Chapter 8

SEICHE IN PORTS

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In his thesis Prof. John S. McNown has given an account of experiments on seiche in port models. Though the fundamental causes of seiche in harbours and ports are more or less known, we have little knowledge of the harbour dimensions for which a particular seiche is likely to occur. A systematic study was therefore initiated by Prof. McNown and our work described in this paper may be considered as its continuation.

We shall recall briefly the essential elements of Prof. McNown's work.

He considered port models with a horizontal bottom and vertical walls ; the ports had a narrow entrance through which waves coming from the sea penetrated. The ports were of geometrically idealized forms - square and circle - and were placed inside a wave-basin, also with a horizontal bottom (fig. 1). Outside the port and in the vicinity of the entrance, fine gravel was deposited forming an absorbing beach to avoid reflections. The incoming wave from the sea was normal to the pass.

In this model Prof. McNown sought to describe both experimentally and theoretically, the agitation inside the port as a function of the conditions in the open sea outside. He considered a regular train of waves coming towards the entrance of the port and he made the simplyfying hypothesis that the wave-crest continues to be uniform at the section of entrance. But because of the reflection inside the port a clapotis is formed in the immediate vicinity of the entrance.

Two types of motion were observed inside the port. In the first, called the resonant motion, the period of the generating wave coincides with a characteristic period of the port regarded as a closed basin. In this case,

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As might be expected, there were no appreciable normal velocities at the entrance section. In the second type of motion, the non-resonant, the normal velocity at the entrance is not zero and forms an integral part of the internal motion. McNown was able to realize experimentally certain characteristic resonant movements. On the basis of the hypothesis of clapotis mentioned in the preceeding paragraph he postulated that in every case when the period of the sea-wave is equal to a characteristic period of the port, resonant motion ensues.

THEORY OF THE PHENOMENA

Profs. Kravtchenko and McNown have given a theoretical account of this phenomenon. With axes of reference Oxyz - axis Oz vertically upwards and Oxy in the plane of mean water level inside the port - the velocity potential of motion in the port can be written as :

$$\phi(x,y,z,t) = A \sin\frac{2\pi}{T} t \; \frac{ch\{k(z+h)\}}{ch\,kh} F(x,y), \tag{1}$$

where T is the period of the generating wave, h is the depth of water, A a constant and F (x,y) satisfies the differential equation :

$$\Delta F + k^2 F = 0 \; ; \tag{2}$$

k is related to the period by the equation :

$$\left(\frac{2\pi}{T}\right)^2 = g\,k\,th\,(kh). \tag{3}$$

In addition to (2), F satisfies the condition :

$$\frac{dF}{dn} = 0 \; , \tag{4}$$

along the vertical walls of the port, where n is the direction of the normal to the boundary of the port. Along the part of the boundary occupied by the entrance we write, on the basis of the clapotis :

$$\frac{dF}{dn} = B \; , \tag{5}$$

B being a constant. Evidently we may generalize the problem by taking the right-hand side of (5) to represent a variable function instead of a constant. Such a generalized problem presents almost insuparable difficulties both for the theoretical solution and its experimental realization.

The constant B in equation (5) is determined by the clapotis outside the port ; the equation (5) expresses the fact that at the entrance the normal velocity depends only on the coordinate z. A resonant movement is a consequence of zero normal velocity, i.e. B = o. In the general case, the value of B will have to be determined experimentally.

LIMITATION OF McNOWN'S EXPERIMENTS

The hypothesis of clapotis was found to be in good accord with the experiments performed by McNown. However there are limitations to both the theoretical and physical aspects of these experiments which were recognized by Prof. McNown himself.

For the hypothesis of clapotis to be approximately correct, the width of the entrance to the port should be small ; this will become evident if one considers the profile of the water-surface at the entrance. If the level of the water-surface across the entrance is to follow that of a clapotis outside the port, the water surface will be approximately a horizontal straight line. Such would definitely not be the case if, for instance, a nodal line of the resonant motion inside the port were to end on the entrance. Nor must a nodal line terminate at a point near the section of entrance. It is interesting to see that in the cases of resonant motion examined experimentally by McNown, the nodal lines were considerably away from the entrance section.

There is yet another difficulty in the formulation of the theory, where the hypothesis of a clapotis, so attractively simple as it is, leads to a contradiction. The potential function for the region occupied by the port which is the potential of a resonant motion and the potential function outside the port which represents a clapotis do not represent the same function, which they should, since the wave-basin and the port are parts of a region described by the analytic continuation of a harmonic function.

Profs. Kravtchenko and McNown have recognized this difficulty and have remarked that the theory can be applied as a first approximation only to ports having narrow entrances. Under such circumstances the deviation of the free surface profile from a horizontal straight line can be neglected in the section of entrance. We have already seen, however, that this approximation even ceases to be valid when there is a nodal line terminating on the entrance.

Table I. Period in seconds.

n \ m	0	1	2	3	4	5	6	7	8
0				1.319	1.063	0.916	0.819	0.750	0.698
1			1.718	1.277	1.046	0.907	0.817	0.747	0.695
2			1.451	1.176	1.0	0.882	0.799	0.738	0.689
3	1.460	1.382	1.220	1.063	0.940	0.848	0.778	0.723	0.679
4	1.164	1.131	1.053	0.962	0.879	0.810	0.752	0.705	
5	0.994	0.977	0.934	0.879	0.823	0.771	0.725		
6	0.883	0.874	0.848	0.812	0.773	0.734			
7	0.805	0.799	0.783	0.758	0.730				
8	0.747	0.734	0.731	0.714					

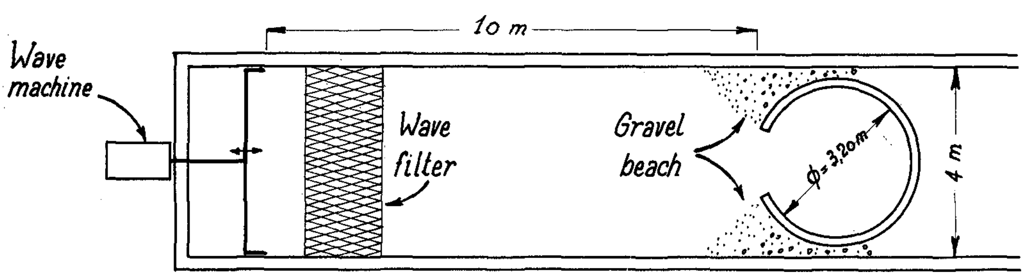

Fig. 1. Port model used by Professor McNown.

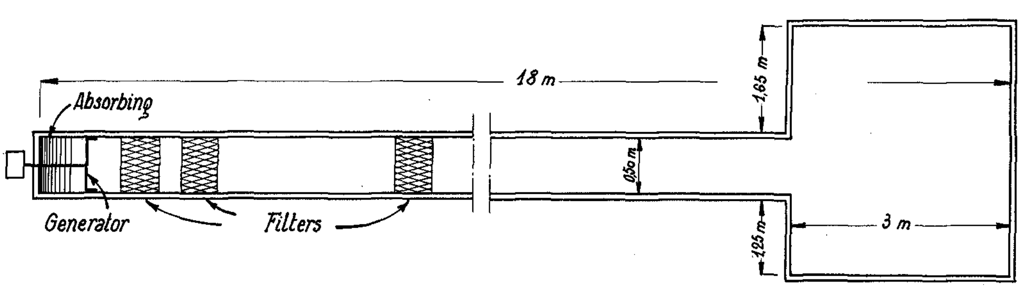

Fig. 2. Port model at the hydraulics laboratory Institut Polytechnique, Grenoble.

It appeared to us that these were the reasons at the origin of certain anomalies noted by Prof. McNown. Considering the practical importance of seiche in the theory of harbour design, it was thought desirable to extend the results of Prof. McNown's work, eliminating at the same time the rather restrictive hypotheses he made.

It is the object of this article to describe the experiments and the theoretical analysis carried out starting from McNown's initial experiments. Our researches are not yet complete, though, we believe that the discrepancies observed by Prof. McNown have been explained and in part removed. We have attempted to give here the present state of our investigation ; for simplicity the following exposition will be restricted exclusively to resonant movements in ports.

We may state the general conclusions of our experiments : As a first approximation the theory developed by McNown gives an excellent interpretation of the phenomenon observed in the laboratory for a large band of periods, with the reservation that the width of the entrance is small with respect to the dimensions of the port. The theory needs modification when the latter restriction is no longer valid or when a nodal line of the resonant movement terminates on the entrance. In either case the existence of a resonant movement is not affected, only the hypothesis of the clapotis is no longer applicable.

DESCRIPTION OF THE MODEL : EXPERIMENTAL RESULTS

We used a rectangular port of width 3.40 m, length 3 m ; the depth could be varied from 30 cm to 40 cm. There was, however, one important difference in our model from the one studied by Prof. McNown : whereas his model was placed in a wave-basin, ours was situated at the end of a wave-canal (fig.2). We chose rather artificial conditions, since it was our objective to have as pure an incident wave as possible. The canal was not symmetric with respect to the port ; however this does not cause appreciable dissymmetry in the agitation observed inside the port. At the other end of the canal, at a distance of 15 m from the port, was placed the wave generator.

With this arrangement a wide range of frequencies was investigated for resonant movements. For all characteristic periods a seiche was observed in the port, excepting a very small number of cases, the reasons of which are still under study. In their article referred to below, Profs. Kravtchenko and McNown have obtained an analytical condition which allows a solution more general than a simple resonant movement, when the corresponding period is a characteristic period of the resonant movement. The condition has not yet been verified experimentally.

For resonant movements the free-surface inside the port is given by the expression :

$$\eta = A \cos \frac{m\pi x}{a} \cos \frac{m\pi y}{b} , \quad (6)$$

where a and b are used to denote the lengths of sides of the rectangular port, and m, n denote the number of nodal lines parallel to each side.

The characteristic values of k in equation (2) are given by the formula :

$$k_{m,n} = \pi \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^{1/2} . \quad (7)$$

The depth of water in our experiments was kept at 30 cm. The table at the end gives the characteristic periods investigated ; the period $T_{m,n}$ was calculated by means of equations (3) and (7) on substituting h = 0.3.

In the entrance of the port the amplitude of oscillation of the water surface is known from (6) :

$$\eta_{m,n}(x,o) = A \cos \frac{m\pi x}{a} , \quad (8)$$

where x varies between x_1, x_2 the abscissae of the extremities of the entrance. Thus if m is small and if a zero of $\cos m\pi x/a$ does not fall between x_1, x_2 we may approximate the profile by a straight line. This presupposes naturally that the width of the entrance is small as compared to the length of the side of the port. More precisely, McNown's hypothesis is valid when the section of entrance contains a maximum (and a minimum) value of the function (8).

When m is large, $\cos m\pi x/a$ in (8) has one or more zeros between x_1, x_2, denoting so many nodal lines terminating on the section of entrance. Even then we have observed, however, that the resonant phenomenon persists ; in fact the amplitude of oscillation inside the port being much greater than that in the wave-canal, the movement in the latter is altered considerably at least for some distance (4 m. approximately) from the section of entrance.

Let us cite, for illustration, a few resonant cases from the Table of periods. For the movement denoted by m = 5, n = o corresponding to a period 0.916 second, there were five nodal lines parallel to the section of entrance ; while at the section we had a perfect clapotis. As against this we observed the movement m = 5, n = 1 (period = 0.907 s), which differs from the previous in that it has a nodal line right on the entrance section. Similar results were observed for the cases (4,1), (1,4) and (1,5), the first of the two numbers in the brackets denotes the number of nodal lines parallel to the entrance of the port and the second denotes the number of nodal lines parallel to the length of the wave-canal.

We were able to observe most of the cases corresponding to the characteristic periods listed. It must not, however, be assumed that the water-level at the positions corresponding to nodal lines was absolutely stationary. Evidently in some cases they had appreciable movement, but it was small compared with the maximum amplitude.

The amplitudes of resonant movement inside the port were very often double that of the incident clapotis. In some cases the ratio of amplitudes was as much as three. This should give an idea of the importance of seiche provoked by a wave of small amplitude. The amplitudes of oscillation inside the port were sometimes as much as 10 cm, i.e. one-third of the depth of water in the port.

THEORETICAL PRESENTATION

We have formulated a theoretical analysis of the phenomena to take account of the complex agitation produced in the wave-canal in the vicinity of the port entrance.

The potential ϕ_2 describing the motion in the wave canal will be taken as the analytic continuation of the potential function ϕ_1 of the resonant motion inside the port. We shall suppose as the boundary condition that at a great distance in the canal from the port the potential ϕ_2 reduces to the potential function of a clapotis in a depth h :

$$\Phi = C \sin \frac{2\pi}{T} t \; \frac{\cosh[k(z+h)]}{\cosh kh} \cos ky \qquad (9)$$

As in equation (1), the potentials ϕ_1 and ϕ_2 define two functions F_1 and F_2, both satisfying the differential equation (2). The function F_1 satisfies the boundary condition (4) along the vertical walls of the port; in place of (5) we have the new conditions :

$$\frac{dF_2}{dn} = \frac{dF_1}{dn} \qquad \text{for} \quad y = 0 , \quad x_1 < x < x_2 , \qquad (10)$$

along the entrance section. Equation (10) expresses the continuity of the normal derivative, which, however, is small since we have in the port principally a resonant movement. Again the equality of water levels at the common section gives us :

$$F_2 = F_1 \quad \text{for } y = 0 \qquad x_1 \leq x \leq x_2 \tag{11}$$

since the water-levels are proportional to F_2 and F_1 in the wave-canal and in the port respectively. The function F_1 is given by the sum of (6) and an expression corresponding to the perturbation in the port :

$$F_1 = \sum_m A_m \cos\frac{m\pi x}{a} \cos\left\{\left(k^2 - \frac{m^2\pi^2}{a^2}\right)^{1/2} (y-b)\right\} \tag{12}$$

The coefficients A_m corresponding to values of m other than that for the resonant movement are small. F_2 can be similarly written as a Fourier series :

$$F_2 = \sum_0^{i<pk/\pi} C_i \cos\frac{i\pi}{p}(x-x_1)\cos\left\{\sqrt{k^2 - \frac{i^2\pi^2}{p^2}}\; y + t_i\right\} + \sum_{i<pk/\pi} C_i \cos\frac{i\pi}{p}(x-x_1)\, e^{\sqrt{i^2\pi^2/p^2 - k^2}\,y} \tag{13}$$

where for brevity we have written $p = x_2 - x_1$; the constants C_i, E_i and A_i can be determined with a sufficient approximation by means of the conditions (10) and (11). Obviously in the canal the principal term corresponding to the generating clapotis is predominant. This term corresponds to i = o in expression (1 3). The importance of other terms depends on the perturbation introduced in the port as also from the modifications of the clapotis in the canal.

Detailed results of calculations for the resonant and non-resonant cases will be published later along with a comparison of experimental data.

We may note that the form of the potential function F_2 explains theoretically the transversal phenomena observed in the canal in the vicinity of the entrance. This phenomenon is provoked by the reaction of the movement inside the port and attenuates as we go farther from the section of entrance.

CONCLUSION

We may note the practical, theoretical and experimental aspects of the study. As regards the practical nature of the experiments, it is unnecessary to emphasize the importance of seiche in port construction. The study is hoped to be of use in model technique and particularly for the effects of scale-distortion.

To resume the theoretical and experimental aspects : The theory of Professors Kravtchenko and McNown is applicable when the profile of the water surface at the entrance of the port can be approximated by a horizontal straight line. This condition, however, limits on the one hand the width of the entrance section and on the other hand, it excludes resonant movements with nodal lines terminating on the entrance.

In general the law of resonance holds when the period of the sea wave coincides with a characteristic period of the port basin. We have extended the range of frequencies so as to remove the restriction on nodal lines mentioned above. The theoretical problem reduces to solving the Neumann problem for the differential equation $\Delta F + k^2 F = 0$, for a polygonal contour.

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RESUME

OSCILLATIONS À PORTUAIRES

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Dans sa thèse, J. Mc Nown a étudié les oscillations de la surface libre des eaux portuaires sous l'action de la houle venant du large. Nous rappelons que l'étude a été poursuivie sur un modèle réduit et idéalisé ; l'ouvrage était à fond horizontal et à parois verticales percées d'une passe étroite à bords verticaux. Le modèle était placé dans un canal à houle beaucoup plus large, à fond horizontal. Des plages d'amortissement étaient disposées à l'amont du modèle. Rappelons les conclusions essentielles du travail de Mc Nown.

On observe dans le port un mouvement résonnant chaque fois que la période de la houle génératrice coïncide avec celle d'une seiche propre du port, supposé fermé. Le mouvement des eaux à l'intérieur du port peut toujours être déterminé à priori si l'on connaît le mouvement des eaux à l'extérieur du port et dans le voisinage immédiat de la passe. Ce dernier mouvement se réduit à un clapotis placé dans le voisinage de l'entrée.

Une théorie, dûe à J. Kravtchenko et J. Mc Nown permet, à partir de ces lois, de calculer le phénomène dans les cas particuliers où le contour du port présente des formes simples (cercle, rectangle, etc...). Toutefois, Mc Nown a signalé des régimes erratiques où sa thèse est en défaut. Nous avons repris la question en opérant sur un modèle de port rectangulaire excité par la houle qui se propage dans un canal débouchant directement dans le port et de largeur égale à celle de la passe. On s'écarte ainsi des conditions naturelles ; mais le phénomène devient plus pur.

Nos principales conclusions peuvent être, à titre provisoire, résumées comme il suit. La loi de résonnance énoncée par Mc Nown, demeure exacte dans tous les cas que nous avons observés. Mais la loi du clapotis à l'entrée ne s'est trouvée justifiée pour les périodes résonnantes que si aucune nodale de la seiche propre du port n'aboutit à la passe.

La détermination de la surface libre des eaux portuaires exige alors le recours à d'autres règles plus compliquées.

Par contre, dans le cas non résonnants, l'hypothèse du clapotis se trouve être bien vérifiée en première approximation. Les méthodes de calcul de Kravtchenko et Mc Nown s'appliquent intégralement ; nous indiquons quelques variantes simplificatrices de leur processus opératoire et nous comparons nos prévisions theoriques avec les mesures faites en laboratoire.