Chapter 10

TWO-DIMENSIONAL SEICHE IN A BASIN SUBJECTED TO INCIDENT WAVES

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I - GENERAL

A - Foreward -

Seiche movements in lakes have for a long time been submitted to the calculations of the most competent scientists .

Studies of seiche in open basins subject to the action of ocean waves are not so numerous and above all they are less rigorous. However the studies of Lord RAYLEIGH, HONDA, TERADA, YOSHIDA, ISITANI[1] and HANSEN can be mentioned, and the important work by PROUDMAN on tide movements (which often have a similar character to seiches)[2], the studies of NEUMANN introducing the notion of hydraulic impedance[3], and those of LAMOËN on the theory of estuaries[4] etc.

More recently MC NOWN made theoretical and experimental studies, at the Laboratoire Dauphinois d'Hydraulique, for circular and square harbours[5].

Formulae have been known for a long time giving the resonance period of long basins : basins which are sufficiently long for the oscillations to be considered only in the direction of the basin's length. By analogy with sound tubes two cases have been considered : that of basins completely closed and that of basins completely open[6].

(1)	Bibliography	5	(4)	Bibliography	6
(2)	"	10	(5)	"	8
(3)	"	9	(6)	"	12

Then :

$$T = \frac{2}{k+1} \int_0^d \frac{dx}{\sqrt{gh(x)}} \quad \text{for closed basins (fig. 1-a)}$$

$$T = \frac{4}{2k+1} \int_0^d \frac{dx}{\sqrt{gh(x)}} \quad \text{for open basins (fig. 1-b)}$$

k being a whole number characterising the harmonic

d being the length of the basin in the direction of the movement on the mediant axis of the free surface

h (x) being the mean depth in a plane perpendicular to the movement of the x abscisse.

These formulae suppose that wave reflections are possible only at the ends of the basin (with or without change of sign according to whether the reflection is from a wall or from a sudden widening or deepening). The transverse sections must therefore vary gradually.

From these formulae it can be seen that a closed basin contains (k + 1) times a half wave length whereas an open basin contains $(\frac{2k+1}{2})$ times this same quantity. It may be said, by acoustic analogy, that a closed basin resonates in 1/2 wave lengths and an open basin in 1/4 wave lengths.

The present note attempts to clarify these notions and to establish resonance laws as a function of the type and the width of the entry (fig. 1-c). The currents in the neighbourhood of the entrance, although locally in three dimensions, very rapidly approach a plane movement when the width of the basin is less than half wave length. JEFFREYS waves are superposed on the plane movement for greater widths.

The theory has been established and tests have been carried out for a rectangular basin of constant depth. The results found and the laws arrived at however, present a more general character. By means of numerical calculations they can be applied, for a harbour with gradually varying sections, to all movements which occur along the axis of the entrance.

First we will define the different types of basin which can occur. The definitions given will be used throughout this study.

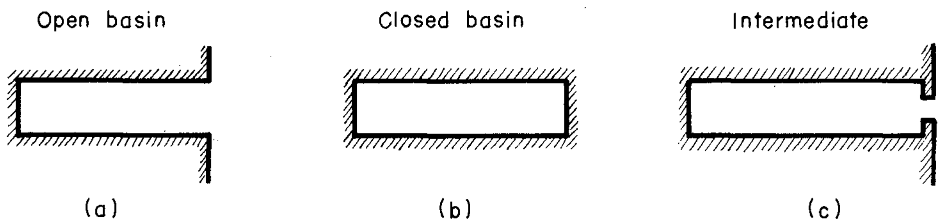

Fig. 1.

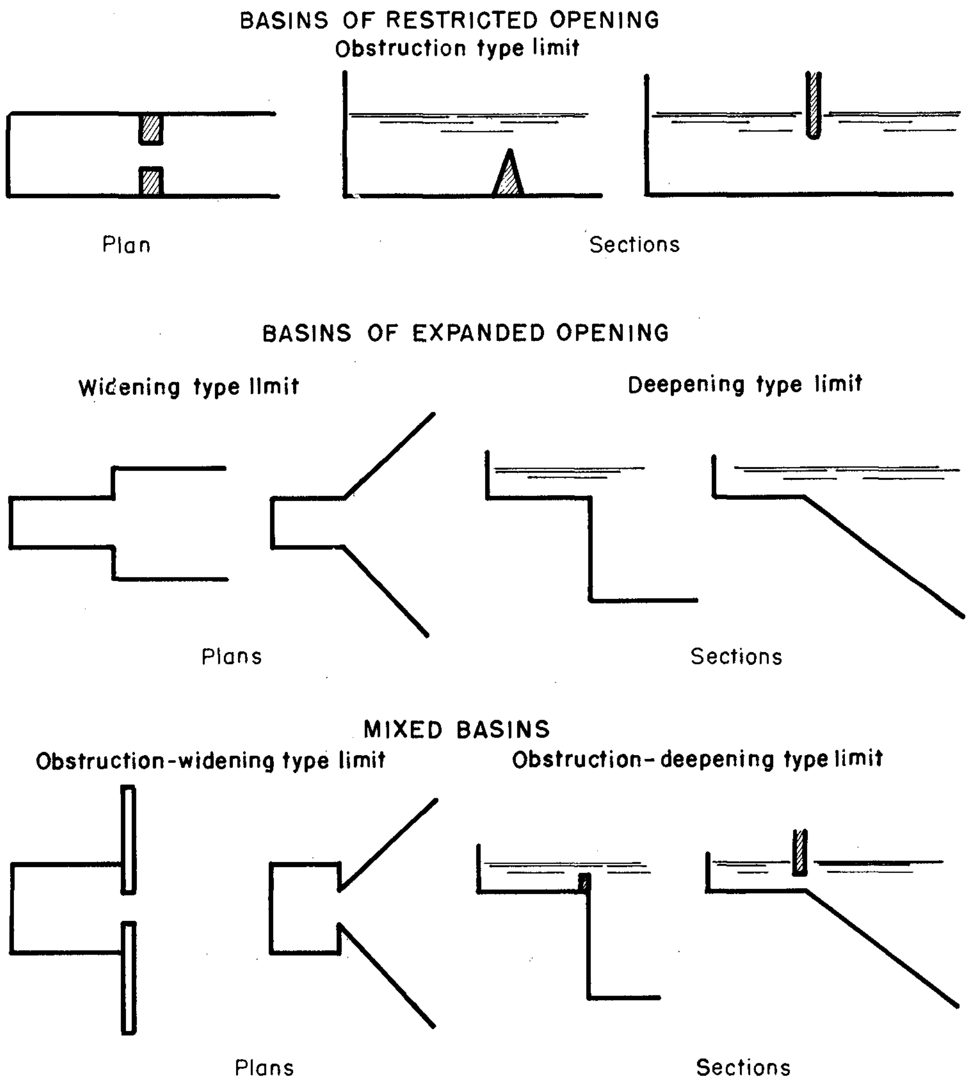

Fig. 2. Different types of basins.

B - Different types of basin -

The definitions given below may seen arbitrary. But in fact they have been dictated by the study of obstacles subjected to ordinary waves, a study whose theoretical results apply directly to the study of seiches (figure 2).

1 - A basin is said to have a "restricted opening" when it is limited by an "obstruction" and it opens into a canal of the same width and depth. The "Obstruction" will cause a restriction of the wave passage. It will be essentially an object placed in the wave passage.

2 - A basin is said to have an "expanded ppening" when it is limited :

a) by a *"widening"* characterised by a constant depth and a sudden increase in section towards the ocean.

b) by a *"deepening"* characterised by a constant width and a sudden increase in depth towards the ocean.

A simple "obstacle" is formed by one only of the above characteristics. A "complex" obstacle consists of several "simple" obstacles brought together in a short distance (generally a distance of the order of magnitude of the depth of the basin). "Complex" obstacles can be considered in 4 categories :

- widening + deepening
- obstruction + deepening
- obstruction + widening
- deepening + widening + obstruction

We shall say that there is resonance when the agitation in the basin attains its maximum for a constant incident amplitude. This agitati· is a function of the period of the exciting wave and the basin length. The "agitation curve" gives the ratio of the maximum vertical amplitude of the agitation to the incident wave amplitude, as a function of the length of the basin expressed in wave lengths

C - Experimental apparatus -

The experiments have been carried out in two different wave canals, according to the practical possibilities that each one offered for the different measurements.

The first canal used was constructed on a slab 24 m by 4 m levelled with an accuracy of approx 1mm. Prefabricated rectangular blocks (40 × 28 × 8 cm.) allowed the necessary basins to be formed very quickly. The wave generator of the type "balançoire" allowed periods of 0,5 to 3 seconds to be realised. A great thickness (3 m.) of grillage wave filter served to purify the wave and to limit the resonance effects proper to the test canal.

The second canal used, with a width of 30 cm, allowed only two dimensional tests to be carried out. On the other hand as it had glass walls the phenomena were more easily observed. The canal wave generator, also of the "balançoire" type, allowed the periods to be varied from 0,4 to 2,8 seconds[1].

The amplitude measurements were effected either by a measuring pointer and cathodic eye, or by a capacitance wave recorder, or by the graphical wave recorder (E.G.H.). These pieces of apparatus are described in "LA HOUILLE BLANCHE".(2) Certain measurements have been made directly on the glass for the tests carried out in the second wave canal.

The rail necessary for the displacement of the sounding element of the E.G.H. was fixed to the concrete blocks bordering the basin of the canal. The periods were measured with a stop watch.

II - STUDY OF OBSTACLES SUBMITTED TO WAVES -

Calculations for establishing exact equations for water movements are often very complex, and are accessible only in a few simple cases. However, when the approximation is limited to linear phenomena the usual mathematical procedures of electricity provide a simple means of obtaining interesting results in wave studies.

The theoretical study of obstacles submitted to waves (3)and its application to seiche theory is a particularly profitable example.

We shall limit ourselves to giving the results of the theory , for the obstacles encountered in seiche studies. The full calculations will be published later in "LA HOUILLE BLANCHE".

(2) Bibliography : -4

(3) The basis of this st dy was presented in a paper by M. BIESEL to the Société Hydrotechnique de FRANCE.

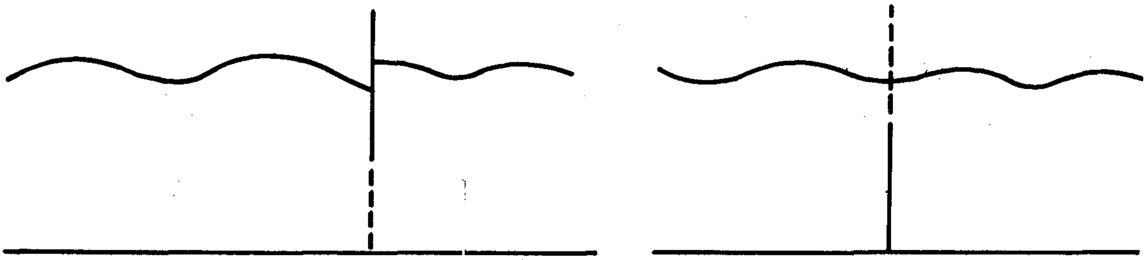

Fig. 3. Localized obstruction.

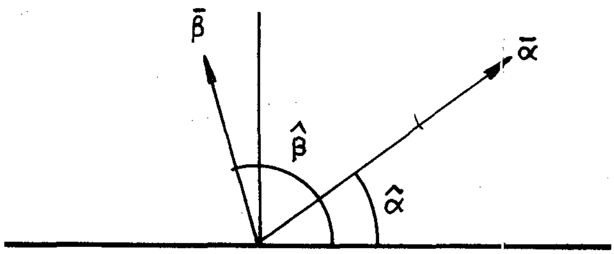

Fig. 4. Vectorial representation of waves.

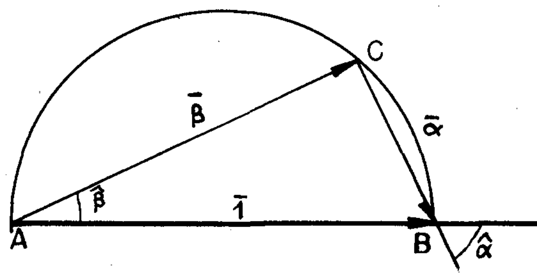

Fig. 5. Vectorial representation of waves on a localized obstacle not absorbing energy.

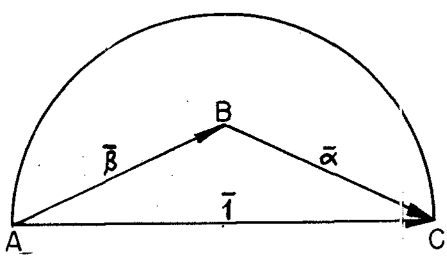

Fig. 6. Vectorial representation of waves on a localized obstacle absorbing energy.

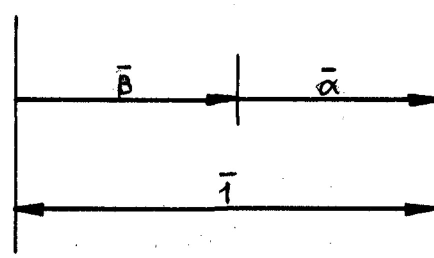

Fig. 7. Vectorial representation of waves on a perfectly resistant localized obstacle.

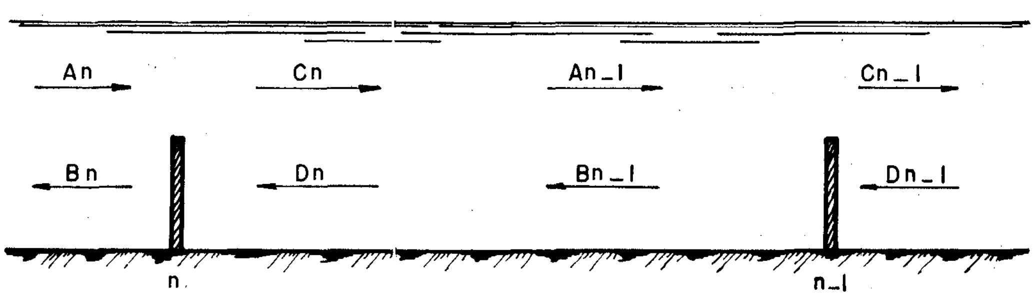

Fig. 8. Combination of obstacles.

A - Isolated obstacle -

The study is limited to periodic irrotational waves of two dimensions in constant depth. We assume that the fluid is perfect and we neglect terms of second or higher powers of the camber of the wave.

The obstacles are such that they allow part of the wave to pass and they reflect part. We shall assume that the wave passes without absorption of energy, except when we shall indicate otherwise, and without deformation of the waves, except in the immediate neighbourhood of the obstacles studied.

The incident waves are both transmitted and reflected, and undergo considerable deformations in the immediate vicinity of the obstacle, but the deformations disappear rapidly with the distance from the obstacle.

The transmitted and reflected incident waves being considered sufficiently far from the obstacle for them to have regained the characteristics of periodic waves in constant depth, it may be assumed that the ratio of the transmitted wave amplitude to the incident wave amplitude has a value α independent of the absolute magnitude of the amplitudes. Similarly it may be assumed that the ratio of the reflected wave amplitude to the incident wave amplitude has a value β. These hypotheses derive from the assumption of a linear theory.

On the other hand the transmitted waves are out of phase with the incident waves by an amount which we shall call $\hat{\alpha}$. Similarly the reflected waves have a certain phase difference $\hat{\beta}$ from the waves which would be reflected by a perfectly vertical wall placed in the same position as the obstacles studied.

In order to be able to establish a relationship between the phases of the incident and reflected waves it is therefore necessary to introduce an imaginary reflecting wall, a reference plane related to the obstacle. It is then possible to designate the phase of the waves as that of their vertical amplitudes in front of this reference plane whatever their direction of propagation (1)

For a plane theoretical barrier it is natural to choose the plane of the obstacle as the reference plane.

For simplicity, when the obstacle possesses a plane of vertical symmetry it is evident that this plane will be chosen as the reference plane.

If for the waves studied we adopt a vectorial representation in the plane of immaginaries (fig.4) (analogous to that used in the study of alternating currents) we can represent the coefficients of transmission

(1) More exactly it is a question of determining the phase of the vertical amplitudes of a periodic wave which is equivalent to the wave considered, sufficiently far from the obstacle. n abstraction is made from the local disturbance caused by the obstacle in order to estimate the phase of the waves a long way from it.

and reflection, by vectors i.e. by the complex numbers $\vec{\alpha}$ and $\vec{\beta}$ which represent these coefficients both by their modules α and β and by their arguments $\hat{\alpha}$ and $\hat{\beta}$. When the phase and amplitude of the incident wave are represented by the complex numb... $\vec{A}$, the transmitted and reflected waves will be represented by $\vec{\alpha}A$ and $\vec{\beta}A$ respectively.

From the theory of the conservation of energy, expressed by $|\alpha|^2 + |\beta|^2 = 1$, the relation $\hat{\alpha} - \hat{\beta} = \pi/2 + K\pi$ is established.

If the obstacle is symmetrical about a plane perpendicular to the direction of propagation of the wave it is clear that the values the coefficients $\vec{\alpha}$ and $\vec{\beta}$ will be independent of the side from which the incident wave comes. Whereas if the obstacle is assymetrical no conclusions can be made beforehand, e.g. an obstacle might reflect more from one side than from the other.

In fact M. BIESEL (1) demonstrated, starting from the theory of conservation of energy, that all obstacles even assymetric have coefficients of transmission and reflection whose values are independent of the direction of the incident wave. This result has also been arrived at by KREISEL, in a different manner (2).

Further, M. MEYER of the Laboratoire Dauphinois d'Hydraulique has been able to show that the phase differences for transmission are independent of the direction of the incident wave.

If now an obstacle of the vertical barrier type is considered, it is observed that when the canal is totally obstructed there is total reflection without change of phase : in other words

$$|\beta| = 1 \quad \hat{\beta} = 0 \quad \text{or } \vec{\beta} = 1 \quad (\text{and } \vec{\alpha} = 0)$$

When on the other hand the barrier is completely removed there is total transmission without change of phase, therefore :

$$|\alpha| = 1 \quad \hat{\alpha} = 0 \quad \text{or } \vec{\alpha} = 1 \quad (\text{and } \vec{\beta} = 0)$$

These observations suggest that for this type of obstacle a relation exists between the phase differences and the absolute values of the coefficients of reflection and transmision. As we shall show, this relation can be effectively defined by simple physical considerations which although hypothetical are nevertheless very plausible.

1) Localised obstacle not absorbing any energy :

We recall that an obstacle in the usual sense, will be called an obstruction when it restricts the wave passage, in a test canal for example. This definition may at first seem useless but in a canal certain types of

(1) Bibliography : - 2
(2) Bibliography : - 6

discontinuity can exist which act as partially reflecting obstacles without being *"obstructions"*. On the contrary they might be a local widening or deepening.

By *"localised"* we mean that the objects forming the obstacle must occupy only a very short length in the direction of wave propagation.

The typical localised obstruction is a plane vertical barrier without thickness.

Our first hypothesis depends on the fact that the obstacle is of the obstruction type ; if there is a phase difference for the transmission it will be in the sense of a delay. By reducing the area of the passage the obstruction causes velocity increases and therefore increases of inertia effects (comparable to self induction effects in electricity). Similarly a reduction of depth over a certain distance (obstacle not localised) produces a phase delay because the celerity is smaller above the obstacle.

Our hypothesis will therefore be :

$$\pi < \hat{\alpha} < 0 \text{ to } 2\,K\pi \text{ approx.} \qquad (1)$$

Our second hypothesis will be due to the fact that the obstacle is localised : it will be that if asymptotic waves (incident, reflected and transmitted) were supposed to extend close to the obstacle, they would satisfy a (fluctuating) discharge continuity condition.

This condition gives :

$$\vec{\alpha} + \vec{\beta} = 1 \qquad (2)$$

The hypothesis of continuity of discharge is natural for if it were not true , a certain volume of water would alternately accumulate and disappear in the region of the obstacle, which seems impossible since the obstacle occupies only a very small area. It is true however that even a very localised obstacle (a thin plate for example) can deform the wave in the neighbouring region which in fact will have an extent of the order of a few depths (1).

The hypotheses (1) and (2) together with that of the conservation of energy allow, as well as the determination of the law

$$\hat{\alpha} - \hat{\beta} = \pm\,\pi/2$$

(1) It can be shown that these deformations are the sum of terms which decrease exponentially very rapidly with the distance from the obstacle, the rate of decrease being proportional to the depth.

Our reasoning will thus be valid in the limit, for waves that are very long in relation to the depth. It will be doubtful for greater relative depths. This pessimistic note is compensated by the fact that the exact analysis of the case with a plane vertical plate in infinite depth shows that the hypothesis of continuity is satisfied (perheps forfuitously). This analysis has been made by ; . URSELL (Bibliography : 11).

128 found before, the determination of the quantities $|\alpha|$, $|\beta|$, $\hat{\alpha}$ and $\hat{\beta}$, as long as one of them is known. (fig. 5)

All the obstacles satisfying hypotheses (1) and (2) are therefore defined by a single parameter for example $\hat{\beta}$ and :

$$\alpha = \sin \hat{\beta}$$
$$\hat{\alpha} = \pi/2 - \hat{\beta}$$
$$\beta = \cos \hat{\beta}$$

2) Localised obstacle absorbing energy :

As the obstacles studied in this chapter are again of the localised obstruction type, we conserve hypotheses (1) and (2) and add the condition of non-conservation of energy :

$$|\vec{\alpha}|^2 + |\vec{\beta}|^2 < 1$$

If a graphical representation is considered analogous to that of figure 5, it can be seen that the apex C of the triangle ABC is inside the circle of diameter AB (fig. 6).

For a given value of α , β or α/β it can be seen that there is a maximum absorption when C lies on the diameter AB (fig. 7).

If the transmission is nil, the reflection takes place without loss of energy and, similarly, if the reflection is nil the transmission takes place without loss of energy.

Thus it is seen that within the limits of the hypotheses :

- a localised obstruction can only absorb a fixed amount of energy and this limiting amount is a function of the ratio of the coefficients of reflection and transmission. In particular :

a) no energy can be absorbed if one of these coefficients α or β is nil (the other consequently being equal to 1) ;

b) the maximum proportion can be attained only if the coefficients α and β are both equal to 1/2. This proportion is then 50 %.

It can be observed incidentally that this confirms and clarifies the known fact that it is not possible to completely absorb a wave by passive means over too short a distance. For example the practical rule of the NEYRPIC Laboratory is that a wave damper must be at least one wave length long before it is acceptable.

When point C falls on AB, we shall say that the obstacle is purely resistant (by analogy with electricity where a pure resistance causes losses of energy and no change of phase). The type obstacle which is a perfectly resistant localised obstruction is a vertical plate of permeable material which blocks the whole canal.

It is essential besides that the forces limiting the water passage be opposed and proportional to the velocities, and that the inertia effects be negligible in the obstacle.

B - Combination of obstacles -

Having established the equations for isolated obstacles we are now going to study the effect of combining the obstacles considered, disposing them one behind the other.

1) Agitation between any two obstacles :

Each obstacle has individual coefficients of transmission and reflection (fig. 8), which are related to the plane of symmetry.

The magnitudes A_n, B_n, C_n, D_n represent waves and their phases relative to the obstacles n ; similarly A_{n-1}, B_{n-1}, C_{n-1}, D_{n-1}, represent waves and their phases relative to the obstacle $n-1$.

Then :

$$B_n = \vec{\beta}_n A_n + \vec{\alpha}_n D_n$$

$$C_n = \vec{\alpha}_n A_n + \vec{\beta}_n D_n$$

$$B_{n-1} = \vec{\alpha}_{n-1} D_{n-1} + \vec{\beta}_{n-1} A_{n-1}$$

$$C_{n-1} = \vec{\alpha}_{n-1} A_{n-1} + \vec{\beta}_{n-1} D_{n-1}$$

$$C_n = \vec{r}_{n,\, n-1} A_{n-1}$$

$$B_{n-1} = \vec{r}_{n,\, n-1} D_n$$

with

$$\vec{r}_{n,\, n-1} = \rho\, e^{i\theta} = e^{i\theta}$$

$$\theta = \frac{2\pi}{L}$$

d being the distance between the two obstacles n, $n-1$.

Which gives as a function of A_n and D_{n-1}

$$D_n = \frac{B_{n-1}}{\vec{r}_{n,\,n-1}} = \frac{\vec{\alpha}_n \vec{\beta}_{n-1} A_n + \vec{r}_{n,\,n-1} \vec{\alpha}_{n-1} D_{n-1}}{\vec{r}^2_{n,\,n-1} \quad \vec{\beta}_n \vec{\beta}_{n-1}}$$

$$A_{n-1} = \frac{C_n}{\vec{r}_{n,\,n-1}} = \frac{\vec{\alpha}_{n-1} \vec{\beta}_n D_{n-1} + \vec{\alpha}_n \vec{r}_{n,\,n-1} A_n}{\vec{r}^2_{n,\,n-1} - \vec{\beta}_n \vec{\beta}_{n-1}}$$

$$C_{n-1} = \frac{(\vec{r}^2_{n,\,n-1} \vec{\beta}_{n-1} - \vec{\beta}^2_{n-1} \vec{\beta}_n + \vec{\alpha}^2_{n-1} \vec{\beta}_{n-1}) D_{n-1} + \vec{r}_{n,\,n-1} \vec{\alpha}_n \vec{\alpha}_{n-1} A_n}{\vec{r}^2_{n,\,n-1} - \vec{\beta}_n \vec{\beta}_{n-1}}$$

$$B_n = \frac{(\vec{r}^2_{n,\,n-1} \vec{\beta}_n - \vec{\beta}^2_n \vec{\beta}_{n-1} + \vec{\alpha}^2_n \vec{\beta}_{n-1}) A_n + \vec{r}_{n,\,n-1} \vec{\alpha}_n \vec{\alpha}_{n-1} D_{n-1}}{\vec{r}^2_{n,\,n-1} - \vec{\beta}_n \vec{\beta}_{n-1}}$$

The agitation between the two obstacles is for example :

$$A_g = \vec{r}^{1/2} C_n + \vec{r}^{-1/2} B_{n-1}$$

and can be calculated theoretically when the difference in phase between A_{n-1} and D_n is known.

It can be seen that the agitation is generally theoretically infinite for :

$$\vec{r}^2_{n,\,n-1} - \vec{\beta}_n \vec{\beta}_{n-1} = 0$$

2) Agitation between two identical obstacles submitted to a single incident wave :

The preceding formulae become with $D_{n-1} = 0$, by posing :

$$\vec{\alpha}_{n-1} = \vec{\alpha}_n = \vec{\alpha}$$
$$\vec{\beta}_{n-1} = \vec{\beta}_n = \vec{\beta}$$

from which

$$D_1 = \frac{\vec{\alpha}\,\vec{\beta}\,A_1}{\vec{r}^2 - \vec{\beta}^2}$$

$$C_1 = \frac{\vec{\alpha}\,\vec{r}^2\,A_1}{\vec{r}^2 - \vec{\beta}^2}$$

At the centre O, the agitation becomes :

$$A_g = C_0 + D_0 = \vec{r}^{1/2} D_1 + \vec{r}^{-1/2} C_1$$

Or after simplifying :

$$A_g = \frac{\vec{r}^{1/2} \alpha A_1}{\vec{r} - \vec{\beta}} \qquad (3)$$

a) Obstacles not absorbing energy :

Still within the stated hypotheses for localised obstacles not consuming energy, the complex expression (3) may be written in terms of the arguments :

$$A_g = \frac{\sin \hat{\beta} A_1}{\sqrt{1 + \cos^2 \hat{\beta} - 2 \cos (\hat{\theta} - \hat{\beta}) \cos \hat{\beta}}} \qquad (4)$$

The agitation will therefore be a maximum when :

$$\hat{\theta} = \hat{\beta} + 2 K \pi$$

The agitation then is :

$$A_m = \frac{\vec{\alpha} A_1}{1 - \vec{\beta}}$$

which gives the amplitude by the expression :

$$A_m = \frac{\sin \hat{\beta} A_1}{1 - \cos \hat{\beta}}$$

This relation gives the resonance amplitude as a function of $\hat{\beta}$

In this case the total reflection for the two obstacles is nil.

It can be seen that when $\hat{\beta}$ tends to 0 (which for the stated hypotheses corresponds to a complete closure) the resonance amplitude tends to infinity.

The theoretical curves (fig.9) given by equation (4) show the increase in selectivity of resonance as $\hat{\beta}$ decreases ; on the other hand the resonance agitation increases simultaneously.

This can be expressed physically by saying that the more the obstacles limit the wave passage, the less the water contained between them is in danger of resonating ; but the greater the possible agitation should full resonance occur. However in practice the agitation will be limited by the losses of energy due to friction, in the basin and in the passage across the obstacle.

The minimum agitation occurs when :

$$\hat{\theta} = \hat{\beta} + (2\ k + 1)\ \pi$$

It is then equal to :

$$A_m = \frac{\vec{\alpha}\ A_1}{1 + \vec{\beta}}$$

The amplitude may be written within the same hypothesis :

$$A_m = \frac{\sin\ \hat{\beta}\ A_1}{1 + \cos\ \hat{\beta}}$$

Interesting relationships :

A few interesting relationships may be singled out :

a) the product $A_M \times A_m$ becomes :

$$A_M \times A_m = \frac{\vec{\alpha}^2}{1 - \vec{\beta}^2} A_1{}^2 = A_1{}^2$$

where :

$$\vec{\alpha}^2 + \vec{\beta}^2 = 1$$

It is necessary to emphasise that the equality $A_M \times A_m = A_1{}^2$ is only valid when there is no loss of energy.

b) $\vec{\alpha}$ and $\vec{\beta}$ can be obtained from the expressions :

$$\alpha = \sin\ \hat{\beta} = \frac{A_M \times A_m}{2\ (A_M + A_m)}$$

$$\beta = \cos\ \hat{\beta} = \frac{A_M - A_m}{A_M + A_m}$$

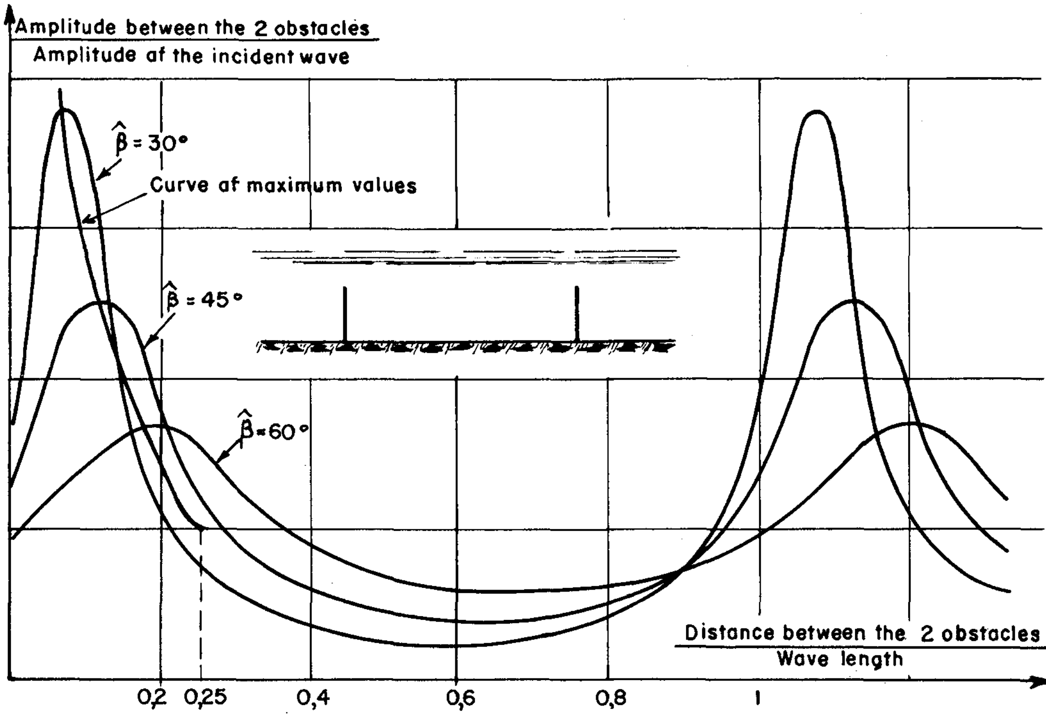

Fig. 9. Agitation between two identical obstacles not absorbing energy.

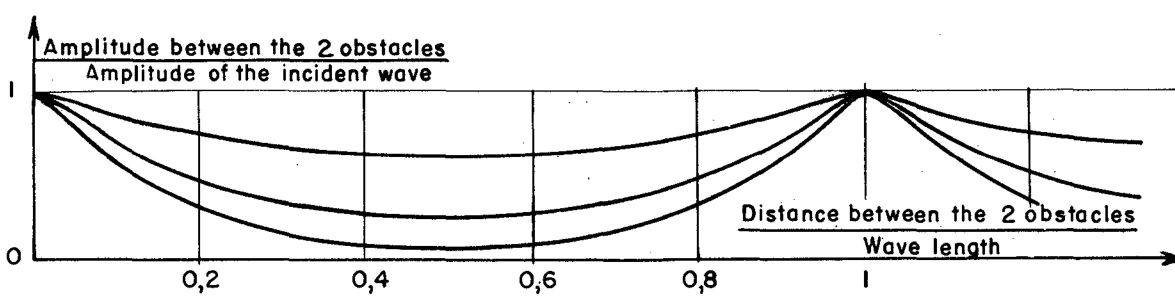

Fig. 10. Agitation between two perfectly resistant obstacles.

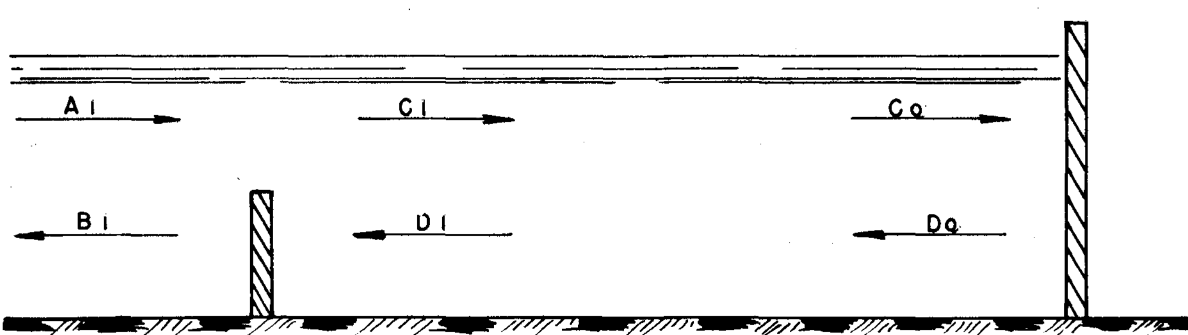

Fig. 11. Schematization of waves in a basin of restricted opening.

These formulae allow $\vec{\alpha}$, $\vec{\beta}$ and $\hat{\beta}$ to be obtained experimentally from A_M and A_m.

b) Perfectly resistant localised obstacles :

If we assume :

$$\alpha + \beta = 1 \tag{5}$$

The expression for the agitation becomes, for the condition that $\hat{\beta} = 0$:

$$A_g = \frac{\vec{r}^{1/2} (1 - \vec{\beta}) A_1}{\sqrt{1 + \vec{\beta}^2 - 2 \vec{\beta} \cos \theta}}$$

From which the theoretical curves in figure 10 are derived giving the value of the agitation as a function of $\vec{\alpha}$ or β. It can be seen that the agitation in the basin is always less than or at least equal to, the agitation at sea. It is thus unlikely (when the relation (5) is satisfied) that the permeability of a breakwater will be the direct cause of an increase in the coefficient of amplification.

III - AGITATION IN A BASIN OF RESTRICTED OPENING

A - Theory -

The preceding theory, used to establish the value of the agitation between two identical obstacles, has a particular interest in the study of seiche. Because it remains valid when the basin is limited an one side by a perfectly reflecting obstacle and an the other by any obstacle whatever (fig. 11).

We have seen that no obstacle, whatever its shape, presents any assymetry to the wave. Therefore the obstacle at the basin entrance has the same characteristics $\vec{\alpha}$ and $\vec{\beta}$ for the incident and reflected waves, at the point O at the end of the basin.

The preceding equations therefore remain valid in every way when the lengths are divided by two and the amplitudes multiplied by two.

In the simple case of a closed basin which opens onto a canal of the same width, although the movement is three dimensional around the entrance it may be considered two dimensional in the basin. It is therefore possible to obtain the expression for agitation by means of the preceding calculations. However for this particularly simple case it is possible to obtain it directly from the equations :

$$C_1 = \vec{\alpha} A_1 + \vec{\beta} D_1$$

$$B_1 = \vec{\alpha} D_1 + \vec{\beta} A_1$$

$$C_1 = \vec{r}^{1/2} C_0$$

$$D_e = \vec{r}^{1/2} D_1$$

$$C_0 = D_0$$

The vertical agitation A_g can be obtained from the expression

$$A_g = C_0 + D_0$$

from which

$$A_g = \frac{2\,\vec{\alpha}\,\vec{r}^{1/2} A_1}{\vec{r} - \beta}$$

The theoretical resonance curves are therefore the same as those found from the theory for the obstacles, with a factor of 2 .

Similarly the following equalities and relations are known :

$$\text{Maximum agitation} = A_M = \frac{2\,\vec{\alpha}\,A_1}{1 - \vec{\beta}}$$

$$\text{Minimum agitation} = A_m = \frac{2\,\vec{\alpha}\,A_1}{1 - \vec{\beta}}$$

$$\frac{A_M \times A_m}{A_1^2} = \frac{4\,\vec{\alpha}^2}{1 - \vec{\beta}^2} = 4$$

The obstacle limiting the basin introduces a phase difference which moves the peak values a distance of $(K^{L/2} - \hat{\alpha}/2\pi\, L)$ away from the perfectly reflecting wall O . This is only valid for the peak values outside the basin, i.e. beyond the obstacle.

When $\hat{\beta}$ varies from 0 to $\pi/2$ resonance takes place according to formula (5) when the length of the basin d expressed in wave lengths varies from $(K\pi)\ L/2\pi$ to $(K\pi + \pi/4)\ L/2\pi$, i.e. from $(K\ L/2)$ to $(K\ L/2 + L/8)$.

The resonance in a closed port is therefore of the 1/2 wave length type, the length that it is necessary to add to L/2 being an increasing function of $\hat{\beta}$, i.e. of the opening.

When the basin tends towards complete opening into a canal of the same width, the resonance length theoretically tends towards :

$$K \frac{L}{2} + \frac{L}{8}$$

The resonance amplitude is then equal to the amplitude of the clapotis.

We will now seek to calculate the wave reflected by the whole unit, in front of the obstacle.

The formulae already established give directly, when $D_{n-1} = 0$ and $n = 2$, the value :

$$R = B_n + C_{n-1} = B_2 + C_1$$

for the reflected wave R in front of the obstacle.

$$R = \frac{(\vec{r}^2 \vec{\beta} - \vec{\beta}^3 + \vec{\alpha}^2 \vec{\beta} + \vec{r} \vec{\alpha}^2) A_1}{\vec{r}^2 - \vec{\beta}^2}$$

from which

$$R = (\vec{\beta} + \frac{\vec{\alpha}^2}{\vec{r} - \vec{\beta}}) A_1$$

It can be verified that :

$$|R| = |A_1|$$

and that :

$$\hat{R} - \hat{A}_1 = \hat{\theta} - \pi + 2 \text{ arc tg } \frac{\sin (\hat{\theta} - \hat{\beta})}{\cos \hat{\beta} - \cos (\hat{\theta} - \hat{\beta})}$$

(localised obstacles without loss of energy).

For resonance $\hat{\theta} = \hat{\beta} + K\pi$ and $\hat{R} - \hat{A}_1 = \hat{\beta} + 2 K\pi$.

In particular when the opening becomes nil, β tends towards zero and R tends to be in opposing phase with A_1. The external clapotis then presents a node at the entrance .

In this latter case it is interesting to notice that the phase of the interior agitation is $A_1 - \pi/2$ while the phase of the clapotis is $A_1 + \pi/2$ at the first peak value away from the entrance. The interior and exterior movements are therefore out of phase by π. The currents in the entry pass will therefore be of considerable magnitude(1).

(1) The currents in the entry pass are always considerable whatever the type of resonance, so it cannot be concluded from this single observation that it is a quarter wave length resonance.

B - Experimental verification -

The first series of experiments was a systematic research into the magnitude of the agitation as a function of the basin length and of its opening.

In order to limit the friction effects the amplitude of the incident wave has been kept small. This is because it is essential to realise test conditions in which the energy losses are kept to a minimum, in order to be able to establish the graph for the phenomena indicated by the preceding theory.

This condition has led to the principal tests being made with a streamlined entry pass whose length was not too small in relation to the wave length, for the purpose of keeping the friction forces small in relation to the inertia forces.

Different types of obstacle have been studied systematically. Here we shall limit ourselves to giving the final results.

1 - Firstly we modified the width of the entry pass by means of two adjustable concrete blocks 20 cm thick, offering to the water movements profiles shaped to reduce head losses (fig. 12).

It has been possible for us to locate the nodes and loops of the movement in the basin and at sea.

Figure 13 gives the results of agitation tests carried out for the following conditions :

- period : $T = 2$ s
- amplitude : $2a = 6.4$ mm
- depth : $h = 140$ mm .

Is abscissa we have taken the ratio of the basin length over the wave length ; and ordinates the ratio of the maximum vertical agitation in the basin to the incident wave amplitude.

The length d of the basin is measured from the internal face of the blocks forming the pass.

There is a striking agreement between the theoretical and experimental results. The curve of maxima which theoretically tends to infinity for zero opening, in effect falls only after the relative opening has been reduced to 0.075 (ratio of opening O to the width of canal). Further, the high selectivity of the agitation curves in the neighbourhood of resonance, for small openings, makes it possible to think that it is possible for the maximum to increase still further ; the regulation of the model is then extremely delicate and for these test conditions we have not been able to exceed a maximum agitation of 6 times the amplitude of the external wave.

Generally measurements have only been made for basin lengths between 0 and 0.7 times the wave length. But we have verified for a few cases that when the length of the basin is increased, resonance conditions are again found every half wave length which agrees with the theory. However the amplitude tends to decrease because of the increased energy losses due to the greater length.

It is surprising that the periodic form of the resonance curves is conserved even for extremely short basins. It might well have been expected that in this case the theory would fail as it is based on wave properties at a considerable distance from the obstacles.

Other measurements have been made with the same obstacles but for periods varying from 0.5 to 3 seconds. The results plotted on an analagous system of axes only confirm the results already obtained, concerning the general form of the agitation curves. In the course of these tests, we have obtained an amplification of 14 in the following conditions :

- period of the wave : 1 sec
- amplitude of the wave: 2a = 8 mm
- ratio of opening to basin width : O/e = 0.075

2 - Tests have been made in identical conditions, on a basin whose entrance was formed by thin plane faced blocks 3 cm thick.

It appeared that the maximum (maximorum) agitation took place for a relative opening of 0.2 and basin length d equal to 0.585 times the wave length.

The agitation in the basin was then equal to 4 times the amplitude of the perfect clapotis.

The curve of maxima seems to begin at d = 0.5 L when O/I = 0 and to end at d = 0.72 L for O/I = 1.

It seems certain that this divergence from the theory for the higher value of d (d = 0.625 L) may be attributed to the influence of friction at the entrance. This friction causes phase differences for the transmitted and reflected waves to be different from those derived from the theory.

The water movement in the pass causes two vertical vortices which form alternatly inside and outside the pass.

These vortices cause an important dissipation of energy which it was possible for us to illustrate by putting two streamlined plates at the end of the 2 walls limiting the entry. The agitation in the basin increased then from 4 times the incident wave amplitude to 4.8 times.

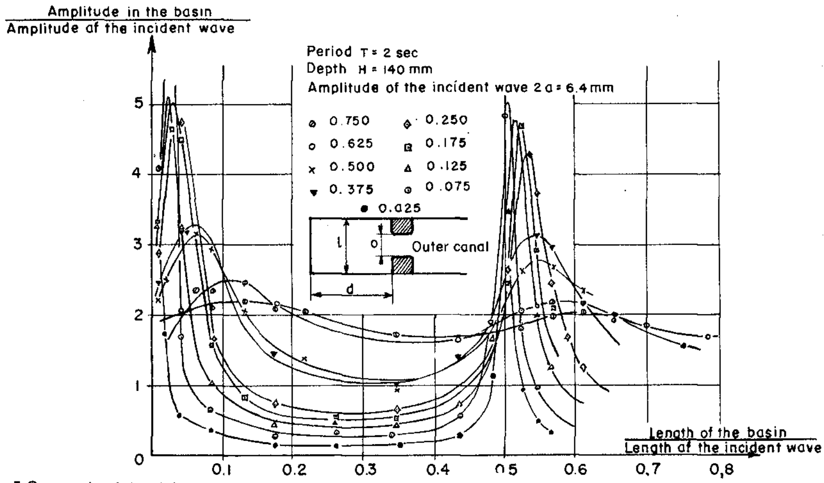

Fig. 12. Agitation in a basin of restricted opening, limited by two jetty heads, as a function of the opening and of the relative length of the basin.

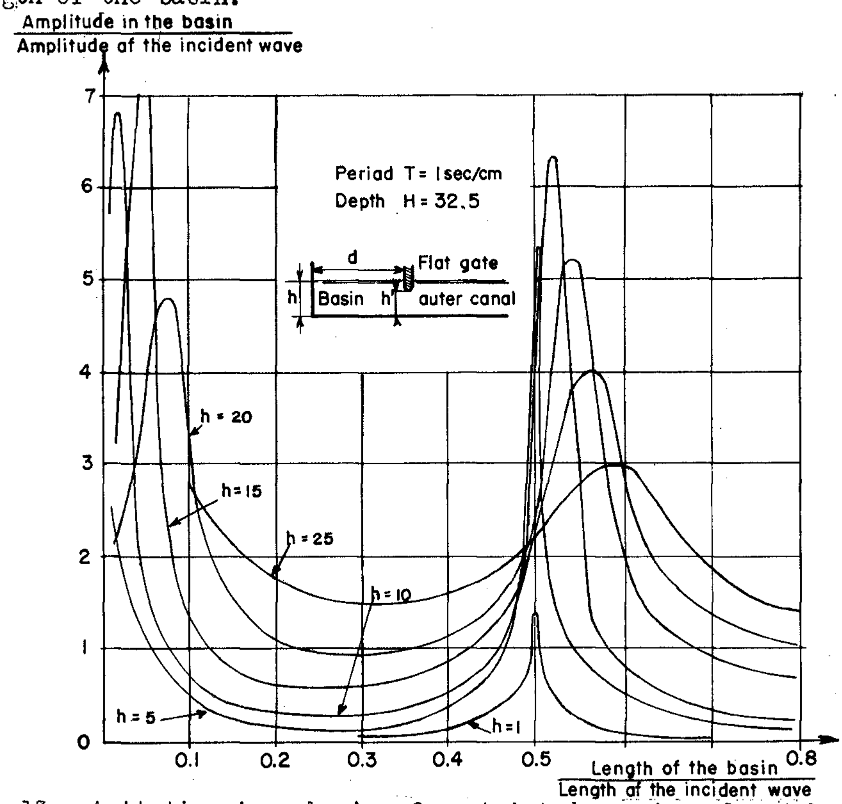

Fig. 13. Agitation in a basin of restricted opening closed by a plane gate, as a function of the opening and of the relative length of the basin.

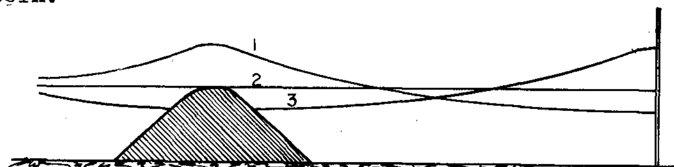

Fig. 14. Particular form of the agitation.

3 – Identical tests have been carried out with a plane gate closing the basin.

The test conditions were then the following :

- period of the wave : T = 1 second
- depth of the canal : h = 32.5 cm .

h' represents the distance between the bottom of the canal and the lower edge of the gate. The gate has a thickness of 2.5 cm and can therefore be considered as a localised obstacle.

The results obtained for these conditions obey the some laws as those obtained before and therefore confirm the value of the theory (fig. 14)

Here again the resonance conditions are particularly interesting as they are very nearly the same for a basin length of $\frac{\hat{\beta}}{2}\frac{L}{2\pi}$ as for a length of $(K\pi + \hat{\beta}/2)\, L/2\pi$.

The magnitude of resonances obtained for relative lengths of basin less than 0.125 indicate the importance of resonaters of length : $\frac{\hat{\beta}}{2} \times \frac{L}{2\pi}$ for the absorption of the incident wave energy. M. VALEMBOIS calls this type *"résonateur en charge"*. Experience shows that the binodal or multinodal swaying taking place in basins of length $K\frac{L}{2} + \frac{\hat{\beta}}{2}\frac{L}{2\pi}$ is replaced by a *"piston-like"* movement of the water, presenting a horizontal free surface, when K is nil.

4 – *Plane obstacles entirely submerged and more or less perfectly streamlined* have also been tried.

It might happen for an obstacle whose height is equal to the depth of water in the basin that a very pronounced 1/2 wave resonance will occur. In this case the crest of the obstacle will be uncovered periodically (fig. 16).

Other lower obstacles give the maximum agitation for a length of basin slighty greater than half a wave length and provoke a periodic breaking on both sides, which is particularly marked in the direction basin-sea. This phenomenon is sufficient to indicate that the amplitude is greater in the basin than at sea. These facts have already been observed from breakwater studies.

Finally, we have verified that the coefficient of amplification is independent of the incident wave amplitude as long as turbulence remains unimportant. (one of the graphs established during this verification is reproduced in M. BIESEL'S paper at this same conference).

(1) Bibliography : 13

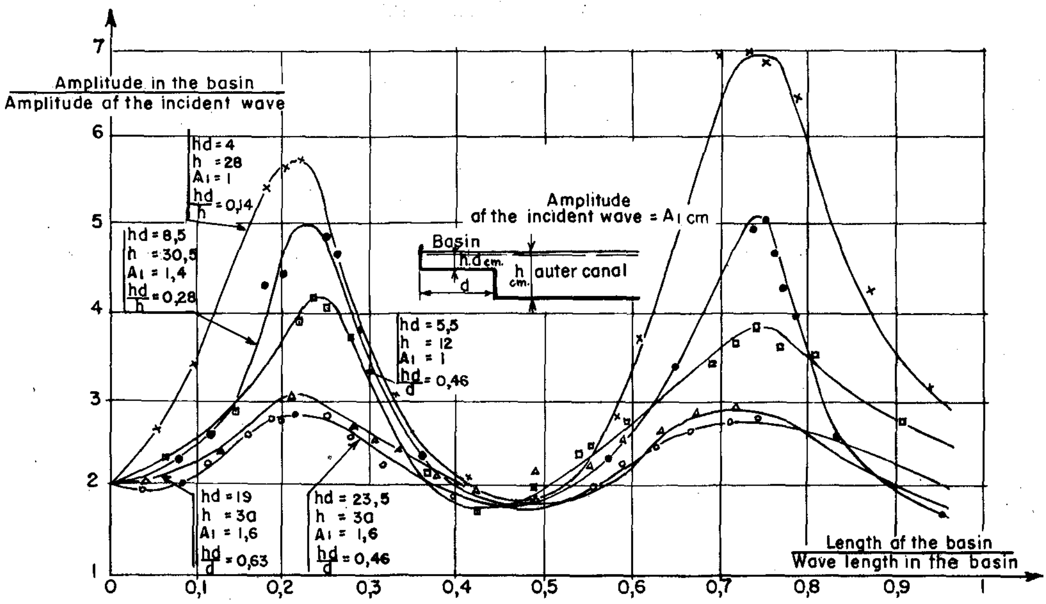

Fig. 15. Agitation in a basin of expanded opening limited by a deepening, as a function of the relative length of the basin.

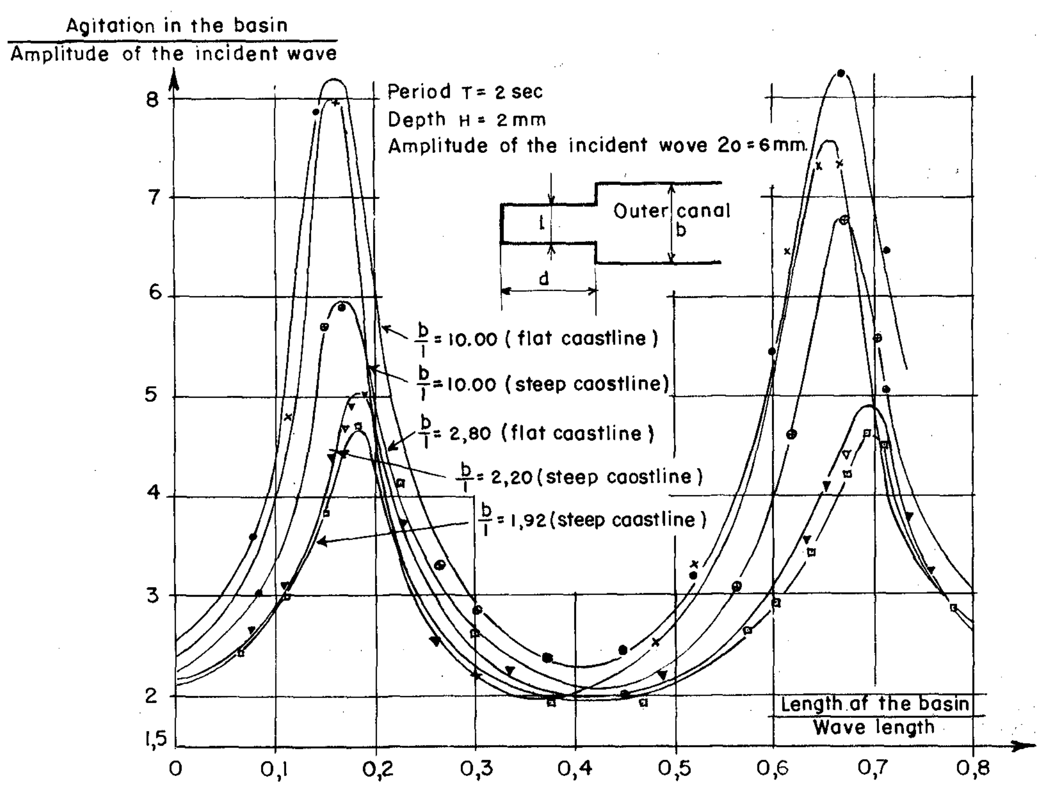

Fig. 16. Agitation in a basin of expanded opening limited by a widening, as a function of the relative length of the basin.

IV - AGITATION IN A BASIN WITH EXPANDED OPENING

In the same way as before, vector theory may be used to obtain the value of the agitation in a basin limited by *"deepening"*.

But as it is valid only for plane movements it cannot be applied to seiche movements in basins limited by *"widening"*.

We shall limit the argument for these two cases to the essential experimental results.

A - Deepening -

A wooden bank the same width as the channel and fixed to the bottom causes a sharp change of depth. A vertical plate which penetrates to the upper level of this bank, is fixed at a distance d from its edge (fig. 17).

This corresponds practically to the case of a shallow basin opening into the sea, or again to the case of a continental shelf giving onto the greater ocean depths.

Tests made with various different incident wave amplitudes and depths of basin show clearly that resonance occurs for basin lengths tending towards (2 k + 1) L/4 (fig. 17).

Curves 17 show that the amplification increases as the ratio of depth in the basin to depth at sea decreases. But the chances of resonance are independent of this value (1).

The minimum value of agitation is approx. $2A_1$ and takes place when the basin has a length : K L/2 . This is true a priori for K = 0.

B - Widening -

Consider now a basin which is fully open onto a bay greater width (fig. 18).

The characteristics of the incident wave are :

- period : T = 2 sec
- depth : H = 20 cm

The results of these tests are represented by the curves (18) these are traced for different widths of bay and show :

(1) This is the principal difference from basins of restricted opening where the chances of resonance diminish when the obstruction and amplification increase.

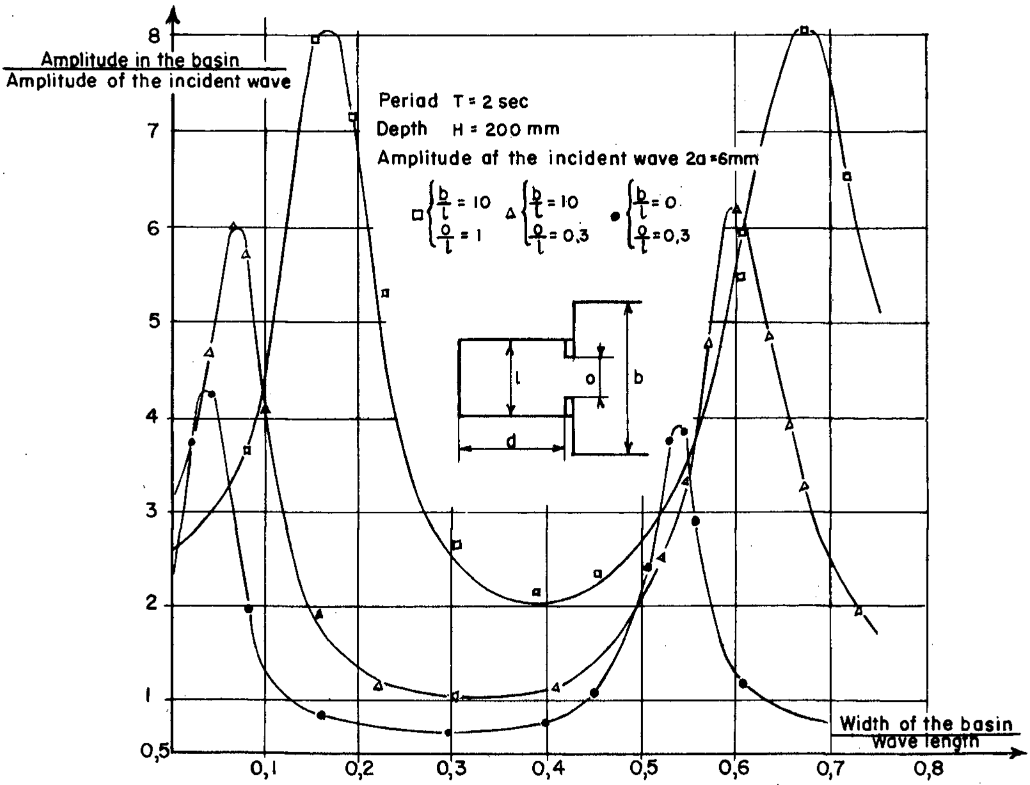

Fig. 17. Agitation in a basin limited by a complex obstacle, as a function of the relative length of the basin.

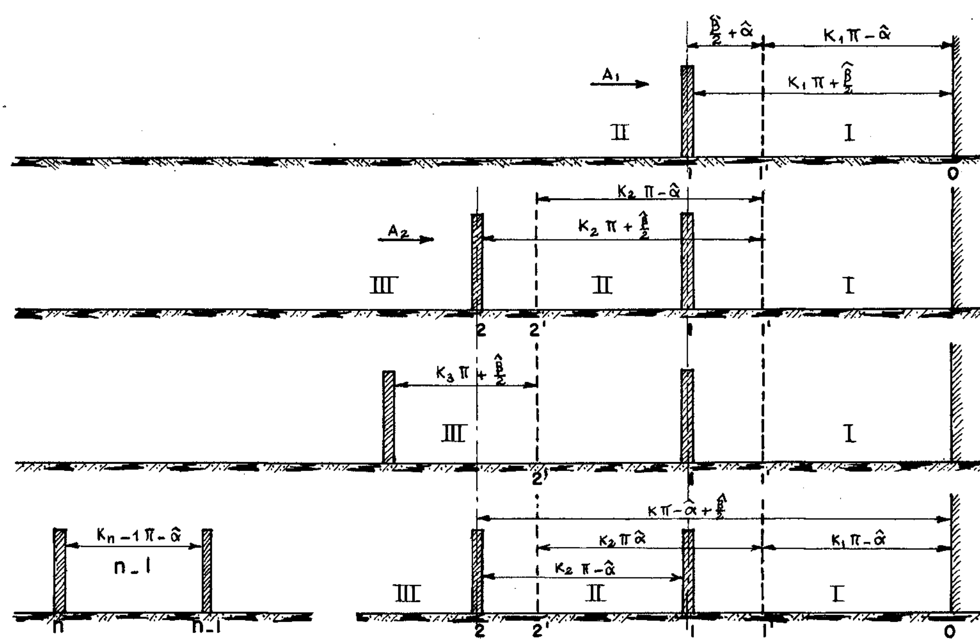

Fig. 18. Basins in series.

1) that the agitation in a basin opening onto a wide water area is never less than that of the total clapotis ;

2) that the chances of resonance remain the same whatever the relation between the width of the bay and the width of the basin (1) ;

3) that the amplitude of the agitation increases with the ratio of the width of the bay to the width of the basin and tends assymptotically towards a limit determined principally by friction ;

4) that from the point of view of resonance in a basin, it makes no difference whether it opens onto a steep coastline or a flat coastline.

To determine the resonance periods for this type of basin NEUMANN proposed using the formula given by BOSANQUET for sound pipes (2) . The experimental coefficient being then 0.346. This formula is expressed in the form :

$$d = (2\,h + l)\,\frac{L}{4} - 0.346\,(l + h)$$

l and h designate respectively the width and depth of the basin at the entrance (h can generally be neglected in relation to l).

Our experiments showed that this experimental coefficient was on the whole equal to 0.4 for this type of basin.

C - Basin limited by a complex obstacle -

A partially closed basin opening onto another basin of different width, corresponds to the condition most frequently met in harbours (eg. Port of LEIXOÉS-PORTUGAL).

It is then difficult to determine whether the basin is limited by an obstacle of restricted opening or expanded opening.

A few experiments have been made on resonance conditions for a basin limited by a complex obstacle, i.e. one terminating in a restriction followed by a widening or a deepening.

A number of results are shown in (20) andare compared with those obtained for the component simple obstacles.

We have seen that the form of the resonance curves differentiates clearly the obstacle of expanded opening from the obstacle of restricted opening. The resonance curve obtained for the complex obstacle is a "mean" curve which approaches one or the other curve according to whether the expanded opening or restricted opening characteristics of the complex obstacle are dominant.

(1) see note (1) page 13
(2) Bibliography : - 9

This "mean" curve, which is difficult to define accurately is true for amplitude and period.

D - Basins in series -

Here we examine basins in series and more particularly cascade resonance. These conditions are obviously very rare in practice but present a certain theoretical interest, illustrating and verifying the method of calculation.

1) Outline of the theory :

The formulae giving the agitation values soon become exceedingly complicated, but the computation methods remain the same.

Let us suppose the basin 1 is in resonance (fig. 13). Its length is therefore :

$$d_1 = (k_1 \pi + \hat{\beta}/2) \frac{L}{2\pi}$$

Because of the phase of the reflected wave, the loops of the clapotis outside the basin are found at a distance $(k_1 \pi - \hat{\alpha}) L/2\pi$ from O.

Thus the conditions will be as if the wave were reflected from a plane 1' at such a distance from O.

Further, the agitation in the basin 1 is greater than the external incident amplitude 1_1 , in a given ratio.

If now we place a second and identical obstacle at a distance $2\text{-}1' = (k_2 \pi + \hat{\beta}/2) L/2\pi$ from the plane 1' determined before, a pseudo-basin II is formed, itself in resonance. The amplitude of the agitation in this basin will be greater than the exterior incident amplitude A_2.

The value of A_1 will therefore be greater than A_2 because of the resonance in the pseudo-basin A-1', or more precisely in the basin 2-1. The value of the agitation will therefore increase in the direction III, II, I.

The distance 2,1 is then $(k_2 \pi + \hat{\alpha}) \frac{L}{2\pi}$. (1)

The wave is reflected again from the wall O , but the obstacles 1 and 2 introduce a phase difference 2 . The loops of the clapotis on the sea side of the obstacle 2 will therefore be at a distance from the wall O equal to :

$$[(k_1 + k_2) \pi + 2 \hat{\alpha}] \frac{L}{2\pi}$$

(1) N.B. $\hat{\alpha}$ is negative

The conditions at sea therefore are as if the wave were reflected from a plane 2' at the above distance from O.

A third pseudo-basin III will be in resonance if the distance 3-2' equals

$$(k_0 \; \pi + \frac{\hat{\beta}}{2}) \; \frac{L}{2\pi}$$

A very small agitation at sea can therefore theoretically cause a large amplitude in the first basin 1.

The distances between the obstacles are respectively between

$$1 \text{ and } 0 \; (k_1 \; \pi + \frac{\hat{\beta}}{2}) \; \frac{L}{2\pi}$$

$$\text{and } (k_{n-1} \; \pi + \hat{\alpha}) \; \frac{L}{2\pi}$$

between the obstacles n and n-1 (n being $\geqslant$ 2).

It is now possible to know the phase difference between A_n and D_{n-1} , all calculations made :

$$K \; \pi + \hat{\beta} + \hat{\alpha}$$

(or $\pi/2$ + $K\pi$ approx.) and consequently the agitation in each basin, knowing the wave amplitude at sea and the boundary conditions.

2) Experimental verification :

We have just seen that it is theoretically possible to have an agitation which increases from basin to basin as the distance from the sea increases.

Friction effects make it difficult for this phenomenon to be put in evidence. However with the shaped obstacles described above, it has been possible for us to verify it for two successive obstacles. The movement had a period of one second, the wave amplitude was 9.3 mm , the depth 13 cm , the width of canal 40 cm and the width of entry 6 cm. The following results were obtained:

1) With one obstacle only :

- length of basin d_1 = 55 cm

Ratio between the amplitude in the basin and the amplitude of the wave at sea :

$$\frac{A_{g_1}}{A_1} = 3.04$$

2) With two obstacles :

Length of the basins : $d_1 = 55$ cm

$d_2 = 60$ cm

$$\frac{A_{g_1}}{A_{g_2}} = 1.67$$

$$\frac{A_{g_2}}{A_1} = 2.58$$

$$\frac{A_{g_1}}{A_1} = 4.31$$

The indices 1 and 2 characterise the basins I and II, and A_g is the amplitude of the agitation.

The agitation with two obstacles is therefore 1.44 times greater than the agitation with a single obstacle. But the agitation in basin II is 1.18 times less than the agitation with a single obstacle.

With three successive obstacles, the friction absorbs too high a percentage of the energy for the agitation in basin I to be increased. However the same result may be obtained as with a single obstacle.

In practice cascade resonance is very rare as it requires improbable conditions. However if the waters of the continental shelf are in resonance thay can induce a resonance of the same period in a harbour. The amplitude in the harbour will be great because of this first amplification.

It might also be mentioned that a basin situated at the end of a harbour might be subject to seiche when the long waves are not absorbed in the far-port (LEIXOËS HARBOUR) (1)

V - CONCLUSION

The agreement between theory and experimental results illustrates the practical interest of the calculus of imaginaries in the analysis of linear periodic movements in two dimensions.

(1) Bibliography (1)

The laws evolved can be used to explain resonance phenomena in harbours, bays, or a continental shelf. They are absolutely independent of the period and can be applied to movements provoked by ordinary waves, long waves (Table Bay, Tamatave) or tides (Gulf of Gabès).

It is important to notice that resonance can only take place when the basin length lies between :

$$K\frac{L}{2} \text{ and } K\frac{L}{2}+\frac{L}{4}$$

whatever the form of the obstacle at the entrance (i.e. between 0 and $\frac{L}{4}$ between $\frac{L}{2}$ and $\frac{3L}{4}$ etc..).

Finally, in nature the exciting waves never have a rigorously constant period ; morever the time the resonance takes to become established in a basin is a direct function of the incident energy i.e. of the entrance width.

These two observations are of great importance for by considering the degree of selectivity of the resonance curves the following can be stated:

1) that a basin with a narrow opening whose resonance curve is very pointed is not likely to attain its maximum degree of agitation. This is because the narrowness of the entry does not allow the resonance to be established sufficiently quickly, for the full resonance to be attained, before the period of the exciting wave changes from the resonance period.

2) that on the other hand resonance may be established quickly in a basin with a large opening. And that the range of frequencies which will produce an agitation close to the maximum resonance amplitude is large. Such basins have therefore a good chance of experiencing resonance.

Thus knowledge of the degree of selectivity of the agitation curves gives an indication of the probability of resonance in a basin, as a function of the size of opening. It is still however necessary to know the probability of the resonance exciting waves.

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RESUME

MOUVEMENTS de SEICHES à DEUX DIMENSIONS
DANS UNE DARSE SOUS L'ACTION d'ONDES INCIDENTES

Bernard Le Mehaute

Cette étude porte sur les lois de résonance des mouvements de seiches à deux dimensions dans une darse rectangulaire sous l'action d'ondes venant du large.

La théorie est établie pour les mouvements plans ou pouvant être considérés comme tels. Elle est basée sur le calcul des nombres complexes. Cette méthode, appliquée aux mouvements périodiques, s'avère particulièrement simple et fructueuse, chaque fois que l'approximation se limite aux phénomènes linéaires.

Les lois de résonance des eaux dans une darse sous l'action d'ondes incidentes dépendent essentiellement de l'ouverture reliant l'intérieur du bassin au large. Elles sont donc fonction du type de l'obstacle qui limite la darse. Le mur obstacle est pris ici dans son sens le plus général et caractérise tout changement brusque de section.

On distingue ainsi plusieurs sortes de darses :

- darse sous ouverte, limitée par un obstacle du type obstruction
- darse sur-ouverte, limitée par un obstacle du type approfondissement ou élargissement
- darse limitée par un obstacle complexe répondant simultanément à ces deux caractéristiques.

L'étude théorique commence par une vue d'ensemble rapide des phénomènes plans liés aux obstacles soumis à la houle (déphasage, amplitudes ...). Ainsi est-il possible de connaître la valeur de l'agitation entre deux obstacles, à partir de leurs caractéristiques propres et de la distance qui les sépare.

Une darse sous-ouverte étant limitée d'un côté par un obstacle parfaitement réfléchissant, il est possible d'établir la valeur de l'agitation en fonction des caractéristiques de la passe d'entrée. La valeur de l'agitation de résonance croît lorsque l'ouverture décroît, mais la courbe de résonance devenant plus sélective, les chances de résonance sont donc moindres. Les essais expérimentaux vérifient remarquablement la théorie.

Les chances de résonance dans une darse sur-ouverte sont indépendantes du degré d'ouverture. La valeur de l'agitation croît avec celui-ci.

Enfin des résonances en cascade peuvent se produire, la valeur de l'agitation dans des bassins successifs augmentant à mesure que l'on s'éloigne du large.

Les lois trouvées permettent d'expliquer l'ensemble des mouvements de résonance dans les cas considérés.