will permit a long enough useful life for obsolescency to require a new design. The problem is to design and install a facility within the approved budget appropriation, bearing in mind the useful intended life of the facility during which time maintenance costs will be kept to an economic minimum.

Following are a few maintenance functions that are practiced in harbor operation.

MAINTENANCE DREDGING

This can vary considerably in operating a small craft harbor. Many factors affect the unit dredging cost, but one of the most important is location and size of disposal areas.

Disposal areas - As harbor property develops, these areas become scarce. Then the dredged material is placed on beaches, where the material finds its way bayward to once again shoal the required water depths. Disposing of material only during high tides,is practiced so that the sea level will act as a dike to prevent the material from flowing back into the cut. This raises the cost as operating time is limited by the fall of the tide. Often the beach is not deep enough in land extent to warrant dike construction with bulldozers. If neither practice is desirable, the material must be barged to sea - which is expensive. For maintenance around individual boat slips, small barges carrying one hundred cubic yards of dredged material per trip are very useful - where the best disposal area is the open sea.

Small grain size and light weight dredged material can cause problems by being very difficult to contain as fill. Much of it is lost in suspension in the pump discharge water through the drainage pipes. Ordinary beach sand, if not contained by a dike or sea level, approximates a one on twenty slope when placed hydraulically. The coarser the grain size, the steeper the slope.

Problems also occur when placing fill on soft muddy ground. With the weight of disposed material increasing, a mud wave begins to flow bayward as the mud is displaced by the fill. This has been used to advantage to rid the filled area of an unstable base, but necessitates eventual removal of the mud - by some method. Where to put the undesirable mud is the main problem. It is easily dredged, but confining it while placing it results in a mud hole that has practically no soil bearing value. In cutting for the desirable beach slope in this material, it is necessary to overcut the slope and backfill with clean sand.

Common shoaling problems - One of the most common and costly of these problems is shoaling of harbor entrances. This is caused by littoral sand drift which is trapped by the breakwaters or jetties. As the trapped sand increases it builds around the entrance and deposits in the breakwater lee or else sifts through the rock breakwater or jett; This shoaling effect can be decreased if the tidal prism of the harbor is sufficient to cause currents that will scour the entrance to keep it clean. Making the breakwater or jetty more impervious to sand infiltration may help, although in time the sand will build around the end into the entrance. Another method is to bypass the sand, past the entrance, by dredging. If the entrance is trapping sand the beach in the down drift direction, without any other source of sand, will erode. All of these are factors to be considered before harbor construction commences. However, in many cases, it has become an operational problem.

Another shoaling problem is caused by pier and float installations. If sand is moving past a point on the shoreline, any structure will tend to cause a lee and the moving material will deposit to form a shoal behind the structure. This is particularly true if the structure is solid in form and parallel to the shore lengthwise. The longer and closer to the shore the structure is, the faster and larger the shoal forms.

If storm drainage must run down a beach slope before entering the water, it will scour out beach material that will build a shoal bayward. To eliminate this, the drain should be carried bayward to a headwall, so that drainage will run directly into the bay waters at all tides.

Types of dredges - Maintenance dredging requires two types of equipment. A cutter head is best to cut to unobstructed channel project depths. However, around slips and piling, the hydraulic suction dredge is preferable. In this case, the cutter head might damage these installations. Also the suction dredge is more maneuverable and less costly to operate on small jobs which are prevalent around boat slips.

Sounding equipment - For short sounding ranges not exceeding 800' and including beach slopes, use of a tally reel line marked off in tens of feet is the handiest way to locate the sounding positions. A 6 lb. lead line, marked off in feet, is used to obtain the depths. If the ranges are longer and channel navigation is heavy, the use of a recording fathometer is recommended. Ratheon produces a portable set, the 1373 type recorder which can be easily mounted on a skiff. Sounding positions can be determined by sextant angles from the sounding boat or with transits from the shore.

REMOVAL OF KELP BEDS FROM CHANNELS

Kelp grows on rocky ground. The kelp holdfasts or fingers attach themselves firmly and have even pulled up their rock base to the surface before breaking loose. I had been advised that if sand was deposited over the holdfasts the kelp would die and disappear. However, this was not the case in the entrance channel to Newport Harbor. Kelp beds had been growing larger each year, so that navigation was being affected. Investigation proved that ordinary kelp cutting 3 or 4' beneath the surface as done commercially would only stimulate the growth. It was deemed advisable to cut the kelp off at ground level, which was over 20' below the water surface. A diver was hired to do this job. As he cut the kelp, he sank his 2' long knife blade into sand without striking rock. Yet the kelp was thriving in the area. Since he completed the job over 6 years ago the kelp has not come back to interfere with navigation.

SUBTERRANEAN INVESTIGATIONS - THEIR USEFULNESS AND LIMITATIONS

The more data that can be obtained of the subterranean stratas, the less will be the contingent cost of a dredging or pile driving project. One type of investigation practiced over the years is by test hole boring. Boring and lab analysis techniques have been perfected, so that a complete picture can be gained of the stratas bored. However, one of the most important phases of determining the underground condition is the selection of the test hole locations and their number. Ground conditions can change from spot to spot and readings of holes bored may be misleading for gaining a comprehensive finding. Boring is expensive and if the area to be surveyed is extensive, other means of determining the soil structures may be advisable. Geophysical seismic surveys can cover much ground in a short time and correlated to a few test holes, can convey a good picture with less cost than many test holes could do. The principal points of seismic surveying are these. Blasting charges are set off at known points at a precise signal. Sound detectors are strung out on known locations to pick up the shock waves rebounding from the underlying stratus. These recorded echoes are timed. The more dense the stratus, the faster will be the return signal. These signals are calibrated as to velocity, to determine the nature of the subsurface. Seismic surveying is adaptable for hydrographic work as well as on dry land.

GROIN INSTALLATIONS - NOT ALWAYS THE ANSWER TO EROSION

Too often when erosion is cutting away the shoreline, the first remedy considered is the installation of groins to hold or build back the beach. However, there are times when groins are not the answer and in fact may increase erosion. Two pertinent facts must be in evidence before any groin will be effective.

1. There must be a source from which comes the beach material to be trapped by the groin, and

2. There must be a predominant direction to the littoral drift of material passing the groin location.

If this is the case, and the groins are properly designed and installed, the erosion should be decreased.

PREVENTION OF CHANNEL ENCROACHMENTS

This problem must be continually watched and prevented if unwarranted. Pierhead and Bulkhead Lines once established should be followed. If they are not practical in application in certain areas, they should be revised so that they can be used as intended - PIERHEAD LINE, to define the bayward extent of any open structure; BULKHEAD LINE to define the bayward entent of any solid fill. To be practical in setting these lines, consideration should be given to the type of use to which the shoreline will be put, as well as the clear width of the channel to be maintained. In the conventional boat slip installation a minimum one and one half feet of water should be under the pontoon at the <u>shore</u> end, to prevent grounding during wave action.

Use of vertical view aerial photographs to desired scale is recommended to study channel encroachment and shoreline changes.

TRAILER BOAT LAUNCHING

The tremendous increase in trailer boat craft has caused a demand for launching facilities. In the case of established harbors that are completely developed, this has presented a real problem, as the necessai space for launching and trailer parking is not available. Crane launching and multiple vertical parking space is the answer here. Where undeveloped space is available, the launching ramp has advantages.

1. Many boats can be launched simultaneously.
2. Ramp launching is less hazardous than crane launching.
3. Maintenance costs are less if the installation is correctly installed.
4. There is no possibility of launching cessation due to a mechanical failure.

A convenient ground slope for a launching ramp is 10 to 1. A durable ramp surface is a mixture of sand and shell that can be packed by truck tires rolling over the surface.

CALIFORNIA'S PROGRESS TO OVERCOME HARBOR SCARCITY

The problems stated above can be remedied by efficient harbor operation. However, the basic problem in California is the scarcity of harbors. Along its 1200 miles of coastline exposed to the Pacific Ocean, only San Francisco Bay and San Diego Bay are naturally protected harbors. The remaining harbors have necessitated expenditures of millions of dollars through dredging and construction of protective breakwaters or jetties before becoming safe havens for vessels of any size. The East Coast of the United States is naturally blessed with hundreds of protected bays and inlets due to its coastline of submergence over the past era. The coastline of California being a shoreline of emergence is lacking these natural barriers to ocean waves. Consequently, harbors of refuge are scattered far apart and harbor facilities here are far behind the public demand. Due to the great expense involved, those harbors that have been constructed have taken years of effort on the part of many people before becoming a reality. In 1947 the Corps of Engineers made a survey, and a "Report on Preliminary Examination of the Coast of Southern California with a View to Establishment of Harbors for Light Draft Vessels", with an attempt to locate such harbors approximately every 30 miles apart along the coast. The Korean War interrupted the survey before completion and it is still unfinished.

It is difficult for local communities to raise the total cost of a protected harbor and it is almost as difficult to obtain aid from the Federal Government for this purpose. Realizing these facts, a large group of harbor minded people, mainly within the organization of the California Marine Parks and Harbor Association, have worked hard for the past several years to organize within the State Government an agency which could negotiate as a central body with the Federal Government to aid localities in developing their feasible harbor sites.

Establishment of California State Division of Small Craft Harbors - This year two great steps were taken toward small craft harbor development in California when Governor Knight signed into law

1. a bill creating a State Division of Small Craft Harbors, and,
2. a bill appropriating $100,000 to be adminstered by the Division, for making loans to local jurisdictions for planning feasible small craft harbors.

General policies for the guidance of the Division are vested in a five-man Harbor Commission appointed by the Governor - with the advice and consent of the Senate. Members of the Commission serve without compensation, but may be reimbursed for actual and necessary expenses incurred in the performance of official duties. A Chief has been appointed by the Commission to head the staff of the Division. A Small Craft Harbors Revolving Fund has been created to support the Division and also to be used for loans to local agencies for planning harbors. For construction loans, specific appropriations must be made by the Legislature.

The enacted law states that money loaned from the Revolving Fund must be repaid in full within 10 years, such repayment to include an interest rate that could be derived by investing the total deferred payment at the interest rate prevailing for legal state investment.

Private enterprise to aid Small Craft Harbor Development - There is evidence that private enterprise, once basic protection and dredging is completed, will undertake to develop the needed facilities such as slips, lockers, boat launching, repair yards, and retail sales business. This development, along with sound State and Local planning, should show definite progress during the next few years, in solving the problem of shortage of small craft harbors in California.

CHAPTER 39

USE OF MODEL EXPERIMENTS IN SOLVING QUESTIONS OF NAVIGATION WITH SPECIAL REFERENCE TO THE ENTRANCE OF ST. ANNA BAY, CURACAO, NETHERLANDS ANTILLES

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GENERAL

For a great many years model experiments with ships have been carried out in towing tanks in order to ascertain the reactive movements of ships under different wave and current conditions. Such experiment do not only pertain to the stability of the ships but also to forces brought to bear on the hulls of the ships.

A recent development in the coastal engineering field includes th use of ship models in questions of navigation.

The reason why such experiments are undertaken is that numerous accidents, such as collisions and run-agrounds, occur when ships enter harbors and inlets. Very often such accidents are caused by adverse wave and current conditions.

In the laboratory it is possible to duplicate the actual wave anc current conditions and studies can be made by using models of ships oi the influence of waves and currents on the navigation of ships approac ing or passing through the harbor.

The importance of these studies is obvious. First, it is possibl for pilots to study the problems in detail and thereby gain more exper ience about how to navigate under certain conditions, and next, recommendations for improvements to the entrance of the harbor can be made.

The following is a description of the conditions found in the harbor at Curacao, Netherlands, Antilles, and a description of correc ive measures taken, including the use of model experiments.

SITUATION

Curacao, the largest island of the Netherlands Antilles, has one of the busiest harbors in the world (Fig. 1). In 1954, 7600 ships pu into the harbor, among which were 5400 tankers.

The ports are situated on the Schottegat, a big inland lake, and on St. Anna Bay, the connection of the Schottegat with the sea. On t Schottegat are situated the landing-stages of the CPIM (Shell Oil) re fineries and a new commercial port. Along St. Anna Bay we find the older quays of the shipping companies which transport both passengers and goods.

USE OF MODEL EXPERIMENTS IN SOLVING QUESTIONS OF NAVIGATION WITH SPECIAL REFERENCE TO THE ENTRANCE OF ST. ANNA BAY, CURACAO, NETHERLANDS ANTILLES

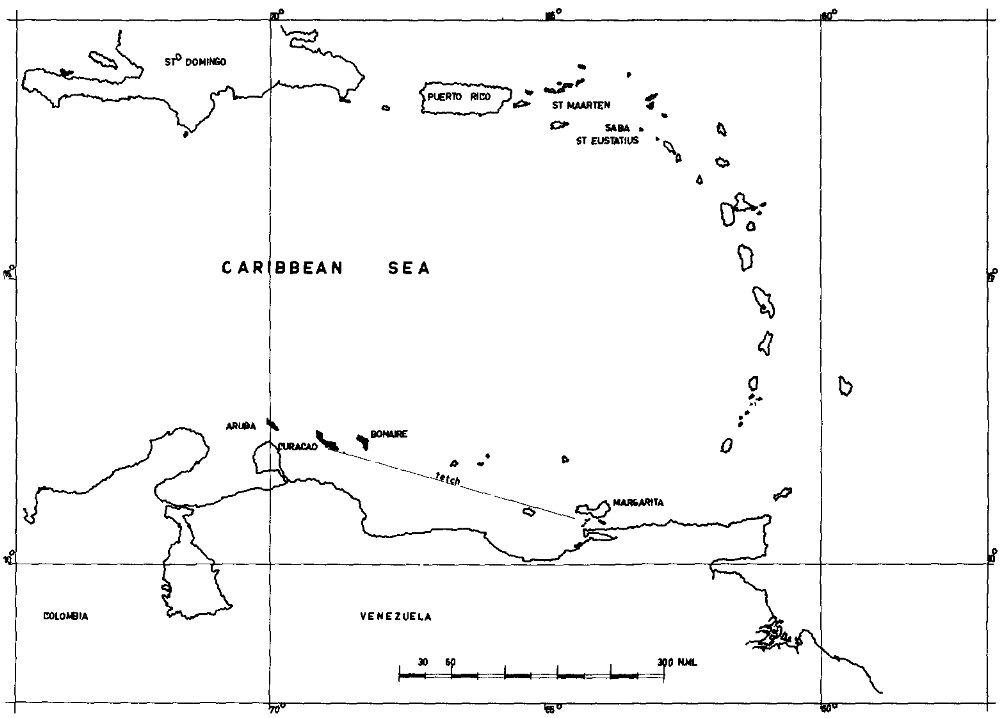

Fig. 1. Netherlands Antilles

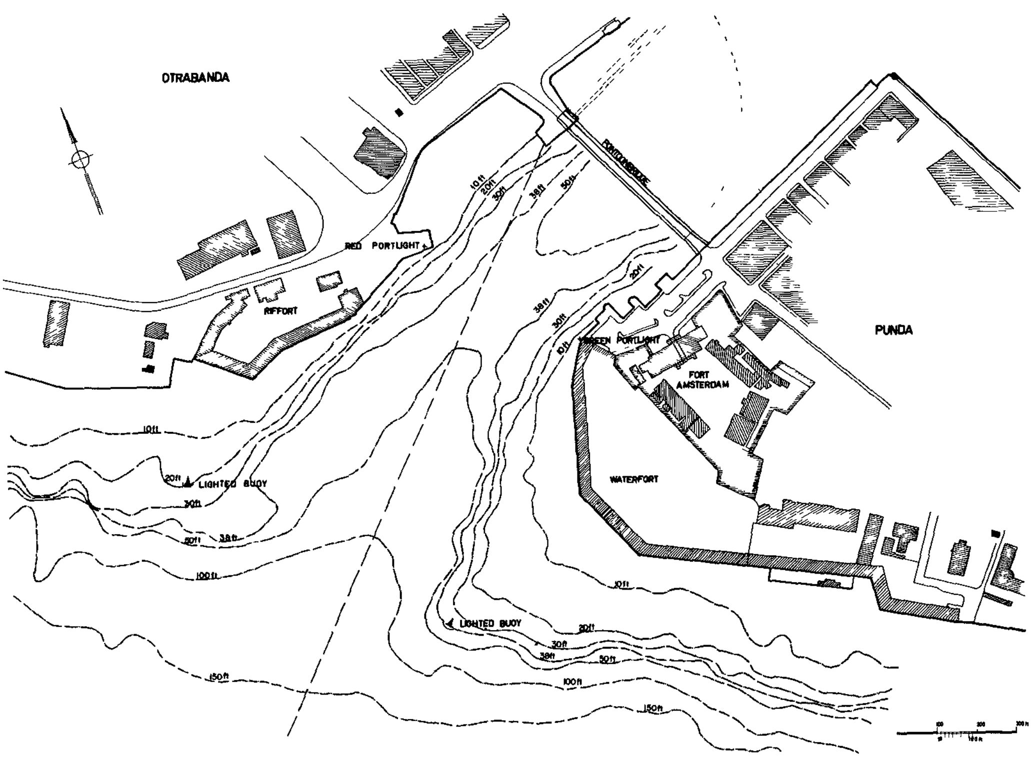

Fig. 2. Entrance St. Anna Bay. Present situation.

Fig. 3. Survey St. Anna Bay. In the foreground.the harbor entrance, on the right-hand side of the entrance Waterfort, on the left-hand side Riffort.

Fig. 4. Entrance St. Anna Bay. The bulkhead of the bridge which runs far into the entrance channel is clearly visible. On the left top the Shell refineries.

USE OF MODEL EXPERIMENTS IN SOLVING QUESTIONS OF NAVIGATION WITH SPECIAL REFERENCE TO THE ENTRANCE OF ST. ANNA BAY, CURACAO, NETHERLANDS ANTILLES

The entrance to the harbor, the mouth of St. Anna Bay, lies in the center of Willemstad, the capital of Curacao. This narrow entrance is bounded by two old fortresses (Waterfort and Riffort). Over a range of 800 ft. the entrance channel narrows from 650 to 300 ft. near the portlights (measured between the drop lines of 30 ft. below low water) (Figs. 2, 3, 4). The underwater slopes of the channel are very steep.

At about 400 ft. inwards from the portlights there is a pontoon bridge which opens in a westerly direction, and which then has a navigable passage of 380 ft.

The mouth of the channel is bounded on either side by coral reefs. From the coast line the surface of those reefs slopes gently down to a 30 ft. depth. Then the slope grows rapidly steeper, and the surface inclines to great depths at an angle of 45°.

The fairway is not only marked by the portlights at the narrowest spot, but also by two buoys at sea.

At the harbor mouth there is, in general, an easterly littoral current. This west-going current is mostly weak, but occasionally it can reach a considerable velocity for a short time. Several times velocities of 4 ft./sec. have been measured near the easterly buoy.

An easterly trade wind blows in Curacao. No data are available concerning the wind at the mouth of St. Anna Bay but only as to the wind on the north side of the island where the airport is situated. From a short series of comparative measurements it has been proved that in the daytime the wind at the airport has reversed itself as compared with the wind near the buoys at sea at the harbor mouth. It also has been proved that the wind velocities are greater near the airport than at the harbor mouth. During the periods of drought an average wind velocity of 14 knots is normal. In contrast to the wind forces near the buoys at sea there is but little wind between the high walls of the fortresses.

The difference between the water lines at low tide and at high tide amounts to approximately 1 ft. The tidal currents in St. Anna Bay are not strong -- the maximal steepness of the local tidal diagram $\frac{dz}{dt}$ appears to be 2.10^{-5} m/sec.

DIFFICULTIES WHEN PUTTING INTO THE HARBOR

When putting into the harbor ships meet with difficulties when there is a strong easterly littoral current.

These difficulties arise when the ships pass from the sea current into the relatively smooth water in the harbor mouth where there is insufficient space for correction after possible sheering.

In most cases, when a maneuver has failed, the ship turns to starboard when passing the easterly buoy as a result of the pressure of the current on the stern. Then the ship threatens to run aground near the green portlight or a little farther on.

In some cases the tendency of running aground can be successfully checked by carrying out a correcting rudder and machine maneuver. Then however, the ship has a tendency to turn to the port side, which cannot be checked. The ship then runs the risk of going aground on the west side near the Otrabanda bridge head.

Finally, there are a number of cases in which the ship, either owing to excessive correction of the expected turning to starboard, or because of too little speed, turns to the port side and is in danger of running aground under Riffort or at the Otrabanda bridge head, or of running into that bridge head.

The risk of sustaining damage is considerably increased by the presence of the pontoon-bridge and of cables lying in the fairway.

In case the pontoon-bridge should be damaged, the vehicular traffi that is using the bridge (8,000 vehicles per day) has to be diverted over a distance of about 16 kms. Pedestrians are transported by ferry. Up to now it has always been possible to repair the damage caused to tl bridge within a few weeks because spare parts and pontoons are availab in Curacao.

The difficulties presented themselves especially just before the Second World War and have continued since that time. This is due to tl greater intensity of the shipping traffic; the fact that the ships are getting bigger and bigger; and to the fact that some post-war types of tankers, which frequently put into port, steer badly.

Whether a change in the staffing of the pilotage service could exert any influence cannot easily be traced.

SURVEY OF THE ADVICE GIVEN AND THE MEASURES TAKEN IN 1954

In 1948 the Hydraulic Laboratory at Delft was charged with the ir vestigation of the possibility of shifting the current farther outsid the mouth.

A model was built on a scale of 1 to 144. From the investigatio carried out in this model it proved to be possible to shift the curre about 250 ft. seaward by building a breakwater about 350 ft. long in the sea. In view of the outlay in money involved, however, there was some doubt as to the question whether the construction of such a brea water would serve any purpose. Consequently it was decided not to pr ceed with the building of this breakwater.

In 1951 the entrance channel was broadened by means of dredging the advice of Dr. ir Ringers, ex-Engineer-in-Chief, Director of the Ministry of Transport and Works, Waterstaat, and ex-Minister of Trans and Waterstaat in the Netherlands. It is remarkable that in his report Dr. Ringers points to the influence of the wind. When coming between the fortress walls a ship comes into an area where there is but a faint current and to a certain extent, this is also the case with the wind.

Owing to a series of difficulties that arose during a highly critical period of strong currents at the harbor mouth in the middle of 1953, during which period there were many collisions, the harbor-master, who is also head of the Pilotage Service, ordered the old 3,000 ton tanker "Susane" to be stationed at about 200 ft. east of the easterly buoy to serve as a temporary breakwater.

By means of this temporary breakwater the current was diverted to a point outside the harbor. According to the pilots this improved the situation. After some weeks the tanker had to be removed because postponement would render salvage impossible.

After the installation of the temporary breakwater the Council of Curacao appointed a technical commission, whose personnel included the Harbor-master, the Director of Public Works and the Marine Superintendent of the C.P.I.M.

The advice given by the commission was practically the same as that of ir J. B. Schijf, Engineer-in-Chief of the Research Division of the Ministry of Transport and Waterstaat, who had in the meantime been appointed adviser, and who had come over from the Netherlands to Curacao in order to give advice.

IMPROVEMENTS PROPOSED BY IR J. B. SCHIJF IN JANUARY 1954

An illustration of the proposed improvements is shown in Fig. 5.

THE CONSTRUCTION OF A BREAKWATER

As has been mentioned before, the difficulties are caused by the transition of the sea current into the relatively smooth water in the harbor mouth. Therefore it is obvious that the situation would be improved by shifting this sudden change in the strengths of the currents as far outside the harbor mouth as possible, which has also been demonstrated in practice by means of the test with the tanker "Susane". Once more the Hydraulic Laboratory was charged with the investigation of the situation in the harbor mouth, as regards the currents, after a breakwater would have been built.

THE EXECUTION OF DREDGING WORK BEFORE RIFFORT

On the west side, the entrance channel can still be broadened considerably, which would facilitate maneuvering.

THE INSTALLATION OF A CURRENT METER

As the velocity of the current at the harbor mouth can increase considerably within a short period, a permanently installed current meter, provided with an indicator and a recorder, was recommended.

Then the indicated strength of the current would always be visible at sea.

THE BROADENING AND DEEPENING OF THE NARROW CROSS-SECTION BETWEEN THE PORTLIGHTS

In order to facilitate steering the ships it was recommended to make the narrow cross-section between the portlights broader and deeper.

THE REMOVAL OF CABLES LYING IN THE FAIRWAY

Between the two banks, near the pontoon-bridge, there are the pulling-cable of the pontoon-bridge, and several electric and telepho cables, which give much trouble if an anchor has to be used. The cab are lying there at the risk of their owners. It was recommended to r move these cables.

THE SHIFTING OR ENLARGING OF THE NAVIGABLE PASSAGE OF THE PONTOON-BRIDGE

The Public Works Service has investigated the possibility of mov ing the bridge farther inward.

This proved to be possible, though only at high expense, but it certainly meant no improvement for the traffic by land. By lengthen the bridge, and by shortening the land-abutment, however, the naviga passage can be broadened by 50 ft.

INVESTIGATIONS CARRIED OUT IN THE MODEL

These investigations have been effected by the Hydraulic Labora in an open-air model in the Northeast Polder of the Netherlands, in pletion of the advice given.

Experiments have been carried out to determine the aspect of th currents in different situations. Further experiments have been mad with sailing models of ships, in order to study the entering into th harbor of different types of tankers.

As it was also necessary to carry out sailing tests, a scale ha been chosen of 1 to 64. So the scale of the velocities of the curre is V 64 = 8. To get enough space to sail with the ship models it wa necessary to put sufficiently far out to sea.

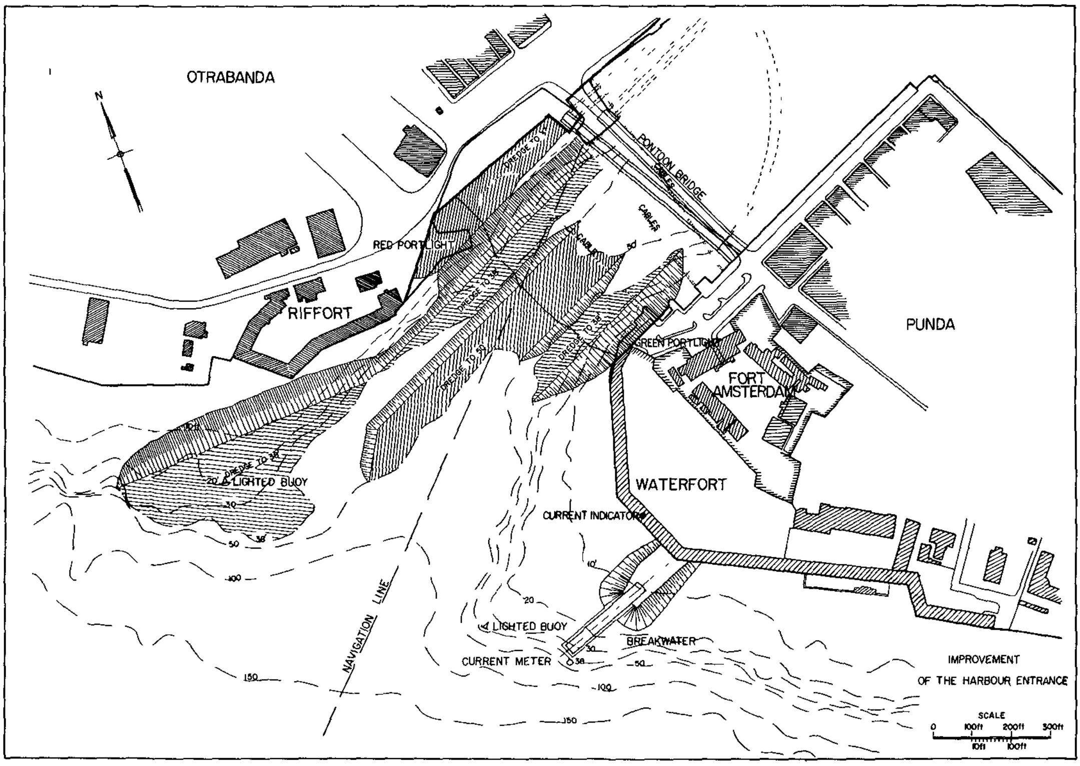

Fig. 5. Improved harbor entrance.

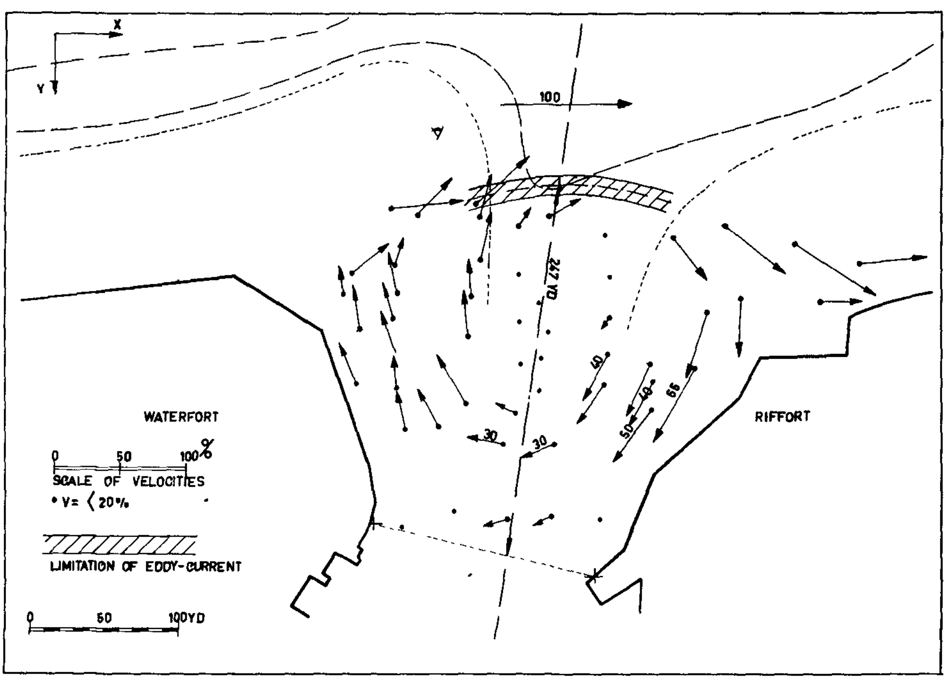

Fig. 6. Aspect of currents in the harbor mouth in the present situation.

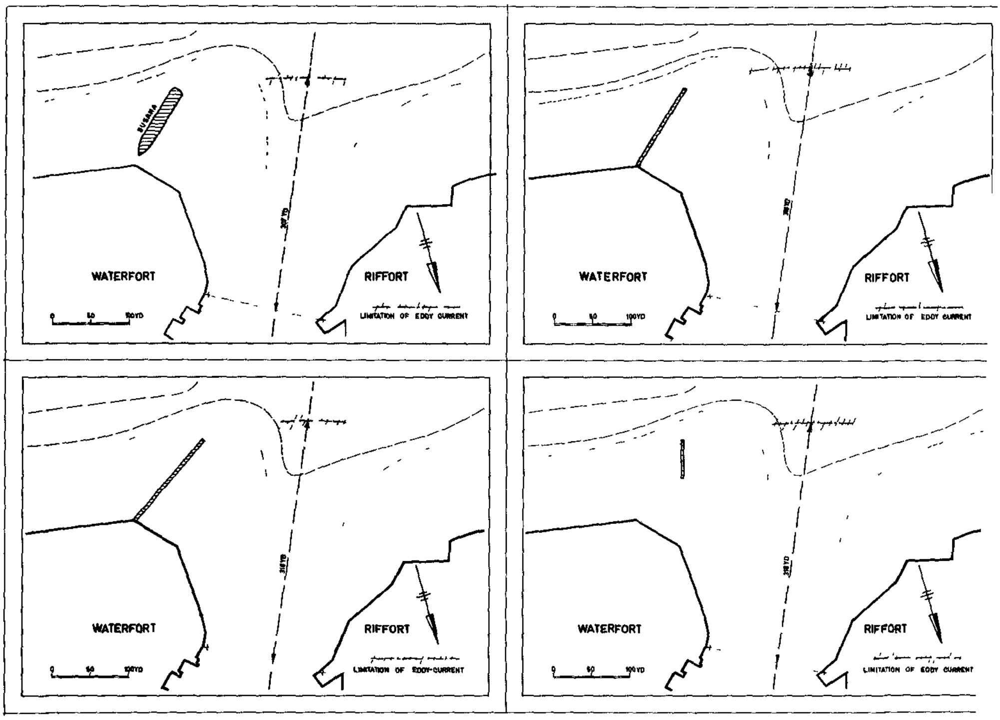

Fig. 7. Limitation of eddy-current with different ground plans of th breakwater.

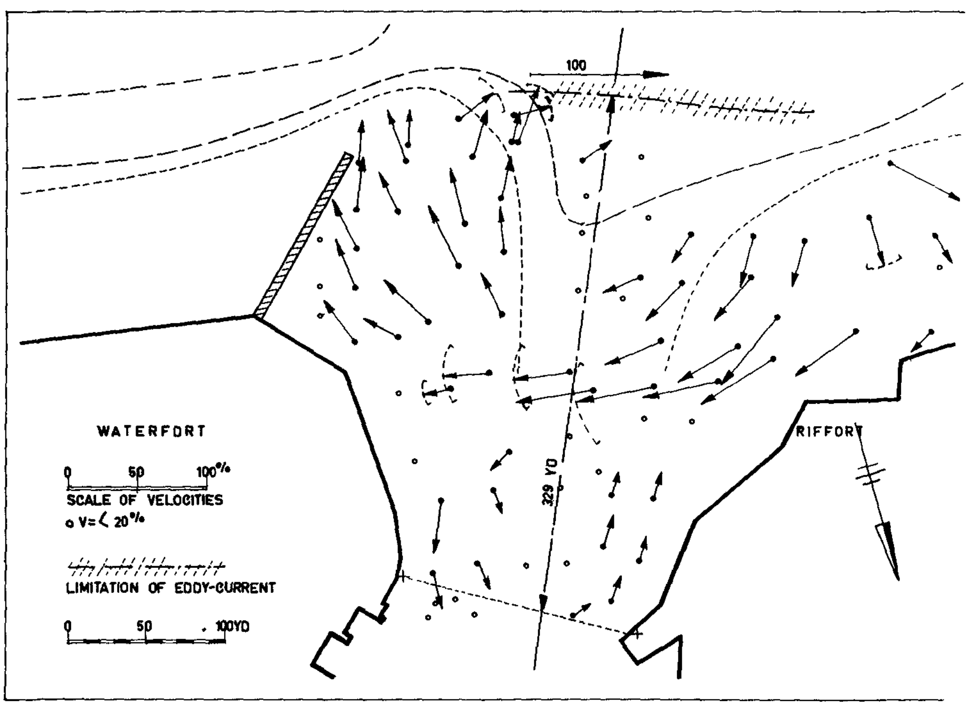

Fig. 8. Aspects of currents in the harbor mouth after the breakwa has been constructed.

USE OF MODEL EXPERIMENTS IN SOLVING QUESTIONS OF NAVIGATION WITH SPECIAL REFERENCE TO THE ENTRANCE OF ST. ANNA BAY, CURACAO, NETHERLANDS ANTILLES

THE PRESENT SITUATION

In order to make it possible to study the present situation, the model has been brought into agreement with the measurements carried out on the spot. This has been done with regard to the directions of the currents as well as with regard to the interrelations of their velocities in the various places, in so far as it was possible to reconstruct the aspect of the currents from the observation material. Of importance is the great velocity-gradient between the main current and the slowly turning eddy current. This transition is not stable and is characterized by small whirling currents between the main current and the eddy, which flow in the direction of the main current.

The result of the detailed measurements of the currents in a model of the present situation is shown in Fig. 6. The influence of the tidal current in St. Anna Bay, near the limits of the eddy current, has also been studied but proved to be imperceptible.

INVESTIGATION OF POSSIBLE IMPROVEMENTS IN THE MODEL

As mentioned in the foregoing, an old tanker was used as a breakwater in 1953. On carrying out measurements with the tanker in the model, the limit of the eddy current was found to be at about the same spot as had been found in reality during the measurements carried out by means of a float (150 ft. seaward from the limits of the eddy in Fig. 6).

Further, various ground-plans of the breakwater have been investigated in the model. The head of the breakwater was always placed at the edge of the reef at a depth of 40 ft., as this head can hardly be placed farther away owing to the steep underwater slope.

The ground-plans shown in Fig. 7 always proved to have the limits of the eddy lying at about 180 or 200 ft. seaward from the limits of the eddy in Fig. 6. Since the ground-plan of the breakwater does not appear to exert any appreciable influence, the recommended plan has been taken as a basis, and has been subjected to a more detailed examination, a survey of which is given in Fig. 8. The currents have also been investigated at various depths in the harbor mouth.

With this ground-plan of the breakwater it appears that the current over the reef head has not become appreciably stronger. Further, it appears that a secondary eddy current has been formed. The return current of the eddy is situated at about 400 ft. outside the connecting line of the portlights, and amounts to 40% to 60% of the main current. This is 30% to 50% more than the return current in the present situation (Fig. 6), but a favorable circumstance is that this return current lies 200 ft. farther seaward so that there is a greater opportunity of regaining the correct course. The cause of the greater intensity of the return current should be sought in the fact that the main current drives the eddy on over a larger surface.

Further, it has been investigated whether there is a possibility o weakening the return current by making an aperture in the breakwater, and by dredging a channel into the reef. This proved indeed to be possible in the model, but not in reality, as the channel would have to be dredged very accurately under cross-section, which is not feasible in practice. Besides, frequent dredging work at sea would be very expensive because there is no dredger available in Curacao which can be usec at sea.

The dredging work carried out at Riffort appears to exert no influence on the limits of the eddy. Nor does it entail an appreciable change in the current system in the harbor mouth.

SAILING TESTS CARRIED OUT IN THE MODEL WITH AND WITHOUT BREAKWATER

A model was made on a scale of 1 to 64 of three current types of tankers.

Type	Number of Screws	Measurements			Measurements		
		Overall length	beam	draught	Overall length	beam	draught
T2 tanker	1	523'6"	68'0"	39'3"	98"	12½"	7½"
G2 tanker	2	405'10"	62'6"	21'6"	76"	12"	4"
Supertanker	1	610'0"	80'6"	45'0"	115"	15"	8¼"

The ship models are driven by an electromotor and are electrical steered. The velocity of the rudder movement has been brought into agreement with that of the tankers. The contact between the shore an the ship consists of a composite cable which, by means of a fishing r is held in such a position that no tensile stress is exerted on the vessel.

The time that is needed for the switching over from full speed ahead to full speed astern is different for each ship. If it is 1 mi ute in reality, it is 8 seconds in the model since it is possible to switch over more quickly in the model. The reaction time of the crew on the bridge, and that of the controller of the model are equal. Cc sequently one has to react 8 times more quickly in the model. Therefore the controller only succeeds in sailing with the models by practice.

As the model had been placed in the open air, the sailing tests had to be carried out when there was very little wind. Wind-screens were put up around the model. Sailing tests were carried out with e of the models of the three types of tankers, during which tests the model was brought into agreement with the present situation in the Curacao harbor.

The behavior of the model tankers and the difficulties in controlling these tankers appeared to correspond very well with reality, since the failing of the maneuvers led to the same consequences.

Further tests were made in the situation where there is a breakwater, and where the dredging work at Riffort has been performed. Some random trial trips of the maneuvers have been recorded in Figs. 9 and 10.

It was of very great importance that the harbor-master and some other navigation experts attended the sailing with the models.

After having carried out the sailing tests the following conclusions have been drawn.

(1) In the present situation the best way of putting into the harbor is to sail dead slow against the stream, parallel to the coast at about a quarter of a mile from it. Just before the leading lights are seen in a line the helm should be put a little to port, and one has to steer direct for the buoy. When the ship has come at about a ship's length from the buoy, the helm must be put entirely to port, so that the stern is pushed against the stream. With supertankers full speed ahead is required for it. When the stern is out of the main current, the helm should be righted.

(2) The presence of the breakwater does not facilitate in itself the maneuvers to put the T2 and G2 tankers into the harbor. It is important, however, that in case the maneuver fails, the tanker grounds further outside the harbor, so that the risk of damage is considerably reduced.

The advantage is most marked, however, when supertankers are concerned. In the present situation full speed ahead has always been necessary, when maneuvering, in order to put into port allright. When there is a breakwater ships can also enter dead slow, which is a great advantage in view of the dangers attendant on the increase in velocity of most of these tankers.

(3) The dredging away of the reef at Riffort is recommended since, in case of successful maneuvers with supertankers, there is risk of running aground on the westerly reef with the stern. With every type of tanker the advantage is that when the ship drifts off to the westerly reef during a failed maneuver, there is a greater possibility of casting anchor in time.

(4) The present leading line is of little use when maneuvering. It is not a matter of keeping a certain course but rather of carrying out a certain maneuver at the proper moment.

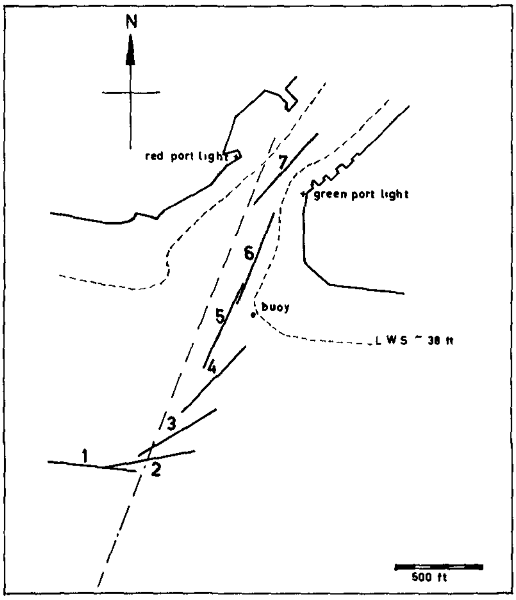

Fig. 9a

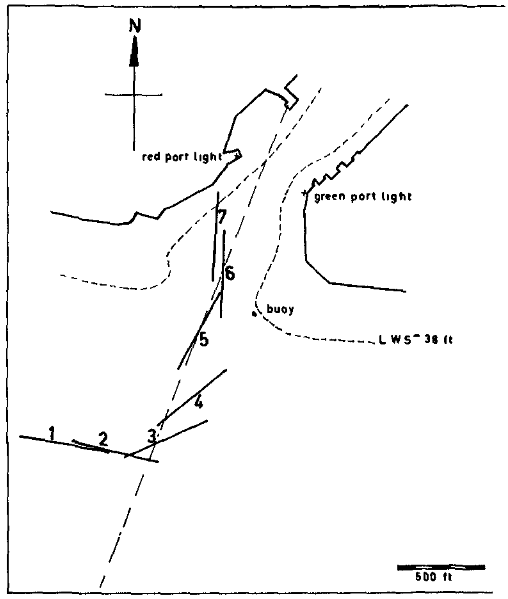

Fig. 9b

Fig. 9a Maneuver T2 Tanker

With the helm turned a little to port and the screw dead slow, the ship makes straight for the buoy, against the stream, at a quarter of a mile's distance from the shore. In position 3 the helm is turned completely to port, and kept so during positions 4 and 5. Then the helm is gradually turned into its former position, in such a manner that it is righted in position 7. During the whole maneuver the screw works dead slow.

Fig. 9b Maneuver T2 Tanker

The ship threatens to pass the buoy on the wrong side. Therefore the helm is already put entirely to port in position 3. Screw is kept dead slow. In position 6, however, the helm is still entirely put to port. This is incorrect. In spite of helm being turned to starboard, and screw full speed ahead, the ship runs into the westerly reef near the red portlight position 7. After position 5 the helm should have been righted gradually.

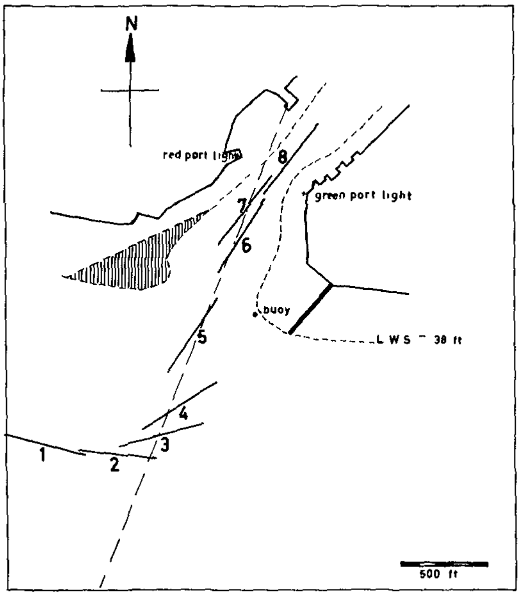

Fig. 9c

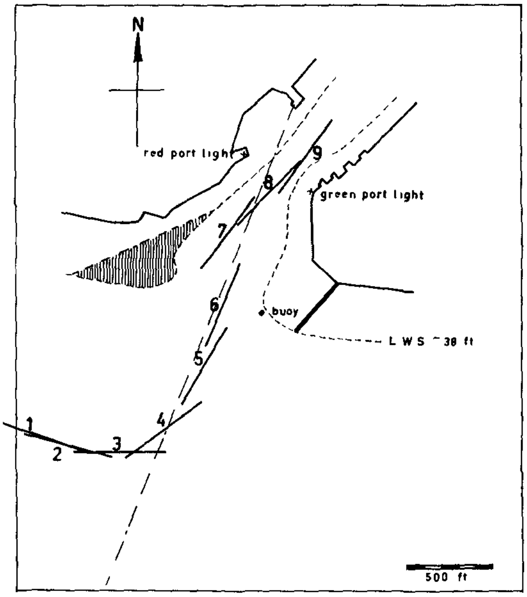

Fig. 9d

Fig. 9c Maneuver T2 Tanker

The tanker sails dead slow against the stream at a quarter of a mile's distance from the shore. During the whole maneuver screw dead slow. In position 2 the helm is turned slowly to port in positions 5 and 6 it is kept entirely to port. Bow does not turn to starboard. In position 7 the helm is righted again. The tanker arrives correctly between the heads of the bridge.

Fig. 9d Maneuver T2 Tanker

At about a quarter of a mile's distance from the shore the tanker sails dead slow against the west going current. During the whole maneuver the screw is kept dead slow. In position 3 helm slowly to port. In position 5, when the ship comes into smooth water, helm entirely to port. Also in position 6 the helm is kept completely to port. Then it is righted and in position 7 put a little to starboard. In positions 8 and 9 the helm is righted, after which the tanker arrives correctly between the heads of the bridge.

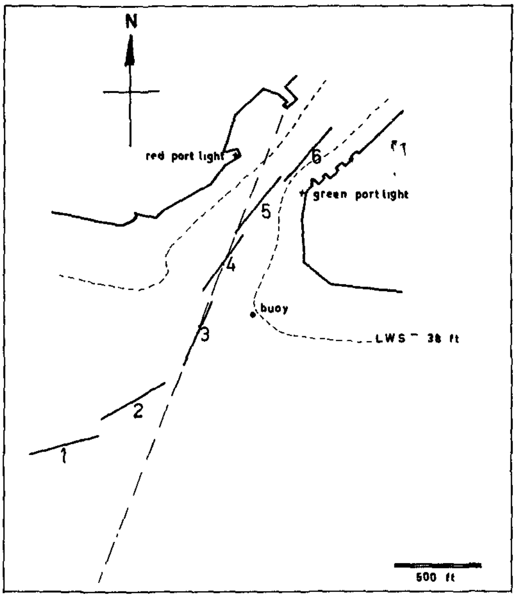

Fig. 10a

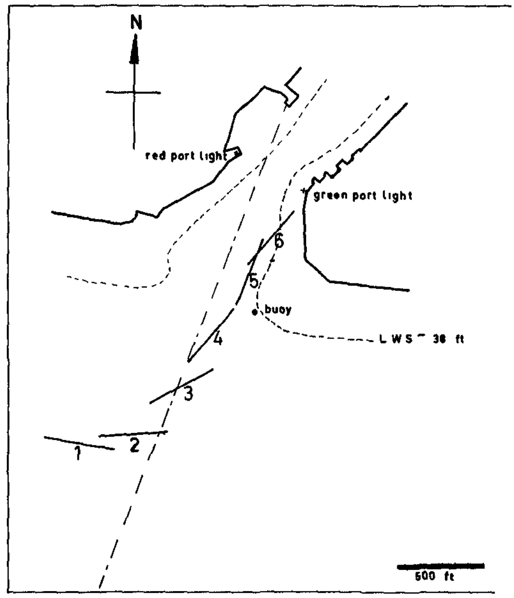

Fig. 10b

Fig. 10a Maneuver G2 Tanker

With helm a little to port the tanker makes straight for the buoy. In position 3 helm completely to port. The bow threatens to turn a little to starboard. This is obviated by sailing full speed ahead between positions 3 and 4, for a short moment. The port screw keeps revolving dead slow ahead. In position 4 both screws again dead slow ahead. The helm is still kept entirely to port. In positions 5 and 6 helm righted.

Fig. 10b Maneuver G2 Tanker

Tanker correctly makes straight for the buoy. Both screws revolve dead slow ahead. In position 1 helm a little to port, in position 4 helm completely to port. Then, before the stern has come into smooth water the helm has already been righted, so that the bow turns to starboard and the tanker runs ashore near the green portlight. In position 5 the helm should still have been put entirely to port, only after position 5 should it have been righted slowly.

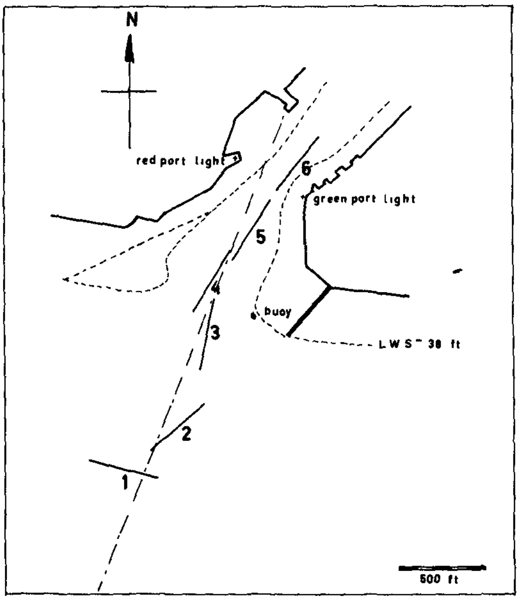

Fig. 10c

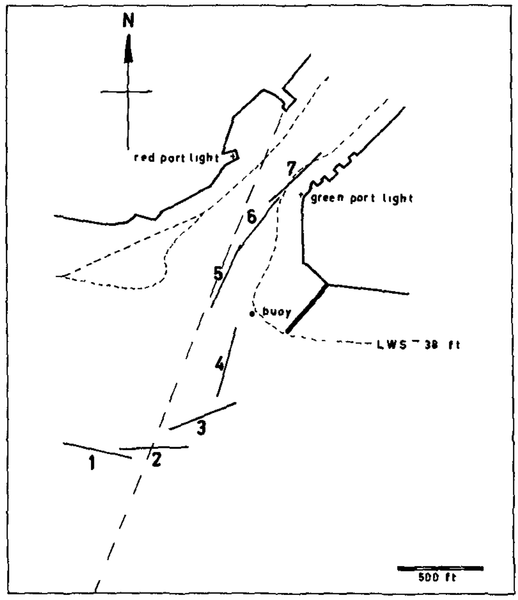

Fig. 10d

Fig. 10c Maneuver G2 Tanker

Tanker sails dead slow against the stream and the helm is put a little to port in position 1. In position 2 the helm is turned further to port. In position 3 the helm is turned entirely to port. If in position 4 the whole tanker, including the stern, has come into smooth water the helm is righted. During the whole maneuver both screws revolve dead slow ahead.

Fig. 10d Maneuver G2 Tanker

Tanker threatens to pass the buoy on the wrong side. Both screws dead slow ahead. Already in position 3 rudder entirely to port in order to get the buoy on starboard. Between positions 4 and 5 the bow threatens to turn to starboard. Therefore maneuvers are carried out with the screws. Starboard screw full speed ahead and port screw full speed astern. In this way running ashore on the easterly reef is successfully avoided. Maneuvers by means of screws cause loss of time. In position 5 again both screws dead slow ahead. The helm is still kept a little to port. The tanker puts into port a little too near the green portlight.

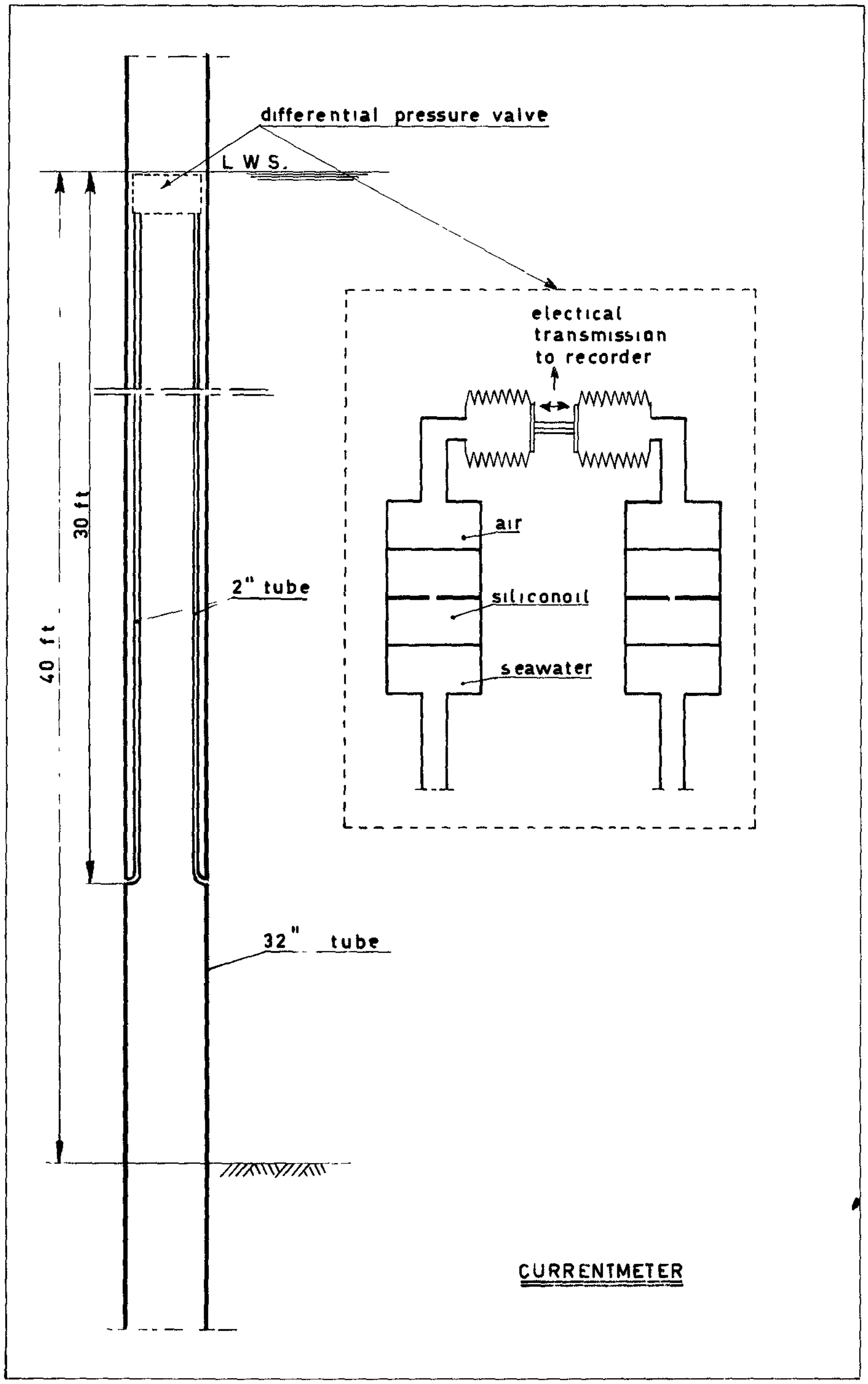
differential pressure valve
L W S.
30 ft
40 ft
2" tube
32" tube
electical
transmission
to recorder
air
siliconoil
seawater
CURRENTMETER

Fig. 11

The sailing tests taken are of very great value. It can be shown how the improvements projected will work in practice; besides, these tests are very instructive for the pilotage staff. Each pilot has his own way of piloting a ship into port. It is impossible to give instructions for the piloting into port of ships. It can only be useful for every pilot to try his way of entering into port in a model.

The small number of ship accidents since the middle of 1955 is due, in large part, to a better insight into the behavior of ships when putting into port as a result of the investigations carried out in the model.

EXECUTION OF THE IMPROVEMENTS PROPOSED

Of the improvements proposed only the current-meter and widening of the narrow cross-section have been realized up to now, whereas a start has been made with the preparations for the construction of a breakwater.

A temporary current-meter, projected as a result of the combined efforts of several Netherlands laboratories, has been put up about 200 ft. east of the easterly buoy, at a spot where the water is 40 ft. deep. This current-meter consists of a 32 ft. tube which has been driven into the ground. At a depth of 30 ft. there are 2 holes, diametrically opposed to each other, and lying in the direction of the current. The velocity of the current can be obtained by measuring the difference in pressure between the two holes. In order to cause the current to detach itself always from the wall at the same spots, two angle sections have been welded on either side of the tube, across the direction of the current.

In order to prevent the wave motion from influencing the results, columns of air and silicon oil have been applied between the points of pressure and the pressure gauge. In the column of silicon oil a plate has been fixed, provided with a narrow hole, which plate acts as a resistor. This resistor serves to eliminate the influence of the differences in pressure caused by the waves. Silicon oil is used because it is chemically inert. Fig. 11 gives a schematic outline of the operation. The measurements are transmitted electrically to the shore and recorded. Signal lamps indicate the velocity of the current at sea and are visible at sea at all times.

No decision has yet been made as to the manner in which the breakwater will be constructed. Since the breakwater has to be built as far seaward as possible it will be necessary that the end of the breakwater be constructed with vertical walls.

Information about waves was not available at the time of research on this paper. At the present time a wave recorder is installed near the harbor entrance to obtain basic information for design.

CHAPTER 40

SHIP WAVES IN NAVIGATION CHANNELS

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INTRODUCTION

Ships moving through water generate surface waves which in many navigation channels cause severe wave-wash damage to the bank: In some waterways, rather extensive protection works have been constructed to reduce this destructive action to levee faces (Hertzberg, 1954). This action, termed "foreshore erosion", has been described by Lewis (1956) for the lower Mississippi River as follows:

> "The attack, interestingly enough, takes place at low-water. It is due largely to the waves created by passing ocean-going ships and is augmented by shallow draft traffic. It may be asked at this point why this attack is new, in view of the fact that ship and barge traffic has long existed on the river in very substantial volume? The answer is that the recent technological advances in ocean-going and river transportation have greatly increased maximum and average speeds, and accordingly, the wave making potentials."

Despite the importance of this problem, very little quantitative data are available on the characteristics of ship waves at a given distance from the sailing line in terms of type of ship, displacement, speed, and water depth. Naval architects have long been interested in the waves generated by ships, but primarily from the standpoint as to how the waves affect the resistance of the ship (Havelock, 1908, 1934, and 1951; Lunde, 1951; Birkhoff, et al, 1954). The pioneer work in the field of ship resistance was done by Rankine (1868) and Froude (1877); however, as discussed below under the section on theory, Lord Kelvin (1887a, 1887b) was perhaps the first to present a theory f such waves by calling attention to the similarity of actual ship waves t the waves generated by a pressure point moving across the water surface. Standard text books on naval architecture (Taylor, 1943; Robb, summarize Lord Kelvin's theory which provides a method of plotting the wave patterns as well as the relative wave heights throughout the pattern. What was apparently the first comprehensive set of measure ments of actual ship waves were the observations of Hovgaard (1909) o the waves generated by several different types of ships. He compared the actual angles of the diverging waves with the theoretical values of

Lord Kelvin for deep water conditions. For restricted waterways numerous investigations have been made to determine the form of canal cross section and system of bank protection to be adopted with a view to resisting the destructive wash of passing tows and motor boats. These studies consisted of model tests (Krey, 1913 and 1929) as well as tests and observations with boats in actual canals (De Bruyn and Maris, 1935; Rosik, 1935; Schijf, 1949; Sum, 1935; Vanderlinden, 1912; Van Loon, 1912; Wortman, 1894). As discussed below there is very little information in any of these various studies on the actual wave heights that occurred during the tests.

THEORY

As originally discussed by Froude (1877), the main features of the entire disturbance behind a ship, which is confined between two straight lines, is that there are two distinct systems of waves. These are the transverse and the diverging waves (Figure 1). Curves of equal phase in the two systems meet on the straight line boundaries of the disturbed region in cusps. These lines often are designated as the "cusp locus". Wave heights are greatest and the crests the sharpest at the cusps, as well as at the point of the disturbance, so that breaking water will be found at these points, if anywhere.

There is a striking similarity between the waves generated by an actual ship moving across the water surface and the theoretical wave pattern resulting from a single moving point as developed by Lord Kelvin (1887a). His solution shows that for deep water the whole pattern of waves is comprised between two straight lines drawn from the bow of the ship and inclined to the wake on its two sides at equal angles of $\alpha = 19° 28'$. Figure 2 shows a plot of a Kelvin wave group caused by a travelling disturbance, and Figure 3 shows a single crest of Kelvin's wave group with relative heights at various points. The height in this instance is the elevation of the wave crest above the undisturbed water level. The difference between the Kelvin wave groups and actual waves is that a Kelvin group is an ideal system resulting from forces applied at a single moving point; whereas an actual wave group is due to forces spread over a ship's hull. Actually, the application of the Kelvin theory perhaps is valid only at a distance of several ship lengths from the stern.

An essential parameter which determines the wave pattern created by a ship on a straight course is according to the Kelvin theory

$$\lambda = \frac{C}{C_0} \qquad (1)$$

in which C is the ship speed and C_o is the velocity of a wave in shall water; that is, $C_o = \sqrt{gd}$, where g = acceleration of gravity, and d water depth. The wave amplitudes, and hence the wave-making resistance of the ship apparently depends on the character of the ship's hull. However, the general shapes of the curves formed by the wave crests and troughs and their spacing depend upon λ and the speed of the ship and not upon the shape of the hull. For $\lambda = 0$, i.e. infinite depth, the character of the wave pattern given by the Kelvin theory is as shown in Figures 2 and 3.

Relationships for wave patterns for a travelling disturbance water of any depth have been developed by Havelock (1908). A simpl fied method of plotting such wave patterns has been presented by Rol (1952). For a finite water depth d, the angle α between the ship's course and the cusp locus increases as λ increases and approaches theoretical value of 90° as λ increases from zero to the critical valu of 1.0. The value of the angle α increases, however, vary slowly with λ until λ is somewhat greater than 0.7 (Figure 12). For finite depth, the spacing of the waves continues to be proportional to the speed, but the constant of proportionality depends on the value of λ. The spacing increases for a given speed with diminishing depth, but this increase is small for λ values less than 0.7. At $\lambda = 1$, $\alpha = 90°$, and the wave pattern consists essentially of a single wave with its crest at right angles to the ship. When the critical value $\lambda = 1$ has been passed, the character of the wave pattern behind the ship chan completely, and there are no longer distinct systems of waves as th transverse system disappears, the disturbed region lies between tw straight lines with the angle α being expressed in terms of λ. The angle α theoretically approaches zero at high speeds (or low depth and the straight line boundary is now a phase curve of the wave system and not a cusp locus of such curves as in the case where $\lambda < 1.0$.

OBSERVED SHIP WAVE CHARACTERISTICS

DEEP-WATER CONDITIONS

Observations on waves generated by actual ships have been made by naval architects in connection with studies on ship resista Most of these investigations have been with models, and only a few systematic observations with full size ships appear to have been m The most extensive observations were those of Hovgaard (1909) wh made measurements on both wave height and wave pattern for vario sized ships and models operating in deep water. A typical example

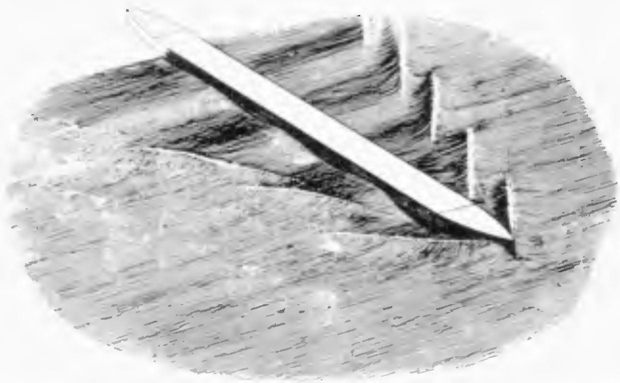

Fig. 1. Froude's sketch of the characteristics of ship waves, Trans. Inst. Naval Arch., 1877.

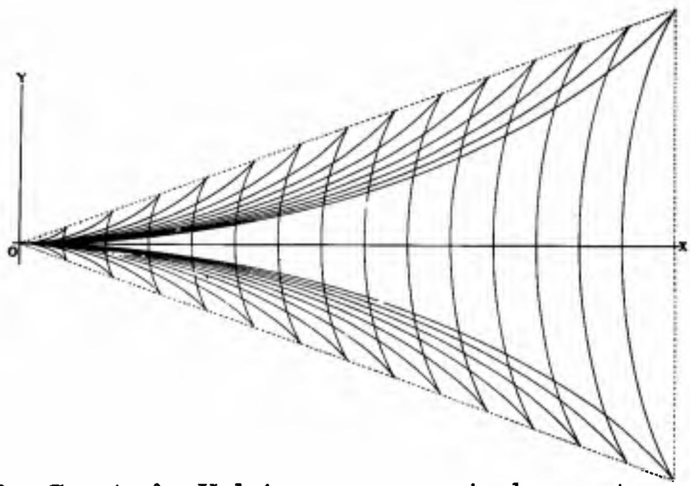

Fig. 2. Crest of a Kelvin wave group in deep water caused by a travelling disturbance at 0 (Taylor, 1943).

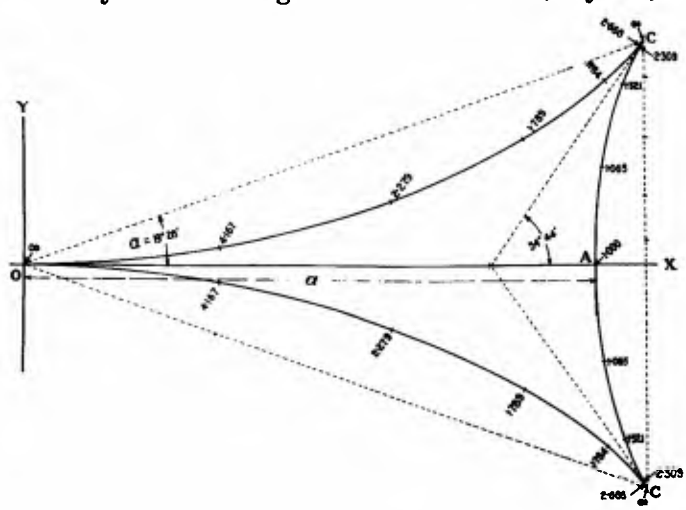

Fig. 3. Single crest of Kelvin wave group in deep water with relative heights at various points given by numbers (Taylor, 1943).

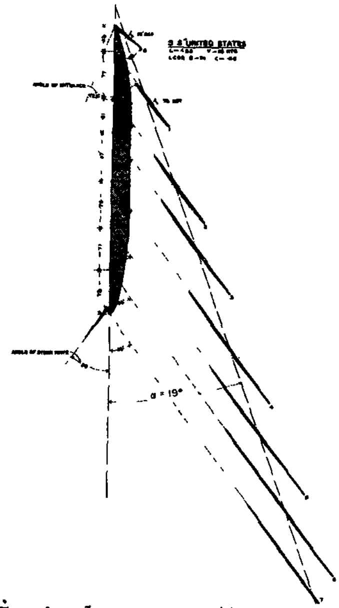

Fig. 4. Typical wave pattern as observed by Hovgaard (Trans. Inst. Naval Arch., 1909).

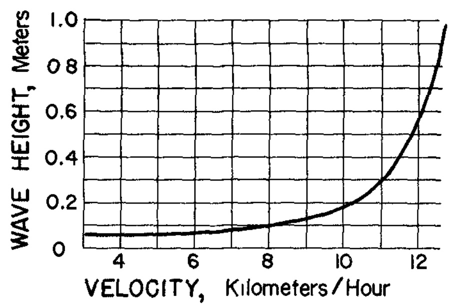

Fig. 5. Relation between velocity travel and wave height at shore of canal under the influence of a sing tugboat (Franzius, 1936).

TABLE I
Summary of Hovgaard's Observations on Diverging Ship Waves

Ship or Model	Displacement or Tonnage	L Length (ft)	V Speed (Knots)	$\frac{V}{\sqrt{L}}$	α Cusp Locus (degrees)	Height of Bow Breaker
SS United States	10,000 tons	493	14 5	0 86	18	3 ft
"	10,000 tons	"	15	0 68	19	-
Danish Patrol Boat	47 tons	84	4 5	0 48	16 5	3-6 inches
"	"	"	6	0 68	17	4-6 "
"	"	"	8	0 87	15-1/2	6-8 "
"	"	"	10	1 09	17	9-12 "
"	"	"	12	1 31	-	9-12 "
Olfert Fisher	3,450 tons	272	14	0 85	17-1/2	7 5 ft
"	"	"	9	0 55	19	1-2 ft
S S Dronning Maud	1,760 tons	271	15	0 91	18-1/2	4 ft
SS Kong Haakon	1,760 tons	271	16	0.97	16	-
Danish Mine Vessel	107 tons	68	7-1/2	0 91	17	12-15 inches
Destroyer Model	505 lbs	20	3 01	0 67	10	-
"	"	"	4.49	1 00	9-1/2	-
"	"	"	6 00	1 47	9	-
"	"	"	8 20	1 83	9	-
"	"	"	8 86	2 23	8	-
Battleship Model	2,820 lbs	20	3 16	71	17	-
"	"	"	3 60	81	16-1/2	-
"	"	"	4 00	.88	15-1/2	-
"	"	"	4 50	1 01	14	-
"	"	"	4 90	1 10	13-1/2	-
Merchant Vessel Model	2,422 lbs	20.9	2 70	0 59	17	-
"	"	"	3 30	0.72	16-1/4	-
"	"	"	3 86	0 89	15-3/4	-
"	"	"	4 10	0 90	15	-
"	"	"	4 63	1 01	15	-
"	"	"	5 06	1 11	14-3/4	-
"	"	"	5 45	1 19	14-1/2	-

of Hovgaard's observations is shown in Figure 4 and a summary of some of his observations are presented in Table I.

The Kelvin theory predicts that the diverging wave system is the same for all hull forms and speeds; however, Hovgaard's measurements lead him to state that:

> "The observations here recorded show that this is not the case, the obliquity of the waves being greatly influenced by speed and form of the ship, and being not ever the same for all waves in the same ship at a given speed."

SHALLOW-WATER CONDITIONS

As a ship moves through relatively shallow water the flow conditions around the hull, as well as the surface waves which are generated, will be different from deep-water conditions, and the ship resistance is increased. The change in flow conditions around the hull probably increases the skin friction resistance slightly, but the major increase in resistance apparently is due to the increased wave heights occurring with shoal-water conditions. Perhaps the best summary of the problem of ship resistance in shallow water is that by Taylor (1943), wherein previous investigations are reviewed and a tentative attempt is made to define the condition at which depth effects cause the resistance to begin to increase. Since the effect of the waves probably is the principal cause of increased resistance, the relationship presented by Taylor is of importance in evaluating the effect of depth on the wave characteristics. For moderate and slow speed vessels, that is, vessels capable of deep-water speeds in knots of $0.9\sqrt{L}$ or less, Taylor shows that the relative minimum depth for no increase of resistance is given by the expression,

$$d_{min} = 10\, D \frac{V}{\sqrt{L}} \tag{2}$$

d_{min} = minimum depth for no increase of resistance - ft.

D = draft - ft.

V = speed in knots

L = length of vessel - ft.

This relationship gives the critical point at which ship resistance starts to increase, but unfortunately no information is available from the data on the actual change in the characteristics of the waves themselves.

For a high speed vessel in water of depth less than the ship's length, Taylor (1943) states that "at a definite speed in a given depth the vessel will begin to experience appreciably increased resistance as compared with the resistance in deep water. The excess of resistance above the normal increases with speed until it reaches a maximum. This maximum appears to be at about a speed such that a trochoidal wave, travelling at this speed in water of the same depth, is about 1-1/4 times as long as the vessel."

From observations on the operation of canal boats in restricted waterways Rosik (1935) made some measurement of wave heights. He stated that:

> "The wave from the bow usually narrows suddenly about half the length of a vessel, whereas behind the stern it is suddenly thrust out. According to the speed at which the craft is travelling, the first wave behind it reaches a height of 2 meters measured from crest to hollow. This wave is followed by several others of an elongated form and two or three grouped waves having 1/2 to 2/3 the height of the first one. The effect of the wave is all the greater on the banks when a vessel runs close alongside them. On the Danube this distance does not drop below 15 to 20 meters. At a distance of 50 meters from the banks the effect is only half, and at a distance of 250 meters it scarcely makes itself felt. The influence of the waves makes itself felt if the ship speed craft exceeds 8 km an hour."

No mention is given by Rosik as to the type or displacement of the vessel or the water depth to cause such wave conditions.

Other wave data of limited extent are given by Franzius (193 in Figure 5. Information is lacking on the size, shape, and displacement of the tugboat and the depth and width of the canal under which these observations by Franzius were made.

As mentioned previously, observations on the movement of tow boats in restricted waterways have been made both by model tes and with actual canal boats. Most of these latter studies were concerned primarily with the maximum speeds that were allowed in various canals. This maximum speed was not based on scientific stuc but provisional speeds were adopted and the extent of bank erosion c served. The maximum speeds usually were expressed as a functior of the ratio of the mid-ship cross sectional area of the boat to the c sectional area of the canal. For example, Vanderlinden (1912) states that where the canal cross-sectional area is eight times that of the boat a maximum speed of 10 to 12 km per hour is possible. Allowable speeds in various canals also have been given by Wortman (1894) and Van Loon (1912). In a discussion of the effect of ship waves on the banks and beds of canals Sum (1935) observes that the greatest damage occurs about one meter above and below normal water level. In narrow canals he recommends a maximum speed of 5 km per hour. Where a river's width is 70-150 meters Sum (1935) states that the waves are from 30 to 80 cm in height. No mention is made, however, of the size or type of boat that produced such waves.

Perhaps the most comprehensive investigation on boats operating in comparatively narrow waterways was that of Schijf (1949) who used both models and actual power boats in his studies. His tests show that in a canal with erodible bed and banks the maximum velocity is dependent on the ratio of the midship cross-section of the boat to the cross-section of the canal, the shape of the hull, and the method of propulsion.

SHIP WAVE MEASUREMENTS

MODEL STUDIES

Equipment and Procedure - As evidenced by the above discussion, relatively few data are available on the wave heights to be expected at a given distance from the sailing line of a ship with a given hull shape and displacement moving through shallow water at a given speed. To obtain such data and better define the importance of the variables involved, a series of model experiments were made at the University of California model basin. In these studies ship models were towed at various speeds in water of various constant depths, and the wave characteristics measured at various distances along a perpendicular to the sailing line. The general arrangement of the test facilities is shown in Figure 6, and the characteristics of the models tested are shown in Figure 7 and Table II. In addition to the geometric characteristics of the various models, Table II also shows values of the "block coefficient" and the "displacement-length ratio." These coefficients commonly used by naval architects are defined as follows:

$$\text{Block Coefficient} = b = \frac{v}{L \times D \times B}$$

where v = volume of displacement

L = waterline length

B = waterline beam at midship

D = draft at midship

$$\text{Displacement-Length Coefficient} = \frac{\Delta}{\left(\frac{L}{100}\right)^3} -$$

where Δ = displacement in tons

L = waterline length in feet

TABLE II

Model Characteristics and Test Conditions
(Fresh Water Tests)

Model	Displacement lbs.	Water Line Dimensions L Length (ft)	B Beam (ft)	D Draft (ft)	b	$\frac{\Delta}{\left(\frac{L}{100}\right)^3}$	Wa dep d (ft)
A_L	9.27	3.32	0.94	0.13	0.37	127	0.
A_H	27.63	3.40	0.98	0.23	0.58	352	0. 1. 1.
B	27.63	3.44	0.82	0.19	0.82	340	0.
C	27.63	2.92	1.00	0.27	0.56	553	0.
D	51.5	4.0	0.67	0.33	0.93	402	1.
E	20.94	3.0	0.50	0.25	0.93	398	1.
F	5.89	2.0	0.33	0.167	0.93	368	0.

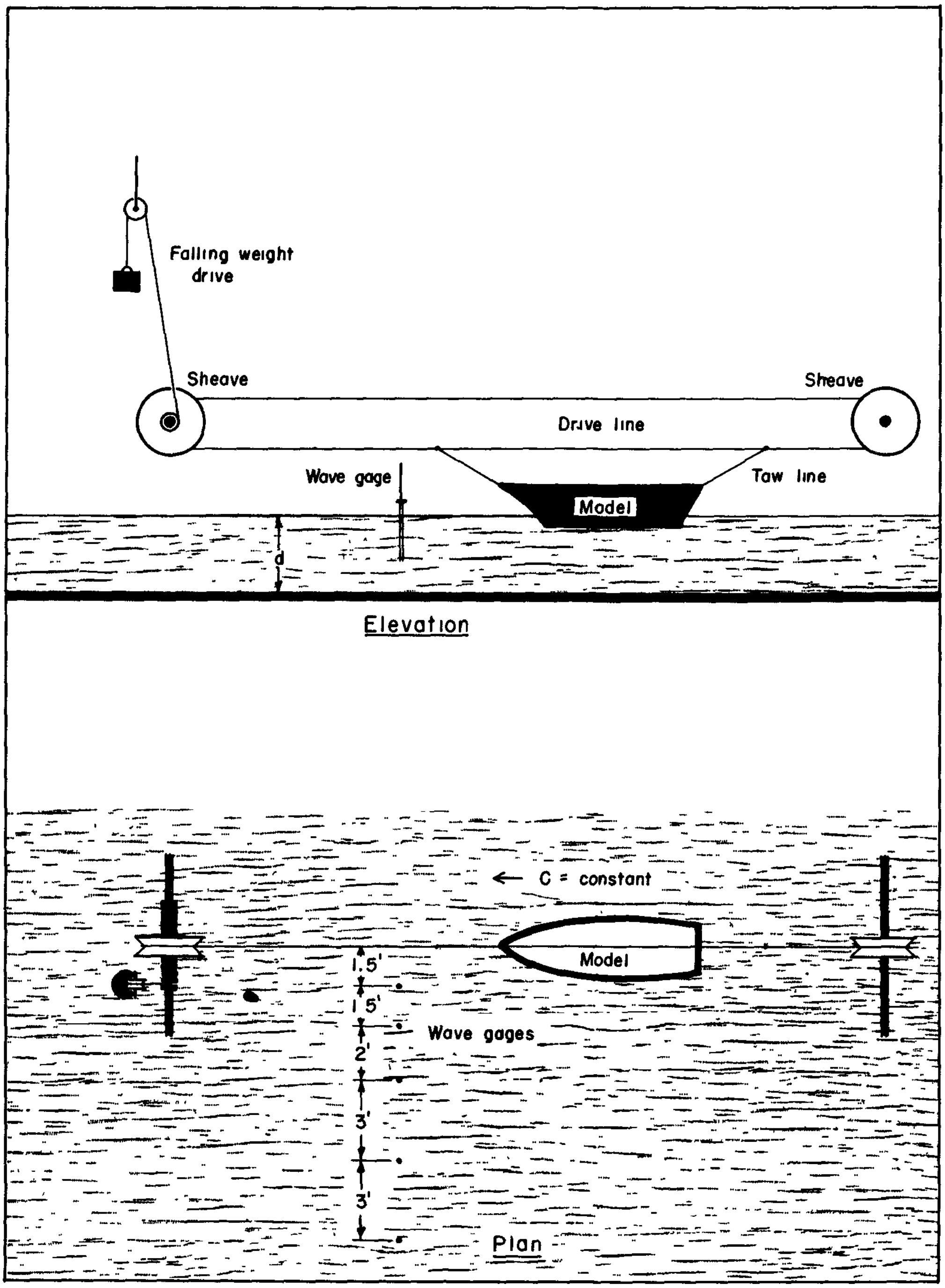

Fig. 6. Equipment used for investigating ship waves by models.

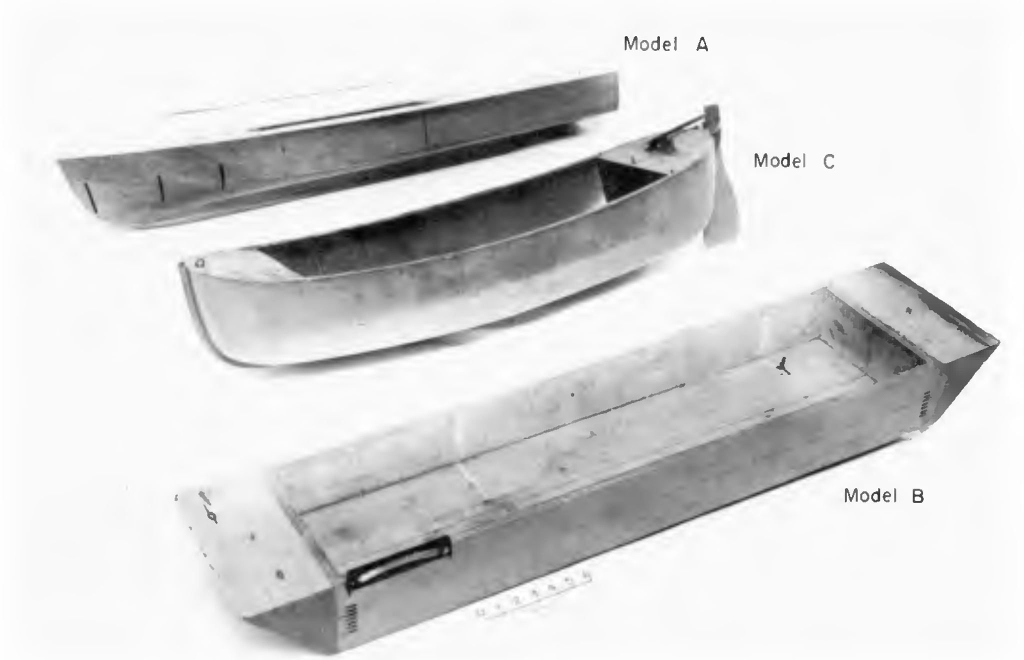

Fig. 7. Photographs of Models A, B, and C used in ship wave studies.

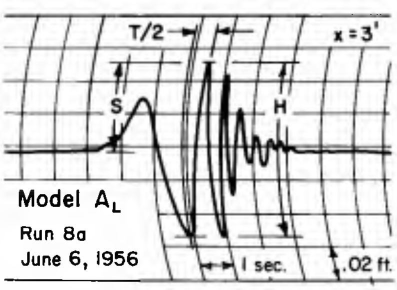

Fig. 8. Typical chart record of ship waves showing the various terms determined in the tests.

MODEL DIMENSIONS, in Inches

Model	L	B	D	e	f	h
D	48	8	4	7	41	8
E	36	6	3	5.25	30.25	8
F	24	4	2	3.5	20.5	5

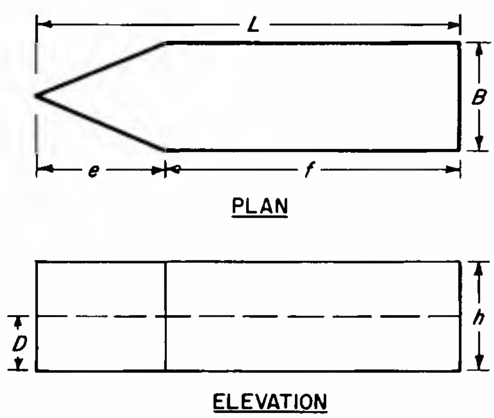

Fig. 9. Idealized models used in determining scale effects in ship wave studies.

The towing system shown in Figure 6 was driven by falling weights with the model being attached fore and aft by strings to a taut line which passed over sheaves at two ends of the model basin. Craft speeds were varied by varying the weights on the driving system. All tests were made with fresh water in the model basin. Wave characteristics were measured in a locality where the model had attained a constant speed. These measurements were made at five positions by parallel-wire resistance elements (Wiegel, 1956), located as shown in Figure 6, and recording on a six channel Brush recorder. The sixth channel of the recorder was used to determine the speed of the craft from timing marks made on the recorder chart by a contact actuated during each revolution of one of the sheaves on the drive system. Measurement of the waves was made at various constant speeds from the lowest speed that waves could be recorded accurately to the highest speed that the driving system could be operated. For a model with "good" lines (Model A_L, Figure 7) the resistance to motion was small enough that sufficient weights could be placed on the driving mechanism to attain a craft speed well beyond the critical speed where the wave height was a maximum. A typical example of a wave height record is shown in Figure 8. Such chart records were analyzed for the maximum crest elevation above still-water level and for the maximum wave height. This maximum wave height usually was the vertical distance of the maximum crest height above the preceding trough (Figure 8). The angle between the cusp locus and the sailing line was computed for each test from the time lag between maximum wave height occurrence at two wave gages, the distance between gages, and the model speed.

To give a measure of the effect of scale on the results from model tests a series of runs were made with an idealized ship form constructed to three different sizes as shown in Figure 9. These three models were tested under conditions in which certain variables were held constant to determine the effect of absolute size of the craft on the wave characteristics.

Test Results - A typical example of the type of wave height data obtained from the tests on a particular model is shown in Figure 10, where for Model A_L the values of maximum wave height, H, and maximum crest elevation, S, as determined at various distances x from the sailing line, are plotted as a function of ship speed. The water depth was held constant during the tests. Also indicated in Figure 10 are scales showing values of $V/\sqrt{L}$ and λ (as defined by equation 1). In the term, $V/\sqrt{L}$, which is commonly used by naval architects in comparing the performance of ships, V is the ship speed in knots, and L is the waterline length in feet. It is of importance to note in Figure 10 how rapidly the wave height increases with speed up to a critical point.

Beyond this critical speed the wave heights decrease and approach a con stant value for relatively high speeds. The rapid increase in wave height with ship speed for speeds below the critical speed confirms the observation of Lewis (1956) as quoted in the Introduction--namely, that increased ship speeds has resulted in greatly increased wave heights and consequently serious wave-wash erosion problems. An item of interest in connection with Figure 10, and similar plots for other model: is that the maximum wave height, H, is approximately twice the value of S, the maximum crest elevation above the still-water level.

In addition to the data on wave heights as measured from the recorder charts the time between the occurrence of the maximum crest height and the preceding trough (see Figure 8) was determined and has been termed the "half-period." Figure 11 shows such data plotted again λ. These data were obtained from the same series of tests for which th height data in Figure 10b is presented. Within the accuracy of measure ment the half-period was independent of the distance, x, from the sailing course.

For the tests from which the wave height data shown in Figure 10 were obtained the angle, α, between the cusp locus, or point of maximum wave height, and the sailing line was computed and is plotted against λ in Figure 12. Also shown on this plot is the theoretical curve of Lord Kelvin. There is fair agreement between theory and experiment.

Wave height data similar to that shown in Figure 10 were obtained for the other models and test conditions listed in Table II. To permit a comparison of the wave generating capacity of the various hull forms represented by the models, dimensionless plots were prepared using the following parameters:

$$\frac{H}{D} = f\left(\frac{V}{\sqrt{L}}, \frac{d}{D}, \frac{x}{L}\right) \tag{5}$$

where

H = maximum wave height, ft.
D = ship draft at mid-section, ft.
L = ship length at water line, ft.
V = ship speed in knots
d = water depth, ft.
x = perpendicular distance from sailing line to point of wave measurement.

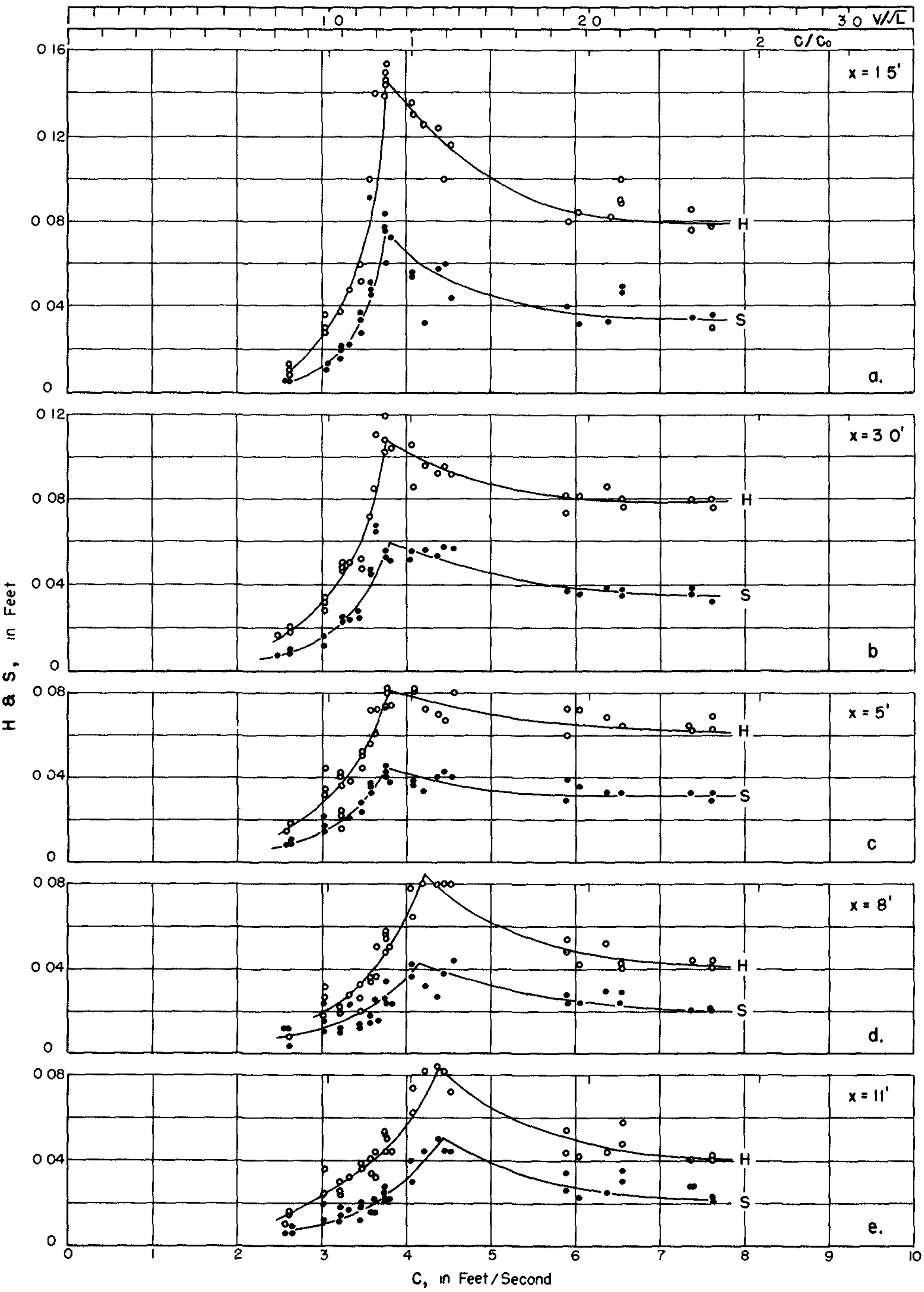

Fig. 10. Typical data on wave height as a function of ship speed and distance of wave recorder from sailing course. Model A_L with a water depth of 0.52 ft. used in tests.

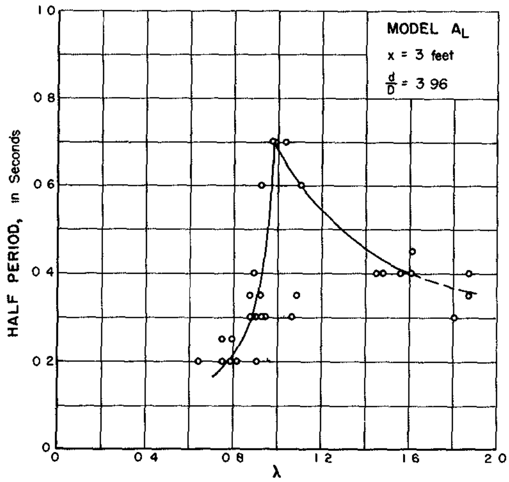

Fig. 11. Relationship between period for maximum wave and λ. Model A_L with a water depth of 0.52 ft. used in tests.

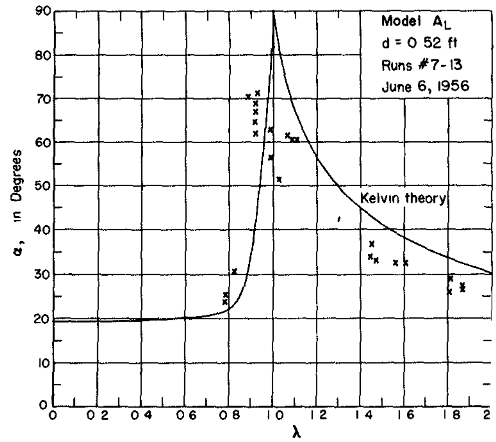

Fig. 12. Relationship between an, ∝, between the cusp locus and sailing line and λ for Model A_L.

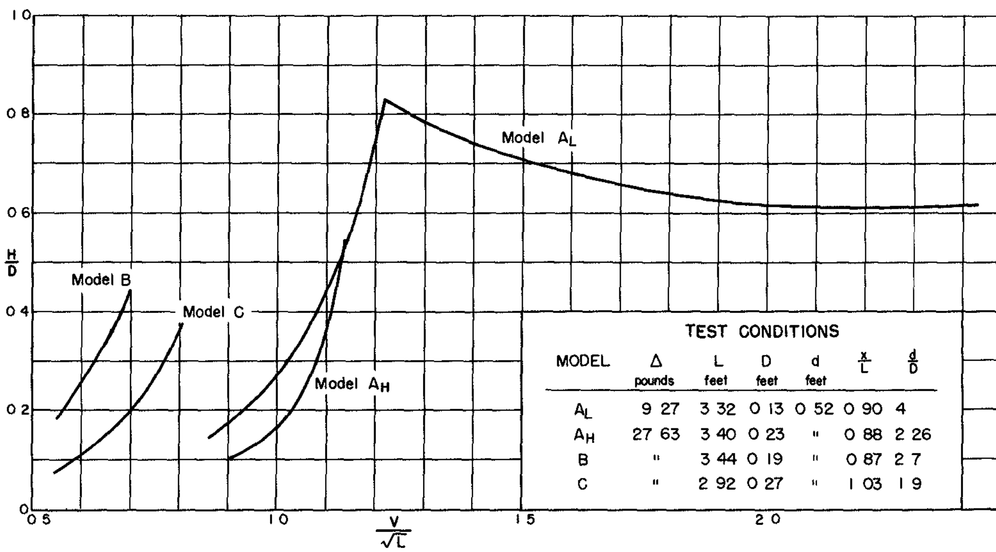

Fig. 13. Relationship between the wave height-ship draft ratio and the relative ship speed, $V/\sqrt{L}$.

Figure 13 shows a plot of H/D as a function of $V/\sqrt{L}$ for a constant water depth and position of wave measurement for models A_L, A_H, B, and C. Due to the slight differences in the length, L, and draft, D, for the various models, there is a slight difference in the values of x/L and d/D between models. The curve for model A_L in Figure 13 was derived from the basic data presented in Figure 10. This model was the only one in which the resistance to motion was low enough that the model could be towed at speeds in excess of the critical condition for maximum wave height. Comparison of the curves for models A_L and A_H gives an indication of the effect of varying the displacement with a particular hull shape. The actual wave heights were greater for model A_H than for model A_L at a particular speed; however, because the draft is less for model A_L than for A_H, the H/D versus speed curve for the lighter model (A_L) plots above the curve for the heavier model (A_H). Comparison of the curves for models A_H, B and C in Figure 13, where the displacement was a constant, shows the general effect of hull form on the relative wave making capacity of the various models; that is, the better the ship lines the lower are the wave heights for a particular speed. To provide information on the variation of wave height with distance from the sailing course Figure 14 has been prepared for one of the models (Model A_H). This figure shows the ratio H/D plotted against the ratio x/L for a constant speed, $V/\sqrt{L}$, with d/D as a parameter. The curves shown are cross plots from such curves as shown in Figure 13. The curves in Figure 14 show that when the water depth is relatively small compared with the draft the wave height decreases rapidly with distance from the sailing line; whereas, for the larger water depth, the rate of decrease in wave height with distance, is considerably less.

Further information on the effect of water depth on the height of ship waves is shown in Figure 15 which is a cross plot from Figure 14. In this figure the wave height-draft ratio, H/D, is plotted against the depth-draft ratio, d/D, for a constant speed of $V/\sqrt{L} = 1.1$ at a constant distance from the sailing line of x/L = 1. In addition to the three values of d/D shown in Figure 15 (namely, 2.26, 4.35, and 7.95) some data were obtained in a relatively narrow towing tank with a d/D value of 22.5. These data were rather limited; hence the point on the curve in Figure 15 at the d/D value of 22.5 is not accurately defined; consequently the curve beyond a value of d/D = 7.95 is shown as a dotted line. Also shown in Figure 15 is the limit for no increase in ship resistance as computed from equation 2. This plot indicates that when the ratio of depth to draft (d/D) is less than about 8, the wave heights increase rapidly with a decrease in depth, that is, when the depth is greater than about 8 to 10 times the draft the waves are essentially those for deep-water conditions.

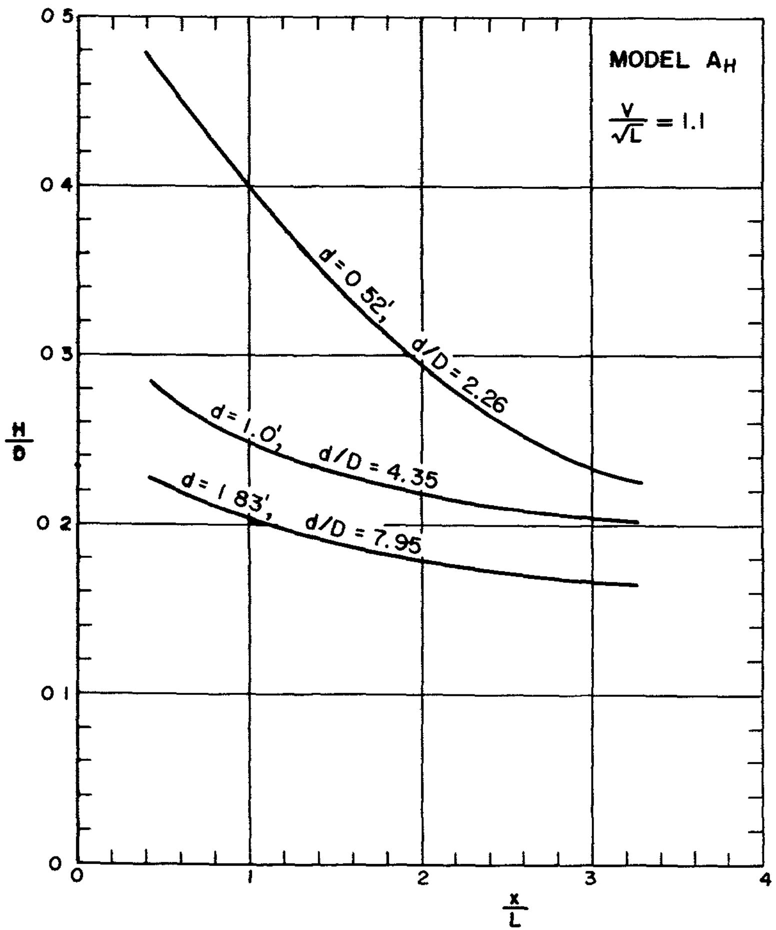

Fig. 14. Relationship between the wave height-ship draft ratio and the relative distance, x/L, from the sailing line with the ratio of water depth to draft as a parameter.

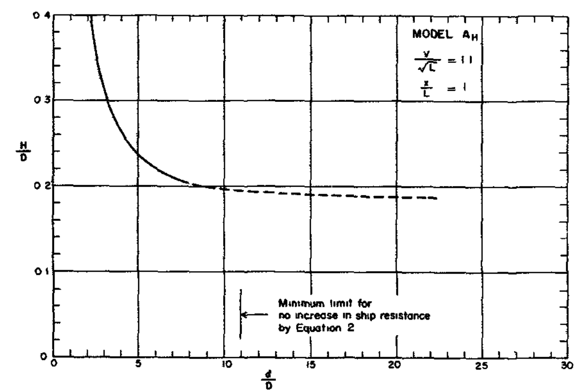

Fig. 15. Relationship between wave height-draft ratio and depth-draft ratio for a constant ship speed and distance from the sailing line.

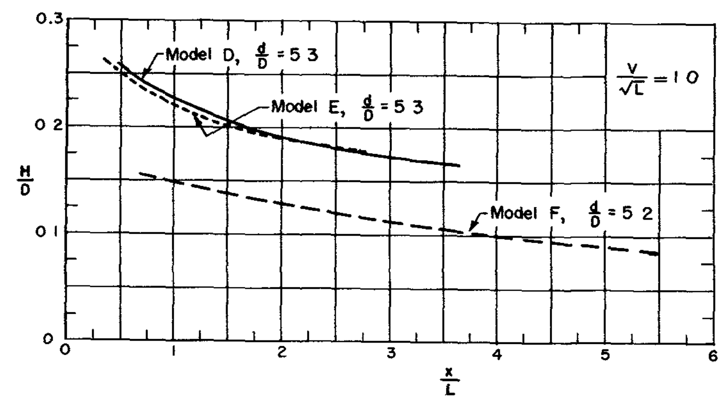

Fig. 16. Relationship between the ratio of wave height to draft and relative distance, x/L, for similar models D, E, and F. Relative speed. $V/\sqrt{L}$. and depth-draft ratio, d/D,

Scale Effects - As mentioned above, a series of tests were made with the idealized ship form shown in Figure 9. The three geometrically similar models were towed at various speeds in water of various depths such that a constant value of depth-draft ratio of 5.3 existed in the tests. The wave heights were measured at the distances from the sailing line as shown in Figure 6. The wave height data for each model were plotted on diagrams similar to Figure 10. From these basic data the dimensionless parameters presented in equation 5 were computed and plotted as shown in Figure 16, which is similar to Figure 14 for Model A_H. Figure 16 shows for each model the ratio H/D plotted as a function of x/L for a constant speed ratio, $V/\sqrt{L}$, of 1.0 and a constant depth-draft ratio of 5.3. Inspection of this graph shows that the scaling law for the larger models (models D and E) agree almost exactly, but the curve for the smallest model (model F) plots considerably below the other curves, thus indicating that there is an effect of absolute size of model. In other words, if models are too small in absolute size the prediction of prototype values from model data is of questionable value. Obviously more experimental data are required to better define the limit of model size for reliable prototype predictions.

POWER BOAT STUDIES

A limited number of observations were made on the waves generated by a 42 ft. power boat of 241 cu. ft. displacement operating at various speeds in water about 7 ft. deep. Measurements were made of wave heights in the tests. In a few instances vertical aerial photographs were made to determine wave patterns. Photographs of wave patterns for values of λ from 0.70 to 1.32 are shown in Figures 17 and 18. A summary of the pertinent data for each of the tests shown in the photographs is presented in Table III. The angle α shown in this table is the angle between the sailing line and a line drawn from the bow through what appears to be the point of maximum wave height.

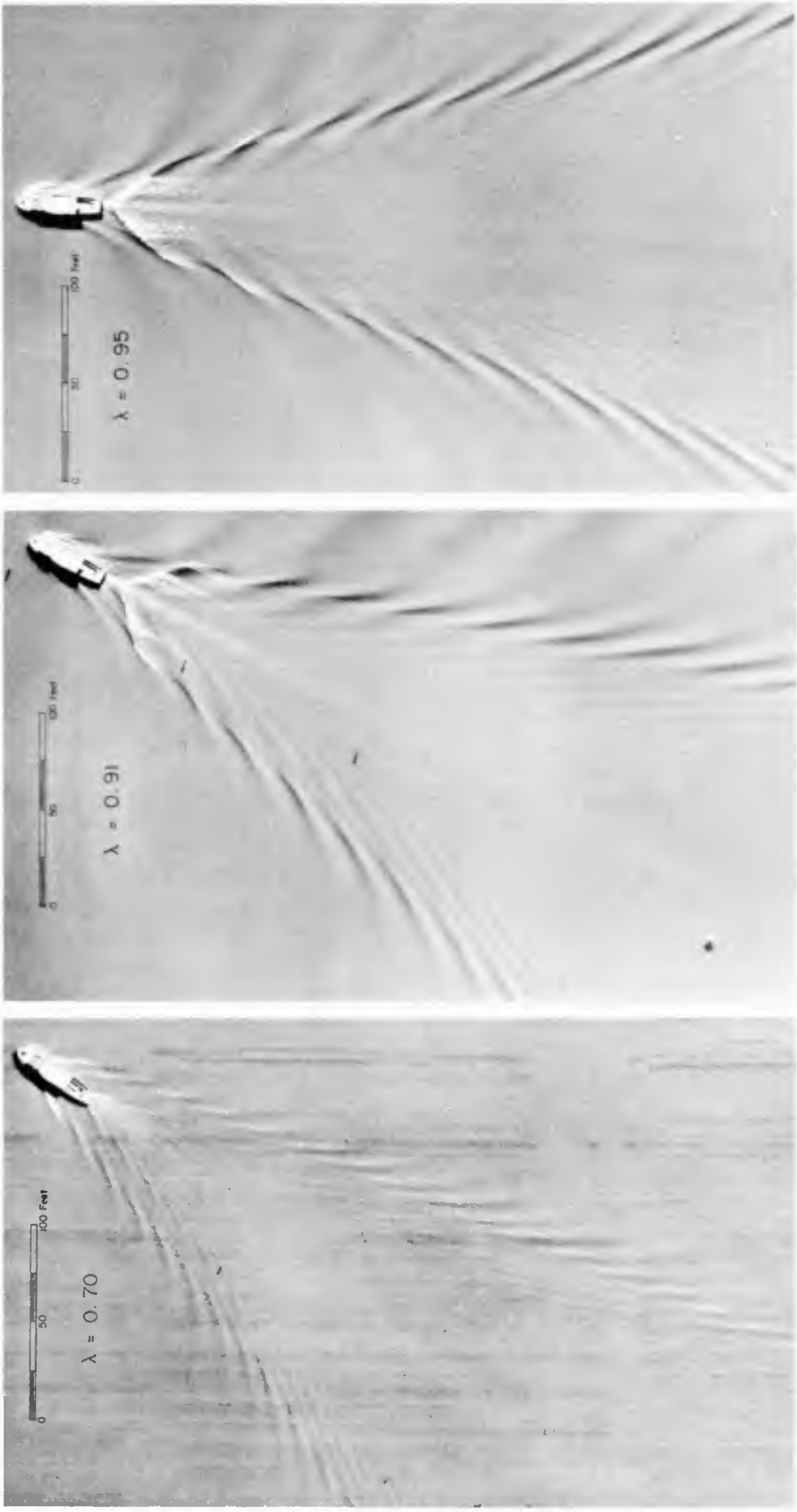

Fig.. 17c

Fig.. 17b

Fig. 17a

Fig. 17. Wave patterns resulting from a 42 ft. power boat moving at various speeds in shallow water.

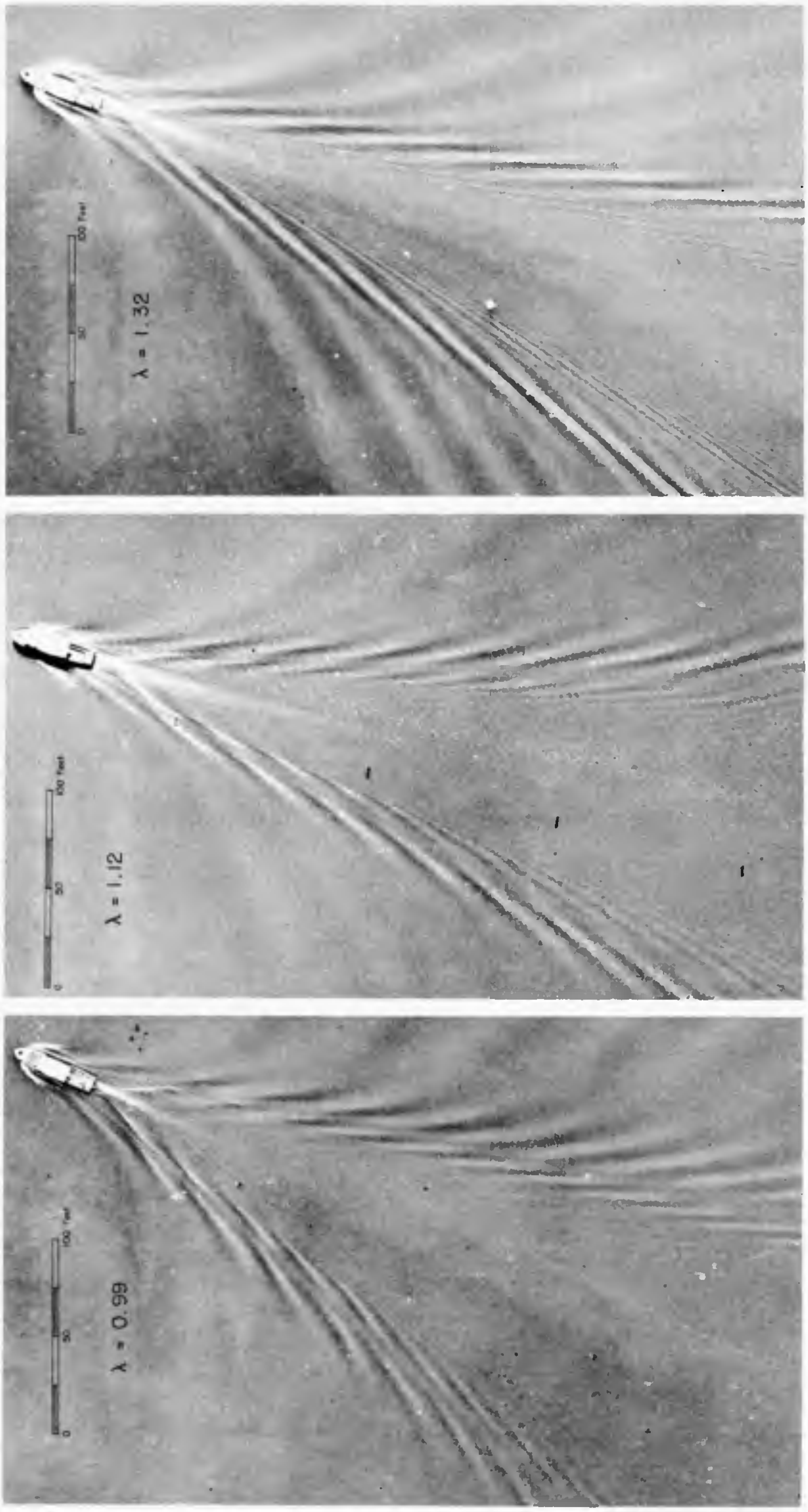

Fig. 18c

Fig. 18b

Fig. 18a

Fig. 18. Wave patterns resulting from a 42 ft. power boat moving at various speeds in shallow water.

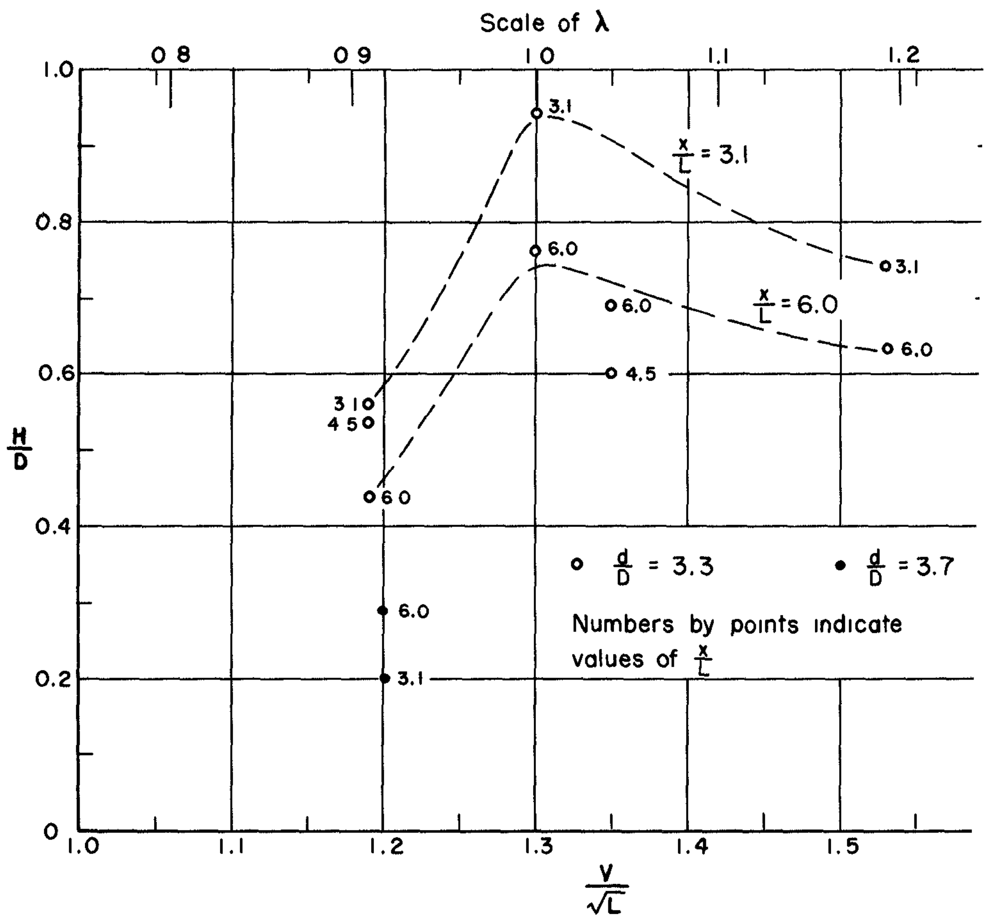

Fig. 19. Relationship between the ratio of wave height and draft and relative speed for a 42 ft. power boat with relative distance x/L and depth-draft ratio as parameters.

TABLE III

Summary of Ship Wave Patterns

Boat length = 42 ft.
Boat displacement = 241 cu. ft.
Boat draft (midship) = 1.9 ft.
Block coefficient = 0.37
Displacement-length coefficient = 101

Figure	C ft/sec	d ft.	$\frac{d}{D}$	C_o ft/sec	λ	α degrees
17a	10.6	7.2	3.8	15.2	0.70	15° 30'
17b	13.5	6.8	3.6	14.8	0.91	17°
17c	14.0	6.8	3.6	14.8	0.95	16° 30'
18a	14.8	7.0	3.7	15.0	0.99	15°
18b	16.6	6.8	3.6	14.8	1.12	13°
18c	19.0	7.1	3.7	15.1	1.32	10°

Examination of the photographs in Figures 17 and 18, and in particular Figure 18a, indicates a discrepancy between the character of the actual ship waves and those expected by theory near the critical value of $\lambda = 1$. The model test data summarized in Figure 12 indicates that the angle α approximates that expected by the Kelvin theory. In Figure 18a, however, where λ has a value close to 1.0, the value of the angle α is only 15° (Table III) instead of about 90° as would be expected by theory. It is possible that the short steep waves which appear to form the cusp locus are in reality lower in height than are the long low steepness waves which are outside of what has been assumed to be the cusp locus.

The measurement of wave height during the tests were made by water-level recorders installed at two distances, x, from the sailing line. From the water-level recorder charts the maximum wave height (distance from maximum crest elevation to preceding trough) was determined. The ratio of wave height to draft was then computed and plotted as a function of the relative speed of the craft, $V/\sqrt{L}$, as shown in Figure 19 with the ratio, x/L, as a parameter. A scale of λ is also shown in this figure. All runs were made with a value of the ratio to depth, d/D, equal to 3.3 except in one run where d/D equaled 3.7. Although the data are limited and some scatter occurs, examination of Figure 19 shows results similar to those determined from the model studies--namely, that for a particular ratio of depth to draft there is a certain speed at which the wave height is a maximum. In relatively shallow water this point occurs in the vicinity of $\lambda = 1.0$.

For values of $\lambda > 1$ the wave heights reduce with increase in speed and appear to approach a constant value at high speeds. As to be expected the wave heights decrease with increasing relative distances, $\frac{x}{L}$, from the sailing line.

CONCLUSION

The data from a series of model tests define the variables involved and their relative importance in determining the characteristic of waves generated by passing ships. It appears that with models of sufficient size reliable prototype prediction can be made of the waves generated by a particular hull shape; however, additional model studie and prototype observations are required to permit such predictions to be made with confidence.

ACKNOWLEDGEMENTS

The tests described were conducted with research funds provic by the University of California. Mr. A. L. Arnold assisted in conducting the tests and analyzing the data, and Miss Margaret Lincoln p pared the illustrations.

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MIAMI BEACH

PART 4

COASTAL STRUCTURES AND RELATED PROBLEMS

MIAMI BEACH

CHAPTER 41

WAVE RUN-UP ON COMPOSITE SLOPES

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ABSTRACT

A method is presented for determining wave run-up on composite slopes from laboratory-derived curves for single slopes. The method is one of successive approximations and involves replacement of the actual composite slope with a hypothetical single slope obtained from the breaking depth and an estimated run-up value. Comparison of predicted values is made with actual laboratory data.

Accurate design data on the height of wave run-up is needed to determine design crest elevations of protective structures subject to wave action such as seawalls, beach fills, and dams. Such structures are normally designed to prevent wave overtopping with consequent flooding on the landward side and, if of an earth type, possible failure by rear face erosion. Wave run-up (the vertical height to which water from a breaking wave will rise on the structure face) therefore, has an important bearing on the final determination of crest elevation or freeboard.

Apart from the safety factor, decisions as to the necessary crest elevation frequently have considerable economic implication also, as for example in the levees presently being designed for protection with the planned raised water levels in Lake Okeechobee (Florida) where it has been estimated that each additional foot of levee elevation required will cost several million dollars.

Much study by models has recently been devoted to the problem of run-up on structures, both in this country and abroad. The problem for smooth impermeable structures of constant slope has been discussed previously (Saville, 1956). Savage (1957), more recently, has given data on run-up on roughened and permeable structures, but still of a constant slope. Some information has also been given (Saville, 1956) for composite slopes made up of a smooth impermeable structure slope rising from a smooth 1 on 10 (beach) slope which is at or below the still water level. Few structures, however, fit exactly the cases reported, and interpolation or extrapolation of the curves is relatively difficult. Consequently resort frequently is still made to an exact model study of a planned structure to obtain the design values of wave run-up. This is particularly the case where more complex composite slope structures, such as those with berms, are being considered.

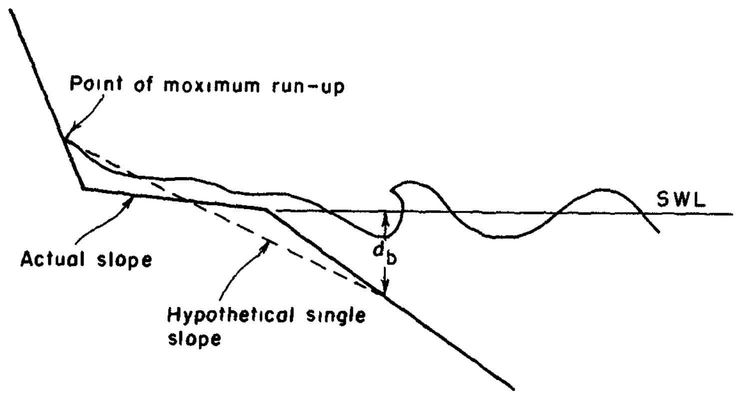

Fig. 1. Schematic of hypothetical single slope for use in determining run-up for composite slopes.

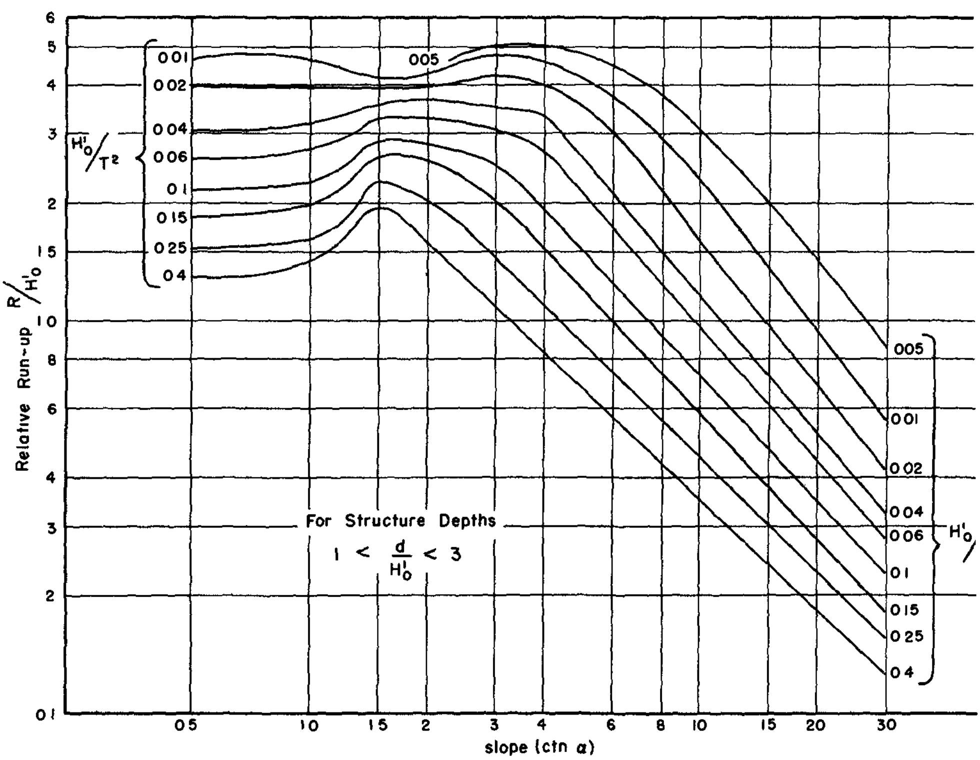

Fig. 2. Run-up on sloped structures.

However, an analysis of existing data shows that this data may be used to predict relatively accurate values of wave run-up for any slope if the actual composite slope is replaced by a hypothetical single constant slope; this hypothetical slope is obtained from the breaking depth and an estimated value of wave run-up. Such a case is shown in Figure 1 where a composite slope consisting of a beach slope, a very gently sloping berm, and a steep structure slope is replaced (dashed line) by a single hypothetical slope extending from the breaking point to an arbitrarily estimated point of maximum run-up. Using this hypothetical slope, a value of run-up may be determined by interpolation from the earlier data. In the general case, the value of run-up determined will be somewhat different from that initially chosen to obtain the hypothetical single slope; the process is then repeated using the new value of run-up to obtain a new single slope value, which in turn determines a new value of run-up. The process is repeated until identical values are obtained for two successive trials.

In order to make the interpolation between the earlier curves somewhat simpler, these curves have been replotted as shown in Figure 2. The case of structure depth, that is depth of water at the toe of the structure, between one and three wave heights is the only one presented, as this range contains the breaking point which would be at the toe of the hypothetical single slope structure used here. The figure shows relative run-up (R/H_o') as a function of structure slope for various values of wave steepness (H_o'/T^2), where R is the wave run-up (the vertical height above still water level to which water will rise on the face of the structure), H_o' is the equivalent deep water wave height, and T is the wave period. It should be noted that the values are given in terms of the deep water wave height (corrected for refraction), H_o'. This value, if not known initially, may be obtained from the non-breaking wave height in any depth of water by using tables of functions of d/L_o (relative depth) as, for example, given by Wiegel (1948) and later reproduced by the Council on Wave Research (1954) and the Beach Erosion Board (1954); or from the breaking depth or breaking height as given by the solitary wave equations (Munk, 1949). These, as rearranged to utilize the generally more available value of wave period, T, rather than the deep water wave length, L_o, are:

$$d_b = 1.28 H_b \quad \text{and}$$

$$H_o' = 1.5 d_b \left(H_o'/T^2\right)^{1/3}$$

where d_b and H_b are respectively the breaking depth and height. These same solitary wave equations may also be used to obtain the breaking depth for use in determining the hypothetical single slope used to replace the actual composite slope.

Although actual verification of this method is not shown until later in the paper, to illustrate the method, an actual design example is worked out below for the Jefferson Parish levee on Lake Ponchartrain outside New Orleans, Louisiana. A schematic diagram of the existing levee is shown in Figure 3. The problem was whether this levee would be overtopped by hurricane waves on Lake Ponchartrain, and, if so, how high the existing levee would have to be raised to prevent overtopping. The wind tide level under hurricane winds for a particular choice of design storm was determined as +8 feet mean sea level, as indicated on the figure. It was estimated that waves 7 feet high and of 6.7-second period would be observed one mile offshore where the water depth is 19 feet. The equivalent deep water wave height H_o' may then be found as 7.38 feet by obtaining a value of H/H_o' from tables of functions of d/L_o. For this particular case there are three values of wave run-up that need to be computed. These are (1) that resulting from the 7-foot incident wave breaking on the 1 on 130 slope, (2) that from the smaller wave propagated over the 20-foot berm in 6 feet of water and breaking on the 1 on 4 slope, and (3) that from the still smaller wave propagate across the 40-foot berm in 3 feet of water and breaking directly on the 1 on 8 levee slope. They will be computed below in that order.

(1) Run-up on 1:8 slope for wave breaking on 1:130 slope:
Compute $H_o'/T^2 = 0.164$
From the solitary wave equation (above) compute the depth of breaking, $d_b = 8.99'$
Assume run-up on the 1:8 slope as any value, say 2'
Compute a hypothetical single slope as a vertical rise from -0.99' msl (breaking depth) to +10' (crest of run-up) in a horizontal distance of 40' (1:8 slope = 8 x 5) plus 40' (berm at 3' depth) plus 12' (1:4 slope) plus 20' (berm at 6' depth) plus 389' (1:130 slope = 130 x 2.99) or slope = 10.99/501 = 1:45.6
From Figure 2 (extrapolated) determine $R/H_o' = 0.115$ and compute $R = 0.85'$
Repeat the above computations assuming $R = 0.8'$; then slope = 9.79/491.4 = 1:50.2
From Figure 2 (extrapolated) determine $R/H_o' = 0.11$; then $R = 0.8$ approximately. As the computed value of 0.8' agrees with the assumed, then this value is the final computed run-up for thes particular assumptions as to breaking condition.

(2) Run-up on 1:8 slope from stable wave on 20-foot berm where d = 6':
From $d_b = 6'$ using the solitary wave equation, and the same wave period, $T = 6.7$ seconds, compute $H_o' = 4.02'$ and $H_o'/T^2 = 0.0895$
Assume the wave breaks just at the toe of 1:4 slope
Assume run-up as any value, say, 4 feet
Compute a hypothetical single slope as a vertical rise from +2' msl (depth of breaking) to +12' (crest of run-up) in a horizontal distance of 12' (1:4 slope) plus 40' (berm) plus 56' (1:8 slope = 8 x 7) or slope = 10/108 = 1:10.8
From Figure 2, $R/H_o' = 0.71$ and $R = 2.85'$
Assume $R = 2.85$ and repeat, computing slope = 8.85/98.8 = 1:11.2 and $R/H_o' = 0.68$ from which $R = 2.74'$
Assume $R = 2.74'$ and repeat, computing R again = 2.74', which becomes the final run-up value.

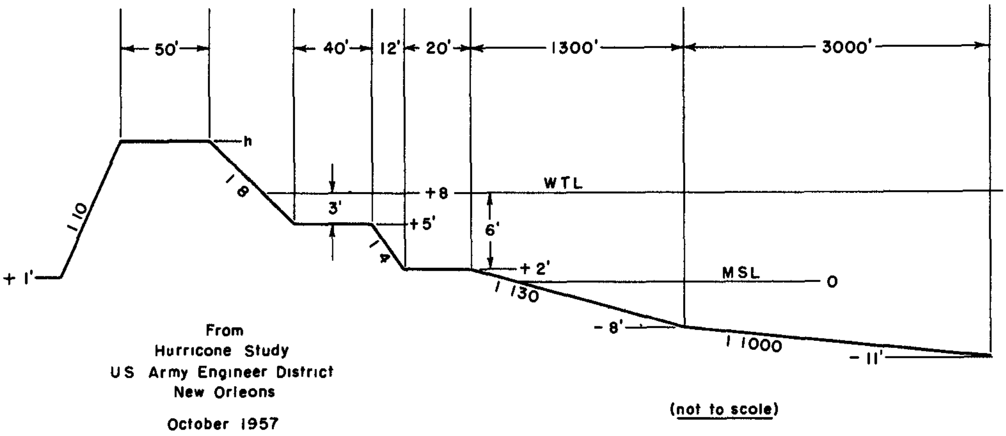

Fig. 3. Schematic of Jefferson Parish levee.

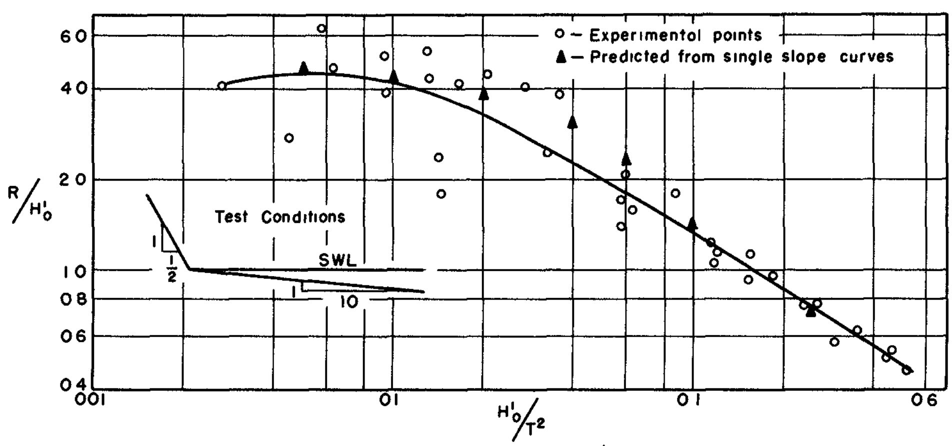

Fig. 4. Wave run-up, composite slope (1: 1/2 above SWL, 1:10 below SWL).

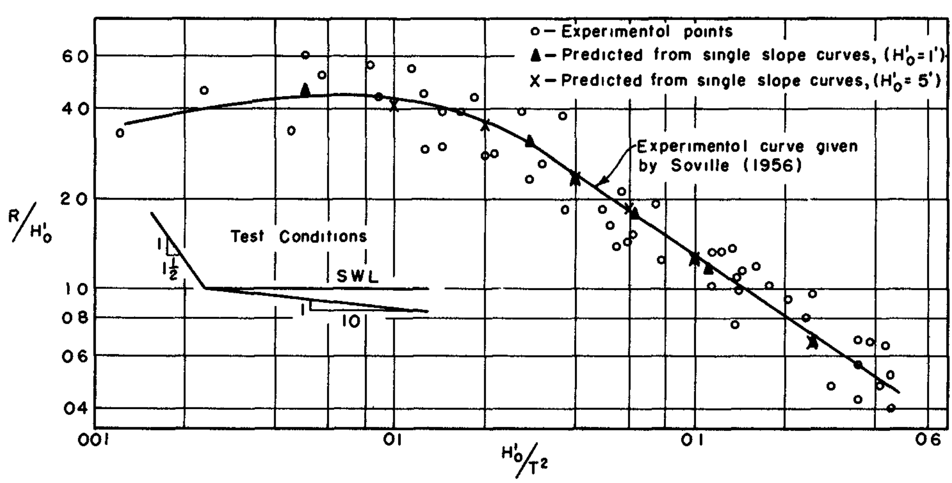

Fig. 5. Wave run-up, composite slope (1: 1 1/2 above SWL, 1:10 below SWL).

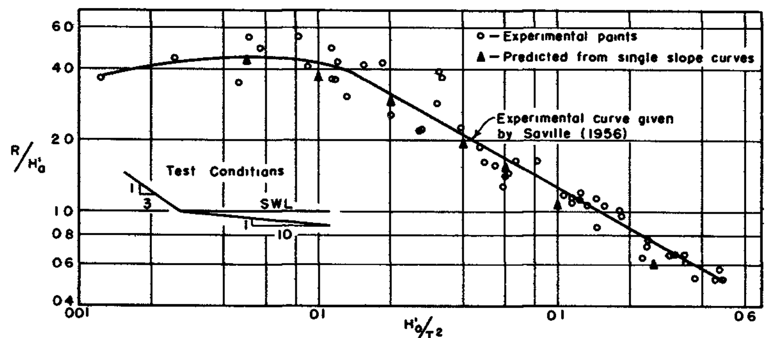

Fig. 6. Wave run-up, composite slope (1:3 above SWL, 1:10 below SWL).

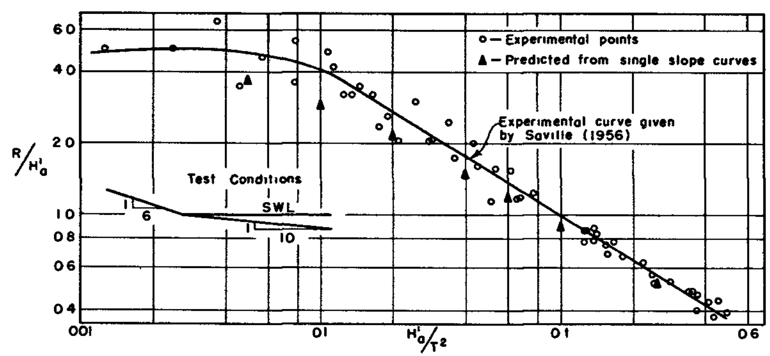

Fig. 7. Wave run-up, composite slope (1:6 above SWL, 1:10 below SWL).

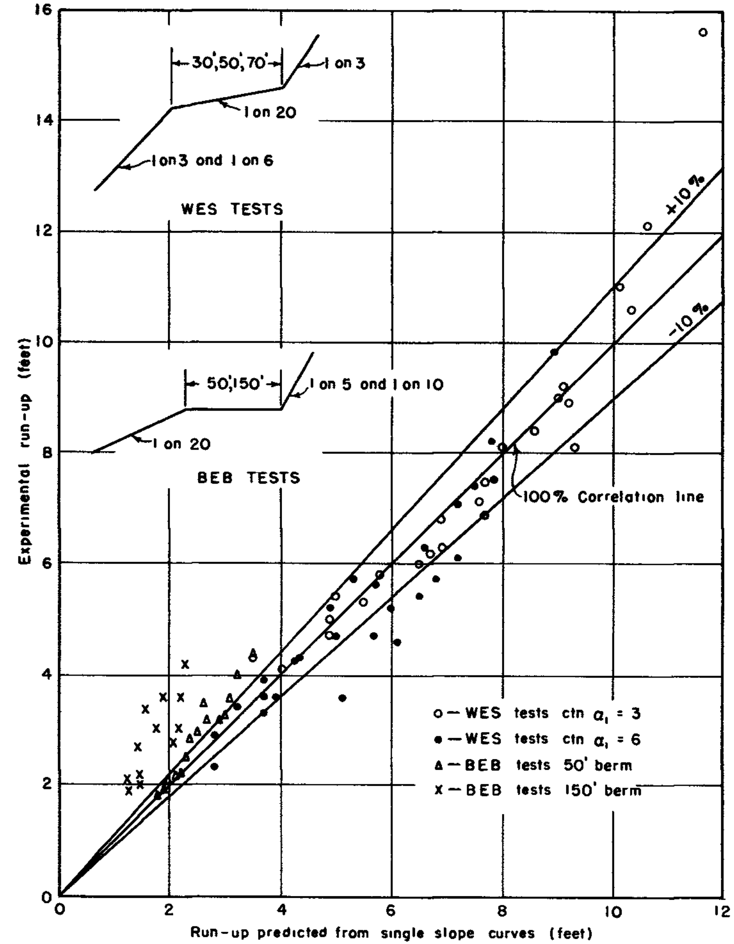

Fig. 8. Comparison with experimental values of run-up for slopes with berms obtained by

(3) Run-up on 1:8 slope from stable wave on 40-foot berm where $d = 3'$:
From $d_b = 3'$ and $T = 6.7$ seconds, compute $H'_o = 1.43'$, and $H'_o/T^2 = 0.0318$
From Figure 2, $R/H'_o = 1.67$ and $R = 2.39'$

The design run-up for this wave condition is then the maximum of these three values or 2.74 feet. It may be noted that this value is not for the full sized hurricane wave breaking on the outer slope, but for a somewhat smaller wave in the spectrum (or a reformed wave) which can propagate as a stable wave over the deeper berm, and break on the 1:4 slope.

Comparisons have been made for values computed by this method with those determined experimentally in wave flumes for certain cases. These are composite slopes made up of smooth constant impermeable slopes of 1 on 6, 1 on 3, 1 on $1\frac{1}{2}$, and 1 on $\frac{1}{2}$ above still water level and a constant beach slope of 1 on 10 below still water level. The comparisons for these cases are shown in Figures 4 - 7. The experimental data and curves for the 1 on 6, 1 on 3, and 1 on $1\frac{1}{2}$ slopes are those previously reported (Saville, 1956) and data for the 1 on $\frac{1}{2}$ slope are additional unpublished data obtained at the Beach Erosion Board; the curves in all cases were drawn by eye through the general center of the experimental points. Actually the points determined by the hypothetical slope method may be connected to form a curve which can be compared with the experimental curve. As the plotted points represent dimensionless quantities the actual wave height used in determining the points makes no difference in the curves obtained. This may be seen in Figure 5 where values determined from waves of both 1 and 5-foot height are shown. The points or curves predicted by this method vary from somewhat above the experimental curve for the 1 on $\frac{1}{2}$ slope, to almost exactly on the curve for the 1 on $1\frac{1}{2}$ slope, to somewhat below the curve for the 1 on 3 and 1 on 6 slopes. In every case, however, the predicted points lie within the scatter pattern of the experimental points. The deviation of the predicted values from the previously drawn experimental curves is generally within 10 percent with a maximum deviation of about 25 percent.

In addition, a comparison was made with run-up data for a number of structures having berms. These comparisons are shown in Figure 8 where the actual prototype value of predicted run-up is compared with the experimentally determined values from model studies for the Lake Okeechobee levee design reported by Hudson, Jackson and Cuckler (1957) and for beach dune design reported by Savage (1957). The former were tests made at the Waterways Experiment Station of the Corps of Engineers in Vicksburg, Mississippi and involved underwater slopes (α_1) of 1 on 3 and 1 on 6, berms of 30, 50, and 70-foot width on a 1 on 20 slope with the toe at still water level, and upper structure slopes of 1 on 3 (see Figure 8). Those reported by Savage were tests carried out at the Beach Erosion Board and involved an underwater beach slope of 1 on 20, horizontal berms of 50 and 150-foot width, and dune slopes of 1 on 5 and 1 on 10 (see Figure 8). There was also an outer bar involved in these latter tests; the values used herein were restricted to the cases where the wave did not break on passing over the bar. Waves breaking on the bar could also have been used, but additional computations would have been necessary to determine whether the run-up was due to the wave breaking on the bar or to the reformed waves generated in the water area shoreward of the bar. Similarly usage was also restricted to the cases of still water depths over the berm of -2, -1, 0, and 1 feet in an attempt to ensure that the run-up was due to the wave breaking on the beach slope rather than to a reformed smaller wave propagating in the water over the berm.

As may be noted from Figure 8, the agreement of the predicted with the experimental values is fairly good except for the case of the 150-foot berm, where experimental values were considerably higher than those predicted. It is interesting to note that for these cases essentially the same value of run-up was obtained for both the 50-foot and 150-foot berm width. The maximum difference between the two was 0.2 feet for run-ups ranging from 1.9 to 4.2 feet. This would seem to imply that after a berm has reached a certain width, further widening has no significant effect in reducing wave run-up - - at least for horizontal berms. This possibility has previously been indicated by researchers in The Netherlands (van Asbeck, Ferguson, and Schoemaker, 1953) in stating that berms wider than about one-fourth of the wave length, while still reducing the wave uprush, do so at a lesser rate. For the tests reported here one-fourth of the wave length is between 40 and 50 feet. This reduction in effect of berm width may be because in the laboratory tests at least, a definite "set-up" of water occurre on the berm. This "set-up" or increase in mean water level is caused by the forward transport of water by the waves and, for these tests, ranged between 0.9 and 2.4 feet with an average value of 1.7 feet and a most frequent value of 1.8 feet. This "set-up" increased the water depth over the berm appreciably, and in many cases the run-up measured may have been due more to reformed waves or surges in this increased depth than to the actual uprush of the wave. This is partially substantiated by the fact that experimental values for the higher berms (at or above still water level) are more nearly approached by th predicted values than are those for the lower berms where a greater water depth is observed. This "set-up" phenomenon appears to be much more apparent for horizontal berms than for sloping berms, where the water pushed forward by the wave may flow back much more readily. No mention of this occurrence was made in the Vicksburg tests, and the difference between predicted andobserved values for these tests did not appear to be affected by the berm width (which varied from 30 to 70 feet).

Referring again to Figure 8, some 72 percent of the experimental values lie within ± 10 percent of the predicted values if the points for the 150-foot berm are ignored; if these points are included, then 61 percent of the experimental values are within ± 10 percent of the predicted.

In conclusion, a method for predicting wave run-up on any type of composite sloped impermeable structure has been presented. The accura of the method, as judged by comparison with experimentally observed values obtained from laboratory tests, is regarded as satisfactory. It is felt that use of the method will simplify design determination of run-up for many structures. However, further tests are needed to define those cases where width of horizontal berm becomes great enough to affect the validity of the method.

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CHAPTER 42

MODEL INVESTIGATION ON WAVE RUN-UP CARRIED OUT IN THE NETHERLANDS DURING THE PAST TWENTY YEARS

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1. INTRODUCTION

This paper gives a summary of the results of the model studi on wave run-up carried out in the Netherlands since 1936. More detailed information on these studies is to be found in publication H.L.D. No. 11 to be issued shortly by the Delft Hydraulics Laboratory.

The object of the first studies on wave run-up was the establishment of a relationship between the wind velocity W, the fetch F, and the waterdepth D on one hand and the height of wave run-up Z on the other.

Apart from the model investigations on wave run-up, a large number of tests were carried out on the growth of wind-generated waves and, when after the war the now well-known studies of Sverdrup and Munk on deep-water waves, ref. 1, were published, these could be compared with and supplemented by the studies of the Delf Laboratory on wind waves in deep-, as well as in shallow water. These were presented by Thijsse to the General Meeting of the Inte national Association on Physical Oceanography, Oslo 1948, ref. 2.

These studies made it possible to determine the characteristics of the waves in shallow water from those in deep water, so that in the tests carried out after 1942 the wave run-up could be related to the wave height in front of the dike.
In addition to the above mentioned model studies, many tests were carried out to determine the influence of the dike facing on the run-up. Only impermeable facings have been tested.

2. ANALYSIS OF THE PROBLEM

The general problem is to determine, by means of statistical methods, the most unfavourable combination of astronomical tide, piling-up of the water level by wind, and run-up of the waves on the talus of the dike, that is likely to occur with a reasonable degree of frequency. From these three determinant factors for the height of the dike, only the wave run-up is the subject of the present paper.

From the beginning it was realized that many of the factors, involved in this problem, are of a statistical character, requiring statistical methods for their analysis and solution. For this reason most of the tests were carried out with wind-generated waves and the results were plotted on probability paper. The critical value of the run-up, to be used as a basis for the design, will be defined as the height which has a probability of n% to be attained or exceeded (Z_n). The design formulas are based on a frequency of 2%. In certain cases, however, this percentage may be too high or too low, dependent on the required safety of the dike and the duration of the high water periods, so that each case has to be considered separately.

The factors governing the run-up are divided into the following two groups.

a) The factors determined by nature

b) The factors determined by the dike.

The factors sub a) are:

1) the characteristics of the waves in front of the dike

2) the direction of the wave attack,

and the factors sub b);

3) the slope of the dike

4) the shape of the dike

5) the character of the dike facing

6) the artificial-foreshore conditions.

The factors sub a) are in turn dependent on W, F, and D, the latter representing the sea bottom conditions.
Since the ultimate object of the model studies on wave run-up is the safety of the dike under various conditions, not only the height of the run-up is of importance, but also the velocities with which the water runs over the talus of the dike and the magnitude of the masses of water involved.
For notations and dimensions see fig. 1.

3. APPARATUS AND PROCEDURE

The tests were carried out in the wind flume of the Delft Laboratory. In this flume waves can be generated by wind, as well as by means of a wave machine, and the tests were carried out with so called "equilibrium waves". These waves were generated by wind and wave machine together, in such a way that finally a state of equilibrium is reached in which the waves no longer grow and the energy supplied by the wind and by the wave machine is fully dissipated by the friction losses. Theoretically the generation of

equilibrium waves requires an unlimited length of fetch, but in practice it can be taken that the waves no longer grow if the length of the fetch is at least 1000 D.
A description of the wind flume with wave-generating equipment and test procedure is given in publication H.L.D. No. 11.

Regarding the transference of the model results to the prototype, it is assumed that Froude's law may be applied, provided the following conditions are fullfilled:

a) geometrical similarity and negligible influence of molecular forces

b) statistical similarity

c) breaking of the waves under influence of the wind.

Concerning the first requirement, it appeared that mechanically-generated waves, having originally a trochoidal shape, become less steep when they are subjected to a wind current, thus obtaini a proportionally larger length and a greater velocity, while the shape becomes sinusoidal. The run-up of these waves is considerabl higher (up to 30%) than that of equivalent waves generated without wind. Since the equilibrium waves are partly generated by wind and partly by the wave machine, these waves are also somewhat too stee resulting in too low values of the run-up.

The second requirement, i.e. statistical similarity, was not fullfilled as, contrary to nature, the direction and the velocity the wind in the model remained practically constant.

4. WAVE RUN-UP AS A FUNCTION OF WIND, FETCH, AND WATER DEPTH, ON A SLOPE OF 1 : $3\frac{1}{2}$

In 1936 tests were carried out with the object of establish a relationship between the 2% run-up and the variables: wind velocity W, fetch F and water depth D, for a dike with straight slope of 1 : $3\frac{1}{2}$, provided with a smooth facing. The tests were carried out with a constant water depth of 0.32 m, so that the wind veloci ty was the only variable. The wind velocity was expressed in the velocity-head of the wind (wind pressure) : s, measured in mm wate column. The tests were conducted with wind pressures of 1.3, 2.0, 3.0, 3.7 and 4.8 mm. The corresponding dimensions of the waves are shown in fig. 2. The average steepness was 0.07.

Based upon these tests the following relationship was derive

$$Z_2 = 41 \; s^{\frac{1}{2}} \qquad \text{(in cm)} \qquad (a)$$

This relationship is graphically shown in fig. 3.

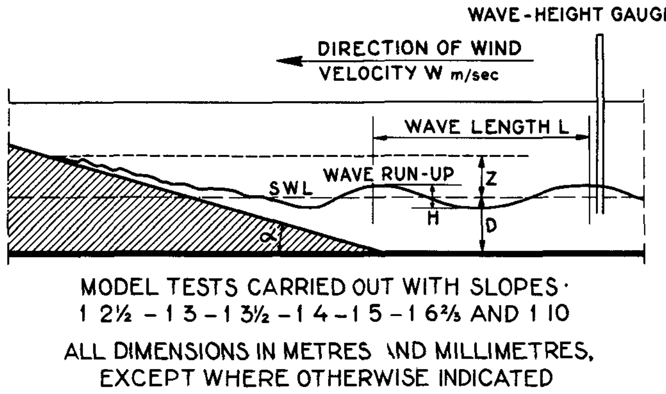


Fig. 1

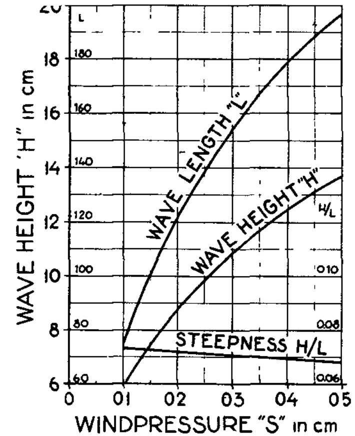


Fig. 2

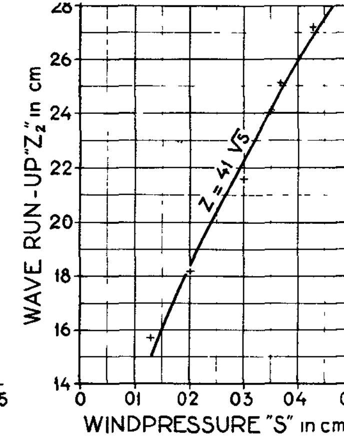


Fig. 3

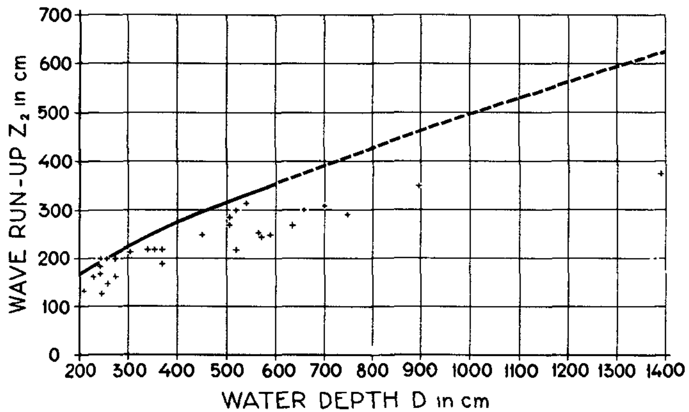


Fig. 4

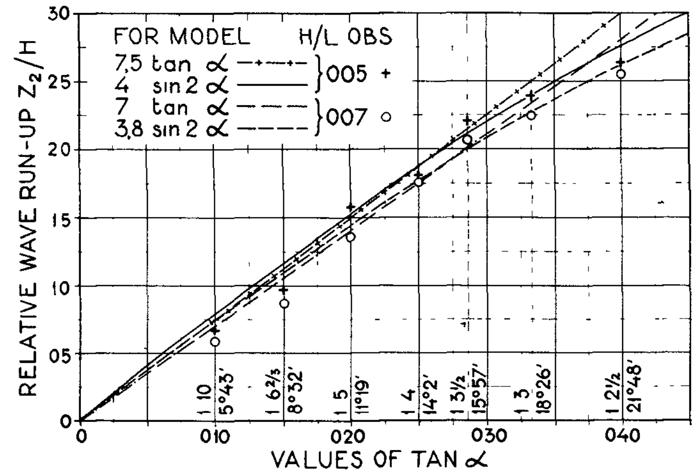


Fig. 5

Fig. 1. Notations and dimensions

Fig. 2. Relationship between wind pressure in model and dimensions of waves

Fig. 3. Relationship between wind pressure in model and wave run-up Z_2

Fig. 4 Comparison of formula: $Z_2 = 4.95\ D_{2/3}$ with observations in nature (Zuiderzee Works)

Fig. 5 Comparison of tangent and sinus formulas for run-up Z_2 with frequency of 2%

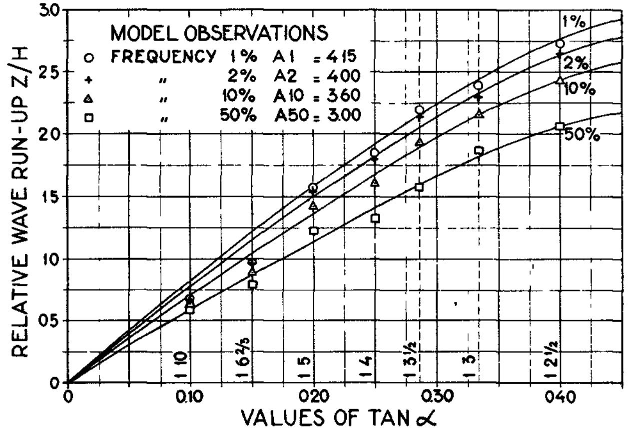


Fig. 6

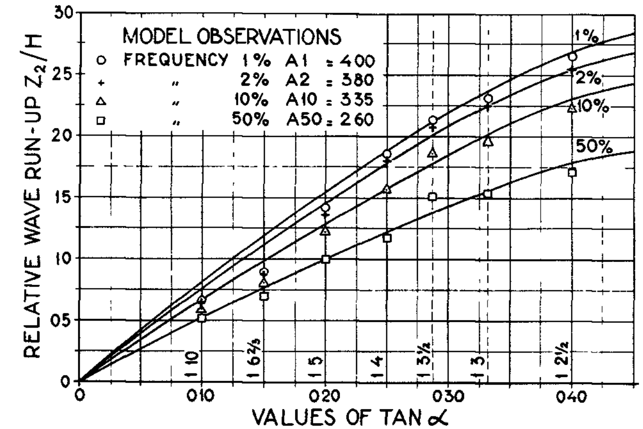


Fig. 7

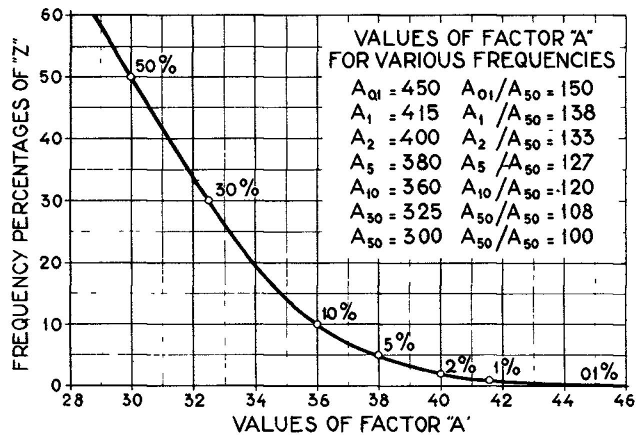


Fig. 8

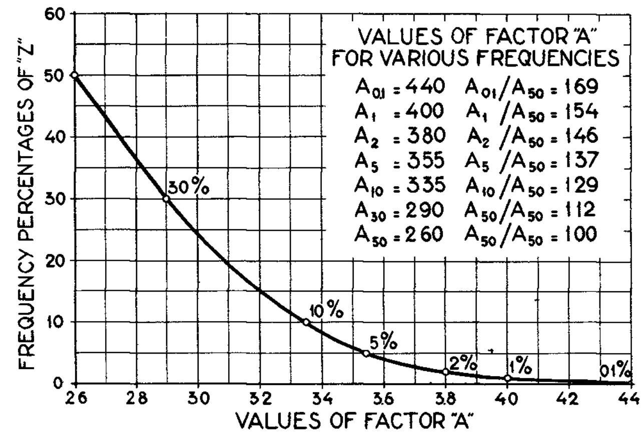


Fig. 9

Fig. 6. Relative wave run-up as a function of slope: $Z/H = A \sin 2\alpha$ for $H/L = 0.05$
Fig. 7. Relative wave run-up as a function of slope: $Z/H = A \sin 2\alpha$ for $H/L = 0.07$
Fig. 8. Relationship between frequency and value of A for $H/L = 0.05$

Using the relationship:

$$s_1/s_2 = y_1^{2/7} \; / \; y_2^{2/7} \qquad \text{(b)}$$

between the wind pressures s_1 and s_2 at heights y_1 and y_2, the following expression was found for the 2% run-up:

$$Z_2 = 7\ D^{2/3}\ s^{\frac{1}{2}}\ y^{-1/7} \qquad \text{(in cm)} \qquad (1)$$

Assuming a wind velocity of: $W = 25$ m/sec at a height of: $y = 15$ m, the wind pressure is 4 cm, so that $s^{\frac{1}{2}} = 2$ and $y^{-1/7} = 1500^{-1/7} = 0.3533$. After substituting these values in (1) the following equation is obtained:

$$Z_2 = 4.95\ D^{2/3} \qquad \text{(in cm)} \qquad (2)$$

Equation (2) is graphically shown in fig. 4, together with the observations in nature used as a basis for the Zuiderzee works and mentioned in the report of the "Staatscommissie Lorentz", ref. 3.

As may be seen from fig. 4, the values of formula (2) are well in accordance with the observations in nature for water depths below 6 m. If the fetch is less than 1000 D, the values of Z_2 must be multiplied by a correction coefficient which was determined by model tests.

5. WAVE RUN-UP ON SLOPES WITH VARYING ANGLES OF INCLINATION, AS A FUNCTION OF THE ANGLE OF INCLINATION AND THE WAVE CHARACTERISTICS IN FRONT OF THE DIKE

In 1942 the model studies on the growth of wind-generated waves were sufficiently far advanced to establish a direct relationship between the wave run-up and the wave height in front of the dike.

For this purpose two series of tests were carried out on dikes with slopes of 1 : $2\frac{1}{2}$, 1 : 3, 1 : $3\frac{1}{2}$, 1 : 4, 1 : 5, 1 : $6\frac{2}{3}$ and 1 : 10, viz: with waves of steepness 0.05 and of steepness 0.07 respectively. The water depth was kept constant at 0.35 m.

For the 2% run-up the results of these tests are shown in fig. 5. Straight lines were drawn through the plotted points expressing the various observations, and from these it appeared that for values of α ranging from 7° to 16°, the relationship between the relative run-up Z_2/H and the angle of inclination α can be expressed by the equations:

$$Z_2/H = 7.5 \tan \alpha \qquad (\text{for } H/L = 0.05) \qquad (3a)$$

and:

$$Z_2/H = 7.0 \tan \alpha \qquad (\text{for } H/L = 0.07) \qquad (3b)$$

In the above formulas the wave height H is the average height of the equilibrium waves in the model which did not vary very much.

Since the waves in the model were proportionally too steep (resulting in too small values of Z_2), the difficulty arose how to transfer the model results to the prototype.
After considering all factors involved, it was decided to increase the factor 7.5 in the model to 8 in the prototype, for the 2% run-up on a dike provided with a stone revetment and waves of a steepness of 0.05. The run-up is thereby expressed in the "significant wave height" $H_{1/3}$. In this way the following formula was obtained:

$$Z_2/H_{1/3} = 8 \tan \alpha \qquad (\text{for } H/L = 0.05) \qquad (4)$$

This formula, which was published for the first time in a paper presented to the 18th International Navigation Congress, Rome 1953, ref. 4, proved to be in good agreement with the prototype observations carried out in the Netherlands for slopes not steeper than 16 degrees.

As may be seen from fig. 5, this was also the case with the model observations. A trial was therefore made to find a better formula for slopes steeper than 16°. Partly based on the theoretical considerations of Miche, ref. 5, and partly on the results of new model tests on a dike provided with a berm, the following formula was developed:

$$Z_2/H_{1/3} = 2.7 \sin \left(\frac{90^\circ}{\alpha}\right)^{\frac{1}{2}} \left(\cos \beta - \frac{b}{L}\right)$$

in which:

- b = width of the berm
- L = wave length
- α = angle of inclination of the upper slope
- β = angle between the direction of the wave crests and the axis of the dike.

The above formula, which was mentioned in a paper presented to the 5th Conference on Coastal Engineering, Grenoble 1954, ref. 6 proved not to be in accordance with observations carried out in nature and should, therefore, be discarded. One of the reasons that the above formula cannot be generalized comes from the fact that the term $(\cos \beta - \frac{b}{L})$ was based on earlier tests on a berm dike, with equal upper and lower slopes, while in the tests on which the above formula is based the upper and lower talus had a different slope. Moreover, the width of the berm was kept constant during the latter tests, so that the factor $(\cos\beta - \frac{b}{L})$ could not be verified.

A fresh trial was made by multiplying tan α by $\cos^2\alpha$, thus obtaining a formula of the following general form:

$$Z_n/H = A \sin 2\alpha \qquad (5)$$

The value of A can be determined in such a way that, for $\alpha = 15^\circ$, with formula (5) the same value for Z_2 is obtained as with formula (3 a). The value of 7.5 tan 15° must then be equal to A sin 30°, or: 7.5 x 0.268 = 0.5 A. so that:

$$A = \frac{7.5 \times 0.268}{0.5} = 4.0$$

Hence:

$$Z_2/H = 4.0 \sin 2\alpha \qquad (\text{for } H/L = 0.05) \qquad (6a)$$

In the same way has been calculated:

$$Z_2/H = 3.8 \sin 2\alpha \qquad (\text{for } H/L = 0.07) \qquad (6b)$$

The curves representing formulas (6a) and (6b) are also shown in fig. 5 and from this it may be seen that they are well in accordance with the model observations for values of $\alpha > 15^\circ$. For values of $\alpha < 15^\circ$, the difference between the tangent and the sinus formulas is so small, that also for values of $\alpha < 15^\circ$ the sinus formula could be used. The max value of α for which the sinus formula was investigated is 22°.
For other frequencies than 2% the values of the factor A have also been determined and the curves representing the relationship between Z_n/H and α, for frequencies of 1%, 2%, 10% and 50%, for H/L = 0.05 and H/L = 0.07 respectively, are shown in figs. 6 and 7. The relationship between the factor A and the frequencies of A is shown in figs. 8 and 9.

The above values of A are for the model observations and again the difficulty arose how to transfer these results to the prototype. For Z_2 this could be done by giving A such a value that, for $\alpha = 15^\circ$, the sinus formula gives the same value for Z_2/H as the tangent formula.
Then is:

$$A = \frac{8 \times 0.268}{0.5} = 4.3$$

so that:

$$Z_2/H_{1/3} = 4.3 \sin 2\alpha \qquad (\text{for } H/L = 0.05) \qquad (7)$$

For other frequencies, however, the values of A for the prototype conditions can only be determined by means of observations in nature, since the frequency distribution in the model is different from that in nature. Taking the same ratio of the prototype values to the model values as assumed for Z_2, then the following values of A are obtained.

Run-up	H/L = 0.05	H/L = 0.07
$Z_{0.1}$	$A_{0.1} = 4.90$	$A_{0.1} = 4.75$
Z_1	$A_1 = 4.50$	$A_1 = 4.30$
Z_2	$A_2 = 4.30$	$A_2 = 4.10$
Z_5	$A_5 = 4.10$	$A_5 = 3.85$
Z_{10}	$A_{10} = 3.90$	$A_{10} = 3.60$
Z_{30}	$A_{30} = 3.50$	$A_{30} = 3.10$
Z_{50}	$A_{50} = 3.25$	$A_{50} = 2.80$

Due to the difference in frequency distributions, however, the above values of A may not be considered as correct, and they should be corrected as soon as more prototype data are becoming available.

Formula (5) can be written in the general form:

$$Z_n/H = A.B \sin 2\alpha \qquad (8)$$

in which formula the value of factor A depends on:
1) the steepness of the waves in front of the dike
2) the value of H in which the run-up is expressed
3) the frequency of Z

and the value of factor B on:

1) the shape of the dike (factor B_1)
2) the character of the dike facing (factor B_2)
3) the foreshore conditions (factor B_3)
4) the direction of the wave propagation (factor B_4)

For a "normal" dike the value of each of the factors B_1, B_2, B_3 and B_4 will be taken as "one". By definition this will be the case for the following conditions:

$B_1 = 1$: dike without berm and straight talus.
$B_2 = 1$: dike facing of neatly-set stone
$B_3 = 1$: if the foreshore conditions have no influence on the run-up
$B_4 = 1$: if the direction of the wave propagation makes an angle of 90° with the axis of the dike.

Hence, for the reference dike the above formula becomes again:

$$Z_n/H = A \sin 2\alpha \qquad (5)$$

6. INFLUENCE OF THE SHAPE OF THE DIKE

One of the first model studies on the influence of the shape of the dike on the run-up was carried out in 1946, in connection with the reconstruction of the sea wall on the island of Walcheren, which was badly damaged by war activities and severe storms. The results of these tests are briefly shown in fig. 10.

Fig. 11 shows a number of schematical cross sections of modern Dutch dikes. From various model tests on berm dikes, it was found that a berm has a benificial influence on the run-up, if it is placed at approximately storm water level and of its width is approximately equal to 1/4 L.

In order to establish the influence of the shape of the dike on the run-up, the various dike profiles have been divided into the following 8 types, shown in the figures 12 and 13.

Type	Fig.	Description
p o	12 A	Straight talus with angle α
p 1	12 B	Convex talus
p 2	12 C	Concave talus
p 3	12 D	Berm dike with equal slopes
p 4	13 A	Ditto, with "stilling basin"
p 5	13 B	Berm dike with unequal slopes
p 6	13 C	Ditto, with "stilling basin"
p 7	13 D	Ditto, with parabolic transition

For the 2% run-up the following reduction factors have been found.

Type	
p 0:	$B_1 = 1$ (Reference)
p 1:	$B_1 = 0.95$ for convexity of 3%
p 2:	Not investigated
p 3:	$B_1 = 0.75$ for equal slopes of $1 : 3\frac{1}{2}$ en $b = 1/4$ L
p 4:	$B_1 = 0.65 - 0.70$
p 5:	The tests showed that changes in slope of the upper talus, as well as in that of the lower one, have a considerable influence on the run-up
p 6:	Not investigated
p 7:	A few tests showed that a smooth transition between berm and upper slope has a benificial influence on the run-up.

7. INFLUENCE OF THE CHARACTER OF THE DIKE FACING

Since 1936 a large number of model tests on the influence of the character of the dike facing on the run-up have been carried

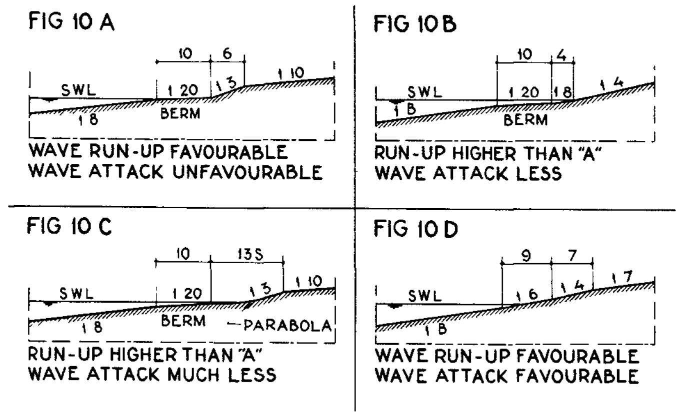


Fig. 10

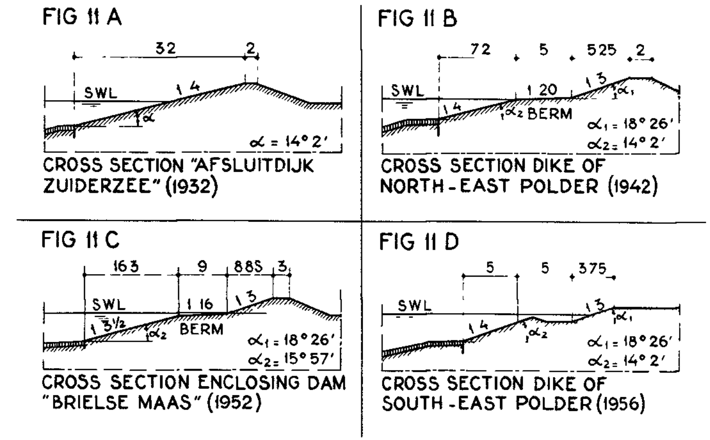


Fig. 11

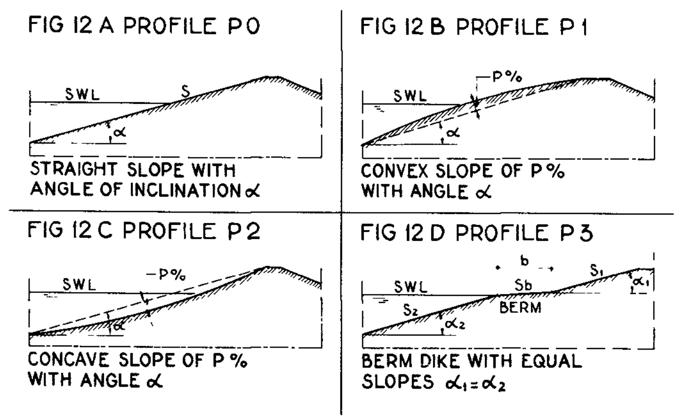


Fig. 12

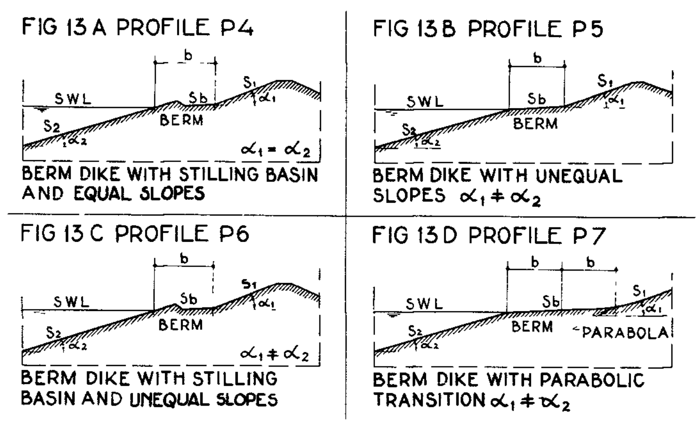
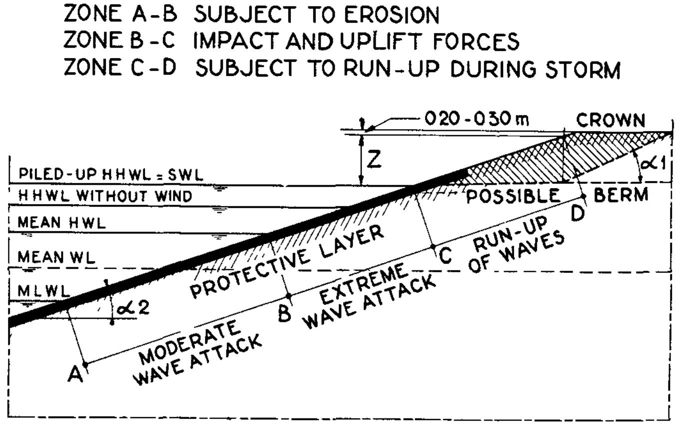


Fig. 13

Fig. 10. Modelstudy of sea wall on the Island of Walcheren
Fig. 11. Cross sections of modern Dutch dikes
Fig. 12. Dike profiles types P 0 to P 3, inclusive



Figs. 14

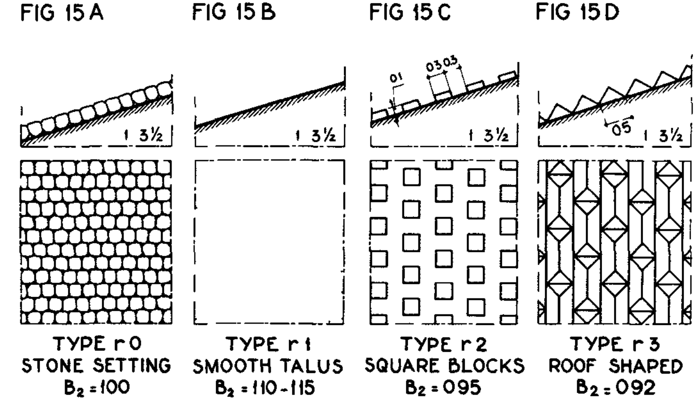


Fig. 15

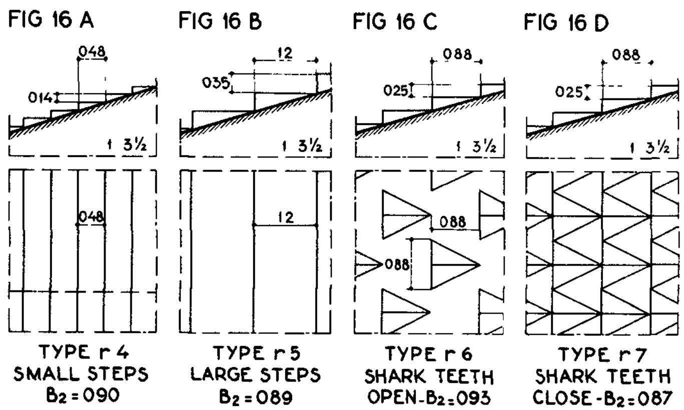


Fig. 16

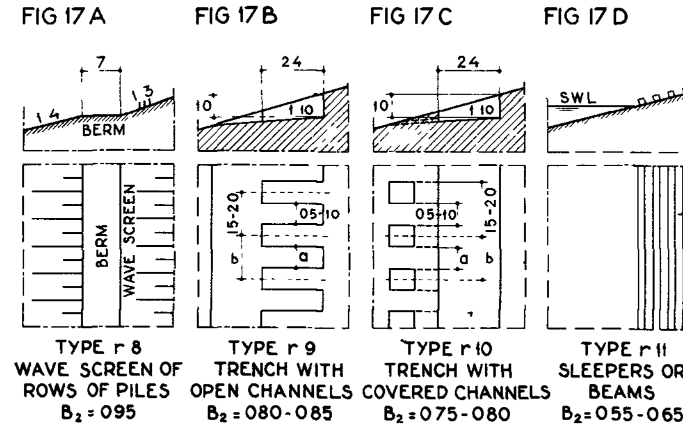


Fig. 17

Fig. 14. Diagram of impermeable sloping sea wall
Fig. 15. Dike facings types R 0 to R 3, inclusive
Fig. 16. Dike facings types R 4 to R 7, inclusive
Fig. 17. Dike facings types R 8 to R 11, inclusive

out. In order to facilitate comparison of the various kinds of facing the following types will be distinguished.
The values of the reduction factor B 2 are for the 2% run-up Z_2.

Type	Fig.	Description	B_2.
r 0	15 A	Revetment of neatly-set stone	1
r 1	15 B	Smooth protective layer	1.10-1.15
r 2	15 C	Square concrete blocks, 0.3 x 0.3 sq.m, protruding 0.1 m	0.95
r 3	15 D	Roof-shaped blocks, 0.5 x 1 sq.m, high 0.25 m	0.92
r 4	16 A	Small steps, 0.14 x 0.48 m	0.90
r 5	16 B	Large steps, 0.35 x 1.2 m	0.89
r 6	16 C	Shark teeth, 0.88 x 0.88 m, high 0.25 m, open setting	0.93
r 7	16 D	Ditto, close setting	0.87
r 8	17 A	Rows of piles, acting as a wave screen	0.95
r 9	17 B	Trench 2.4 m wide and 1 m deep, provided with open drainage channels	0.80-0.85
r10	17 C	Ditto, provided with covered drainage channels	0.75-0.80
r11	17 D	Beams or sleepers on the talus of the dike	0.55-0.65

As may be seen from the diagram shown in fig. 14, the zone above S.W.L. is only subject to the run-up of waves during storm a for this reason this part of the dike needs only a light protectio for which in most cases a grass cover will be sufficient. Unfortunately, however, it is just in this region that the artificial roughness must be placed to be efficient.

8. INFLUENCE OF THE FORESHORE CONDITIONS

In the foregoing tests the models were arranged in such a wa that the conditions of foreshore and sea bottom had no influence o the wave run-up, so that the waves were breaking on the dike.

Tests on the influence of the foreshore conditions on the ru up are at present in progress.

9. INFLUENCE OF THE DIRECTION OF THE WAVE ATTACK

For a dike without berm the following formula was derived, based upon a limited number of model tests:

$$B_4 = \frac{1 + \sin \beta}{2}$$

where: β = angle between the direction of the wave propagation and the axis of the dike. For a frontal wave attack is $\beta = 90^{\circ}$ and $B_4 = 1$.

For values of $\beta > 45^{\circ}$, the values calculated by means of the above formula are higher than the ones observed in the tests. Due to reflection of the waves in the wind flume these observations are not fully reliable and additional tests are required to investigate this influence.

The attempt made in point 5 to express the influence of the wave direction, for a berm dike with varying width of the berm, by means of the factor

$$(\cos \beta - \frac{b}{L})$$

has to be discarded for the reasons mentioned before.

For the investigated case of b = 1/4 L and upper and lower talus of 1 : $3\frac{1}{2}$, the following formula is in agreement with the model tests

$$B_4 = 0.0075\ \beta + 0.325\ .$$

However, to generalize this formula additional model tests should be carried out.

10. PROGRAM OF FURTHER INVESTIGATIONS ON WAVE RUN-UP

It is desirable to carry out the following additional investigations on the influence of the various factors on the wave run-up.

a. Slope of the dike

1) Additional investigations on the influence of the upper slope, as well as that of the lower one, for a dike provided with a berm.
2) Influence of the steepness H/L on the run-up.
3) Relationship between the values of the factor A and the frequency of the wave height, for equal frequencies of run-up and wave height.

b. Shape of the dike (B_1)

1) Influence of convexity and concavity, for dikes without berm.

2) Influence of a parabolic transitien between berm and upper talus, for a dike provided with a berm.

3) Influence of the berm width.

c. Facing of the dike (B_2)

Beams or sleepers on a berm dike.

d. Foreshore conditions (B_3)

Influence of foreshore and sea bottom conditions.

e. Direction of the wave attack (B_4)

1) Verification of the reduction factor: $B_4 = (\frac{1 + \sin \beta}{2})$ for a dike without berm.

2) Influence of the wave direction on the run-up for a berm dik with varying width of the berm.

f. Safety of the dike

Investigation of the velocity with which the water runs over th talus and of the magnitude of the masses of water involved.

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CHAPTER 43

AN EXPERIMENTAL STUDY OF HYDRAULIC BREAKWATERS

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INTRODUCTION

A hydraulic or water-jet breakwater is formed by forcing water through a series of nozzles mounted on a pipe which is installed perpendicular to the direction of the incident waves. The jets create a surface current which results in breaking of the incident wave. Apparently, this effect is primarily responsible for attenuation of the incident wave. An earlier development, the pneumatic breakwater, operates on a similar principle with a horizontal surface current induced by rising air bubbles.

It may be noted in Fig. 1 that the pneumatic breakwater generates two surface currents. One opposes the incident waves, and the second or leeward current has no appreciable detrimental effect on the waves with the result that about half of the energy supplied to the system is wasted. This suggested the possibility of utilizing horizontal water jets in which all of the surface current could be directed against the incident wave. Of primary interest was the relative power requirements of the hydraulic and pneumatic types. This paper presents the results of both small- and large-scale tests of the hydraulic type. Large-scale tests of the pneumatic type are also planned but results are currently not available for comparison.

The concept of using a portable device for protection against waves is not new. Philip Brasher (1907) patented a method of attenuating waves by forcing compressed air through a submerged, perforated pipe. Prototype installations at Million Dollar Pier, Atlantic City, (1908) and at El Segundo Pier, California, (1915) were described as successful in damping waves. Other reports published on pneumatic breakwaters include Bogolepoff (1930), Platzer (1938), Schijf (1940), Taylor (1943 and 1955), Carr (1950), Evans (1954 and 1955), Wetzel (1955), Hensen (1955 and 1957), Kyushu University (1955), Herbich et al (1956), and Snyder (1957); but some are conflicting and controversial. Carr, Evans, and more recently, Snyder conducted tests on the hydraulic breakwaters in addition to the studies reported herein.

EXPERIMENTAL STUDIES

GENERAL COMMENTS

The major part of the study was essentially two-dimensional in character and a single manifold was used in most of the tests.

The primary objective was the procurement of information concerni the effect of various parameters on wave attenuation, which is defined as $1 - H_T/H_I$ (H_I = incident wave height and H_T = transmitted wave height). Parameters which were varied include the wave length L, th wave height H, jet submergence y, jet area a_j, and jet discharge q. The horsepower and discharge required under various test condition were expressed in the form of dimensionless ratios as follows:

$$\phi = \frac{\text{Horsepower per lineal foot of breakwater}}{\rho\, g^{3/2}\; L^{5/2}}$$

$$\psi = \frac{\text{Discharge per lineal foot of breakwater}}{L\,\sqrt{gd}}$$

where ϕ is dimensionless power ratio, ψ is dimensionless discha ratio, ρ is density of water in slugs per cubic foot, g is acceleration due to gravity in feet per second^2, and d is water depth.

The horsepower was normally computed at the jets; for design purposes it would be necessary to add losses in the supply lines and manifold.

The two wave channels employed in these studies were, for practi purposes, geometrically similar. The small channel is 2 ft wide, 1 f 3 in. deep, and 50 ft long and the large channel is 9 ft wide, 6 ft deep, and 253 ft long (Fig. 2). Both channels were equipped with goo wave absorbers. Capacitive wave-profile recorders were employed to measure the wave characteristics.

EXPERIMENTAL RESULTS

The data indicated that the power requirements of a hydraulic breakwater are primarily dependent upon wave length, water depth, wave steepness, submergence of the nozzles, spacing and size (or area of nozzles, and the number of manifolds.

Effect of Wave Length and Water Depth

Experimental data were obtained in the large channel for L/d values up to about 4.2 and in the small channel up to 5.6. Considerable scatter occurred for values less than 1.0. Excluding these values, it appears that ϕ, the horsepower ratio, is fairly constant for L/d values up to about 2.0 and that it increases quite rapidly for L/d values in excess of 2.0. Figure 3 illustrates typical data for an attenuation of 100 per cent. In this instance th power requirements for a single manifold increase by a factor of 7 as the L/d value is increased from 2.0 to 5.0.

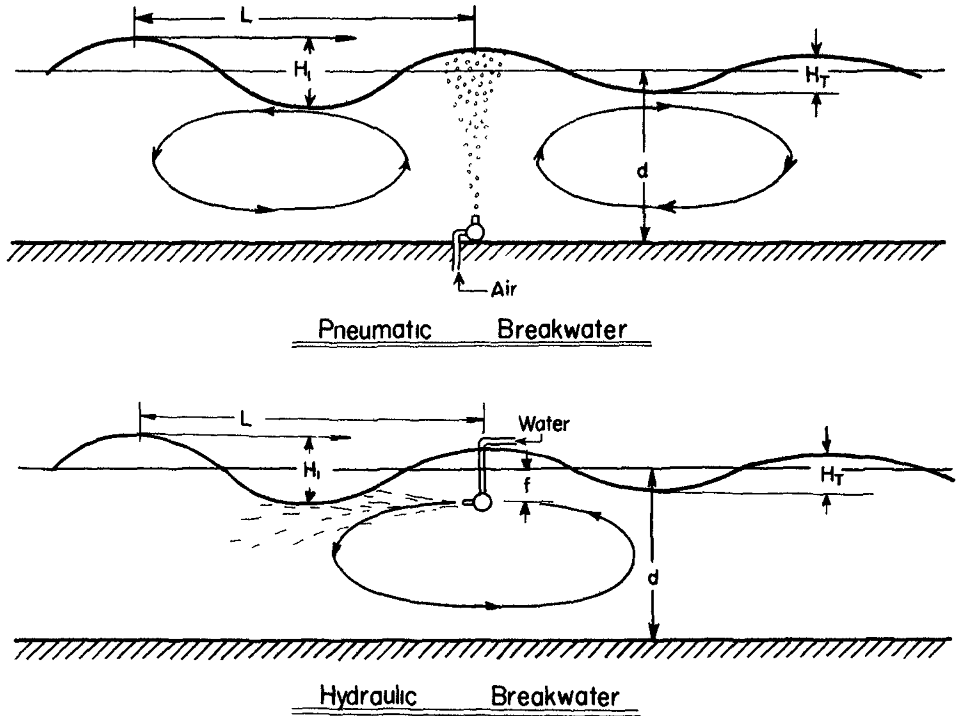

Fig. 1. Sketches Illustrating Pneumatic and Hydraulic Breakwaters.

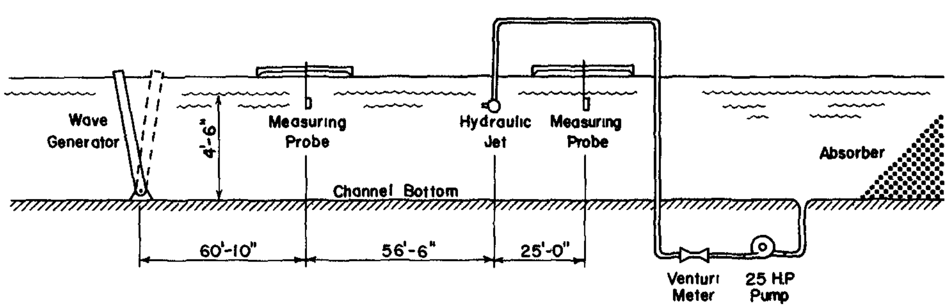

Fig. 2. Sketch of Test Installation in the Large Wave Channel.

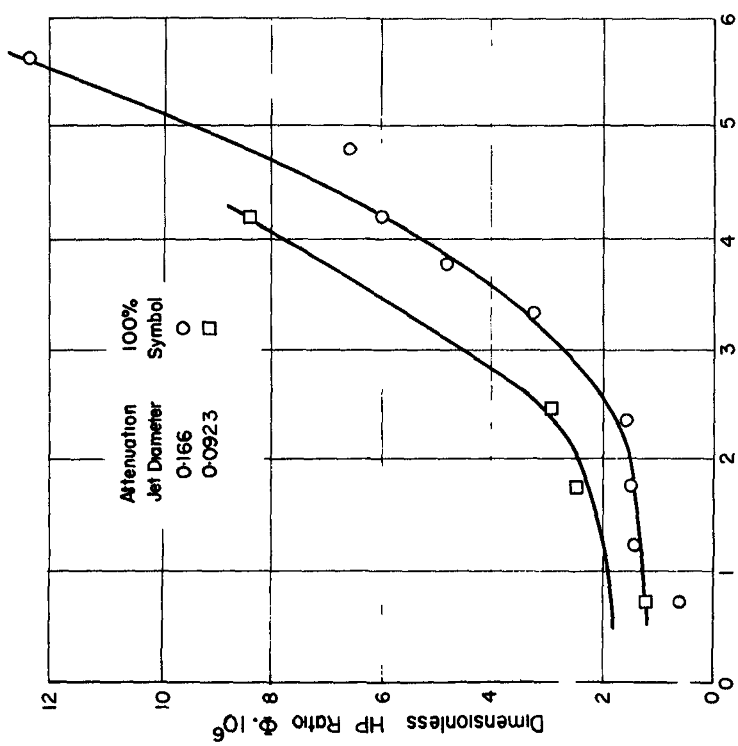

Fig. 3. Horsepower Ratio as a Function of Length-to-Depth Ratio.

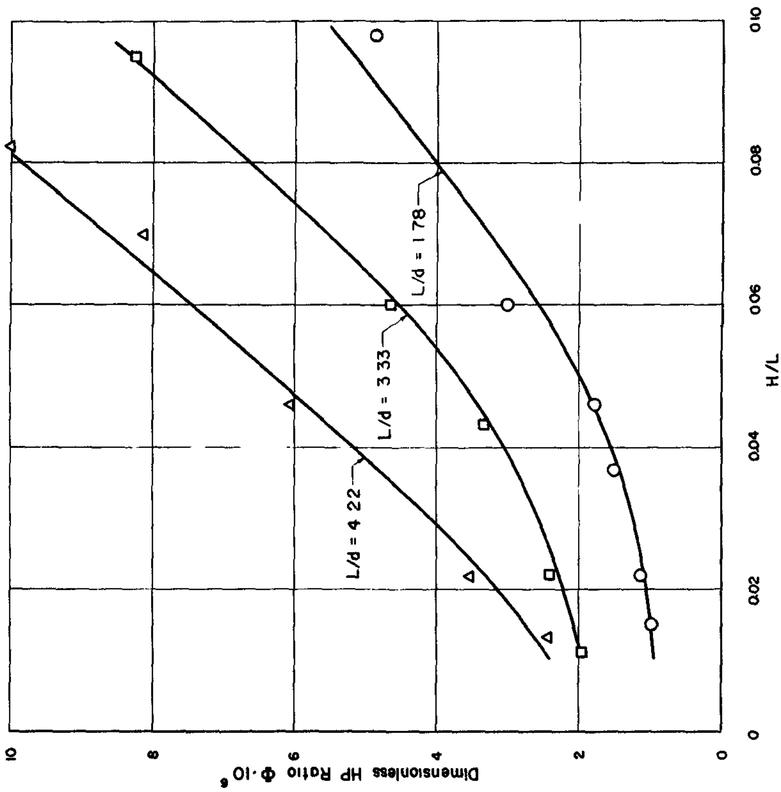

Fig. 4. Horsepower Ratio as a Function of Wave Steepness.

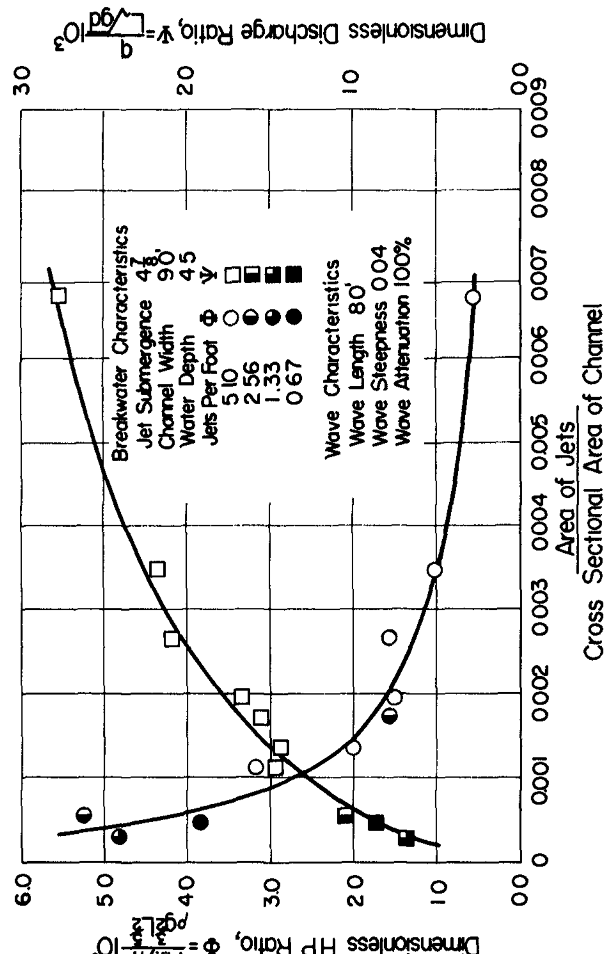

Fig. 6. Typical Graph of Horsepower and Discharge Requirements as a Function of Jet Area.

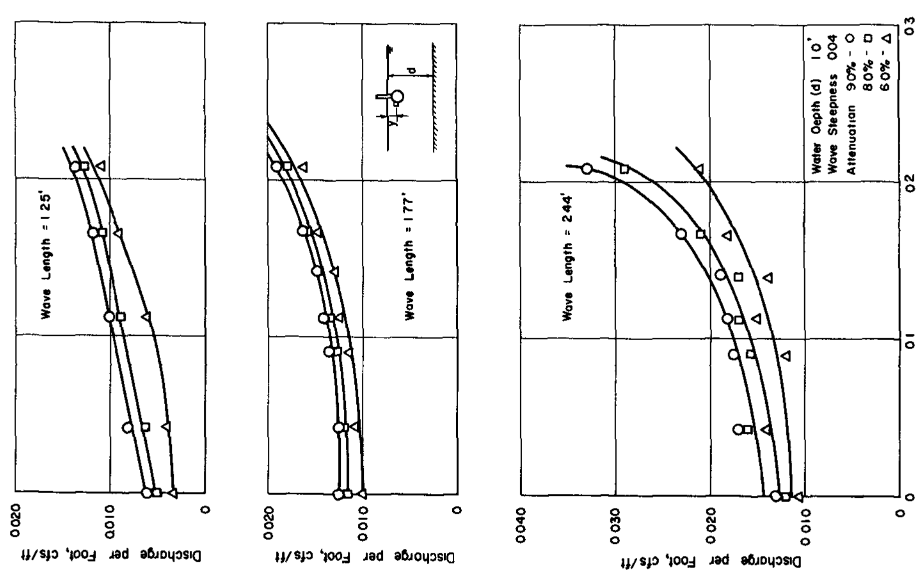

Fig. 5. Effect of Jet Submergence on Discharge Requirements.

Effect of Wave Steepness

Wave steepness has an important effect on power requirements. Figure 4 presents typical data for three L/d values based on the small-scale tests. Considering the curve for an L/d value of 3.33, it may be noted that for an increase in wave steepness from 0.02 to 0.08, a factor of 4, the required horsepower ratio ϕ is increased by a factor of about 3. However the true efficiency, that is, the ratio of attenuated wave energy to the jet energy, is considerably higher for the steep waves than it is for the shallow waves.

Effect of Jet Submergence

The results presented in Fig. 5 indicate that the zero submergence condition is the most efficient; however there may be objections to a prototype installation using this value, and the majority of tests were based on a relative submergence value y/d of 0.091. A slight correcti can be applied to such data by use of Fig. 5 if zero submergence is required.

Effect of Jet Area

One of the most important parameters affecting both the discharge and horsepower requirements is the jet area per lineal foot of breakwater. Jet area is dependent on two variables--jet spacing and jet size. During this study both the jet spacing and jet area were varied; large-scale tests covered spacings of 1.34, 2.56, and 5.11 jets per ft and a number of jet diameters between 0.422 and 1.047 inches. Four L/d ratios (L = wave length, d = water depth) were selected for the tests: 0.72, 1.22, 1.78, and 2.44. A typical summary graph for L/d = 1.78 and 100 per cent attenuation is presented in Fig. 6. In this case the dimensionless horsepower ratio ϕ and dimensionless discharg ratio ψ were plotted against the dimensionless jet area which is defined as

$$\frac{\text{area of jets}}{\text{cross-sectional area of channel}}$$

which is equal to

$$\frac{a_j}{a_c} = \frac{\text{area of jets per foot}}{\text{1 x depth of channel}}$$

Figure 6 indicates that the discharge and power requirements are strongly dependent on the jet area. Low values of jet area are associa with a requirement for high power, high jet velocities, and low discharges. It appears that power requirements at the jets or nozzles wil probably decrease as the jet area is increased until the latter is equa to the cross-sectional area of the required surface current. Apparentl

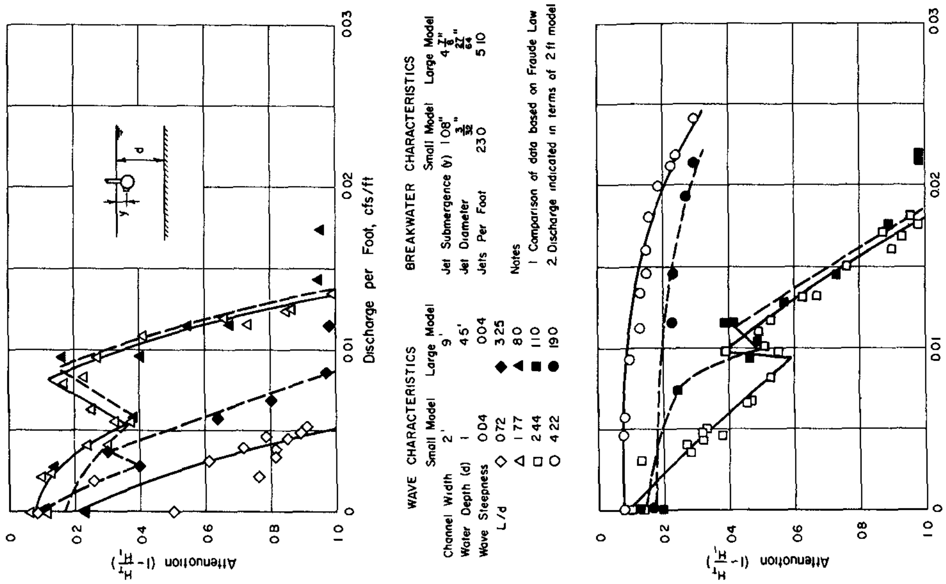

Fig. 8. Wave Attenuation as a Function of Discharge. Comparison Between Small- and Large-Scale Data.

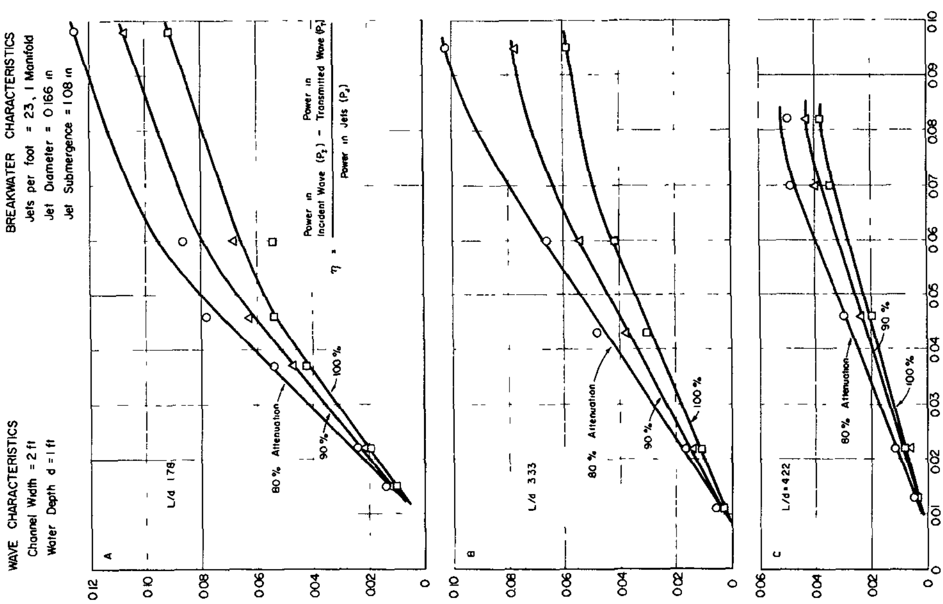

Fig. 7. Efficiency as a Function of Wave Steepness.

the turbulent losses in the jets decrease as the jet velocity decreases, resulting in better efficiencies. However, as the jet area is increased the required discharge is increased; as a result, it would be necessary to use larger manifolds and supply pipes or provide for larger losses in these components.

Efficiency

As noted in the section on wave steepness, more power is required to attenuate relatively steep waves than flat waves; however the efficiency of the system η is much higher for the steep waves than it is for the flat waves. The efficiency is defined as

$$\eta = \frac{P_I - P_T}{P_j}$$

where P_I is the power in incident wave, P_T is the power in transmitted wave, and P_j is the power in the hydraulic jets.

Figure 7 illustrates experimental data on efficiency. For the one manifold system, the efficiency varies with steepness, attenuation, and relative wave length. For attenuations of 80 to 100 per cent, the maximum efficiency obtained was only about 12 per cent.

Scale Effect

The small and large wave channels, being geometrically similar, were ideally suited for a scale-effect study. For the purpose of comparison of small- and large-scale models, it was assumed that gravity and inertia forces are of primary importance in relating the two models and, consequently, that Froude's law governs. The Froude number may be expressed as

$$F = \frac{C}{\sqrt{gd}}$$

where C = wave celerity, g = acceleration of gravity, and d = depth of water.

Consequently, the length ratio for the two models is $L_r = 1/4.5$, the discharge ratio is $Q_r = L_r^{5/2}$, and the power ratio is $P_r = L_r^{7/2}$

Figure 8 illustrates comparative discharge data expressed in terms of the small model. The preceding formulas were used to reduce the large-scale data to the appropriate conditions. It may be noted that data in the low-attenuation region exhibited considerable scatter and that the curves had a pronounced discontinuity; the latter may have resulted from a variation in the mechanism of energy loss as the jet

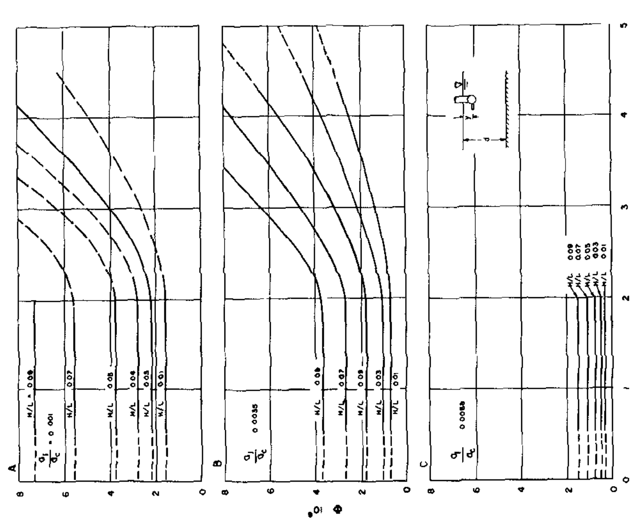

Fig. 9. Summary Curves--Attenuation 100 Per Cent.

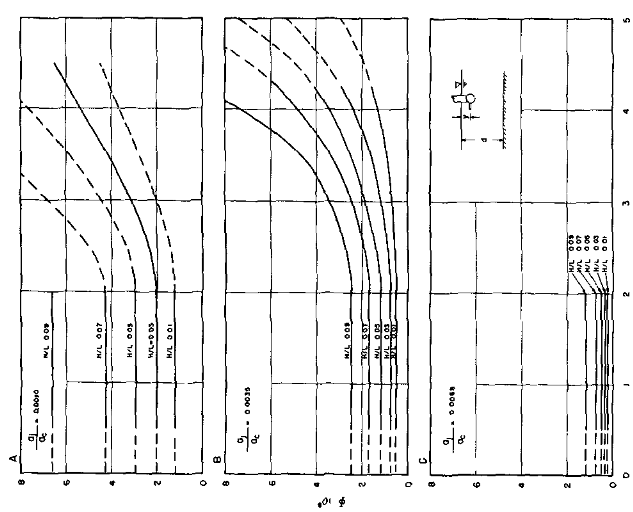

Fig. 10. Summary Curves--Attenuation 80 Percent

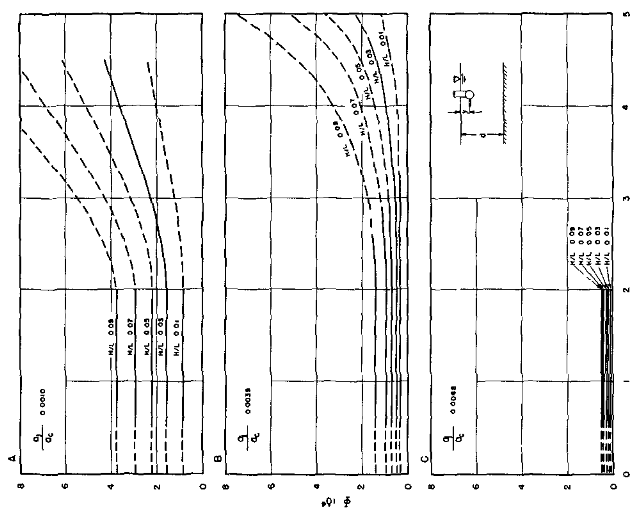

Fig. 11. Summary Curves--Attenuation 60 Percent.

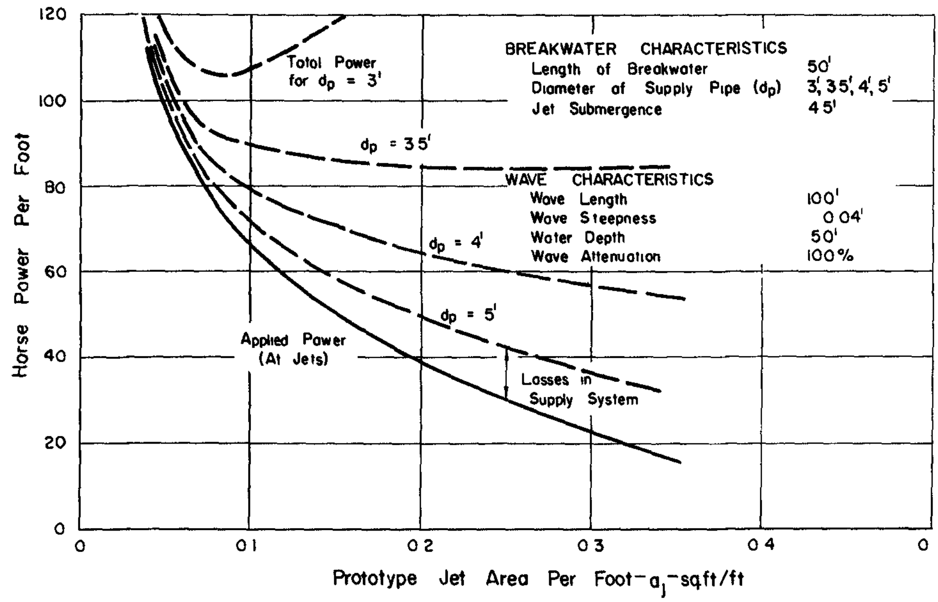

Fig. 12. Horsepower Requirements for a Typical Hydraulic Breal water.

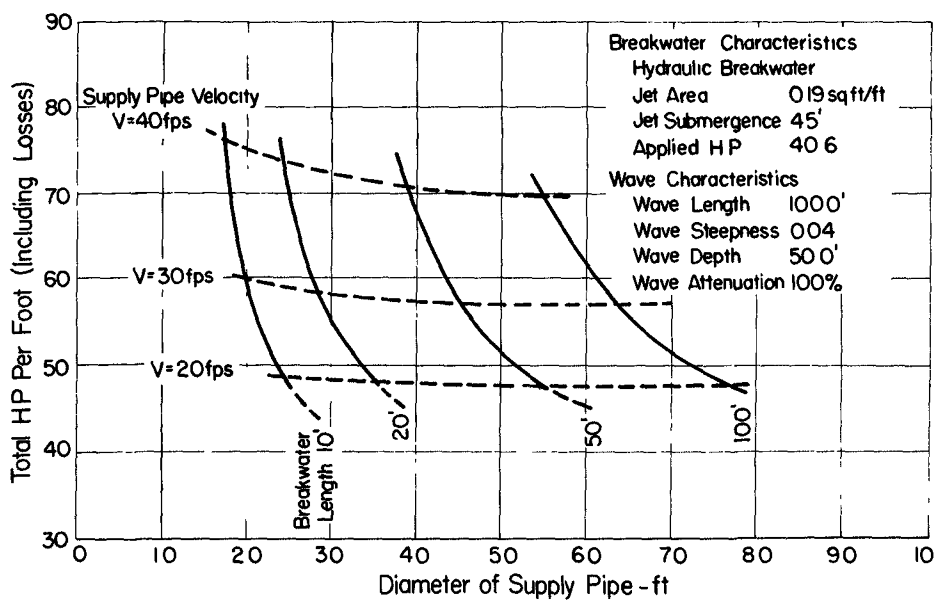

Fig. 13. Typical Relationship Between Power Requirements and Diameter of Supply Pipe.

discharge was varied. Considering the data for attenuations in excess of 50 per cent and L/d values in excess of 1.0, agreement between the data for two models is quite good. It was concluded that little, if any, evidence of scale effect existed for the 1:4.5 scale ratio of the two sets of tests. It is believed that these are the first tests on two different sizes of models of the hydraulic breakwater.

SUMMARY CURVES

Power requirements at the jets are summarized in Figs. 9 through 11 in dimensionless form. The faired curves were based primarily on the large-scale tests in the region of $1 < L/d < 2.5$ and upon small-scale tests for $L/d > 2.5$. The dimensionless power ratio ϕ can be determined for selected values of L/d, H/L, a_j/a_c, and attenuation.

The required discharge per lineal foot of breakwater can be computed by the following formula:

$$q = 58.58\, a_j^{2/3}\, \phi^{1/3}\, L^{5/6} \quad \text{in cfs per ft}$$

where 58.58 is a constant $= 1100^{1/3}\, g^{1/2}$ and L is the wave length.

The value of a_j can be computed from the equation

$$a_j = \frac{a_j}{a_c} d$$

after the depth is selected. The ratio a_j/a_c is given in each graph.

As noted before, the value of ϕ is the applied power at the end of the nozzles. The total power of the system will involve losses in the manifold and supply system, in addition to the applied power. To illustrate the comparative importance of losses in the supply system and the effect of jet area, computations were made for an arbitrary set of conditions involving a wave length of 100 ft, water depth of 50 ft, and a wave steepness of 0.04. Various pipe sizes were considered. As a hypothetical case, it was assumed that the pump or compressor was located about 10 ft above the water surface and 20 ft to the lee side. Several values of breakwater length to be supplied from one compressor were considered. Piping losses were computed for various discharges using the equation

$$H_L = \left(f\frac{\ell}{d_p} + k\right)\frac{V^2}{2g}$$

where H_L is head loss in feet, f is friction coefficient (=0.01), ℓ is length of pipe, d_p is pipe diameter, V is velocity of flow in pipe, and k is a total coefficient loss for bends, nozzles, and tees (k was assumed = 1.0). It was also assumed that the supply pipe had an effective length of 60 ft, including an assumed allowance for the manifold.

Figure 12 presents the curve for applied horsepower at the nozzle as a function of jet area and curves of total horsepower including losses in the supply system for pipe and manifold diameters of 3.0, 3.5, 4.0, and 5.0 ft; the losses in the supply system were computed for a breakwater length of 50 ft per supply pipe, and the result divided by 50 to give horsepower per unit length of breakwater. Similar curves could be computed for various effective lengths of breakwater to assist in the selection of optimum jet area.

Figure 13 was prepared as a second illustration of the effect of pipe size and length of breakwater supplied by a single pipe. A jet area of 0.19 sq ft per ft was used, which is near the minimum total power requirement for a 3.5-ft pipe and 50-ft length of breakwater. The solid curves indicate total power as a function of pipe diameter for various lengths of breakwater supplied by a single pipe. The dotted curves indicate the resulting velocity in the supply pipe.

CONCLUSIONS

1. Horsepower and water-discharge requirements for the hydraulic breakwater are dependent upon wave characteristics such as length, height, and water depth, and breakwater characteristics such as spacing and size of nozzles, submergence of nozzles, and the number of manifolds.

2. A single-manifold hydraulic breakwater is quite effective for deep-water waves ($L/d < 2$), but its effectiveness decreases with increasing L/d ratios.

3. For high values of attenuation, more power is required to attenuate the steep waves; however, the efficiency of the system based on the ratio of the difference between incident and transmitted wave energy to the jet energy is much higher for the steep waves than it is for the flat waves.

4. Zero submergence of the nozzles appears to be most efficient for the range of wave lengths tested; however, the differences in discharge requirements are not large for values of y/d between 0 and 0.

5. Power and discharge requirements at the nozzles are dependent upon the area of jets per unit length of manifold. As the area is increased, the power decreases, and the discharge increases. As losses in the pumping and supply system are dependent on the discharge, the supply system must be analyzed along with the manifold and jet system in order to determine the optimum jet area.

6. Comparative data on the hydraulic breakwater obtained in the large and small channels (scale ratio 1:4.5) agree quite well when compared on the basis of Froude's law for values of L/d between 1.22 and 1.78. This would indicate that little scale effect exists over this range and tends to substantiate extrapolation of the data to the prototype condition.

7. There are preliminary indications that a several-manifold hydraulic breakwater producing a thicker surface current might require less power at the jets and higher discharges than a breakwater with a single manifold for large L/d values, but additional tests would be required to determine the optimum efficiencies of several-manifold breakwaters for a given range of L/d values.

8. Limited comparisons with Taylor's theory indicated the theoretical value of surface-current velocity usually occurred at a distance of one or two wave lengths from the breakwater for an experimental wave steepness of 0.01. The comparison is somewhat arbitrary as the theory does not consider wave steepness.

9. Snyder's experiments show fairly good agreement with the tests described herein for high values of attenuation. For low values of attenuation Snyder's experiments resulted in somewhat larger discharges than St. Anthony Falls data indicate.

ACKNOWLEDGMENT

This investigation was sponsored by the Office of Naval Research and conducted at the St. Anthony Falls Hydraulic Laboratory.

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CHAPTER 44

LABORATORY TESTS OF PERMEABLE WAVE ABSORBERS

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INTRODUCTION

Laboratory studies involving surface waves usually require the use of absorbers at some boundaries of the test facility to prevent objectionable reflections of the test waves. Such absorbers frequently consist of sloping beaches with or without a layer of permeable material. These are sometimes used in conjunction with other devices which absorb part of the wave energy. If sufficient space is available, a long absorber with a low surface slope can be used, resulting in very low reflections. However, space limitations sometimes necessitate the use of an absorber of minimum length with a steeper surface slope. In such cases permeable materials are highly beneficial to assist in absorbing the wave energy.

While absorbers have been utilized in most laboratories performing wave studies, very little quantitative information has been published on the subject. The majority of data that are available have been obtained in connection with studies of beaches, breakwaters, and other field installations, but they are of interest in the design of laboratory absorbers.

The studies described herein had as their objective the procurement of additional basic information to assist in the design of an absorber for a specified range of wave conditions.

EARLIER STUDIES

A review of earlier work revealed one theoretical development by Miche (1944) which was considered of interest in this study. Considering the ideal case of a smooth barrier or beach forming an angle α with the horizontal, he developed analytically a formula for the maximum wave steepness (in deep water) which will be totally reflected by such a barrier.

$$\delta_m = \sqrt{\frac{2\alpha}{\pi}} \; \frac{\sin^2 \alpha}{\pi}$$

From this he deduced that waves steeper than δ_m would be partially reflected and that the theoretical reflection coefficient R' would be

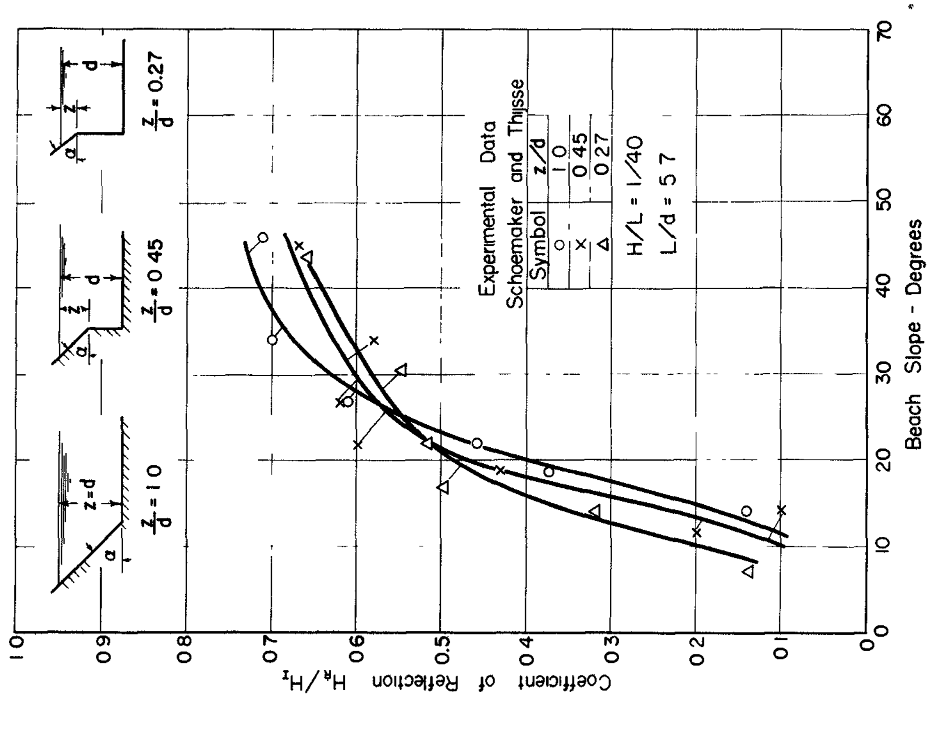


Fig. 1. Experimental Data on Reflections from Impermeable Surfaces--Schoemaker and Thijsse

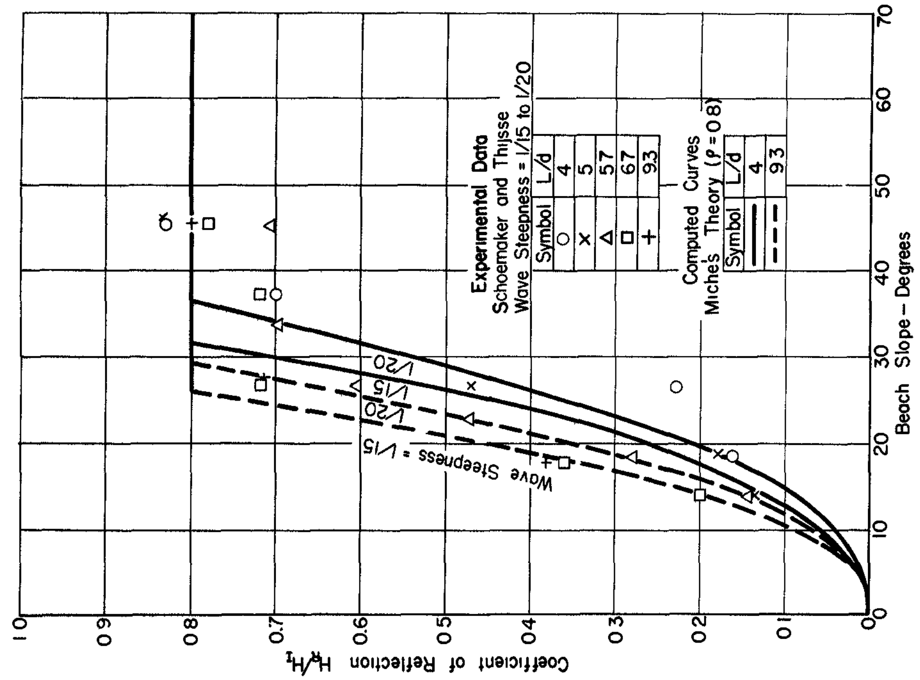


Fig. 2. Comparison of Experimental and Theoretical Results for Continuous Imper-

$$R' = \frac{\delta_m}{\delta_o}, \quad \text{with} \quad R' \leq 1,$$

where δ_o is the incident wave steepness in deep water.

One of the boundary conditions for the theoretical development was based on the assumption of zero normal velocity at the surface of the barrier or that the barrier was impermeable. To account for permeability as well as roughness of actual installations, Miche introduced an intrinsic coefficient ρ which was assumed to be independent of the beach slope. The actual reflection coefficient was then given as

$$R = \rho R' = \rho \frac{\delta_m}{\delta_o} = \frac{H_R}{H_I},$$

where H_R is the reflected wave height and H_I the incident wave height.

In a subsequent publication Miche (1951) indicated that ρ might vary from 0.68 to 1.0 for rough and smooth impermeable barriers, respectively. A value of $\rho = 0.32$ was indicated for a rubble mound structure.

Schoemaker and Thijsse (1949) published the results of a series of experimental studies on walls with both continuous and discontinuous slopes. The discontinuous slopes were formed by a lower vertical wall and an upper sloping section; the point of juncture was positioned at various distances below the still water surface.

Figure 1 illustrates data obtained by Schoemaker and Thijsse for one value of wave steepness and one wave length. The curves for the three types of models tested are fairly close together, but greater differences can probably be expected for shorter waves.

Figure 2 illustrates additional data for length-to-depth ratios from 4 to 9.3; it was indicated that the wave steepness varied from 0.05 to 0.067. Curves based on Miche's theory have been plotted on the same figure for comparative purposes; a coefficient ρ of 0.8 appeared to give the best agreement between the experimental data and the computed curves. The experimental data are for a continuous slope ($z/d = 1$).

The Beach Erosion Board (1949) published data on the reflective characteristics of various simple structures based on tests with solitary waves. Both permeable and impermeable structures were tested. The porosity of some of the structures was varied; it was concluded that the best absorption was obtained with a porosity of 60 to 80 per cent. In tests of a porous rock wall backed by open water with both faces of the wall vertical, it was found that the maximum absorption was obtained when the length of permeable material was 2.5 times the water depth.

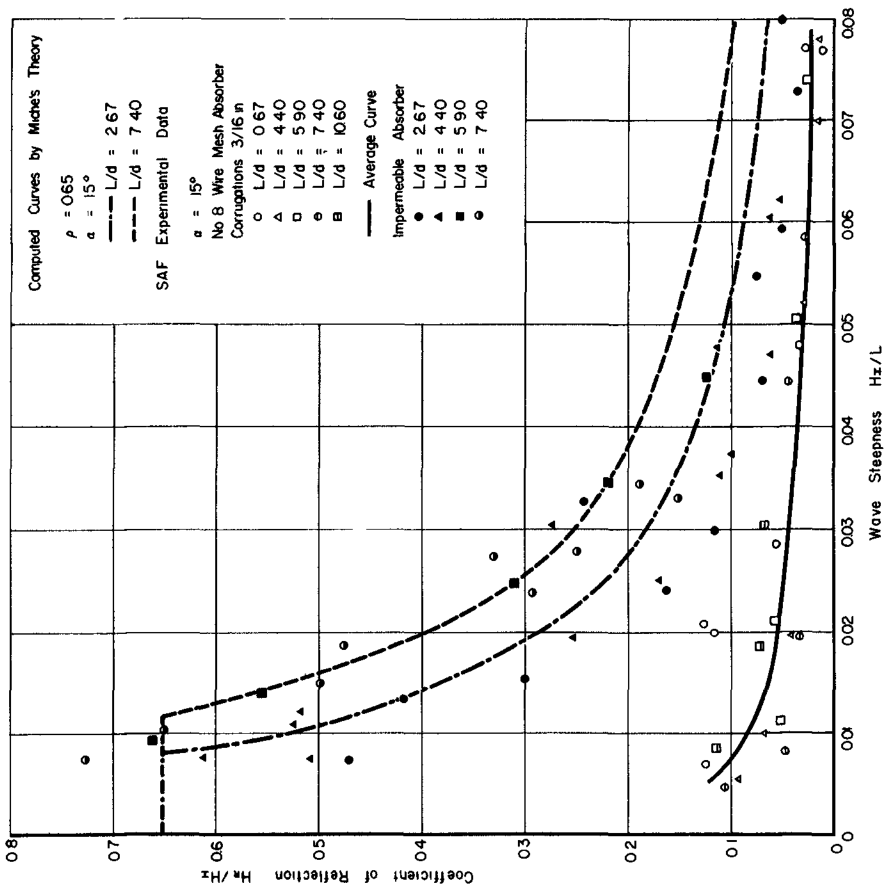


Fig. 4. Comparison of Experimental and Theoretical Results

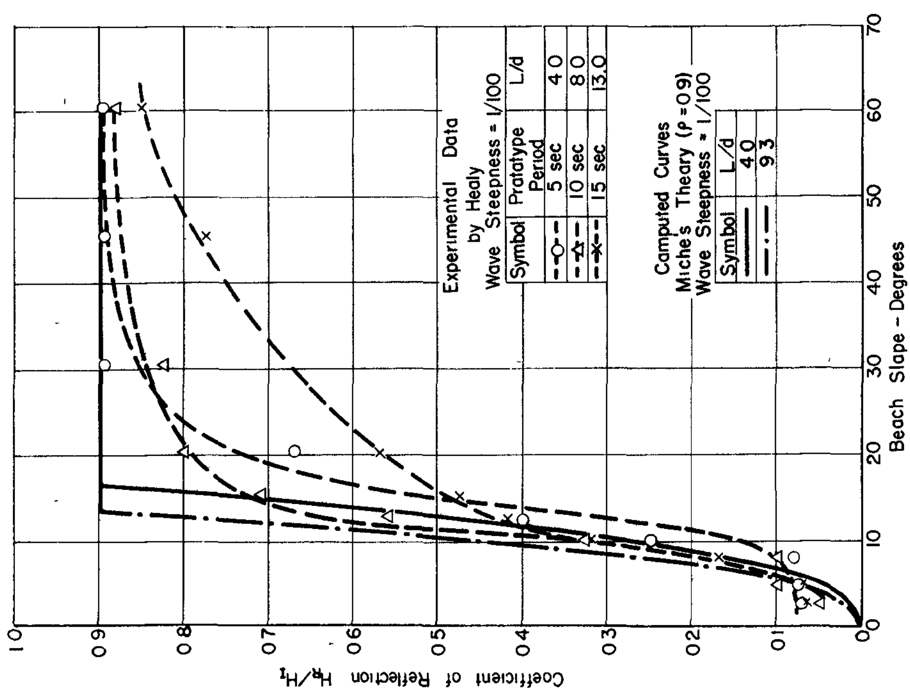


Fig. 3. Comparison of Experimental and The-

Laurent and Devimeux (1951) published the results of studies of reflections from several types of seawalls. They concluded that the reflecting capacity of a seawall varies inversely as a function of the wave steepness and that reflections decreased with a decrease in wall slope.

Healy (1953) presented experimental data on reflections from sloping impermeable beaches made of smooth plywood. Values of wave steepness from 0.005 to 0.025, beach slopes from 2 degrees to 60 degrees, and prototype wave periods of 5, 10, and 15 sec were used in the tests. Figure 3 illustrates some of the data obtained by Healy with two comparative computed curves based on Miche's theory. Other data indicated a decrease in reflection coefficient from 0.67 to 0.16 as the wave steepness increased from 0.003 to about 0.017 for a 10-degree slope.

While the preceding experiments were very helpful in the study of impermeable structures, the lack of data on the reflections of oscillatory waves from permeable structures necessitated additional experimental work. As a result a series of tests was conducted on permeable materials of various types with a few additional check tests on impermeable sloping surfaces.

TEST EQUIPMENT AND PROCEDURE

The major portion of the experimental studies were conducted in a wave channel 6 in. wide, 15 in. deep, and 40 ft long. Subsequently some check tests were performed in a larger facility with a width of 9 ft, a depth of 6 ft, and a length of 253 ft.

Waves were generated by a pendulum-type generator in the small facility and by a hinged-plate generator in the large facility.

Wave heights were measured by a capacitive wave-profile recorder. The probe of this recorder was traversed over a distance of at least one-half wave length, giving a record of the envelope of the standing wave or partial clapotis which resulted from addition of the incident and reflected waves. The reflection coefficient R and incident wave height H_I were obtained from the following formulas:

$$R = \frac{H_R}{H_I} = \frac{H_\ell - H_n}{H_\ell + H_n}$$

and

$$H_I = \frac{H_\ell + H_n}{2},$$

where H_ℓ = height at loop

H_n = height at node of standing wave

These can be developed by addition of equations for the surface profile of the incident and reflected waves, using an assumed sine profile.

The reflections were measured over a length of about 3d to 3d + L/2 from the toe of the absorber; preliminary tests indicated that the maximum reflections were obtained by measuring in this zone.

EXPERIMENTAL PROGRAM AND DATA

The experimental studies included measurements of wave reflection for variations in the slope, shape, and porosity of the absorber and variations in the length-to-depth ratio and steepness of the incident waves.

Initially, data were obtained on a permeable material consisting of corrugated wire mesh with a porosity of about 93 per cent. This was followed by tests on absorbers of gravel, crushed rock, and perforated plates. Finally, limited tests were performed on absorbers made up of transverse square rods, spaced to produce a porosity of about 70 per cent. Brief tests were also conducted on impermeable surfaces for comparison with theory and the data of others; this was considered desirab: as a check on the test procedure.

Values of wave steepness ranging from 0.005 to 0.08 and length-to-depth ratios ranging from about 0.6 to 10.0 were used during the variou: phases of the tests, although this range was restricted in some instanc

Figure 4 illustrates experimental data for permeable and impermeab] absorbers with a 15-degree surface slope. Two curves based on Miche's theory have been included for comparative purposes. Good agreement between theory and experiment for the impermeable surface was obtained for wave steepness less than 0.03. However, for values of wave steepness on the order of 0.06 to 0.08 the measured reflections were about half the theoretical values.

As may be noted in Fig. 4, the permeable absorber was considerably superior to the impermeable unit, although the total length required for the two types was the same.

During the tests of both continuous and discontinuous slope absorbe it was found that the curves of reflection as a function of steepness, slope, or wave length frequently were not smooth curves due to secondary variations. Some of these variations were apparently caused by additior and cancellation of reflections from two or more parts of the absorber. With the impermeable absorber, some variation resulted from waves generated by water returning to the normal level following the initial uprush due to breaking of the wave. The resultant wave or reflection was sometimes of an appreciable magnitude even though complete breaking of the incident wave occurred. A relatively thin layer of permeable material was sometimes quite beneficial in alleviating this effect.

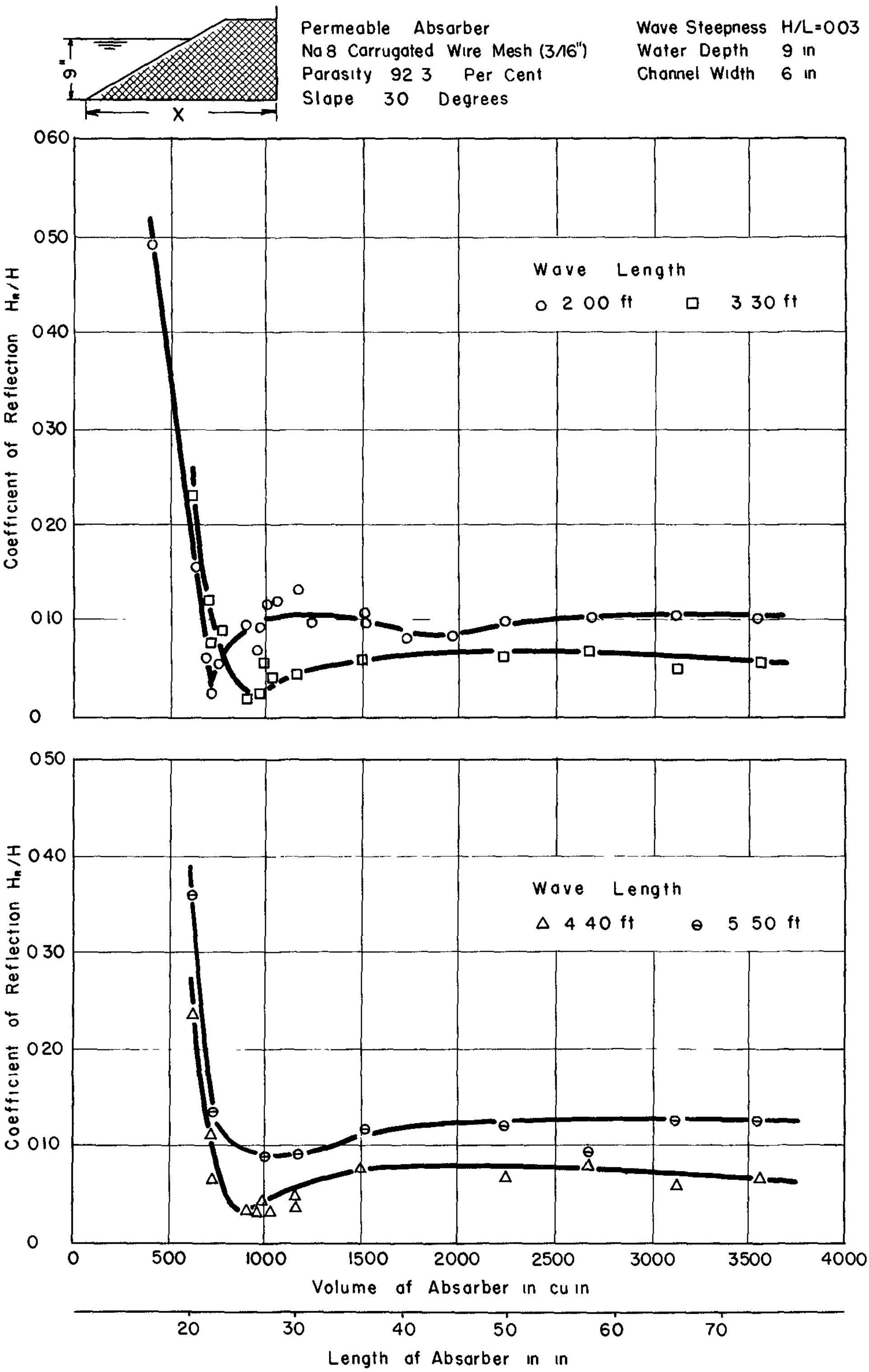
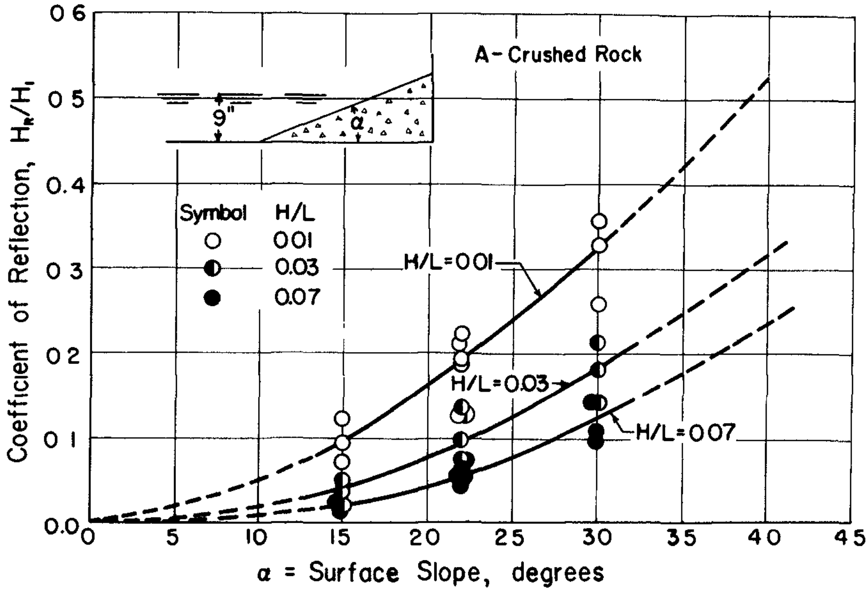


Fig. 5. Effect of volume on coefficient of reflection for a wire-mesh absorber.



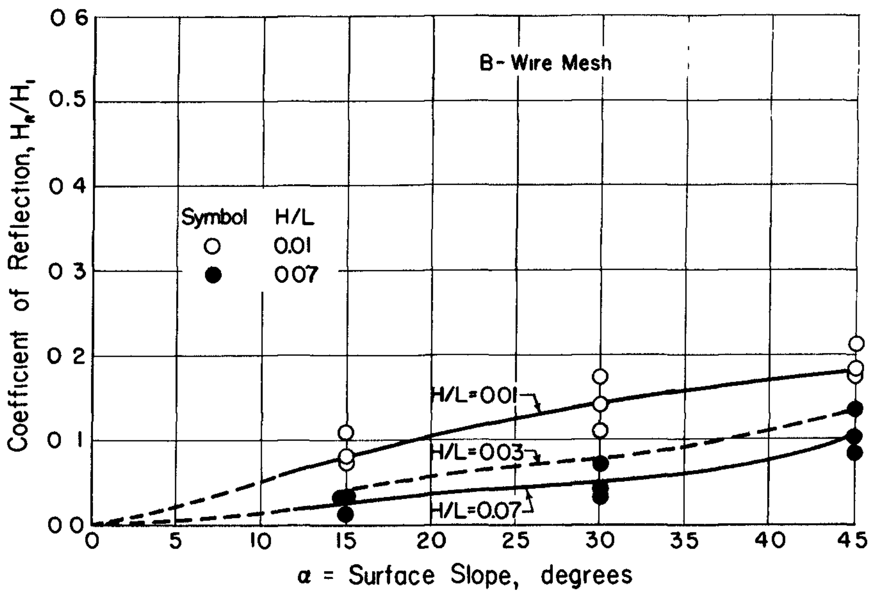


Fig. 6. Reflection coefficients for crushed-rock and wire-mesh absorbers.

During the tests of wire-mesh absorbers, which had a high porosity, it was noted that an appreciable orbital motion occurred well back in the permeable material. As a check on the effect of variations in the length or volume of permeable material, a brief series of tests was performed with absorbers of various length or volume with a constant surface slope. A steep slope of 30 degrees was selected for the tests. Figure 5 illustrates typical data on the effect of variation in length or volume of the absorber for a wave steepness of 0.03. The minimum allowable length was well defined by a very steep portion of the curve of reflection as a function of absorber length. With a further increase in length, the reflections increased slightly and then remained relatively constant in spite of large increases in volume of permeable material. The low point in the curve may have been caused by cancellation of reflections from the sloping surface and vertical backing plate of the absorber. The data are of limited value as the minimum length or volume will vary as a function of the slope and porosity of the absorber, but they are indicative of the effect of variations in volume. Also, the data indicated that low absorption coefficients could be obtained for relatively high values of L/d with a short absorber.

Figure 6(a) illustrates experimental data for crushed-rock absorbers with surface slopes from 15 to 30 degrees. The length of the absorbers varied with the slope, but the volume of material used was a constant. Several sizes of rock were used in the tests: 1/4 in. to 3/4 in., 1 in. to 1-1/2 in., and 1-1/2 in. to 2 inches. The narrow size gradation resulted in porosities on the order of 50 per cent. Initial tests were performed with gravel (porosity 40 per cent) and pit-run crushed rock (porosity 45.8 per cent). The reflections for the latter two materials were fairly low, but further improvement was obtained by screening the crushed rock to produce a narrow size gradation and a higher porosity. Accordingly, most of the tests were performed on the screened rock.

In this series of tests the size of rock apparently had little effect on the average reflection coefficient. However, placement of the rock was very important with the result that subsequent tests with the same material produced variations in the reflection coefficients up to 100 per cent. These variations were more pronounced with the larger size ranges.

The data in Fig. 6(a) were obtained with test wave lengths ranging from 2 to 4.4 ft ($L/d = 2.67$ to 5.85). Variations as a function of wave length were noticeable in this range for a single absorber; however, such variations were less than those due to placement of the rock. As a result, an average curve is shown. With a discontinuous absorber the variations in wave length have a much greater effect.

The coefficient ρ in Miche's formula was computed for the crushed-rock absorbers. It varied from about 0.11 with a wave steepness of 0.01 to about 0.19 with a wave steepness of 0.07. For comparative purposes, the coefficient for the impermeable absorbers varied from about 0.65 with a wave steepness of 0.01 to about 0.35 with a wave steepness of 0.07.

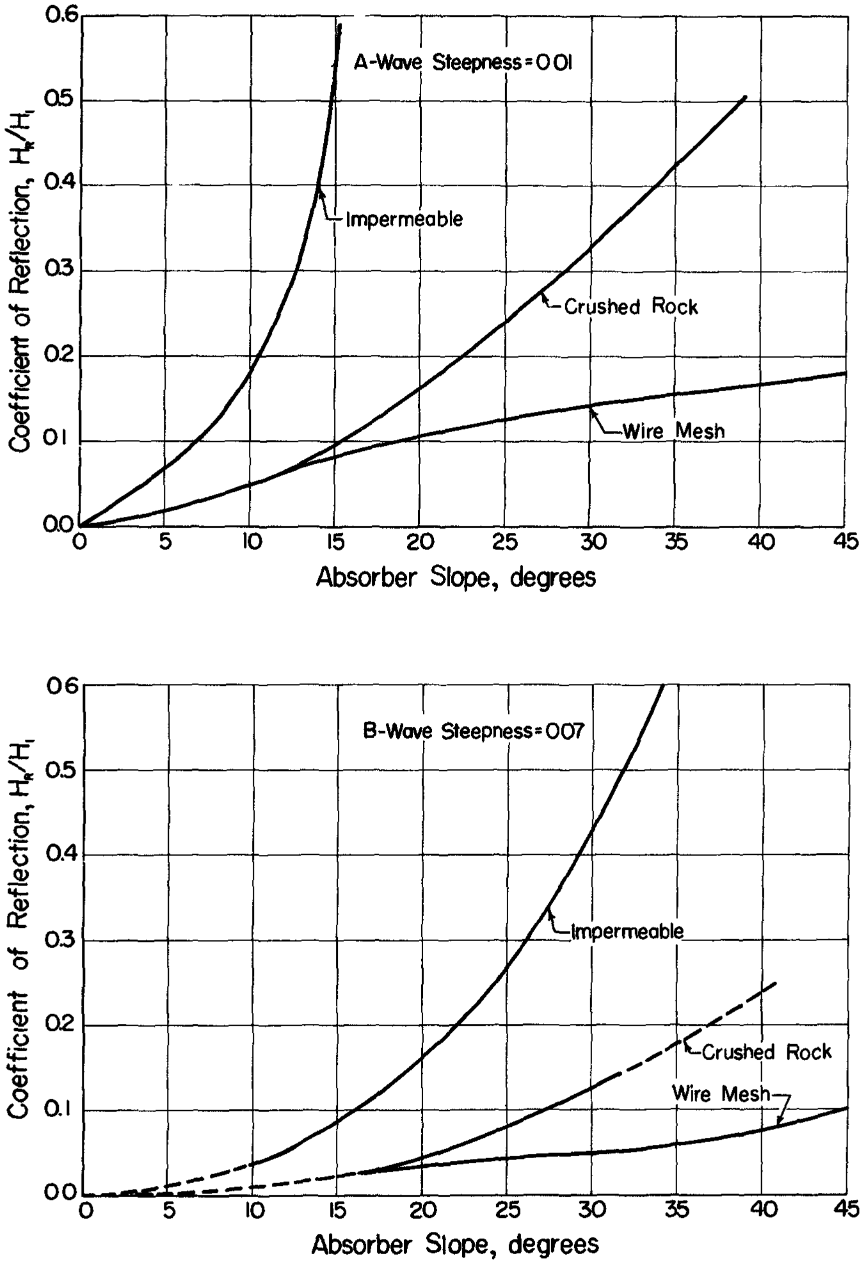


Fig. 7. Comparison of impermeable, crushed-rock, and wire-mesh absorbers.

Reflection coefficients for continuous-slope wire-mesh absorbers are plotted in Fig. 6(b) for two values of wave steepness. Average values are shown for wave lengths ranging from 2.0 to 4.4 ft. The absorbers were constructed of alternate layers of corrugated and plain wire mesh. The corrugations had a total height of 3/16 inches. The mesh had a spacing of 8 wires per inch. The porosity of this arrangement was about 93 per cent.

The curves of reflection as a function of absorber slope are similar to those for crushed rock for slopes less than 15 degrees. For slopes in excess of 15 degrees the wire-mesh absorbers were considerably more efficient than the crushed-rock absorbers. The coefficient ρ varied from about 0.09 with a wave steepness of 0.01 to about 0.19 with a wave steepness of 0.07 for a surface slope of 15 degrees.

Figure 7 illustrates comparative curves of reflection coefficient as a function of slope for the three types of continuous absorbers: impermeable, crushed rock, and wire mesh.

Following the above tests, studies were made of absorbers consisting of a layer of permeable material over an impermeable surface. With water depths of 9 and 12 in. and wave lengths ranging from 2 to 4.4 ft, reflection coefficients of about the same magnitude as those shown in the preceding graphs were obtained for a thickness of permeable layer of about 3 inches. A thickness in excess of this value was not beneficial; a lesser thickness usually resulted in higher reflections. However, this thickness cannot be considered as a fixed value as it depends to a considerable extent on the range of wave lengths of interest.

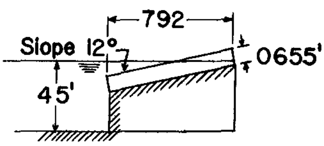
On the basis of these studies, it was concluded that a material with a high porosity was desirable if an absorber of minimum length was needed. Wire mesh is only one of the possible materials which might be used. Others include perforated plates and metal shavings. However, the cost of such materials would be prohibitive in many installations, and cheaper materials, such as crushed rock or cast concrete products, may be desirable. With low surface slopes crushed rock with a narrow size gradation is almost as good as the mesh or other metal products except for variations which result if the material is packed or laid so as to produce a low porosity. In an effect to reduce cost and still achieve uniformity, tests were performed on a system of square, concrete bars, spaced to produce a porosity of about 70 per cent. The bars were laid with their main axis horizontal and transverse to the direction of wave propagation. Square bars were selected, in preference to other shapes, to reduce variations in the drag coefficient due to viscous effects. This was considered desirable to avoid serious scale effects. Tests were performed on a series of discontinuous absorbers designed for shorter wave lengths than the tests described above. The reflection coefficients were much more consistent than those obtained with crushed rock, and average coefficients were somewhat superior to crushed rock.

The majority of data presented in this paper have been restricted to continuous absorbers or beaches with the thought that this configuration would be of primary interest. However, one illustration of a discontinuous absorber may also be of interest. Absorber A in Fig. 8

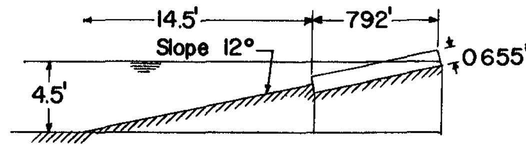
ABSORBER CHARACTERISTICS

Absorber A
Discontinuous, Permeable
Thickness of Permeable Layer 7 85 in
Seven Layers of Bars
Slope 12 Degrees
Porosity 67 Percent
Channel Width 9 ft

Absorber B
Continuous, Partly Permeable
Same as Absorber A Except
Impermeable Beach Added in Front



Absorber A



Absorber B

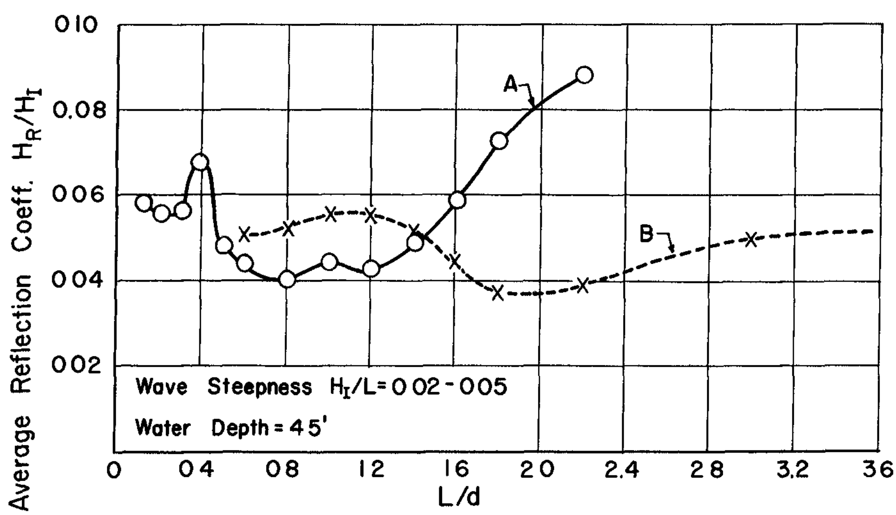


Fig. 8. Comparative reflection coefficients for continuous and discontinuous absorbers.

illustrates a discontinuous absorber developed for a range of L/d values from 0.15 to 2.0. The maximum allowable length of the absorber was 1.8d. Initial tests were performed in the small wave channel; following a preliminary selection of the basic geometry, tests were performed in the large channel with various thicknesses of permeable material over an impermeable surface. Crushed rock and square bars were used as permeable material.

The reflection curve shown for Absorber A is an average of the range of wave steepnesses of primary interest for the square-bar construction. For L/d values less than 1.6 some variation in the reflection coefficient occurs, probably due to addition of multiple reflections. For L/d values above 1.6 the curve has a pronounced upward trend indicating increased reflections from the lower part of the absorber. Addition of a vertical layer of permeable material on the lower face was slightly beneficial but did not warrant the increased cost. For comparative purposes a section was added to produce a continuous absorber--Absorber B of Fig. 8. The average curve for this unit has some minor variations but can be considered relatively independent of wave length for L/d values up to about 3.6. For L/d values less than 1.6 this unit was comparable to the discontinuous unit even though it had triple the length of the latter.

Figure 9 illustrates reflection coefficients obtained in small- and large-scale model tests of the discontinuous absorber plus data obtained by the Navy on a prototype section. The square-bar permeable material was used in all three models. Agreement was considered very good and indicated little, if any, scale effect.

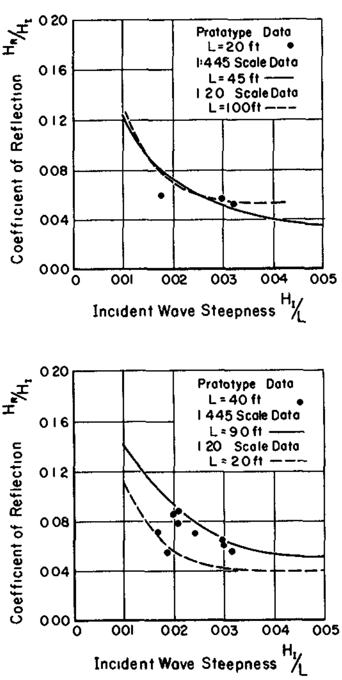


Fig. 9. Comp rison of Reflection Data Obtained in Two Models and a Prototype Absorber.

ACKNOWLEDGMENT

The tests described herein were sponsored by the David Taylor Model Basin, Department of the Navy.

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CHAPTER 45

MODEL STUDY ON THE IMPACT OF WAVES

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1. INTRODUCTION

The problem to be investigated is the structural strength of a sluice gate under the influence of wave attack. The gate was designed in view of the hydraulic forces in quasi permanent conditions.

The fact, however, that the gate is exposed to wave attack, necessitates an investigation of:

1) the transient forces due to impact,

2) the possibilities of modificating the shape of the gate in order to avoid, or at least to diminish, the chance on the occurrence of impacts.

As the mechanism of wave attack is influenced by the hydrodynamic properties of the oncoming waves, the tests are being carried out in a flume in which the waves are generated by wind in order to simulate the expected extreme natural conditions as close as possible.

Since the effects of impact are determined by the elastic properties of the structure, the final tests are being carried out by means of models with an elastic behaviour in accordance with the model scales. These tests have been designed in close cooperation with the laboratory of mechanics of solid materials.

The first phase of the model study deals with the way in which the impact is influenced by the angle of inclinatiom of the face of the gate.
The model tests have been carried out with three different positions of the gate, viz:

a) with forward inclined face,

b) with backward inclined face,

c) with vertical face.

Although the model study has not yet been concluded, it may be of interest to present already at this stage a brief report on the results obtained sofar.

The three positions of the gate are shown in figs. 1, 2 and 3 and part of the observations, recorded with each of these positions, are shown in figs. 4, 5 and 6. Fig. 7 gives four stages of a brea-

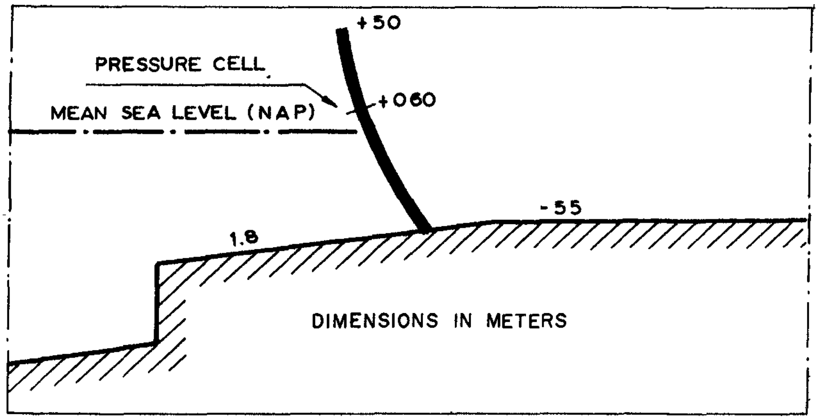

Fig. 1. Forward inclined face.

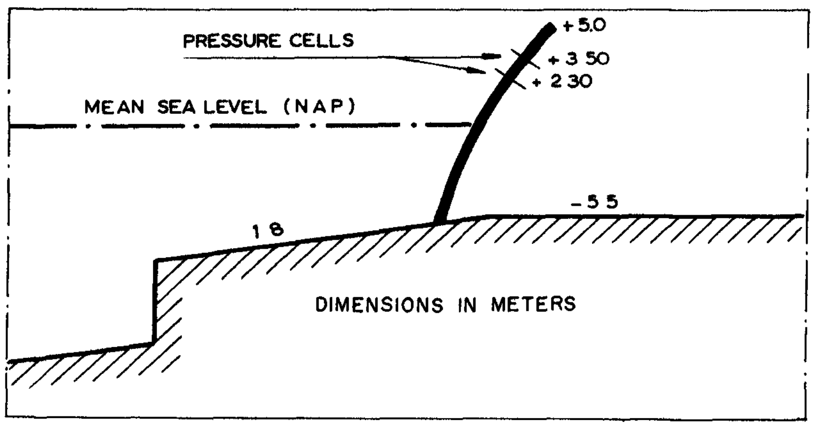

Fig. 2. Backward inclined face.

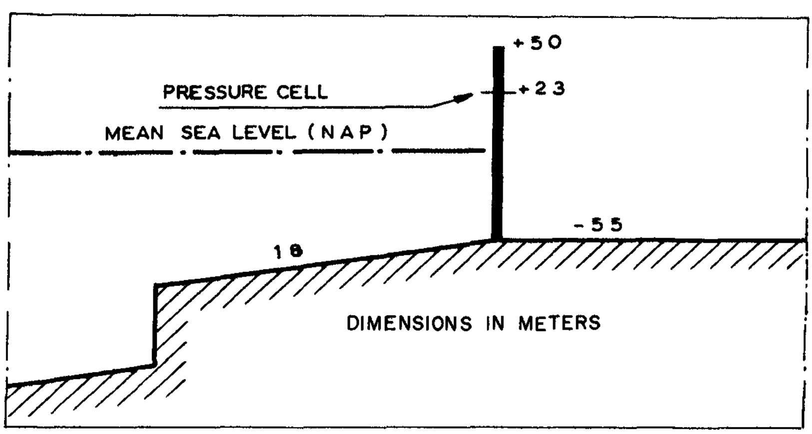

Fig. 3. Vertical face.

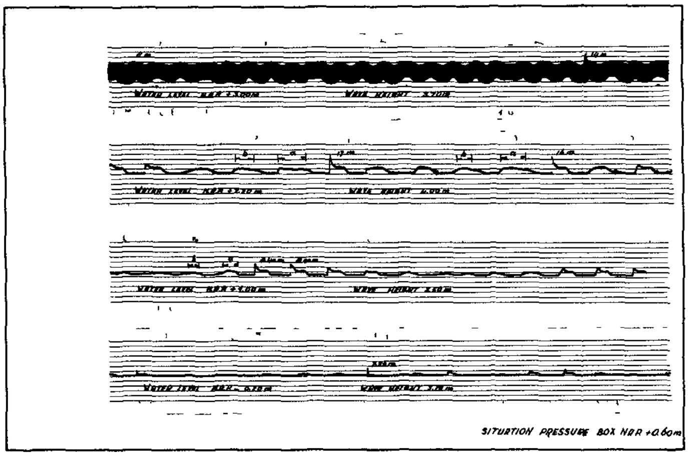

Fig. 4. Pressures on forward inclined face.

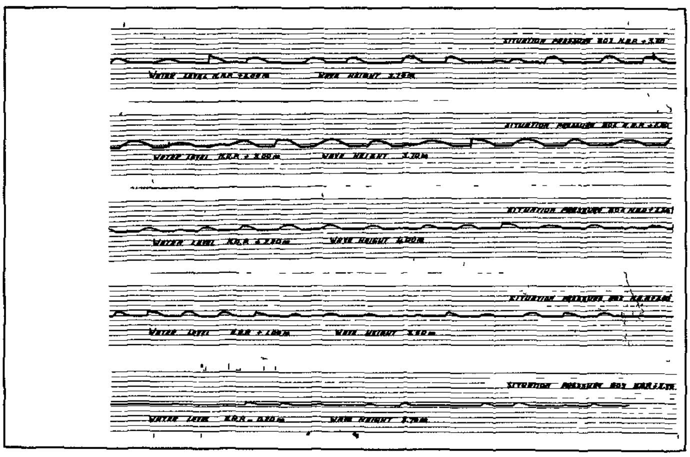

Fig. 5. Pressures on backward inclined face.

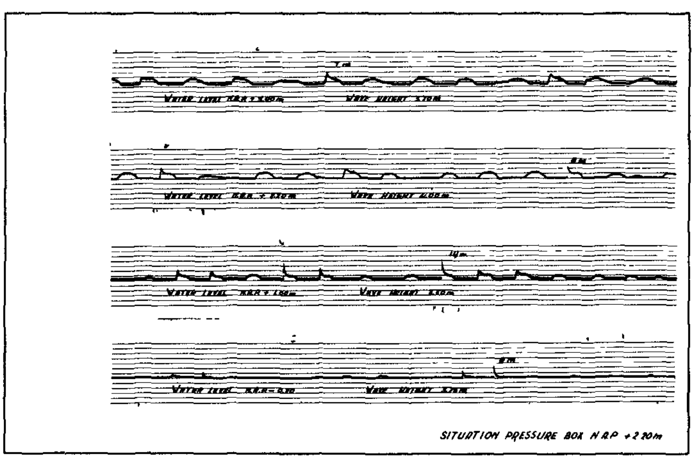

Fig. 6. Pressures on vertical face.

king wave in the case of a gate with forward inclined face.

During each of the three positions the height of the waves varied from 3.5 to 4 metres, while the water level varied from 0.2 m below to 3 m above Mean Sea Level (N.A.P.).

The tests have not advanced sufficiently far to determine th zone of impact and the magnitude of the areas struck by the pressu shocks.

2. APPARATUS AND PROCEDURE

The tests are being carried out in the 100 m long and 4 m wi wind and stream flume of the open-air laboratory at De Voorst (a subsidiary of the Delft Hydraulics Laboratory), in which flume wav can be generated by means of an air current blowing over the water surface, as well as by means of a wave machine.
A model of the sluice gate, on scale 40, was placed at the lower end of the flume and exposed to waves produced by the combined action of wind and wave machine.

Since a wave is a flow phenomenon with free surface, and sin the wave height can be expressed as a function of the square of th wind velocity, wave period and wind velocity in the model were tak in accordance with Froude's law.

The pressure fluctuations were measured by means of a pressu cell placed at different heights in the outer face of the gate. In order to eliminate the possibility of a reduction in magnitude of the pressure shocks resulting from a yield of the face of the gate the pressure cells were fixed in a block of concrete, thus obtaini a surface as rigid as possible.

As the pressure shocks are of a very short duration (increas from zero to maximum), viz. in the order of 1/500 sec., it is important that the natural frequency of the membrane of the pressure cell is very high compared to that of the forced displacement in order to minimize disturbance of the measurements by resonance. Fo this reason the membrane was given a frequency of 1.400 Hz.
The pressure cells are of the capacitance type. Displacement of th membrane causes a change in capacitance which is recorded on a film by means of an oscilloscope.

In the wave troughs the water level falls below the elevatio of the pressure cell, thus indicating on the film the zero positio (atmospheric pressure) of the pressure observations.

3. THE TEST RESULTS

a) GATE WITH FORWARD INCLINED FACE

During the model tests with forward inclined face (figs. 1 and 4) the pressure cell was placed at a height of 0.6 m above M.S.L.. The pressure shocks occurred with wave heights exceeding 2.5 m, independent of the water level.

As may be seen from fig. 4, pressure shocks were also recorded, with the pressure cell at + 0.6 m, at high water levels (3 m above M.S.L.). This is not much higher than the level of the trough of the wave. Evidently pressure shocks may be expected over the full height from wave trough to wave crest. The maximum pressure recorded was 1.8 x 10^5 newton/m^2 (25.5 psi).

Visually it was observed that the waves showed a beat phenomenon caused by the wind. The first breaking wave fills up a trough; the second one may cause a pressure shock. This is shown in fig. 4 by a prolongation of the wave pressure preceding the pressure shock ("a" is longer than "b").

Each pressure shock is followed by a "pressure ridge", which indicates that the shock occurs at the front of a wave: the pressure ridge is the remainder of the wave which is partly reflected and partly overtopping the gate.

Fig. 7 gives four pictures of such a breaking wave. The simultaneously recorded pressure did not show a shock, nor did the wave break in a trough (the pressure cell did not reach below the trough preceding this wave).

b) GATE WITH BACKWARD INCLINED FACE

During the model tests with backward inclined face (figs. 2 and 5) the pressure cell was placed at heights of 2.3 m and 3.5 m above M.S.L.

As may be seen from the pressure fluctuations shown in fig. 5, not a single shock was recorded. As a result of lesser reflection, the waves were in this case less steep than in the case of a gate with forward inclined face.

c) GATE WITH VERTICAL FACE

During the model tests with vertical face (figs. 3 and 6) the pressure cell was placed at a height of 2.2 m above M.S.L.

The pressure diagram of fig. 6 shows that the pressure fluctuatuions are about the same as in the case of a gate with forward inclined face.

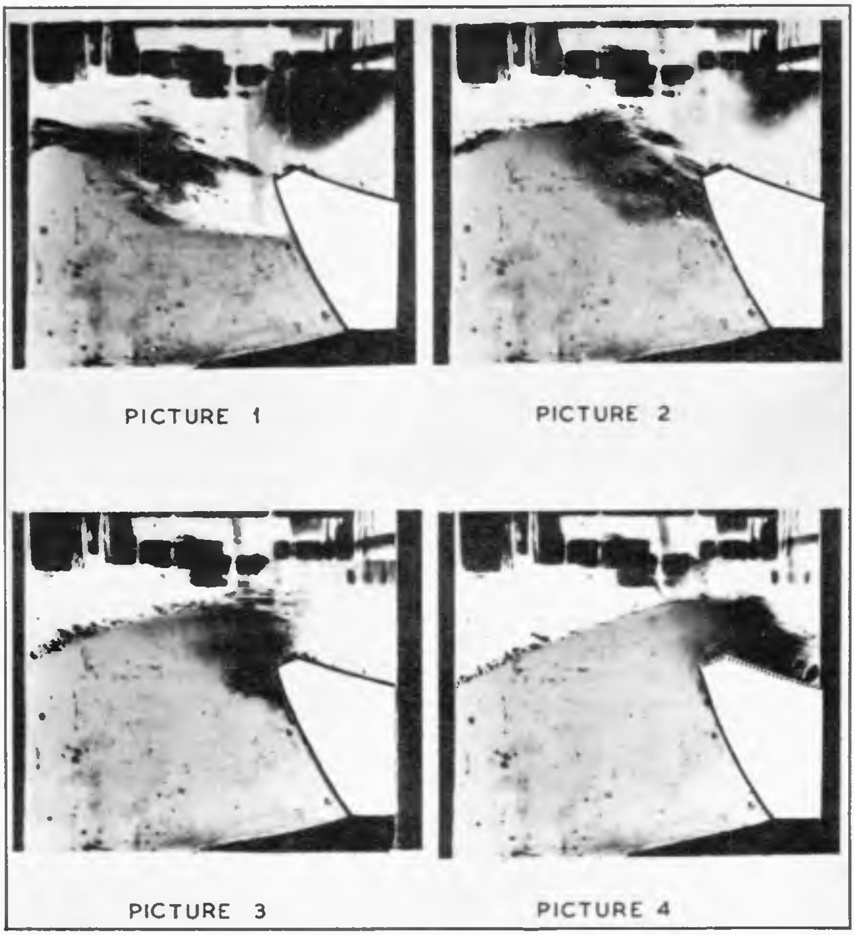

Fig. 7. Photograph of breaking wave.

The observations indicate a tendency of the pressure shocks to decrease in strength with increasing height of the water levels (3 m above M.S.L.). The crests of the breaking waves overtop the gate, resulting in less reflection, less steepness, and less breaking of the waves.

However, the possibility should not be excluded that (though with a lesser degree of frequency) the combination mentioned under a) may occur.

ACKNOWLEDGMENT

The model study is being carried out on the request of the "Rijkswaterstaat". Arrangement of the model, conducting of the tests, and interpretation of the test results, occurred in close cooperation with W.C. Bisschoff von Heemskerk and W.A. Venis of the Hydraulics Division of the "Deltadienst".

CHAPTER 46

CORRELATION OF WATER LEVEL VARIATIONS WITH WAVE FORCES ON A VERTICAL PILE FOR NONPERIODIC WAVES

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ABSTRACT

This paper describes the design and application of numerical transforms for the estimation of the field of motion associated with irregular, nonperiodic surface waves from measured serial sequences of water level at a fixed point. The design of these transforms is based upon the linear theory for long-crested waves. The method is applied in the analysis of wave forces exerted upon a vertical circular cylinder, where the measured reaction is considered to be expressible as a linear combination of two independent functions of time. One of these functions depends (nonlinearly) upon the velocity field, the other depends (linearly) upon the acceleration field. The covariance of these functions with the measured reaction allows a direct means of evaluation of the drag and inertial coefficients for the cylinder.

1. INTRODUCTION

In the analysis of records of forces exerted upon structures by ocean waves it is desirable to have an accurate and objective means of deducing the field of fluid motion so as to provide the necessary kinematic information for a reliable evaluation of the drag and inertial coefficients associated with such forces. In field tests, direct measurement of the distribution of fluid motion associated with waves is not as yet a feasible means of providing the detailed information desired. Even in laboratory wave tests the direct measurement of particle velocities is difficult. On the other hand, direct measurement of water level variations at a fixed point can be carried out with relative ease, in both laboratory and field tests. If the waves are simple harmonic and periodic, or closely approximate this condition, then the amplitude and period of the surface variations, together with the known depth of water, will allow the estimation of the desired particle velocity and acceleration field through the use of classical wave theory. The orbital currents so deduced plus the simultaneous records of wave forces on the object in question will allow an estimation of the drag and inertial coefficients. In

[1] Contribution from the Department of Oceanography and Meteorology, Agricultural and Mechanical College of Texas, Oceanography and Meteorology Series No. 101.

effect one is really correlating a record of water level variations with wave force variations through the medium of the wave theory and deducing therefrom two linearly independent regression coefficients. This method has precedent in the studies carried out in the laboratory at the University of California (Morison, et.al., 1950) and has been utilized in a number of later laboratory studies. Controlled conditions of wave generation allow the attainment of nearly simple harmonic waves and the foregoing method of analysis is therefore ideally suited to measurements carried out in the laboratory.

In most field studies on the other hand it is generally the case that the waves are neither simple harmonic nor periodic. Instead the waves are characterized by a continuous spectrum which covers a broad range of periods The resulting water level variations and serial sequences of forces are highly complicated and constitute what might be termed filtered noise. With the exception of certain cases of very regular swell, as may occur at times on the west coast of the United States or the Atlantic coast of Europe during the northern summer, it is virtually impossible to pick out a characteristic period and amplitude for the waves, other than from a statistical standpoint. The statistical mean "period" and mean amplitude for a given wave train are useful from the standpoint of gross classification of the waves, but these quantities are hardly sufficient from the standpoint of the details or even the statistics of the fluid motion or pressure field associated with the wave train in a given depth of water. It is known for example that the mean "periods" of waves as deduced from pressure measurements at the bottom in shallow water do not coincide with mean "periods" as deduced from direct surface measurements. Furthermore, if the mean amplitude of the pressure variatio is converted to an equivalent amplitude of water level variation based upon the mean "period" of the pressure variations, the equivalent amplitude is not the same as the mean amplitude of the measured surface waves, unless the waves possess a very narrow spectrum. The difference in wave statistics deduced from pressure gages and from direct surface measurements is borne out strikingly in the recent studies at Berkeley (Wiegel and Kukk, 1957).

Much of the discrepancy can be accounted for on the basis of the continuous nature of the spectrum of the waves and proper utilization of the wave theory in the conversion of pressure records to equivalent surface records or visa versa [see for example Fuchs (1952)]. The conversion of water level variations to pressure variations at the bottom can be effected by means of a special numerical filter which is designed on the basis of the linear wave theory. If we regard the surface profile as the resultant of many simple harmonic waves of different amplitudes, periods and relative phases, then the filter when properly constructed acts upon each of those components simultaneously and adjusts the amplitude according to the period of each individual component without altering the relative phase. The output of the filte is the resultant of all the adjusted wave components. The numerical filter for pressure simply simulates the hydrodynamic filtering as predicted by the linear theory. The advantage of the system is that it can be utilized for the most complex wave records.

CORRELATION OF WATER LEVEL VARIATIONS WITH WAVE FORCES ON A VERTICAL PILE FOR NONPERIODIC WAVES

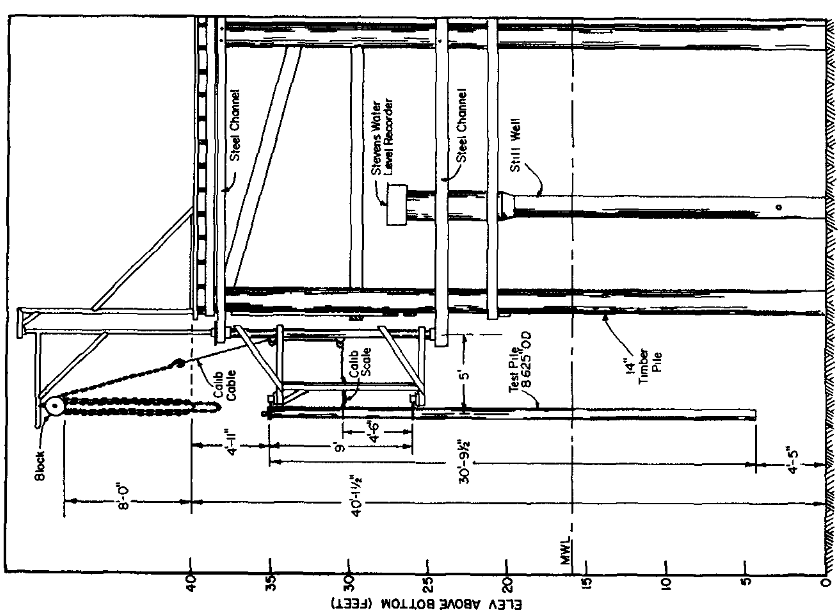

Fig. 1. Scaled drawings of wave force installation at Sun Oil Company Pier, Caplen, Texas. The steel test pile was located on the southeast corner of the platform one-half mile from shore. The directional orientation of the pile support could be varied through an angle of 180°.

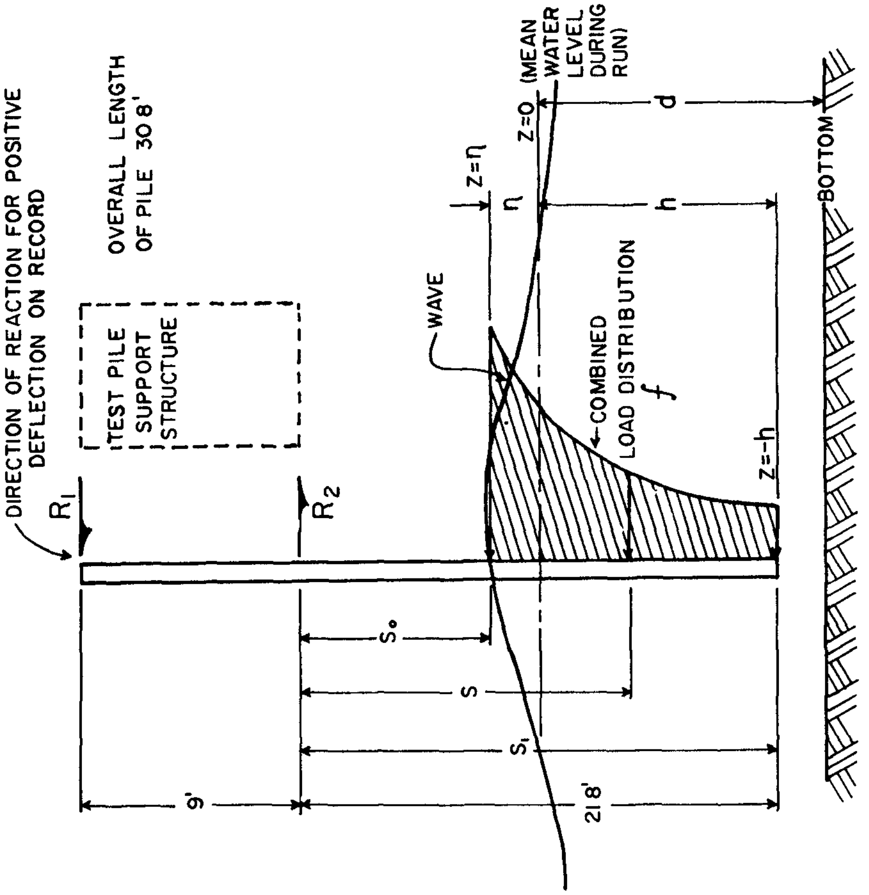

Fig. 2. Schematic of loads on test pile.

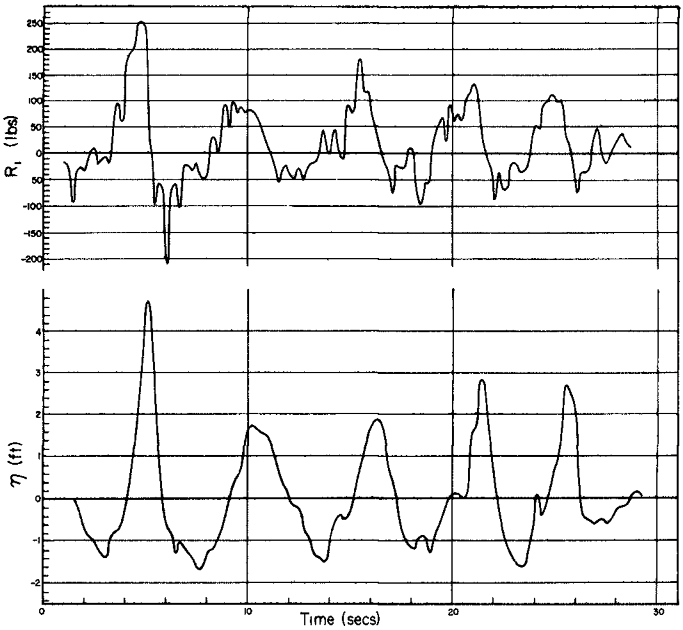

Fig. 3. Sample of serial sequences of η and reaction R_1 (Run 7).

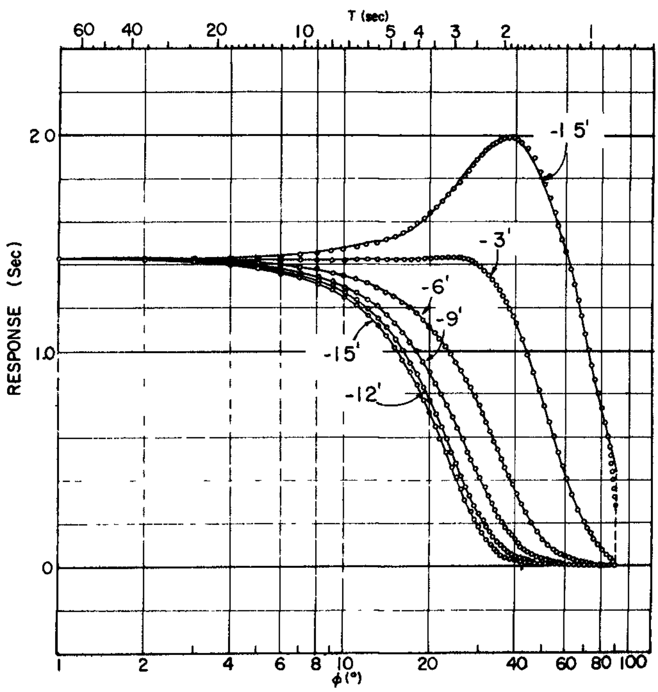

Fig. 4. Response Diagram for u at Subsurface Levels. Full curves are response from linear wave theory, circled points are derived from Eq. (36) using the a_n values from Table II. Vertical dashed line is the design "cut-off" position (ϕ=90° or T=0.8 sec). For a simple harmonic wave of period T and amplitude A, the amplitude of velocity u at depth z is equal to A times the response factor.

Numerical filters or transforms can also be constructed for ascertaining the serial sequence of velocity and acceleration of fluid at a given depth using the measured water level variations as input. In the case of acceleration the numerical transform must be such as to allow for a shift in phase for each component in the water level sequence. The following report discusses the design and application of such transforms. A specific set of measurements of waves and wave forces on a vertical cylinder in the Gulf of Mexico is utilized to illustrate the method of analysis of the field of motion and of the drag and inertial coefficients deduced from this field of motion and the measured forces.

2. WAVE FORCE FIELD EXPERIMENT

Measurements of wave forces on a smooth vertical pile of 8.625 inches diameter associated with irregular waves of about 2 to 4 feet significant height and about 3.5 to 5 seconds mean "period" were carried out at the Sun Oil Company pier at Caplen,[2] Texas, as part of a project sponsored by the Bureau of Yards and Docks. A scaled drawing of the installation is given in Fig. 1 . The test pile was supported at two positions nine feet apart by means of "U" bolts attached to flexure bars. The pile was submerged to a nominal depth of about 12 feet in sea water of about 16 feet total depth at mean tide level. Measurements of the reaction R_1 , at the upper level (see Fig. 2) were obtained simultaneously with movie film records of the water level variations at the pile position, the pile being marked in one foot intervals. The pile support could be rotated into the waves so as to obtain the maximum thrust normal to the instrumented flexure bars. Details of the measuring system, the calibration of the system and the listings of basic data are contained in technical reports of the project (Reid, 1954, 1956). Essentially the reaction was measured by means of the calibrated output of SR-4 strain gages mounted on the flexure bars which supported the pile. Unfortunately only the upper level measurements were satisfactory so that it was not possible to ascertain experimentally both the total wave load and the effective center of action of the wave load by two separate reactions R_1 and R_2 as originally planned. However, the measurement of R_1 alone can be utilized in the estimation of the drag and inertial coefficients. Measurements of wind velocity, wave direction, tide elevation and mean surface currents were obtained as supplementary information.

A typical sequence of measurements of water level and reaction are shown in Fig. 3. Positive R_1 represents reaction in the direction of wave propagation (see Fig. 2). The water level anomaly, η , was estimated from the film records to the nearest 0.1 foot. The relative error in water level anomaly is estimated as about $\pm$ 0.05 foot and that of R_1 as about $\pm$ 5 lbs.

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[2] Caplen is located about 30 miles east of Galveston on the Bolivar Penninsula. The installation was located at the end of the pier which extends about 1/2 mile into the Gulf of Mexico.

However, the zero reference for R_1 is subject to a much larger error, and is considered as one of the unknowns in the analysis. All tabulations of η and R_1 were carried out at intervals of 0.2 second from the original records Time checks were provided in the film records to insure proper interpolation and alignment with the records of R_1 . A total of 570 seconds of record consisting of 18 separate runs were analyzed. The longest single run was about 46 seconds. The range of wind speeds was 10 to 30 mph during the different series of runs.

A schematic of the loads on the test pile accompanying the passage of a wave is indicated in Fig. 2. In the absence of vibration of the pile and its supporting platform, a quasi-static balance of the moments of load on the tes pile must exist. Taking moments about the position of the lower support give:

$$b R_1 (t) = \int_{s_0}^{s_1} s\, f(s,t)\, ds \qquad (1)$$

where s is the vertical distance below the lower support, b is the vertical distance between the two supports and f is the wave load per unit length of pile at position s and time t. It is assumed that the moment induced by the "U" bolt connection at each support is negligible. The static balance of moments should be adequate as long as the spectral energy associated with η (t) is confined to periods in excess of the natural periods of vibration of th test pile and/or supporting structure, such that resonant conditions are not excited. If this is not the case then the platform and pile accelerations can b(come significant and should therefore be taken into account if the records of R_1 are used directly. An alternative is to apply Eq.(1) to records of R_1 from which the energy associated with vibrational resonance or near resonar has been effaced, provided that the same range of periods are suppressed in the estimated wave load f. In the measurements utilized here, vibrational periods were present and the suppression of these vibrations was carried ou objectively by use of a numerical filter which is described in a later section. The vibrational periods were approximately 0.5 and 1.1 seconds. Evidence of these periods can be seen in the unfiltered record of R_1 shown in Fig. 3. The amplitudes associated with these periods in the R_1 record are disproportionately large as compared with the relative energy associated with thes same periods in the simultaneous water level record. Consequently unless these vibrations are effaced from the record, it is apparent that significant errors in the estimates of drag and inertial coefficients associated with f can result.

3. THE WAVE LOAD REGRESSION FORMULA

Following Morison, et.al. (1950), it is presumed that the wave load per unit length on the vertical cylinder can be expressed in the form

$$f = C_D \frac{w}{2g} D\,|v|\,v + C_M \frac{w}{g} \frac{\pi}{4} D^2 \dot{v} \tag{2}$$

where v and $\dot{v}$ are respectively the horizontal components of fluid velocity and acceleration in the vicinity of the pile at level s and time t, w is the specific weight of sea water (64 lbs/cu ft), D the pile diameter, g the acceleration due to gravity, and C_D and C_M are the dimensionless drag and inertial coefficients respectively. The latter coefficients are regarded as constants for any particular sequence of waves. In this sense Eq.(2) is really a regression formula to which the observed data are to be fitted in such a way as to give the best estimate of f in a least squares sense. However, Eq.(2) is not directly applicable since both f and v are unknown.

The field of velocity and acceleration can be deduced from the observed sequence of water level anomaly and a knowledge of the steady currents upon which the waves are superimposed. The measured reaction on the other hand gives an estimate of the moment of the total wave load according to Eq.(1). Using relations (1) and (2) jointly it is possible, in the absence of vibrations, to represent the reaction R_1 in the form

$$R_1(t) = C_D F_1(t) + C_M F_2(t) \tag{3}$$

where $F_1(t)$ and $F_2(t)$ are defined by

$$F_1 \equiv \frac{wD}{2g} \left\{ \left[\frac{L-h}{b} - 1\right] \int_{-h}^{\eta} |v|\,v\,dz - \frac{1}{b} \int_{-h}^{\eta} z\,|v|\,v\,dz \right\} \tag{4}$$

and

$$F_2 \equiv \frac{\pi}{4} \frac{wD^2}{g} \left\{ \left[\frac{L-h}{b} - 1\right] \int_{-h}^{\eta} \dot{v}\,dz - \frac{1}{b} \int_{-h}^{\eta} z\,\dot{v}\,dz \right\} \tag{5}$$

where L is the pile length, h is the depth of submergence of the pile below still water level, η is the instantaneous elevation of the sea surface above still water level, and z is the vertical coordinate taken positive upwards from still water level. The value of h of course depends upon the state of the tide.

The velocity v can be expressed in the form

$$v = U + u \tag{6}$$

where U is the steady current at level z and u is the component of motion at z, t associated directly with the waves. It is evident that there is no contribution of the steady current to the acceleration so that $\dot{v} = \dot{u}$. For simplicity in notation we will hereafter replace the velocity product $|v|\,v$ by p. Thus

$$p \equiv |U + u| \cdot (U + u) \quad , \tag{7}$$

which is directly related to the drag pressure, but has the units ft^2/sec^2. Note that p has the sign of the sum $U + u$, and that the mean value of p over a long time interval is not zero even though the average of u, $\dot{u}$ and η is zero. Because of this we should expect to find that the mean value of R_1 differs from zero.

In view of the fact that u enters in a quadratic manner in Eq.(4) it is necessary to evaluate u at several levels and employ an appropriate summati to approximate the integrals. Let the constant h_o represent a nominal depth of submergence (12 feet for the present example) and consider the range 0 tc $-h_o$ divided into four equal intervals of size Δz. The integrals in Eq.(4) cai then be approximated as follows:

$$\int_{-h}^{\eta} |v|\,v\,dz \doteq \frac{\Delta z}{3}\,[p_o + 4p_1 + 2p_2 + 4p_3 + p_4]$$

$$+ (h - h_o)p_4 + \eta p_o + \frac{1}{2}\,\eta^2\,\frac{\Delta p_o}{\Delta z} \tag{8}$$

and

$$- \int_{-h}^{\eta} z\,|v|\,v\,dz \doteq \frac{4}{3}(\Delta z)^2\,[p_1 + p_2 + 3p_3 + p_4]$$

$$+ \frac{1}{2}(h^2 - h_o^2)\,p_4 - \frac{1}{2}\,\eta^2\,p_o - \frac{1}{3}\,\eta^3\,\frac{\Delta p_o}{\Delta z} \tag{9}$$

where the subscripts indicate the elevation in the sense that p_j is the value o p at $z = -j\Delta z$ (relative to still water level), and Δp_o is defined by

$$\Delta p_o \equiv p\,[\frac{\Delta z}{2}, t] - p\,[-\frac{\Delta z}{2}, t] \quad . \tag{10}$$

Simpson's rule has been applied for the interval $-h_o$ to 0 in the above approximations. A generalization of this procedure for any even number of intervals is easily made.

In the case of Eq.(5), the acceleration enters linearly and it is possible to evaluate the major portion of the integrals (for the range $-h_o$ to 0) directly from the water level variations as we shall see presently. Consequently the complete integrals in Eq.(5) can be approximated as follows:

$$\int_{-h}^{\eta} \dot{v}\,dz \doteq \int_{-h_o}^{\eta} \dot{u}\,dz + (h - h_o)\,\dot{u}_4 + \eta\,\dot{u}_o + \frac{1}{2}\,\eta^2 \left[\frac{\partial \dot{u}}{\partial z}\right]_o \tag{11}$$

$$-\int_{-h}^{\eta} z\,\dot{v}\,dz \doteq \int_{-h_o}^{\eta} |z|\,\dot{u}\,dz + \frac{1}{2}(h^2 - h_o^2)\,\dot{u}_4 - \frac{1}{2}\,\eta^2\,\dot{u}_o - \frac{1}{3}\,\eta^3 \left[\frac{\partial u}{\partial z}\right]_o \tag{12}$$

where the subscripts have the same meaning as in Eqs.(8) and (9). Thus $\dot{u}_4$ is the acceleration at $z = -h_o$. The last term in each of the Eqs.(8) to (12) is a secondary correction term to take into account the effect of the gradient of p and $\dot{u}$ near the surface.

Eq.(3) is a linear regression equation for R_1 in terms of the linearly independent functions F_1 and F_2. The function F_1 can be expressed as a linear combination of p_j functions with the coefficients of some of the terms being polynomials in η. The function F_2 can be expressed as a linear combination of $\dot{u}_n$ and linear integral operations on $\dot{u}$ but again the coefficients of some of the terms are polynomials in η. Since u and $\dot{u}$ can be expressed in terms of the sequence $\eta(t)$, it follows that F_1 and F_2 depend primarily upon the sequence η. In addition F_1 depends upon U(z) and both functions depend upon the slowly changing value of h. The functional dependence of F_1 and F_2 on the sequence $\eta(t)$ is nonlinear and in the case of the "drag" function, F_1, the dependence on $\eta(t)$ is strongly nonlinear. This implies that the spectrum of the function $\eta(t)$ cannot be converted to the spectrum of $F_1(t)$ by a simple linear transformation. Some of the spectral energy at and near frequency ω in the record of $\eta(t)$ will show up as energy with frequency at or near 2ω and zero frequency in the spectrum of $F_1(t)$. Furthermore there will be interaction of the spectral components such that frequencies of absolute value ω_1 and ω_2 in the record of $\eta(t)$ can produce frequencies of $\omega_1 + \omega_2$ and $|\omega_1 - \omega_2|$ in the spectrum of $F_1(t)$. This is also true in respect to the "inertial" function $F_2(t)$, but the amount of nonlinear dispersion of energy in the spectrum is less pronounced since the primary contribution to the function is from linear transformations of η, through $\dot{u}$. The possibility of producing low frequencies $|\omega_1 - \omega_2|$ in either function, and particularly in $F_1(t)$, from high frequencies of nearly the same value, is a point to be borne in mind in respect to the final analysis of $F_1(t)$ and $F_2(t)$.

It is clear that once $F_1(t)$ and $F_2(t)$ are determined, the regression coefficients C_D and C_M in Eq.(3) can be evaluated by a suitable least squares fit procedure employing the measured sequences of R_1 . This matter is discussed in some detail in section 8.

4. NON-PERIODIC WAVES

Any wave record of finite duration, extending from time t_1 to t_2, can be represented in the form of a Fourier integral as follows

$$\eta(t) = \int_0^\infty M(\omega)\cos\left[\omega t - \Theta(\omega)\right] d\omega \tag{13}$$

where the functions $M(\omega)$ and $\Theta(\omega)$ can be evaluated from the relations

$$M\cos\Theta = \frac{1}{\pi}\int_{t_1}^{t_2} \eta(t)\cos\omega t\, dt \tag{14}$$

$$M\sin\Theta = \frac{1}{\pi}\int_{t_1}^{t_2} \eta(t)\sin\omega t\, dt \quad . \tag{15}$$

The quantities $M(\omega)$ and $\Theta(\omega)$ are real functions of the frequency parameter ω and jointly characterize the finite sequence of η at some fixed location. The quantity $E = M^2(\omega)/(t_2 - t_1)$ represents the energy spectral function for the finite η sequence and has the important property

$$\int_0^\infty E(\omega)\, d\omega = \overline{\eta^2} \tag{16}$$

where the bar indicates a time average for the period t_1 to t_2 . This is a direct result of Parseval's theorem in connection with Fourier Integrals. It follows that one system of evaluating the energy spectrum is to subject the record $\eta(t)$ to narrow band pass filters [3] and evaluate the mean square value of the output of each filter.

The waves represented by (13) are not periodic. However, in the spec case where the major portion of the spectral energy is concentrated in a nar band centered at some modal frequency ω_o , the disturbance η manifests itself in the form of an amplitude modulated wave train with a quasi-periodic carrier wave of mean frequency ω_o . The statistical properties of waves

[3] It is implied here that the filter leaves the energy unaffected for those fre quencies in a small band $\Delta\omega$ centered at frequency ω and eliminates the energy associated with all other frequencies.

whose spectrum is narrow, and for which the phase parameter Θ is random, has been studied analytically by Longuet-Higgins (1952). However, in many cases the wave spectrum is not narrow; this is particularly true of wind waves in the process of generation. The records for waves possessing a broad spectrum resemble filtered noise and do not possess any distinct periodicity (see Fig. 3). However, the record can always be represented by an equation of the type (13).

If the waves are long-crested, and of small amplitude then it follows [4] from the linear theory of irrotational motion associated with waves in water of constant depth that

$$u(z, t) = \int_0^\infty M(\omega)\left\{\omega \frac{\cosh k(z+d)}{\sinh kd}\right\}\cos[\omega t - \Theta(\omega)]\, d\omega \qquad (17)$$

and

$$\dot{u}(z, t) = -\int_0^\infty M(\omega)\left\{\omega^2 \frac{\cosh k(z+d)}{\sinh kd}\right\}\sin[\omega t - \Theta(\omega)]\, d\omega \qquad (18)$$

where k is the wave number and is related to the frequency ω by the formula

$$\omega^2 = gk \tanh kd \ . \qquad (19)$$

The evaluation of k in terms of ω is facilitated by the use of Wiegel's Tables (1954). There the notation $T = 2\pi/\omega$ and $L = 2\pi/k$ for period and wave length is employed.

Formulas (17) and (18) hold provided that the mean square slope which is specified by

$$\int_0^\infty k^2 E(\omega)\, d\omega \qquad (20)$$

is sufficiently small compared with unity, and provided that the beam width of the actual directional spectrum of the waves is small. This is likely the case for swell but may be somewhat doubtful for wind waves. For waves or swell near shore the directional spread of the spectra is narrowed by refraction but steepness is enhanced. There is no general way of taking the nonlinear effects associated with large steepness into account for irregular waves, except perhaps by solving the hydrodynamic equations numerically for the particular case at hand. Directional effects of the spectra associated with short-crested waves can be taken into account in the linear theory but in order to be of any use it is required that supplementary information in

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[4] See for example Lamb (1945). Eqs.(17) and (18) apply at the position where η is measured.

regard to water level variations be known. A two-dimensional grid of wave gages could provide the necessary information required in the detailed analy of short-crested waves. However, in the present analysis we limit our considerations to deductions from $\eta(t)$ at a single position. It is therefore cle: that we are limited to the theory of long-crested waves.

Three quantities of concern in the evaluation of F_1 and F_2 in additio to u and $\dot{u}$ are the integrals of $\dot{u}$, $|z|\dot{u}$ and the gradient of the acceleration at the surface [see Eq.(11) and (12)]. The last of these quantities is given l

$$\left[\frac{\partial \dot{u}}{\partial z}\right]_o = - \int_o^\infty M(\omega) \left\{\omega^2 k\right\} \sin[\omega t - \Theta(\omega)]\, d\omega \tag{21}$$

and the integrals in question can be shown to be given by

$$\begin{aligned} I_1 &\equiv \int_{-h_o}^{o} \dot{u}(z,t)\, dz \\ &= - \int_o^\infty M(\omega) \left\{ g\, \frac{\sinh kd - \sinh k(d - ho)}{\cosh kd} \right\} \sin[\omega t - \Theta(t)]\, d\omega \end{aligned} \tag{22}$$

and

$$\begin{aligned} I_2 &\equiv \int_{-h_o}^{o} |z|\dot{u}(z,t)\, dz \\ &= - \int_o^\infty M(\omega) \left\{ g\, \frac{\cosh kd - kh_o \sinh k(d - h_o) - \cosh k(d - h_o)}{k \cosh kd} \right. \cdot \\ &\qquad \sin[\omega t - \Theta(t)]\, d\omega. \end{aligned} \tag{23}$$

It is possible to utilize Eqs.(17), (18), (21), (22) and (23) directly in tl evaluation of the pertinent quantities. However, this is difficult because of t nature of the integrals, but even more important is the fact that for each wa record the two integrals defining $M(\omega)$ and $\Theta(\omega)$ must be evaluated. Fortunately a more direct approach exists which bypasses the necessity of the detailed evaluation of the spectral functions, yet is capable of yielding essentially the same results as those indicated implicitly above. However, the foregoing material is an essential step in arriving at the results to follo The only information required in regard to the wave spectra is an estimate (the effective range of frequencies containing the majority (say 95 percent) o the spectral energy.

5. NUMERICAL TRANSFORMS OF $\eta(t)$

For the practical evaluation of the quantities u, $\dot{u}$, I_1, I_2 or $\partial \dot{u}/\partial z$ we can make use of one or the other of the following linear transforms of $\eta(t)$:

$$G_s[\eta(t)] \equiv a_o \eta(t) + \sum_{n=1}^{N} a_n [\eta(t+n\tau) + \eta(t-n\tau)] \tag{24}$$

$$G_a[\eta(t)] \equiv \sum_{n=1}^{N} b_n [\eta(t+n\tau) - \eta(t-n\tau)] \tag{25}$$

where n and N are integers, τ is a fixed time interval at which discrete values of η are known and a_n and b_n are coefficients which depend upon the type of output G(t) desired. We will refer to the operation $G_s[\eta(t)]$ as a symmetrical linear transform of order N; while $G_a[\eta(t)]$ is an anti-symmetrical transform of order N. It is possible of course to construct an asymmetrical transform by combination of the above two operations but in the present development this general type is not needed. It will be noted that the output depends not only upon the coefficients but is also dependent upon the order and the size of the mesh interval τ.

Suppose the input $\eta(t)$ is given by Eq.(13). The output for operation $G_s[\eta(t)]$ is readily shown to be

$$G_s(t) \equiv \int_0^{\infty} R_s(\omega) M(\omega) \cos[\omega t - \Theta(\omega)] d\omega \tag{26}$$

where

$$R_s(\omega) \equiv 2\left[\frac{a_o}{2} + \sum_{n=1}^{N} a_n \cos n\omega\tau\right] \tag{27}$$

On the other hand the output of operation $G_a[\eta(t)]$ for the same input is

$$G_a(t) = -\int_0^{\infty} R_a(\omega) M(\omega) \sin[\omega t - \Theta(\omega)] d\omega \tag{28}$$

where

$$R_a(\omega) = 2\left[\sum_{n=1}^{N} b_n \sin n\omega\tau\right] \tag{29}$$

It is therefore evident that the symmetrical numerical transform produces no phase distortion in the output, relative to the input. On the other hand the anti-symmetrical operation alters the phase of each component in the spectrum of the input by $\pi/2$ radians, so that the output leads the input (if b_n are positive). In both cases the amplitude spectrum is altered compared with the spectrum of the input, the amount of alteration being specified by the spectral response factors R_s or R_a. These response factors are functions of ω as determined by the parameters τ and N and the coefficients a_n or b_n.

It is evident that the symmetrical transform operation $G_s[\eta(t)]$ can be useful in the estimation of u(z, t) provided that the coefficients a_n can be so chosen that the response factor $R_s(\omega)$ will approximate the desired response according to Eq.(17). It is also evident that the antisymmetrical operation $G_a[\eta(t)]$ can be of value in estimating $\dot{u}(z, t)$ and the other quantities closel associated with the acceleration, provided that the coefficients b_n can be appropriately chosen so as to produce the desired responses.

Consider the problem of matching $R_s(\omega)$ with an even function $R_s'(\omega)$ for $|\omega| \leqslant \pi/\tau$. Since the response factor $R_s(\omega)$ is expressed as a finite series of cosine functions which are orthogonal in the interval $-\pi/\tau \leqslant \omega \leqslant \pi$ it is readily shown that $R_s(\omega)$ will represent the best approximation of $R_s'($ in the least squares sense for $|\omega| \leqslant \pi/\tau$ if

$$a_n = \frac{1}{\pi}\int_0^{\pi} R_s'(\omega)\cos n\phi\, d\phi \quad , \tag{30}$$

where $\phi = \omega\tau$ radians and n = 0, 1, 2, ...N. The coefficients a_n are therefore simply the Fourier coefficients (up to n = N) in the cosine expansion of the function $R_s'(\omega)/2$ for the interval $-\pi/\tau \leqslant \omega \leqslant \pi/\tau$.
It is evident that the accuracy of the approximation of $R_s'(\omega)$ by $R_s(\omega)$ for $|\omega| \leqslant \pi/\tau$ increases as N increases. Furthermore, the range of representation of $R_s'(\omega)$ by $R_s(\omega)$ is increased by allowing τ to decrease. It will be noted of course that the operational response $R_s(\omega)$, as given by Eq (27) is periodic in respect to ϕ with a period equal to 2π radians. If $G_s[\eta(t)]$ is to be an exact predictor of a function whose amplitude spectrur is $R_s'(\omega) M(\omega)$, then τ should be chosen so that $M(\omega)$ is negligible for $|\omega| > \pi/\tau$ and N should be very large. Furthermore, the range of influence of the numerical operator, $2N\tau$, should be large in order that the low frequencies in the spectrum of $\eta(t)$ are adequately sampled by the numerical operator.

In a similar way $R_a(\omega)$ as defined by Eq.(29) will approximate an od function $R_a'(\omega)$ in the least squares sense for the range $-\pi/\tau < \omega < \pi/\tau$ provided that

$$b_n = \frac{1}{\pi}\int_0^{\pi} R_a'(\omega)\sin n\phi\, d\phi \tag{31}$$

where n = 1, 2, 3 ...N. As in the case of the symmetrical transform resp the accuracy of the representation of $R_a'(\omega)$ by $R_a(\omega)$, using the coefficie given by Eq.(31), increases as N increases, and the resolution in respect t frequency is increased if τ is decreased.

6. THE VELOCITY AND ACCELERATION PREDICTORS

In the above discussion it was tacitly assumed that no errors exist in the record of $\eta(t)$. Actually it is known that the tabulations of $\eta(t)$ in the present experiment can be in error by $\pm$ 0.05 foot due to rounding off of values to the nearest 0.1 foot. Such errors are random and tend to show up at all frequencies in the energy spectrum for $\eta(t)$. The highest detectable frequency in a discrete sequence with time interval τ is π/τ, which corresponds to the limit in range of meaningful response in regard to the transforms $G_s[\eta(t)]$ or $G_a[\eta(t)]$. A high frequency $\omega > \pi/\tau$ will show up in the discrete sequence as the lower frequency $\omega' = (2\pi/\tau) - \omega$ and because of the periodic nature of the response functions of the numerical operators, the response for ω is equivalent to the response for ω'. For this reason we can confine our attention to frequencies less than π/τ.

If the desired response is such that it approaches zero with increasing ω then there is no difficulty encountered in respect to high frequency "noise" created by errors in the input. All that is required is that τ is sufficiently small so that the response function is nearly zero for frequencies at or near π/τ. The response function for $u(z, t)$, as given by the quantity in braces in Eq.(17), behaves in the above manner for $z < 0$. However, for $z \geqslant 0$, the response function increases without limit as ω increases, and it is difficult to simulate this response accurately even over a finite range of frequencies, unless N is taken very large and τ very small. On the other hand, if the response indicated by the hydrodynamical theory is accurately reproduced at high frequencies, then the error "noise" is amplified beyond reasonable bounds and masks the meaningful part of the output. This unwanted amplification of noise can be subdued by filtering out high frequencies, but only at the expense of eliminating some of the meaningful output and thereby introducing error associated with loss of detail. As in many problems of this sort (e.g., communication theory) a compromise in the separation of signal from noise is necessary. The optimum filter would be that for which the combined error in the output has a minimum mean square value. However, the selection of the optimum filter requires a knowledge of the spectrum of the noise as well as that of the signal [see for example, Wiener (1950)].

The procedure employed in the present analysis is much less sophisticated and suffers from being somewhat arbitrary. A cut-off frequency ω_c is defined such that the design response is zero for all frequencies in the range $\omega_c < \omega < \pi/\tau$. This implies that a_n and b_n are to be evaluated from the relations

$$a_n = \frac{1}{\pi} \int_0^{\omega_c \tau} R_s'(\omega) \cos n\phi \, d\phi \qquad (32)$$

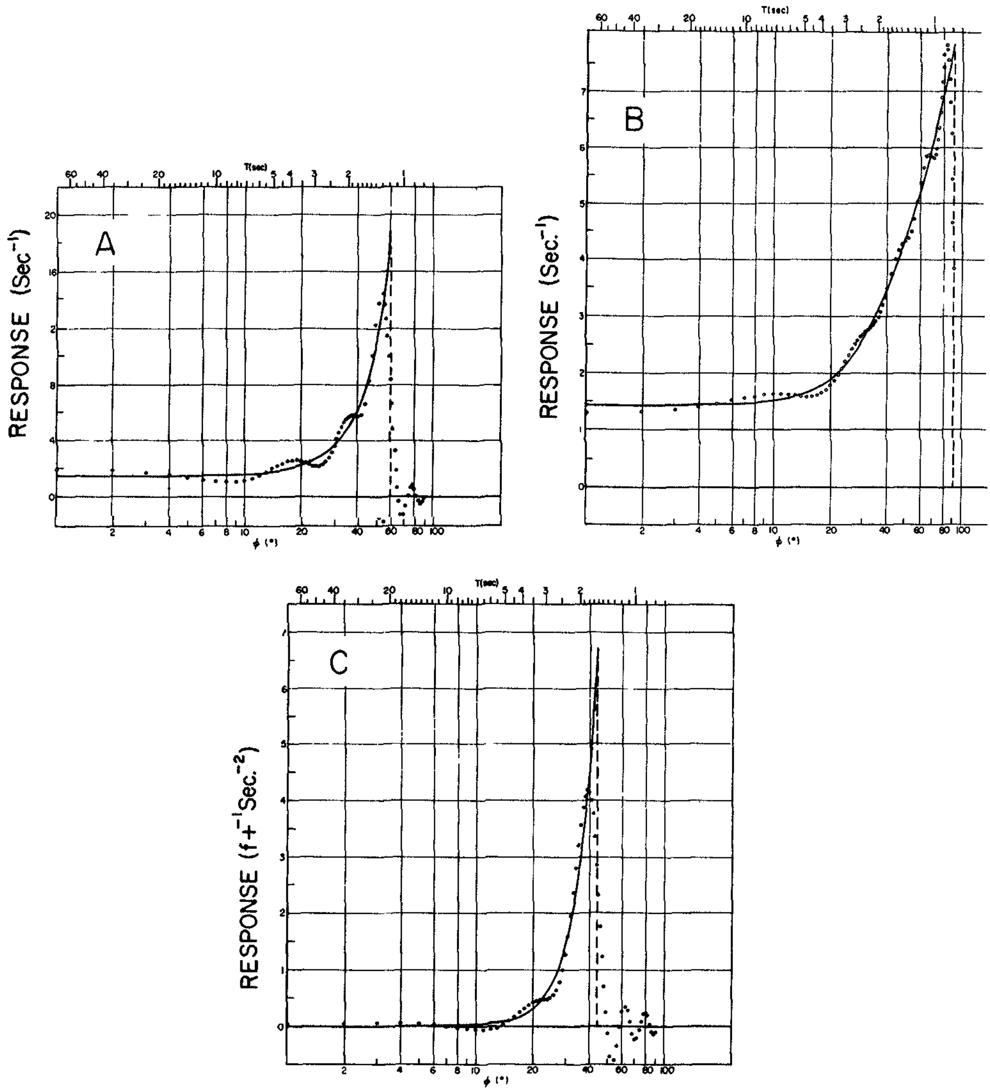

Fig. 5. Response Diagrams for u at + 1.5 feet (A), u at mean water lev (B), and $\partial u/\partial z$ at mean water level (C). Full curves are from linear wave theory, circled points are from Eq. (36) for velocities and Eq. (3 for graph (C). Design "cut-off" shown by vertical dashed line.

$$b_n = \frac{1}{\pi} \int_0^{\omega_c \tau} R_a'(\omega) \sin n\phi \, d\phi \tag{33}$$

where it is understood that $\omega_c < \pi/\tau$. We stipulate that the selection of ω_c for a particular design response in the frequency range $|\omega| \leqslant \omega_c$ should satisfy the following conditions:

(A) The mean square value of the fitted response, as given by (27) or (29), should not deviate from the mean square value of the design response by more than five per cent, for the range $0 \leqslant \omega \leqslant \omega_c$;

(B) The contribution of that portion of the energy spectrum of $\eta(t)$ for which $\omega > \omega_c$ should not exceed five per cent of the total spectral energy.

The above conditions also place some restraint upon the selection of τ and N.

Condition (B) can be stated more specifically in the form

$$\int_{\omega_c}^{\infty} M^2(\omega)d\omega < 0.05 \int_0^{\infty} M^2(\omega)d\omega \quad . \tag{34}$$

This condition can be tested by comparing the mean square value of η with the mean square value of a filtered counterpart of η, where the filter passes only those frequencies less than ω_c. We will return to a further discussion of this in section 7.

In the evaluation of the functions, $F_1(t)$ and $F_2(t)$ in Eq.(3) we need u(t) at seven different levels, $\dot{u}(t)$ at two levels, $\partial\dot{u}/\partial z$ at the surface and the integrals $I_1(t)$ and $I_2(t)$, as defined in Eqs.(22) and (23). Consequently seven different transforms of type $G_s[\eta(t)]$ are required for estimating the seven velocity functions and five different transforms of type $G_a[\eta(t)]$ are required for the accelerations and the gradient and integrals thereof. The desired responses for these transforms are given by the expressions in braces in Eqs.(17), (18), (21), (22) and (23). These functions are given in column three of Table I. It will be recalled that k is related to ω by Eq.(19); this has been employed in arriving at the particular expressions for the response functions given in Table I. The final values of cut-off period, $2\pi/\omega_c$, used in the evaluations of the transform coefficients are indicated in the table. These correspond to $\phi_c = \omega_c\tau$ as indicated in the last column (expressed in degrees) for $\tau = 0.2$ second.

The values of a_n for the seven different velocity predictors and b_n for the five different acceleration predictors were evaluated numerically by Simpson's rule from Eqs.(32) and (33), using an interval $\Delta\phi$ of one degree. The values of the pertinent parameters utilized in the computations are as follows:

$\tau = 0.2$ second	$h_o = 12$ feet
$d = 16$ feet	$\Delta z = 3$ feet

The order N was chosen as 20 for all transforms. The 21 values of a_n for each of the seven velocity predictors are given in Table II. Each column is labeled according to the z value to which the coefficients correspond. The units of a_n are sec^{-1}, such that with η in feet the outputs of the $G_s(j)$ predictors are in feet/sec.

The 20 values of b_n for each of the five antisymmetrical transforms are given in Table III. The units of each set of b_n are indicated. It will be noted that the b_n values for the predictors of KI_1 and KI_2 are given in place of those for I_1 and I_2, where K is simply a constant defined by

$$K = \frac{\pi}{4} \frac{w}{g} D^2 \tag{35}$$

which is one of the factors in the equation for F_2. Taking $g = 32.2$ ft/sec^2, 64 lbs/cu ft and D = 8.625 inches (0.719 feet) leads to the value 0.806 lb sec^2/ft^2 for K.

The response factors for the twelve different predictors, as evaluated by the relations

$$R_s = a_o + 2 \sum_{n=1}^{N} a_n \cos n\phi \tag{36}$$

$$R_a = 2 \sum_{n=1}^{N} b_n \sin n\phi \tag{37}$$

for each degree in the range $0 < \phi < 90°$, are indicated by the circled points in Figs. 4, 5 and 6. These responses represent simply the output of the numerical transforms for a simple harmonic input of unit amplitude and frequency ω, or period $T = 2\pi/\omega$. The scale for T is related to ϕ by the simple formula

$$T = \frac{360\,\tau}{\phi} = \frac{72}{\phi} \tag{38}$$

where ϕ is expressed in degrees and $\tau = 0.2$ seconds. It will be noted the scales for T and ϕ are logarithmic. The design response functions are indicated by the full curves in Figs. 4, 5, and 6. The vertical dashed lines indica the arbitrary cut-off position.

Fig. 4 contains the response functions for velocity at the subsurface levels. The curve for $z = -15$ feet was added as a matter of interest but was not utilized in the computations of F_1. The fit of the predictor responses to the desired response is remarkably good owing to the nature of the response curves. The limiting value of response for $\phi = 0$ (i.e. zero frequency or unlimited T) is given by

$$R_s(0) = a_o + 2 \sum_{n=1}^{N} a_n \tag{39}$$

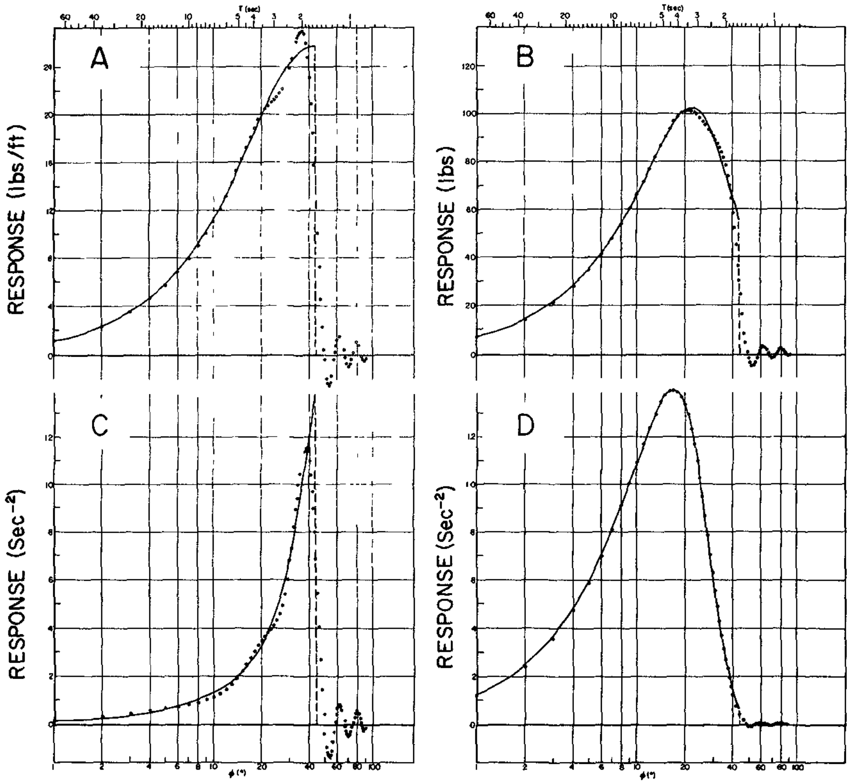

Fig. 6. Response Diagrams for KI_1(A), KI_2(B), u at mean water level (C), and u at 12 feet below mean water level (D). Full curves are from linear wave theory, circled points from Eq. (37) using b_n values in Table III. Design "cut-off" shown by vertical dashed line.

TABLE I

PERTINENT INFORMATION FOR DESIGN OF THE VELOCITY AND ACCELERATION PREDICTORS

Predictor Operation	Design Output	Design Response	$2\pi/\omega_c$	ϕ_c
$G_s^{(j)}[\eta(t)]$	$u(t)$ at $z=-j\Delta z$	$\omega \frac{\cosh k(d-j\Delta z)}{\sinh kd}$ ($j=-\frac{1}{2}, 0, \frac{1}{2}, 1, 2, 3, 4$)	0.8 sec for $n \geq 0$ 1.2 sec for $n=-\frac{1}{2}$	90° 60°
$G_a^{(1)}[\eta(t)]$	$\dot{u}(t)$ at $z=0$	gk	1.67 sec	44°
$G_a^{(2)}[\eta(t)]$	$\dot{u}(t)$ at $z=-h_0$	$\omega^2 \frac{\cosh k(d-h_0)}{\sinh kd}$	1.67 sec	44°
$G_a^{(3)}[\eta(t)]$	$\partial\dot{u}/\partial z$ at $z=0$	$\omega^2 k$	1.67 sec	44°
$G_a^{(4)}[\eta(t)]$	$I_1 = \int_{-h}^{0} u\,dz$	$\frac{\omega^2}{k} - g\frac{\sinh k(d-h_0)}{\cosh kd}$	1.67 sec	44°
$G_a^{(5)}[\eta(t)]$	$I_2 = \int_{-h_0}^{0} \lvert z \rvert u\,dz$	$\left\{\frac{g}{k}\left[1-\frac{\cosh k(d-h_0)}{\cosh kd}\right] - gh_0\frac{\sinh k(d-h_0)}{\cosh kd}\right\}$	1.67 sec	44°

TABLE II

COEFFICIENTS a_n FOR VELOCITY PREDICTORS

(Units of a_n are sec^{-1})

η	3(ft) 1.5	0	-1.5	-3	-6	-9	-12
0	1.8495	2.0426	0.7217	0.4162	0.2460	0.1911	0.1674
1	1.3013	0.9864	0.5134	0.3482	0.2273	0.1820	0.1614
2	0.0623	-0.7209	0.1029	0.1925	0.1785	0.1570	0.1446
3	-0.9752	-0.9402	-0.1400	0.0468	0.1168	0.1220	0.1198
4	-1.1343	0.0638	-0.1187	-0.0276	0.0606	0.0845	0.0913
5	-0.4531	0.4929	-0.0215	-0.0382	0.0211	0.0510	0.0632
6	0.4128	-0.0396	0.0011	-0.0236	0.0002	0.0254	0.0387
7	0.7676	-0.3485	-0.0148	-0.0094	-0.0069	0.0087	0.0197
8	0.4274	0.0338	-0.0004	-0.0004	-0.0066	-0.0004	0.0066
9	-0.2029	0.2853	0.0206	0.0040	-0.0040	-0.0040	-0.0010
10	-0.5439	-0.0105	0.0093	0.0042	-0.0016	-0.0046	-0.0045
11	-0.3499	-0.2238	-0.0085	0.0026	-0.0002	-0.0038	-0.0053
12	0.1281	0.0129	-0.0008	0.0017	0.0004	-0.0025	-0.0046
13	0.4255	0.1929	0.0114	0.0016	0.0004	-0.0015	-0.0033
14	0.2987	-0.0083	0.0023	0.0007	0.0003	-0.0007	-0.0019
15	-0.0873	-0.1673	-0.0094	-0.0005	0.0002	-0.0002	-0.0008
16	-0.3466	0.0055	-0.0022	-0.0005	0.0000	0.0001	-0.0001
17	-0.2591	0.1464	0.0076	-0.0000	-0.0000	0.0003	0.0004
18	0.0593	-0.0054	0.0008	-0.0003	-0.0000	0.0003	0.0005
19	0.2900	-0.1319	-0.0076	-0.0007	-0.0000	0.0002	0.0005
20	0.2265	0.0038	-0.0013	-0.0003	-0.0000	0.0002	-0.0004
[illegible]	1.9470	1.2972	1.4101	1.4178	1.4190	1.4191	1.4186

TABLE III

COEFFICIENTS b_n FOR ACCELERATION PREDICTORS

η	Predictor for	$\dot{u}_o$	$\dot{u}_{-12}'$	$\partial \dot{u}/\partial z$	KI_1	KI_2
	Units of b_n	(sec^{-2})	(sec^{-2})	($ft^{-1}\ sec^{-2}$)	(lbs/ft)	(lbs)
1		0.6588	0.0579	0.1969	1.9890	7.1693
2		1.0731	0.1057	0.3107	3.3616	12.4422
3		1.0497	0.1355	0.2729	3.6439	14.3793
4		0.7619	0.1450	0.1635	3.1107	13.4307
5		0.2177	0.1349	-0.0237	1.8254	9.8450
6		-0.2990	0.1109	-0.1830	0.4189	5.3314
7		-0.5903	0.0803	-0.2488	-0.6075	1.4142
8		-0.5737	0.0501	-0.2013	-0.9808	-0.9105
9		-0.3070	0.0253	-0.0729	-0.7339	-1.4478
10		0.0519	0.0082	0.0706	-0.1463	-0.7208
11		0.3254	-0.0014	0.1626	0.4139	0.3915
12		0.3991	-0.0055	0.1665	0.6665	1.1023
13		0.2672	-0.0063	0.0909	0.5319	1.0452
14		0.0221	-0.0057	-0.0193	0.1362	0.3535
15		-0.2005	-0.0047	-0.1068	-0.2808	-0.5071
16		-0.2925	-0.0038	-0.1311	-0.5002	-1.0394
17		-0.2230	-0.0027	-0.0870	-0.4318	-0.9823
18		-0.0459	-0.0014	-0.0034	-0.1441	-0.4204
19		0.1363	-0.0001	0.0735	0.1881	0.2998
20		0.2286	0.0009	0.1051	0.3852	0.7815

The values of this quantity for each of the velocity predictors are given at the bottom of Table II. The theoretical response at zero frequency (i.e. for very long waves) is simply $\sqrt{g/d}$ for all levels and has the value 1.4186 sec^{-1} for the present case (d = 16 feet). The subsurface velocity predictors give $R_s(0)$ values well within one per cent of this value. However, the predictors for u at and above the mean water level are much less accurate, as should be expected from the form of the response functions (Figs. 5A and 5B). However, even for the least accurate of the predictors (that for u at +1.5 ft elevation) the mean square value of R_s is less than two per cent different from the mean square value of the design response R_s' for the interval $0 \leqslant \omega \leqslant \omega_c$, which is therefore consistent with condition A stipulated earlier.

The response function for $\partial \dot{u}/\partial z$ at mean water level (Fig. 5C) and $\dot{u}$ at mean water level (Fig. 6C) behave in a manner similar to u at or above the surface, except for one important difference: the response is zero at $\phi = \omega = 0$. In fact, this latter condition holds for all five of the antisymmetrical transforms used in the prediction of the acceleration and functions thereof. The response factors for the case of KI_1, KI_2, and $\dot{u}$ at -12 feet (Figs. 6A, 6B and 6D, respectively) are certainly satisfactory but evidently an improvement could be made by selection of a smaller cut-off period. However, it will be apparent in the discussion which follows that any improvement in response of the acceleration transforms for $T < 1.6$ seconds will have little influence on the final results in respect to the inertial coefficie On the other hand the accuracy of the response of the velocity predictors for periods less than 1.6 seconds does affect the evaluation of drag coefficient, an accounts for the selection of the lower cut-off period in the case of those trans forms. This is true in spite of the fact that both of the forcing functions $F_1(t)$ and $F_2(t)$ are filtered to eliminate all periods less than 1.6 seconds. As discussed earlier, the nonlinear dependence of $F_1(t)$ on $u(t)$ implies that the low frequency end of the spectrum of $F_1(t)$ is partially dependent upon the high frequency end of the spectrum of $u(t)$.

7. THE VIBRATIONAL FILTER

It was pointed out that the structure and test pile were not free of vibrations. The primary ranges of resonant periods, 1.1 ± 0.1 second and 0.5 ± 0.1 second, were evaluated analytically (Wilson and Reid, 1955) and verified in the experimental records. The effects of these vibrations can be eliminated from the recorded reaction $R_1(t)$ by employing a symmetrical filter operation of the type

$$F^*(t) \equiv c_0 + 2 \sum_{n=1}^{N} c_n \left[F(t + n\tau) + F(t - n\tau) \right] \qquad (40)$$

where $F(t)$ is the particular time sequence to be filtered [e.g. $R_1(t)$]. The desired characteristics of this filter are: unit response for $T > T_c$ and zero response for $T < T_c$ where T_c is a nominal cut-off period which will assure the elimination of all vibrational effects. The operation (40), of course, is free of any phase distortion for $T > T_c$. The above response characteristics are approximated by taking

$$c_n = \frac{1}{\pi} \int_0^{2\pi\tau/T_c} \cos n\phi \, d\phi \tag{41}$$

which yields

$$c_o = \frac{2\tau}{T_c} \tag{42}$$

and

$$c_n = \frac{1}{n\pi} \sin \frac{2\pi n \tau}{T_c} \tag{43}$$

for $n = 1, 2, \ldots N$. The amplitude response factor of the filter for simple harmonic input is

$$R_s = c_o + 2 \sum_{n=1}^{N} c_n \cos n\phi \tag{44}$$

A continuous graph of this function for $T_c = 1.6$ seconds and $N = 20$ is shown in Fig. 7. It can be seen that this response assures almost complete suppression of the vibrational periods.

This filter has two important functions: (a) application to the sequences of $R_1(t)$, $F_1(t)$, and $F_2(t)$ to assure suppression of a common band of high frequencies in all three records and (b) application to the sequence $\eta(t)$ in order to gain some information in regard to the spectrum of this sequence.

The primary function of the filter of course is the suppression of vibrations in the $R_1(t)$ sequence. If the filter had perfect unit response for $T > 1.6$ seconds and if the sequences $F_1(t)$ and $F_2(t)$ contained no spectral energy for $T < 1.6$ seconds then there would be no need of filtering these time series since the output would be the same as the input. However, the filter is not perfect; there is some amplitude distortion for $T > 1.6$ seconds as is evident in Fig. 7. Furthermore, the sequence $F_1(t)$ will definitely contain spectral energy for $T < 1.6$ seconds, as provided by the relatively small cut-off periods in the design of the velocity predictors. In addition, high as well as low frequencies are generated by the nonlinear transformation leading to $F_1(t)$. This is the case to a lesser degree in regard to $F_2(t)$. In view of these considerations it is apparent that each of the functions $R_1(t)$, $F_1(t)$, $F_2(t)$ should be subjected to the same filtering operation if they are to be analysed on a comparable basis.

Figure 8 illustrates the degree of smoothing accomplished by the above filter when applied to a record of water level variations. Here $\eta(t)$ is the original record and $\eta^*(t)$ is the filtered output. It will be noted that an interval of $N\tau$ (4 seconds) is lost at each end of the finite record in the filter process. From these two sequences it is possible to ascertain the relative amount of spectral energy of water level variations for all periods less than the cut-off period of 1.6 seconds. This relative spectral energy is given by

$$1 - \overline{(\eta^*)^2} / \overline{\eta^2} \tag{45}$$

where the averages are taken over the same time interval for both sequences. The value of this quantity was found to be 0.038 based upon a total of 280 seconds of filtered record (sampled from all runs). Thus the net effect of all periods less than 1.6 seconds in the spectrum of $\eta(t)$ contributes less than four per cent to the total spectral energy, for the data utilized in the present study. The cut-off period for the acceleration predictors was taken as 1.67 seconds which is only slightly greater than the cut-off for the vibrational filter. It therefore appears that condition B of section 6, as expressed by the inequality (34), is satisfied for the mean conditions.

8. ANALYSIS OF THE DATA

The application of the numerical transform operations $G_s[\eta(t)]$ and $G_a[\eta(t)]$ is illustrated schematically in Figs. 9A and 9B; here the input, weighting factors a_n or b_n, and the output are shown diagramatically. The output curve is generated by shifting the product and summing operation progressively along the t axis [see Eqs.(24) and (25)].

As an illustration of the type of vertical distribution of currents obtained from the wave records, some sample results of the velocity transforms for a particular run are shown in Fig. 10A. The distributions of current at five different times are shown, each curve terminating at an elevation dependent upon the instantaneous value of η. In each case the velocities at $z = 0$ and $z = 1.5$ ft were used in estimating the shape of the curve near the surface. The appropriate portion of the water level record from which the velocity distributions were obtained is shown in Fig. 10B.

The sequence of evaluation of $F_1(t)$ and $F_2(t)$ and finally C_D and C_M by use of the regression formula (3) is indicated in the schematic flow diagram of Fig. 11. The entire program of computations was carried out by an electronic digital computer. The seven transforms of type (24) and five transforms of type (25) with coefficients as stipulated in Tables II and III were utilized in the evaluation of the velocities $u(z, t)$ and the acceleration functions respectively. The mean current $U(z)$ was estimated from measurements of the surface drift (Reid, 1956). The mean surface current ranged

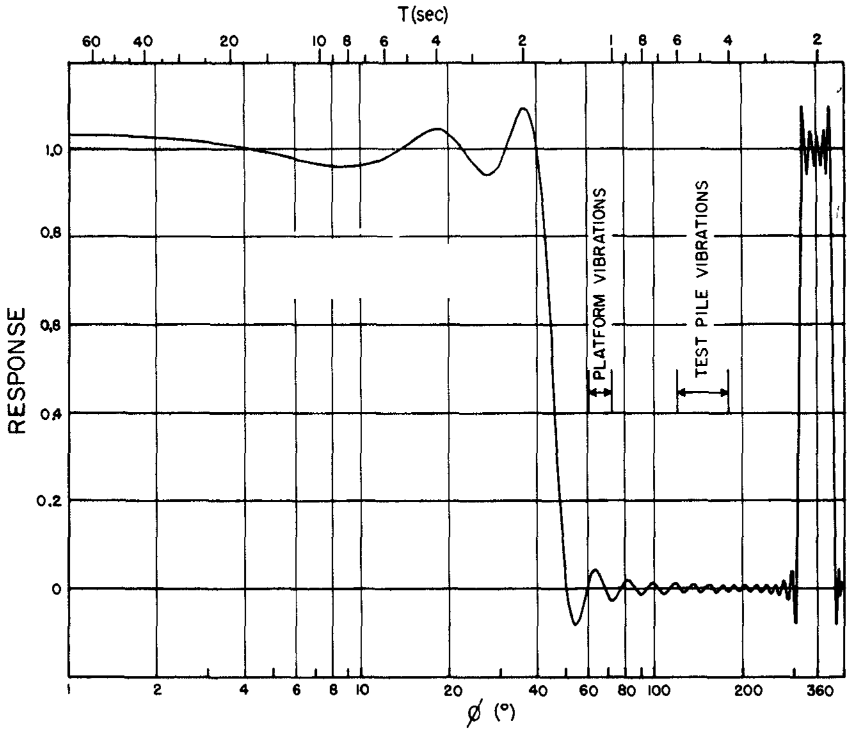

Fig. 7. Response diagram for vibration filter with nominal "cut-off" at 1.6 seconds. Based upon Eqs.(42), (43) and (44) with $\tau = 0.2$ seconds, $N = 20$.

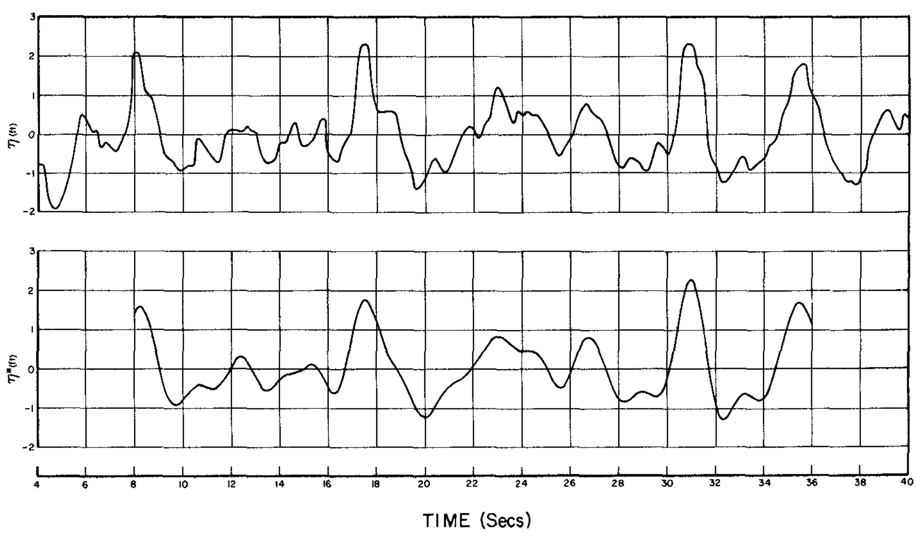

Fig. 8. Illustration of original and filtered sequence of water level variations (Run) 2; $\eta^*(t)$ is the output of filter operation (40) with input $\eta(t)$ and spectral response as depicted in Fig. 7.

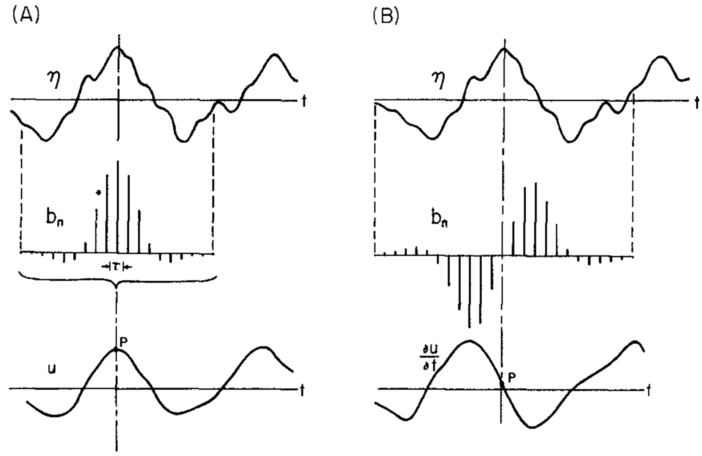

Fig. 9. Schematic of numerical transform operation for symmetrical (A) and antisymmetrical (B) transforms, showing input, weighting coefficients and output.

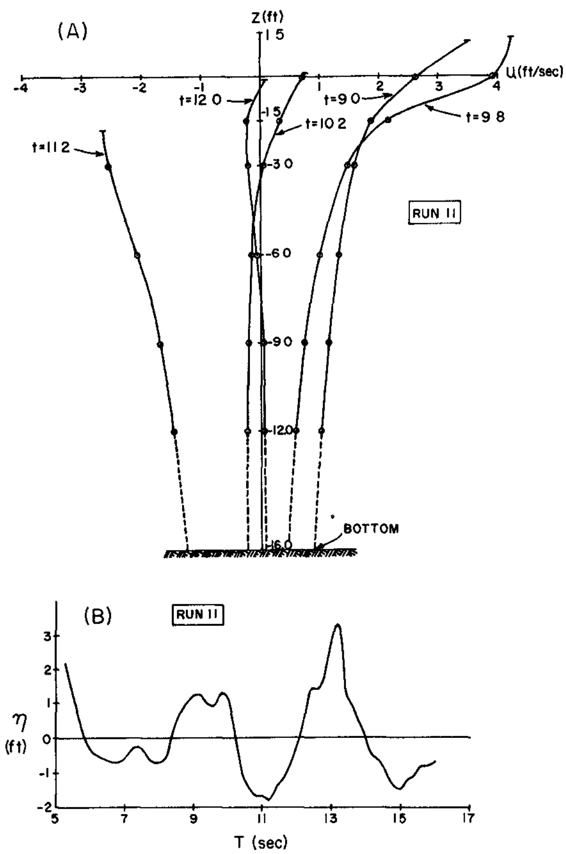

Fig. 10. Illustration of the vertical distribution of velocity u for five different times (A) as deduced by numerical transformation of the water level sequence (B) for run 11.

TABLE IV

SAMPLE LISTINGS OF ORIGINAL AND COMPUTED SEQUENCES FOR RUN 11

Run	t (sec)	u(1.5) (ft/sec)	u(o) (ft/sec)	u(-1.5) (ft/sec)	u(-3) (ft/sec)	u(-6) (ft/sec)	u(-9) (ft/sec)	u(-12) (ft/sec)	η (ft)	F_1 (lbs)	F_2 (lbs)	R_1 (lbs)	η* (ft)	F_1^* (lbs)	F_2^* (lbs)	R_1^* (lbs)
11	14.8	2.668-	2.351-	2.140-	1.887-	1.573-	1.353-	1.212-	1.4-	30.1-	11.4-	29.0-	1.20-	13.3-	13.2-	56.1-
11	15.0	4.379-	2.444-	2.162-	1.925-	1.659-	1.482-	1.366-	1.5-	36.8-	3.7-	72.0-	1.23-	7.7-	7.0-	48.4-
11	15.2	4.449-	2.292-	1.919-	1.797-	1.627-	1.503-	1.421-	1.2-	35.5-	2.6	58.0-	1.24-	15.8-	1.1-	39.2-
11	15.4	2.822-	1.231-	1.520-	1.567-	1.506-	1.439-	1.393-	1.1-	29.1-	8.7	14.0-	1.23-	31.7-	6.1	26.6-
11	15.6	.933-	.327-	1.188-	1.315-	1.326-	1.311-	1.300-	.8-	22.2-	14.6	0.0	1.15-	43.9-	14.7	10.0-
11	15.8	.241-	.619-	1.037-	1.082-	1.117-	1.146-	1.162-	.8-	16.0-	19.0	38.0	.96-	42.8-	22.2	8.1
11	16.0	1.056-	1.444-	.936-	.855-	.902-	.962-	.995-	.7-	11.0-	21.0	5.0	.69-	26.2-	25.9	23.7
11	16.2	2.104-	1.281-	.675-	.612-	.700-	.775-	.810-	.6-	6.8-	19.6	10.0	.38-	1.0-	23.8	31.8
11	16.4	1.789-	.043-	.264-	.378-	.526-	.590-	.609-	.2-	3.4-	15.6	38.0	.12-	20.2	16.8	32.1
11	16.6	.257	.793	.034	.221-	.381-	.403-	.391-	.1-	.4-	9.5	19.0	.00	26.4	8.4	27.0
11	16.8	2.519	.435	.007	.191-	.254-	.201-	.149-	.0	.1	5.0	29.0	.02-	14.0	3.6	21.8
11	17.0	2.883	.511-	.277-	.248-	.107-	.035	.123	.1-	.0	7.2	14.0	.14-	9.1-	6.6	22.2
11	17.2	.842	1.275-	.557-	.268-	.108	.323	.430	.2-	.2	17.2	14.0	.22-	26.5-	18.4	33.3
11	17.4	1.889-	1.389-	.560-	.092-	.439	.669	.763	.1-	2.8	32.2	62.0	.09-	22.3-	35.5	54.0
11	17.6	3.038-	.683-	.113-	.395	.900	1.058	1.100	.2	11.0	47.4	86.0	.28	9.1	51.7	79.3
11	17.8	1.715-	.199	.819	1.212	1.453	1.450	1.407	.8	28.7	59.8	95.0	.91	65.3	59.1	101.0
11	18.0	1.847	1.448	2.180	2.229	2.003	1.785	1.642	1.4	64.5	60.8	106.0	1.64	132.6	52.4	110.7
11	18.2	5.983	3.866	3.697	3.170	2.414	1.992	1.764	2.1	151.4	43.4	101.0	2.25	191.0	30.4	101.7
11	18.4	8.563	6.770	4.758	3.680	2.557	2.014	1.740	2.8	312.6	3.9	91.0	2.53	221.7	2.2-	74.5
11	18.6	8.267	7.502	4.668	3.487	2.353	1.823	1.560	2.9	333.6	47.3-	48.0	2.36	213.7	37.5-	34.6
11	18.8	5.214	4.655	3.254	2.569	1.812	1.431	1.236	1.8	122.4	78.4-	5.0-	1.74	168.7	65.9-	9.9-
11	19.0	.665	.252	1.124	1.193	1.031	.891	.808	.7	17.1	83.7-	86.0-	.82	100.5	80.2-	49.4-
11	19.2	3.544-	2.446-	.806-	.228-	.169	.288	.330	.5-	1.2-	70.1-	86.0-	.16-	29.3	77.6-	77.7-
11	19.4	5.879-	2.741-	2.020-	1.349-	.610-	.284-	.134-	.9-	10.1-	54.3-	53.0-	.97-	25.6-	60.6-	90.7-
11	19.6	5.743-	2.506-	2.565-	2.006-	1.182-	.747-	.526-	1.4-	25.6-	32.9-	115.0-	1.46-	53.2-	35.3-	87.5-
11	19.8	3.844-	3.074-	2.651-	2.195-	1.485-	1.047-	.803-	1.5-	28.3-	12.2-	58.0-	1.58-	53.5-	9.1-	70.2-
11	20.0	1.499-	3.313-	2.328-	2.006-	1.526-	1.164-	.943-	1.4-	25.0-	6.1	38.0-	1.40-	35.8-	11.8	42.4-
11	20.2	.074	1.996-	1.645-	1.578-	1.355-	1.113-	.947-	1.1-	20.8-	20.7	29.0-	1.07-	13.1-	24.7	8.8-
11	20.4	.355	.073-	.888-	1.070-	1.047-	.930-	.837-	.7-	12.1-	29.5	58.0	.70-	3.4	29.6	25.8
11	20.6	.522-	.510	.403-	.604-	.671-	.666-	.648-	.1-	4.4-	32.3	34.0	.36-	9.1	28.9	57.1
11	20.8	1.488-	.297-	.209-	.202-	.286-	.374-	.425-	.3-	1.4-	28.4	62.0	.08-	5.9	25.3	78.9
11	21.0	1.512-	.981-	.016	.183	.061	.104-	.209-	.1-	.3	22.8	170.0	.15	.0	20.5	87.7
11	21.2	.191-	.092-	.548	.577	.330	.103	.036-	.2	3.2	15.5	53.0	.35	2.4-	14.6	81.3
11	21.4	1.898	1.931	1.227	.896	.479	.219	.069	.7	14.7	6.8	91.0	.49	1.3	7.2	60.2
11	21.6	3.404	3.128	1.563	.987	.481	.232	.100	.9	28.9	4.0-	29.0	.54	9.5	1.9-	30.3
11	21.8	3.228	2.280	1.224	.749	.332	.152	.063	.6	15.2	15.4-	10.0-	.45	16.9	11.7-	.5-
11	22.0	1.333	.136	.349	.230	.070	.006	.021-	.2	1.3	23.0-	19.0-	.21	18.7	19.6-	23.5-
11	22.2	1.233-	1.488-	.609-	.390-	.236-	.163-	.123-	.4-	1.0-	23.5-	62.0-	.12-	13.5	22.8-	31.0-
11	22.4	3.019-	1.873-	1.290-	.907-	.503-	.309-	.210-	.7-	4.1-	18.3-	24.0-	.47-	3.8	19.3-	21.2-
11	22.6	3.136-	1.925-	1.590-	1.171-	.653-	.389-	.254-	.8-	6.8-	8.4-	5.0	.73-	4.3-	9.2-	2.9
11	22.8	1.610-	2.172-	1.514-	1.132-	.647-	.378-	.236-	.8-	6.0-	4.7	53.0	.79-	7.3-	4.9	34.5
11	23.0	.425	1.856-	1.071-	.822-	.483-	.270-	.154-	.6-	3.2-	18.2	58.0	.64-	6.3-	19.2	63.2
11	23.2	1.611	.435-	.372-	.339-	.199-	.085-	.019-	.2-	.2-	29.7	77.0	.30-	2.1-	29.4	80.7
11	23.4	1.404	1.026	.324	.194	.145	.141	.143	.2	3.2	34.9	77.0	.13	5.3	32.7	82.4
11	23.6	.430	1.225	.819	.677	.481	.364	.302	.6	8.7	31.3	72.0	.56	14.0	27.8	69.5
11	23.8	.050	.762	1.163	1.054	.743	.537	.423	.6	12.1	17.9	29.0	.86	20.8	15.9	45.4
11	24.0	.918	1.185	1.486	1.285	.880	.623	.479	.7	18.0	1.2	34.0	.96	23.3	.4	16.5
11	24.2	2.372	2.502	1.692	1.311	.863	.600	.451	1.0	29.5	16.0-	.0	.86	21.2	14.7-	11.1-
11	24.4	3.105	2.906	1.498	1.079	.688	.466	.338	1.1	31.2	28.3-	19.0-	.60	15.6	26.2-	33.6-
11	24.6	2.090	1.244	.800	.606	.380	.238	.150	.2	6.6	30.8-	77.0-	.26	8.9	32.2-	47.4-
11	24.8	.356-	.935-	.102-	.003	.009-	.054-	.089-	.0	.0	31.3-	53.0-	.08-	2.9	32.7-	57.7-
11	25.0	2.897-	1.527-	.811-	.579-	.419-	.368-	.347-	.5-	2.7-	28.6-	29.0-	.38-	1.6-	29.2-	50.8-

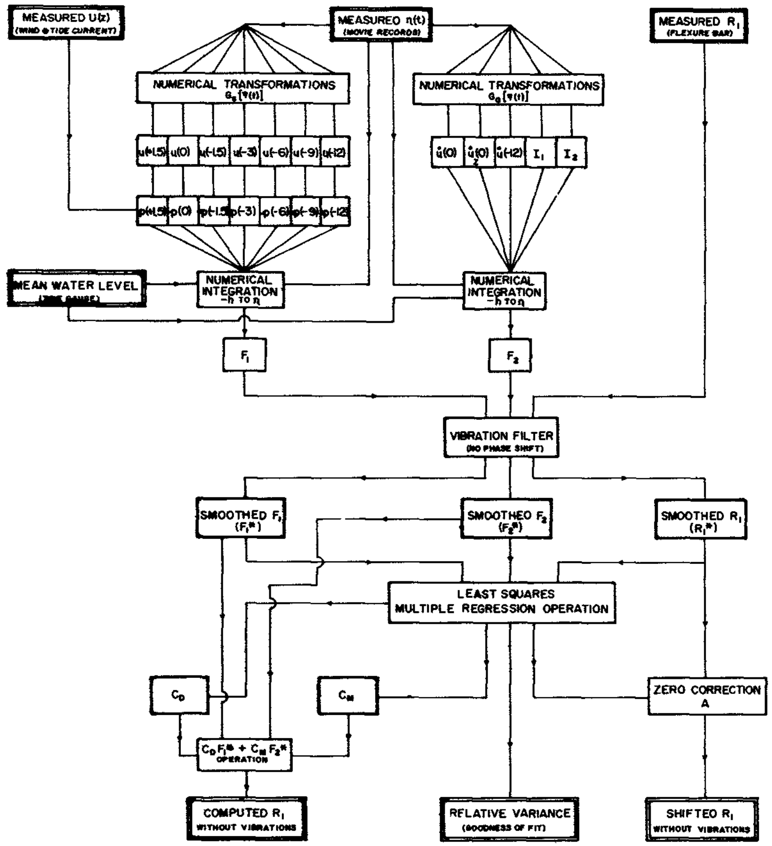

Fig. 11. Schematic flow diagram in the numerical analysis of the water level and reaction data, lead-ing to the evaluation of drag and inertial coeffi-

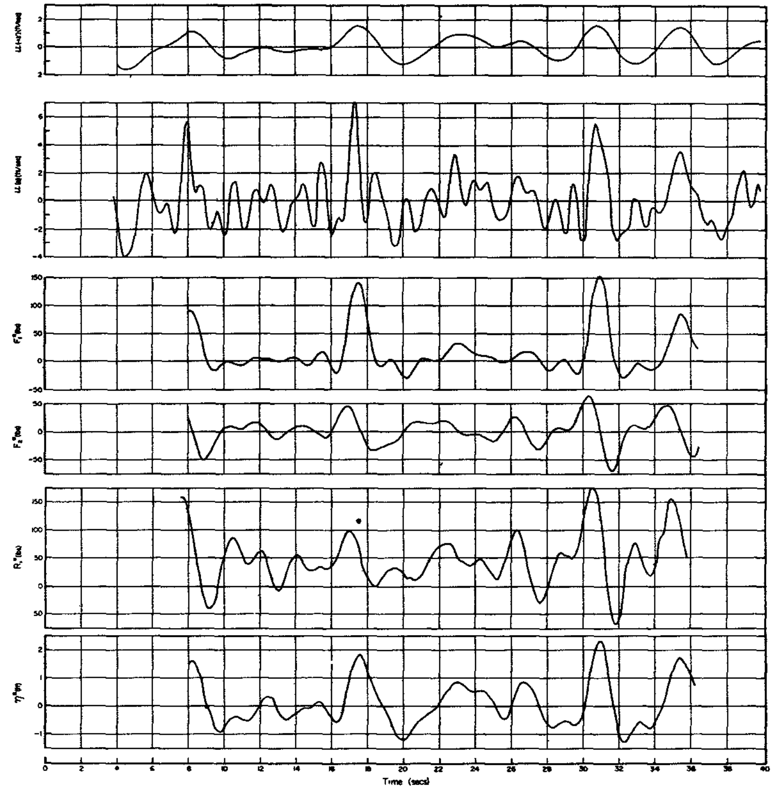

Fig. 12. Samples of simultaneous serial sequences of u at -12 feet, u at mean water level, F_1*, F_2*, R_1*, and η* as computed from the basic sequences R_1 (t) and

from about 0.5 to 0.7 ft/sec, with much smaller values at subsurface depths. In contrast with the steady current, the values of u at the surface cover a range from about -5 to +10 ft/sec, considering all runs. For the most part, the effect of the steady component of current constituted a second order correction in respect to the evaluation of p as defined by Eq.(7).

The time sequences of p_1, $\dot{u}$, $\partial\dot{u}/\partial z$, I_1, and I_2 were utilized together with $\eta(t)$ and h (as inferred from the tide gage) in the evaluation of $F_1(t)$ and $F_2(t)$ using definitions (4) and (5) together with the auxiliary formulas (8) to (12). The values of h ranged from 11.1 to 12.0 feet for the three different series of runs. The values of L and b in Eqs.(4) and (5) were taken as 30.8 feet and 9.0 feet respectively.

The measured sequence $R_1(t)$ and the computed sequences $F_1(t)$ and $F_2(t)$ for each run were filtered to eliminate all periods in the vibrational range. The outputs of the filter operation are designated as $R_1{}^*(t)$, $F_1{}^*(t)$ and $F_2{}^*(t)$ respectively. A sample plot of these sequences for one run is given in Fig. 12. These are compared with a filtered version of the water level sequence from which $F_1{}^*(t)$ $F_2{}^*(t)$ were derived. As a matter of interest, the sequences of u at mean water level and -12 feet are also included in the figure. A sample listing of the calculations of u (at all seven levels), η, F_1, F_2, R_1, and the filtered sequences η^*, $F_1{}^*$, $F_2{}^*$, $R_1{}^*$ is given in Table IV.

As stipulated earlier, Eq.(3) is valid only in the absence of vibrations or if the functions involved are interpreted as the sequences from which vibrations have been effaced. In addition, it was stipulated that the true zero reference in the measurements of R_1 could not be ascertained with certainty. Consequently, allowance for this should be made by incorporating a constant correction term A such that $R_1 - A$ is the true reaction. Because of the possibility of zero drift in the measuring equipment, we must expect differences in A from one run to another. Thus there are really three coefficients A, C_D and C_M which are to be evaluated by least squares multiple regression procedure for each run.

With the above changes, the appropriate regression equation becomes

$$R_1'(t) = C_D F_1{}^*(t) + C_M F_2{}^*(t) \tag{46}$$

which is to be fitted to the sequence $R_1{}^*(t) - A$. Here $R_1'(t)$ is the predicted value of filtered reaction for a particular set of constants A, C_D, C_M. We seek those values of A, C_D, and C_M which make the quantity

$$S_e{}^2 \equiv \frac{1}{P} \sum_{n=1}^{P} [A + C_D F_1{}^*(t_n) + C_M F_2{}^*(t_n) - R_1{}^*(t_n)]^2 \tag{47}$$

a minimum, where P is the total number of points in each sequence. The

TABLE V

SUMMARY OF RESULTS OF WAVE FORCE ANALYSIS

Sun Oil Co. Pier, Caplen, Texas, d = 16 feet, D = 8.625 inches.

Supplementary Data	Run	P total points	2 (ft)	N_r 10^5	S_e (lbs)	S_R (lbs)	C_D	C_M	r
Series I.									
Date: Feb. 27, 1954									
Time: 1704-1708^h CST	1	89	0.80	0.60	25	46	0.73	1.41	0.84
Wind: 21 mph SW	2	139	0.81	0.61	19	45	0.53	1.39	0.90
U_o = 0.5 ft/sec	3	149	0.69	0.52	18	49	0.32	1.96	0.93
h = 12.0 ft									
H_s = 2.3 ft, T_m = 3.5 sec									
Series II.									
Date: May 11, 1954	5	65	0.63	0.48	12	23	0.27	1.63	0.93
Time: 1630-1642^h CST	7	57	1.03	0.76	23	56	0.63	1.60	0.91
Wind: 18 mph SE	8	71	0.76	0.56	14	45	0.82	1.92	0.95
U_o = 0.5 ft/sec	9	88	1.01	0.86	18	54	0.42	1.72	0.94
h = 11.4 ft	10	92	0.78	0.58	17	43	0.88	1.45	0.92
H_s = 2.6 ft, T_m = 4.3 sec	11	114	1.07	0.78	22	62	0.54	1.45	0.94
Series III.	12	71	1.24	0.91	32	63	0.88	0.41	0.86
Date: May 11, 1954	13	55	1.32	0.97	25	71	0.80	0.80	0.94
Time: 1852-1906^h	14	84	1.62	1.18	35	86	0.33	1.37	0.91
Wind. 22 mph SE	15	95	0.74	0.56	14	29	0.37	1.16	0.87
U_o = 0.7 ft/sec	16	95	0.93	0.70	20	51	0.33	1.70	0.92
h = 11.1 ft	17	82	1.09	0.81	15	56	0.44	1.57	0.97
H_s = 3.3 ft, T_m = 4.8 sec	18	46	1.01	0.76	11	47	0.44	1.36	0.97
	Total	1392	Weighted averages[2]		21	58	0.53	1.47	
							Overall correlation	=	0.93

1 The wind speed and directions are averages for preceding 6 hour period.

2 The overall values of S_e and S_R are root mean square values weighted according to P. The overall C_D and C_M are averages weighted according to P

quantity S_e represents the standard deviation of the measured reaction from the predicted or fitted reaction. The requirement of minimum S_e leads to three equations from which the three coefficients are derived. The resulting solutions are symbolically:

$$C_D = \frac{1}{Q}\left\{[R_1^*, F_1^*]\,[F_2^*, F_2^*] - [R_1^*, F_2^*]\,[F_1^*, F_2^*]\right\} \tag{48}$$

$$C_M = \frac{1}{Q}\left\{[R_1^*, F_2^*]\,[F_1^*, F_1^*] - [R_1^*, F_1^*]\,[F_1^*, F_2^*]\right\} \tag{49}$$

where

$$Q = [F_1^*, F_1^*]\,[F_2^*, F_2^*] - [F_1^*, F_2^*]^2 \tag{50}$$

and

$$A = \overline{R_1^*} - C_D\,\overline{F_1^*} - C_M\,\overline{F_2^*} \quad . \tag{51}$$

In the above equations the following special notation is employed for the covariance of any pair of sequences $f_1(t)$ and $f_2(t)$ having non-zero means:

$$[f_1\,,\,f_2] \equiv \overline{f_1\,f_2} - \overline{f_1}\;\overline{f_2} \tag{52}$$

where the bar indicates an average taken over a total of P discrete values. It is of interest to note that in the special case where F_1^* and F_2^* possess zero mean value and zero covariance then the above relations reduce to the remarkably simple form:

$$C_D = \frac{\overline{R_1^*\,F_1^*}}{\overline{(F_1^*)^2}} \tag{53}$$

$$C_M = \frac{\overline{R_1^*\,F_2^*}}{\overline{(F_2^*)^2}} \tag{54}$$

$$A = \overline{R_1^*} \tag{55}$$

These relations would be directly applicable to simple harmonic waves such as are obtained approximately in the laboratory studies, assuming that U is zero.

In the complex records analysed it was found that the above simplification was not applicable since the covariance $[F_1^*, F_2^*]$ was small but not negligible and also the mean value of F_1^* was finite. Consequently the general relations (48) through (52) were employed in the numerical evaluation of the coefficients. The results of the calculations for each run are summarized in Table V. Supplementary information for each series of runs is included.

The parameter $S_R^{\,2}$ is the variance of R_1^* from its mean value, i.e.

$$S_R^{\,2} = \overline{(R_1^*)^2} - \left[\overline{R_1^*}\right]^2 \tag{56}$$

and r is the correlation coefficient, characterizing the goodness of fit of the functions $F_1^*(t)$ and $F_2^*(t)$ to $R_1^*(t)$. It is defined by

$$r \equiv \sqrt{1 - \frac{S_e^{\,2}}{S_R^{\,2}}} \tag{57}$$

perfect correlation being unity.

The quantity N_r is a root mean square value of the Reynolds number defined by

$$N_r = \frac{D\, v_r}{\gamma} \tag{58}$$

γ being the kinematic viscosity of water (taken as 1×10^{-5} ft^2/sec) and v_r is the root mean square value of the total current (averaged over the depth and the duration of the run). Note that all values of N_r given in the table a to be multiplied by 10^5.

The quantity H_s given with the supplementary data is an estimate of significant wave height as computed from the formula

$$H_s = 2\sqrt{2}\,\overline{\eta^2} \tag{59}$$

which follows from the analysis by Longuet-Higgins (1952, p.254). The value of $\overline{\eta^2}$ in this relation is the mean for the particular series of runs. The quantity T_m is a mean "period" evaluated from the filtered sequence $\eta^*(t)$. It is defined as twice the mean time interval between successive zero values of η^*, and is closely related to the significant period of the waves. Series III has the largest significant wave height and largest mean period of the three series. This series followed about two and one-half hou after series II, during which time the wind was steadily gaining in speed ou of the southeast. In the series I data the mean wind speed was nearly the same as for series III, but the wind was from the southwest and hence mor nearly parallel with the shore line.

It can be shown from the Eq.(47) to (52) that the variance of R_1^* car be expressed in the form

$$S_R^{\,2} = [R_1^*, F_1^*]\, C_D + [R_1^*, F_2^*]\, C_M + S_e^{\,2} \tag{60}$$

where the symbolic covariance notation (52) is employed on the right. This

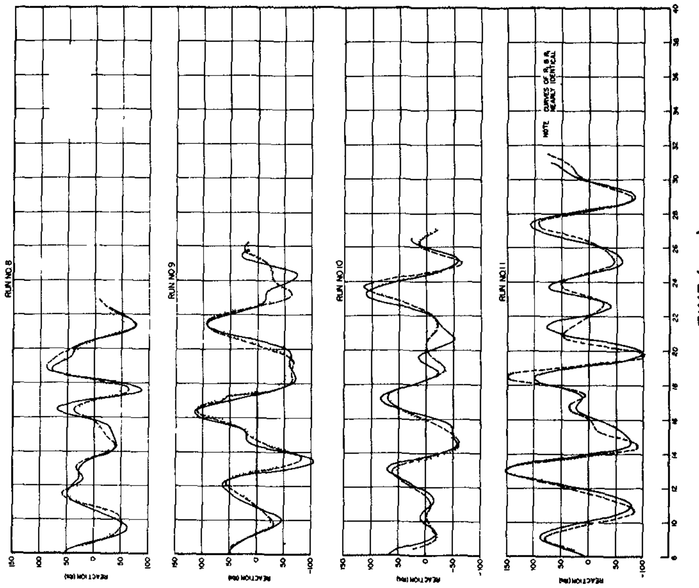

Fig. 14. Graphs of R_1^* -A (full curves), Regression Function R_1' (dashed curves) and Predicted Reaction R_1^* (dotted curves) for runs 8, 9, 10, and 11.

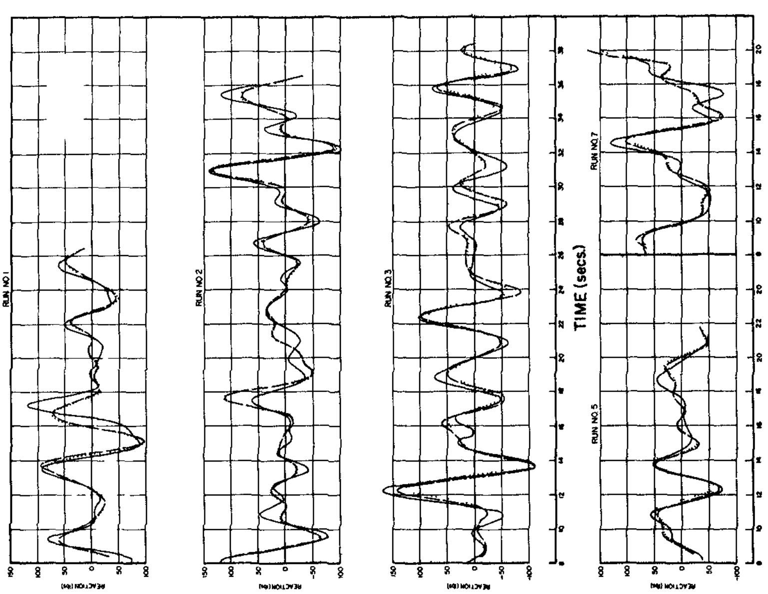

Fig. 13. Graphs of R_1^* -A (full curves), Regression Function R_1' (dashed curves), and Predicted Reaction R_1'' (dotted curves) for runs 1, 2, 3, 5, and 7. The dashed curves are based upon the best fit C_D and C_M values for the individual runs. The dotted curve is based upon the overall mean values of C_D and C_M (0.53 and 1.47 respectively).

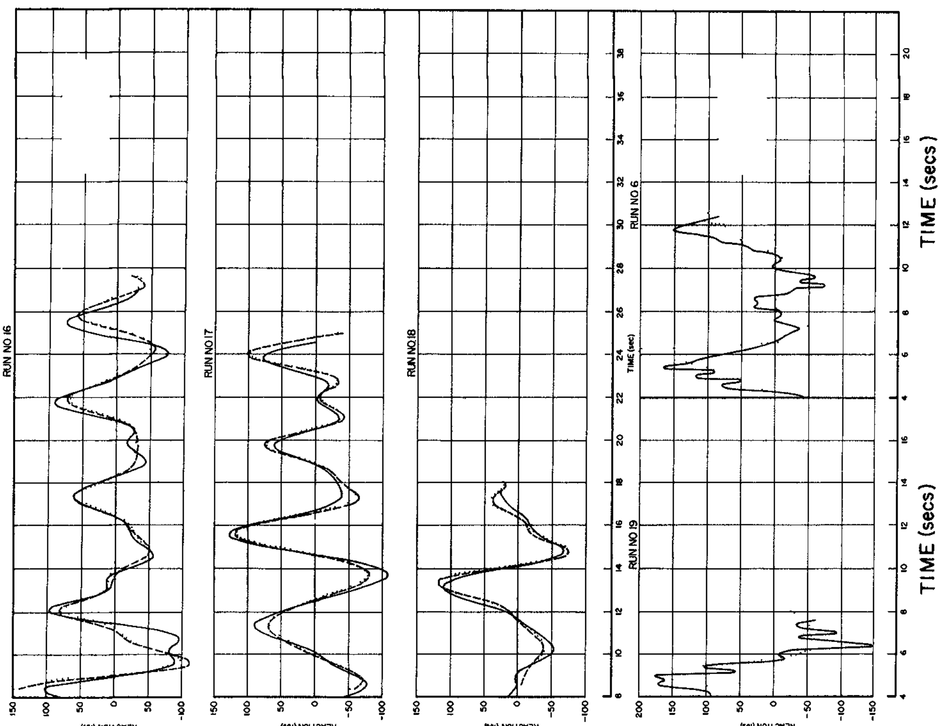

Fig. 16. Graphs of $R_1^* - A$ (full curves), Regression Function R_1' (dashed curves), and Predicted Reaction

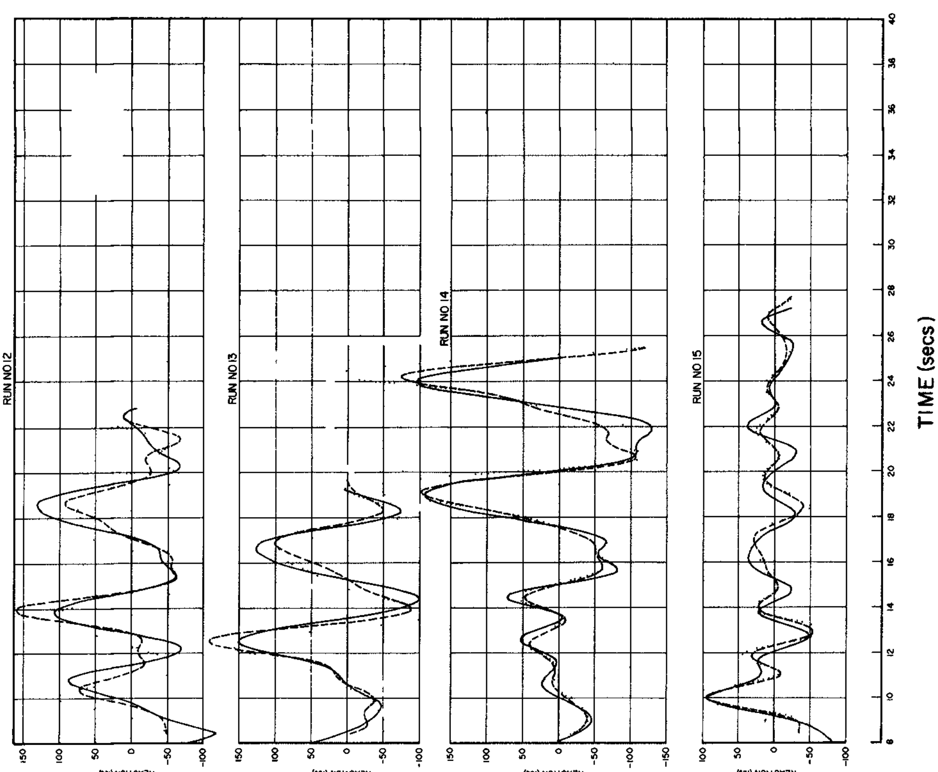

Fig. 15. Graphs of $R_1^* - A$ (full curves), Regression Function R_1' (dashed curves), and Predicted Reaction

relation is useful in the interpretation of the spectral composition of R_1^*. The sum of the first and second terms on the right represent that part of the total spectral energy of the R_1^* sequence which is accounted for by the hydrodynamic forces associated with the waves, while the last term is the unaccountable part. The fractional contributions to the energy spectrum of R_1^* by drag force and inertial force can be evaluated separately from the expressions

$$\frac{[R_1^*, F_1^*]\, C_D}{S_R^2} \quad \text{and} \quad \frac{[R_1^*, F_2^*]\, C_M}{S_R^2} \tag{61}$$

respectively.

The overall root mean square values of S_e and S_R and the mean values of C_D and C_M (all weighted according to the number of points in each run) are indicated in Table V. The overall correlation coefficient of 0.93 was evaluated from (57) using the weighted mean square values of S_e and S_R. The set of C_D values possess a standard deviation of 0.20 from the mean value 0.53, and the set of C_M values have a standard deviation of 0.36 from the mean value 1.47. The weighted mean values of $[R_1^*, F_1^*]\, C_D$ and $[R_1^*, F_2^*]\, C_M$ indicate that the drag force contributes about 27 per cent to the variance of R_1^* while the inertial force contributes about 60 per cent, based upon all runs.[5] The remaining 13 per cent, corresponding to S_e^2/S_R^2 (or $1 - r^2$), is unaccountable insofar as the present hypothesis in regard to the nature of the fluid forces and field of motion is concerned. The fact that the inertial force contribution is of greater importance in the present tests is not surprising in view of the rather small mean "periods" of the waves. It is evident that a proportionately greater degree of reliability exists in respect to the estimates of C_M than in C_D. This may partly account for the greater relative standard deviation of C_D (38 per cent of the mean) as compared with that of C_M (25 per cent of the mean).

The values of S_e and r give a quantitative measure of the degree of compatability of the fitted reaction $R_1'(t)$ with the sequences of R_1^*-A. However, a visual comparison of these sequences is quite helpful. Such a comparison is given for each run in Figs. 13 to 16. The full curve in each graph represents the smoothed and adjusted sequence $R_1^*(t)-A$, derived from the measurements. The dashed curves represent the fitted reaction $R_1'(t)$ as given by Eq.(46) using the individual regression coefficients C_D and C_M from each run. The dotted curves are plots of the relation

[5] The percentages given in the earlier technical report (Reid, 1956, p.46) were found to be in error.

$$R_1''(t) = \bar{C}_D F_1^*(t) + \bar{C}_M F_2^*(t) \tag{62}$$

where $\bar{C}_D$ and $\bar{C}_M$ are the overall mean values 0.53 and 1.47 respectively

Runs 6 and 9 which are depicted in the lower graph of Fig. 16 were not included in the summary of Table V. The sequences of measured R_1 were too short to subject to the filtering procedure, and consequently no attempt was made to estimate the individual C_D and C_M values for these runs. However, adequate water level records were available which permitted the evaluation of the $\bar{F}_1$ and $\bar{F}_2$ functions. As a test of the regression equatio the coefficients $\bar{C}_D$ and $\bar{C}_M$ determined from the other runs were employe to compute $R_1''(t)$ for runs 6 and 9. Thus the dotted curves in the graphs fo these runs actually represent predictions of the force from the measured wat level. Note that the full curves in these two runs are the unfiltered sequences of measured reaction, unadjusted for true zero reaction.

9. CONCLUSIONS

The regression equation (46), using the numerical transformations of $\eta(t)$ to simulate the field of motion of the fluid, as predicted by the linear wa theory for long crested waves, and assuming drag and inertial coefficients which are independent of velocity and acceleration, allows a reasonably good fit of the measured irregular reactions from which vibrations have been effac The variation of the individual C_D and C_M values, deduced by least square regression techniques for each run, vary considerably from one run to anothe However, even when the overall mean values of these coefficients are utilizec to predict the reactions the agreement is still surprisingly reasonable. If the individual C_D and C_M values are employed for each run, then all but 13 per cent of the variance of R_1^* can be explained by the drag and inertial forces. It can be shown that S_e^2 is approximately doubled when the overall mean values of the coefficients are used to predict the reaction (dotted curves of Figs.13 to 16), and the correlation consequently drops from 0.93 to 0.85 for all runs as a whole. This correlation is about the same as that obtained for the individual regression curves for runs 1, 12 and 15.

It would appear that some amount of freedom exists in the possible com binations of C_D and C_M which will lead to nearly the same prediction for total load. It may be noted that the sum of the mean C_D and C_M values is 2.00 . An analysis of the values of $C_D + C_M$ for each run indicates a standard deviation from the mean which is only 13 per cent of the mean. This may be compared with the standard deviations of the values of C_D and C_M separately, which are 38 per cent and 25 per cent of the mean C_D and C_M respectively. It is particularly interesting, though undoubtedly somewhat accidental, that the mean sum of the coefficients is 2.00 , for this is the value of C_M which should exist for accelerated irrotational flow around a circular cylinder in the complete absence of a turbulent vortex wake i.e., for $C_D = 0$ (see Lamb, 1945, pp. 75-77). The implication of the above line of reasoning is of course purely conjectural at this stage, but it would appear that for general flow conditions in the presence of turbulence the sum of C_D and C_M is more nearly conserved than are the individual coefficients. It would be of interest to test this hypothesis, or some modification thereof in further studies.

The scatter of C_D and C_M values obtained from the individual runs are bound to exist in the presence of errors in measurement of η and/or R_1, or errors in simultaneity of the time sequences, or errors in the estimated subsurface values of steady current. It is also possible and quite likely for the tests reported here that a major source of scatter in the regression coefficients results from the short-crestedness of the waves. The source of difficulty stems from the fact that we have attempted to evaluate a vector force from the measurement of a scalar quantity η. This is legitimate if the waves are long-crested and the reaction which we are attempting to predict is aligned with the wave direction. However, in the presence of short-crested waves there can exist variations in η at the test pile which are related to waves approaching normal to the direction of the predominant waves. These waves could produce little if any reaction in the direction of the predominant waves, and consequently the functions, F_1 and F_2 deduced from such variations in η would be in error. In addition, it was assumed that no reflection of wave energy occurs at the test pile where the measurements of η were made. The effect of the presence of the pile on the waves should be small for long wave lengths, but may have a significant effect for waves of 5 feet in length or shorter (corresponding to one second period or less). The spectral components of 1.6 second and less were filtered from the records in the final analysis, however there could still be some error introduced in the F_1 function by the nonlinearity.

It is considered that the majority of the scatter in the C_D and C_M values is a result of the short-crestedness of the waves. In view of the possible errors introduced by short-crestedness it is all the more surprising that the overall mean C_D and C_M (0.53 and 1.47 respectively) can lead to a reproduction of the measured reactions with a correlation as high as 0.85 .

10. ACKNOWLEDGEMENTS

This work represents the results of research carried out under the sponsorship of the U.S. Naval Civil Engineering Research and Evaluation Laboratory of the Bureau of Yards and Docks, Contract NOy-27474, through the Texas A.& M. Research Foundation. The Gulf Coast Division of the Sun Oil Company provided space on the Sun Oil pier at Caplen, Texas and cooperated in many other respects in the measurement phase of the program. The instrumentation and measurements of wave forces were made possible through the efforts of C.L. Bretschneider, G.L. Huebner, and W.H. Clayton. The author is particularly indebted to R.L. Smith for programing and analysis of the numerical data by means of an IBM 650 digital computer, and to R.G. Dean for his assistance in the overall analysis work. Finally, the encourage ment and many valuable suggestions of B.W. Wilson must be noted; and certainly Randolph Blumberg deserves much of the credit for the method of analysis of the complex records.

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CHAPTER 47

SUCCESS AND FAILURE OF COAST PROTECTION WORKS IN CEYLON

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INTRODUCTION

The Island of Ceylon, which is often referred to as the Pearl of the Indian Ocean, is 25,481 square miles in extent. It lies between 0 and 10 degrees North Latitude, and between 79 and 82 degrees East Longitude. The Island is mango-shaped. Its length from north to south is 272 miles, and its breadth from east to west is 140 miles. (Fig.1).

Geologically, Ceylon belongs to what is called Peninsular India: that is India south of the Indo-Gangetic plain, an area which forms a compact tableland of a triangular shape projected into the Indian Ocean, with Ceylon as a semi-detached pendant at its apex. Owing to this salient position as regards the Indian Ocean, the coasts of India and Ceylon are constantly laved by currents resulting from the deflection of the monsoon drift against them, the more powerful currents occurring during the south-west monsoon, when the wind blows over a wide expanse of water to reach the land mass, and the weaker during the north-east monsoon, when the waters of the Arabian Sea and Bay of Bengal are blown south-wards.

CHANGES IN COASTAL TOPOGRAPHY

In the perennial struggle between sea and land, there is evidence that the frontiers of Ceylon , on the west, south-west and the south are being constantly pushed back. However, it is not possible to estimate reliably, even within the period of recorded history of the Island, to what extent the sea has encroached upon land. No comparison is possible between the earliest and present maps of Ceylon. The former were prepared when cartography was only little developed, consequently these plans lack precision that is so essential for comparative study.

Ceylon was known in ancient times as Sri Lanka Pura. It has been calculated that Sri Lanka Pura, the legendary capital of King Ravana (circa 1500 B.C.), through which the meridian of the Brahamins' passed, must have been in 75 degrees 53 minutes east longitude, whereas the present western extremity of Ceylon barely reaches 80 degrees.

In the legendary traditions of the Island are also found the extent of this country in former times. According to chronicles the original circumference of the Island was 5120 miles. It is said to have been reduced by successive inundations to 938 miles, which is not far from the present size. The first of these inundations is said to have taken place in 2387 B.C. The second in 504 B.C. and finally, the extensive submergence near Colombo, the present capital of Ceylon, in 300 B.C.

In 1908 Commander Sommerville of the Royal British Navy whilst er gaged in a marine survey discovered a submerged plateau surrounding the Island, the edge of which, he has stated, is strongly marked. He has further stated that it extends far out to sea on the west coast, and is much less prominent on the east coast. Whence it may reasonably be assum that the west coast of Ceylon has suffered much denudation due to the erosive action of sea throughout the ages.

Within more recent times, the sea has made many inroads into lanc in the coastal stretch from Negombo to Weligama, and continues to do so abated at several points on this coast-line. Due to this incursion of tl sea much damage is being done to public and private property: roads and railways are threatened with destruction; parks and pleasure beaches are gradually disappearing; and many houses and coconut plantations are bei obliterated. All this involve considerable loss to the national economy and diminution of the beautiful coastal scenery of the Island.

THE PROBLEM OF SEA EROSION

Although sea erosion had taken toll of the Island from time immemorial, yet it attracted serious attention of the islanders only ab fifty years ago, when it started attacking works of human construction a cultivated lands. During the last century the maritime lands on the west south-west and the south have been vastly developed by the construction roads, railways, industrial and residential buildings, and the plantati of coconut estates. In course of time the sea had advanced so rapidly tl it started attacking these works of utility and national economy.

The total length of the Island's coast lines subject to extensive erosion is about 245 miles, made up as follows: north of Colombo about 100 miles; south of Colombo about 125 miles and between Trincomalee and Batticaloa about 20 miles.

In recent years a fast expanding economy of the maritime province has focussed much attention on the problem of sea erosion. It has been realized that coast protection and shore-line development should be a distinct charge under suitable administration. Accordingly, Parliament has vested the Ministry of Local Government and Cultural Affairs with authority for the protection of the coast against erosion and encroachm by the sea. What is now aimed at is to build up a Department under the Commissioner of Local Government, which would be able to amass all the necessary data on sea erosion, to study them steadily in sequence, and t deal with individual problems as an organic unit in a comprehensive sch of coastal economy.

COAST PROTECTION WORKS

The Government which is obliged to afford protection to public a private property has spent considerable sums of money on protective worl Hitherto such works have been carried out by the Public Works Departmen The Irrigation Department and the Harbour Engineer's Department. They h been executed sporadically. In the absence of organized studies and

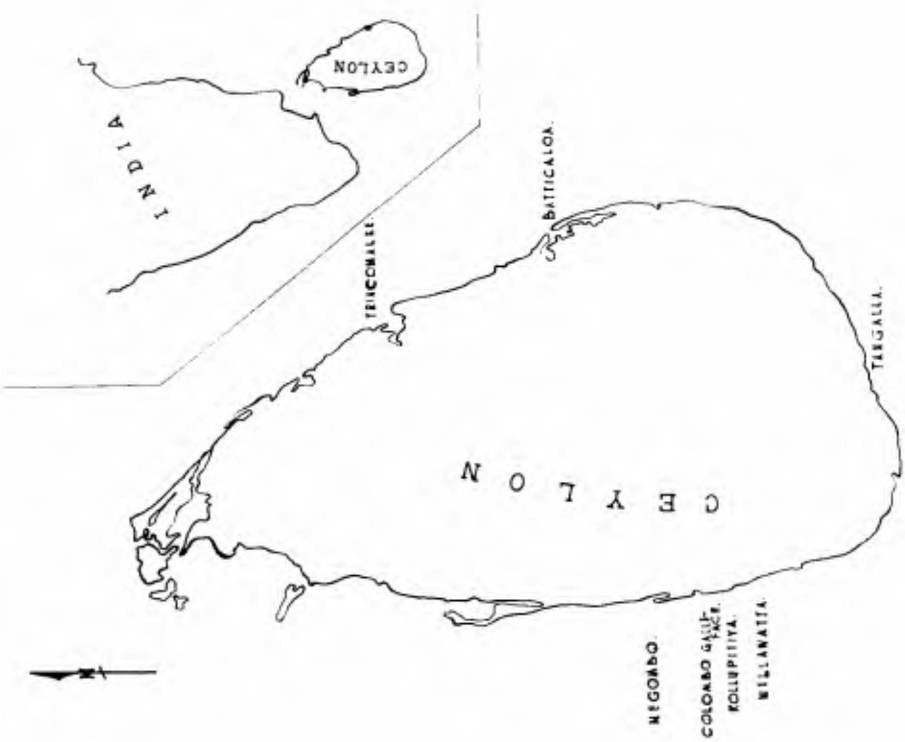

Fig. 1. Map of Ceylon and the adjacent coast of India.

Fig. 2. Southern jetty at Wellawatta Canal outlet in the process of construction. Note the absence of a sandy beach.

Fig. 3. Southern jetty at Wellawatta Canal outlet after completion. Note the beach re-claimed on the southern side.

investigations, of the several problems of sea erosion, around the Isla coast protection works were laid down by engineers on the basis of the trial and error method. Some of these works have stood the test of time others have failed, whilst still others have created fresh problems.

In contrast with western countries, where a great variety of protective structures, representing the ideas of several generations of coastal engineers has been tried out, generally speaking, in Ceylon,the usual method of coast protection consists of the lining of the shore wi tipped rock stones, each weighing from about 100 to 1000 pounds. These rock stones are gneiss of Archaen Age and are plentiful in the Island.Tl are very resistant to erosion by wave action and other forms of weather The typical masonry sea-wall and coconut log revetments are the other expedients for coast protection in the Island.

In 1952, two experimental jetties, each 325 feet in length, were constructed at Wellawatta Canal out-let, in the vicinity of Colombo,(fi primarily with the object of preventing the blocade of this important flood out-let with a sand bar. Whilst these jetties now provide a bar-f out-let for the canal, the southern jetty has, incidentally, acted as a groyne in reclaiming a vast beach which was lost several years ago due sea erosion. (fig.3). It is significant, however, that groynes as struc ures for beach reclamation and coast protection have not yet been tried out in this country. In the future planning of coastal works the provis of groynes on sandy coasts will receive important consideration.

At the present time an experimental project is being considered the protection of a sandy beach by laying a carpet of bituminous sand. is expected that work on this will be started early next year.

STRUCTURES

<u>Rock Stone Lining</u> - A total length of about 30 miles of coast li has been lined with rock stones. This is an endeavour to prevent damage wave action of roads, railway lines and buildings. Transport of rock st to sites along the railway lines is carried out by the railway itself. other sites they are transported either by bullock carts or lorries. Us ly unloading and placing operations are carried out by workmen, but in special circumstances elephants are employed for this work. The rock st so placed form a barrier between the sea and land. Sometimes they are p up to heights of twelve and fifteen feet.

In certain sections where this type of protection has been provi as for instance, along the coast-line railway from Kollupitiya to Wellawatta, a distance of about three miles, it has been effectve in checking the advance of the sea, (fig.4). The rock stones in this reac have been placed to a maximum height of about ten feet and are supporte at the bottom by a coral reef. Storms sometimes displace the stones. Th displaced stones are then replaced with new ones. There are a few other sections along the western coast-line where this type of protection has effective. In all these sections the rock stones are resting on coral r Of the total length of about 30 miles of coast lines provided with rock stone protection only a total length of about 8 miles can be said to be effectively protected.

Fig. 4. Coast lined with rock stones supported at the bottom by a coral reef.

Fig. 5. Coast lined with rock stones supported at the bottom by a sandy beach. Most of the stones have disappeared under the sand.

In all other places where the rock stones are resting on the sandy beach this form of protection has proved ineffective. (fig.5).Not long after the stones are placed on a sandy foundation the pumelling action of waves undermines the sea bed. This scour being greatly induced by the stones themselves they are gradually lost in the sand by a continuous process of sinking. With the disappearance of these stones the waves advance inland. When due notice is taken of this incursion a second line of defence is erected behind the first. Ultimately, this too disappears as before, leaving the problem of coast protection to an indefinite solution of dumping rock stones.

Rock stone lining of a coast line hinders the activities of local fishermen who require an open space to beach their boats. When a rock stone barrier is erected to prevent the erosion of a beach there is a spate of opposition from the local fishremen because their means of livelihood is thereby interfered with.

Sea-walls - There are about 3000 feet of sea-walls in the Island. The most important of all is the promenade seawall at Galle Face in Colombo. Its total length is 2200 feet, of which, 1400 feet was constructed in 1856, and consisted of random rubble set in mortar. In later years it was surmounted by a concrete cap. Quite recently the Galle Face promenade was extended and a battered cement concrete sea-wall 800 feet in length has been constructed. The base of this wall is well-proportioned and protected from scour by steel sheet piling. The entire sea-wall at Galle Face is not exposed to heavy seas as most of the wave energy is dissipated by a chain of natural rock out-crops situated in the sea, close to the sea-wall almost throughout the length of the promenade.

The other sea-walls are at Dodanduwa and Tangalla. The aggregate length of these walls is about 800 feet. Both these sea-walls are of the random rubble type set in cement mortar and founded on firm ground. They have stood up to severe storm conditions and proved very efficient.

Coconut Log Revetment - The coconut palm grows abundantly along the coast line . There is a natural tendency to use the sturdy trunks of these trees to fight the intruding sea. This is evident in certain sections of the coast line where revetments of coconut logs are constructed by fisher-folk to keep off the sea from their door steps.

In the section to be protected a trench of about six feet in depth is excavated in the sand, parallel to the coast-line. The coconut logs, each about ten feet in length, are planted perpendicularly one next to the other. The trench is then re-filled with the excavated material. Often on the landward side of the revetment coconut logs are placed longitudinally one on top of the other, against the revetment, with a back-fill of sand. The strusture is then complete.

This type of coast protection work lasts only for a few seasons. In due course the waves and currents carry away the sand from the foot of the coconut logs. The revetment is then undermined and finally collapses.

SUCCESS AND FAILURE OF COAST PROTECTION WORKS IN CEYLON

CONCLUSION

The frontiers of Ceylon on the west, south-west ang south are being constantly pushed back by the sea. The land which suffers erosion and is being submerged by the sea is for the most part valuable land.The efforts made to counteract sea forces by lining the coast line woth rock stones is not a universal solution to the problem of erosion. Although this method of protection has been effective in a few places, generally, it affords only temporary relief and entails heavy re-current expenditure. Besides, the placing of rock stones along the coast line is an impediment to the fishing industry.

Almost all of the perennial rivers, with the exception of a few, enter the sea on the west, south-west and south of the Island. These rivers are a rich source of supply of beach material in the form of sand. Wave action and current forces carry away the sand from the beaches leaving the land exposed to the onslaught of the south-west monsoon. In the future planning of coast protection works, it would be necessary, therefore, to consider suitable designs to promote beach formation and its maintenance under extreme weather conditions. Little or no scientific data is available today on sea erosion in the Island. It is essential to collate information and secure systematic observations on a long term plan before planning coast protection works.

Coast protection works and their maintenance involve heavy capital and re-current expenditure. In each case it would be necessary to consider whether it is a business proposition, having regard to the works to be saved. Such saving has been done with good results along the coast line railway, where, if it had not been for the rock stone lining of the shore, not only would the railway have long disappeared, but also most of the immensely valuable building land along the Colombo South sea front.

As erosion is often in the last analysis due to bodily subsidence of land, it is impossible to say how long coast protection measures will last, but subsidence is slow and a life of many decades may be looked for which is quite worthwhile where important interests are at stake. At all events, and this is most important, coast protection is effective in saving works of human art which otherwise would be lost, even where there is no nett loss of land over a series of years, in the periodic rhythms of erosion and accretion. Such works lost in an erosive phase are not set up again by the return of accretion and coast protection, by tiding them over the erosive phases, saves them in a relatively permanent manner.

CHAPTER 48

THE EFFECT OF SEEPAGE ON THE STABILITY OF SEA WALL

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INTRODUCTION

Vertical bulkheads, or retaining walls of the sheet-pile type, a often used as sea walls at locations not subjected to continuous or severe wave action. Many miles of this type sea wall have been con structed along the Florida coast and coastline of the United States ar have given satisfactory service. However, the failures of vertical s walls which continue to occur during mild storms indicate that the d sign procedures available, or actually used, may be inadequate.

The stability of vertical sea walls placed in cohesionless soil pends upon the relations between forces which tend to overturn the v and those which provide a restraining moment. Static forces on the wall are produced by the soil and water pressure of the backfill whi tend to overturn the wall, by water and passive soil pressures on th seaward side of the wall, and by anchor loads. Dynamic forces are also applied to the wall by direct wave action and by the forces deve oped in the soil masses due to seepage flow. The soil rebound afte wave impact on the wall increases the soil pressure of the backfill . requires the development of temporarily larger passive soil and an loads for continued wall stability. Seepage forces reduce the passi pressure that can be developed on the seaward side of the wall and thereby threaten wall stability.

The stability of a sea wall thus depends directly upon the cap ity of the soil to develop sufficient passive pressure at regions of designed restraint. Any factor which reduces the available passive soil resistance of loaded regions causes a reduction in the stability the wall.

Conditions such as waves overtopping the sea wall, rain wate falling behind the wall, or the accumulation of water run-off from higher ground may result in complete or partial saturation of the b fill and cause a water level differential between the opposite sides the wall. This head difference results in a seepage flow through tl backfill and under the wall. The vertical component of the attenda seepage pressures causes a change in effective soil density and a responding change in soil pressures, such that the stability of the is changed under seepage conditions.

THE EFFECT OF SEEPAGE ON THE STABILITY OF SEA WALLS

Instability of sea walls may also develop in a progressive fashion due to extensive scour at the wall face with the resulting decrease in passive earth pressure resistance. Scour may occur due to high water velocity alone, or it may occur at lower water velocity if the effective density of the cohesionless soil is reduced by upward seepage flow. A uniform rate of upward flow in front of the wall results from seepage through the backfill and under the wall. In addition, a transient upward seepage flow is developed near the wall face due to differential water pressures on the sea bed caused by wave action. The combined effect of these two seepage flows increases the probability of scour near the face of the sea wall.

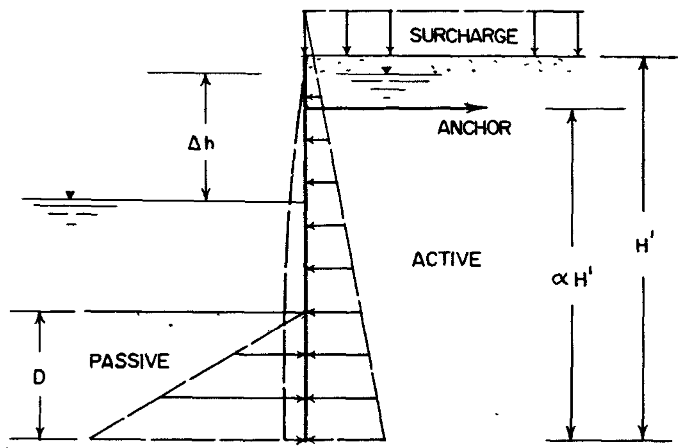
One object of this paper was to determine quantitatively the influence of seepage through the backfill on the factor of safety against wall rotation about the anchor point. The graphical flow net procedure was used to compute the additional loads, caused by seepage flow, which act on the sheet-pile walls. The results of these computations were incorporated into diagrams which permit a rapid computation of these additional wall loads and the resulting changes in the factor of safety against wall rotation. These diagrams include a sufficient range of the variables involved to be useful as design aids.

The second object of this paper was to evaluate the potential effects of seepage on the important problem of scour in front of the wall. It is demonstrated in this paper that an upward seepage gradient at the surface of the soil in front of the wall can be a major factor influencing the potential scour of this zone. Such a vertical gradient is developed when backfill seepage occurs. Furthermore, this steady gradient can be strongly reinforced by a transient gradient developed from wave action in front of the wall. The equations and diagrams presented in this paper permit evaluation of the contribution of these seepage effects to scour at the face of the wall. Small depths of scour cause appreciable changes in the factor of safety against wall rotation about the anchor point.

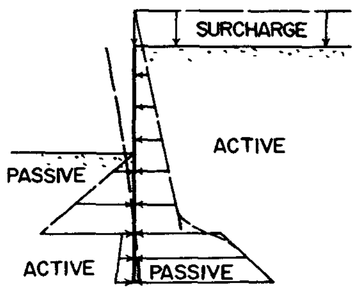
REVIEW OF LOADS ACTING ON VERTICAL SEA WALLS

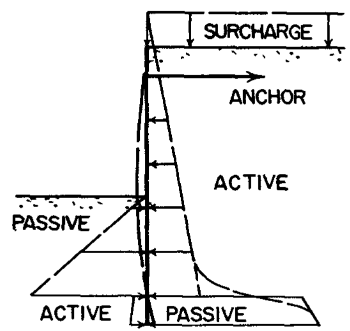
The use of the classical earth pressure theories permits a simplified evaluation of the magnitude and distribution of active and passive earth pressures along the height of a vertical sheet-pile wall. These pressure distributions are illustrated in Fig. 1 as they occur along cantilever and anchored bulkheads.

The classical pressure distributions have generally been used as the basis for the design of bulkheads and sheet-pile walls although it is known that important modifications of the passive pressure distribution, in particular, may result from wall flexibility. Methods for including



(A) Anchored sea wall with free earth support.





(B) Cantilever sheet-pile sea wall

(C) Anchored sea wall with fixed earth support

Fig. 1 - Classical concepts of earth pressure dis-

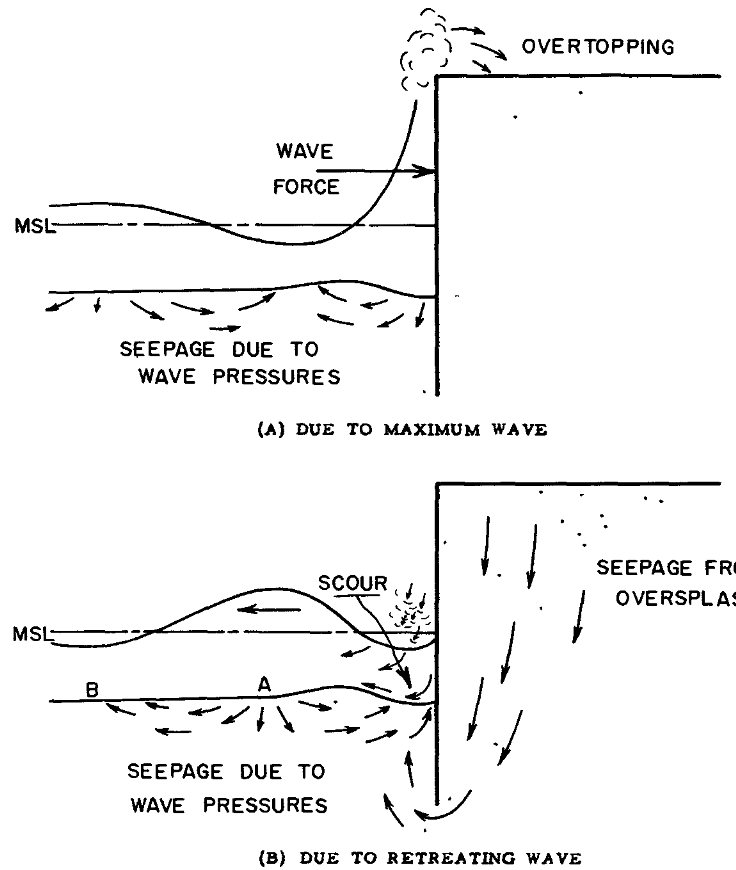


Fig. 2 - Effects of wave on vertical sea wall.
(A) Due to maximum wave
(B) Due to retreating wave

the effect of wall flexibility into design procedures have been presented by Terzaghi (1954), based on the results of model tests by Rowe (1952) and Tschebotarioff (1949).

ACTIVE EARTH PRESSURE

When a wall moves outward relative to the soil mass it confines, the soil mass produces active earth pressure on the wall. For cohesionless backfills bearing against the rear face of sea walls, the active earth pressure, P_A, at any depth, z, in soil of effective unit weight γ is given by the expression:

$$p_A = K_A \gamma z \tag{1}$$

in which K_A is the coefficient of active earth pressure, as computed from Coulomb's Equation (Taylor, 1948).

PASSIVE EARTH PRESSURE

The expression for the limiting passive earth pressure which can be developed at any depth, z, as a wall is moved into cohesionless soil is,

$$p_P = K_p \gamma z \tag{2}$$

in which K_p is computed from the Coulomb equation for passive earth pressure coefficient (Taylor, 1948). Design procedures often are based on the assumption that the angle of wall friction is zero, since this provides a conservative design. Table I gives values of the active and passive coefficients of earth pressure, K_A and K_p, for the conditions corresponding to zero angle of friction between the backfill and a vertical wall, and for which the surface of the backfill is horizontal. Terzaghi (1954) has given values of K_A and K_p, based upon values of angle of wall friction obtained from test data, which may also be used for design purposes.

Passive earth pressure provides direct restraint to the embedded portion of cantilever and anchored sheet pile sea walls, and provides indirect restraint to the upper end of anchored sea walls through the anchor system. Figure 1 shows the zones of soil developing passive resistances to motions of the wall.

TABLE I

Cohesion-less Soil	Angle of Internal Friction ϕ	Coefficient of Active Earth Pressure K_A	Coefficient of Passive Earth Pressure K_P
Dense	38	0.24	4.2
Medium	34	0.28	3.5
Loose	30	0.33	3.0

EFFECT OF STATIC WATER PRESSURE

Whenever static water pressure exists in soil adjacent to a sea wall, it causes an increase of pressure on the wall by an amount of 64 lb. per ft.^{2} for each foot of water depth, and at the same time causes a reduction of the earth pressure on the wall. The active and passive earth pressures are decreased because the submerged unit weight of the material now causes the horizontal soil force on the wall and $\gamma' = \gamma - \gamma_w$, must be used in Eqs. 1 and 2 in place of the total unit weight (γ).

When the backfill is placed hydraulically behind the sea wall, it possible for the total load on the wall to exceed the design load. Unde these conditions, water pressure loads the wall over its entire height addition to the active pressure exerted by the submerged backfill. Su a construction procedure amounts to one type of overload test of the structure and may constitute the greatest static load the wall must sustain.

EFFECTS OF WAVE ACTION

In addition to static loads, a sea wall must resist the attack of waves during storms. Vertical sea walls should not be used at locations subjected to violent, breaking waves because of the large impac forces which may develop, but are often used where moderate wave action may occur. Even moderate wave action contributes dynamic loads directly to a sea wall and the surrounding soil. In addition tem porarily induced water motion in the soil may cause significant chang in earth pressures.

Figure 2 illustrates the factors, described by Bruun (1953) con tributed by wave action, which may have important effects upon the stability of vertical sea walls. Waves acting directly against the wal produce pulsations of horizontal load which are resisted by soil forc developed as the wall moves. Occasional high waves overtop the wal and dump water onto the surface of the backfill. If the backfill material is pervious, this water finds its way back to sea level by percolating through the backfill and under the bottom of the sea wall. Finally, as the wave is reflected from the wall, water rushes down the face of the wall and produces scour of the bottom material near the face of the sea wall. The process of scour near the face of the sea wall is assisted by the upward hydraulic gradient developed in the pervious bottom material by the wave pressures on the bottom and by seepage through the backfill.

Saturation of the backfill can also occur due to direct rain water, or accumulation of rain water runoff, as well as by wave oversplash. Accumulation of water behind the wall causes an increased outward pressure and at the same time reduces passive soil resistance as a result of seepage pressures. A detailed discussion of the methods for evaluating the effects of seepage pressures on the design of sea walls is given in a following section of the paper.

EVALUATION OF SEEPAGE EFFECTS

THE SEEPAGE FLOW NET

The convenient graphical flow net construction for handling seepage problems was developed by Forscheimer (1930) and has been used extensively for many years. The flow net is established by trial sketching. Thus, it is an approximate procedure; but a flow net accurate enough for engineering purposes can be made rapidly after some practice. In addition to the numerical information obtained from such a diagram, the flow net also gives an over-all picture of the flow conditions in the region considered.

The flow net represents a steady state, two dimensional flow condition in which Darcy's Law is assumed valid. It consists of two sets of lines, flow lines and lines of equal total head. If the soil is isotropic with respect to permeability, then these lines everywhere intersect each other at right angles. For sketching and computational convenience, the flow net is generally drawn with a square as the basic element of the net and with an equal rate of flow between any two adjacent flow lines in the net. With the flow net drawn, the rate of seepage flow = q, the hydraulic gradient = i, and the water pressure = p_w, may be computed at any point within the net.

Flow net construction and analysis can be modified to handle more complicated conditions such as cases where the permeability of the soil in the horizontal and vertical directions are quite different, transient flow problems, some three-dimensional flow problems, and flow systems through layers of different permeabilities. Since a detailed discussion of the development and use of flow nets, as well as treatments of

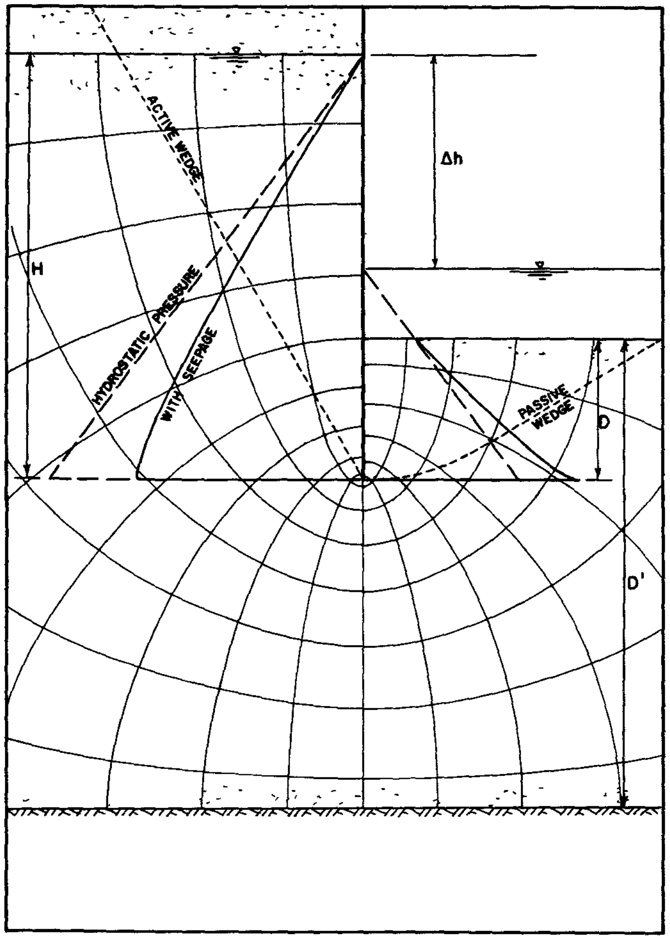
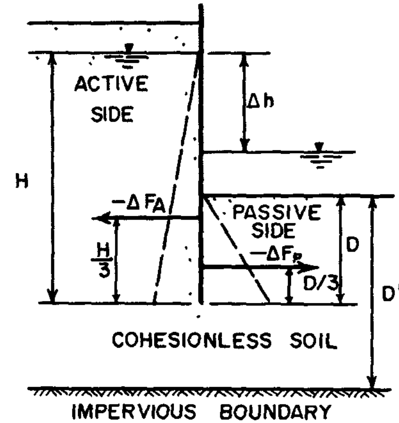


Fig. 3 - Flow net for seepage through the



HOW TO USE THIS FIGURE

1. Compute no-seepage soil and water pressures on active and passive sides of wall by any chosen method.
2. Determine (D/D'). If within 0.1 - 0.7 this figure may be used.
3. Determine (H/D), K_A and K_P values.
4. Use charts below to determine "A" and "P" values.
5. Compute:

$$\Delta F_A/_{\text{unit width}} = -(D\gamma_w \Delta h)A$$
$$\Delta F_P/_{\text{unit width}} = -(H\gamma_w \Delta h)P$$

6. Algebraic sum of above (1) and (5) forces equals the forces on the wall with seepage effects included.

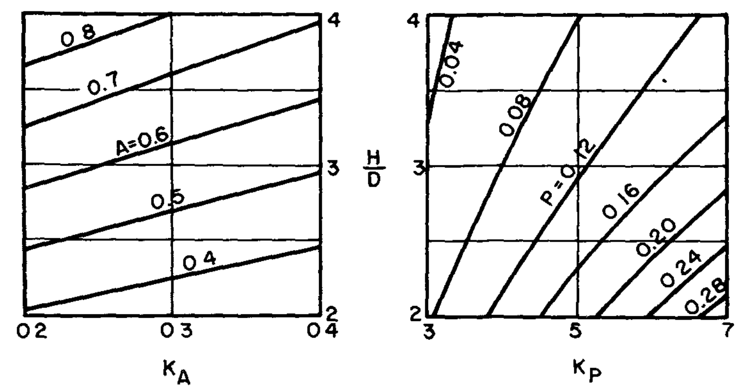


Fig. 4 - Computation of seepage correction forces.

special problems, was given by Casagrande (1937) and by Terzaghi (1943), and is available in recent books on soil mechanics, it will not be repeated here.

EFFECTS OF SEEPAGE ON THE PRESSURE DISTRIBUTION AROUND VERTICAL WALLS

As shown by Eqs. 1 and 2, the active and passive soil pressures are directly related to the unit weight of the soil. The effective unit weight of the soil changes with the development of seepage flow and the associated seepage pressures exerted on the soil. Downward seepage flow behind the wall increases the effective unit weight of the soil and thus increases the active pressure pushing the wall seaward. Upward flow in front of the wall decreases the unit weight of this soil and thus reduces the passive soil resistance to any outward movement of the toe of the wall.

The above effect can be calculated from the formula,

$$\Delta\gamma' = i_v \gamma_w \tag{3}$$

where $\Delta\gamma'$ is the change in the submerged unit weight of the soil,

γ_w is the unit weight of water,

and i_v is the vertical component of the hydraulic gradient.

Thus, in order to obtain the average change in effective unit weight due to seepage flow, it is only necessary to obtain the average vertical component of the hydraulic gradient in either the active or passive failure wedges, and to multiply this by the unit weight of water. Using the flow net of Fig. 3, the average vertical components of hydraulic gradient have been determined for the active and passive earth failure wedges indicated in Fig. 3. These gradients then change the effective unit weight of the soil by the following amounts:

$$\begin{aligned} \Delta\gamma'_{ACTIVE} &= (i_v)_{ACT}\,\gamma_w = +0.21(64) = +13\ \text{lb/ft}^3 \\ \Delta\gamma'_{PASSIVE} &= (i_v)_{PASS}\,\gamma_w = -0.30(64) = -19\ \text{lb/ft}^3 \end{aligned} \tag{4}$$

Since a typical value for the submerged unit weight of a sand is 60 lbs. ft.^{3}, it may be seen that seepage can cause appreciable earth pressure changes in the direction of wall instability.

On the other hand, seepage flow also has an effect which increase the wall stability. The water pressure has a more favorable distributic against the wall when seepage flow exists than the hydrostatic distribution for the same water levels, as shown on Fig. 3. It may be seen thа the effect is one of increased stability of the wall due to reduction of water pressures on the active side, and additional water pressure on tl passive side.

The net effect of the simultaneous two changes in pressure distri bution must be evaluated when studying the effects of seepage on the stability of a wall. Figure 4 was prepared in order to permit a rapid estimate of the change in horizontal forces on a sheet-pile type wall wl the hydrostatic pressure condition is changed by seepage flow under th wall. The seepage force correction method presented in Fig. 4 is bas on the following assumptions:

1. The wall is placed in a homogeneous, isotropic, cohesic less soil which overlies an impervious layer.

2. All changes in soil and water pressures due to seepage effects are assumed to vary linearly with depth, which permits the two pressure change effects to be incorporated into one ΔF computation.

3. The "A" and "P" charts are only valid with wall peneti tion ratios (D/D') between 0.1 and 0.7. Within this range it has been determined that neglecting the individual D/D' ratio involv a maximum error of less than 10%.

Any errors involved in the use of assumptions 2 and 3 are probа minor compared with the potential errors in assumption 1; soils place by man or nature in horizontal strata are not likely to be homogeneou and isotropic. Therefore, in many instances the use of the seepage force correction procedure suggested herein must be considered as a preliminary computation to determine if the pressure changes due to potential seepage flow are significant in the wall design.

SEEPAGE RESULTING FROM WAVE ACTION

Surface water waves occurring in a finite depth of water produc underwater pressures which can be measured, or may be estimated l use of an appropriate wave theory. The pressure at the sea bottom r be expressed as,

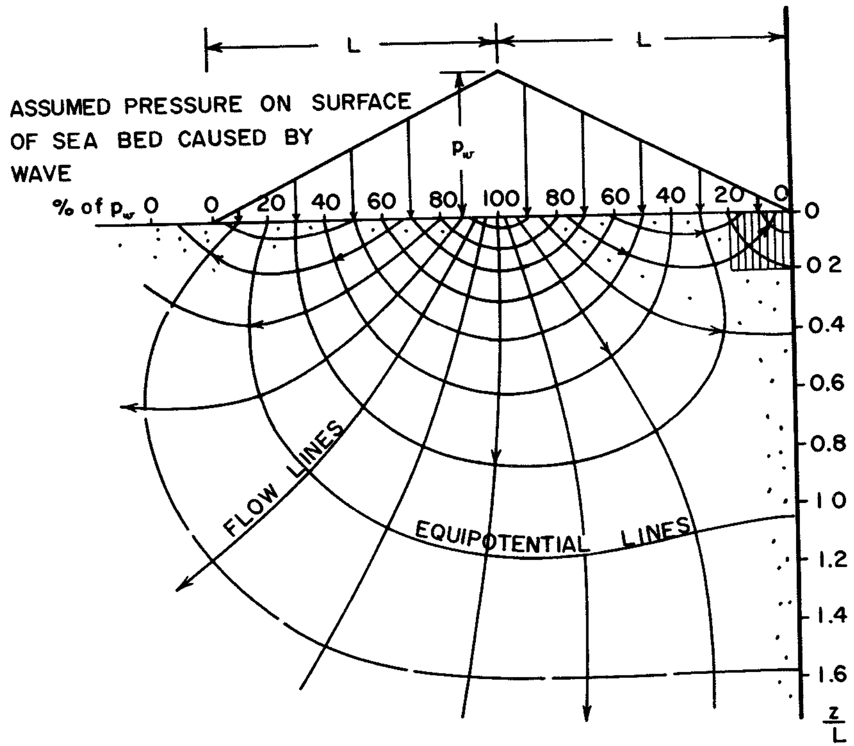
$$p_w = K \gamma_w \psi(x,t) \tag{5}$$

where y (x, t) represents the elevation of the wave surface measured from the still water level, γ_w is the density of water, and K represents a "sub-surface pressure response factor."

At any instant of time a difference in pressure exists between two points separated by a distance x along the sea bottom. If the bed material is permeable this pressure difference on its surface will cause seepage flow. In his study of the damping effect on gravity waves contributed by permeable sea bed material, Putnam (1949) considered that this seepage caused by gravity waves is governed by Darcy's law for steady flow. Recently, Reid and Kajiura (1957), also investigated the effect of a permeable sea bed on the damping of gravity waves by treating the problem as a two-layer, coupled system. They included the effects of acceleration of flow in the permeable layer, but found the effects of acceleration to be negligible for practical cases.

In the immediate vicinity of a sea wall the seepage caused by the differential wave pressures along the bottom is considerably affected by the presence of the wall. As the wave runs into a sea wall, the water height at the wall reaches at least two times the unobstructed wave height and produces a corresponding increase in pressure on the bottom. As the water falls along the wall to develop a retreating wave, a trough is formed adjacent to the sea wall. Figure 2 (a) and (b) illustrate the seepage flow in a permeable sea bed resulting from pressures developed by these two conditions of wave motion at the sea wall face. The impermeable boundary formed by penetration of the sea wall into the permeable material will force seepage flow to become vertical at the wall face as indicated in Fig. 2 (b). For the wave position as indicated in Fig. 2 (b), the upward seepage forces near the wall face due to the wave will reinforce the upward seepage forces developed from oversplash. At the point A, the wave seepage forces are down and in this region they will tend to counteract the oversplash effects.

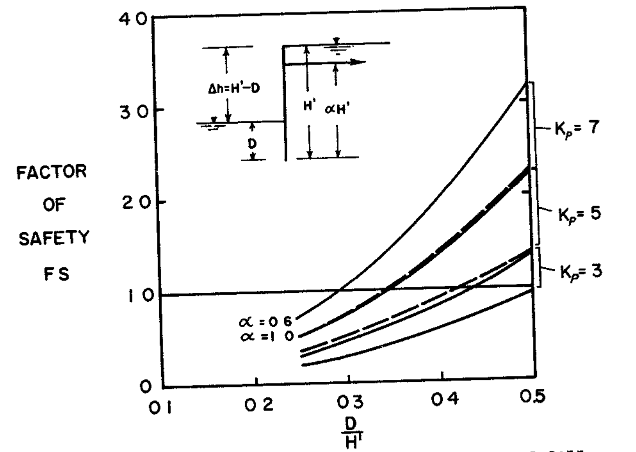
In order to evaluate the importance of the seepage due to wave action near a sea wall, a triangular distribution of pressure on the sea bottom was assumed to represent the transient pressure beneath a retreating wave. By considering this pressure distribution as static at a particular instant of time, the steady state seepage flow was established by means of the "Relaxation" procedure, and the flow net shown in Fig. 5 was obtained. The hydraulic gradient which causes upward flow at the face of the sea wall depends directly upon the hydraulic gradient along the sea bottom, $p_w/\gamma_w L$. The maximum value of hydraulic gradient at the face of the sea wall and at the sand surface is, $i_{max} \approx 1.6\, p_w/\gamma_w L$, and the average value over an area 0.2L deep and extending 0.2L from the wall face is, $i_{ave} \approx 0.66\, p_w/\gamma_w L$. The values of hydraulic gradient at other points beneath the wave can be obtained from Fig. 5.



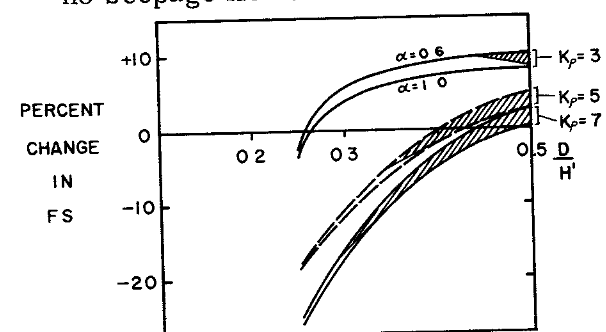
MAXIMUM EXIT HYDRAULIC GRADIENT – AT WALL FACE, $i_{max} = 1.6 \frac{p_w}{\gamma_w L}$

AVERAGE EXIT HYDRAULIC GRADIENT – SHADED AREA, $i_{av} = 0.66 \frac{p_w}{\gamma_w L}$

Fig. 5 - Flow net in sea bed caused by transient wave pres-



(A) Factor of safety - saturated backfill and no seepage flow.



(B) Percent change in factor of safety due to seepage flow.

Fig. 6 - Change in factor of safety of sheet-pile wall - due to seepage.

Upward forces near the sea wall face resulting from wave seepage pressures reduce the effective unit weight of the sea bed material by an amount equal to $i_v \gamma_w$. Figure 5 permits an evaluation of the hydraulic gradient in the sea bed in terms of the wave pressure gradient on the sea bottom, resulting from any given surface wave shape. Thus an estimate can be obtained of the contribution toward scour of the sea bed material which is produced by wave seepage pressures.

The effects of acceleration of flow due to the time rate of change of pressure distribution on the surface of the sea bed were neglected in this study. However, it might be anticipated that the upward hydraulic gradients near the sea wall face would be increased somewhat by accelerative flow.

CHANGES IN WALL STABILITY DUE TO SEEPAGE

In order to evaluate the effect of seepage on the stability of a vertical sheet-pile type sea wall, consider an anchored wall with free earth support as illustrated in Fig. 1 (a). The factor of safety with regard to rotation of the wall as a rigid body about the anchor point will be used as the criterion for evaluating the stability of the wall.

In order to simplify the equations for the factor of safety, it was assumed that the submerged unit weight of the soil, γ', is equal to the unit weight of water, γ_w. The factor of safety with and without seepage flow was determined for a head difference, Δh, as shown on Fig. 1 (a), assumed to be equal to its maximum value of H' - D. For the condition of hydrostatic pressure of soil and water acting on the wall, the factor of safety against rotation about the anchor point is,

$$F.S. = \frac{(D/H')^2 K_P (3\alpha - D/H')}{(1+K_A)(3\alpha-1) - (D/H')^2(3\alpha - D/H')} \qquad (6)$$

in which D is the depth of pile penetration, H' is the total pile length, and α H' represents the distance from the tip of the pile to the anchor point.

Equation 6 involves three geometrical variables, D, H', and α, and two quantities, K_A and K_p, which depend primarily upon the angle of internal friction of the cohesionless soil. The coefficient of active earth pressure, K_A, has a value of 0.3 for loose, clean sand, and a slightly lower value for sand in a more dense condition. In order to reduce the number of variables involved, K_A = 0.3 was used in Eq. 6. Then a diagram was prepared using the dimensionless ratio D/H' as abscissa and factor of safety, F.S., as ordinate. Values of α of 0.6 and 1.0 and K_p of 3, 5, and 7 were used as parameters to prepare the families of curves shown on Fig. 6 (a). Thus for a wall having particu lar geometrical ratios D/H' and α, the factor of safety will depend up‹ the allowable value of the coefficient of passive earth pressure, K_p. B definition, a factor of safety of 1.0 or greater is required for stability of the wall, consequently the curves shown on Fig. 6 (a) which extend below F.S. = 1.0 represent unstable conditions.

When seepage occurs, still with Δh maintained as the maximum value of H' - D, a change in the factor of safety of the wall occurs, or the factor of safety under seepage conditions is given as

$$F.S. = \frac{(3\alpha - D/H')\left[(D/H')^2 K_P - 2P(1 - D/H')\right]}{(3\alpha - 1)\left[1 + K_A - 2A\left\{\frac{D}{H'} - \left(\frac{D}{H'}\right)^2\right\}\right] - \left(\frac{D}{H'}\right)^2\left(3\alpha - \frac{D}{H'}\right)} \qquad (7)$$

Then the change in factor of safety as a result of the seepage flow can be determined from,

$$\Delta F.S. = \frac{\text{Eq. 6} - \text{Eq. 7}}{\text{Eq. 6}} \qquad (8)$$

Figure 6 (b) shows the percent change in factor of safety due to seepa flow. The shaded portions of the diagrams represent the portion of practical significance, for which the wall is stable under conditions o complete backfill saturation and no seepage flow. When seepage flow occurs, the factor of safety may be decreased or increased, however the effect is generally less than 10 percent different from that for the flow condition.

From this simplified treatment of the stability of sheet-pile typ sea walls it is evident that if such a wall is designed to withstand full water pressure difference and retain an adequate factor of safety und these conditions, that the additional effects of seepage flow will prod only small changes in the factor of safety.

EFFECTS OF CHANGES IN D/H'

From Fig. 6 (a) it is seen that the factor of safety depends ma edly upon the ratio D/H', when α and K_p are maintained constant. I order to study the relative importance of seepage compared to chang of D/H' as would occur as a result of decreasing D by scour, a spec example was chosen.

Figure 7 (a) shows the dimensions and soil characteristics chosen to represent approximately a sheet-pile type sea wall which failed during a mild storm. When the backfill is completely saturated, and the water level on the outside of the wall is just at the sand surface, the unbalanced water head, Δh, is 8.5'. For this condition and for no seepage flow, the factor of safety against rotation about the anchor point is 1.62. When seepage flow occurs the factor of safety is reduced to 1.57, or a reduction of about 3 percent. Consequently, the effect of seepage flow alone is insignificant.

When the depth of embedment, D, is varied, the effect on the factor of safety is as shown in Fig. 7 (b). The wall becomes unstable when D is reduced just slightly more than one foot, for the condition including seepage flow, and for slightly less than 1.5' for no flow. This magnitude of scour has been observed at the face of sea walls that failed, and it is probable that backfill saturation and toe scour were important factors in the failures.

The horizontal forces developed during seepage flow thus appear to be of small importance compared to changes in depth of embedment as each contributes to a reduction in the factor of safety of sheet-pile type sea walls with free earth support.

EFFECT OF SEEPAGE ON SCOUR AT WALL FACE

HYDRAULIC GRADIENTS AND SCOUR VELOCITY

The preceding section has illustrated the importance of relatively small reductions in the depth of wall embedment upon the factor of safety. The depth of material which restrains the toe of the wall against outward motion is reduced when scour occurs in this region; scour is particularly important when it occurs adjacent to the wall face. Thus it becomes necessary to estimate the effects contributed by seepage flow toward increasing the probability of scour at the face of the wall.

From laboratory studies, such as those presented by Ippen and Verma (1953), it has been shown that scour is a complex phenomena, even in a controlled laboratory flume. The scouring action of waves reflected from a vertical wall, with appreciable air and soil contained in the turbulent water represents an even more complex problem. However, the contribution of vertical seepage toward increased scour can be estimated by considering only its effect on the unit weight of the soil particles.

Figure 8 shows a surface cohesionless soil particle being subjected to potential scour by water moving across the particle with velocity = V. The forces acting on this particle are as follows:

F_D = drag force = B $V^{3/2} \approx B_1 V^2$ (Ippen and Verma, 1953)

F_L = lift force = C V^2 (Ippen and Verma, 1953)

W' = effective weight of the particle

F_f = maximum friction force = $(W' - F_L) \tan \phi$; where ϕ is the friction angle and includes both the true friction and particle interlocking effects.

An upward hydraulic gradient, i_v, through the soil bed on which the soil particle is resting reduces the effective weight of the soil particle thus reducing the friction force.

The maximum non-scour velocity, V, is attained when $F_D = F_f$, and $i_v = 0$, and may be expressed as:

$$B_1 V^2 = \left(W'_{i_v=0} - CV^2\right) \tan \phi \tag{9}$$

When upward seepage is occurring, ($i_v \neq 0$), W' is reduced and the maximum non-scour velocity is reduced to V'. Equation 9 then becomes:

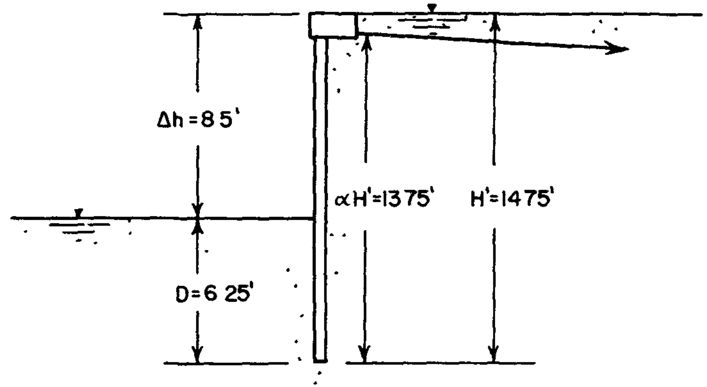
$$B_1 V'^2 = \left(W'_{i_v \neq 0} - CV'^2\right) \tan \phi \tag{10}$$

Dividing equation 10 by 9, and simplifying, gives:

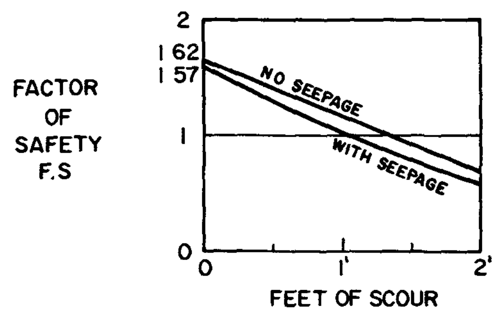
$$\frac{V'}{V} = \left[\frac{W'_{i_v \neq 0}}{W'_{i_v = 0}}\right]^{\frac{1}{2}} \tag{11}$$

The ratio V'/V represents the factor by which the horizontal velocity has to be reduced to prevent scour after an upward hydraulic gradien has developed. This ratio will be called the "scour velocity reductio factor" and be given the symbol R.

Any percent reduction in the effective weight of each soil partic results in a similar reduction in the intergranular pressures within t



(A) Dimensions of example sea wall (saturated backfill, and $K_A = 0.3$, $K_p = 7$)



(B) Variation of F. S. with seepage and scour

Fig. 7 - Example of the effect of seepage and scour on the stability of a sheet-pile type sea wall.

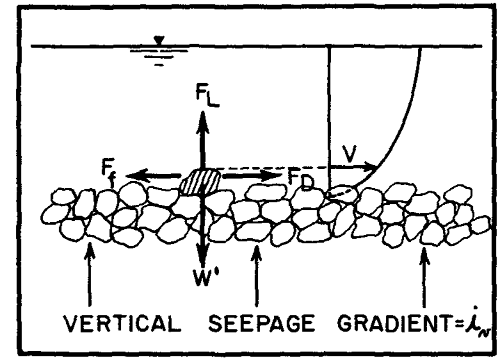


Fig. 8 - Forces on sand particle subjected to scouring action.

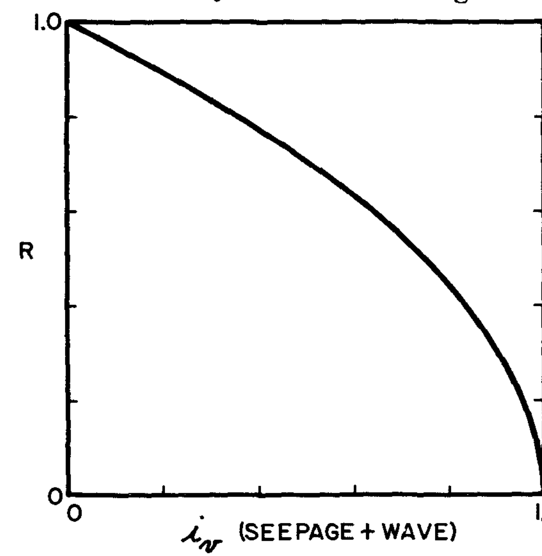


Fig. 9 - Scour velocity reduction factor vs. vertical hydraulic gradient.

soil mass. An expression for ratio of vertical intergranular pressure, $\bar{p}$, and therefore also of individual particle weight, W', at any depth z in a submerged soil mass, before and after upward seepage flow is:

$$\frac{\bar{p}_{i_v \neq 0}}{\bar{p}_{i_v = 0}} = \frac{W'_{i_v \neq 0}}{W'_{i_v = 0}} = \frac{z\left[\left(\frac{G-1}{1+e}\right)\gamma_w - i_v \gamma_w\right]}{z\left(\frac{G-1}{1+e}\right)\gamma_w} = \left[1 - i_v\left(\frac{1+e}{G-1}\right)\right] \quad (12)$$

The specific gravity values, G, of the three most common sand grain minerals are quartz = 2.66, calcite = 2.72, and feldspar = 2.56. A common range of void ratio values, e, for uniform sand is 0.55 to 0.7 Therefore, unless G and e are materially different from the above, equation 12 may be written with sufficient accuracy as:

$$\frac{W'_{i_v \neq 0}}{W'_{i_v = 0}} \approx (1 - i_v) \quad (13)$$

From equations 11 and 12, one can obtain:

$$\frac{V'}{V} = R = \left[1 - i_v\left(\frac{1+e}{G-1}\right)\right]^{\frac{1}{2}} \quad (14)$$

Or, from equations 11 and 13 one can obtain the more approximate fo

$$\frac{V'}{V} = R = (1 - i_v)^{\frac{1}{2}} \quad (15)$$

Equation 15 has been used to prepare a graph of the relationshi between the scour velocity reduction factor, R, plotted against the ve tical hydraulic gradient, i_v. This graph is presented in Fig. 9. At instant of time, i_v represents the summation of the gradients due to steady seepage from the backfill and the transient seepage condition to wave action.

EVALUATION OF EXIT GRADIENT DUE TO BACKFILL SEEPAGE

The flow nets constructed for the purpose of evaluating the hoi zontal forces on the wall due to seepage also provide values of the e hydraulic gradient at the soil surface. For this study, flow nets we constructed for a D/H range from 0.25 to 0.50, and for a D/D' ran of 0.1 to 0.7. By expressing the exit gradient as

$$i_e = S\left(\frac{\Delta h}{D}\right) \tag{16}$$

the extreme values of S were found to be 0.21 and 0.28 for values of the parameters D/D' = 0.7, D/H = 0.25, and D/D' = 0.1, D/H = 0.5, respectively. Values of "S" obtained from data given by McNamee (1949) for the same range of geometrical parameters were found to be 0.22 and 0.30, respectively. In comparing the results of eleven flow nets with McNamee's results, the deviations in individual values of exit hydraulic gradient, i_e, varied from zero to 7 percent. This indicates that the graphical flow nets were uniformly accurate.

Since the values of S were found to have a fairly small variation, it is suggested that an average value of 0.25 be used in Eq. 16, or that the exit hydraulic gradient caused by seepage through the backfill be taken as,

$$i_e = \frac{1}{4}\left(\frac{\Delta h}{D}\right) \tag{17}$$

The use of S = 1/4 involves a maximum error of 17% for the cases studied, but this error is no doubt much smaller than those arising from the differences between actual soil conditions and the homogeneous, isotropic, conditions assumed as a basis for the flow net construction. The use of S = 1/4 is a slight refinement of the upper limiting value of S = 1/3 which was suggested by Terzaghi (1954).

EXAMPLE

The example of Fig. 7 (a) is here continued to investigate the importance of seepage gradients on potential scour in front of this wall. Consider the case of this wall during a storm, when rainfall and over-splash have completely saturated the backfill and waves are striking and being reflected by the wall.

For the maximum value of Δh of H' - D used with Eq. 17, the exit hydraulic gradient due to seepage through the backfill for the example, Fig. 7 (a), is

$$i_e = \frac{H'-D}{4D} = 0.34 \tag{18}$$

In addition to the steady seepage, wave pressures on the surface of the sea bed produce a maximum value of vertical hydraulic gradient at the face of the wall of,

$$i_{max} = 1.6 \frac{p_w}{\gamma_w L} \tag{19}$$

and an average value over a distance 0.2L deep and 0.2L away from th wall face, of,

$$i_{ave} = 0.66 \frac{p_w}{\gamma_w L} \tag{20}$$

For a value of $p_w/\gamma_w L = 0.4$, the maximum and average value of vertical hydraulic gradient due to wave pressures are,

$$\begin{aligned} i_{max} &= 0.64 \\ i_{ave} &= 0.26 \end{aligned} \tag{21}$$

These temporary gradients are added to the steady value due to backf seepage to give

$$\begin{aligned} (i_v)_{TOTAL_{MAX}} &= 0.98 \\ (i_v)_{TOTAL_{AVE}} &= 0.60 \end{aligned} \tag{22}$$

When entering Fig. 9 with the above results, it may be seen tha the scour velocity reduction factor, R, is almost zero for the maximu gradient immediately adjacent to the wall, and is about 0.6 for the av age condition over the 0.2L distance from the wall. Thus, the backfi seepage and wave conditions assumed in this example are certain to result in scour immediately adjacent to the wall, and there is a great increased likelihood of scour for a distance of at least 0.2L from the wall.

METHODS OF ELIMINATING OR MINIMIZING SCOUR

Scour at the face of a vertical sheet-pile type sea wall may cau a marked reduction in factor of safety of the wall, as was demonstra in the study of the example shown on Fig. 7. Therefore, precautions should be taken to minimize the possibilities of scour at the most cri cal regions.

Since upward seepage forces contribute to the probability of sc methods of preventing or controlling seepage flow through the backfil should be incorporated into the design of the wall. Paving the surfac of the backfill is one obvious method of preventing water from enteri

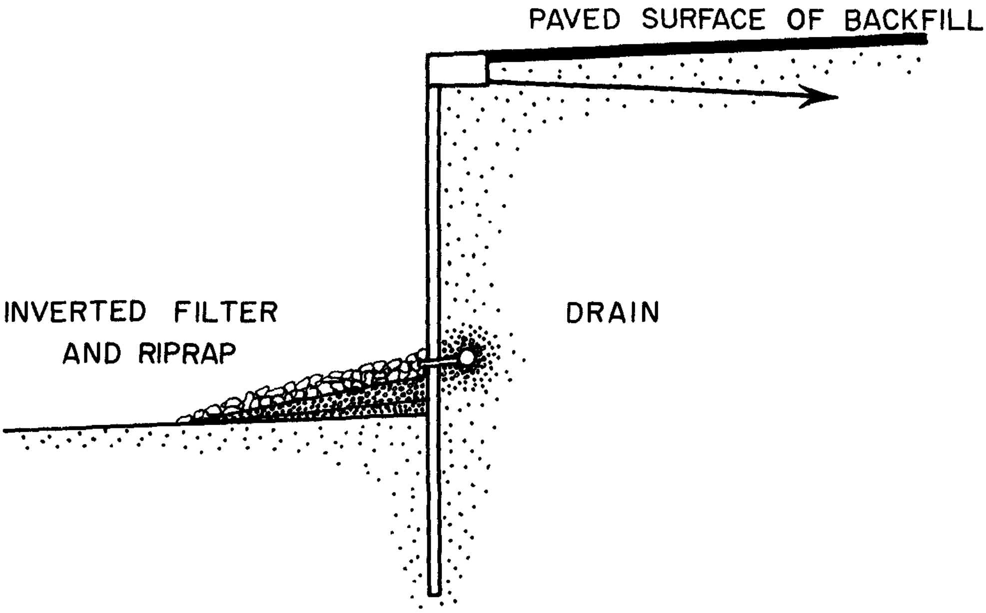


Fig. 10 - Methods of minimizing effects of seepage.

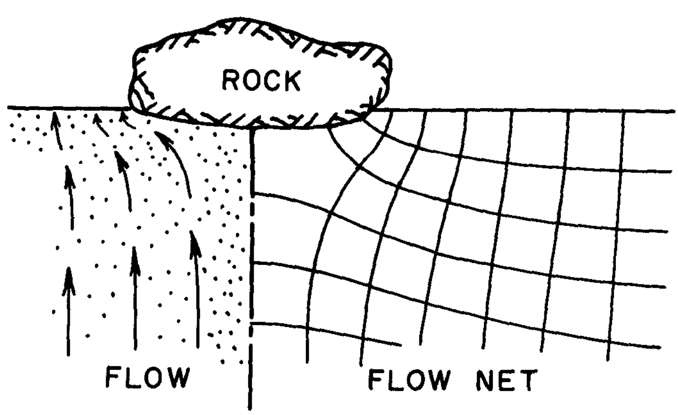


Fig. 11 - Effect of impermeable body on vertical seepage flow.

the backfill by oversplash or rain run-off, and has been found effectiv in increasing the wall stability. Drains are also desirable to intercep water which leaks through cracks in the paving, or to equalize the unbalanced water pressure developed during a rapid drawdown of the me sea level. By intercepting the water flow, the drains prevent flow beneath the toe of the wall and do not allow vertical hydraulic gradients t develop in the zone of passive soil pressure. Figure 10 illustrates an effective location for a drain system; the drain is low enough that larg unbalanced water heads cannot develop.

Upward seepage forces on the sea bed can be counteracted by pr viding static weight near the wall face. Riprap or large rocks must b used as cover for this loaded region to maintain the load at the prope location, even during severe action by longshore currents and breakir waves. Criteria established for breakwater design, such as those given by Hedar (1953), may be used to select the proper size of cover material. However, it is not satisfactory to place large rocks direct upon beach material, since the finer material may be carried througl the voids in the larger material by water flow.

Figure 11 illustrates the disturbance of vertical flow by an impermeable object resting on the surface of the permeable bed. The flow of water must detour around the obstacle, thereby crowding the flow lines together and increasing the hydraulic gradient near the boundary of the rock. In this region the effective weight of the bed material has been reduced by the upward flow. Erosion will occur beneath the edges of the rock as a result of horizontal water velocities caused by waves acting on the bed material. Progressive undermini of the edges of the rock will eventually cause it to sink into the sand and become ineffective as scour protection.

More than forty years ago it was recognized that a permeable surcharge placed over the region of upward seepage flow would prev the occurrence of "piping" beneath structures due to seepage flow through permeable foundations. Terzaghi has made extensive use of such a permeable surcharge constructed in the form of an "inverted filter," in which the layers of cohesionless materials increased in grain size toward the top. He also established relations for the gra size variations of successive layers. These rules for grain size va ations were later modified slightly as a result of extensive tests by Waterways Experiment Station, Vicksburg, Mississippi, and the re mended design procedures are summarized in the paper by Posey (1

Filters should also be placed around collector pipes in the bac fill drain system to prevent the backfill material from flowing out through the drains. A saturated cohesionless backfill will flow rea through relatively small holes or cracks in a retaining wall and can cause a cave-in of the backfill surface. Several cave-in type failures have been reported by Gebhard (1949) which were caused by sand flowing through small holes or cracks in the structure. When a cave-in type failure of a sea wall backfill occurs during a storm, the soil resistance to wave forces is eliminated at this location and the wall is knocked over, landward, by repeated wave impacts. Thus the stability of the sea wall against this type of failure depends upon maintaining continuity of the backfill.

CONCLUSIONS

The effects of seepage flow on the stability of vertical sheet-pile walls were considered in this study. Stability was evaluated in terms of the factor of safety against rigid body rotation of the wall about the point of anchor attachment.

Seepage flow through the backfill and under the wall causes horizontal forces on the wall as a result of the changes in water and soil pressure distributions from those corresponding to the hydrostatic condition. The net effect on the factor of safety produced by these changes in water and soil pressures was found to be unimportant for the cases studied.

A study of the importance of the geometrical parameters of a sea wall demonstrated that the factor of safety changes significantly with small changes in the embedded depth of the wall. Removal of material at the outer face of the wall by scour changes the embedded length of the sea wall and in this way scour may change the factor of safety of the wall appreciably.

One effect of seepage through the backfill is to reduce the effective density of the cohesionless material in front of the sea wall. This steady state reduction in soil density is reinforced by a transient effect resulting from seepage flow induced by pressure gradients developed along the sea bottom by water waves. A diagram is included which illustrates the relation between the exit hydraulic gradient due to seepage flow, which determines the effective soil density, and the reduction in value of horizontal water velocity required to produce scour.

A brief discussion is also included of methods for reducing or eliminating scour of material at the face of the sea wall.

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APPENDIX

LIST OF SYMBOLS

A = coefficient used to calculate ΔF_A
B = constant
C = constant
D = depth of penetration of sheet pile wall (ft.)
D' = depth from dredge level to first impervious soil layer (ft.)
e = void ratio of soil
F_D = drag force due to water flowing across part surface soil particle (lbs.)
F_f = friction force resisting scour movement of soil particle (lbs.)
F_L = lift force due to water flowing across part surface soil particle (lbs.)
F.S. = factor of safety
ΔF_A = net change in force on active side of wall, due to backfill seepage effects (lbs.)
ΔF_P = net change in force on passive side of wall, due to backfill seepage effects (lbs)
G = specific gravity of soil solids
H = height from tip of wall to water level in backfill, but not to exceed H' (ft.)
H' = total height of wall backfill, from embedded tip of sheet piles (ft.)
Δh = height difference between water level behind and in front of wall (ft.)
i = hydraulic gradient (ft. /ft.)
i_e = vertical exit gradient, immediately seaward of wall, due to backfill seepage
i_v = vertical component of hydraulic gradient
K = sub-surface pressure response factor
K_A = coefficient of active earth pressure
K_P = coefficient of passive earth pressure
L = assumed distance between points of maximum and 0 wave pressure on a horizontal sea bed (f)
P = coefficient used to calculate ΔF_P
p_A = active earth pressure (lbs. /ft.^{2})
p_P = passive earth pressure (lbs /ft.^{2})
p_w = water pressure (lbs. /ft.^{2})
q = rate of seepage flow (ft.^{2}/sec.)
R = scour velocity reduction factor V'/V
S = coefficient used to determine i_e
t = time
V = maximum water velocity across top of soil particle, without scour movement of particle (ft. /sec.)
V' = value of V when upward seepage is occurring (ft./sec)
W' = effective submerged weight of individual soil particle (lbs.)
x = distance along the sea bottom (ft.)
z = depth from soil surface (ft.)
α = ratio of height to anchor point/total backfill height
γ = unit weight of soil (lbs. /ft.^{3})
γ' = submerged unit weight of soil (lbs. /ft.^{3})
γ_w = unit weight of water (lbs. /ft.^{3})
ϕ = angle of internal friction of soil

CHAPTER 49

JETTY FOUNDATIONS ON FINE SEDIMENTS

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Mexico City, Mexico

SYNOPSIS

The Secretaría de Marina Nacional of Mexico, contemplates the construction of two long jetties at the mouth of the Grijalva River to permit safe navigation into the port of Frontera in the state of Tabasco, Fig 1. The port of Frontera is located in the estuary of the Grijalva River 9Km. from its mouth. The proposed jetties should reach into the sea to a depth of water of 6 mts. This requires a length from the mouth of the river of about 2000 mts, Fig 2. Rock fill jetties constructed in the past in this area on the fine sediments have failed by spreading and penetration into the fine cohesionless sediments encountered at the sea bottom. Heavy structures cannot be constructed on account of the low shearing strength of the submarine delt clay deposits that may be encountered at the mouth of the Grijalva River.

Subsoil investigations were performed by the author to learn the mechanical properties of the materials at the mouth of the Grijalva River, and made possible the design of light-weight and strong structural jetties to resist the sea and river forces to which these structures will be subjected. The problem involved and stability considerations of the jetties are explained by the author in this paper.

GEOTECHNICAL STUDIES

Subsoil investigations at the mouth of the Grijalva River indicated a surface deposit of about 5 mts thick of fine sand with frequent diameter ranging between 0.15 and 0.20 mm. This material is the product of the present sedimentation of the river and covers uniformly all the mouth and extends in the sea at least 3 Km. from the mouth of the river. The typical subsoil profile obtained from one of the continuous 4 inches undisturbed sample cores taken up to a depth of 37 mts is shown in Fig 2. The thickness of the upper sand deposit at the river mouth changes because of deposition and erosion by the currents of the river during the different seasons of the year, Fig .

Overlain by the fine sand deposits it may be encountered a stratificat of gray clayey-silt with small shells and average thickness of 1.2 mts. The water content of this material assumes a value of 60-70%. Consolidated undrained tests in this material give 0.17 Kg/cm^2 for cohesion and 22°50' for the ϕ_{cq} angle of internal friction. This stratification containing small ells is a good marker to correlate the stratigraphy from one bore hole to a

Fig. 1. Picture of mouth of Grijalva River.

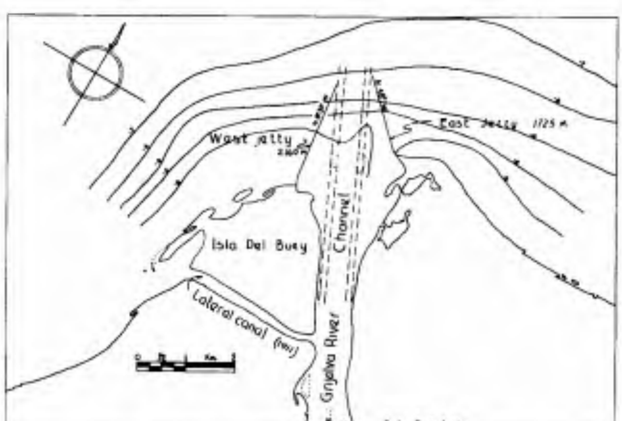
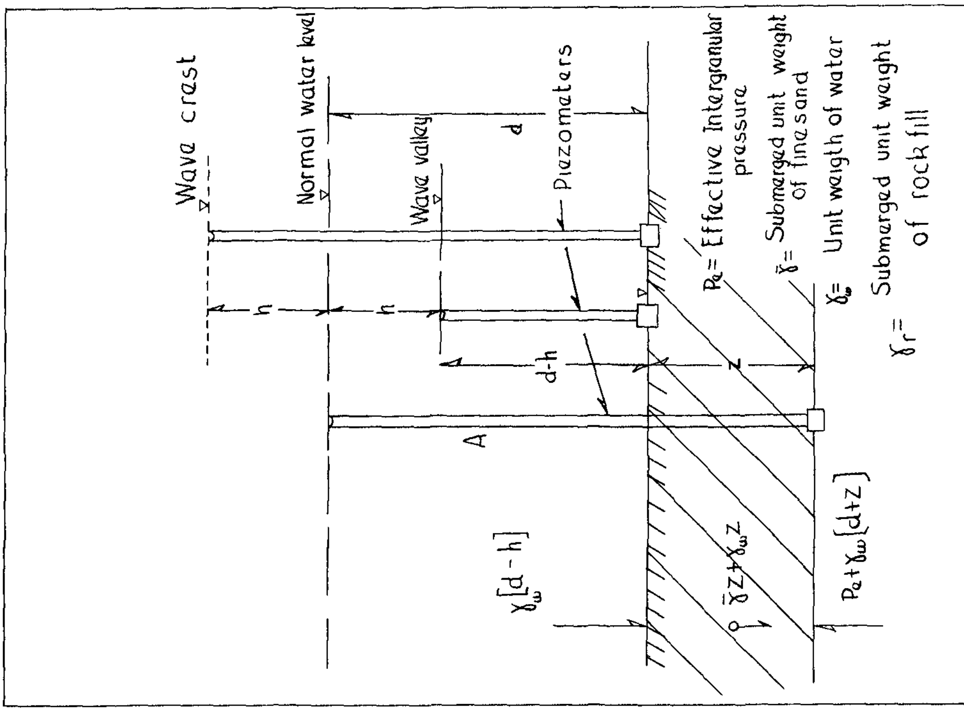
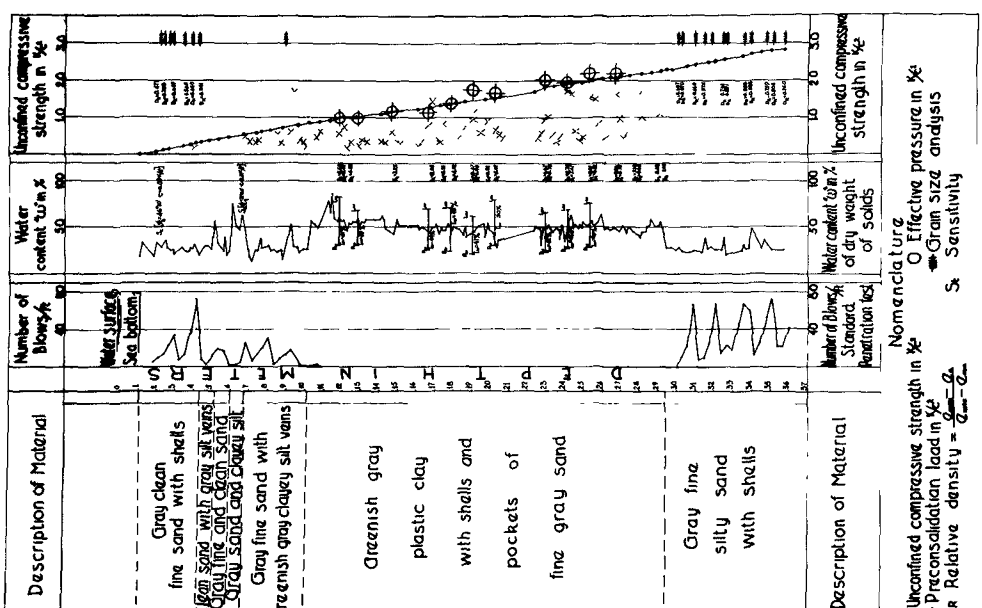


Fig. 2. Proposed location of jetties.



Wave crest
Normal water level
Wave valley
Piezometers
d
h
d-h
A
z
$\gamma_w(d-h)$
$\bar{\gamma}z+\gamma_w z$
$P_e+\gamma_w(d+z)$
P_e = Effective Intergranular pressure
$\bar{\gamma}$ = Submerged unit weight of fine sand
γ_w = Unit weigth of water
γ_r = Submerged unit weight of rock fill



Description of Material
Number of Blows/ft
Water content 'w' in %
Unconfined compressive strength in k/ft^2
Water surface
Sea bottom
Gray clean fine sand with shells
Clean sand with gray silt veins
Gray fine and clean sand
Gray sand and clayey silt
Gray fine sand with greenish gray clayey silt veins
Greenish gray plastic clay with shells and pockets of fine gray sand
Gray fine silty sand with shells
Number of Blows/ft Standard Penetration test
Water content 'w' in % of dry weight of solids
Nomenclature
X Unconfined compressive strength in k/ft^2
Preconsolidation load in k/ft^2
O Effective pressure in k/ft^2
Grain size analysis
D_R Relative density
S_t Sensitivity

other. Following this stratum a uniform deposit of silty sand with little clay may be found with a thickness of 3.1 mts correlating very well in all the bore holes. The average water content is 23% and the unconfined compressive strength as low as 0.2 Kg/cm^2.

From 10 to 29.5 mts depth it may be encountered a thick greenish-gray clay deposit with microscopic shells, and underlain by gray silty sand with small shells. This deposit is the product of the sedimentation of the finer alluvial sediments deposited in the sea. This fines coming into the sea flocculated and settled forming the submarine delta of the river. However, the very fine submarine sediments found in the present at the mouth of the river were undoubtedly formed when the mouth was several kilometers behind its actual position. The recent advance of the mouth of the Grijalva River toward the sea has produced the upper coarser sediments underlain by the clay deposit.

The average water content of the clay deposit is about 55% and shows practically constant with depth Fig 3. The clay is normally consolidated. This fact may be demonstrated by the good agreement shown between the break in the compressibility curves and the computed overburden pressures.

The shearing strength of the clay deposit between 10 and 29.5 mts depth obtained from unconfined compression tests shows a very erratic value, probably because of the strong difference in the salinity of the water during the process of sedimentation and flocculation of the fine grains. This fact may have had an important influence in the shearing strength of the clay; However, the minimum value of the shearing strength is of 0.15 Kg/cm^2 at 10 mts depth and increases only to 0.2 Kg/cm^2 at 29.5 depth.

From numerous consolidation tests on laterally confined specimens it was found an average value of $m_v = 0.033$ to 0.038 cm^2/Kg based on a period of 100 years, when the secondary settlement is taken into account (1). Primary consolidation takes place in only 20 years after load application.

Using the settlement analysis (1)(2) equations:

$$S = m_{v_1} F(T_v)\,\Delta p \cdot H \quad ; \qquad t < t_a$$

$$S = \left[m_{v_1} F(T_v) + m_t \log \frac{t}{t_a} \right] \Delta p \cdot H \; ; \quad t > t_a$$

The corresponding values of the parameters are:

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(1) Ecuación Completa de Consolidación para Depósitos de Arcilla que Exhiben Fuerte Compresión Secundaría, by Leonardo Zeevaert, June 1957 Published in Revista Ingenieria.

(2) Consolidation of Mexico City Volcanic Clay by Dr. Leonardo Zeevaert, Proceedings Joint Meeting of ASTM and SMMS, Mexico City , Dec.9-13-57

$$m_{vi} = 0.027 \quad \text{to} \quad 0.035 \quad cm^2/kg$$

$$m_t = 0.011 \quad \text{to} \quad 0.012 \quad cm^2/kg$$

$$T_{va} = 1.0$$

$$C_v = 0.00159 \quad \text{to} \quad 0.00146 \quad cm^2/sec$$

$$T_v = \frac{4C_v}{H^2} \cdot t \quad ; \quad t_a = \frac{T_{va}}{4C_v} \cdot H^2$$

$$t_a \doteq 20 \text{ years.}$$

The total thickness of the deposit may be taken as 19.5 mts drained ‹ top and bottom.

SUBSIDENCE PROBLEM

When a heavy rock fill is constructed on these fine sand, silt and c sediments to form a jetty or any water wave protection, it is necessary t consider in the design the following phenomena:

a.- penetration of the rock into the fine cohesionless sediments because of the possibility of spontaneous liquefaction.

b.- spreading of the slopes of the rock fill because of material ero sion in the foundation.

c.- subsidence of the fill because excessive shearing stresses in th underlying clay deposit.

d.- large settlement of the fill because of excessive compression.

Items (a) and (b) are produced by wave action and (c) and (d) becau of exseeding, respectively, the allowable mechanical properties of sheari strength and compressibility of the soft clay deposit.

(a) Penetration of the rock into the fine cohesionless material is ‹ used by the reduction of shearing strength in the sand at the passage of water waves. That is to say, as a water wave passes over certain point ‹ the slope of the jetty the hydrostatic pressures in the soil mass cannot just themselves to the instantaneous hydrostatic excess water pressures. erefore, an important hydrodynamic lag may be created in the sand suppor the rock material of the jetty, thus the hydrostatic excess pressure cre momentarily an spontaneous liquefaction condition of the sand permitting rock to penetrate into the fine sand. This effect is a function of the wave height. To demonstrate this phenomenon let us consider Fig 4 a wave coming along the jetty. The crest rises from the normal water level in approximately "h" and as the valley of the wave passes along, the water level drops from the average level in approximately "h". One piezometer A installed at depth "z" into the very fine sand deposit will preserve the normal or average water level as the water wave passes rapidely. Therefore, an important water uplift pore pressure in the soil will take place as the valley of the wave passes over the point considered. Taking in consideration, Fig 4, the forces acting for equilibrium in the upper part of the deposit of thickness "z" may be obtained the following equilibrium equation:

$$p_e + \gamma_w (d+z) = \bar{\gamma} z + \gamma_w z + \gamma_w (d-h) \quad \text{------} \quad (1)$$

thus:

$$p_e = \bar{\gamma} z - \gamma_w h \quad \text{------} \quad (2)$$

The effective intergranular pressure at depth "z" is equal to the weight $\bar{\gamma} z$ of the submerged sand, minus the uplift pressure $\gamma_w \cdot h$ produced by the wave height. The equilibrium is unstable up to a depth where: $p_e = 0$

hence; $z = \dfrac{\gamma_w}{\bar{\gamma}} \cdot h$, since: $\dfrac{\gamma_w}{\bar{\gamma}} \doteq 1$, then $z \doteq h$

This implies that the waves produce an unstable condition in the cohesionless fine sand and silty sediments of the bottom of the sea to a depth approximately equal to the semi-height of the waves.

A heavy rock fill will penetrate into the fine sediments to a depth where p_e starts to be larger than zero.

Therefore, if the submerged weight of the rock fill is $\bar{\gamma}_r$ and its thickness above the sand is D then:

$$\bar{\gamma}_z - \gamma_w h + \bar{\gamma}_r D = 0 \quad \text{------} \quad (3)$$

from which:

$$z = \frac{\bar{\gamma}_r \cdot D - \gamma_w h}{\bar{\gamma}} \quad \text{------} \quad (4)$$

The value of "z" represents the theoretical depth at which the rock fill will penetrate into the sand. Practically no penetration will take place for depths of fill:

$$D \geq \frac{\gamma_w}{\bar{\gamma}_r} \cdot h \qquad \text{------ (5)}$$

However, at the foot of the slope there is always the tendency to have a penetration equal to that given by formula (2).

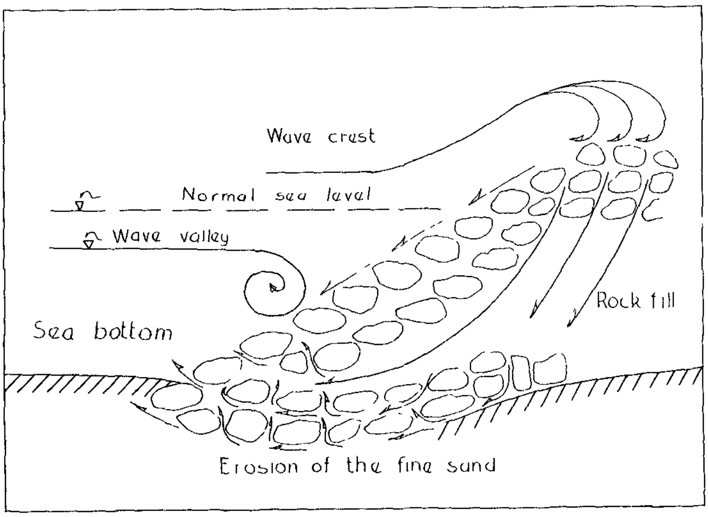

Fig. 5. Erosion in rock fill.

(b) The spreading of the rock fill because of erosion is a very impor tant phenomenon that may take place because of erosion at the foot of the slope of partially submerged rock fills on fine cohesionless sediments, lik fine sand. This phenomenon is facilitated as the fine sand becomes loose during spontaneous liquefaction as explained above (a). Let Fig 5 be the slope of a partially submerged rock fill, as the crest of the water wave co mes along the slope, and the water fills up the voids left in the rock fil Following the valley of the wave, because of the lower water level, the wa ter flows strongly out from the inside of the rock fill, producing a stron erosion in the fine sand at the base, Fig 5. Therefore, the stability of the slope may be lost and spreading may take place. Furthermore, the combined effect of uplift pressure, as explained before, and the horizontal f ce produced by the rapid drawdown as the valley of the wave passes along m produce eventually a total spreading of the fill. The shearing strength a the base of the rock fill may be not enough to counteract the internal wat force developed in the rock fill. A spreading failure of this type in con junction with penetration in the fine sand was originated in old jetties c structed in this location in 1911. The observations made in 1950 to find the section of this rock jetties showed they had spread and penetrated str ly into the sand, demonstrating this type of failure.

(c) Breaking of the rock fill into the ground may be controlled by not exceeding the shearing stresses in the soft clay deposit.

(d) Excessive settlement may be avoided by not overloading the clay deposit. Since the clay deposit is not of high compressibility it appears from settlement analysis that the computed settlements with safe bearing loading capacity will produce reasonable settlements that can be taken safely by the structure.

STABILITY OF PROPOSED SECTION

The decision of the proposed section was obtained after carefull study of the subsoil mechanical properties in conjunction with the theoretical studies concerning the possibility of spontaneous liquefaction of the fine sand sediments, erosion and the phenomenon of possible spreading of the partially submerged rock fill. Furthermore, since the shearing strength of the clay is only 0.15 Kg/cm^2 it was decided to abandon, for safety, the idea of a partially submerged rock fill jetty. After studying other possible sections the author decided to recommend a structure of reinforced concrete boxes built with a system of sheet piles driven previously and forming a cofferdam-like structure, Fig 6. The structure, thus formed is confined laterally against strong erosion of the fine sediments by means of a minimum rock fill placed at the bottom of the sea against the proposed structure. With the purpose of resisting the large dynamic lateral forces because of the impact of the breaking waves against the structure, it was necessary to anchor properly the sheet-piles into the clay deposit. In this fashion it was possible to make a very favorable use of the shearing strength of the clay.

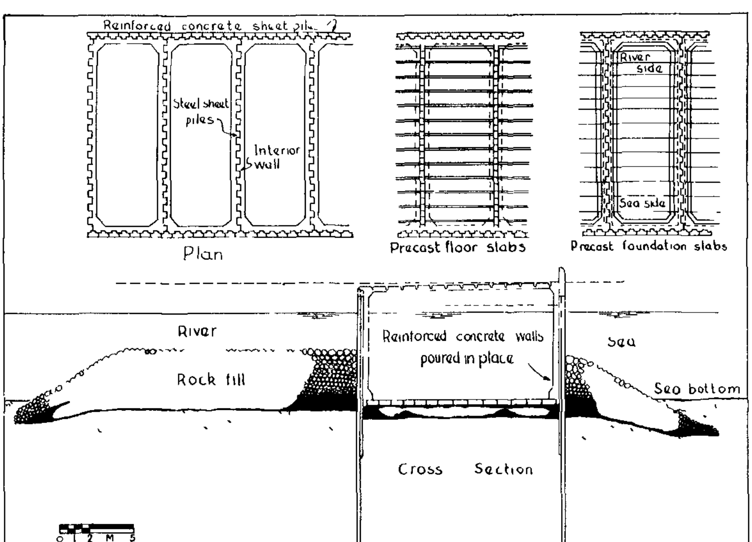

Fig. 6. Proposed section.

However, the coffer-dam formed of hollow reinforced concrete boxes Fig 6, is alone not enough to hold the large thrust of the breaking waves on the structure. Therefore, to increase the safe bearing capacity it was necessary to confine the bottom of the sea close to the reinforced concret structure by means of a rock fill, thus increasing also the stability of tl structure as a whole and reducing the dimensions and cost of the central p tion.

On the river side a larger rock fill is provided to avoid any possibl sliding shear failure either along the clay or through the fine sand. The cross section of the proposed jetty for 6 mts depth is shown in Fig 6. Th central structural part of the jetty shows perfectly fixed by the sheet-pi to the subsoil.

The transverse walls formed by sheet-piles of steel "Z" section are tl basic elements of strength in the jetty. The depth and spacing of these diaphragms was computed in order not to exceed the allowable shearing stre of the clay during the large transient forces applied by the waves on the side of the jetty. The dynamic force is transmitted to the diaphragm and transmitted by shear into the subsoil. The reinforced concrete, Fig 6 box type structure above the bottom of the sea makes the entire structure to w as a unit.

The rock fill used to confine the structure was computed following co clusions on the spontaneous liquefaction and erosion as described before a taking into account that on a long term basis the rock fill at the foot of the slope will penetrate one-half the wave height into the fine sand, because the dynamic action of the waves, Fig 6.

Using the properties of compressibility obtained from consolidation t as reported in the first part of this paper, it was found that the settlem of the jetty would be on the order of 17.4 cm in 100 years. From which 15. cm. will be primary consolidation taking place in approximately 20 years. The compressibility of the clay deposit is rather uniform in a large exten therefore differential settlements will be of minor importance to the beha and maintenance of the jetties.

Pre-stressed concrete was recommended by the author for the construct of all the pre-cast reinforced concrete elements as the lateral sheet-pile foundation slab and floor slab. The walls will be poured in place. The f slab is designed to hold truck loads up to H-15.

Acknowledgement is due to the Secretaría de Marina Nacional of Mexico for allowing the author, to release the information given in this paper. To Ingeniero Alfonso Peiré Ruelas, Sub-Secretario, for his enlightening su gestions during the time the author acted as consulting engineer for the d partment in connection with the studies made to solve this problem. To Pr fesor Richard Foster Flint for valuable help given to the author during th geological studies pertaining the problems of erosion and sedimentation in the vicinity of the mouth of the Grijalva River.

CHAPTER 50

EFFECT OF CLAY CONTENT ON STRENGTH OF SOILS

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(Supported by a contract with Office of Ordnance Research
Ordnance Corps, U. S. Dept. of Army)

ABSTRACT

Engineers have worked greatly on measuring the strength of soils but relatively little on the fundamental geologic causes of strength. Strength depends principally upon the content of (1) water, (2) clastic materials and (3) plastic materials. Soils are primarily of two types (1) cohesionless soils in which the strength is produced mainly by the friction of clastic particles against one another, and (2) cohesive soils in which the strength, among other things, is influenced by forces between clay particles. The present investigation is a study of the effect of clay content upon the strength of cohesive soils. The strength was measured by a shear vane device working upon synthetic mixtures of clays of known composition. In each mixture strength varies inversely with water content in a straight line relationship when strength is plotted logarithmically and water arithmetically. Mixtures of glycerine with vol-clay (a montmorillonite) give a curvilinear relationship. For given water content the strength increases with respect to type of clay from kaolin through illite, ball clay to montmorillonite. Strength also increases progressively with increasing clay-sand ratio for all types of clay. In clay-sand mixtures of given clay composition strength increases with increasing fineness of grain of the sand mixed with clay. The liquid limit likewise increases regularly with increasing clay concentration and varies with clay type in the same way as does strength. Strength varies inversely with temperature to a slight extent, changing less than one percent per degree Centrigrade. Hydrogen kaolin clay, for given water content is several times stronger than sodium clay.

INTRODUCTION

The Office of Ordnance Research of the Department of the Army, through a contract with the Institute of Engineering Research of the University of California at Berkeley, has been supporting an investigation of the fundamental geologic causes of strength in soils. Engineers are primarily interested in determining quantitatively the strength of soils, rather than the basic factors that impart strength. The purpose of this investigation is to study the effects of some of the basic factors influencing strength of soils. Strength is caused principally by three factors (1) water, (2) clastic particles, and (3) plastic particles. Soils and sediments are of two types with respect to strength: (1) cohesionless soils, composed mainly of clastic particles and (2) cohesive soils, which contain substantial quantities of plastic particles and varying quantities of clastic particles. In the cohesive soils composed of clastic particles, that is, broken or transported particles of sand or silt size, the strength is caused primarily by the friction of the particles against one another and water is of relatively little effect. In the cohesive soils the strength is influenced by forces of attraction between particles of clay size. It is convenient to think of such particles as plastic particles because they impart plasticity and cohesiveness to soils. If clastic particles are present in cohesive soils they modify the strength. As the causes of strength in cohesive soils are less well understood than the causes of strength in cohesionless soils, the present investigation has been devoted primarily to the causes of strength in cohesive soils, and in particular to the effect of clay content upon strength. The variables that have been studied are (1) clay type, (2) clay-sand proportion, (3) grain-size of the clastic particles mixed with the clay, (4) base exchange effects, (5) temperature, (6) thixotropic effects, and (7) glycerine content in glycerine-clay mixtures. Numerous other variables affect the strength of soils but these are not considered in the present investigation.

METHODS OF ATTACK

MATERIALS STUDIED

The method of approach has been to use clays of essentially pure composition and sands of differing grain-size. The clays that have been used are. (1) Edgar China clay from Georgia, which is nearly pure kaolin about midway in composition between a hydrogen sodium clay (Fig. 11); (2) Illinois grundite, which consists of about 80 percent illite and 20 percent of clastic material, which X-ray and Differential Thermal Analysis curves indicate to be mainly quartz;

(3) Kentucky Old Mine No. 4 Ball clay, a mixture of about 85 percent kaolin and 15 percent silica with an unknown content of organic matter, called Ball clay No. 1 in this report, (4) Kentucky Mine Special, a ball clay estimated to contain 80 percent kaolin and the remainder quartz, called Ball clay No. 2, (5) Wyoming bentonite, a clay estimated to be 97 percent montmorillonite and 3 percent quartz; and (6) Vol clay, a nearly pure bentonite. The median diameter of the clays range mainly between 1 and 2 microns, though precise figures for illite and montmorillonite are not available.

Each of the six types was first tested in the pure condition without the addition of sand. Subsequently, varying amounts of sand were mixed with the clay. Ten grades of sand were used. The median (average) grain diameters of these sands are 1680, 945, 725, 350, 180, 136, 80, 55, 16 and 1.2 microns, respectively. Sedimentary parameters of these sands are given in Table 1. The 16 micron sand is known in the trade as Silica No. 2 sand and the 1.2 micron as DMAF sand. These two sands are made commercially by crushing in a ball mill. They have coefficients of sorting ranging between 1.8 and 2.0. The other eight samples are California beach sands, or mixtures of beach sands, whose sorting coefficients range principally between 1.1 and 1.4. Though mixtures of all these sands with the various types of clay were studied, experiments on clay-sand properties generally were made with 16 micron sand. Four concentrations were used, namely, 20, 50, 80 and 100 percent clay, The corresponding sand contents of these mixtures are 80, 50, and 20 and 0 percent sand. For montmorillonite, which is highly plastic even in small proportions, mixtures of 90 percent sand and 10 percent montmorillonite were studied. The other clays do not have sufficient plasticity to make cohesive soils when only 10 percent clay is present. These clays, when mixed in a proportion of 10 percent clay and 90 percent sand exhibit dilatancy, or increasing shear-resistance with increasing stress, and give anomalous results.

The sands cannot be studied in the pure state, because they are too dilatant. The only investigations of pure sand were on artificial mixtures of 135 micron sand with slightly larger and smaller sized sands to give different coefficients of sorting (Fig. 2). The strength of such samples was measured in a conventional direct shear box under a normal load of 6 pounds per square inch.

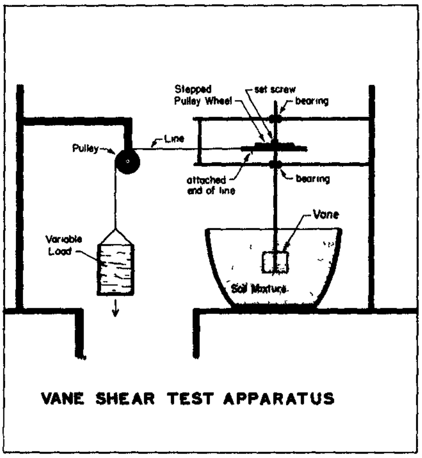

Fig. 1

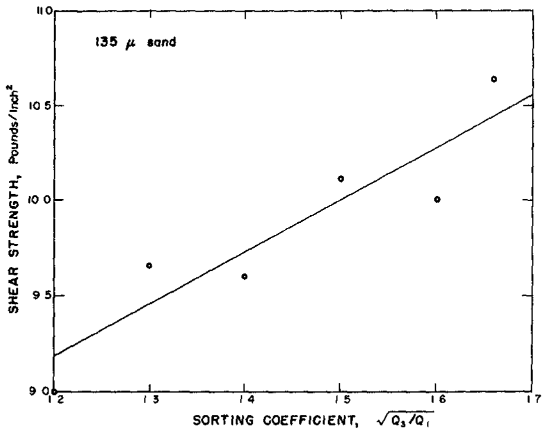

Fig. 2. Effect of sorting on shear strength of sand.

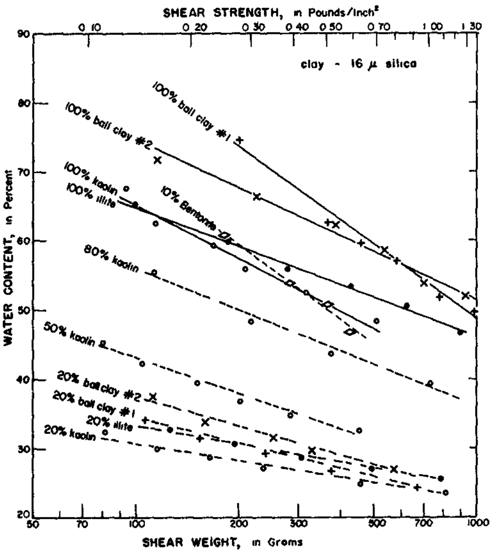

Fig. 3. Relation of clay type and clay concentration to shear strength and water content.

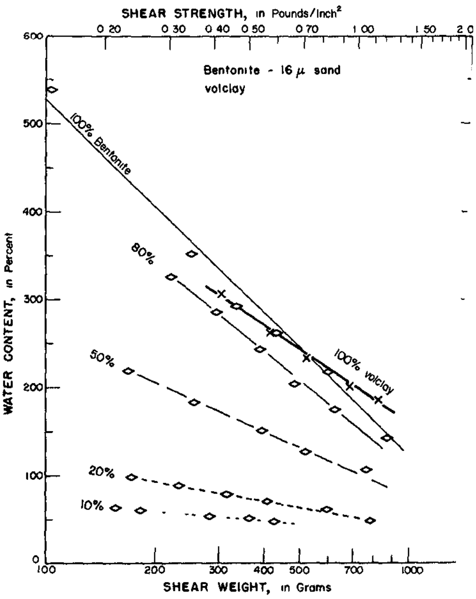

Fig. 4. Relation of Bentonite coı centration to shear streı and water content.

TABLE I

SEDIMENT PARAMETERS OF SAND USED IN STRENGTH STUDIES

M	Q_1	Q_3	D_{10}	D_{90}	So	Log SK
1.15	0.60	2.18	--	--	1.91	-.002
16.2	7.2	33.8	--	--	2.17	-.008
55	41.5?	73	--	100	1.33?	.001?
80	52	110	37	135	1.45	-.048
136	87	175	42	224	1.42	-.043
180	168	190	150	200	1.06	-.003
350	295	425	250	500	1.20	.005
725	555	890	365	1000	1.27	-.016
945	700	1150	505	1310	1.28	-.023
1680	1460	1870	1050	2220	1.13	-.007

M - Median diameter
Q_1, Q_3 - First and third quartiles
D_{10}, D_{90} - Ten and ninety percentiles
So--coefficient of sorting $= \sqrt{Q_3/Q_1}$
LogSK - logarithm of skewness to base 10 $= \log\sqrt{Q_1 . Q_3/M^2}$

METHODS OF MEASURING STRENGTH

The strength of the other samples was measured in a mortar bowl about 8 inches in diameter with a shear vane and pulley apparatus shown in Fig. 1. Water is poured in the container at the end of the pulley system until the weight of the water causes the sediment to fail. The results are reported in the illustrationsin this paper both in terms of grams of water added and the corresponding shear strength in pounds per square inch computed according to the formula of Capper and Cassie (1953, p. 112). This formula is

$$T = wr = C \pi \left(\frac{D^2h}{2} + \frac{D^3}{6}\right)$$

where T is the torque, w the weight of water to cause failure of the sediment, r the radius of the pulley wheel, C the shear strength, D the diameter of the shear vane and h the height of the vane. In the present investigation the measurements generally were made for quantities of water ranging between 100 and 1000 grams. As the height and diameter of the vane and pulley radius of the torque wheel are the same in all tests, the weight in grams is directly proportional to the strength in pounds per square inch. The factor for converting grams to pounds per square inch is 1.37×10^{-3}. The height of the vane is 0.764 inch, the diameter 1.312 inch, and the radius of the pulley wheel 2.02 inches The vane has 4 blades of equal size placed at right angles to one anothe

In each experiment clay or a clay-sand mixture is mixed with a quantity of water close to the liquid limit of the soil, and the shear strength is measured with torque vane. A sample for determination of water content is taken immediately after the strength is measured. If the sample for water content is not removed immediately, water evaporates from the sediment, which causes anomalous results. After the shear strength is measured, additional material is added to the sampl and thoroughly mixed into it. The shear strength is again recorded and the water content measured. More sediment is added until five or six determinations have been made. The results are then plotted on semilogarithmic paper with shear strength plotted logarithmically and the water content arithmetically. The experiments with given types of clays consistently give smooth curves as shown by Figs. 3 and 4. An effort was made to keep the sands saturated. The consistency of the results suggests that the samples were saturated. In preparing the samples, mixtures of desired proportions of clay and sand were made before adding water. In interpreting the effect of water content one should bear in mind that the water is measured according to the practice of engineers, which is the ratio of the weight of the water to the weight of the dried sediment expressed as percent. It is not the percent of water in the sediment as normally used by geologists.

When the investigation was first started water was added in suc cessive increments to the samples, but this procedure proved less practicable than starting with dilute mixtures and increasing the concentration by adding sediment.

The sediments failed at fairly sharp end points. Measurement were made of strain before failure. As a rule the sediments failed before 5 grams had been added after first trace of noticeable movem Thus the error in measurement is less than 5 percent and for most d terminations is less than 3 percent. Satisfactory duplicates were readily obtained.

LIQUID LIMIT DETERMINATIONS

A number of liquid limit determinations were made. As the liquid limit is a rough measure of strength it was thought desirable to measure the effects of (1) clay type, (2) clay-sand ratio and (3) grain size upon liquid limit (Figs. 9 and 10). The liquid limit was measured in the customary Atterberg hemispherical brass cup in which a pat of soil is molded into the hemisphere and a groove of given dimensions made in the sediment. The liquid limit is the water content at which 25 blows are required to cause the sediment on the two sides of the groove to flow together. Thus it is a rough measure of the strength of a sediment for a given water content. Experiments were made with different water contents for each of the given mixtures to determine the relationship between water content and the number of blows to cause flowage of the sample. Straight line relationships were found when blow count was plotted logarithmically and water content arithmetically (Fig. 9).

BASE EXCHANGE AND OTHER MEASUREMENTS

The effect of base exchange was measured on kaolin clay. In two experiments the clays were converted to hydrogen and to sodium clays by repeated washings with hydrochloric acid and sodium hydroxide solutions respectively until all the bases had been replaced by hydrogen and by sodium. Clay-water mixtures were then made with the hydrogen kaolin and with the sodium kaolin. The results are presented in Fig. 11.

The effect of temperature was also investigated by using water of different temperature to test the strength. Inasmuch as the strength is influenced by the water content, the strength had to be converted to the strength at a given water content in order to make the temperature results comparable. This water content for purposes of computation has been taken as 39 percent. The temperature measurements were made only on 100 percent kaolin.

The sediments tended to increase in strength with time of standing, that is, they showed a slight thixotropic effect. The strength of mixtures of illite with different amounts of water were measured after different intervals of time. As with the temperature studies, it was necessary to correct the strength for water content so that the results could be presented on a comparable basis. A water content of 50 percent was used. As the strength did not change greatly with time, errors due to thixotropic effects are believed to be relatively small.

The Land Locomotion Laboratory of the Ordnance Corps at the Detroit Arsenal has been experimenting with the strength of mixtures of glycerine and vol clay - a montmorillonite clay. Glycerine is used because it does not evaporate and gives a constant strength with respect t(time, whereas water evaporates causing the strength to increase gradu· ally with time. In order to make the results of the present investigatioı useful to the people at the Land Locomotion Laboratory, the strength of mixtures of vol clay with glycerine and with water has been determined with the shear vane devices illustrated in Fig. 1. The results are presented in Fig. 12.

RESULTS

COHESIONLESS SOILS

The results of this investigation are presented in two parts (1) cohesionless and (2) cohesive soils. Cohesionless soils tested with the shear vane device gave anomalous results owing to the effect of dilatancy, and after preliminary experimentation were not studied furthe The effect of sorting upon shear strength was tested on a conventional direct shear device under 6 pounds per square inch normal load. The results as shown in Fig. 2 indicate that the shear strength increases with increasing poorness of sorting. All tests were made with artificially mixed sands of 135 microns median diameter. The shear strength increased regularly from 9.2 pounds per square inch for sediments having a coefficient of sorting of 1.2 to 10.6 pounds per square inch for a coefficient of sorting of 1.7. The data are not particularly consistent but they exhibit a general trend.

COHESIVE SOILS

Effect of clay type - The results of the studies of cohesive soils are shown in Figs. 3 to 12. In Figs. 3 and 4 the shear strength is plotted logarithmically and the water content arithmetically for mixtures of different clays with 16 micron sand. In these two figures the shear strength varies inversely with the water content. Samples of pure illite and kaolin have essentially the same shear strength, but kaolin shows a steeper relationship between water and shear strength than does illite. The two ball clays for given water content are slightly stronger than kaolin and illite. Wyoming bentonite is very much stronger. A sample of Wyoming bentonite with 500 percent wat has essentially the same shear strength as a kaolin clay with 70 percent water. The vol clay shown in Fig. 12 has approximately the sar shear strength with respect to water as the Wyoming bentonite as is indicated by Fig. 4. The differences in strength between kaolin, illite and ball clays is not great, whereas the difference between thes clays and montmorillonite is large. Mixtures of 10 percent bentonite and 90 percent sand have about the same strength as pure clays of other types. Mixtures of 20 percent bentonite and sand are stronger than any of the other pure clays. To avoid crowding on Fig. 3, mixtures of 50 and 80 percent clay with sand are not shown for illite and the two ball clays. The strength data however were determined for these clays. They are similar to the data shown for kaolin of comparable sand content.

Effect of clay sand concentration - When water content is plotted arithmetically and shear strength logarithmically, an inverse straight-line relationship results (Figs. 3 and 4). The slope of this straight-line can be expressed mathematically:

$$M = \frac{\delta W}{\delta (\ln S)} = S \frac{\delta W}{\delta S}$$

where M is the slope of the line and is a dimensionless constant, W is water content in percent, and S is shear strength. When the clay content of a sand-clay mixture decreases the slope becomes progressively less, in a more or less regular fashion. On plotting these slopes M, against the corresponding clay content, C, a direct proportionality results, as illustrated by Table 2. The slope of this line, $\delta M/\delta C$ is a constant, and is characteristic of the clay type, at least in the water content range investigated. Viewed practically, M is a measure of the sensitivity of soil strength with respect to changes in water content and $\delta M/\delta C$ constitutes a measure of the change of this sensitivity as clay content is varied. The following table lists M and $\delta M/\delta C$ values for the clays investigated, in mixtures with 16 micron sand.

TABLE 2

RELATIONSHIP OF SLOPE OF WATER CONTENT - SHEAR STRENGTH CURVES TO CLAY TYPES

Clay type	M values at different clay contents Percent clay					$\frac{\delta M}{\delta C}$
	100	80	59	20	10	
Wyo Bentonite	-167	-145	-80	-32	-14	-1.700
Edgar ASP (kaolinite)	-11.5	-9.5	-5.8	-2.9	-	-0.118
Ky. Ball #1	-14.5	-	-8.9	-5.2	-	-0.116
Ky. Ball #2	-11.2	-	-7.5	-5.8	-	-0.068
Grundite (Illite)	-7.8	-7.4	-5.9	-4.0	-	-0.049

M = slope of water content-shear strength relationship.

$\frac{\delta M}{\delta C}$ = change in slope, M with respect to changing clay concentration C

Effect of grain size - The relationship of grain size of sand in kaolin-sand mixtures is shown in Figs. 5 and 6. In Fig. 5 mixtures of different sands in proportions of 20 percent sand to 80 percent clay are shown. The sands range in median diameter from 1.2 to 350 microns. It is clear that for given water content the strength increases progressively as the grain size decreases. Sands with diameters ranging between 55 and 350 microns have relatively little effect upon strength but the finer sands show distinctive differences in strength effects. Th slope of the water content-shear strength relationship for each mixture essentially constant. The shear strength of the 1.2 micron sand mixtur is considerably greater than for the other sands.

Clay-sand mixtures of 20 percent kaolin and 80 percent sand for these same sand sizes were also studied. The 1.2 micron size could not be used owing to effects of dilatancy. Sands of 725, 945 and 1680 microns were also studied in this latter series of tests. The results a shown in Fig. 6. The relationship of shear strength to grain size for water contents of 17, 18 and 19 percent is indicated in this figure. Dat for the 16 micron size are extrapolated from the slope of the water con tent-shear strength curve. In these tests, the lowest concentration of water used with the 16 micron sample is 23 percent. The data show that for grain sizes above 200 microns, the size has relatively little effect on shear strength but as the size decreases to 16 microns the sh strength increases progressively with increasing fineness of grain

The sands used in this series of experiments have coefficients of sorting ranging from 1.06 for the 180 micron sand to 2.17 for the 16 micron sand (Table 1). Most of the sands of sand size had coefficients of sorting under 1.4, whereas the sands (clastic particles) of silt and clay size (16 and 1.2 microns) had coefficients of around 2.0, and thus were less well sorted. Hence though Fig, 6 purports to show the effect of median grain size on shear strength, it should be realized that each sand, because of the variation in size of constituent about the median as represented by the coefficient of sorting, contains significant quantities of constituents finer than the median. Hen these fine constituents may have a significant effect on strength. How ever, since Figure 6 shows increasing strength with increasing finene of median diameter it would follow that the content of constituents fine than the median would increase more or less proportionally with increasing fineness of median diameter. Thus the graphs suggest that increasing content of fine particles increases strength.

This relationship is shown in another way by comparing Figs. 5 and 6. Extrapolation of the strength data in Fig. 5 to a water content 17 to 19 percent underwater materially greater shear strength for th water content in the 80 percent clay-20 percent sand mixture than in percent clay-80 percent sand mixture shown in Fig. 6. The content

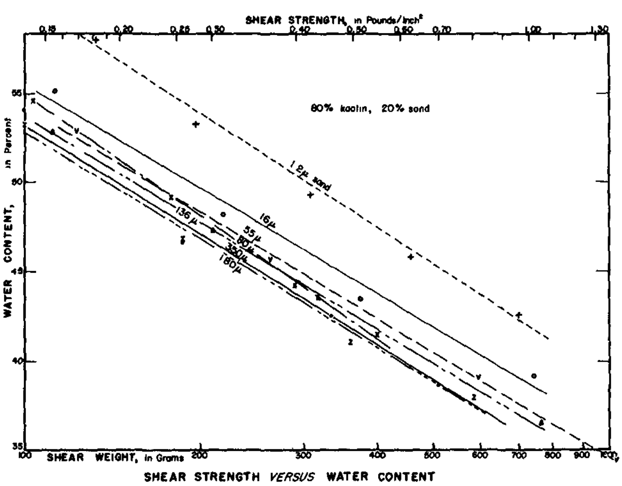

Fig. 5. Relation of median diameter of sand component to shear strength and water content.

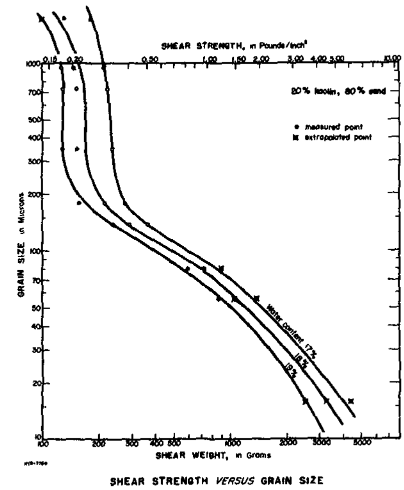

Fig. 6. Relation of water content to shear strength and median diameter of sand component.

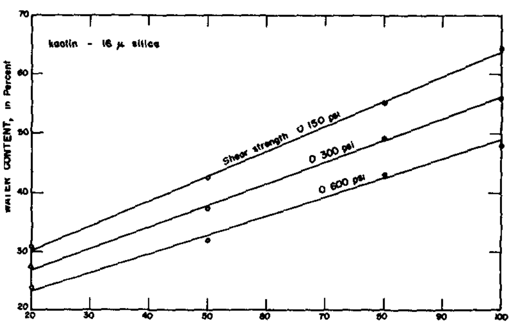

Fig. 7. Relation of shear strength to clay concentration and water content.

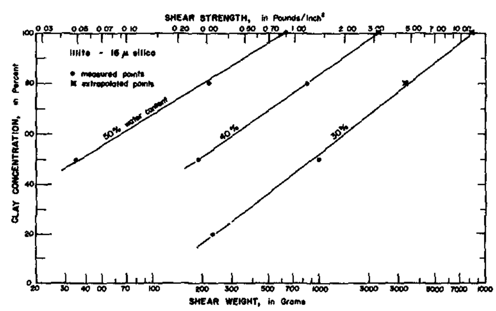

Fig. 8. Relation of water content to shear strength and clay concentration.

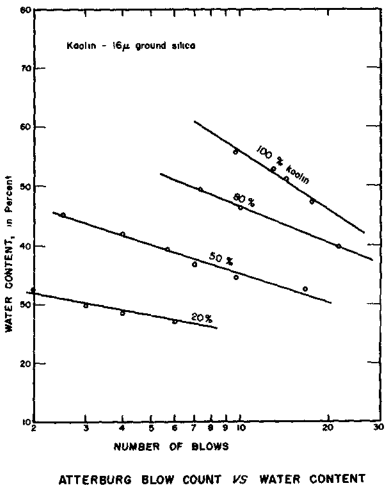

Fig. 9. Relation of clay concentration to Atterberg blow count and water content.

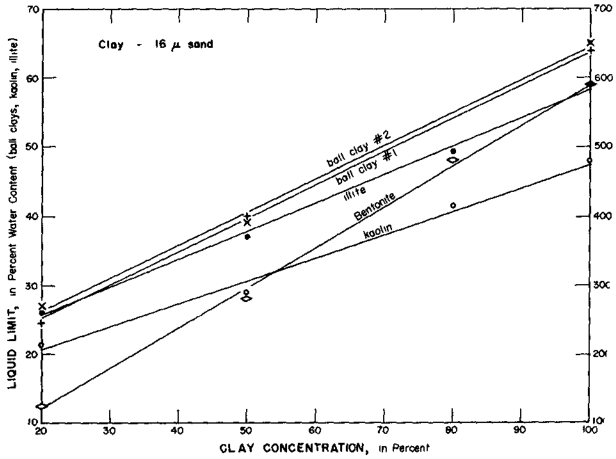

Fig. 10. Relation of clay type to clay con centration and liquid limit.

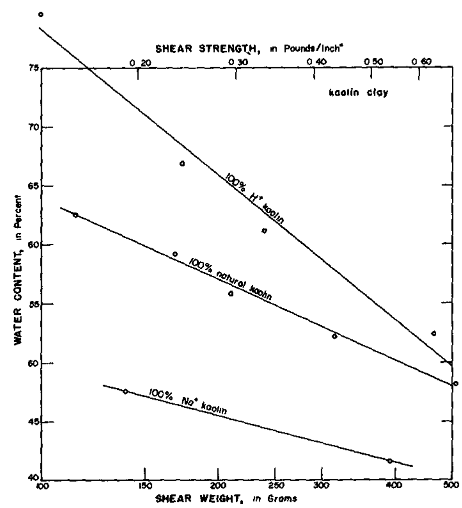

Fig. 11. Relation of exchange-able ion to shear strength and water content.

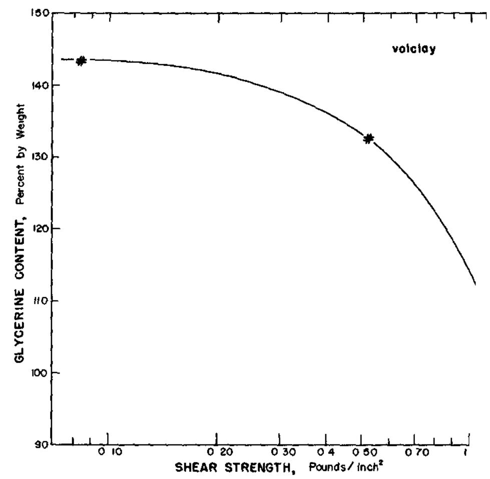

Fig. 12. Effect of glycerine cor on shear strength.

of fine particles is much greater in an 80 percent clay mixture than in a 20 percent clay mixture, since the average diameter of the clay particles is 1.2 microns compared with 16 microns or more in the sand fraction. The effect of course cannot be due entirely to grain size because a mixture of sand of 1.2 microns median diameter with bentonite in the proportion of 90 percent 1.2 micron sand and 10 percent bentonite has a shear strength of only 0.5 pound per square inch for a water content of 75 percent compared with a water content of 300 percent for 100 percent bentonite with the same shear strength of 0.5 pound per square inch. Thus as shown in Figs. 3 and 4, increasing content of clay increases the shear strength even for mixtures of sand of clay size as represented by the 1.2 micron sand. This greater strength for clay could be ascribed to the plastic nature of the clay compared with the clastic character of the sand. The effect of water content upon strength is shown in a different way in Figs. 7 and 8. Figure 7 shows relationships between water content and clay-sand concentration for mixtures of kaolin and 16 micron sand for constant shear strength. The water content increases in a regular manner as the clay concentration increases for each of the three shear strengths shown. Fig. 8 indicates how shear strength increases with increasing concentration of clay for three mixtures of constant water content in mixtures of 16 micron sand with illite. It is perfectly obvious from Figs. 7 and 8 that shear strength increases with concentration of clay in clay-sand mixtures.

Effect of clay content upon liquid limit · The liquid limit is a rough measure of the strength of clays. It represents the water content of a clay at the consistency at which 25 blows in the Atterberg testing device causes the clay to flow together in a groove of standard width. The liquid limit is thus a measure of the lower boundary of the plastic state of the soil. The plastic limit is a measure of the water content of the upper limit of the plastic state of the soils, and represents the water content at which thin threads of the clay begin to break when folded. In the present investigation only the liquid limit was investigated. The liquid limit of the several soils tested in the present investigation was determined by the standard means of measuring the water content. In making the test one mixes the soil in varying water concentrations and determines the blow count to cause closure of the groove. These blow counts plot on a straight line as shown in Fig. 9 which gives data for mixtures of kaolin and 16 micron silica. The liquid limit is taken where the 25 blow count ordinate intersects the water content-blow count curve. Fig. 9 shows that these water content-blow count curves plot as straight lines on semilogarithmic paper and that the water content for any given blow count increases progressively with increasing concentration of clay. The slopes of the curve likewise progressively steepen with increasing concentration of of clay, just as the slope of the water content-shear strength curves increase in the shear vane experiments illustrated in Figs. 3 and 4. As the strength varies directly with the number of blows required to cause flowage, this graph indicates increasing strength with increasing clay concentration.

The effect of clay concentration on the liquid limit of the clay type is shown in Fig. 10. Here the water content at 25 blows is plotted agains the clay concentration for the different clays. The liquid limit increase with increasing clay concentration in a regular manner, and the liquid limits vary in the same manner with respect to clay type as do the water content-shear strength relationships shown in Figs. 3 and 4.

The effect of grain size upon the liquid limit and blow count was studied for one clay, the Wyoming bentonite. Mixtures of 20 percent bentonite and 80 percent of 1.2 micron and 16 micron sand, respectively, were made. The water content for given blow count was found to be approximately two times greater in the 1.2 micron mixture than in the 16 micron mixture. That is, decreasing the grain size of the clastic particles from 16 to 1.2 microns in montmorillonite-sand mixtures increases the water content 100 percent for given blow count. For blow count of 25, which represents the liquid limit, the mixture of 1.2 micron silica had a water content of 250 percent and of 16 micron silica, 125 percent.

Effect of base exchange - The effect of base exchange is shown in Fig. This figure presents data for shear strength and hydrogen and sodium bentonite and of raw bentonite as received from the manufacturer. Hydrogen clays are seen to be relatively strong, sodium clays are relatively weak and the raw bentonite is intermediate between hydrogen and sodium bentonite. Similar relationships have been reported by Sullivan and Gr (1940) for base exchange effects on other clays. The hydrogen clays for given water content are 5 to 8 times stronger than sodium clays. For example at 50 percent water the shear weight is 0.14 pounds per square inch for sodium compared with 0.75 pounds for hydrogen.

Effect of temperature - The strength increases slightly with falling temperature. In general the increase is less than one percent per decrease of one degree centigrade. In view of the fact that water becom more viscous as the temperature drops, it could be presumed that the strength would increase with lowered temperature, because of the greater viscosity of the water.

Effect of time - The strength of pure illite was measured after different intervals of standing. The data were corrected to a constant water content. The strength is 0.70 per square inch after one minute standing, 0.79 after three minutes, and 0.93 pounds per square inch after 100 minutes. Most of the strength is thus attained in a short time, but addidional strength is slowly gained with time, evidently owing to thixotropic effects.

Effect of glycerine - The strength of glycerine mixtures was measured on vol clay, a type of bentonite or montmorillonite. The results are shown in Fig. 12 which indicates that glycerine does not give the straight line effect of water mixtures. The strength increases from about 0.1 pound per square inch at 143 percent glycerine through 0.5 pounds per square inch for 132 percent glycerine to 1.3 pounds per square inch for 90 percent glycerine. The strength thus increases very rapidly as the glycerine content decreases below 100 percent. Mixture of water and vol clay as shown in Fig. 4 closely approximate the water-shear strength relation for Wyoming bentonite. As glycerine is non-polar, whereas water is polar, the forces that cause strength in glycerine mixtures evidently are different than those in water.

CONCLUSIONS

The strength of clays varies with clay type. For given water content illite is slightly stronger than kaolin. Ball clays, which are a mixture of kaolin, silica and organic matter are slightly stronger than illite. Montmorillonite is much stronger than any of the other clays. For given shear strength the water content increases progressively from kaolin through illite to montmorillonite. Montmorillonite, for given strength contains much more water than the other clays. It is interesting that the base exchange capacity of the clays varies in much the same manner as as the water content-strength relationships. The strength in some way is related to the inherent properties of the clay.

For given water content the strength increases with increasing clay-sand ratio and for given strength the water content likewise increases with increasing clay-sand ratio. Similarly with increasing fineness of grain of admixed sand particles, the strength and water content increase in the same way.

It is beyond the scope of this paper to explain the fundamental physical chemistry of the causes of strength in clay-water mixtures. For good discussions of this subject see Hauser, 1955, Norton, 1952 and Langston and Pask, 1956. In clay-water mixtures, the water occurs essentially in two ways; namely, bound and unbound water. Bound water is water associated with molecular and electrical forces surrounding clay particles and for given clay minerals has more or less similar dimensions. Unbound water is water between particles and is located outside the limits of bound water. For soils of given water content, when the content of clay increases in sand-clay mixtures and when the size of the particles diminishes, the size of the pores between particles diminishes with the result that the ratio of bound to unbound water increases, because the thickness of the bound water layer presumably does not change materially and the dimensions of the unbound water necessarily must diminish causing the bound-unbound water ratio to increase. Since the strength among other things is related to the forces that bind the water to the clay particles, the strength for given water content should likewise increase as the relative proportions of bound water increases.

With respect to increase in water content for clays of given strength, if the strength remains constant the ratio between bound and unbound water should likewise remain reasonably constant. If the surface areas of the particles increases or if the thickness of the bound layer increases, the content of unbound water must likewise increase if the ratio of bound to unbound water is to remain constant. Thus with increasing clay content in clay-sand mixtures, or with increasing fineness of grain with resulting increase in content of bound water, the total content of water both bound and unbound must increase for clays of constant strength.

With respect to liquid limit, since the liquid limit test is essentially a measure of the water content for constant strength conditions, the liquid limit should follow the same relationships as do the strength relation ships described above. Even though the liquid limit may not be an exact measure of strength, it represents the energy required to close a furrow with 25 blows under standard conditions, and as such is a sort of measure of strength. The liquid limit thus can be expected to vary with (1) clay type, (2) clay-sand ratio and (3) grain size.

The effect of base exchange upon strength illustrated in Fig. 11 is similar to the findings of Sullivan and Graham. They found that for given water content hydrogen clays are considerably stronger than sodium clays The effect of temperature upon strength is probably in part caused by increased viscosity of the water with lower temperature. Since the increase in strength with drop in temperature is very slight, temperature cannot be regarded as a major factor affecting strength.

The different relationship for glycerine compared with water, as indicated by Fig. 12, perhaps is caused by different molecular effects. Glycerine is non-polar, whereas water is polar, and according to Langston and Pask, 1956, polar effects are a partial cause of strength in clay water mixtures.

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CHAPTER 51

MODEL STUDIES OF THE DYNAMICS OF AN LSM MOORED IN WAVES

by

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ABSTRACT

Presented in this report are the results of model tests performed t determine the motions of a moored LSM (Landing Ship Medium) and the associated mooring cable forces when subjected to the action of uniform periodic water waves. Details are given of the design and construction of the 1:80 scale model LSM and the dynamic balancing of the model. Pr ciples of the designs of the model mooring cables and of the meter for measuring the cable forces are also presented. The use of the laborato equipment, such as the wave-towing tank, the model basin, the photograp equipment, and the force and wave height meters are described, and the testing procedure is outlined. The data are presented in terms of the prototype in graphical form. Resonance conditions in the surging and heaving motions, and the associated high cable forces, are shown to exi for some initial cable tensions within the range of wave periods normal encountered in many ocean areas. The effect of a roll damping device i shown.

INTRODUCTION

One type of mobile platform which has possibilities in the offsho oil program for core boring or well drilling operations is an existing suitably-modified vessel. Such a vessel must be moored within narrow limits or boring and drilling operations must be modified to allow for considerable oscillatory movements; probably the solution will be a co bination of both. The solution of the problem from a mathematical sta point is complicated by the fact that the mooring cables have non-line characteristics.

In order to work at sea it is necessary to moor a vessel so that will not drift under the action of wind, currents or the net motions associated with wave action. Further, it would simplify the operatior coring or drilling through a well in the ship if the oscillatory motic associated with waves could be minimized. In any case, the magnitudes these motions have to be determined. At the same time, the forces exe on the mooring cables have to be predicted. In addition, devices for damping any induced motions should be tested to rate their effectivene

A model of the dynamics of a moored LSM was made by the Wave Rese Laboratory, University of California, Berkeley, California, for the Ca ornia Research Corporation.

MODEL STUDIES OF THE DYNAMICS OF AN LSM MOORED IN WAVES

MODEL LAWS

Gravity, inertia, elastic, surface tension and viscous forces exist in a moored floating body system, neglecting compressibility effects. From a practical standpoint it is not possible to model the system in compliance with all modeling laws simultaneously. The vessel and the waves were modeled in accordance with Froude's modulus. The effect of Reynolds' modulus was neglected in the development of the model (however, turbulence existed in the flow) in conformity with standard naval architectural practice. The effect of surface tension was assumed to be of little importance as the model was to be made large enough that the waves would be well into the "gravity regime" and the cables were to be large enough that surface tension forces would not interfere with their motions.

The most serious question in a model of this sort is the neglecting of the effect of Reynolds' modulus (viscous forces). However, a brief analysis by Dr. Kitter (California Research Corporation, La Habra) indicated that the order of magnitude of viscous forces on the mooring cable would be small compared to gravity and inertia forces, and for the range of variables to be considered, the drag coefficient would be relatively insensitive to Reynolds number.

The following model laws were used in the design of the ship model, model mooring cable and the cable force meter:

Let A = cable area (cross-section).
C = a constant of the system; for the case considered herein it is related to the angle the mooring cable makes with the force meter.
E = modulus of elasticity of cable.
g = acceleration of gravity (32.2 ft./sec.^{2}).
I = moment of inertia of cable (cross-section).
L = cable length.
P = applied force.
S = geometric scale ratio (1:80).
W = weight of a quantity.
$-_m$ = refers to a model quantity.
$-_p$ = refers to a prototype quantity.
$-_r$ = refers to a ratio between model and prototype quantities.
ΔL = axial deflection of cable.
γ = unit weight of a quantity.
ϕ = force ratio.

For Froude similitude, set the dimension of gravity force equal to the dimension of inertia, axial elastic and bending elastic forces, and obtain:

(1) Gravity forces

$$\frac{\gamma_m L_m^3}{\gamma_p L_p^3} = \phi_G = \gamma_r L_r^3 \tag{1}$$

(2) Inertia forces

$$\phi_I = \frac{\gamma_m L_m^3 g L_p T_m^2}{\gamma_p L_p^3 g L_m T_p^2} = \gamma_r T_r^2 L_r^2 = \phi_G \qquad (2)$$

also $\qquad \gamma_r T_r^2 L_r^2 = \gamma_r L_r^3$

therefore $\qquad \sqrt{L_r} = T_r \quad ,$

(3) Axial elongation forces in the cable

$$\Delta L = \frac{PL}{AE}\,; \quad AE = \frac{P}{\Delta L/L}\,; \text{ let } K = AE \qquad (3)$$

$$\frac{K_m}{K_p} = \phi_{AE} = \phi_G = \gamma_r L_r^3; \text{ let } AE = K, \text{ then } K = \gamma_r L_r^3 \,.$$

(4) Forces causing bending in the cable

$$\Delta = \frac{CPL^3}{EI}\,, \quad \frac{P_m}{P_p} = \phi_B = \frac{\Delta_m (EI)_m L_p^3}{\Delta_p (EI)_p L_m^3} = \gamma_r L_r^3 = \phi_G \qquad (4)$$

$$\frac{\Delta_m}{\Delta_p} = L_r, \text{ and } (EI)_r = \gamma_r L_r^3 \,.$$

These relationships will be referred to in the following paragraphs on the design and construction of the ship model, the mooring cable model and the mooring cable force meter.

SHIP MODEL

DESIGN

Before designing the ship model, it was necessary to choose a scale ratio that would satisfy three criteria:

1. The effects of surface tension would not be important.
2. The model would be small enough for use in the wave-towing tank, yet large enough that minor variations, when magnified to prototype, would not prove to be excessive.
3. The model would be large enough that a proper scale mooring system could be constructed and the expected mooring cable forces would be of sufficient magnitude that they could be easily measured.

A scale ratio of 1:80 was chosen. The model was scaled from a U. Navy drawing (Bureau of Ships, LSM (1)-S0701-112485, ALT. and LSM (1)-S0103-112380, ALT. 5), and from California Research Corporation Drawing (Nos. LC7990-7999, 9000-9007). Transverse cross section drawings for n stations along the ship were prepared to assure accurate ship dimension (Figure 1). The superstructure, except for the foreward superstructure deck, was left off the model, as were the several decks, and the model was designed to built to its own freeboard. It was necessary to appro: mate successively sections C-C from station 0 on, as drawings for these sections were unavailable. For the region near the bow where the shape changes considerably, many cross sections were drawn. The skegs and rudders were drawn up separately, since they were left off the basic model shell. Because the model hull form was to built up in the manner shown in Figure 1, a series of drawings were prepared of horizontal cross sections through the model at 3/4-inch intervals above the bottom tangent.

The weight of the completely balanced model was to be 3.78 pounds while the model shell was designed to weigh 1.20 pounds, leaving 2.58 pounds to be added for static and dynamic balancing.

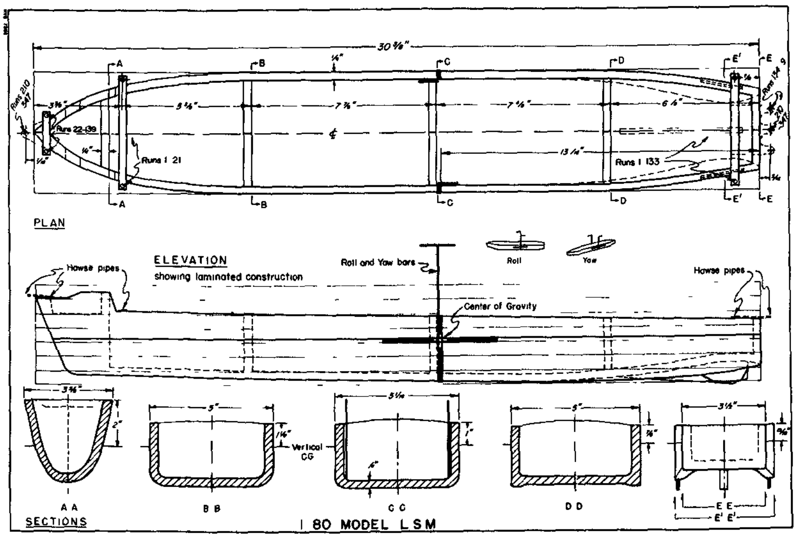


Figure 1.

CONSTRUCTION

The model LSM was constructed of clear soft pine. The wood was planed smooth to a 3/4-inch thickness. Then individual horizontal cross sections (at vertical distance 3/4 inch below each water line plane) were scaled off the drawings and a section of pine was cut to approximate this water plane area. The sections were glued together to make a rough hull form which was finished to the outside dimensions of the ship and to a 1/4-inch wall thickness. Bulkheads were added to approximately the quarter points of the hull to give the model strength to resiste the lateral forces which were likely to be encountered in handling. The model was then covered with several coats of spar varnish.

The points of attachment of the mooring cables (through bars) are shown in Figure 1. Originally these bars were extended completely across the ship model in order to guard against failure of the side wall of the model due to excessive mooring cable loads. However, after the first series, this system was modified as shown, eventually leading to center-line mooring both bow and stern.

BALANCING

After the basic model shell was completed, weights were added to bring the model up to its correct scale weight. These weights were place in such a manner that the center of gravity of the model was properly located, and so that the natural periods of roll and pitch would be correct This static and dynamic balancing was performed as follows: The model wa suspended fore and aft by suspension bars as shown in Figure 2a. The weight of the foreward suspension was counterbalanced on the scale prior to placing the model on the scale. Since the distances from the aft suspension bar to the center of gravity, from the aft suspension to the fore ward suspension bar, and the total weight of the model were known, it was possible to calculate the weight needed to balance the model when the longitudinal center of gravity was correctly positioned; this was done by taking moments about the aft suspension bar. Small pieces of lead were used as weights for adjusting the balance.

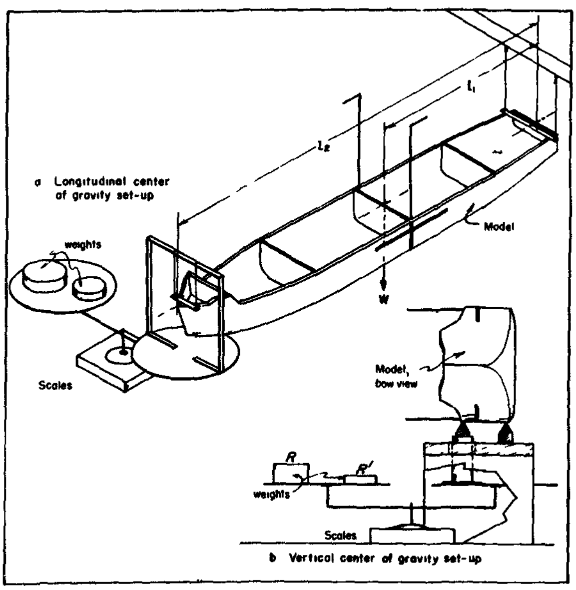


Figure 2. Balancing proceedure.

The model was placed on the device illustrated in Figure 2b and moments were taken about a given point (varied for each trial to minimize the effects of human error) and the necessary weight was calculated and set on the scale. The movable weights then were a justed until the vertical center of gravity was prope located; this was completed after about twenty trials. After locating the vertical center of gravity the perio of roll was adjusted.

The model was placed in the model basin, inclined and released. The natural period of roll was observec and recorded. The transverse position of the cente of gravity was obtained by adjusting weights until ze list was observed. Then lead weights were moved in a direction parallel to the axis of pitch, until the correct period of roll was obtained. By moving the weights parallel to the axis of pitch, the weights remained i the same position with respect to the vertical, so that the vertical cen ter of gravity was unchanged. A few trials with the device used to loc the vertical center of gravity confirmed the fact that the vertical cen of gravity had not shifted.

The last step in the dynamic balancing of the model was the adjustment of the period of pitch. The experimental method used is illustrat in Figure 3, and the calculations required for this method are presente below. The period of pitch of a vessel is proportional to the radius of gyration of the vessel in the pitching direction, and is given by the equation (Rossell and Chapman, 1939)

$$T_n = \frac{1.108\ K_1}{\sqrt{GM}} \tag{5}$$

where T_n = natural period of pitch,
K_1 = radius of gyration of the model,
GM = longitudinal metacentric height of the model, and
1.108 = $2\pi/\sqrt{g}$ (where $g = 32.2$ ft/sec.^{2}).

Severe damping of the pitching motion of the structure precluded accurate direct measurement of the period of pitch so that it was necessary to determine the radius of gyration of the model about the pitching axis by using the bifilar pendulum method as given by Timoshenko (1937). By hanging the model as shown in Figure 3, it can be shown that the period of oscillation of the bifilar pendulum so set up is a function of the radius of gyration of the vessel about the yaw axis. In addition, the radius of gyration about the yaw axis, and the radius of gyration about the pitch axis are essentially equal (personal communication, Dr. A. D. K. Laird, University of California, Berkeley). From Timoshenko (1937) it can be shown that

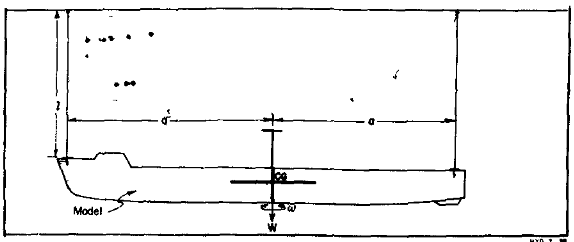


Figure 3. Bifilar pendulum setup.

$$T_p = \frac{2\pi K}{a}\sqrt{\frac{l}{g}} \tag{6}$$

where T_p = natural period of oscillation of suspended model in air,
l = length of suspending lines, and
a = equal distance from CG to suspending lines.

In the experimental adjustment of the period of pitch $l = 7.125$ feet, $a = 1.0625$ feet, $g = 32.2$ ft./sec.^{2}, and $k_1 = 63.4$ feet/80 = 0.792 foot. Substituting these values in Equation 6, it was found that the period of the pendulum would have to be 2.2 seconds for the radius of gyration in the pitching direction to be properly located. The weights in the model were then adjusted until this period of the pendulum was obtained. The longitudinal metacentric height was adjusted by other means.

Following this final adjustment of the weights, the complete balancing procedure was repeated, and the model was found to be balanced, and thus both geometrically and dynamically similar to the prototype, within the limits discussed in the introductory remarks.

MOORING CABLE MODEL

Inspection of the model relationships previously derived indicates that for geometric similarity the diameter of the model cable must be scaled according to the relationship

$$d_m = \frac{d_p}{80} \quad . \tag{7}$$

In the same unit weight properties are to be maintained, it is necessary to scale the cable according to the relationship

$$d_m = \frac{d_p}{(80)\ (b)} \tag{8}$$

where $b>1$ since the cable is stranded and the resulting unit weight for a given diameter is less than that of a solid wire. If the axial deflection forces of the mooring system are to be maintained in scale, t diameter of the mooring cable must scale according to the rule

$$(AE)_r = \gamma_r L_r^2, \quad \text{but} \quad E_r = \frac{\text{elastic modulus of model}}{\text{elastic modulus of prototype}}$$

$$= \frac{3\times 10^7}{10^7} = 3 \tag{9}$$

$$(AE)_r = 3\ d_r^2 = \gamma_r\ L_r^3\ ; \qquad \therefore\ d_r = \sqrt{\frac{\gamma_r}{3}}\quad L_r^{3/2} \quad .$$

If the bending properties of the cable are to be in scale, it is necessary to scale the bending resistance of the cable cross section accordi to the relationship

$$(EI)_r = \gamma_r L_r^5 \quad \text{or} \quad d_r^4 = \gamma_r L_r^5$$

or (10)

$$d_r = \sqrt[4]{\frac{\gamma_r}{3}}\ L_r^{1.25} \quad .$$

It was not possible to satisfy the four model criteria by choosin a proper cable diameter. Therefore, the cable diameter was chosen so t the moment of inertia of the section would correspond to and be in sca with that of the prototype stranded cable. This meant it was necessar choose a diameter somewhat smaller than the outside diameter of the pr type stranded cable, since a geometric scale would mean that the model cable extrapolated to prototype conditions would correspond to a steel rod in bending stiffness. Split lead shot were added to the mooring cable at one-inch intervals to bring the cable up to the proper weight requirements. An arbitrary spacing of 1 inch between shot having been assumed, the required shot diameter was 0.050 inch, which was readily available. The axial deflection characteristics of the cable were calculated and found to be less than those required; hence, some axial flexibility of the cable system was built into the force meters. This kept the total mooring system deflection in scale. The only criteria not satisfied was that of geometric scale of the diameter, which would result in the viscous drag and inertia forces being even more out of scale. However, as was assumed in the case of the ship model, these forces would be small compared to other forces acting on the cable.

FORCE METER

The force meter served two purposes: it was used as a sensing element in a force recording system, and it was used in the modeled mooring system to satisfy scale factor requirements for cable elongation due to axial loads. In designing the meter it was necessary to satisfy the similitude requirements for cable deflection (as determined from model laws) and to insure sufficient sensitivity so that the meter could measure the smallest expected forces.

In order to provide the proper deflection sharacteristics it was necessary to use other than a simple structural shape (Figure 4). The deflection characteristics of the beam used were calculated through application of the principles of virtual work. The force measuring system was designed to provide maximum possible recorder sensitivity to applied cable force. Strain gages were used for the force measuring elements; force was measured indirectly by measuring the strains induced in the meter beam by bending moments caused by the force in the mooring cable. The strain measurements were recorded with a Brush Electronic Company recording system.

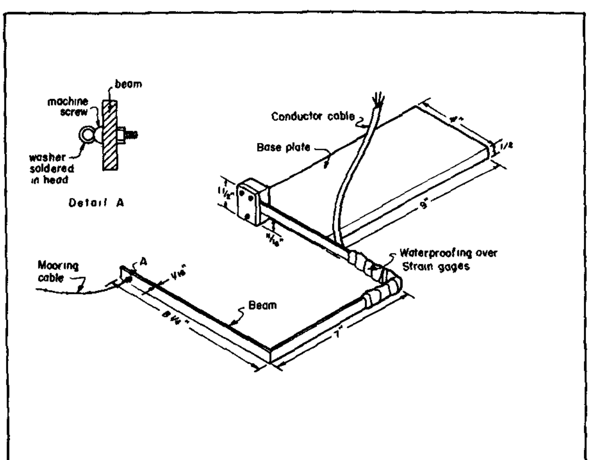
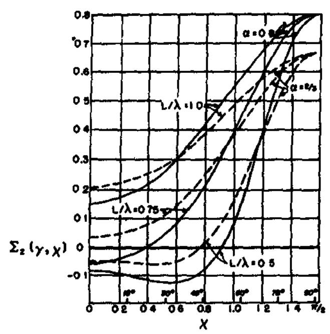


Figure 4. Force meter

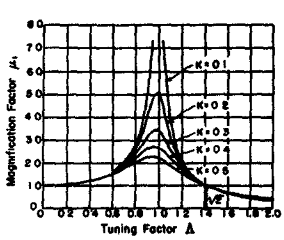
Details of the force meter design, construction and calibration will not be included as they have been published elsehwere (Beebe, 1956).

GENERAL CONSIDERATIONS

The motion of a freely-floating vessel in a seaway is extremely complex. Studies have been made in the past (Froude, 1861; Kriloff, 189ε of the motion of rolling and pitching of such vessels. Heaving has been studied by Haskin (1946). Other investigators have studied these proble recent summaries have been given by Weinblum and St. Denis (1950) and Weinblum (1955). They present equations and graphs of various ship moti functions, among which are the magnification (response) factors (for the various degrees of freedom) which are shown to be functions of the ratio of the wave period to the natural period of the vessel, and the exciting factors which are shown to be functions of the ratio of the wave length to the ship length (see Figure 5 for example). Both of these sets of factors are, of course, dependent upon several other wa and vessel characteristics. The author state that little is known of the surge, sway and yaw motions of floating vessel and that knowledge of the total motion of a vessel, including phase relationships between the various motions, is a most nil, although it is known that certain couplings between the motions in t different degrees of freedom exist. Of particular importance to studies of moo ing problems are the possibilities of induced roll, sway and yaw even when th vessel is encountering only head seas (Grim, 1952).



a Heaving force function $\Sigma_z(y, \chi)$ for two wall-sided vessels plotted against heading angle χ with L/λ as parameter. Waterline coefficient α = 2/3 and 0.8



b Magnification factor $\mu_1 = \frac{\sqrt{1+\kappa^2\Lambda^2}}{\sqrt{(1-\Lambda^2)^2+\kappa^2\Lambda^2}}$

From Weinblum & St Denis

Figure 5.

The above-mentioned studies were concerned with periodic waves of unifor amplitude. Recently, a few studies of ship motion in non-uniform waves have been published (St. Denis and Pierson, 1953; Fuchs, 1955; Sibul, 1955).

The details of ship motion are beyond the scope of this report and th reader is referred especially to the paper by Weinblum and St. Denis (1950) for the necessary background.

In considering a moored vessel the problem becomes more complicat especially as the elastic restraining force (the mooring system) is no linear. The motions of surge, sway and yaw become of prime importance

It appears that only a few studies have been made of the motion c a moored vessel (Wilson, 1950; Carr, McGraw and Snapiro, 1953; Beebe, 1955 a,b; O'Brien, 1955; Wilson and Abramson, 1955). These studies, w the exception of Beebe's, are of a greatly simplified problem, primari that of the longitudinal motion of a vessel moored alongside a pier, t motion being induced by relatively long period harbor seiches of small amplitude. All motions excepting surging are neglected, although Wilson (1950) commented upon the transverse motion.

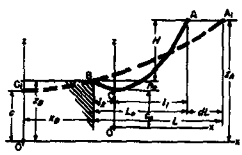


Fig. 6. Geometry of mooring rope suspension.

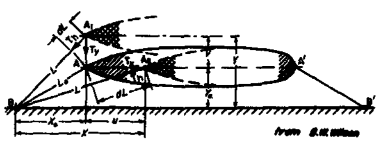


Fig. 7. Geometry of longitudinal and transverse ship motion.

The pioneering effort on the mooring problem was made by Wilson (1950). The generalized geometry of the dockside mooring rope suspension is shown in Figures 6 and 7. Referring to Figure 6 for an explanation of the symbols (where in the generalized case the lowest point of cable sag is at C_1 which is outside of A_1 B) the equations of the catenary were given as

$$z = c \cosh\left(\frac{x}{c}\right) \tag{11a}$$

$$s = c \sinh\left(\frac{x}{c}\right) \tag{11b}$$

where s is the length of the hypothetical cable from point C_1 to any point (x, z) on the catenary. Further

$$z_B + H = c \cosh \frac{x_B + L}{c} \tag{12a}$$

$$z_B = c \cosh \frac{x_B}{c} \tag{12b}$$

and thus

$$H = c \left(\cosh \frac{x_B + L}{c} - \cosh \frac{x_B}{c}\right) \tag{13a}$$

$$S = c \left(\sinh \frac{x_B + L}{c} - \sinh \frac{x_B}{c}\right) . \tag{13b}$$

After certain simplifications were made, Wilson (1950) gave the horizontal component of rope tension T_h, at points A_1 and B as

$$T_h = \frac{W_c L^2}{\sqrt{12\left[S^2 - (H^2 + L^2)\right]}} \tag{14}$$

where W_c is the weight per unit length of the cable.

The next step taken by Wilson (1950) was to determine the relationships between cable tension and elongation for several types and sizes of standard mooring cables (coir rope and steel wire rope). In order to utilize these data, a further simplification was made wherein it was shown that for common dockside mooring the following equation was of sufficient accuracy

$$L = \sqrt{S^2 - H^2} \quad . \tag{15}$$

Utilizing this approximation, and referring to Figure 7 for an explanatic of the symbols, the relationship between the components of cable tension and the components of ship movements were developed. These relationship are shown in Figure 8. Also shown in Figure 8 are the best-fit expenent curves, which are of the form

$$T_{x,y} = k \,(u \;\text{ or }\; v)^n \tag{16}$$

where k is a constant and n is a numerical exponent. For the case o a ship with many mooring lines, this was expressed for the longitudinal directions as

$$\Sigma T_x = C\, u^n \tag{17}$$

where C is a constant which depends upon the number, size and conditio of the cables and n depends upon the tension in the cables.

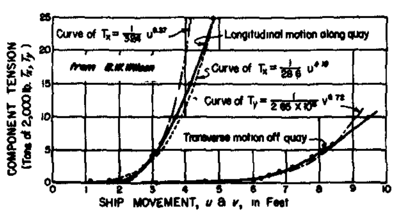


Figure 8. Relationship between rope tension and ship movement.

A harbor seiche has a relatively long period (usually in the range of a minute or more — sometimes much longe and usually has a low amplitude. It i possible to describe the water particl velocities and accelerations rather simply compared with the case of seas and swell. These approximations were used by Wilson (1950), together with ε approximation by Havelock (1940) which expresses the wave force on a rigid vessel which extends to the bottom. : is not necessary here to go through the various steps, but merely to present the final result

$$\frac{d^2u}{dt^2} + \frac{K}{2M} \cdot \frac{du}{dt} + \frac{C}{2M} u^n = \frac{KV}{2M} \cos pt - \frac{Vp}{2} \sin pt \tag{18}$$

where K is a constant which depends essentially upon the size and sha of the vessel, M is the mass of the ship and p is the angular frequency of the surge. V is given by

$$V = \frac{Ap}{qd} \sin qx \tag{19}$$

where A is the seiche amplitude, q is the nodal frequency of the seiche, and d is the water depth. It should be noted that the phase angle has been neglected and the damping is shown as being proportional to the first power of the velocity rather than the square of the veloci

The natural frequency of the system thus depends upon the amplitude of the forcing function as well as upon the spring and mass characteristics of the system.

After certain other simplifications were made, Wilson (1950) showed the relationships between resonant period of ship oscillation, τ, and maximum individual cable tension, T_x, versus seiche amplitudes, A, for several conditions of cable tension (Figure 9). The effect of cable tension on the periodicity of the system is apparent: the higher the tension the lower the resonant period. The effect of amplitude is relatively unimportant for cables with high tension, but very important for cables with low tension: the greater the seiche amplitude the lower the resonant period of the system.

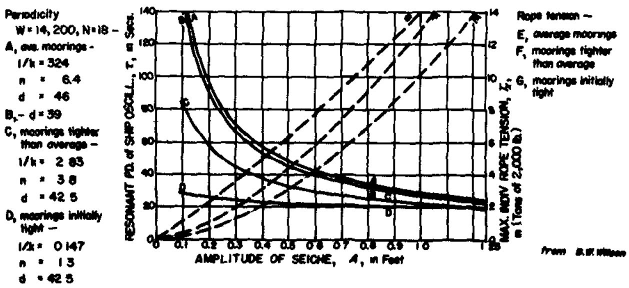


Figure 9. Influence of rope tightness on the resonance in longitudinal ship motion.

Recently Equation 18 has been treated by Wilson and Abramson (1955) using the Ritz method (which is useful for certain non-linear differential equations). Typical results are shown in Figures 10 and 11. In Figure 10 is shown the relationship between the maximum amplitude of horizontal ship displacement, u, and η^2 for various values of seiche amplitude. η is

$$\eta = \frac{p}{\omega} \tag{20}$$

where p is the angular frequency of the seiche. ω may be considered as the non-linear equivalent of the natural frequency

$$\omega^2 = \frac{C}{2M} = \frac{Nk}{4M} \tag{21}$$

although the non-linearity, entering through k results in a departure from the ordinary dimensions of frequency.

The above information on mooring has been presented although it is a great simplification compared with the problem of the LSM moored at sea. It does point to the great importance of initial cable tension and to the possible importance of wave amplitude.

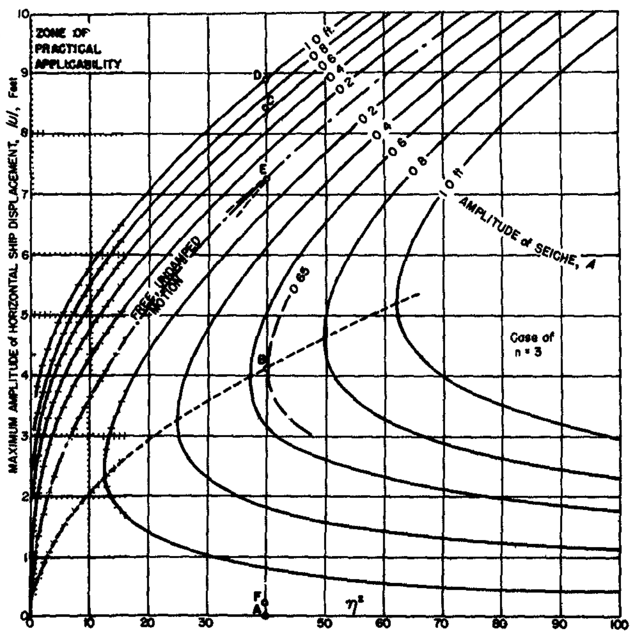


Figure 10. Typical response curves of longitudinal ship displacement

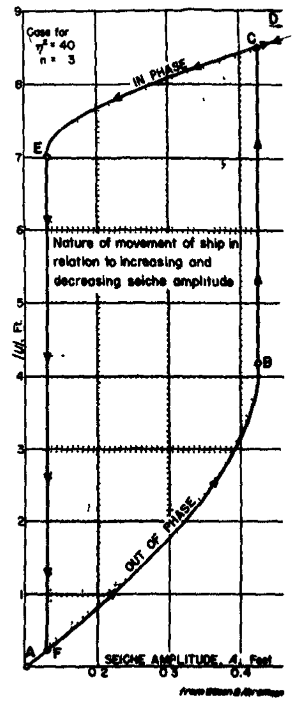


Figure 11.

LABORATORY EQUIPMENT

The laboratory equipment consisted of:

1. Four mooring cable force meters (Figure 4).
2. A six-channel Brush Electronic Company recording oscillograph, and six Brush Universal Analyzers.
3. A 6-foot by 8-foot by 200-foot wave-towing tank, with bulkhead type wave generator (Figure 12).
4. A 2.5-foot by 64-foot by 150-foot model basin with a flapper-type wave generator.
5. Two 35mm. Bell and Howell movie cameras.
6. Two clocks and a neon glow tube.
7. A camera box and a camera base plate.

Before the force meters were used their deflection characteristic and their sensitivity to applied cable forces were measured.

Two experimental set-ups were used, one in the wave-towing tank, other in the model basin. The wave-towing tank was used to determine effect of moored vessel dynamics in head seas and the model basin was used to investigate the moored vessel dynamics in quartering and beam

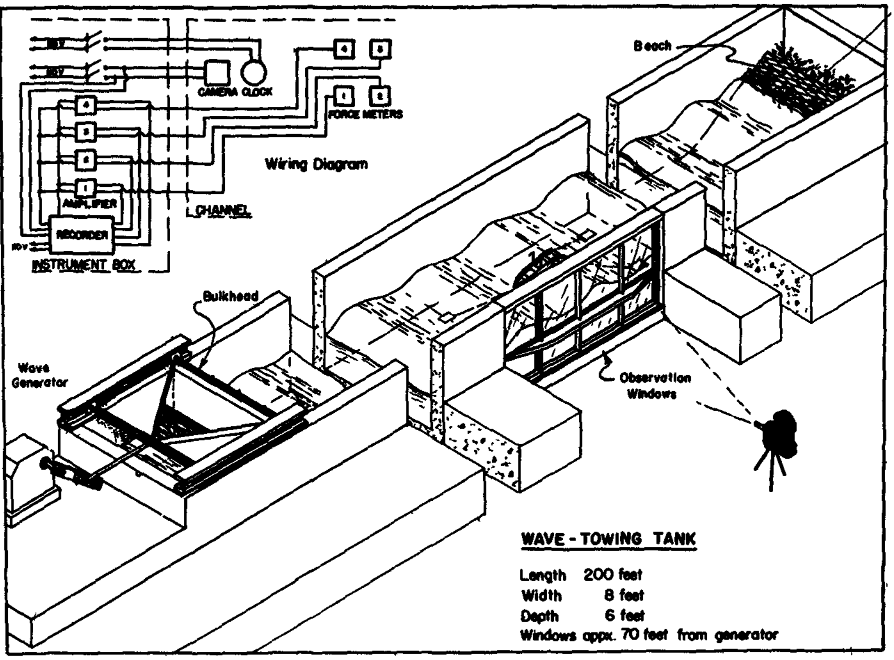


Figure 12.

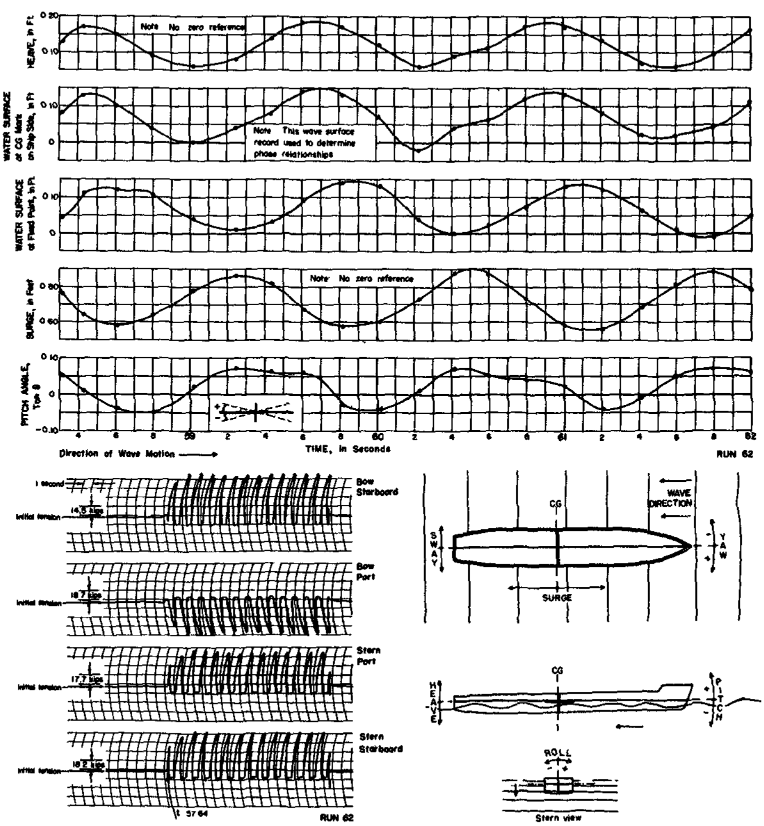


Figure 13.
Sample data with model LSM in head seas.

A few tests also were run in the madel basin for head seas in order to correlate the results obtained by use of the two facilities. The wave-towing tank would have been used for all tests because of the superior control possible with it; however, its transverse dimensions were not ad quate to permit the use of the necessary mooring cable patterns for quar tering and beam seas.

The use and disposition of the cameras and timing equipment (clocks are discussed in the section on Testing Procedure.

TESTING PROCEDURE

Three testing procedures were followed: one for the head sea tests in the wave-towing tank, one for the quartering sea, beam sea and head s tests in the model basin, and one for the tests in the wave-towing tank of the effectiveness of the large bilge keels.

WAVE-TOWING TANK TESTS

The following procedure was used:

1. The depth of water was adjusted to the desired value.
2. A grid was hung from supports across the top of the channel in a plane through the location of the longitudinal axis of model LSM during the tests. Photographs of the grid were taken with 35mm. movie camera.
3. The grid was removed and the model LSM, mooring cables, and mooring cable force meters were placed in the channel. The mooring cables were attached to the force meters and to the ship model.
4. A desired initial tension was set into the mooring cable system and the force meters were oriented so that the deflection of the entire system would be in scale.
5. A clock was placed just above the model.
6. The wave generator was adjusted for a pre-determined wave height and period and the wave generator started.
7. About eight waves were allowed to pass the leeward force meter, then the camera was started, and about two seconds later both the clock and the force meter recording system were started.
8. After the forces resulting from about ten to twelve waves were recorded on the force-meter recorder, the recorder and clock were stopped. The camera was stopped two seconds later.
9. Steps 7 and 8 were repeated for each wave condition to be tested at one particular mooring condition (scope, water depth, initial tension).

The clocks and force meter recorders were started and stopped at same time while the camera was running for the purpose of synchronizat Although the clock took about 0.25 second to come up to speed, the fra speed of the camera was known, so that any desired phase relationship tween motions and associated mooring cable forces could be determined.

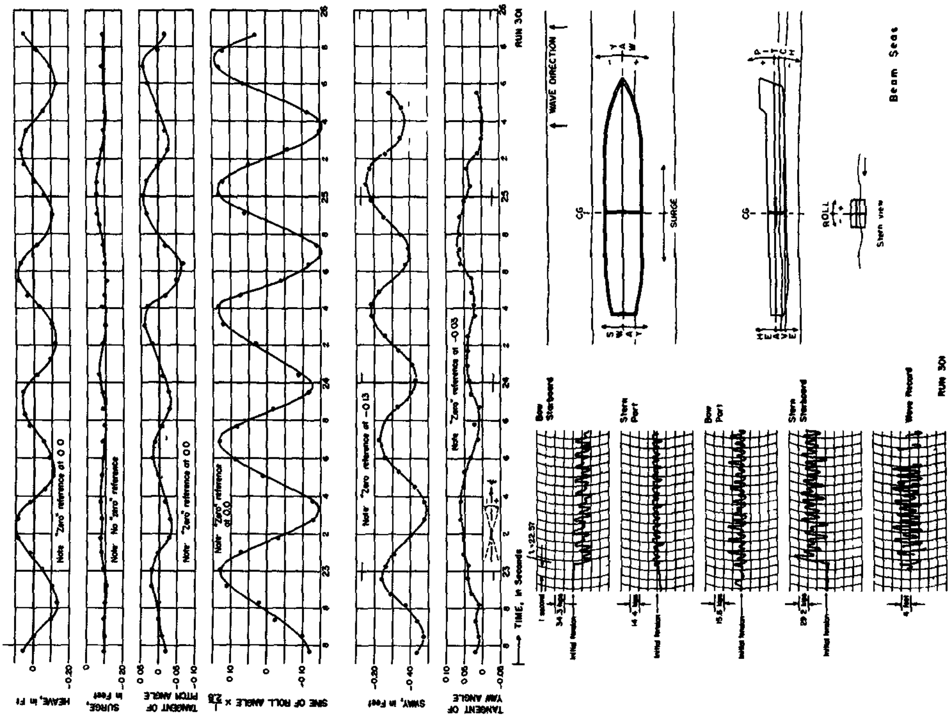


Figure 15. Sample data with model LSM in beam seas.

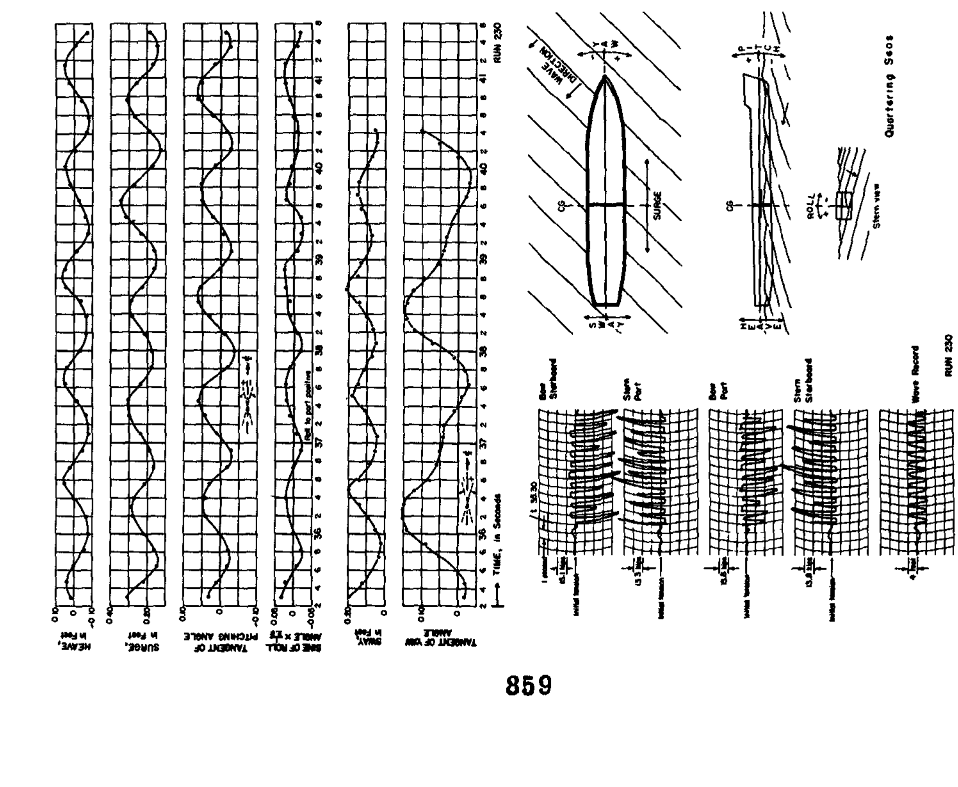


Figure 14. Sample data with model LSM in quartering seas.

The reference for the grid coordinate system consisted of two strip of black tape, one horizontal and one vertical, pasted to the glass of the channel wall.

MODEL BASIN TESTS

The model basin tests were performed in the same manner as the wave towing tank, with the exception that two cameras were used. A grid was placed in the plane through the center of the model before the model was placed in the water and a few feet of film were exposed.

ROLL DAMPING TESTS

The effectiveness of an enlarged bilge keel was tested in the wave-towing tank under "freely floating" conditions; that is, no mooring line were attached. The model was centered in the tank in such a manner that it was under the action of beam seas. The data were recorded on 35mm. movie film.

EXPERIMENTAL RESULTS AND DISCUSSION

DATA INTERPRETATION

Since all experimental data were recorded on either 35mm. movie fi or on six-channel recording oscillograph paper, it was necessary to reduce this data to such a form that it could be effectively plotted.

The data obtained from the film were plotted as shown in Figures 1 14 and 15. In Figure 13 are shown data for Run 62 for the model LSM in head seas; in Figure 14 are shown data for Run 230 in quartering seas; in Figure 15 are shown data for Run 301 in beam seas. The ranges of cc ditions tested are shown in Table I.

FIRST SERIES OF TESTS

The first series of tests were conducted to determine the mooring forces and ship motions of a model LSM (1:80 scale) moored in head sea: These tests were made in the wave-towing tank. The parameters studied this series of tests were initial cable tension, the cable scope, the water depth, the wave height and the wave period.

Two geometric configurations were used in regard to "hawse pipe" cations at the ship's bow. This was because it was noticed that the w spacing of the take-off points of the forward cables was responsible f an induced roll of the model vessel when the vessel was at even a very small angle to the waves; hence, centerline mooring was introduced.

The results of the tests for the centerline-bow, outboard-stern hawse pipe locations are given in Figures 16 and 17. Data were obtair for wave periods between 6 and 15 seconds and for wave heights betweer 4 and 12 feet (prototype). The "least-count" accuracy of the data was about plus or minus five percent. The scatter of data was considerabl

TABLE I. MOORING CONDITIONS FOR MODEL LSM

Runs	Mooring	Cable Scope	Model: Water Depth ft.	Model: Cable Length ft.	Model: Initial Cable Tension lbs.	Prototype: Water Depth ft.	Prototype: Cable Length ft.	Prototype: Initial Cable Tension kips
1- 9*	outboard bow and stern	6.0	2.5	15	0	200	1200	0
11- 21	" "	6.0	2.5	15	0.0195	200	1200	10
22- 39	℄-bow, outboard stern	6.0	2.5	15	0.0058	200	1200	3
40- 53	" "	6.0	2.5	15	0.0012	200	1200	6
54- 64*	" "	6.0	2.5	15	0.0195	200	1200	10
66- 81	" "	15.0	1.0	15	0.0058	80	1200	3
82- 96	" "	9.0	1.0	9	0.0058	80	720	3
97-119	" "	6.0	2.5	15	0.00585	200	1200	3
124-127	bow starb'rd line dropped	6.0	2.5	15	—	200	1200	—
128-130	℄-bow, outboard stern	6.0	2.5	15	0.0195	200	1200	10
131-133	bow, starb'rd line dropped	6.0	2.5	15	—	200	1200	—
134-136	℄ bow, ℄ stern	6.0	2.5	15	0.0195	200	1200	10
137-139	bow starb'rd line dropped	6.0	2.5	15	—	200	1200	—
140-151	freely floating roll tests	—	2.5	—	—	200	—	—
152-161*	bilge keel roll tests	—	2.5	—	—	200	—	—
172-174	freely floating roll tests	—	2.5	—	—	200	—	—
175* **	natural period of roll							
210-235	direct ℄ mooring bow & stern	6.0	2.0	12	0.0195	160	960	10
236-250	" "	6.0	2.0	12	0.0117	160	960	6
251-265	" "	6.0	2.0	12	0.0058	160	960	3
266-277	" "	6.0	2.0	12	0.0058	160	960	3
278-289	" "	6.0	2.0	12	0.0117	160	960	6
290-301	" "	6.0	2.0	12	0.0195	160	960	10
302-313	" "	6.0	2.0	12	0.0117	160	960	6
314-326	" "	6.0	2.0	12	0.0058	160	960	3
327-339	" "	6.0	2.0	12	0.0195	160	960	10
339-347	" "	6.0	2.0	12	0.0058	160	960	3

* Runs 10, 65, 162-171 and 200-210 were test runs and are not included.
** Runs number 176-199 were not used.

and much of this was due to the difficulty imposed by the least-count when working with a model of this scale. However, the trends were apparent, and the scatter of data of heave, surge and pitch were probably within the necessary limits from an operational standpoint.

A sample of the data on the horizontal component of cable force at the meter for each cable have been tabulated in Table II, together with the averages of the bow cables, the stern cables and all four cables. T data presented for the individual cables are the maximum recorded forces as measured from the initial tension, i.e., the actual horizontal component of force is the tabulated force plus the initial tension. The data on the force records did not show much scatter from wave to wave; thus the scatter is due to the least-count errors and to the fact that the sy tem is rather unstable. Some of the variation in cable force was due tc the inability of the investigators to adjust all four cables to the same initial tension. This difficulty, and the result of this difficulty, wa particularly apparent in Runs 40 to 53. It is expected, however, that similar difficulties would be encountered in prototype; hence, the varia tion of data (particularly in regard to the largest forces measured) should be of considerable value.

The most important fact that was evident in these data was in regai to the relationship between surging motion (and hence cable force) and the wave character-istics. The data showed a resonance condition occurring for certain combina tions of wave and mooring characteris tics. A check of t surging characteris tics of the test sy tem was undertaken. It was found that t natural period of surge (for the 2 mi wire with lead shot was about 16 seconc for the case of a cable scope of 6, a water depth of 200 feet, and an initia tension of 10 kips (Figure 18). As tl determination of tl natural period was done with a new se of model cables, a due to the difficu in obtaining exact initial tensions, natural period of surge during Runs l

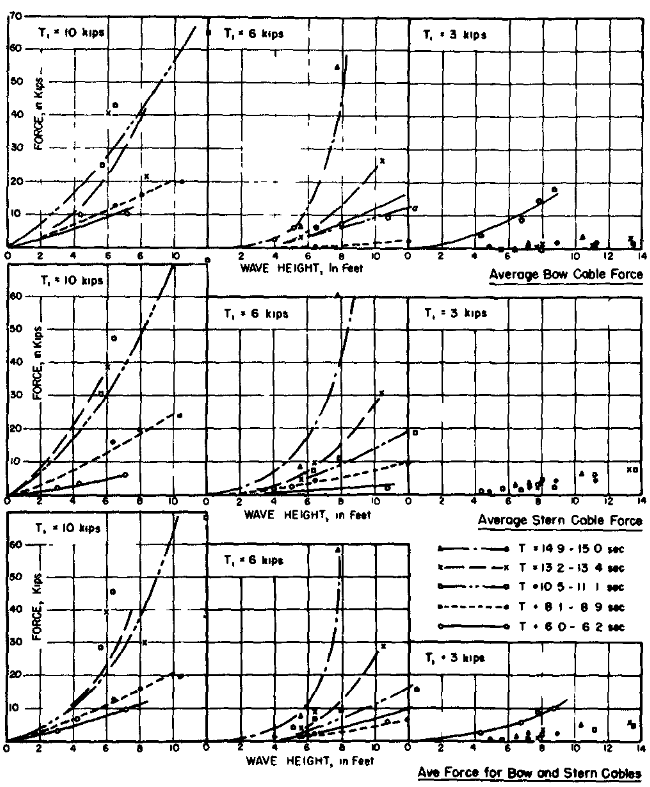


Figure 16. Mooring cable forces in head seas, Runs 22-64.

TABLE II. SAMPLE DATA OF MOTIONS AND MOORING FORCES FOR MODEL LSM IN HEAD SEAS

Run No.	Wave Period sec.	Wave Height ft.	Surge ft.	Heave ft.	Pitch deg.	F_{BP} kips	F_{BS} kips	F_{SP} kips	F_{SS} kips	$F_{B_{ave}}$ kips	$F_{S_{ave}}$ kips	F_{ave} kips
Runs 22–39: scope = 6, water depth = 200', initial cable tension = 3 kips												
22	6.0	4.4	1.5	0.8	1.6	2.7	4.6	0.4	1.6	3.7	1.0	2.4
23	6.0	6.8	1.6	0.8	2.3	8.9	9.0	1.5	2.3	8.9	1.9	5.4
24	6.0	7.8	2.0	2.4	4.0	15.2	13.7	2.1	4.4	14.5	3.2	8.9
25	6.0	8.8	2.0	2.4	4.3	15.2	21.1	1.4	3.0	18.1	2.2	10.2
26	8.1	4.8	6.9	3.2	2.3	0.3	0.3	1.2	1.2	0.3	1.2	0.8
27	8.1	7.2	8.7	5.2	2.9	1.0	0.5	2.3	1.9	0.8	2.1	1.5
28	8.1	8.8	7.6	6.4	4.0	2.0	1.1	4.0	3.5	1.5	3.8	2.7
29	8.1	11.2	8.8	8.0	4.6	1.0	1.6	5.5	4.4	1.3	5.0	3.2
30	10.5	5.6	6.0	4.0	1.7	-1.1	0.3	2.2	2.0	-0.4	2.1	0.9
31	10.5	8.0	7.2	8.0	2.3	-0.3	0.9	5.3	3.2	0.3	4.3	2.3
32	10.5	11.2	8.8	8.8	3.4	0.5	1.7	7.2	4.6	1.1	5.9	3.5
33	10.5	1.36	10.4	11.2	3.4	1.5	2.6	9.2	6.8	2.1	8.0	5.1
34	13.4	7.8	8.3	6.6	1.7	0	1.0	3.7	2.0	0.5	2.9	1.7
35	13.4	8.0	9.6	7.6	2.6	2.4	0.3	3.9	2.7	1.3	3.3	2.3
35a	13.4	—	—	—	—	2.4	-0.4	4.5	3.3	1.0	3.9	2.5
36	13.4	13.3	14.4	11.0	2.9	5.0	2.9	9.1	6.4	4.0	7.7	5.9
37	15.0	6.4	6.8	6.0	0.9	1.7	-2.2	3.4	2.6	-0.3	3.0	1.4
38	15.0	7.2	6.8	6.0	1.0	2.0	1.1	4.0	3.0	1.6	3.5	2.6
39	15.0	10.4	11.2	9.6	1.5	3.6	2.9	7.4	5.2	3.3	6.3	5.0
Runs 40–53: scope = 6, water depth = 200', initial cable tension = 6 kips												
40	6.2	4.0	1.2	3.2	1.5	-1.5	4.9	1.4	1.1	1.7	1.3	1.5
41	6.2	5.2	1.6	2.4	2.9	3.4	8.4	2.8	1.3	5.9	2.1	4.0
42	6.2	10.8	3.2	3.6	4.3	7.2	11.0	2.3	1.4	9.1	1.9	5.5
43	8.8	6.4	5.6	5.6	2.9	-1.2	1.4	6.6	2.1	0.1	4.3	2.2
44	8.8	8.0	8.0	6.4	3.4	—	—	—	—	—	—	—
45	8.8	12.0	8.8	7.6	4.9	0.2	5.8	13.2	6.4	3.0	9.8	6.4
46	10.9	6.4	7.2	6.0	1.5	5.2	7.7	11.0	3.4	6.5	7.2	6.9
47	10.9	8.0	9.6	7.2	2.3	2.7	11.6	16.6	6.4	7.1	11.5	9.3
48	10.9	12.4	13.6	5.2	3.2	5.7	18.7	26.0	11.9	12.2	18.9	15.6
49	13.2	5.6	5.6	3.6	1.1	0.4	6.2	7.6	1.5	3.3	4.6	4.0
50	13.2	6.4	8.8	6.4	1.4	2.6	10.4	14.4	4.7	6.5	9.6	8.1
51	13.2	10.4	19.2	9.6	2.4	13.8	39.1	42.1	19.7	26.5	30.9	28.7
52	14.9	5.5	7.7	4.4	1.0	2.2	10.8	13.0	4.0	6.5	8.5	7.5
53	14.9	7.7	33.5	7.6	1.4	50.2	59.0	61.8	61.2	54.6	61.5	58.1

Note: F_{BP}, force measured for port bow cable; F_{BS}, for starboard bow cable; F_{SP}, for port stern cable; and F_{SS}, for starboard stern cable.

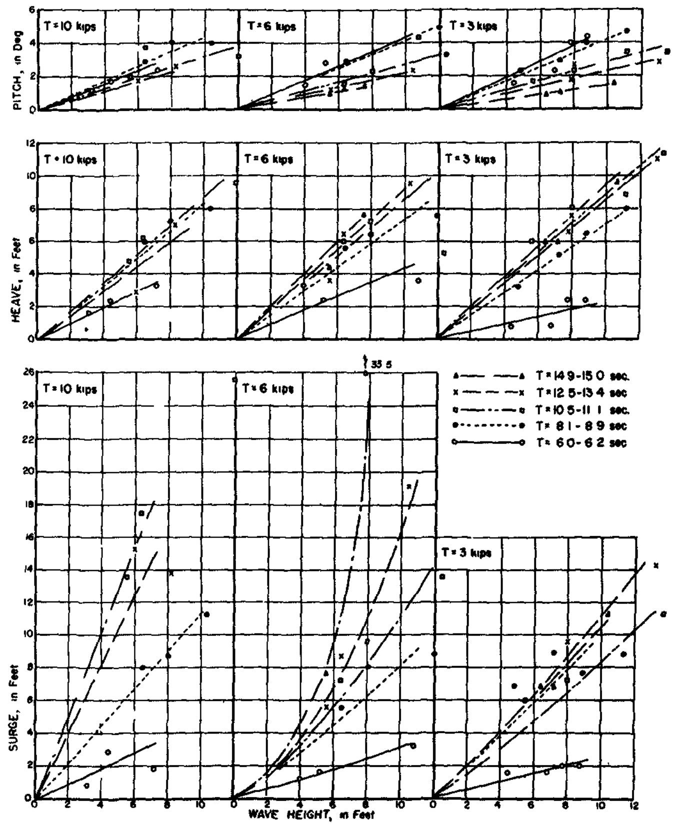


Figure 17. Pitch, heave and surge in head seas, Runs 22-64.

might well have been a few seconds less. It was evident from the data in Table II and Figures 16 and 17 that the relationshi between both surging and cable force and the wave height was non-linear, at least when near resonance. Both the surging motion and the cable force increased quit rapidly for the greater wave heights

It is apparent that this difficulty would not be encountered when operation were conducted in an area where the wave period was in the vicinity of 5 to 8 seconds, nor would i become apparent for low waves (say, in t neighborhood of 3 fe due to the inabilit of a person to judg the extent of such small motion.

It was found that for the case of low initial cable tension, the highest mooring cable forces were associated with the shortest wave period; however, as the initial tension increases, the highest forces were associated with the longer periods — those nearer, or at, resonan

In regard to the vessel motions, what was really needed was its spatial location as a function of time. The tremendous amount of calcu lations necessary to present this information precluded obtaining it on this investigation, and instead the individual motions were shown in re lation to the wave height and period. These data have been presented i Figure 17. It was found that the pitching angle was nearly a linear fu tion of wave height with the shortest period (within the limits of the experiments) being responsible for the greatest pitch. The pitching angle appeared to be independent of the initial cable tension. Heave w greatest for the longest wave periods and appeared to be independent o the initial cable tension.

Surging motion was dependent upon initial cable tension to a grea degree. In addition, it was not linearly related to wave height; rath it increased with wave height at an increasing rate, at least where the period was in the vicinity of resonance.

SECOND SERIES OF TESTS

The model cable first used (2 mil diameter wire with lead shot attached for weight) was found to deteriorate very rapidly in use; hence, very small forces would break the cables during tests. California Research Corporation and University of California personnel agreed that a 10 mil wire should be used if tests indicated that the result would correspond with those obtained using the 2 mil-with-shot wire, and providing swivels were used at points of attachment, and kinks in the wire avoided.

In order to determine the difference in effects in mooring dynamics due to the use of two types of mooring wires, a series of tests were run. At the same time the effect of "breaking" one cable was studied, as was the effect of introducing a centerline mooring "hawse pipe" at the stern of the model. The data were compared with the results of previous tests and were found to be in fair agreement provided the differences in natural period of surge are kept in mind. For example, in the case of an initial cable tension of 10 kips, a resonance, or near-resonance, condition for the 2 mil-with-shot cable occurred for both Runs 63 and 18 (12.5 and 13.0 second wave, respectively) while for the 10 mil wire this condition occurred for Run 129 (a 9.2-second wave period). This can readily be understood by examining the curves of natural period of resonance versus initial cable tension in Figure 18.

As a part of this same series of tests data were obtained which showed the effect of one mooring cable breaking. The tension in the diagonally opposing cable dropped off considerably and the vessel yawed slightly, slacking off the remaining two cables (from an "initial tension" standpoint). This caused an increase in the natural period of surge. Hence, the resonance occurred at a greater wave period. For example, a resonance

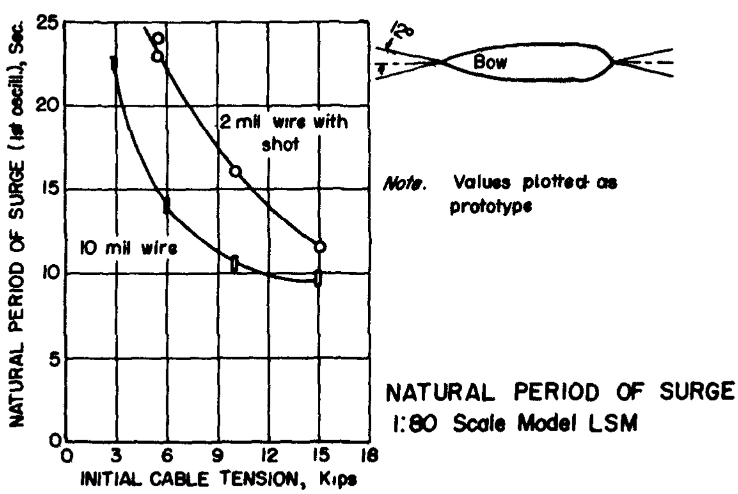


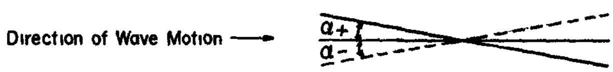
Figure 18.

or near-resonance condition occurred for a wave period of 9.2 seconds fo the 10-kip initial cable tension with four mooring lines, but shifted to 13.5 seconds when the starboard bow cable was dropped. It was not possi ble to tell conclusively from these data whether dangerously high forces will be experienced when one cable breaks, because of the spread of wave periods tested, with the resulting probability that the peak resonance condition might have been encountered for the 13.5-second wave but not f the 9.2-second wave, vice versa, or neither.

Data were also obtained for a centerline stern mooring. The effect of this type of mooring was not apparent unless there was an induced rol

THIRD SERIES OF TESTS

In order to test the effect of roll-damping devices, it was necessa to use the wave-towing tank, which in turn prevented the testing of a properly-moored vessel. Because of this, the effect of the device was studied for the freely-floating condition. The data have been given in Table III. The roll was considered in two parts, as shown in the follo ing sketch:



$$\text{Roll} = \frac{\alpha + + \alpha -}{2}$$

$\alpha+$ was the angle of roll as measured upwards on the up-wave side of th vessel, while $\alpha-$ was the angle of roll as measured downwards on the u wave side of the vessel. It can be seen in Table III that $\alpha+$ was con sistently larger than $\alpha-$. This was a dynamic characteristic of the interaction between the waves and the vessel and was probably a result diffraction. It did not occur during tests for natural period of roll; hence, it was not caused by some unbalance in the vessel. This "dynami list" was most apparent for the case where bilge keels were added. The bilge keels were very effective (especially for the short period waves) when one considered the mean roll from its dynamic equilibrium positior (i.e., $\alpha+ + \alpha-/2$); however, it was not nearly so effective when the maximum roll ($\alpha+$) from the horizontal was considered.

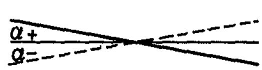
FOURTH SERIES OF TESTS

The fourth series of tests was performed to determine the vessel motions and mooring forces in quartering and beam seas. In addition, tests were made of the vessel in head seas. This was done so that a c parison could be made between the tests in the model basin and the tes in the wave-towing tank. Due to the limitations of the model basin it was possible to moor the vessel in no more than 160 feet of water (pro totype). 10-mil model mooring cables were used. Some of the data hav been plotted in Figures 19 to 21. In addition to a least count accura of ±5% the data obtained in the model basin are subject to variations as the wave generator motion is not as consistent as in the wave-towin tank and because it was not possible to have complete protection from wind effects on the model.

TABLE III. EFFECT OF ROLL DAMPING DEVICES FOR FREELY FLOATING MODEL LSM IN BEAM SEAS

Run No.	Wave Period sec.	Wave Height ft.	Ave. $\alpha+$ deg.	Ave. $\alpha-$ deg.	Range of Roll deg.
Runs 140-151, no roll damping devices					
140	6.3	1.2	5.0	4.2	3.3 - 4.5
141	6.3	2.1	10.0	8.0	8.5 - 9
142	6.3	6.3	12.3	10.2	11.0 - 11.5
(143)					
144	7.8	4.6	5.0	3.7	4.3 - 4.5
145	7.8	7.9	8.2	7.0	6.3 - 8
(146)					
147	7.8	7.1	7.5	6.7	7 - 7.3
148	7.8	11.6	11.5	10.0	10.5 - 11
149	12.5	3.2	2.5	1.7	1.5 - 2.0
150	12.5	5.8	2.5	1.5	1.8
151	12.5	8.2	3.9	3.2	2.3 - 3.6
Runs 152-161, with bilge keel					
152	natural period of roll = 11.9 seconds				
153	6.3	5.4	4.3	1.0	1.5 - 3.5
154	6.3	10.1	10.2	0.9	3 - 7
155	6.3	1.6	3.0	0	1.3 - 1.8
156	7.9	3.8	4.6	0	1.8 - 3.3
157	7.9	7.7	6.5	0.3	2.5 - 4.0
158	7.7	11.0	8.4	0.7	3.5 - 6.5
159	12.9	3.5	2.0	1.0	1.3 - 1.8
160	12.9	6.6	2.7	2.0	2.3
161	12.9	9.5	3.5	2.5	3.0
Runs 172-175, no roll damping devices					
172	6.1	1.4	9.4	8.0	7.3 - 9.5
173	6.1	4.6	12.3	11.8	11 - 15
174	6.1	7.5	20.6	18.6	19 - 22
175	natural period of roll = 5.1 seconds				

Note:

1. All values tabulated are for the prototype.
2. $\text{Roll} = \frac{\alpha+ + \alpha-}{2}$, where 

3. Maximum variation of ±1 degree for roll angle for no damping device. The variation was up to ±2 degrees for the case of the bilge keel; near resonance conditions may be larger.
4. The time history records of the roll motion of the freely floating model LSM can be considered to be of two types: 1. the rolls were nearly uniform; and 2. the roll were irregular. The data presented in Table III

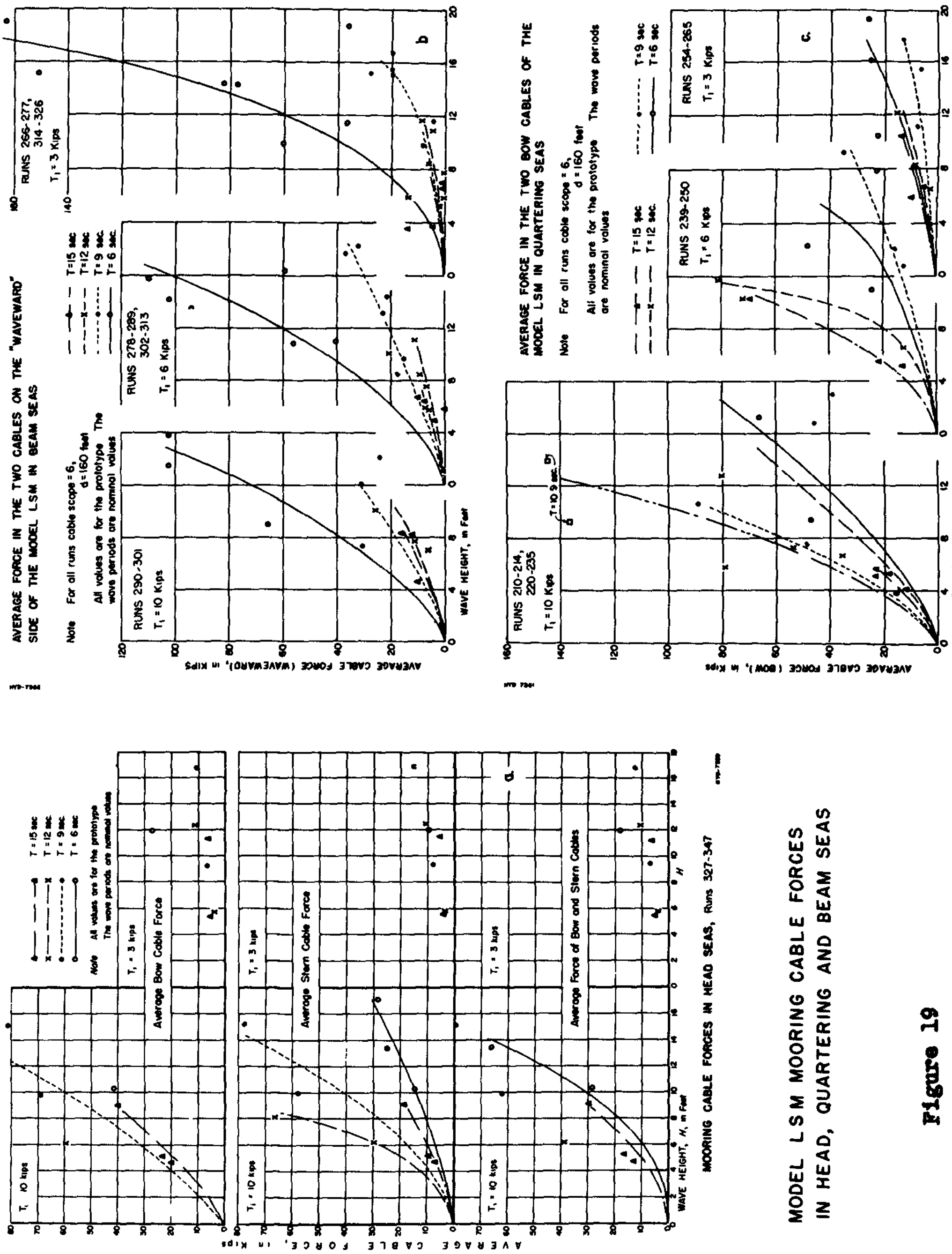
TABLE III. Note (cont.)

are of two types: 1. $a+$ and $a-$ which are the values of straight lines put on the records by eye to represent the average position of the roll up and the roll down; and 2. range of roll, which shows the range of roll angles as measured from the roll down position to the following roll up position.

An examination of the forces in the mooring cables for head seas as obtained in the wave-towing tank and the model basin for the same initial cable tensions and cable scopes, but different water depths, (Fi ure 16 and 21a) showed that they compared reasonably well considering th difference in model cables. This was also true of the pitching and hea ing motions (Figures 17, 20a and 21a). Considering the fact that ocean waves are non-uniform in both amplitude and period and the possibility exists of a particular wave period component forcing the vessel in resonance, it would appear that the comparisons of the two sets of data wit respect to maximum probable vessel motions and mooring cable forces are good.

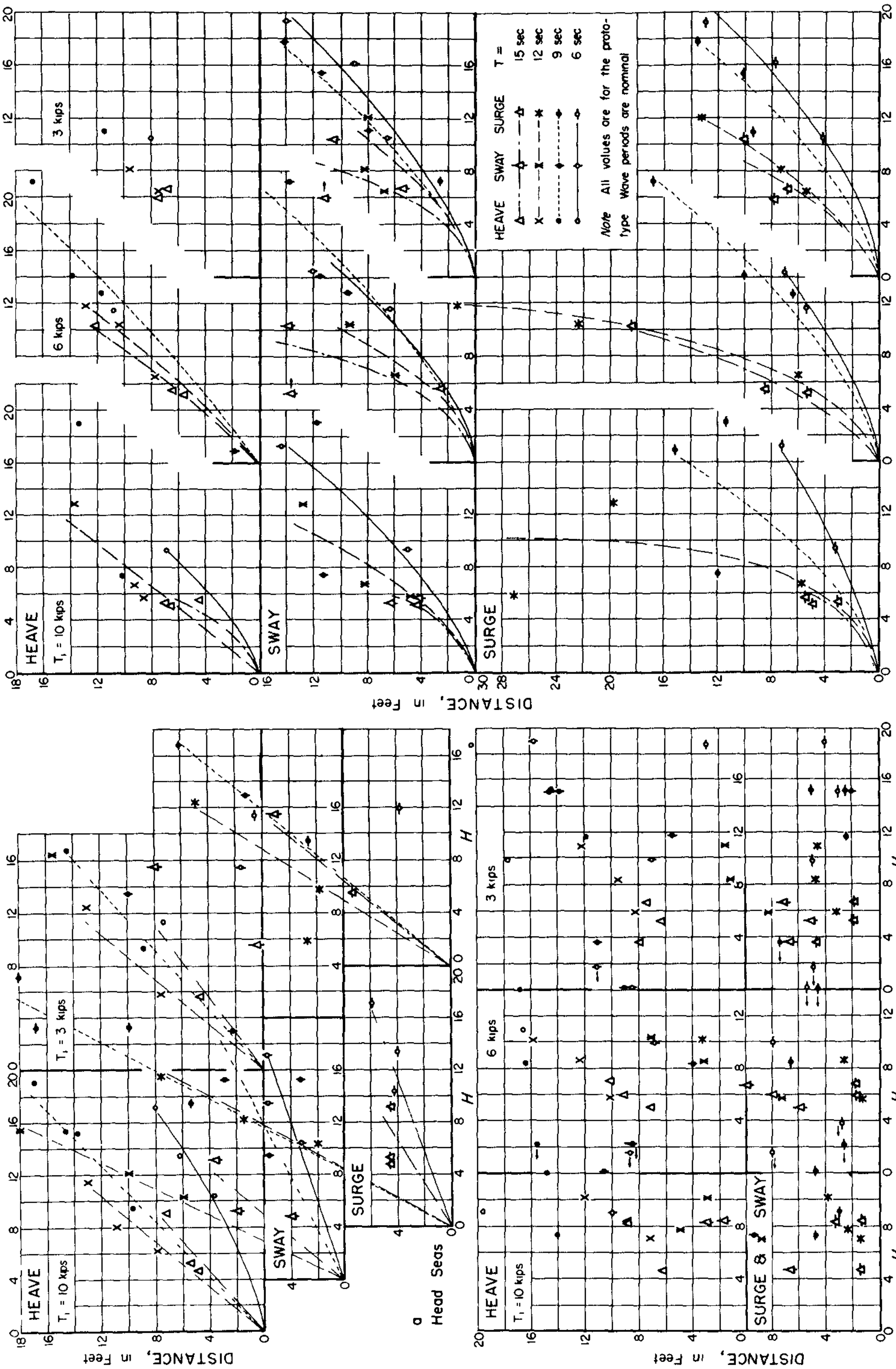
It can be seen that the vessel rolled, yawed and swayed even thoug it was in "head seas". It is not possible to fix the vessel heading exactly, and any induced motion would be emphasized by the elastic mooring cables. In addition, there may be an inherent instability as discussed by Grim (1952).

In regard to motions in quartering seas, it can be seen that the surge and pitch amplitudes are of the same order of magnitude as for th head seas tests in the model basin. The difference in water depth (160 feet rather than 200 feet) should not be of any significance as both depths are relatively deep as far as the water particle velocities and accelerations within the surface layer of the same thickness as the ves sel's draft. The heaving amplitude is greater than for head seas; in fact, the heaving exceeded the wave height on many occasions. A portic of this can be attributed to the effect of angular distortion; however, it is mainly due to the fact (Weinblum and St. Denis, 1950) that the heaving force function increases with increasing angle between the shi bow and the wave direction (see Figure 5a). This, combined with the ma nification factor (Figure 5b) for a damping coefficient of 0.4 (approx mately the value for an average ship), leads to a heaving amplitude wh can be in excess of the wave amplitude. In addition, there is considerable sway, roll and yaw. An example of the type of data obtained ha been given in Figure 14. It was apparent from the tests that the sway and yaw records may be either fairly uniform or quite non-uniform. Th record in Figure 14 cannot be said to be typical; rather, there was an entire spectrum of variations. These two motions appeared to get in a out of phase for certain wave periods; maximum values of sway and yaw often as much as 25 percent greater than the average values reported i the tables. The mooring cable forces were of the same order of magnit as in head seas and the trends appeared to be the same as in head seas



MODEL L S M MOORING CABLE FORCES
IN HEAD, QUARTERING AND BEAM SEAS

Figure 19

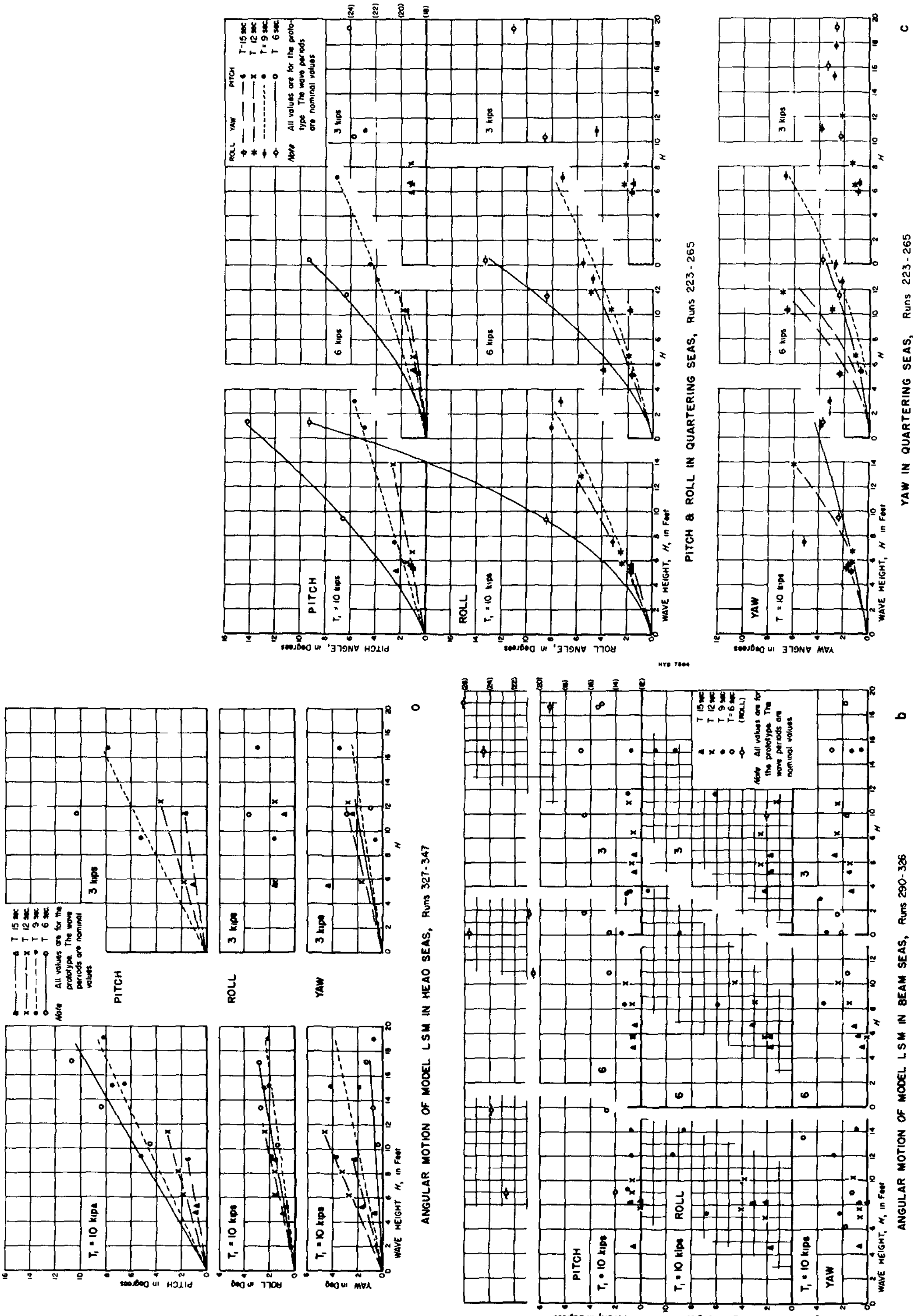


b Beam Seas

c Quatering Seas

MODEL LSM HEAVE, SWAY AND SURGE IN HEAD, QUARTERING AND BEAM SEAS

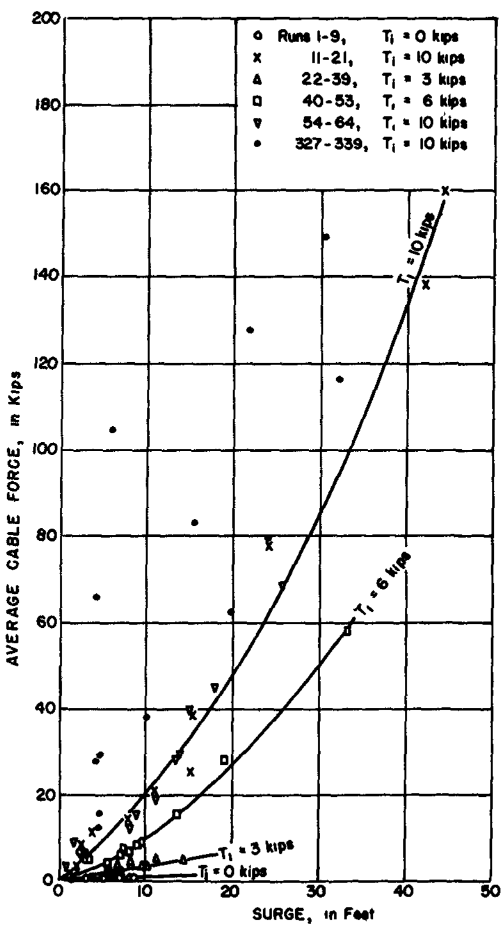
HYDS 7359, 7362 7363, 7674



MODEL LSM PITCH, ROLL AND YAW IN HEAD, QUARTERING AND BEAM SEAS

Figure 21

In beam seas the surging motion decreased considerably, although it did not go to the zero. This was possibly due to a small error in the positioning of the vessel, unequal initial cable tensions, or both. The same was true for pitching. The yawing was appreciable, although the magnitude was not as great as for the case of quartering seas. The roll· ing motion appeared to be of about the same order of magnitude for beas ‹ for quartering seas. The sway increased. It was evident that large motions could be expected for certain combinations of wave heights and periods. The reason for the apparent "resonance" condition occurring fo sway in beam seas was not apparent considering the long natural period. It was evident from Figure 21b that it was nearly independent of initial cable tension. However, it well may be explained if curves similar to Figure 5 were available. Certainly the "swaying force function" must be very large. The ratio of the maximum values of sway to wave height neve approach the maximum value of the ratios of surge to wave height.



RELATIONSHIP BETWEEN AVERAGE MOORING CABLE FORCE AND SURGE IN HEAD SEAS

Figure 22.

The maximum cable forces occurred for lower wave periods in beam seas than in eithe head or quartering seas. Thu it might be necessary to head the vessel differently, depen ding upon the wave period (say, local seas or swell).

GENERAL

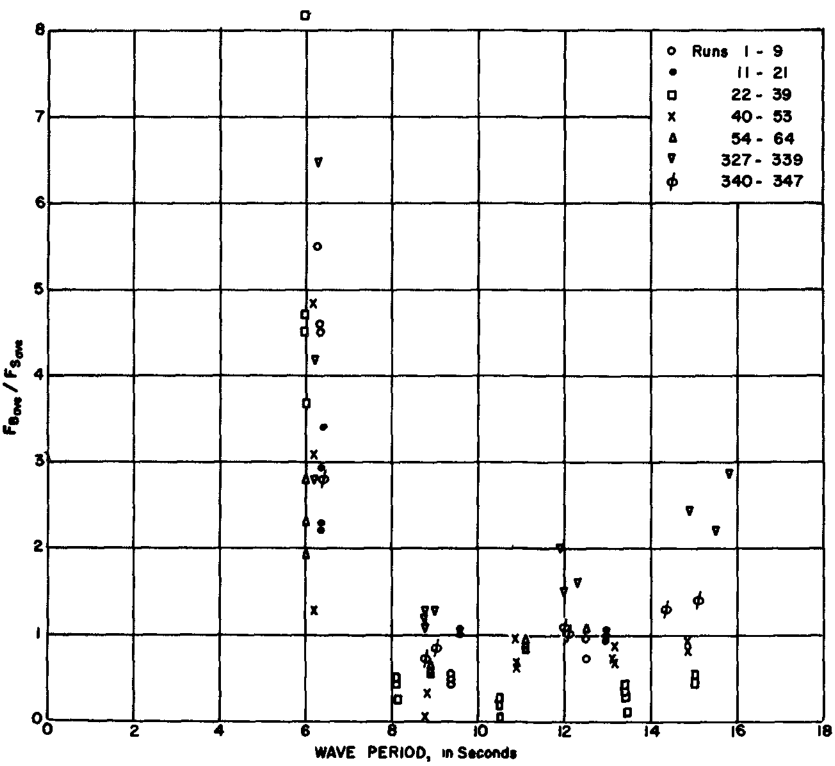
The average cable force data have been plotted in Figure 22 in relation to the vessel surge for three initia cable tensions for the vessel in head seas. The data taker in the wave-towing tank show a marked degree of correlati In addition, the data taken the model basin (for an init cable tension of 10 kips) we plotted. Some of the data were not consistent. Some waves used in the model basi were considerably higher tha was the case for the tests i the wave-towing tank and the data which were most out of agreement were those associa with the highest waves and t accompanying large pitching and heaving. It is believed that for waves under, say, 12 feet in height, the relationship shown in Figure 22 should be valid.

The bow cable forces for the model in head seas were considerably higher than the stern cable forces for the shortest wave periods used. For longer wave periods this relationship was reversed, while the longest waves tested showed the relationship often reversed again. The ratio of the average bow cable forces to the average stern cable forces have been plotted in relationship to the wave period in Figure 23. The curve was found to drop abruptly at about $6\frac{1}{2}$ seconds to a minimum value and then to increase with increasing wave period.

The data could have been plotted as a function of the dimensionless parameters λ/L (ratios of vessel length to wave length) and H/L (ratio of wave height to wave length). This was done for a few cases for checking purposes. However, it does not clarify the understanding of the phenomena as only one ship model length, λ, was used; hence, the dimensionless numbers would have no significance.

CONCLUSIONS

The problem of the prediction of motions of a freely floating vessel in a seaway has not been solved completely. The motions of a moored



RELATIONSHIP BETWEEN THE WAVE PERIOD AND THE RATIO OF THE AVERAGE BOW MOORING CABLE FORCE AND AVERAGE STERN MOORING CABLE FORCE

Figure 23.

vessel are much more complicated and require an empirical solution at the present time. From a practical standpoint the most important difference between the motions of a freely floating and a moored vessel is the possibility of resonance conditions occurring in the surging, swaying and yawing motion of a moored vessel, as sell as in the heaving, pitching and rolling motions.

Resonance conditions were found to occur in the surging motion of the model in both head and quartering seas; resonance conditions were found to occur in swaying and heaving motion in both quartering and beam seas (especially in beam seas), although the magnification factor for swaying was not nearly as high as for the case of surging. The greatest forces in the mooring cables occurred at these resonance conditions.

The natural periods of surge, sway and yaw were found to be critically dependent upon the mooring cable configuration and initial tension, with the greater the initial tension the shorter the natural period. Heaving, pitching, and rolling appeared to be essentially independent of the initial cable tension within the limits of the experimental range of conditions; if they were not independent the relationship was masked by the experimental scatter. The natural periods of sway and yaw for the range of initial cable tension used in the tests were found to be much greater than the periods of waves normally encountered in the ocean. The resonance condition for surging for the higher initial cable tensions used in the tests was found to occur within the range of wave periods normally expected along the Pacific Coast of the United States. The relationship between the surging motion (and mooring cable forces) and wave height was non-linear, at least when near the resonance condition. The surging motions and cable forces were found to increase at increasing rates with increasing wave height.

It was found that the higher the initial cable tension the greater were the maximum cable forces experienced when the vessel was subjected to wave action in head or quartering seas. This was because resonance conditions occurred for wave periods in the range of 10 to 15 seconds. However, in the case of beam seas the cable forces were found to be relatively independent of the initial tension. In this case the maximum cable forces occurred for the shortest wave period tested (about 6 seconds). These occurred in the cables on the "waveward" side of the model. At the same time, the forces on the "downwave" cables were relatively small. It would appear that the high forces on the "waveward" cables were due to a combination of diffraction and second order effects for this case.

The data on heaving, pitching, rolling and yawing were relatively consistent and it should be possible to determine these motions for the prototype with an acceptable degree of accuracy.

Considering the difficulties inherent in obtaining quantitative results using a small model, and where no prototype measurements are available to use as a guide, the results of the mooring cable force measurements are not too inconsistent. Certainly the trends can be relied upon as can the order of magnitude of the motions and the cable forces.

This is especially true when the non-uniformity of ocean waves is considered.

None of the motions or cable forces appear to be excessive for a wave height in the neighborhood of 5 feet, regardless of wave period. Waves in excess of 8 feet (more or less), on the other hand, might result in high mooring cable forces and large surging, swaying or heaving motions, depending upon initial cable tension, wave period and vessel heading. If the characteristics of the waves in a certain location were known, it should be possible to use the curves presented in this report to predict the dangerous conditions.

In regard to roll-damping devices, it was found that the bilge keels decreased the periodic roll angle considerably (especially for the short period waves); however, when bilge keels were used, a "dynamic list" was developed.

ACKNOWLEDGEMENTS

The authors wish to express their appreciation to the California Research Corporation for making this study possible, and to Drs. G. A. Schurman, L. W. Thrasher and R. E. Kitter of the California Research Corporation for their suggestions and encouragement during the study; to R. C. Wiley for his workmanship in construction of the model and force meters; to A. L. Arnold for his help in conducting the experiments; to Dr. R. W. Clough for his comments on the model laws, the theoretical consideration and the design of the force meters; to John Williams, Jaak Kukk, S. K. Mihara, E. Bennett and Stephen Whisenand for helping to analyze the data; and to M. M. Lincoln for preparing the manuscript.

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CHAPTER 52

FREE OSCILLATION IN SURGE AND SWAY OF A MOORED FLOATING DRY DOCK

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SUMMARY

Measurements are presented of the free period of oscillation in surge and sway of the AFDL-20 (floating dry dock with 2100 long ton displacement) and of the forces and movements induced. The Dock is spread moored fore and aft, respectivel by one 1-1/2 inch die lock chain about 260 feet long with rise of about 35 feet and scope of 8. These measurements are compared with those obtained from oscillating a 1 to 40 linear scale model and from analytics and the agreement is pronounced good.

INTRODUCTION

For many years the Bureau of Yards and Docks has been concerned with resear on the forces induced on moored vessels by waves. In such a study -- unlike a larg number of those in the field of hydrodynamics which involve consideration of signifi cantly free or fixed objects -- the concern is with objects which are forced to move against the restraint of elastic type moorings.

Since, as in the hydrodynamics field in general, it is very rewarding to study at a reduced scale in the laboratory, it is necessary frequently to model the charac teristics of the ships moorings as well as of the ship itself, where although the technique for the latter seems well established, that for the former is not.

To provide correlated data with which to evaluate the ability to model ships' mooring characteristics, a relatively small floating dry-dock, was spread moored in a simple manner, and a 1 to 40 linear scale model of it were caused to oscillate significantly in surge and sway in sensibly still water and the period of the free oscillations was measured. The results obtained from the model, as extrapolated to the Prototype by means of the Froude Model Law since inertial forces seem dominan were compared with those obtained from the Prototype.

Because an analytical approach is desired in general, considerable attention was given also to the application of basic mechanics to provide a comparision with results obtained from both the Prototype and the Model.

TEST FACILITIES AND PROCEDURES

PROTOTYPE

The vessel used is a floating drydock [AFDL-20 of 2100-long tons displacem

(Figure 1) as moored in the harbor of Port Hueneme, California in about 35 feet of water by one chain , 1-1/2 inch die lock of scope 8, respectively fore and aft (Figure 2). Strain gage type dynamometers as described by O'Brien and Jones (1955) were used to measure chain tension at the Dock end.

By means of a tug temporarily attached to it, the Dock was displaced particular amounts in surge or sway as the case might be and then permitted to oscillate freely. The output from the chain dynamometers was recorded as a funtion of time so that direct measurement of both the chain tension and period of oscillation could be made. Movement of the Dock was determined as a function of time by means of direct reading by surveyors of the positions of scales attached to the Dock.

The initial tension in the chains was varied during the experiments to provide a variation in the restoring force. This was done either by waiting for the tide to vary the still water level or by changing the length of the chain. (Table I)

An attempt was made to conduct the experiments only when the wind, currents and waves were at a negligible level. In the case of the latter persistent surges-those with about 1, 3, and 12 minute period — had to be tolerated but these, like the locally generated wind waves and the other environmental disturbances, were not considered to have affected adversely the results obtained. In no case was it possible to obtain pure surge or sway, so that coupling at what is considered a low level had to be tolerated.

Better results were obtained with surge where the restoring forces were relatively high and period and amplitudes short than with sway where the amplitudes were very long — of the order of 15-feet — and the motion died down after an oscillation or two due to the large form resistance and low restoring force involved.

MODEL

A 1 to 40 linear scale Model was constructed by the David Taylor Model Basin and balanced dynamically and moored by the University of California under a contract with the Bureau of Yards and Docks as described by Wiegel et al (1956). The Dock ends of the lines were fitted with dynamometers whose design and installation entailed considerable effort.

As in the Prototype the Model was displaced in either surge or sway and then permitted to oscillate freely with force data recorded as a function of time so that the period could be deduced. Movements were not measured rigorously; in some cases estimates of initial displacement were made.

Table I

Data on the characteristics of the mooring chains and free oscillations in surge and sway

Run No	Type of Motion	I T	Period of free oscil-lation *	MAX. AMPL of MOTION					MAX RESTORING FORCE					CHAIN GEOMETRY					
				Initial Disp	Oscillation Number 1	2	3	4	Initial Disp	Oscillation Number 1	2	3	4	$2b_B$	$2b_s$	$2L_B$	$2L_s$	S_B	S_s
		KIPS	SEC	FT	FT	FT	FT	FT	KIPS	KIPS	KIPS	KIPS	KIPS	FT	FT	FT	FT	FT	FT
1	Surge	4 9	45 2	4 7	2 7	2 4	1 4	2 3	31.8	8 3	6 3	4 8	5.9	38.0	35 2				
2	↑	4 5	44 5	4.6	3 5	3.2	3 2	1 7	27 4	9.7	9 7	7 9	5 2	38 0	35.2				
3		4 3	45 2	4.5	4.0	3 8	3 1	3 2	26 5	11 9	9 8	9 6	9 0	37 9	35.2				
4		4 3	45.2	4.2	3 3	2 6	2 2	2 3	17 2	9 9	6 0	5 3	4 9	37 8	35 0				
5		4 4	46 0	4.6	3 4	3 3	2 5	1 5	15.4	8 8	7 4	5 6	3.8	37.8	35.0				
6		4 0	46 4	4.3	2.4	2 2	1 5	1 0	16 9	4 0	5 5	6 0	4 0	36 9	34 1				
7		4.8	44.4	4 6	2 9	2 8	0 8	1 3	25 2	14 0	8 2	7.0	4 5	36.8	34 0	266 00	264.70	258 55	257 65
8		5 5	42 0	4 0	3.7	3 1	2 6	1 5	31 0	13 0	10 5	10 5	8.6	36 8	34 0				
9		5 0	43 2	4.7	2 4	2 1	1 6	1 2	22 0	7 0	6.0	4 0	3 0	38.1	35 3				
10		5.0	49.0	4.5	3.0	2.1	1 9	2.1	24 2	9 4	5 4	4 7	5 6	38.2	35 4				
11	↓	5.0	46.6	4.3	3 4	3.0	2 0	1 6	18 4	10 9	7.8	5.0	3 8	38 2	35 5				
12	Surge	5.0	46.2	4.8	3.3	2.7	1 8	1.6	24.0	10 7	6 9	5 3	4 0	38.4	35 6				
13	Sway	5 0	261 0	17	8	-	-	-	0 8	0 3	-	-	-	38 4	35 6				
14	↑	4.0	313.0	15	2	-	-	-	0 7	0 1	-	-	-	38 2	35 4				
15	↓	4.5	300 0	21	11	-	-	-	1 1	0 3	-	-	-	38 2	35 4				
16	Sway	4.5	326.0	20	12	-	-	-	1 0	0 3	-	-	-	38 0	35 2				
17	Surge	6.5	35 0	2.5	2 3	1 3	1 3	0.9	22 4	10 6	6.7	5.3	4.0	38.0	35.2				
18	↑	6.5	33 6	2.8	2.7	2 0	1 7	1 4	29 0	16 0	11 5	7.9	6 4	38.1	35 3				
19		6 8	33 4	2 6	2 5	1 9	1 8	1 9	25 3	13 4	10 5	9 5	9.3	38 2	35 3				
20		7 0	33 4	2.7	2.6	1 9	1 8	1 7	25 1	15 1	11 6	9 5	8 3	38 4	35 6				
21		-	-	-	-	-	-	-	-	-	-	-	-	38 6	35 8				
22		7 0	33 4	3 2	2 7	2 1	1 9	1 8	26 6	13 8	9 9	8 9	8 0	38 4	35 6				
23		6 8	33 8	3 2	2 3	2.0	1.9	1 7	19 8	12 9	9 0	8.8	7 1	38 4	35 6				
24		6 8	33 4	3 3	2 8	2.4	2 0	1 5	25 9	13 9	10 7	9 0	5 9	38 4	35 6				
25		7 2	32 8	2 7	2 1	1 7	1.6	1 5	18 4	8 5	8 4	6.7	7 2	37 1	34 3				
26	↓	6 9	34 8	1 4	0.7	0 4	-	-	-	-	-	-	-	37.0	34 2	264.65	263 35	258 55	257 60
27	Surge	7 0	33 0	3.0	2 9	2.1	1.7	1 7	22 9	12 1	9.6	8 8	6.9	37.0	34 2				
28	Sway	7 5	217 0	16	10	-	-	-	0.4	0.1	-	-	-	37.0	34 2				
29	↕	7 5	247 0	22	11	10	-	-	1 8	0.1	0.1	-	-	37 0	34.2				
30	Sway	7 5	261 0	25	6	2	-	-	2 0	0	0	-	-	37 1	34 3				
31	Sway	7 5	250 0	20 0	2 0	-	-	-	1.8	0	-	-	-	37 5	34 7				
32	Sway	7.5	254 0	27 0	3 0	-	-	-	2 5	0	-	-	-	37 8	35 0				
33	Surge	8 0	30 6	2 3	2 0	1 4	1.2	0.9	20 0	11 1	9.1	5 8	4.8	38 4	35 6				
34	↕	7 9	31 2	2.1	1.7	1 3	0 8	0 8	20 0	12.9	8 5	4.9	4 7	38 5	35 7				
35	Surge	8 5	30 0	2 2	1 9	1 5	1 1	0 8	22 4	11 7	10 0	8.0	6 0	38 6	35 8				
36	↑	8 5	244 0	20 0	5 0	0 0	-	-	2 0	0 1	0	-	-	38 6	35 8				
37		8 5	266 0	20 0	5 0	-	-	-	2 0	0 1	-	-	-	38 7	35 9				
38		8 0	30 0	2.4	1 5	1 3	-	-	16 7	9 0	8 0	-	-	38 8	36 0				
39		8 0	30 8	2 3	1.6	1 7	1 0	0 9	15 0	11 3	8.6	6 6	5.4	38 7	35 9	264.25	262 97	258 55	257 60
40		8 0	31.8	2 0	2 0	1 5	1 0	0 6	12 0	10 0	7 4	3 7	3 4	38 7	35.9				
41		7 0	32.4	2 8	2 2	2 3	1 2	1 4	23 0	10 0	10 0	8 2	5 8	35 1	32.3				
42		9 5	26 0	1 9	1 6	1 0	0 7	0 7	22 0	13 9	10 0	6 6	5 5	34 8	32.0				
43		9 5	26 4	1 8	1 5	1 0	0 9	0 8	26 9	14 3	10 1	7 3	5.8	34 8	32 0				
44		9 7	28 2	1 9	1 1	0 8	0 7	0 3	30 3	12.0	9 5	6 9	6 0	34 8	32 0				
45		9.8	26.6	2 2	1 5	1 3	1 0	0 8	24 9	11 9	9 1	6.8	5 9	35.1	32 3				
46	↓	9 8	26.6	2 0	1 4	1 2	0 9	0 8	23 8	12 0	8.3	7.5	5 3	35 3	32 5				
47	Surge	10 4	25.8	2 1	1 7	1 3	1 1	0 8	26 9	14 4	10.7	8 1	6 8	35 6	32 8				
48	Sway	10 0	200 0	20 0	12 0	6 0	3 0	-	2 5	1 0	0 4	0.2	-	36.1	33 3				
49	↑	10 0	222 0	15 0	10 0	-	-	-	1 5	0 7	-	-	-	36 7	33 9				
50	↓	10 5	224.0	20 0	15.0	3 0	-	-	2 5	1 5	0 2	-	-	36 9	34 1				
51	Sway	10 5	218 0	25.0	9 0	-	-	-	5 0	0 6	-	-	-	36.9	34 1				
52	Surge	10 2	24 8	1 9	1 0	0.8	1.0	0.3	24 2	11 4	9.6	7.2	4 9	37 5	34 7				
53	Sway	10 2	224.0	15.0	7.0	-	-	-	1 5	4	-	-	-	35 2	32.4				
54	↑	10 5	213.0	20 0	9.0	-	-	-	2.5	5	-	-	-	35 4	32.6				
55		12 0	228 0	20 0	15.0	9 0	-	-	2 5	1 5	.5	-	-	38 4	35 6	263 25	262 00		
56		12 5	221 0	30.0	17.0	9 0	-	-	10 0	2.0	5	-	-	38 3	35 5				
57	↓	12 5	223 0	30.0	18 0	10.0	-	-	10 0	2 0	5	-	-	38 3	35 3				
58	Sway	12.0	205 0	30 0	18 0	10 0	-	-	10 0	2 0	5	-	-	38 1	35.3				
59	Surge	12.0	25 8	2 3	1 6	1.3	1.1	1.0	23 7	15 5	10 5	7 9	7 6	37 8	35 0				
60	↓	12.0	26 2	2 2	1 0	0 9	0 6	0 7	29.5	7 4	6 1	3 5	4.6	37 7	34 9				
61	Surge	11 0	26 0	2 3	1.7	1 3	1 0	1 0	34 7	15 0	9 2	8 3	8 6	37 6	34 8				
62	↑	11 0	26 4	1 4	1 2	0 8	0 7	0 7	21 0	12 4	8.2	5 5	5 0	37.5	34 7				
63		21 0	13 2	0 5	0.4	0 3	0	0 1	21 0	10 0	6 4	1 0	0.7	38 5	35 7				
64		21 0	13 8	0 6	0 5	0 3	0 2	0 1	29.2	12.6	7 2	5 3	2.6	38.4	35 6	261 50	261 45	257 90	258 25
65		20 0	13 6	0 5	0 4	0 3	0.2	0 1	21.9	10.4	6 6	3 6	2.8	38.5	35 7				
66		20 5	13 0	0 8	0 6	0 3	0 3	0.2	32 0	14 3	9 6	9 8	6 6	38 3	35 5				
67	↓	20 0	13 6	0 9	0 6	0 4	0.1	0 1	31 3	18 1	10 1	2 9	1 6	38 3	35 5				
68	Surge	20 0	13 8	0 8	0.8	0 4	0 4	0 5	32 9	17 5	10.1	7 3	4 4	38 3	35 5				
69	Sway	20 0	180 0	17.0	12 0	9 0	5.0	-	5 0	2.5	1 5	0.5	-	38 2	35.4				
70	Sway	20 0	186 0	18 0	12.0	9 0	5 0	-	6 0	2.5	1 5	0 5	-	38 0	35 2				
71	Surge	17.5	16 2	1.0	0 5	0 3	0 4	0 2	23 4	15 5	8 1	3 4	5.6	36.5	33 7				
72	Sway	19 0	196 0	17.0	11 0	6.0	-	-	5 0	2 0	1.0	-	-	36 9	34 1				
73	Sway	18 0	182 0	20.0	14.0	10 0	-	-	8 0	3.0	1 8	-	-	37 2	34 5	261 10	261 30		
74	Surge	22 0	12 6	0.6	0.5	0.4	0 2	0 2	28 0	14 1	10.6	7 7	5.7	37 1	34.3				
75	↑	22 0	12 6	0 8	0 6	0 4	0 3	0 2	35 3	21.2	14.0	9 6	7.6	37 3	34.5				
76		22 5	12.2	0 8	0 4	0.2	0 3	0 2	34.6	17 2	10 6	9 5	6.0	37 2	34.4				
77		23 0	11 6	0 8	0.5	0 2	0.2	0 2	36 2	19 4	11 0	8.8	6 4	37 4	34.6				
78	↓	24 0	11 0	1 0	-	-	-	-	24 5	-	-	-	-	37 7	34 9			257.70	258.50
79	Surge	24 0	9.8	1 0	-	-	-	-	15 1	-	-	-	-	37 8	35 0				
80	Sway	22 0	176.0	17.0	12 0	9 0	-	-	5.0	2.5	1 5	-	-	36 3	33 5				
81	↕	20.0	177 0	17 0	12 0	9.0	-	-	5.0	2 5	1 5	-	-	36.3	33 5				
82		20 0	184 0	17.0	10 0	8.0	-	-	5 0	1 8	1 4	-	-	36 4	33.6				
83	Sway	21.0	174 0	17 0	12 0	8 0	-	-	5 0	2 5	1 4	-	-	36 6	33.8				
84	Surge	19 0	14 6	0 7	0 6	0 2	0 2	-	34.8	10 5	7 4	3.4	-	36 7	33.9				
85	↑	20 0	13 8	0 5	0 5	0 5	0.5	0	23 0	12 2	4.0	5 6	1.2	36 7	33 9	261 00	261 45		
86		19 0	13 0	0 6	0 5	0 4	0.3	0 1	24 7	19 2	13 0	7 2	5 8	36 7	33 9				
87		20 0	-	-	-	-	-	-	-	-	-	-	-	36 7	33 9				
88		20 0	13 4	0.5	0 3	0 3	0 1	-	34 1	19 2	9 3	3 0	-	37.0	34 3				
89		21.0	13.0	0 5	0.4	0 2	0	-	29 3	11 5	6.3	0	-	37 1	34.3				
90	↓	22 0	13 6	0.7	0 6	0 5	0.5	0.2	35.7	23.0	14 4	13.8	9 7	37 1	34 3				
91	Surge	22 0	12.8	0 7	0.3	0 5	0 2	-	31 3	9 7	6.1	1 6	-	37 3	34 5				

* Average of first four oscillations in surge, usually average of first two oscillations in sway since motion died down quickly in this direction

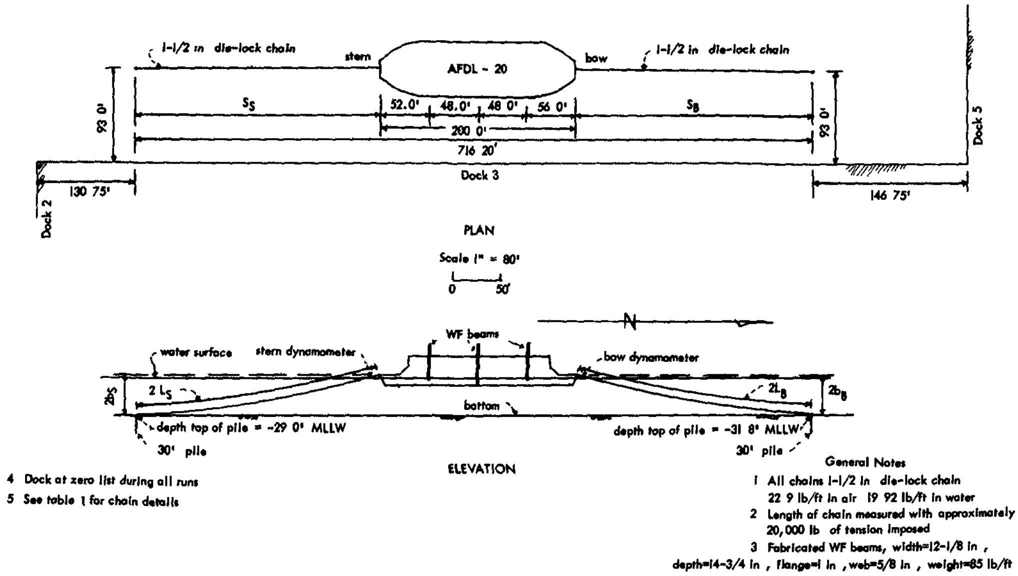

Fig. 1. AFDL-20 as moored during study.

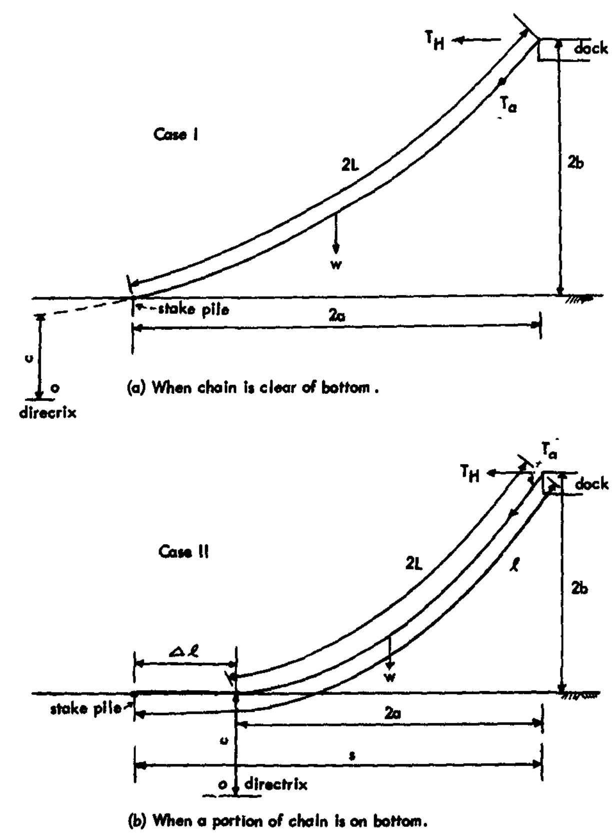

Fig. 2. Basic geometry of mooring chains.

ANALYTICAL CONSIDERATIONS*

The condition of force equilibrium on the Dock in either surge or sway is considered to be:

$$\text{inertia} + \text{damping} + \text{restoring force} = 0 \qquad (1)$$

where the sign for each term is determined by the performance of the system as a who

The inertial reaction is made up of two parts: that of the Dock and that of the hydrodynamic mass of the Dock — added mass effect — where in the case of surge (x):

$$\text{Inertia force} = m\,(1+C_m)\; d^2x/dt^2$$

C_m is the inertial coefficient (ratio of hydrodynamic mass to the displaced mass). In most cases the term $(1+C_m)$ is written as C_M and is termed the mass factor. Thus

$$\text{Inertia force} = C_M\, m\, \ddot{x} \qquad (2)$$

The damping reaction is based on the well known expression:

$$\text{damping force} = C_D\, A\, \rho\, (\dot{x})^2/2 \qquad (3)$$

where in the case of laminar flow the velocity term ($\dot{x}$) may be treated as linear rather than squared. A is the projected underwater area of the vessel for form drag and the wetted area of the vessel for frictional drag.

The restoring force F(x) consists only of that due to the weight of the chains, since in no case are the movements of the Dock large enough to cause the chain to elongate. While in all cases the chain obviously hangs in a catenary, it is necessa to consider two definite attitudes which are dictated mainly by the initial tension namely: Case I (Figure 2-a) where the chain is suspended entirely between the anchor and the Dock and the low point of the curve describing the system is locate below the bottom of the Harbor with a run from the directrix which is greater than that of the chain proper; Case II (Figure 2-b) where some portion of the chain rests on the bottom such that the low point is located there with a run equal to that of tl suspended portion.

For Case I the suspended length (2L) of the chain, its run (2a) rise (2b) and unit weight (w) are measured. In Case II in addition to 2b and w the total length (ℓ) is measured and the run (2a) is assumed. This run is substracted from the horizontal distance (s) between anchor and Dock ends of chain to permit an estimc of the portion of the chain ($\Delta\ell$) considered to be resting on the bottom. The estimated length (2L) of the suspended portion of the chain is obtained from the expression: $2L = \ell - \Delta\ell$. This assumption is checked analytically as outlined below and by such "cut and try" procedure a firm value for 2L is obtained.

The restoring force in either surge or sway is the algebraic sum of the horiz components of the tension in the Dock end of the chain. In Case I the determina

* See list of symbols p.893

of the component (T_H) through use of the catenary equations as outlined by Marks (1930) is straightforward as follows:

$$(\sinh z)/z = (L^2 - b^2)^{1/2} / a \quad \ldots\ldots (4)$$

where z is a parameter as defined in equation (4) and determined by cut and try procedure using the measured values of L, b and a. Then:

$$c = a/z \quad \ldots\ldots (5)$$

where c is the vertical distance from the low point on the chain to the directrix (Figure 2–a). With this value the horizontal component T_H can be obtained simply as:

$$T_H = w\, c \quad \ldots\ldots (6)$$

Because the chain tension (T) and not its horizontal component is measured it is necessary as a check to calculate this from the relationship:

$$T = w\,(c + y_o + b) \quad \ldots\ldots (7)$$

where y_o is the vertical distance from the midpoint of the line connecting the anchor and Dock ends of the chain to the low point on the catenary system (Figure 2–a) such that:

$$y_o = (L/\tanh z) - c \quad \ldots\ldots (8)$$

The horizontal distance x_o corresponding to y_o is given by:

$$x_o = c \tanh^{-1} (b/L) \quad \ldots\ldots (9)$$

For Case II, it is necessary after computing the distance from the directrix to low point on chain (c) on the basis of equation (5) to calculate the run (2a) to determine whether or not this checks the assumed value. This is done on the basis of the relationship:

$$2a = c \sinh^{-1} 2L/c \quad \ldots\ldots (10)$$

When the assumed and calculated runs agree then the chain tension at the Dock end and its horizontal component are calculated as in Case I.

In general, as indicated by a consideration of the catenary equations, the relationship between restoring force and movement will be non-linear such that for surge:

$$F(x) = \text{restoring force} = k\, x^n \quad \ldots\ldots (11)$$

$$= T_H \text{ bow} - T_H \text{ Stern} \quad \ldots\ldots (12)$$

where k is the spring factor, x the movement in surge and n an exponent defining the non-linearity of the moorings.

In the case of sway:

$$F(x) = T_H \text{ bow } (\cos\alpha) + T_H \text{ stern } (\cos\alpha) \ldots\ldots\ldots (13)$$

where α is the horizontal angle between the chain and the direction of motion (sway).

The natural period of oscillation in surge or sway is obtained by a solution of equation (1) in the form of an expression for movement as a function of time such that the period — time between two successive peaks — can be evaluated. Becaus the restoring force is in general non-linear with movement, such a method demands the solution of a non-linear differential equation which is best accomplished by numerical methods utilizing machine computation.

If damping and added mass effects are neglected, graphical methods as outlin for example by Timoshenko (1937) may be used to obtain the natural period (T_n) whic will vary of course with amplitude of initial displacement. Thus from equation (1):

$$m\ddot{x} = F(x) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where graphical methods or the equivalent must be used when F(x) is non-linear. However, when F(x) is linear then the natural period from equation (14) is the very familiar:

$$T_n = 2\pi (m/k)^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

ANALYTICAL RESULTS

The relationships between restoring force and movement was calculated by u of equations (12), (13) and (4) through (10) for three initial tensions with results of the type shown in Figures 3 and 4. The relationship is found to be non-linear alth depending upon the initial tension, there is in all cases a range over which this non-linearity is very weak.

The natural periods of oscillation for particular conditions of restoring force versus movement were calculated by means of graphical intergrations of restoring force-displacement curves such as those in Figures 3 & 4 on the basis of equation (It was assumed that the added mass effect is negligible — this is substantiated by experiment — and that the damping can be neglected in a consideration of natural period. Such an analysis (Figures 5 and 6) indicates that for any particular initial tension the natural period varies inversely with the initial displacement in a non-linear manner.

If the nearly linear restoring force-displacement range is considered then it rewarding to study the relationship between the natural period (T_n) and initial ten (IT) only. The results of such a study are presented in Figures 7 and 8 where the

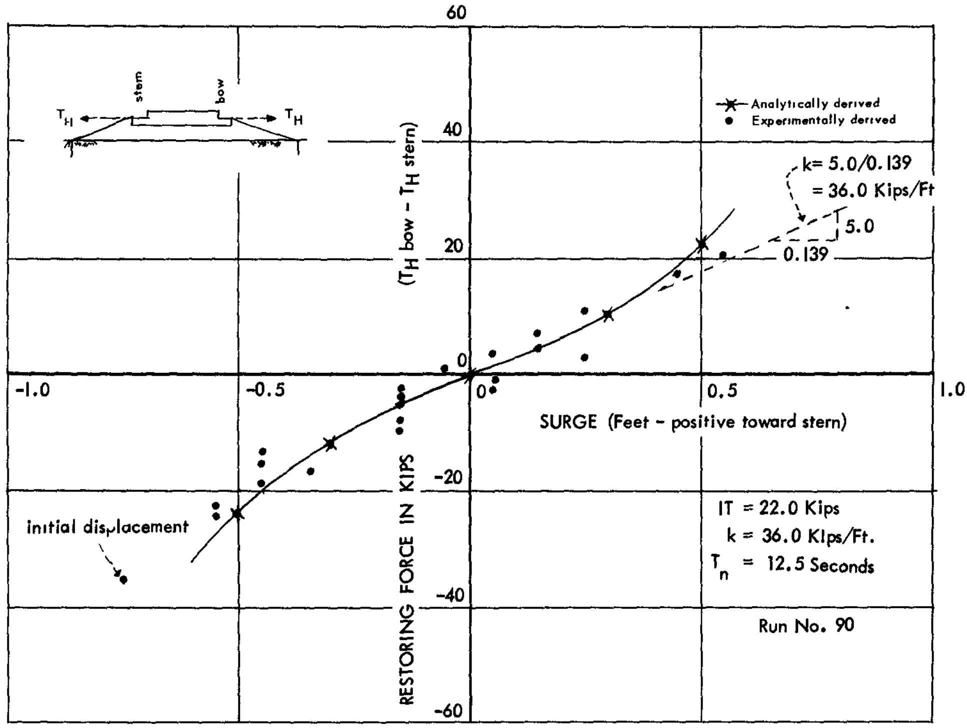

Fig. 3. Restoring force versus surge for a high initial tension.

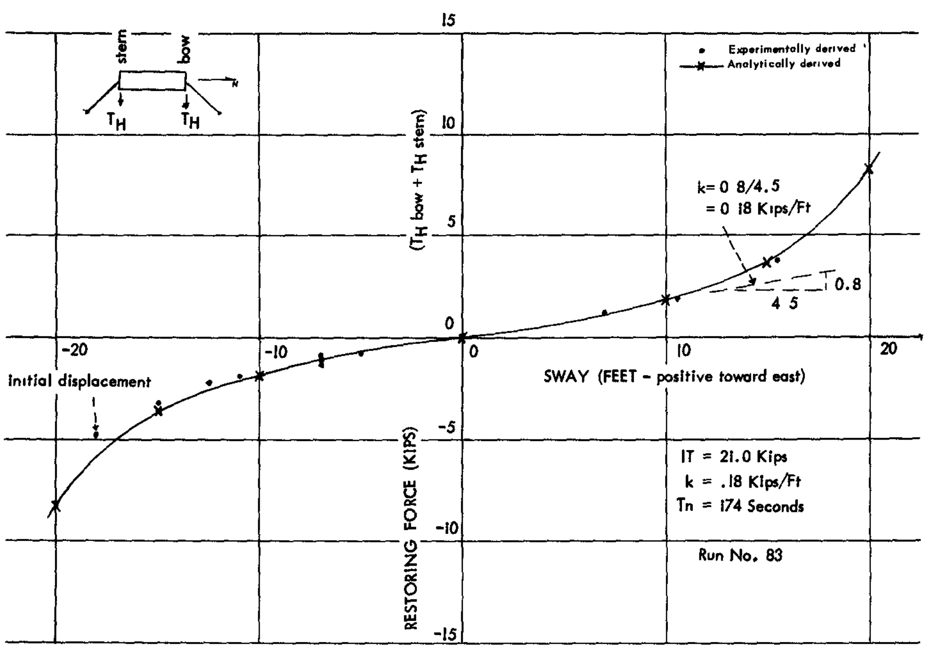

Fig. 4. Restoring force versus sway for a high initial tension.

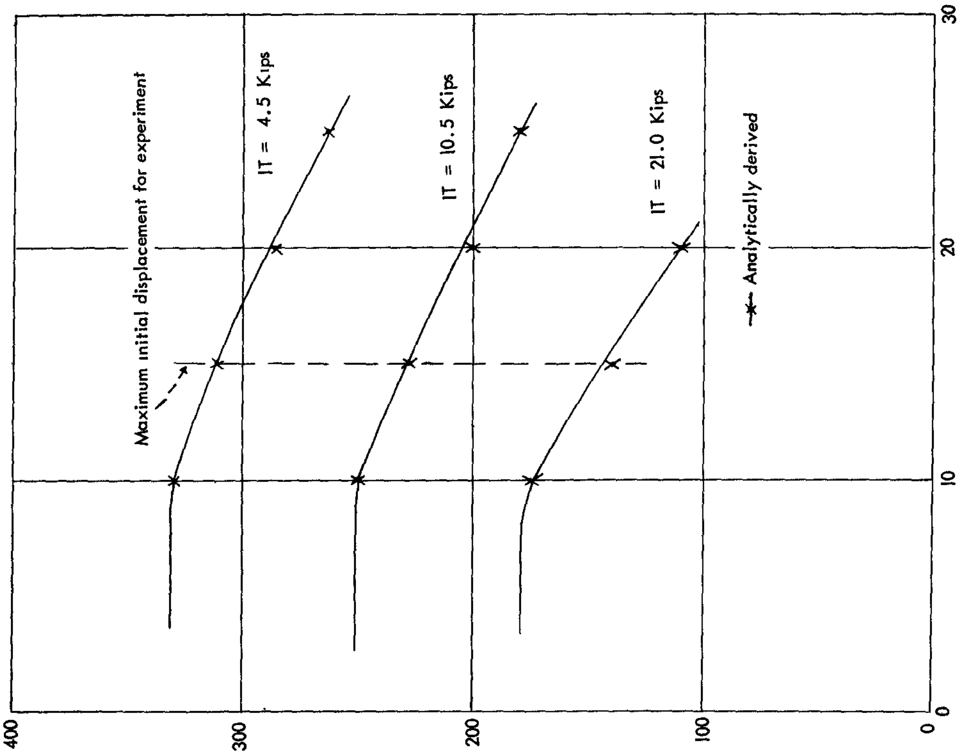

Fig. 6. Free period of oscillation in sway versus

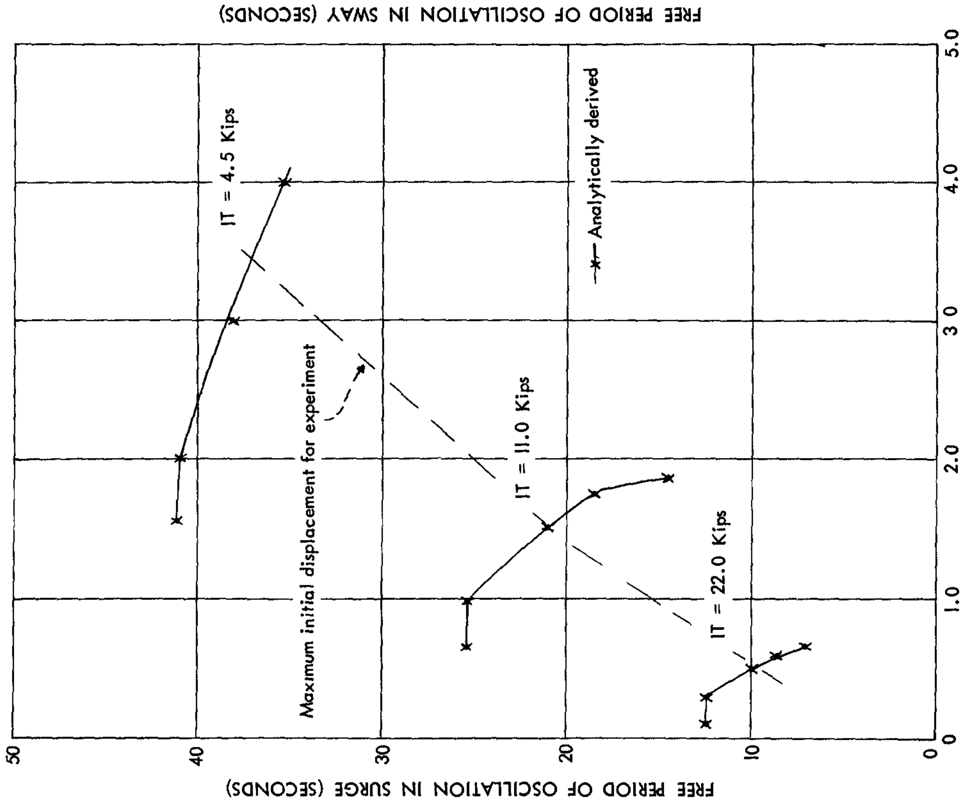

Fig. 5. Free period of oscillation in surge versus

used to define the curves is taken from the nearly linear portion of the curves in Figure 5 (T_n of 42, 25.5 and 12.5 seconds for IT of respectively 4.5, 11 and 22 kips) and Figure 6 (T_n of 315, 225 and 175 seconds for IT of respectively 4.5, 10.5 and 21 kips). It is indicated that, within such a linear restoring force-displacement range, the natural period varies inversely with initial tension such that:

$$T_{nx} = 55.5/1.07^{IT} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$T_{ny} = 530/IT^{0.366} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (17)$$

where T_n is in seconds and IT is in kips and x and y refer respectively to surge and sway. Equations (16) and (17) are not proper when initial displacements are into the definitely non-linear range.

It is interesting to note that by linearizing the restoring force data over a reasonable range of initial displacements and then introducing the slope of the restoring force-displacement line as the spring factor (k) in equation (15) it is possible (Table II) to obtain values for the natural period which agree well in many cases with those obtained by more elaborate means for the true condition which is non-linear.

Table II. Natural periods computed using the true and linearized curves of restoring force versus movement (Figures 3 & 4)

Dir.	Run No.	Initial Tension (kips)	Initial Disp. (feet)	Curve Used	Natural Period (seconds)
surge	90	22.0	0.5	true (non-linear)	10.0
				linearized	12.5
			0.3	true	12.5.
				linearized	12.5
sway	83	21.0	15	true	136
				linearized	174
			10	true	175
				linearized	174

That reasonable agreement is obtained is due to the fact that in this study the restoring force-displacement relationship is not strongly non-linear over a considerable range of movements. Linearizations permit natural periods to be computed relatively easily where the results obtained in many cases may be well within the accuracy desired in ordinary engineering applications.

A more complicated mooring system for the Dock, consisting of eight chains (1 each fore and aft and 3 each port and starboard) each with a large concentrated load as described by Wiegel et al (1956) was studied analytically using equations (4) through (15). The comparison between the natural periods of oscillation in both surge and sway as calculated, on the basis of the developed restoring force-displacement curves, and as measured on the Model is indicated to be very good.

EXPERIMENTAL RESULTS FROM THE PROTOTYPE

The oscillations of the Dock occurred in the range where restoring force varied in a nearly linear manner with displacement. This range varied with initial line tension being up to 0.6 and 2 feet in surge for initial tension of respectively 22 an 4.5 kips and up to 15 feet in sway for the same initial tensions.

A very marked increase in restoring force occurred when the Dock moved into the definitely non-linear region where, although the tug used to produce initial displacement had the capability to move the Dock into the lower part of the definitely non-linear range, the free oscillations were never sustained in this region due likely to the high rate of damping involved.

The results obtained are in the form of oscillograms of oscillations in force (those from the chain dynamometers) as a function of time. A facsimile of two suc oscillograms is presented in Figure 9. The oscillations appear to be approximately sinusoidal and to exhibit a dying down typical of a system with greater than critic damping. In the case of surge the amplitudes appear to decrease in nearly equal decrements as an arithmetical series (.7, .5, .3, .1, etc. feet for maximums in th case of run #90, Figure 10). This seems to be characteristic of a Coulomb type damping — an apparent anomaly in a hydrodynamic system -- where the non-linearity is located in the damping system.

The chain tensions, measured at the Dock end of the chain, when resolved into their horizontal components by use of measured chain slopes, give the restori forces as expressed by equations (12) and (13). The comparison between these values and those obtained analytically is considered in the main to be good — th results presented in Figures 3 and 4 are examples -- although there is considerabl scatter in the data in places. This is considered to reflect both the non-linear an coupled nature of the Dock movement which was not purely in either surge or swa and to certain vagaries in the measurement of this movement by surveyors readin from some distance on scales attached to the moving Dock.

The magnitude of the added mass effect was evaluated at the extremes of the oscillations where the acceleration ($\ddot{x}$) was maximum and the velocity and thus th damping was zero. For surge equations (1), (2) and (12)

$$C_M = F(x)/m\,\ddot{x}$$

$$= (T_H \text{ bow} - T_H \text{ stern})/m\,\ddot{x} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

Values for the maximum acceleration were obtained by plotting the measured movements of the Dock versus time and performing the necessary two differential

FREE OSCILLATION IN SURGE AND SWAY OF A MOORED FLOATING DRY DOCK

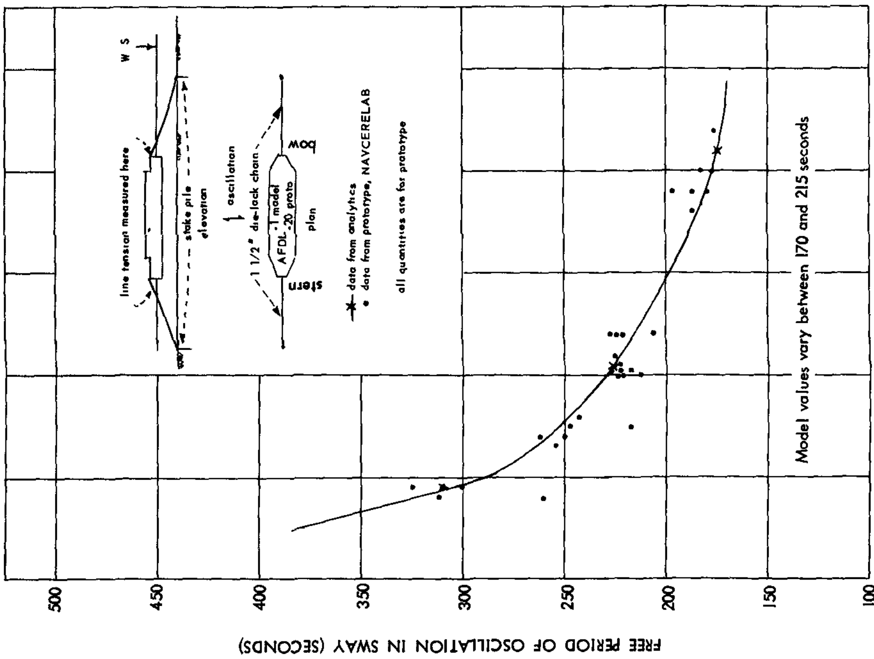

Fig. 8. Free period of oscillation in sway versus initial tension.

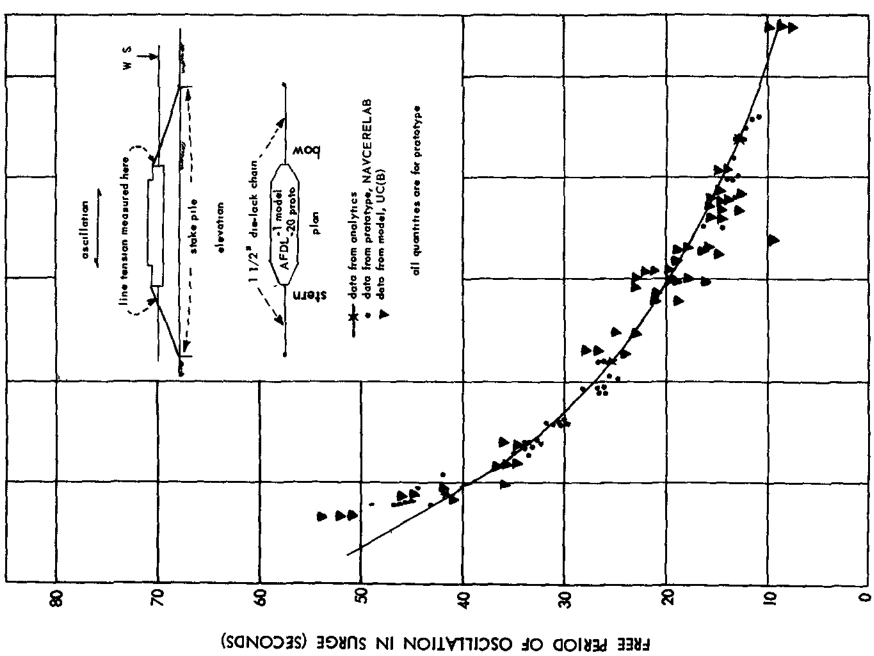

Fig. 7. Free period of oscillation in surge versus initial tension.

of this curve graphically (Figure 10). These values along with values for the measured restoring force and mass (m) when substituted in equation (18) gave values for C_M close enough to unity to permit the added mass effect to be declared negligible in this study.

The oscillations tend to maintain themselves at a nearly constant period even though the amplitude of the motion tends to die down in the manner discussed previously. This linearity of period with movement as well as time permits consideration of the measured periods as a function only of the initial tension. An inverse relationship is indicated (Figures 7 and 8) as predicted by the analytics. The comparison between the experimental and analytically derived periods on this basis is considered good with the greatest differences occurring at the lowest initial tensions.

EXPERIMENTAL RESULTS FROM THE MODEL

The principal effort was devoted to obtaining the natural period of oscillation in surge as a function of initial tension and displacement. The data is contained on oscillograms of force -- that is the Dock end of the lines -- versus time. From these data the periods were measured.

The Model values for the period were extrapolated to the Prototype by means c the Froude Law -- a kinematical relationship which states that for identical conditi of gravity the length ratio is equal to the square of the time ratio -- which in this study means that the natural periods obtained in the Model were multiplied by 6.3 -- the square root of the length ratio of 40 prototype to 1 model -- to obtain th corresponding value in the Prototype.

Movement of the Dock was not measured directly. It is recognized that this movement can be obtained by indirect means from the force oscillograms -- this wa done in three cases of particular interest by use of calculated curves of restoring force versus movement — but such an effort on a general basis was considered beyond the scope of this paper. However, from the limited indirect studies of movement which were made, it appears that in general the free oscillations occurr in a range of amplitudes within which the variation of restoring force with moveme was close enough to linear to permit the Model results for period to be reviewed as functions of initial tension only.

Such a comparison is made in Figure 7 where, although the scatter is consider with the mid initial tension, the comparison with the Prototype and analytically derived values is thought to be good with the usual inverse relationship between period and initial tension indicated. The scatter, besides being due to the inabil of the experimenters to cause the Dock to oscillate purely in surge, is attributed to the fact that the restoring force-displacement relationship is not truly linear

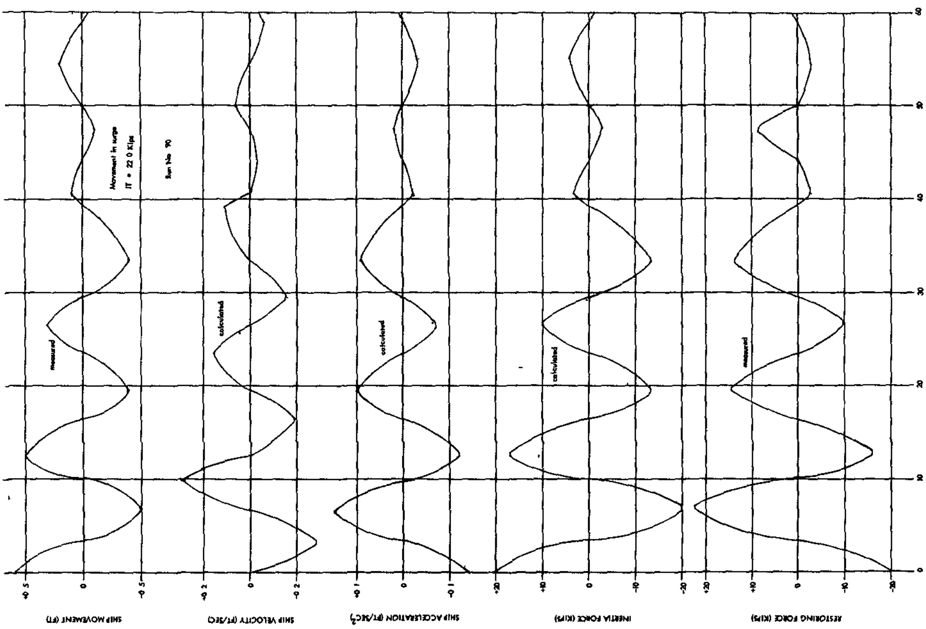

Fig. 10. Variation of measured restoring force and calculated inertia force with time.

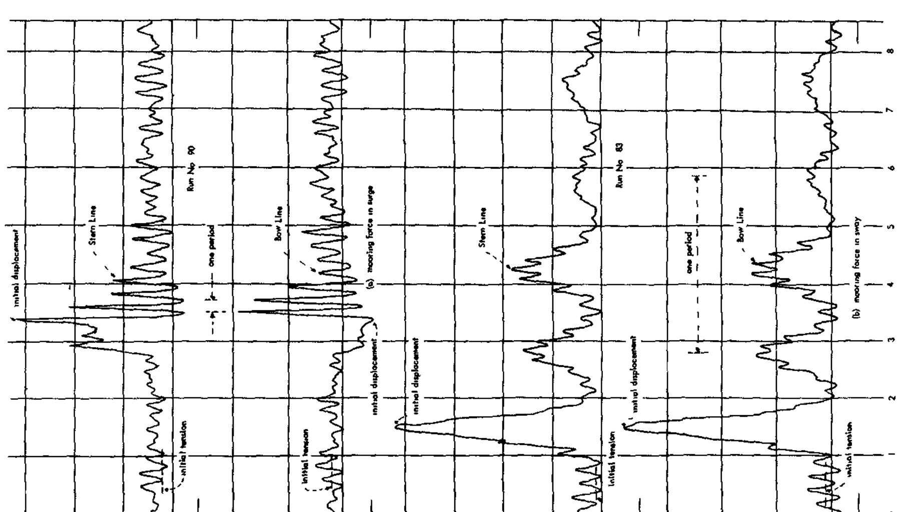

Fig. 9. Facsimile of oscillogram of mooring force in surge and sway versus time for prototype.

although assumed so for this presentation and therefore for any particular initial tension there could exist many natural periods of oscillation depending upon the initial displacement given to the Dock. This same criticism is of course applicable to the Prototype and analytically derived data as presented in Figure 7 where for the latter case a series of curves each for a particular initial displacement could be drawn as a function of period and initial tension.

The measurement of the natural period in sway in the Model proved unrewarding because the unwanted damping contributed particularly by the winds in the outdoor model basin was enough to override effectively the very small restoring force provided by the chains and therefore no significant free oscillation of the Dock in this direction could be sustained for a long enough period to obtain a significant recording. Visual observations indicated that for a range of initial tensions of about 10 to 20 kips the natural periods varied between 215 and 170 seconds which is considered to confirm roughly those obtained from the Prototype and analytics (Figure 8).

On the basis of the results obtained in surge (Figure 7) it is considered that it has been demonstrated that it is possible to model successfully not only the characteristics of the Dock itself but also of the moorings in the manner described by Wiegel et al (1956).

CONCLUSIONS

For the Dock as spread moored by a single chain of 8 scope, respectively, fore and aft and oscillating in either surge or sway it is concluded that:

1. The natural period of oscillation calculated as a function of amplitude and initial tension by use of the catenary equations and graphical intergration of the well known spring-mass equation, checks the experimentally determined values to a sensible degree of accuracy where damping and the added mass effect are neglected. Such calculations are suitable for design purposes; much more complicated mooring systems can be treated in the same manner.

2. Restoring force varies with movement in a non-linear manner. For small movements -- maximum of 1/2 and 6 feet for initial tensions of respectively 22 and 4.5 kips -- this relationship can be linearized without significant loss of accuracy for use in the equation:

$$T_n = 2\pi\ (m/k).^{1/2}$$

3. The natural period (T_n) in surge (x) and sway (y) varies inversely in a non-linear manner with initial tension (IT) such that for relatively small oscillations $T_{nx} = 55.5/1.07^{IT}$ and $T_{ny} = 530/IT^{0.366}$ where T_n is in seconds and IT in

kips. This period varies also with the amplitude of oscillation so that for relatively large oscillations the expressions given do not hold.

4. The natural periods as obtained from oscillating a 1 to 40 linear scale Model in surge confirm to sensible degree those obtained from the Prototype and by analytics; the Model results in sway are not as extensive as in surge but also tend to confirm the Prototype results. On the basis of the surge data it is concluded that the characteristics of a ships mooring lines can be modeled satisfactorily.

ACKNOWLEDGEMENTS

Appreciation is expressed to the U. S. Naval Civil Engineering Research and Evaluation Laboratory of the Bureau of Yards and Docks for permission to publish this paper.

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SYMBOLS

A = Project underwater area of vessel for form drag

= Wetted area of vessel for frictional drag

2a = Run of chain

2b = Rise of chain

c = Vertical distance from low point on the chain to the directrix in the catenary system

C_D = Coefficient of drag

C_m = Inertial coefficient

C_M = Mass factor

$F(x)$ = Restoring force

g = Acceleration due to gravity

k = Spring factor = ratio of restoring force to movement

$2L$ = Suspended length of chain

ℓ = Total length of chain

$\Delta\ell$ = Portion of chain resting on the harbor bottom

m = Mass of vessel

n = Exponent defining the non-linearity of the moorings

s = Horizontal distance between anchor and Dock ends of mooring chains

T = Chain tension

T_H = Horizontal component of chain tension

T_n = Period of oscillation

IT = Initial chain tension

w = Unit weight of chain

y = Movement in sway

x = Movement in surge

x_o = Horizontal distance corresponding to y_o

y_o = Vertical distance from the midpoint of the line connecting anchor c vessel ends of the chain to the low point on the catenary system

z = Catenary parameter = $a \sinh z / (L^2 - b^2)^{1/2}$

α = Horizontal angle between the chain and the direction of motion

d^2x/dt^2 = Acceleration of vessel = $\ddot{x}$

dx/dt = Velocity = $\dot{x}$

ρ = Density of water

CHAPTER 53

STABLE CHANNELS*

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ABSTRACT

The form and size of a channel in cohesionless material, stable against erosion for a definite discharge, Q, is studied.

The angle of internal friction ϕ and the limiting tractive force τ_{max} are taken as known. Distribution of shearing stresses τ is assumed to be such that they are proportional to the distance between bottom and water surface, measured at right angles to the bottom. In addition to the action of gravity and shearing stress τ the grains are acted upon by a hydro-dynamic lift force, proving to be proportional to τ. The differential equation of the bottom form is established and integrated numerically; the form depends on ϕ.

Based on the logarithmic law of velocity distribution and the assumed distribution of shearing stresses, the velocities in all parts of the cross section can be found, and the total discharge is found by numerical integration.

A profile consisting of a curved bank-part of the above mentioned cross section and a middle part of indefinite width and a constant depth y_{max} would be stable for the same tractive force. On the assumption, however, that nature will produce that cross section which has a minimum of area, only one definite solution, viz. the equilibrium profile, is found. The dimensions depend not on ϕ alone but also on the relative roughness of the bottom k/y_{max}. Provided that the hydraulic roughness k is assumed to be in conformity with that of natural watercourses, it is found that the area of the equilibrium profile varies slowly with ϕ and must be proportional to $(Q/\sqrt{\tau_{max}})^{0.9}$.

The above assumptions are checked by calculation of a complete set of isovels.

Three model tests, carried out in Vienna in 1916 are studied and compared with profiles calculated according to this theory. The values of ϕ vary from 14° to 20°.

On the same basis a study is finally made of the relation between mean and maximum velocities v_m/v_{max}, resulting in a simple diagram giving v_m/v_{max} as a function of H, the "degree of fullness" of the profile, and also as a function of y_{max}/k, the reciprocal relative roughness. Methods for estimating k are given.

* To be published separately by the Coastal Engineering Laboratory, University of Florida, Gainesville, Florida.

CHAPTER 54

CHANGING SITE REQUIREMENT FOR PORT OPERATIONS

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ABSTRACT

Site selection and site adaptation for construction of port termina require evaluation of the natural or man-made environment of each specific project. Common to most projects, however, is the need for analysing present and anticipating future operational requirements. Recent developments in vessel design and cargo handling methods have brought about changes not only in navigational criteria for channels and harbor basins, but also in space requirements for shore facilities. In this pape an attempt is made to define some aspects of land area requirements fo future port operations.

There has been a change in general approach toward problems o site planning for ports; the emphasis given in the past to the number of vessel berths that could be accommodated, has gradually shifted to the amount of land area available to "support" each berth. This has been caused, in part, by the increased rate and volume of cargo movement b modern methods, leading to greater requirements for in-transit storage and a larger scale of transfer operations at the terminals. High costs c vessel operation and waterfront construction dictate design of most bert with provisions for ultimate maximum capacity of cargo transfer. Acco ing to present trends, for a given tonnage movement the required numbe of berths will decrease and the total required land area at the terminal will increase.

A distinction is made between transit-storage areas and transit-handling areas adjacent to berths; different criteria govern the amount r quired for each. The effects of the type of cargo, handling methods and other characteristics of the traffic have been analysed by recent studies some standards have been developed by the Port of New York Authority and data on a number of large terminals have recently been compiled by the American Association of Port Authorities. It is of interest to compare these area requirements with the first known data on container and roll-on roll-off operations.

The relative location of shore facilities to vessel berths, anothe aspect of site planning, is also affected by the type of cargo traffic and handling methods used. While proximity of transit areas to berths is e sential at general cargo terminals, it is of little significance at many ty of bulk facilities and may not be of great importance to some of the future container operations. The effectiveness of site development for a port depends on the recognition of present trends and on finding means of financing construction now for the requirements of the future.