

CHAPTER 16
THE USE OF THE STOKES-STRAUIK APPROXIMATION
FOR WAVES OF FINITE HEIGHT

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ABSTRACT

A formal approximate solution is derived for the profile and velocity components of a wave with permanent form of finite height in moderate water depths. The approximation is carried to the third order, sufficiently far to represent all except the very high "design" waves. The relationship of the formulas to others found in the literature is discussed.

The wavelengths and the coefficients in the third-order series for the wave profile, and the water particle velocities and local accelerations are tabulated for approximately 2000 waves. The depths, heights, and periods for the listed wave conditions vary respectively from 10 to 500 feet, 5 to 40 feet, and 4 to 20 seconds. The range of applicability of the theory is discussed and approximate limits estimated.

As an aid in calculations, tables of the trigonometric and hyperbolic sines and cosines for integral multiples of the argument are included.

DERIVATION OF FORMULAS

We wish to construct potential flows satisfying Bernoulli's theorem on the free surface $\eta(x)$ whose shape is one of the unknowns. The problem is one of some difficulty mathematically because of the non-linear boundary condition. The usual general theorems of existence of solutions for linear problems are not valid and an *ad hoc* verification of existence of a solution must be made to be sure that the formal solution (Stokes, 1847), which can be rather easily obtained, exists. The existence proof was first carried out by Struik (1926) following the method of Levi-Civita (1925) for deep water waves. Struik also obtained formulas for the profile, velocity components, and the necessary auxiliary functions to relate the parameters in these formulas to the usual physically observed wave characteristics.

Unfortunately all these formulas have algebraic errors, as was first pointed out by Hunt (1953) and then independently by Tanaka (1953). However the results of Hunt and Tanaka do not, at first sight, agree. It seemed advisable to make a new and simpler derivation of these results:

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in a slightly different way, in order to clear up these discrepancies. A more recent calculation by De (1955) has checked the results of Hunt and extended them to the fifth approximation.

We introduce the device of a moving coordinate system whose velocity, C , is the celerity of the waves. In this coordinate system, the motion is steady and two-dimensional. It is then convenient to use complex variables (Lamb, 1932) and a complex potential w , whose real and imaginary parts are the velocity potential ϕ and stream function ψ ,

$$w(z) = \phi(x, y) + i\psi(x, y), \quad (1)$$

and whose derivative is related to the velocity components v_x and v_y

$$\frac{dw}{dz} = -v_x + iv_y. \quad (2)$$

The mathematical problem may be described briefly. We wish to construct a function w of complex variable z which has the properties that dw/dz has a real period L ,

$$\frac{dw}{dz}(z) = \frac{dw}{dz}(z + L), \quad (3a)$$

that there is no flow through the bottom,

$$\text{Im } w = 0 \quad \text{on} \quad \text{Im } z = 0, \quad (3b)$$

and that Bernoulli's equation is satisfied

$$\frac{1}{2} \left| \frac{dw}{dz} \right|^2 + g \text{Im } z = \text{Const.}, \quad (4a)$$

on the free surface

$$\text{Im } w = lC. \quad (4b)$$

We will assume that the solution to the problem has the form

$$w = C \left[z + \frac{L}{2\pi} \sum_{n=1}^{\infty} a^n A_n \sin nkz \right], \quad (5)$$

where $k = 2\pi/L$. We note that equations (3a) and (3b) are satisfied if A_n are real. The success of this assumption will presently justify the form. We know that Bernoulli's theorem must hold on the line given by putting the imaginary part of w equal to a constant lC in equation (5), whose real and imaginary parts are then parametric equations for the profile. The parameter is ϕ , the real part of the potential. We see that we will need to invert equation (5) to obtain z as a function of w , in order conveniently to satisfy Bernoulli's theorem, equation (4a). It is also necessary to obtain dw/dz as a function of w . We will indicate formally how these steps can be carried out, although to justify each step mathematically we would have to investigate the convergence of the assumed solution after we have calculated the values of the coefficients, A_n .

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We differentiate equation (5) with respect to z and obtain

$$\frac{dw}{dz} = C \left[1 + \sum_{n=1}^{\infty} a^n A_n \cos nkz \right], \quad (6)$$

We assume z as a function of w is given by

$$Cz = w + C \frac{L}{2\pi} \sum_{n=1}^{\infty} a^n B_n \sin \frac{nkz}{C}, \quad (7)$$

the justification being that we are able to substitute equation (7) into equation (5) and solve successively for B_n as functions of A_n .

Then we substitute equation (7) in equation (6) and obtain

$$\frac{dw(w)}{dz} = C \left[1 + \sum_{n=0}^{\infty} C_n \sin \frac{nkz}{C} \right], \quad (8)$$

where C_n are functions of a and A_n .

We put $\text{Im } \dot{w} = iLC$ in equation (8) and equation (7) and substitute in equation (4a). We find

$$\frac{1}{2} \left| 1 + \sum a^n C_n \cos \frac{nk}{C} (\phi + iLC) \right|^2 + \frac{gL}{2\pi C^2} \text{Im } z(\phi + iLC) = \text{Const.} \quad (9)$$

We can also put

$$C^2 = \sum_{n=0}^{\infty} D_n a^{2n}. \quad (10)$$

since the celerity may be a function of wave height and depth. Bernoulli's theorem, equation (9), must be valid for all ϕ and a , and we put the separate coefficients of $a^n \cos m\phi$ equal to zero. When this is done, the resulting equations may be solved successively to obtain

$$A_1 = 1$$

$$A_2 = - \frac{3}{4(\cosh 2kl - 1)}$$

$$A_3 = - \frac{2(\cosh 2kl - 11)}{16(\cosh 2kl - 1)^2}.$$

$$D_0 = \frac{\sinh kl}{\cosh kl} \frac{gL}{2\pi}$$

$$D_2 = \frac{(2 \cosh^2 2kl + 2 \cosh 2kl + 5)}{4(\cosh 2kl - 1)} \frac{gL}{2\pi} \frac{\sinh kl}{\cosh kl}.$$

The profile $\eta(x)$ is found by eliminating ϕ from the real and imaginary parts of equation (7), evaluated on $w = \phi + iLC$. Then we obtain the water level d by integration:

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$$d = \frac{1}{L} \int_0^L \eta(x) dx = l + \frac{L}{8\pi} a^2 \sinh 2kl, \quad (13)$$

and the wave height H by some algebra,

$$H = \eta\left(\frac{L}{2}\right) - \eta(0) \\ = \left[2a \sinh kl + 3a^3 \frac{4 \cosh^3 2kl + 4 \cosh^2 2kl + 1}{8(\cosh 2kl - 1)^2} \sinh kl \right] \frac{L}{2\pi}. \quad (14)$$

Finally, the celerity is given by equation (10),

$$C^2 = \frac{\sinh kl}{\cosh kl} \left[1 + \frac{2 \cosh^2 2kl + 2 \cosh 2kl + 5}{4(\cosh 2kl - 1)} \right] \frac{gL}{2\pi}, \quad (15)$$

and the particle velocities from minus the real part and the imaginary part of equation (6),

$$v_x = -\operatorname{Re} \frac{dw}{dz} = -C \left[1 + a \cos kx \cosh ky - \frac{3 \cos 2kx}{2(\cosh 2kl - 1)} \cosh 2ky \right. \\ \left. - \frac{3(2 \cosh 2kl - 1)}{16(\cosh 2kl - 1)^2} \cos 3kx \cosh 3ky \right], \quad (16)$$

and

$$v_y = \operatorname{Im} \frac{dw}{dz} = -C \left[a \sin kx \sinh ky - \frac{3 \sin 2kx}{2(\cosh 2kl)} \sinh 2ky \right. \\ \left. - \frac{3(2 \cosh 2kl - 1)}{16(\cosh 2kl - 1)^2} \sin 3kx \sinh 3ky \right]. \quad (17)$$

We can transform back into a stationary coordinate system by the substitution

$$x = x' - ct, \\ v_x = v'_x - c, \quad (18)$$

and obtain, on dropping the primes,

$$v_x = C \left[a \cos k(x - ct) \cosh ky - \frac{3 \cos 2k(x - ct)}{2(\cosh 2kl - 1)} \cosh 2ky \right. \\ \left. - \frac{3(2 \cosh 2kl - 1)}{16(\cosh 2kl - 1)^2} \cos 3k(x - ct) \cosh 3ky \right], \quad (19)$$

and

$$v_y = -C \left[a \sin k(x - ct) \sinh ky - \frac{3 \sin 2k(x - ct)}{2(\cosh 2kl - 1)} \sinh 2ky \right. \\ \left. - \frac{3(2 \cosh 2kl - 1)}{16(\cosh 2kl - 1)^2} \sin 3k(x - ct) \sinh 3ky \right]. \quad (20)$$

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The set of three equations (13), (14), and (15) have as unknowns the three auxiliary parameters a , k , and l . We see that, given the wave height, the water depth, and the wave period, they may be solved for these auxiliary parameters. The parameter k has a simple interpretation as seen from these equations since

$$\frac{2\pi}{k} = L. \quad (2)$$

Also, l and a are related to the depth and height, respectively.

The equations here given check with those of Hunt and Tanaka. To compare with Hunt and De it is necessary to obtain a as a function of μ , which may be done from the equations for the depth and height. It is then found that a is an odd function of μ ,

$$a = \sum_{n=0}^{\infty} \mu^{2n+1} M_n(l). \quad (3)$$

To compare with Tanaka, it is only necessary to identify his parameter d with our l . His statement that his d is the depth is misleading, since clearly his d is not the mean water level. Finally, it is relevant to notice that the equations pertaining to the Stokes-Struik theory in the Beach Erosion Board Technical Memoranda 1 and 2 are incorrect, as is pointed out by Hunt.

APPLICATION OF THE THEORY

After a theory is explicitly stated, normally two steps remain to be completed before the Engineer or Oceanographer can readily apply the theory to an actual problem. First, some procedure must be given to overcome the computational difficulties involved in the use of the theory, and second, the range of applicability must be indicated so that it is possible to determine when the theory should be used.

COMPUTATIONAL FORMULAS

The Stokes-Struik theory as presented can be put in a somewhat more convenient form for computations as follows. We multiply eq. (13) by eq. (15) and ignore terms containing powers of a greater than the third. Then we divide both sides of the resulting equation by L^2g to obtain

$$\frac{d}{gT^2} = \frac{\tanh kl}{4\pi^2} \left\{ kl + \frac{a^2}{4} \left[\sinh 2kl + \frac{kl (\cosh 4kl + 2 \cosh 2kl + 6)}{\cosh 2kl - 1} \right] \right\}.$$

Similarly we multiply eq. (14) by eq. (15) and divide by L^2g ignoring powers of a greater than the third, and obtain

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$$\frac{H}{gT^2} = \frac{a \tanh kl}{2\pi^2} \left\{ \sinh kl + \frac{a^2}{8} \left[\frac{\sinh kl (2 \cosh 4kl + 3 \cosh 2kl + 10)}{\cosh 2kl - 1} + \frac{\sinh 3kl (3 \cosh 4kl + 4 \cosh 2kl + 2)}{2 (\cosh 2kl - 1)^2} \right] \right\}. \quad (24)$$

Normally at the start of the problem, the depth d , wave height H , and wave period T are known. The symbols π and g are the mathematical constant 3.1415... and the acceleration due to gravity respectively. Hence kl and a are the only two unknown quantities in equations (23) and (24), and the problem becomes one of solving the two nonlinear equations in two unknowns. While this is difficult by hand, it can be solved easily by iteration on an electronic computer. Once kl and a are known, the other properties of the wave follow immediately. An equation for the wavelength in same general form we have been using is obtained by dividing eq. (15) by gL and remembering that $C = L/T$. Hence

$$\frac{L}{gT^2} = \frac{\tanh kl}{2\pi} \left[1 + \frac{a^2 (\cosh 4kl + 2 \cosh 2kl + 6)}{4 (\cosh 2kl - 1)} \right]. \quad (25)$$

The wavelength is thus determined. The wave profile* and the water particle velocities and local accelerations (equations 19 and 20) can be reduced to the convenient form

$$\begin{aligned} \eta &= d + \eta_1 \cos \theta + \eta_2 \cos 2\theta + \eta_3 \cos 3\theta \\ v_x &= v_1 \cos \theta \cosh ky + v_2 \cos 2\theta \cosh 2ky + v_3 \cos 3\theta \cosh 3ky \\ v_y &= v_1 \sin \theta \sinh ky + v_2 \sin 2\theta \sinh 2ky + v_3 \sin 3\theta \sinh 3ky \\ a_x &= \frac{\partial v_x}{\partial t} = a_1 \sin \theta \cosh ky + a_2 \sin 2\theta \cosh 2ky + a_3 \sin 3\theta \cosh 3ky \\ a_y &= \frac{\partial v_y}{\partial t} = -a_1 \cos \theta \sinh ky - a_2 \cos 2\theta \sinh 2ky - a_3 \cos 3\theta \sinh 3ky \end{aligned} \quad (26)$$

where $\eta_1, \eta_2, \eta_3, v_1, v_2, v_3, a_1, a_2, a_3$ are functions solely of L, T, kl and a , and where

$$\theta = 2\pi \left[\frac{x}{L} - \frac{t}{T} \right], \quad k = \frac{2\pi}{L}.$$

Explicitly

$$\begin{aligned} \eta_1 &= \frac{La}{2\pi} \left[\sinh kl + \frac{a^2}{64} \frac{(9 \sinh 5kl + 15 \sinh 3kl + 6 \sinh kl)}{\cosh 2kl - 1} \right] \\ \eta_2 &= \frac{La^2}{16\pi} \frac{(\sinh 4kl + 4 \sinh 2kl)}{(\cosh 2kl - 1)} \end{aligned} \quad (27)$$

*Your attention is drawn to the fact that the wave height computed from $H = \eta(0) - \eta(\pi)$ may occasionally be slightly different from the initial wave height chosen at the start of the problem. This discrepancy is due to the approximations made.

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$$\eta_3 = \frac{La^3}{256\pi} \frac{(3 \sinh 7kl + 15 \sinh 5kl + 27 \sinh 3kl + 39 \sinh kl)}{(\cosh 2kl - 1)^2}$$

$$v_1 = \frac{La}{T}$$

$$v_2 = \frac{3La^2}{2T(\cosh 2kl - 1)}$$

$$v_3 = -\frac{3La^3}{16T} \frac{(2 \cosh 2kl - 11)}{(\cosh 2kl - 1)^2} \quad (2)$$

$$a_1 = \frac{2\pi v_1}{T}$$

$$a_2 = \frac{4\pi v_2}{T}$$

$$a_3 = \frac{6\pi v_3}{T}.$$

As an examination of the equations will show, η/L , $v_x T/L$, $v_y T/L$, $a_x T^2/L$ and $a_y T^2/L$ are all dimensionless and depend only on kl and a or in turn only on d/gT^2 and H/gT^2 . The same dimensionless form extends to the coefficients.

In all of the preceding formulas, a coordinate system with its origin at the sea floor directly beneath the wave crest when $t = 0$ was used. The x axis is horizontal and positive in the direction of wave propagation while the y axis is positive upwards.

When the quantity d/L becomes very large, the hyperbolic functions of ky in the equations become unmanageably large and exceed most tables of hyperbolic functions. In order to avoid these large numbers, a different coordinate system is used whenever $d/T^2 > 2.56$. This is approximately the same as changing the coordinate system whenever $d/L > 0.5$. The new coordinate system has its origin at the sea surface directly below the wave crest when $t = 0$, and has its y axis positive downward. The x axis remains the same as before. The equations for the velocities and accelerations assume a slightly different form under this transformation. Let y' be the new vertical axis and y be the old one. Then

$$y = d - y' \quad (2)$$

We substitute this into the previous formulas, dropping the prime, and obtain v_x , v_y , a_x , and a_y as:

$$\begin{aligned} v_x &= v_1 e^{-ky} \cos \theta + v_2 e^{-2ky} \cos 2\theta + v_3 e^{-3ky} \cos 3\theta \\ v_y &= v_1 e^{-ky} \sin \theta + v_2 e^{-2ky} \sin 2\theta + v_3 e^{-3ky} \sin 3\theta \end{aligned} \quad (2)$$

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$$\begin{aligned}
 a_x &= a_1 e^{-ky} \sin \theta + a_2 e^{-2ky} \sin 2\theta + a_3 e^{-3ky} \sin 3\theta \\
 a_y &= -a_1 e^{-ky} \cos \theta - a_2 e^{-2ky} \cos 2\theta - a_3 e^{-3ky} \cos 3\theta
 \end{aligned}
 \tag{29}$$

TABLES OF COEFFICIENTS

Although all coefficients in the equations for η , v_x , v_y , a_x , and a_y can be stated in a dimensionless form as functions only of d/gT^2 and H/gT^2 , a table of the coefficients versus the three variables, wave depth d , wave height H , and wave period T , is more convenient for most computations. Accordingly, the coefficients for approximately 2000 different wave conditions are tabulated in Appendix I. Water depths from 10 feet to 200 feet are listed in 10-foot intervals, and from 200 feet to 500 feet in 20-foot intervals. Wave heights from 5 to 40 feet in increments of 5 feet, and wave periods from 4 seconds to 20 seconds in increments of 2 seconds are given wherever the Stokes-Struik theory is not demonstrably false. In some cases it was convenient to measure the elevation y , from the water surface positive downward instead of from the sea floor positive upward and to use equations (29). The waves for which y should be measured from still water level are indicated in the tables by a minus sign on the wave period (T). The negative sign is used here merely as a convention to indicate the transformation of z .

A heavy preceding dot has been attached to the wave periods on certain waves. This dot indicates that we do not actually believe the Stokes-Struik theory is applicable to those particular waves. However, since a frequently used method for computing the velocities and accelerations in waves for which no theory is available consists of interpolation between the several most nearly applicable theories, the coefficients for the wave are included in the table even though the theory does not apply.

The position of the decimal point is indicated at least twice on each page of the table. The decimal point for any other number in the column can be located by comparison.

CIRCULAR AND HYPERBOLIC SINES AND COSINES FOR INTEGRAL MULTIPLES OF THE ARGUMENT

To facilitate computation of the three-term series, several auxiliary tables were prepared. Table 1 in Appendix II gives $\sin \theta$, $\sin 2\theta$, $\sin 3\theta$, $\cos \theta$, $\cos 2\theta$, and $\cos 3\theta$ as a function of θ for each degree of θ between 0° and 360° . Table 2 in Appendix II gives $\sinh 2\pi y/L$, $\sinh 4\pi y/L$, $\sinh 6\pi y/L$, $\cosh 2\pi y/L$, $\cosh 4\pi y/L$, and $\cosh 6\pi y/L$ as a function of y/L in increments of 0.01 from 0 to 1.0. These two tables reduce the labor of looking up the functions for a given computation from six entries in a standard table to only two entries.

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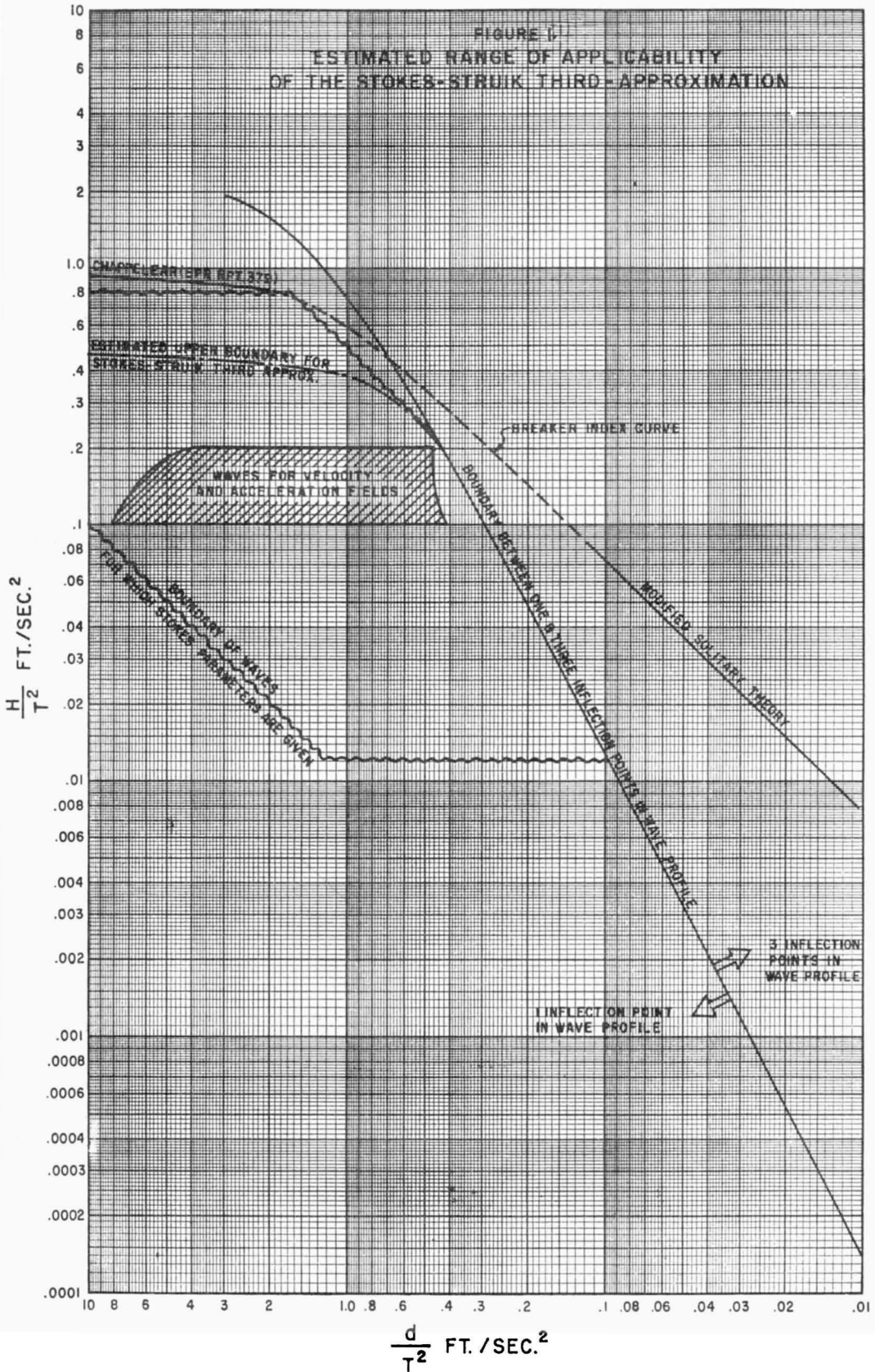
RANGE OF APPLICABILITY

Our estimate of the range of applicability of the theory is given Figure 1. For each value of d/T^2 there is a limiting value of H/T^2 such that waves will break before H/T^2 grows larger. A line connecting these limiting points is called the breaker index curve. In Figure 1, for the estimation of the breaker index curve, the modified solitary theory was used for $d/T^2 < 0.1$ while the results of an unpublished study made by J. E. Chappellear were used for $d/T^2 > 2.0$. The position of the line in between was estimated and is shown by dashes. This breaker index curve is an obvious upper limit to the use of any wave theory. A somewhat smaller upper boundary can be developed from the wave profile equation. From wave tank studies and from intuition, we are fairly sure that waves have only one maximum per wave cycle, i.e., the wave crest. Even more stringently, we suspect that the waves do not have any jogs or semi-steps in the profile. This is the same as saying that the wave profile should have only one inflection point between the wave crest and trough. The boundary line between those waves, as computed by the Stokes-Struik third approximation, that have three inflection points between their crests and troughs and those that have only one is shown in Figure 1 as a solid line. Although this criterion strictly is intuitive, it seems reasonable to the author. The inflection point boundary and the breaker index curve cross at about $d/T^2 = 0.7$. Hence the inflection point criterion is unsuitable for larger d/T^2 values, and a different criterion was developed.

The percent of the wave height above still water level for maximum waves where $d/T^2 > 2.0$ was known by a previous unpublished study. Graph of the percent of wave height above still water level were plotted versus H/T^2 holding d/T^2 at a constant value both for the Stokes-Struik third approximation and Chappellear's limiting case. The Stokes-Struik values departed from a smooth curve through the points at about $H/T^2 = 0.45$. The point of departure decreased somewhat with a decrease of d/T^2 . This departure line is shown as a dash-dot line in Figure 1. Reid and Bretschneider (1953, p. 12) estimate the second approximation of the Stokes-Struik theory to be accurate in deep water for approximately $H/T^2 < 0.3$. Hence the boundary line reaching $H/T^2 = 0.45$ for the third approximation is reasonably consistent with their estimate.

In the preceding work, the basic formulas were derived by Chappellear while the section on the application of the theory together with the various tables were prepared by Borgman. Both authors wish to express their appreciation to the Shell Oil Company and the Shell Development Company for permission to publish the paper.

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REFERENCES

- De, S. C. (1955). Contr. to the theory of Stokes' waves: Proc. Camb. Phil. Soc., vol. 51, pp. 713-736.
- Hunt, J. N. (1953). A note on gravity waves of finite amplitude: Quart. Journ. Mech. and Applied Math., vol. VI, Pt. 3, pp. 336-343.
- Lamb, Sir Horace (1932). Hydrodynamics, Dover (VI Edition), pp. 66 ff. and 375 ff.
- Levi-Civita, Tullio (1925). Determination rigoureuse des ondes permanentes d'ampleur finie; Math. Ann., vol. 43, pp. 264-314.
- Reid, R. O., and Bretschneider, C. L. (1953). Surface waves and offshore structures: Texas A. and M. Research Foundation, College Station, October 1953.
- Stokes, Sir George (1847). On the theory of oscillatory waves: Trans. Camb. Soc., vol. VIII, p. 441.
- Struik, D. J. (1926). Determination rigoureuse des ondes irrotationnelle periodiques dans un canal a profondeur finie: Math. Ann., vol. XCV, pp. 595-634.
- Tanaka, Kiyoshi (1953). On sea waves: Tech. Rep. of Osaka Univ., #65, March 1953.

LIST OF SYMBOLS

- | | | |
|-----------------|---|---|
| a | = | a mathematical parameter in the Stokes-Struik third approximation. The physical meaning is given by the formulae. |
| A_n | = | expansion coefficients to be calculated. |
| a_x, a_y | = | horizontal and vertical components of the local accelerations of water particles, i.e., $a_x = \partial v_x / \partial t$, $a_y = \partial v_y / \partial t$; (ft/sec ²). |
| a_1, a_2, a_3 | = | Stokes-Struik third approximation coefficients for the local accelerations. |
| B_n | = | expansion coefficients to be calculated. |
| C | = | wave celerity, or speed of advance of the waveform (ft/sec). |
| C_n | = | expansion coefficients to be calculated. |
| d | = | still water depth, or depth the water would assume if the waves were calmed down and the water was still (ft). |
| D_{2n} | = | expansion coefficients to be calculated. |
| g | = | acceleration due to gravity (nominally taken as 32.16 ft/sec ²). |
| H | = | wave height, or vertical distance between a crest and the preceding trough (ft). |
| k | = | $2\pi/L$ (ft ⁻¹). |
| l | = | a mathematical parameter in the Stokes-Struik third approximation. The physical meaning is given by the formulae. |
| L | = | wavelength, or horizontal distance between two succeeding troughs or crests in the direction of wave propagation (ft). |

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- t = time measured from the instant when the wave crest was above the origin of the coordinate system (seconds).
 T = wave period, or length of time for two succeeding crests to pass a fixed point (seconds).
 v_x, v_y = horizontal and vertical components of the water particle velocities (ft/sec).
 v_1, v_2, v_3 = Stokes-Struik third approximation coefficients for the water particle velocities (ft/sec).
 $w(z)$ = complex potential, $\phi + iy\psi$
 for T marked plus - x, y = horizontal and vertical coordinate axis, respectively. The origin is at the sea floor directly below the wave crest when $t = 0$. x is positive in the direction of wave propagation and y is positive upwards.
 for T marked minus - x, y = horizontal and vertical coordinate axis, respectively. The origin is at still water level directly beneath the wave crest when $t = 0$. x is positive in the direction of wave travel and y is positive downwards.
 z = complex variable, $x + iy$
 $\eta(x)$ = vertical elevation of the wave profile above the sea floor, (ft). η is a function of x or of θ depending on what horizontal measure of distance is being used.
 η_1, η_2, η_3 = Stokes-Struik third approximation coefficients in the equation for the wave profile (ft).
 θ = angular phase position or distance from crest measured so as to make
 $\theta = 0$ at the crest and $\theta = 180^\circ$ at the preceding trough.
 In general,
 $\theta = 2\pi(x/L - t/T)$, but usually t is set to zero and the simpler formula
 $\theta = 2x/L$ is used (dimensionless)
 $\phi(x, y)$ = velocity potential
 $\psi(x, y)$ = stream function

APPENDIX I

STOKES-STRIUK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS

Column Headings:

- d = water depth (ft)
 H = wave height (ft)
 T = wave period (sec)
 L = wavelength (ft)
 η_1, η_2, η_3 = profile coefficients (ft)
 v_1, v_2, v_3 = velocity coefficients (ft/sec)
 a_1, a_2, a_3 = local acceleration coefficients (ft/sec²)

(The reader is referred to the list of symbols for more detailed definitions, and to the text for the mathematical series in which the coefficients are to be used.)

The decimal point position is indicated in the first line of each depth group and on the first line of each page. The decimal positions of the other numbers in a given column are the same as those where the position is indicated.

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d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	O_1	O_2	O_3
18	3	4	640	2344	4583	2075	3100	1195	-0.188	4870	-6136	2086
20	3	4	782	2431	5030	2307	3400	1245	-0.008	4278	2010	-0.009
20	3	4	1377	2368	6134	1441	2954	2205	443	2037	1848	710
20	3	4	838	4823	9915	2939	3237	3450	16	5084	1445	1580
28	10	4	1440	4632	14044	6055	4494	3452	445	4766	7187	1460
30	5	4	830	2472	2236	4330	1792	3005	0.000	3344	0016	-0.000
30	5	4	1565	2475	2386	259	1686	180	58	1765	377	8
30	5	4	2532	2428	4531	749	2195	1013	104	1579	90	10
30	5	12	3623	2393	3990	1273	2243	2128	528	1175	1228	827
30	5	14	4284	2317	7497	2006	2233	2987	1090	1002	2481	1467
30	10	6	1607	4851	8506	1774	3322	3511	2	2690	97	5
30	10	6	1607	4851	8506	1774	3322	3511	2	2690	97	5
30	10	8	2326	4751	11674	2869	3953	2829	317	3059	744	20
30	10	8	2036	4573	13441	4683	4092	4641	1392	2371	6084	2624
30	10	8	2401	4878	13190	9135	5644	5372	1126	4634	8752	2855
40	5	4	842	2474	2172	4293	1386	1000	0.000	4506	0000	0.000
40	5	4	2318	2483	2078	131	1534	238	4	1281	95	2
40	5	10	3914	2472	2760	296	1847	359	33	1161	703	66
40	5	12	4091	2450	3750	451	1941	974	119	1016	1020	187
40	5	14	4620	2383	5691	1192	1993	1482	201	991	1321	379
40	5	18	6380	2339	7056	1721	1982	2355	915	692	1782	958
40	5	20	7137	2265	8076	2307	1938	3064	1370	609	1925	1291
40	10	6	1721	2265	6781	1003	2841	192	4	1369	6	12
40	10	6	2556	4881	7787	1320	5241	954	36	2246	1459	85
40	10	8	3367	4806	10346	2111	3575	2167	279	2246	2724	526
40	10	8	4984	4538	16500	3685	5213	3809	1803	1931	3809	1387
40	15	6	1768	7280	12616	2922	3685	485	111	3639	1015	2435
40	15	6	2609	7112	16126	3952	4754	2166	138	3734	5405	326
40	15	8	3520	6878	20461	5690	4332	407	98	3237	9915	1768
40	20	8	1820	9518	10461	5690	4332	407	98	3237	9915	1768
40	20	8	2673	9383	26339	6298	6200	3907	376	4470	6134	806
50	5	4	845	2476	2183	4283	1386	1000	-0.009	4279	0021	-0.009
50	5	4	2790	2400	1344	100	1353	99	1065	135	135	13
50	5	10	3614	2488	1937	147	1562	270	6	994	339	15
50	5	12	4080	2450	3229	224	1702	507	34	991	531	53
50	5	14	4530	2454	4071	362	1803	1130	186	700	807	219
50	5	18	6201	2430	4970	828	1821	1501	356	634	1048	352
50	5	20	7047	2430	6045	1149	1812	1872	537	569	1176	306
50	10	6	1807	2265	5845	1149	1812	1872	537	569	1176	306
50	10	6	1793	4937	5045	717	1807	34	1	1892	113	211
50	10	6	2788	4930	5712	721	2684	401	2	2108	429	5
50	10	8	3452	4859	7503	1094	3108	1067	65	1938	1341	122
50	10	8	4842	4859	9503	1696	4241	1222	282	2508	441	1354
50	10	14	6298	4643	14377	3396	5391	2999	643	1522	2632	648
50	10	14	5422	4735	11895	2396	3391	2999	643	1522	2632	648
50	15	6	1858	7307	16786	2250	3335	5223	2519	1171	3647	2324
50	15	6	1858	7307	16786	2250	3335	5223	2519	1171	3647	2324
50	15	8	2781	7301	12092	2312	3984	424	11	3128	1488	37
50	15	8	3705	7201	13310	3367	4555	2374	22	2108	629	5
50	15	12	4552	7030	15850	4122	4790	4241	862	2508	441	1354
50	15	12	4552	7030	15850	4122	4790	4241	862	2508	441	1354
50	20	6	1689	5712	10101	4305	3725	2779	2010	3900	584	278
50	20	6	1689	5712	10101	4305	3725	2779	2010	3900	584	278
50	20	8	2833	5608	20102	4957	5251	1691	38	4124	2656	89
50	20	8	2833	5608	20102	4957	5251	1691	38	4124	2656	89
50	25	6	1642	1207	24635	7209	5924	4180	536	3722	1049	252
50	25	6	1642	1207	24635	7209	5924	4180	536	3722	1049	252
50	25	8	2890	11875	29392	8771	6493	2732	99	5101	4291	264
50	25	8	2890	11875	29392	8771	6493	2732	99	5101	4291	264
60	5	4	846	2475	2125	4283	1386	1000	-0.001	4129	0017	-0.005
60	5	4	1804	2494	1201	83	1653	285	10	1125	44	4
60	5	8	2853	2494	1170	66	1125	44	2	862	63	784
60	5	10	3487	2484	1442	86	1272	142	3	962	129	17
60	5	14	4885	2468	2368	158	1593	471	13	1119	450	174
60	5	14	6720	2477	2954	297	1637	687	13	715	550	86
60	5	18	7644	2456	3592	432	1660	926	137	500	646	143

THE USE OF THE STOKES-STRIJK APPROXIMATION FOR WAVES OF FINITE HEIGHT

APPENDIX I
STOKES-STRIJK THIRD-APPROXIMATION COEFFICIENTS FOR AN ASSEMBLAGE OF WAVE CONDITIONS (CONTINUED)

d	H	T	L	η_1	η_2	η_3	V_1	V_2	V_3	q_1	q_2	q_3	$\frac{q_1}{V_1}$	$\frac{q_2}{V_2}$	$\frac{q_3}{V_3}$	$\frac{q_1}{V_1}$	$\frac{q_2}{V_2}$	$\frac{q_3}{V_3}$	$\frac{q_1}{V_1}$	$\frac{q_2}{V_2}$	$\frac{q_3}{V_3}$	
80	80	20	9749	2484	2571	0220	1445	0834	0036	2791	434	0082	0348	0348	0082	100	90	20	10317	9313	26168	7570
80	80	6	1998	4897	6897	1586	14509	2	0	2791	434	0	0348	0348	0	100	20	20	10317	9313	26168	7570
80	80	10	3076	4959	3507	206	1269	44	0	1233	65	1-	1233	1233	65	100	20	20	10317	9313	26168	7570
80	80	10	3473	4959	3461	302	2112	189	12	1327	280	1-	1327	1327	280	100	20	20	10317	9313	26168	7570
80	80	10	5427	4959	5445	404	421	431	12	1267	482	1-	1267	1267	482	100	20	20	10317	9313	26168	7570
80	80	10	7652	4919	6784	828	2711	1165	117	1065	915	138	1065	1065	915	100	20	20	10317	9313	26168	7570
80	80	18	8743	4894	8213	1180	2780	1626	239	971	1135	230	971	971	1135	100	20	20	10317	9313	26168	7570
80	80	15	9545	4754	9257	1816	2085	2113	417	1329	392	372	1329	1329	392	100	20	20	10317	9313	26168	7570
80	80	6	1922	7384	8287	1447	1100	94	2-	1132	148	4-	1132	1132	148	100	20	20	10317	9313	26168	7570
80	80	6	3109	7416	7135	925	2263	9	2-	1836	178	4-	1836	1836	178	100	20	20	10317	9313	26168	7570
80	80	15	4442	7976	9740	1398	3397	848	18	1074	65	4-	1074	1074	65	100	20	20	10317	9313	26168	7570
80	80	15	6495	7532	12028	1855	3632	1700	136	1789	209	65	1789	1789	209	100	20	20	10317	9313	26168	7570
80	80	13	7712	7293	14988	2686	3897	485	387	1366	2008	485	1366	1366	2008	100	20	20	10317	9313	26168	7570
80	80	15	9922	7059	20194	4921	4042	4307	250	2158	2828	1237	2158	2158	2828	100	20	20	10317	9313	26168	7570
80	80	20	8	1974	9857	13417	2942	1535	7	1607	173	10-	1607	1607	173	100	20	20	10317	9313	26168	7570
80	80	20	3150	9827	12121	2030	3154	177	4-	2485	218	10-	2485	2485	218	100	20	20	10317	9313	26168	7570
80	80	20	4658	9621	20410	4121	5041	2968	367	2282	2766	494	2282	2282	2766	100	20	20	10317	9313	26168	7570
80	80	20	6684	9240	24850	7656	5138	5410	189	3797	4444	1084	3797	3797	4444	100	20	20	10317	9313	26168	7570
80	80	20	8029	8240	24850	7656	5138	5410	189	3797	4444	1084	3797	3797	4444	100	20	20	10317	9313	26168	7570
80	80	25	8	2059	12266	19152	4950	2010	14	2104	30	100	2104	2104	30	100	20	20	10317	9313	26168	7570
80	80	25	3156	12212	17995	3652	3974	295	8-	3121	463	100	3121	3121	463	100	20	20	10317	9313	26168	7570
80	80	25	4624	12255	24492	6759	6222	3854	159	3624	4056	544	3624	3624	4056	100	20	20	10317	9313	26168	7570
80	80	25	5983	12035	29224	5010	5849	2478	186	3061	2805	313	3061	3061	2805	100	20	20	10317	9313	26168	7570
80	80	25	7873	11392	36101	10377	6319	6181	4559	716	2774	4092	4559	4559	716	100	20	20	10317	9313	26168	7570
80	80	30	8	3246	14578	24615	5810	4795	455	3766	711	33-	3766	3766	711	100	20	20	10317	9313	26168	7570
80	80	30	4440	14466	28435	6606	6185	1760	128	3886	2211	32	3886	3886	2211	100	20	20	10317	9313	26168	7570
80	80	30	5624	14255	34492	8759	6222	3854	159	3624	4056	544	3624	3624	4056	100	20	20	10317	9313	26168	7570
80	80	35	8	3297	16936	31861	8510	5628	659	4421	1035	31-	4421	4421	1035	100	20	20	10317	9313	26168	7570
80	80	35	4495	16752	57107	9850	7171	2448	55	4505	3076	104	4505	4505	3076	100	20	20	10317	9313	26168	7570
80	80	35	5690	16495	49319	13114	7957	5246	196	4172	1494	236	4172	4172	1494	100	20	20	10317	9313	26168	7570
80	80	40	8	3591	19293	39661	11757	6473	920	5068	1445	275-	5068	5068	1445	100	20	20	10317	9313	26168	7570
80	80	40	4583	19016	46506	13854	8146	3271	101	5118	4110	191	5118	5118	4110	100	20	20	10317	9313	26168	7570
80	80	40	5760	18609	56177	18421	8988	6853	950	4708	7187	1460	4708	4708	7187	100	20	20	10317	9313	26168	7570
90	90	5	846	2475	2122	0283	1625	0000	0000	15978	0000	0000	15978	15978	0000	100	15	4-	846	2475	2122	0283
90	90	5	1890	2484	1058	67	245	0	0	287	0	0	287	287	0	100	15	4-	1890	2484	1058	67
90	90	5	4403	2497	780	32	946	3	0	540	7	0	540	540	7	100	15	4-	4403	2497	780	32
90	90	5	5639	2563	948	37	1109	71	3	581	74	0	581	581	74	100	15	4-	5639	2563	948	37
90	90	5	6984	2498	1139	51	1210	132	3	543	118	4	543	543	118	100	15	4-	6984	2498	1139	51
90	90	5	9132	2485	1729	105	1321	298	16	461	208	17	461	461	208	100	15	4-	9132	2485	1729	105
90	90	5	10288	2488	2066	144	1349	398	30	424	250	28	424	424	250	100	15	4-	10288	2488	2066	144
90	90	10	1698	4997	6837	1886	14310	0	0	2793	2	0	2793	2793	2	100	10	4-	1698	4997	6837	1886
90	90	10	3140	4976	2997	248	1317	19	0	1084	30	0	1084	1084	30	100	10	4-	3140	4976	2997	248
90	90	10	4421	4977	3130	232	1875	109	4	1178	137	0	1178	1178	137	100	10	4-	4421	4977	3130	232
90	90	10	6658	4969	4707	589	2501	291	4	1057	294	7	1057	1057	294	100	10	4-	6658	4969	4707	589
90	90	10	8033	4945	5545	361	2524	819	59	931	643	132	931	931	643	100	10	4-	8033	4945	5545	361
90	90	10	9192	4923	6689	788	2602	1164	127	908	812	132	908	908	812	100	10	4-	9192	4923	6689	788
90	90	15	1056	7335	13276	3035	2085	1946	231	3762	972	215-	3762	3762	972	100	15	4-	1056	7335	13276	3035
90	90	15	1930	7388	8110	1398	8602	2	0	848	2	0	848	848	2	100	15	4-	1930	7388	8110	1398
90	90	15	3172	7428	6486	785	1986	44	0	1560	72	0	1560	1560	72	100	15	4-	3172	7428	6486	785
90	90	15	4416	7416	8871	944	3288	639	15	1720	665	23	1720	1720	665	100	15	4-	4416	7416	8871	944
90	90	15	6659	7375	9940	1290	3564	1165	72	1599	1046	97	1599	1599	1046	100	15	4-	6659	7375	9940	1290
90	90	15	8081	7259	12069	1808	3723	1815	198	1466	1425	293	1466	1466	1425	100	15	4-	8081	7259	12069	1808
90	90	15	9192	7175	16068	5401	3973	5345	744	1216	2104	701	1216	1216	2104	100	15	4-	9192	7175	16068	5401
90	90	20	1012	9835	18829	6829	26894	7	4-	42244	23	18-	42244	42244	23	100	20	4-	1012	9835		

COASTAL ENGINEERING

APPENDIX II

Table 1 Circular Sines and Cosines for Integral Multiples of θ

Table 2 Hyperbolic Sines and Cosines for Integral Multiples of $2\pi z/L$

The decimal point position is indicated by the vertical lines.

THE USE OF THE STOKES-STRIJK APPROXIMATION FOR WAVES OF FINITE HEIGHT

APPENDIX II
TABLE I CIRCULAR SINES AND COSINES FOR INTEGRAL MULTIPLES OF θ

(Degrees)	Sin θ		Cos θ		Sin 2θ		Cos 2θ		Sin 3θ		Cos 3θ		Sin 4θ		Cos 4θ	
	θ	(Radians)	θ	(Radians)	θ	(Radians)	θ	(Radians)	θ	(Radians)	θ	(Radians)	θ	(Radians)	θ	(Radians)
0	0	0	10000	0	10000	0	10000	0	10000	0	10000	0	10000	0	10000	0
1	0.0175	0.0175	9998	0.0349	9994	0.0698	9986	0.1045	9976	0.1572	9954	0.2167	9926	0.2811	9876	0.3493
2	0.0349	0.0349	9986	0.0698	9976	0.1396	9954	0.1743	9932	0.2434	9892	0.3125	9840	0.3816	9780	0.4497
3	0.0524	0.0524	9976	0.1045	9954	0.1743	9932	0.2434	9892	0.3125	9840	0.3816	9780	0.4497	9719	0.5178
4	0.0698	0.0698	9954	0.1396	9932	0.2167	9892	0.2811	9840	0.3493	9780	0.4174	9719	0.4855	9654	0.5539
5	0.0873	0.0873	9926	0.1743	9892	0.2434	9840	0.3125	9780	0.3816	9719	0.4497	9654	0.5178	9573	0.5800
6	0.1047	0.1047	9892	0.2167	9840	0.2811	9780	0.3493	9719	0.4174	9654	0.4855	9573	0.5539	9452	0.6461
7	0.1222	0.1222	9840	0.2591	9780	0.3125	9719	0.3816	9654	0.4497	9573	0.5178	9452	0.5800	9331	0.7122
8	0.1396	0.1396	9780	0.3016	9719	0.3493	9654	0.4174	9573	0.4855	9452	0.5539	9331	0.6183	9210	0.7783
9	0.1571	0.1571	9719	0.3440	9654	0.3816	9573	0.4497	9452	0.5178	9331	0.5800	9210	0.6461	9089	0.8344
10	0.1745	0.1745	9654	0.3865	9573	0.4174	9452	0.4855	9331	0.5539	9210	0.6183	9089	0.6823	8968	0.8905
11	0.1920	0.1920	9573	0.4290	9452	0.4497	9331	0.5178	9089	0.6461	9331	0.6823	8968	0.7384	8847	0.9466
12	0.2094	0.2094	9452	0.4715	9331	0.4855	9210	0.5539	9089	0.6823	9210	0.7122	8968	0.7845	8726	1.0027
13	0.2269	0.2269	9331	0.5140	9210	0.5178	9089	0.5800	9089	0.7122	9089	0.7384	8968	0.8106	8607	1.0588
14	0.2443	0.2443	9210	0.5565	9089	0.5539	9089	0.6183	9089	0.7384	9089	0.7545	8968	0.8367	8488	1.1149
15	0.2618	0.2618	9089	0.5990	9089	0.5278	9089	0.6461	9089	0.7545	9089	0.7706	8968	0.8628	8269	1.1710
16	0.2793	0.2793	8968	0.6415	9089	0.4977	9089	0.6740	9089	0.7706	9089	0.7867	8968	0.8889	8050	1.2271
17	0.2967	0.2967	8847	0.6840	9089	0.4674	9089	0.7019	9089	0.7867	9089	0.8028	8968	0.9150	7831	1.2832
18	0.3142	0.3142	8726	0.7265	9089	0.4371	9089	0.7278	9089	0.8028	9089	0.8187	8968	0.9411	7612	1.3393
19	0.3316	0.3316	8607	0.7690	9089	0.4068	9089	0.7537	9089	0.8187	9089	0.8346	8968	0.9672	7393	1.3954
20	0.3491	0.3491	8488	0.8115	9089	0.3765	9089	0.7796	9089	0.8346	9089	0.8505	8968	0.9933	7174	1.4515
21	0.3665	0.3665	8369	0.8540	9089	0.3462	9089	0.8055	9089	0.8505	9089	0.8664	8968	1.0194	6955	1.5076
22	0.3840	0.3840	8250	0.8965	9089	0.3159	9089	0.8314	9089	0.8664	9089	0.8823	8968	1.0455	6736	1.5637
23	0.4014	0.4014	8131	0.9390	9089	0.2856	9089	0.8573	9089	0.8823	9089	0.8982	8968	1.0716	6517	1.6198
24	0.4189	0.4189	8012	0.9815	9089	0.2553	9089	0.8832	9089	0.8982	9089	0.9141	8968	1.0977	6298	1.6759
25	0.4363	0.4363	7893	1.0240	9089	0.2250	9089	0.9091	9089	0.9141	9089	0.9300	8968	1.1238	6079	1.7320
26	0.4538	0.4538	7774	1.0665	9089	0.1947	9089	0.9350	9089	0.9300	9089	0.9459	8968	1.1499	5860	1.7881
27	0.4712	0.4712	7655	1.1090	9089	0.1644	9089	0.9609	9089	0.9459	9089	0.9618	8968	1.1760	5641	1.8442
28	0.4887	0.4887	7536	1.1515	9089	0.1341	9089	0.9868	9089	0.9618	9089	0.9777	8968	1.2021	5422	1.9003
29	0.5061	0.5061	7417	1.1940	9089	0.1038	9089	1.0127	9089	0.9777	9089	0.9936	8968	1.2282	5203	1.9564
30	0.5236	0.5236	7298	1.2365	9089	0.0735	9089	1.0386	9089	0.9936	9089	1.0095	8968	1.2543	4984	2.0125
31	0.5411	0.5411	7179	1.2790	9089	0.0432	9089	1.0645	9089	1.0095	9089	1.0254	8968	1.2804	4765	2.0686
32	0.5585	0.5585	7060	1.3215	9089	0.0129	9089	1.0904	9089	1.0254	9089	1.0413	8968	1.3065	4546	2.1247
33	0.5760	0.5760	6941	1.3640	9089	0.0000	9089	1.1163	9089	1.0413	9089	1.0572	8968	1.3326	4327	2.1808
34	0.5934	0.5934	6822	1.4065	9089	0.0000	9089	1.1422	9089	1.0572	9089	1.0731	8968	1.3587	4108	2.2369
35	0.6109	0.6109	6703	1.4490	9089	0.0000	9089	1.1681	9089	1.0731	9089	1.0890	8968	1.3848	3889	2.2930
36	0.6283	0.6283	6584	1.4915	9089	0.0000	9089	1.1940	9089	1.0890	9089	1.1049	8968	1.4109	3670	2.3491
37	0.6458	0.6458	6465	1.5340	9089	0.0000	9089	1.2199	9089	1.1049	9089	1.1208	8968	1.4370	3451	2.4052
38	0.6632	0.6632	6346	1.5765	9089	0.0000	9089	1.2458	9089	1.1208	9089	1.1367	8968	1.4631	3232	2.4613
39	0.6807	0.6807	6227	1.6190	9089	0.0000	9089	1.2717	9089	1.1367	9089	1.1526	8968	1.4892	3013	2.5174
40	0.6981	0.6981	6108	1.6615	9089	0.0000	9089	1.2976	9089	1.1526	9089	1.1685	8968	1.5153	2794	2.5735
41	0.7156	0.7156	5989	1.7040	9089	0.0000	9089	1.3235	9089	1.1685	9089	1.1844	8968	1.5414	2575	2.6296
42	0.7330	0.7330	5870	1.7465	9089	0.0000	9089	1.3494	9089	1.1844	9089	1.2003	8968	1.5675	2356	2.6857
43	0.7505	0.7505	5751	1.7890	9089	0.0000	9089	1.3753	9089	1.2003	9089	1.2162	8968	1.5936	2137	2.7418
44	0.7679	0.7679	5632	1.8315	9089	0.0000	9089	1.4012	9089	1.2162	9089	1.2321	8968	1.6197	1918	2.7979
45	0.7854	0.7854	5513	1.8740	9089	0.0000	9089	1.4271	9089	1.2321	9089	1.2480	8968	1.6458	1699	2.8540
46	0.8029	0.8029	5394	1.9165	9089	0.0000	9089	1.4530	9089	1.2480	9089	1.2639	8968	1.6719	1480	2.9101
47	0.8203	0.8203	5275	1.9590	9089	0.0000	9089	1.4789	9089	1.2639	9089	1.2798	8968	1.6980	1261	2.9662
48	0.8378	0.8378	5156	2.0015	9089	0.0000	9089	1.5048	9089	1.2798	9089	1.2957	8968	1.7241	1042	3.0223
49	0.8552	0.8552	5037	2.0440	9089	0.0000	9089	1.5307	9089	1.2957	9089	1.3116	8968	1.7502	823	3.0784
50	0.8727	0.8727	4918	2.0865	9089	0.0000	9089	1.5566	9089	1.3116	9089	1.3275	8968	1.7763	604	3.1345
51	0.8901	0.8901	4799	2.1290	9089	0.0000	9089	1.5825	9089	1.3275	9089	1.3434	8968	1.8024	385	3.1906
52	0.9076	0.9076	4680	2.1715	9089	0.0000	9089	1.6084	9089	1.3434	9089	1.3593	8968	1.8285	166	3.2467
53	0.9250	0.9250	4561	2.2140	9089	0.0000	9089	1.6343	9089	1.3593	9089	1.3752	8968	1.8546	0	3.3028
54	0.9425	0.9425	4442	2.2565	9089	0.0000	9089	1.6602	9089	1.3752	9089	1.3911	8968	1.8807	0	3.3589
55	0.9599	0.9599	4323	2.2990	9089	0.0000	9089	1.6861	9089	1.3911	9089	1.4070	8968	1.9068	0	3.4150
56	0.9774	0.9774	4204	2.3415	9089	0.0000	9089	1.7120	9089	1.4070	9089	1.4229	8968	1.9329	0	3.4711
57	0.9948	0.9948	4085	2.3840	9089	0.0000	9089	1.7379	9089	1.4229	9089	1.4388	8968	1.9590	0	3.5272
58	1.0123	1.0123	3966	2.4265	9089	0.0000	9089	1.7638	9089	1.4388	9089	1.4547	8968	1.9851	0	3.5833
59	1.0297	1.0297	3847	2.4690	9089	0.0000	9089	1.7897	9089	1.4547	9089	1.4706	8968	2.0112	0	3.6394
60	1.0472	1.0472	3728	2.5115	9089	0.0000	9089	1.8156	9089	1.4706	9089	1.4865	8968	2.0373	0	3.6955

Vertical lines indicate the decimal point position

COASTAL ENGINEERING

APPENDIX II
TABLE I CONTINUED

(Degrees) θ	Sin θ		Cos θ		Sm. 2 θ		Cos. 2 θ		Sm. 2 θ		Cos. 2 θ		Sin 3 θ		Cos 3 θ	
	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)	(Degrees) θ	(Rachons)
121	21118	3572	121	10000	0	7071	7071	0	7071	0	7071	0	7071	0	7071	0
122	21133	3576	122	9994	0349	7431	6691	196	34208	2588	2419	9703	4695	5000	8660	8660
123	21148	3587	123	9976	0696	7771	6293	197	34383	2924	2756	3290	7771	6293	3290	7771
124	21162	3599	124	9958	1045	8090	5878	198	34558	3266	3590	3590	9111	5878	8090	8090
125	21177	3611	125	9939	1392	8387	5446	199	34732	3617	3935	3935	6157	5446	8387	8387
126	21191	3623	126	9920	1736	8680	5000	200	34905	3976	4201	4201	3266	3976	8680	8680
127	21206	3635	127	9900	2078	8968	4546	201	35077	4344	4476	4476	2756	4344	8968	8968
128	21220	3647	128	9880	2419	9251	4090	202	35248	4719	4761	4761	2249	4719	9251	9251
129	21235	3659	129	9860	2756	9529	3636	203	35418	5097	5045	5045	1743	5097	9529	9529
130	21249	3671	130	9840	3071	9802	3171	204	35587	5477	5319	5319	1231	5477	9802	9802
131	21264	3683	131	9820	3392	10000	2707	205	35755	5862	5592	5592	719	5862	10000	10000
132	21278	3695	132	9800	3710	0	2243	206	35922	6252	6000	6000	207	6252	0	0
133	21293	3707	133	9780	4026	0349	2588	207	36088	6646	6395	6395	1564	6646	0349	0349
134	21307	3719	134	9760	4340	0696	3071	208	36253	7044	6794	6794	1050	7044	0696	0696
135	21321	3731	135	9740	4651	0945	3546	209	36417	7446	7194	7194	535	7446	0945	0945
136	21335	3743	136	9720	4959	1192	4019	210	36580	7852	7599	7599	20	7852	1192	1192
137	21349	3755	137	9700	5265	1436	4490	211	36742	8262	7999	7999	171	8262	1436	1436
138	21363	3767	138	9680	5569	1678	4960	212	36903	8675	8400	8400	142	8675	1678	1678
139	21377	3779	139	9660	5870	1918	5429	213	37063	9091	8799	8799	113	9091	1918	1918
140	21391	3791	140	9640	6169	2156	5896	214	37222	9510	9199	9199	84	9510	2156	2156
141	21405	3803	141	9620	6466	2373	6361	215	37380	9932	9600	9600	55	9932	2373	2373
142	21419	3815	142	9600	6761	2588	6825	216	37537	10357	9999	9999	26	10357	2588	2588
143	21433	3827	143	9580	7054	2801	7288	217	37693	10785	10499	10499	0	10785	2801	2801
144	21447	3839	144	9560	7345	3012	7750	218	37848	11216	10999	10999	1045	11216	3012	3012
145	21461	3851	145	9540	7634	3221	8211	219	38002	11650	11500	11500	1045	11650	3221	3221
146	21475	3863	146	9520	7920	3428	8671	220	38155	12087	12000	12000	1045	12087	3428	3428
147	21489	3875	147	9500	8204	3633	9130	221	38307	12527	12500	12500	1045	12527	3633	3633
148	21503	3887	148	9480	8486	3836	9588	222	38458	12969	12999	12999	1045	12969	3836	3836
149	21517	3899	149	9460	8766	4038	10045	223	38608	13414	13000	13000	1045	13414	4038	4038
150	21531	3911	150	9440	9044	4239	10500	224	38757	13861	13000	13000	1045	13861	4239	4239
151	21545	3923	151	9420	9320	4438	10955	225	38905	14311	13000	13000	1045	14311	4438	4438
152	21559	3935	152	9400	9596	4635	11410	226	39052	14763	13000	13000	1045	14763	4635	4635
153	21573	3947	153	9380	9870	4830	11865	227	39198	15217	13000	13000	1045	15217	4830	4830
154	21587	3959	154	9360	10142	5024	12320	228	39343	15673	13000	13000	1045	15673	5024	5024
155	21601	3971	155	9340	10412	5216	12775	229	39487	16131	13000	13000	1045	16131	5216	5216
156	21615	3983	156	9320	10680	5406	13230	230	39630	16591	13000	13000	1045	16591	5406	5406
157	21629	3995	157	9300	10946	5594	13685	231	39772	17053	13000	13000	1045	17053	5594	5594
158	21643	4007	158	9280	11210	5780	14140	232	39913	17517	13000	13000	1045	17517	5780	5780
159	21657	4019	159	9260	11472	5964	14595	233	40053	17983	13000	13000	1045	17983	5964	5964
160	21671	4031	160	9240	11732	6146	15050	234	40192	18451	13000	13000	1045	18451	6146	6146
161	21685	4043	161	9220	11990	6326	15505	235	40329	18921	13000	13000	1045	18921	6326	6326
162	21699	4055	162	9200	12246	6504	15960	236	40465	19393	13000	13000	1045	19393	6504	6504
163	21713	4067	163	9180	12500	6680	16415	237	40600	19867	13000	13000	1045	19867	6680	6680
164	21727	4079	164	9160	12752	6854	16870	238	40734	20343	13000	13000	1045	20343	6854	6854
165	21741	4091	165	9140	13002	7026	17325	239	40867	20821	13000	13000	1045	20821	7026	7026
166	21755	4103	166	9120	13250	7196	17780	240	41000	21301	13000	13000	1045	21301	7196	7196
167	21769	4115	167	9100	13496	7364	18235	241	41132	21783	13000	13000	1045	21783	7364	7364
168	21783	4127	168	9080	13740	7530	18690	242	41263	22267	13000	13000	1045	22267	7530	7530
169	21797	4139	169	9060	13982	7694	19145	243	41393	22753	13000	13000	1045	22753	7694	7694
170	21811	4151	170	9040	14222	7856	19600	244	41522	23241	13000	13000	1045	23241	7856	7856
171	21825	4163	171	9020	14460	8016	20055	245	41650	23731	13000	13000	1045	23731	8016	8016
172	21839	4175	172	9000	14696	8174	20510	246	41777	24223	13000	13000	1045	24223	8174	8174
173	21853	4187	173	8980	14930	8330	20965	247	41903	24717	13000	13000	1045	24717	8330	8330
174	21867	4199	174	8960	15162	8484	21420	248	42028	25213	13000	13000	1045	25213	8484	8484
175	21881	4211	175	8940	15392	8636	21875	249	42152	25711	13000	13000	1045	25711	8636	8636
176	21895	4223	176	8920	15620	8786	22330	250	42275	26211	13000	13000	1045	26211	8786	8786
177	21909	4235	177	8900	15846	8934	22785	251	42397	26713	13000	13000	1045	26713	8934	8934
178	21923	4247	178	8880	16070	9080	23240	252	42518	27217	13000	13000	1045	27217	9080	9080
179	21937	4259	179	8860	16292	9224	23695	253	42638	27723	13000	13000	1045	27723	9224	9224
180	21951	4271	180	8840	16512	9366	24150	254	42757	28231	13000	13000	1045	28231	9366	9366

THE USE OF THE STOKES-STRAIK APPROXIMATION FOR WAVES OF FINITE HEIGHT

APPENDIX II
TABLE I CONTINUED

(Degrees) (Radians)	θ	Sin θ	Cos θ	Sm 2 θ	Cos 2 θ	Sin 3 θ	Cos 3 θ	θ	Sin θ	Cos θ	Sin 2 θ	Cos 2 θ	Sin 3 θ	Cos 3 θ
241	42062	8746-	46848	8480	5299-	0523	9986	301	52534	8572-	3150	8828-	0523	9986-
242	42237	8829	46493	8580	5299-	1043	9987	302	52709	8450-	3299	8828-	1043	9986-
243	42412	8910	45843	8090	5878	1564	9877	303	52883	8387-	3446	8713-	1564	9877-
244	42586	8988	45384	7880	6157	2079	9781	304	53058	8300-	3592	8546-	2079	9811-
245	42761	9063	44228	7660	6428	2598	9699	305	53233	8192-	3738	8376-	2598	9859-
246	42935	9135	43067	7431	6691	3117	9611	306	53407	8080-	3883	8200-	3117	9911-
247	43110	9205	42000	7193	6947	3640	9518	307	53582	7966-	4027	8020-	3640	9961-
248	43284	9272	41036	6947	7193	4067	9419	308	53756	7849-	4170	7849-	4067	9911-
249	43459	9336	40284	6691	7431	4487	9315	309	53930	7729-	4312	7729-	4487	9861-
250	43633	9397	39640	6428	7660	4907	9206	310	54105	7606-	4453	7606-	4907	9811-
251	43808	9455	39099	6157	7880	5326	9092	311	54280	7481-	4594	7481-	5326	9761-
252	43982	9511	38653	5878	8090	5744	8973	312	54454	7354-	4734	7354-	5744	9711-
253	44157	9563	38300	5592	8290	6162	8849	313	54629	7225-	4873	7225-	6162	9661-
254	44331	9613	38031	5299	8480	6579	8721	314	54803	7094-	5011	7094-	6579	9611-
255	44506	9659	37848	5000	8660	7000	8586	315	54977	6961-	5148	6961-	7000	9561-
256	44680	9703	37754	4695	8829	7413	8436	316	55152	6827-	5284	6827-	7413	9511-
257	44855	9744	37744	4384	8988	7721	8271	317	55326	6692-	5420	6692-	7721	9461-
258	45029	9781	37816	4067	9135	8020	8098	318	55501	6556-	5555	6556-	8020	9411-
259	45204	9816	37960	3744	9272	8268	7946	319	55676	6419-	5689	6419-	8020	9361-
260	45379	9848	38176	3420	9397	8500	7851	320	55851	6281-	5821	6281-	8020	9311-
261	45553	9877	38450	3090	9511	8713	7746	321	56026	6142-	5952	6142-	8020	9261-
262	45728	9903	38782	2756	9613	8903	7631	322	56201	6002-	6082	6002-	8020	9211-
263	45902	9925	39176	2419	9703	9079	7506	323	56376	5861-	6211	5861-	8020	9161-
264	46077	9945	39620	2079	9781	9251	7380	324	56551	5719-	6339	5719-	8020	9111-
265	46251	9962	40116	1736	9848	9419	7254	325	56726	5576-	6466	5576-	8020	9061-
266	46426	9976	40660	1392	9903	9579	7129	326	56901	5432-	6592	5432-	8020	9011-
267	46600	9986	41254	1045	9945	9721	7004	327	57076	5287-	6717	5287-	8020	8961-
268	46775	9994	41898	0698	9976	9849	6879	328	57251	5141-	6842	5141-	8020	8911-
269	46949	9998	42592	0349	9984	9946	6754	329	57426	5000	6966	5000	9984	8861-
270	47124	10000	0	0	10000	10000	0	330	57596	5000	6966	5000	10000	0
271	47298	9995	0175	0349	9994	9986	0523	331	57770	4848-	7099	4848-	9986	0523
272	47473	9984	0349	0698	9976	9945	1045	332	57945	4695-	7240	4695-	9945	1045
273	47647	9956	0523	1043	9945	9277	1564	333	58119	4540-	7379	4540-	9277	1564
274	47822	9906	0698	1392	9903	8781	2079	334	58294	4384-	7516	4384-	8781	2079
275	47997	9842	0872	1736	9848	8268	2598	335	58469	4226-	7651	4226-	8268	2598
276	48171	9845	1045	2079	9781	9511	3090	336	58643	4067-	7784	4067-	9511	3090
277	48346	9823	1219	2419	9703	9336	3584	337	58818	3907-	7915	3907-	9336	3584
278	48520	9800	1392	2756	9613	9135	4067	338	58992	3746-	8044	3746-	9135	4067
279	48695	9877	1564	3090	9511	8910	4540	339	59167	3584-	8177	3584-	8910	4540
280	48869	9848	1736	3420	9397	8660	5000	340	59341	3420-	8308	3420-	8660	5000
281	49044	9816	1908	3744	9272	8387	5446	341	59516	3256-	8436	3256-	8387	5446
282	49218	9791	2079	4067	9135	8098	5821	342	59690	3090-	8561	3090-	8098	5821
283	49393	9761	2250	4384	8988	7711	6213	343	59865	2924-	8683	2924-	7711	6213
284	49567	9723	2420	4695	8829	7431	6579	344	60039	2756-	8803	2756-	7431	6579
285	49742	9679	2592	5000	8660	7000	6946	345	60214	2587-	8919	2587-	7000	6946
286	49916	9633	2764	5299	8480	6579	7321	346	60388	2419-	9032	2419-	6579	7321
287	50091	9583	2924	5592	8290	6162	7694	347	60563	2250-	9142	2250-	6162	7694
288	50265	9531	3090	5878	8090	5744	8069	348	60737	2079-	9249	2079-	5744	8069
289	50440	9475	3256	6157	7880	5326	8446	349	60912	1908-	9353	1908-	5326	8446
290	50615	9415	3420	6428	7660	5000	8821	350	61087	1736-	9454	1736-	5000	8821
291	50789	9356	3584	6691	7431	4594	9190	351	61261	1564-	9552	1564-	4594	9190
292	50964	9292	3746	6947	7193	4067	9564	352	61436	1392-	9647	1392-	4067	9564
293	51138	9225	3907	7193	6947	3584	9936	353	61610	1219-	9739	1219-	3584	9936
294	51313	9155	4067	7431	6691	3090	10300	354	61785	1045-	9828	1045-	3090	10300
295	51487	9063	4226	7660	6428	2598	10629	355	61959	0872-	9913	0872-	2598	10629
296	51662	8958	4384	7880	6157	2079	10958	356	62134	0698-	9994	0698-	2079	10958
297	51836	8850	4540	8090	5878	1564	11287	357	62308	0523-	10000	0523-	1564	11287
298	52011	8829	4695	8290	5592	1045	11616	358	62483	0349-	9998	0349-	1045	11616
299	52185	8746	4848	8480	5299	0523	11945	359	62657	0175-	9994	0175-	0523	11945
300	52360	8660	5000	8660	5000	0	12000	360	62832	0	9998	0	0	12000

Vertical lines indicate the decimal point position

COASTAL ENGINEERING

APPENDIX II

TABLE 2. HYPERBOLIC SINES & COSINES FOR INTEGRAL MULTIPLES OF $\frac{\pi}{L}$

$\frac{x}{L}$	$\text{Sinh } 2\pi\frac{x}{L}$	$\text{Cosh } 2\pi\frac{x}{L}$	$\text{Sinh } 4\pi\frac{x}{L}$	$\text{Cosh } 4\pi\frac{x}{L}$	$\text{Sinh } 6\pi\frac{x}{L}$	$\text{Cosh } 6\pi\frac{x}{L}$
0	0	1.000000	0	1.000000	0	1.000000
0.010000	0.062873200	1.019746	0.12599470	1.0079061	0.18961375	1.0178180
0.020000	0.12599470	1.0079061	0.25398165	1.0317494	0.38598460	1.0719068
0.030000	0.18961375	1.0178180	0.38598460	1.0719068	0.59611035	1.1641940
0.040000	0.25398165	1.0317494	0.52409080	1.1290134	0.82477900	1.2979682
0.050000	0.31935255	1.0497553	0.67048400	1.2039721	1.0883357	1.4779962
0.060000	0.38598460	1.0719068	0.82747905	1.2979683	1.3897762	1.7106952
0.070000	0.45414095	1.0982915	0.99755825	1.4124881	1.7370783	2.0043555
0.080000	0.52409075	1.1290134	1.1834111	1.5493424	2.1480830	2.3694431
0.090000	0.59611035	1.1641940	1.3797762	1.7106952	2.6356363	2.8189777
0.100000	0.67048400	1.2039721	1.6144881	1.8990976	3.2171151	3.3689489
0.110000	0.74790000	1.2495049	1.89665236	2.1175290	3.9132348	4.0389380
0.120000	0.82747900	1.2979682	2.1480830	2.3694431	4.7488076	4.8529551
0.130000	0.91072030	1.3525574	2.4636031	2.6588231	5.7536090	5.8398640
0.140000	0.99755820	1.4124880	2.8180782	2.9902449	6.9634450	7.0348820
0.150000	1.0883357	1.4779968	3.2171134	3.3689492	8.4214295	8.4805945
0.160000	1.1834110	1.5493423	3.6701719	3.8009236	1.0179521	1.0228521
0.170000	1.2831599	1.6268065	4.1749059	4.2929989	1.2300367	1.2300367
0.180000	1.3879762	1.7106952	4.7488091	4.8529556	1.4859548	1.4859548
0.190000	1.4982736	1.8013394	5.3977990	5.4896480	1.7948262	1.7948262
0.200000	1.6144881	1.8990976	6.1321410	6.2131440	2.1676580	2.1676580
0.210000	1.7370783	2.0043555	6.9634455	7.0348825	2.6177362	2.6177362
0.220000	1.8665286	2.1175290	7.9048570	7.9678580	3.1610996	3.1610996
0.230000	2.0033500	2.2390648	8.9712615	9.0268725	3.8171119	3.8171119
0.240000	2.1490830	2.3694431	10.179521	1.0228521	4.6091501	4.6091501
0.250000	2.3040000	2.5091785	11.620571	1.1591955	5.5654400	5.5654400
0.260000	2.4686201	2.6588231	13.30571	1.3138682	6.7200590	6.7200590
0.270000	2.6436363	2.8189677	14.859550	1.4893160	8.1141535	8.1141535
0.280000	2.8290782	2.9907449	16.853490	1.683131	9.7674035	9.7674035
0.290000	3.0111488	3.1733393	19.113920	1.9140061	11.729792	11.729792
0.300000	3.2117131	3.3689499	21.676592	2.1696336	14.283747	14.283747
0.310000	3.4352819	3.5778711	24.511097	2.4602329	17.446716	17.446716
0.320000	3.6701716	3.8009233	27.676107	2.7894038	20.824285	20.824285
0.330000	3.912343	4.0379854	31.610999	3.1626812	25.143949	25.143949
0.340000	4.1749055	4.2929985	35.845728	3.5859674	30.396338	30.396338
0.350000	4.4530639	4.5639652	40.647261	4.0659560	36.657222	36.657222
0.360000	4.7488076	4.8529551	46.091510	4.6012357	44.261121	44.261121
0.370000	5.0633055	5.1611105	52.264565	5.2274135	53.442300	53.442300
0.380000	5.3977985	5.4896475	59.264035	5.9272475	64.527945	64.527945
0.390000	5.7536090	5.8398640	67.200600	6.7208040	77.913105	77.913105
0.400000	6.1321410	6.2131440	76.199750	7.6206310	94.074765	94.074765
0.410000	6.5348890	6.6109590	86.403780	8.6409570	113.98887	113.98887
0.420000	6.9634450	7.0348820	97.74045	9.79279145	137.15081	137.15081
0.430000	7.4195005	7.4865875	111.09349	11.109799	165.06025	165.06025
0.440000	7.9048560	7.9678570	125.96956	12.597353	199.95099	199.95099
0.450000	8.4214295	8.4805945	142.83749	14.283749	241.42718	241.42718
0.460000	8.9712605	9.0268215	161.96308	16.196707	291.90685	291.90685
0.470000	9.5652055	9.6086985	183.65148	18.365420	351.97460	351.97460
0.480000	10.179520	10.228520	208.24290	20.824290	424.98532	424.98532
0.490000	10.842720	10.888736	236.12708	23.612708	513.14075	513.14075
0.500000	11.548740	11.591954	267.74495	26.774682	619.98240	619.98240
0.510000	12.300367	12.340949	303.59644	30.359809	748.10350	748.10350
0.520000	13.100570	13.138661	344.24944	34.424989	903.26385	903.26385
0.530000	13.952508	13.988298	390.34376	39.034504	1090.536	1090.536
0.540000	14.859548	14.893158	442.61125	44.261238	1316.8994	1316.8994
0.550000	15.825269	15.856833	501.87745	50.187845	1590.0548	1590.0548
0.560000	16.853488	16.883129	569.09020	56.908020	1918.8815	1918.8815
0.570000	17.948262	17.976098	645.27960	64.528040	2318.1268	2318.1268
0.580000	19.113918	19.140059	731.68315	73.168385	2798.9783	2798.9783
0.590000	20.355055	20.379604	829.65115	82.965115	3379.5766	3379.5766
0.600000	21.676580	21.699634	940.74785	94.074835	4080.0098	4080.0098
0.610000	23.083706	23.105356	1066.71148	106.671153	4927.0549	4927.0549
0.620000	24.581995	24.602327	1209.5488	120.95488	5949.0650	5949.0650
0.630000	26.177359	26.196453	1371.5083	137.15087	7183.1100	7183.1100
0.640000	27.876105	27.894036	1555.1544	155.51547	8673.1190	8673.1190
0.650000	29.684934	29.701773	1763.3910	176.33913	10472.193	10472.193
0.660000	31.610996	31.626809	1999.5105	199.95108	12644.464	12644.464
0.670000	33.661894	33.676744	2267.2465	226.72467	15267.333	15267.333
0.680000	35.834525	35.859671	2570.8326	257.08328	18434.252	18434.252
0.690000	38.171119	38.184216	2915.0691	291.50693	22258.110	22258.110
0.700000	40.67253	40.689552	3305.9991	330.59993	26875.133	26875.133
0.710000	43.33912	43.295462	3747.9946	374.79947	32449.902	32449.902
0.720000	46.191501	46.12348	4249.8540	424.98541	39181.020	39181.020
0.730000	49.211116	49.091302	4818.9128	481.89129	47308.427	47308.427
0.740000	52.462555	52.274125	5464.1690	546.41690	57121.665	57121.665
0.750000	55.954400	55.663380	6198.8260	619.88260	68970.940	68970.940
0.760000	59.684020	59.272460	7025.4520	702.54520	83277.255	83277.255
0.770000	63.670700	63.115620	7966.1655	796.61655	100551.94	100551.94
0.780000	67.920585	67.208025	9032.8410	903.28410	121409.18	121409.18
0.790000	72.458865	71.568555	10242.346	1024.2346	146593.20	146593.20
0.800000	77.299735	76.206295	11613.806	1161.3806	177001.37	177001.37
0.810000	82.41535	81.147695	13168.907	1316.8907	21371.695	21371.695
0.820000	87.803760	86.409550	14932.224	1493.2224	25804.873	25804.873
0.830000	93.470725	92.012655	16931.665	1693.1665	31157.605	31157.605
0.840000	99.427405	97.79135	19198.834	1919.8834	37620.697	37620.697
0.850000	105.67775	104.3252	21769.557	2176.9557	45424.443	45424.443
0.860000	112.23348	110.9798	24684.525	2468.4525	54848.880	54848.880
0.870000	119.09948	118.0215	27989.810	2798.9810	66223.895	66223.895
0.880000	126.28955	124.9752	31737.646	3173.7646	79960.785	79960.785
0.890000	134.81363	134.4236	35987.851	3598.7851	96547.230	96547.230
0.900000	143.73747	142.84097	40806.098	4080.6098	116574.13	116574.13
0.910000	153.00037	152.01366	46270.080	4627.0080	140755.34	140755.34
0.920000	162.63995	161.69704	52465.645	5246.5645	169952.56	169952.56
0.930000	172.6714	171.74004	59490.850	5949.0850	205206.00	205206.00
0.940000	183.1144	182.5146	67456.740	6745.6740	247772.34	247772.34
0.950000	194.0699	193.56355	76489.195	7648.9195	299168.02	299168.02
0.960000	206.54285	206.24525	86731.190	8673.1190	361225.11	361225.11
0.970000	220.74708	221.74933	98344.590	9834.4590	436154.42	436154.42
0.980000	236.82704	236.12916	111513.04	11151.304	526626.90	526626.90
0.990000	254.83947	254.4146	126444.64	12644.464	635865.70	635865.70
1.000000	274.74489	276.74676	143375.71	14337.571	767764.70	767764.70

Vertical lines indicate the decimal point position