Zuider Zee Dam

Part 1

WAVE THEORY AND MEASUREMENTS

Zuider Zee Dam

CHAPTER 1

WIND WAVES AND SWELL

R. L. Wiegel
Associate Professor of Civil Engineering
University of California
Berkeley, California

Winds blowing over the water surface generate waves. In general the higher the wind velocity, the larger the fetch over which it blows, and the longer it blows the higher and longer will be the average waves. Waves still under the action of the winds that created them are called wind waves, or a sea. They are forced waves rather than free waves. They are variable in their direction of advance (Arthur, 1949). They are irregular in the direction of propagation. The flow is rotational due to the shear stress of the wind on the water surface and it is quite turbulent as observations of dye in the water indicates. After the waves leave the generating area their characteristics become somewhat different, principally they are smoother, losing the rough appearance due to the disappearance of the multitude of smaller waves on top of the bigger ones and the whitecaps and spray. When running free of the storm the waves are known as swell. In Fig. 1 are shown some photographs taken in the laboratory of waves still rising under the action of wind and this same wave system after it has left the windy section of the wind-wave tunnel. It can be seen that the freely running swell has a smoother appearance than the waves in the windy section. The motion of the swell is nearly irrotational and non-turbulent, unless the swell runs into other regions where the water is in turbulent motion. Turbulence is a property of the fluid rather than of the wave motion. After the waves have travelled a distance from the generating area they have lost some energy due to air resistance, internal friction, and by large scale turbulent scattering if they run into other storm areas, and the rest of the energy has become spread over a larger area due to the dispersive and angular spreading characteristics of water gravity waves. All of these mechanisms lead to a decrease in energy density. Thus, the waves become lower in height. In addition, due to their dispersive characteristic the component wave periods tend to segregate in such a way that the longest waves lead the main body of waves and the shortest waves form the tail of the main body of waves. Finally, the swell may travel through areas where winds are present, adding new wind waves to old swell, and perhaps directly increasing or decreasing the size of the old swell.

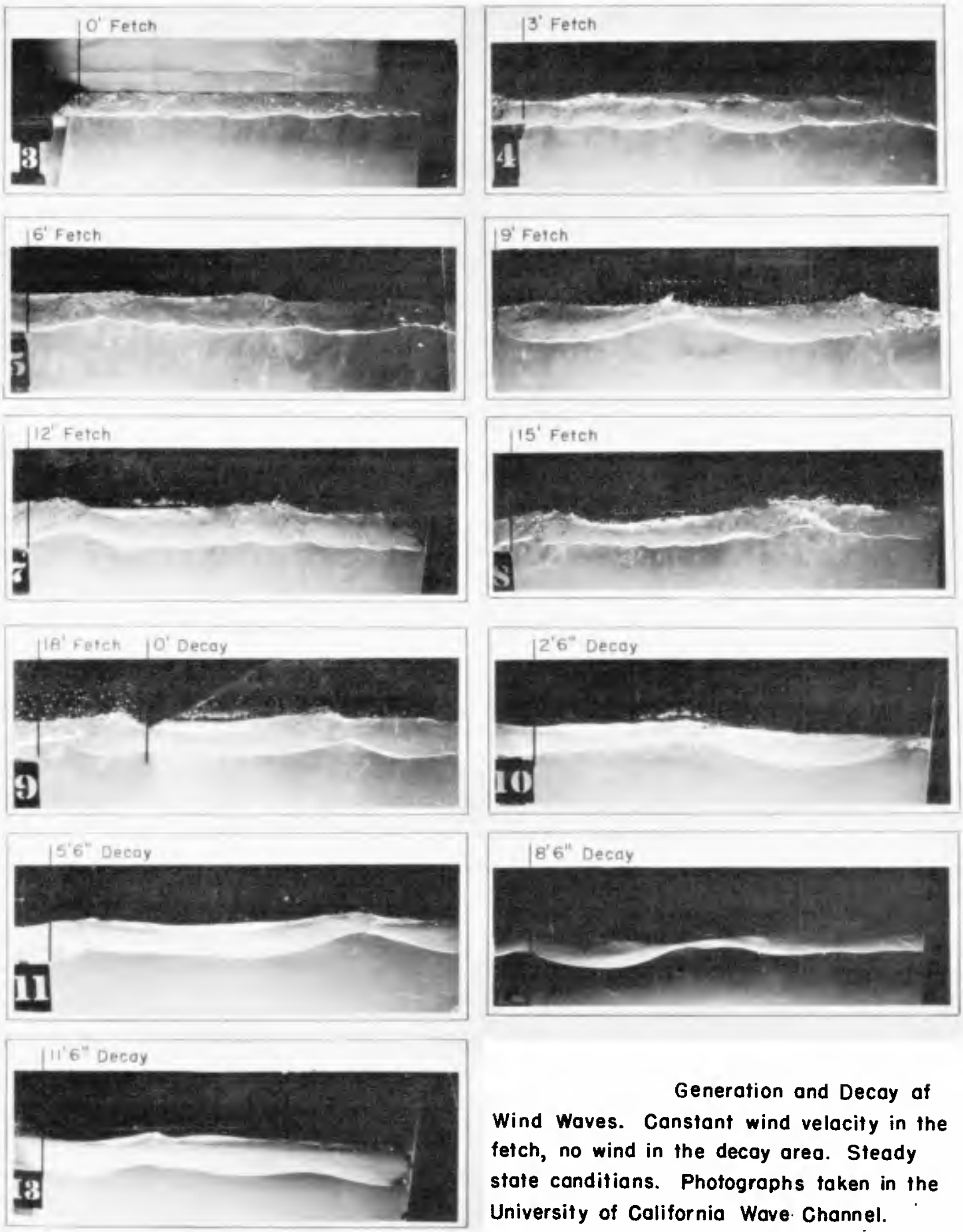
1. Wave Characteristics

An observer stationed high above the area in which wind waves pass from the fetch into an area of calm (called the decay area) would notice that the waves in both regions vary in heights, lengths, and breadths. If he were to follow a particular wave crest he would notice that it would gradually disappear; he would also notice new crests form.

An observer watching the crests leaving the fetch and measuring the time intervals between the successive crests as well as the heights would have a true picture of the surface waves at a particular point without having a true picture of the phenomenon. This is because the surface phenomenon is a result of other complex phenomena. Because there is a spectrum of lengths (or periods) and heights present there must be some sort of a group phenomenon; that is, there are no permanent wave forms. Instead, wave crests and troughs gradually appear and disappear. The longer wave components of the group, traveling with greater speeds than the shorter wave components, gradually move ahead, with the shortest wave components dropping behind. Hence, a spreading of the wave system occurs.

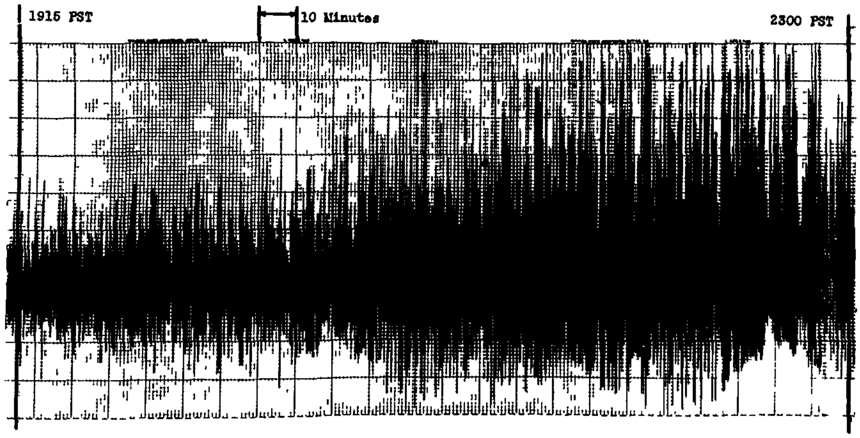
In order to understand what happens in an actual case, where the generating areas vary in size, the winds vary in speed and direction and exist for different lengths of time, it is necessary first to consider the simplified case of a stationary storm of constant dimensions with winds that immediately spring up to a constant speed and remain at that speed for a short time, long enough to generate a considerable number of waves and then die down immediately. In addition, the decay distance must be long enough for complete segregation of wave components to take place. Shortly after the wind starts blowing over the entire generating area the waves will be short, but probably close to maximum steepness ($H/L = 1/7$ in deep water). At some distance downwind from the start of the fetch the waves will gradually grow in height and length as time increases. The maximum wave dimensions at this point will be obtained when all of the waves generated upwind of this point have reached it, this time depending upon the group velocity of the waves. When this has occurred a quasi-steady state condition has been reached.

An observer traveling along with the wave system in the decay area would notice that the long wave components would gradually move in front of the system and short wave components would drop to the rear of the system. The longer the wave system has traveled the greater would be this segregation. If the group were to travel many thousands of miles, and were there no other disturbances, this segregation and stretching of the system would become complete. An observer at a fixed distance from the storm would notice a steady decrease in wave period with increasing time.



Generation and Decay of Wind Waves. Constant wind velocity in the fetch, no wind in the decay area. Steady state conditions. Photographs taken in the University of California Wave Channel.

Fig. 1



Notes: Averages of periods (T) and significant breaker heights $((H_{1/3})_B)$ are for fifteen minute interval on each side of indicating lines.

1915 PST. $T = 12.0$ seconds, $(H_{1/3})_B = 4.0$ feet.
2300 PST. $T = 10.3$ seconds, $(H_{1/3})_B = 8.0$ feet.

Fig. 2. Mark V, No. 1A, record for 1915 to 2300 PST, 18 October, 1949, Camp Pendleton, Oceanside, California.

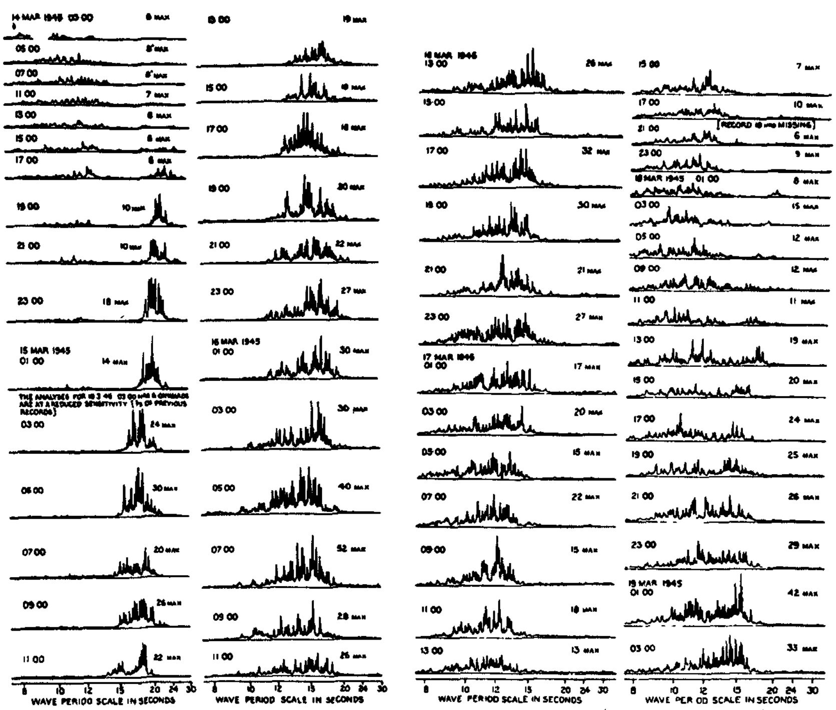


Fig. 3. Wave spectra at Pendeen, 16 to 18 March, 1945.

Actually, the duration of the storm and the relatively short decay distances (even several thousands of miles) are such that complete segregation never takes place. In local storms almost no segregation takes place and the lengths and periods are relatively short, even if the winds were great, the fetch long and the duration long; stretching of wave system does not occur so that the energy density is high. As the decay distance increases the segregation increases and so the long waves, often called "forerunners of storms", reach the coast before the main body of waves. For a particular storm the longer the decay distance the greater is the time between these forerunners and the main body of waves (high energy density).

The normal case is more complicated. For example, suppose the storm lasts for two days and that it takes only one day for the medium length wave component to reach the section of coast under consideration. The longer waves are being continually generated as are all the shorter wave periods. Thus, even as the first of the shortest waves are arriving at the coast the longer waves which were being formed after the shorter waves have left the generating area, overtake the shorter waves and arrive at the same time.

It can be seen then that a section of coast a considerable distance from a storm will be subject to long, low waves first with the mean "period" of the waves (with the "period" being defined as the time necessary for two successive crests to pass a fixed point) decreasing with time but with the period spectrum width about the mean value increasing. The wave "heights" (with the "height" being defined as the vertical distance between a trough and the following crest) will be increasing because the greatest energy density is concentrated in the waves with medium periods. An example of such a "wave front" reaching the coast can be seen in Fig. 2. As the last of the waves reach the coast an abrupt decrease in wave period and height will be observed. The average period of even these short waves, at the coast, will always be longer than the period observed at the end of the fetch because of the spreading phenomenon and because the smallest waves will have been either captured by the large waves or dissipated.

The actual phenomenon is more complex than has been described because the winds gradually rise to a maximum speed, then decrease again, always fluctuating, with the wind field varying in size and the storm moving.

As an example, consider the wave spectra at Pendeen, England, during 14 to 15 March 1945, which has been presented in Fig. 3 (Barber and Ursell, 1948). The wave record at Pendeen, the bottom pressure type, was analyzed by a frequency analyzer so that the component period spectra was obtained. It can be seen that the forerunners

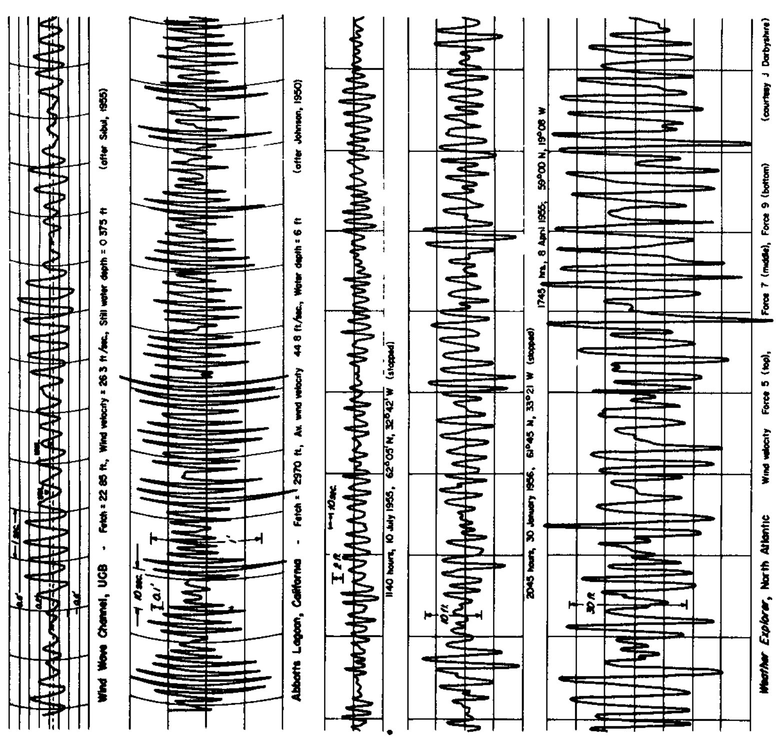


Fig. 4. Sample records of waves unaffected by refraction.

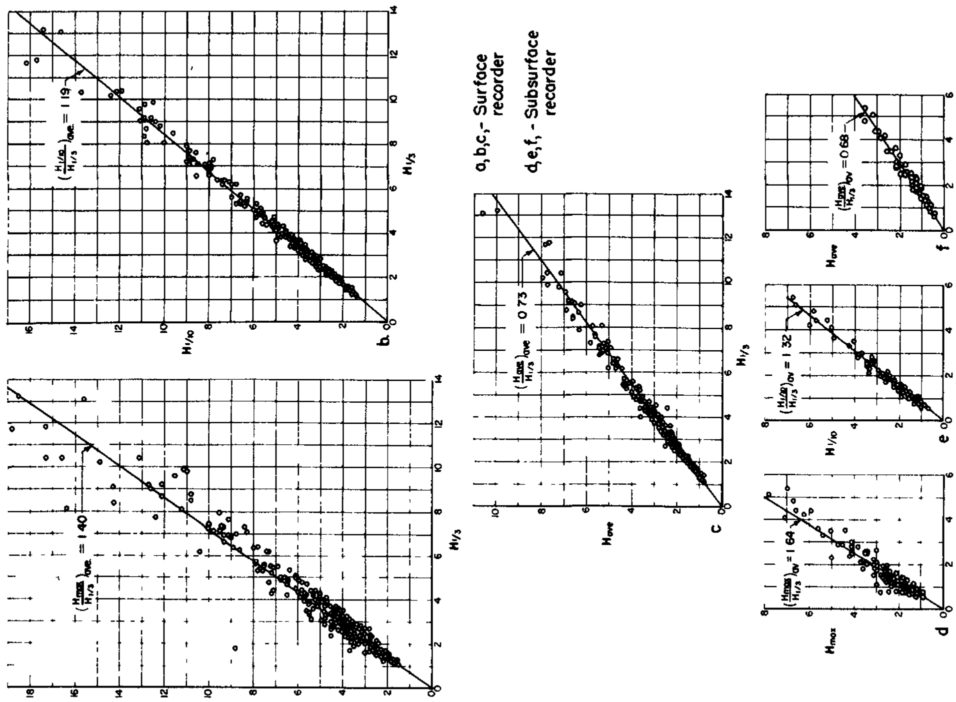


Fig. 5. Wave height relationships (after Wiegel and Kukk, 1957)

of a new wave system first appeared at 1300 on 14 March 1945. The mean of the periods gradually decreased while at the same time the width of the spectra increased.

In the literature on ocean waves the terms gaussian and random are often used when referring to the heights and periods of wind waves and swell. These terms must be used with caution as they oversimplify the true nature of waves and they are not descriptive to the layman of the many groups of nearly periodic waves that exist. That these groups exist can be seen in Fig. 4 in which records of waves in the generating area are presented for laboratory, lakes and ocean conditions. The statistical techniques used in obtaining the information that will be presented in the following sections neglect the time sequence of the phenomena. Thus, one entire class of information is often thrown away in the analysis of waves. From an engineering standpoint it is the groups of several periodic waves, which are almost always the highest waves in a wave system, that are the most effective in causing structural damage. Donn and McGuinness (1959) report that the waves they measured in a relatively open ocean area occurred in striking sinusoidal groups in contrast to the far more irregular patterns observed in the shallow water off coasts in the area.

One of the main problems connected with describing waves is that of defining what we will call a wave. Should every small bump be considered a wave? For some purposes, such as scattering of radar or the reflection and scattering of light, the answer should be yes. For many purposes the answer should be no. One concept that is useful in this respect is to neglect the very small waves and to measure the highest one third of the remaining waves, the average of the highest one third being called the significant or characteristic wave height. This concept was developed during studies of landing craft operations in the surf in World War II. It was found that the wave height estimated by observers corresponded to the average of the highest 20 to 40 percent of the waves (Scripps Institution of Oceanography, 1944). Originally, the term significant wave was attached to the average of these observations, the highest 30 percent of the waves, but has evolved to become the average of the highest 33-1/3 percent (designated as H_S or $H_{1/3}$).

It can be seen in Fig. 4 the higher waves often occur in groups which are nearly periodic. The average period of these high groups in a record was termed the significant period (designated at T_S or $T_{H_{1/3}}$).

In order for the concept of significant wave height and period to be of more value studies were made of the distribution of wave heights and periods about their mean values (Ehring, 1940; Seiwell, 1948; Wiegel, 1949, Rudnick, 1951; Munk and Arthur, 1951; Darbyshire, 1952;

TABLE I. MEASURED STATISTICAL RATIOS FOR OCEAN WAVES

Location and Type Wave Recorder	Statistical Ratios							Remarks
	$\frac{H_{max}}{H_{1/10}}$	$\frac{H_{max}}{H_{1/3}}$	$\frac{H_{max}}{H_{mean}}$	$\frac{H_{1/10}}{H_{1/3}}$	$\frac{H_{1/10}}{H_{mean}}$	$\frac{H_{1/3}}{H_{mean}}$	$\frac{H_{3/10}}{H_{mean}}$	
1a. Davenport, Calif. surface rec'dr		1.40	1.90	1.19	1.61	1.37		11 mos., 12-20 min. interval every 12 hrs water depth 46' MLLW (WIEGEL & KUKK, '57)
1b. Davenport, Calif. bottom-pressure recorder		1.64	2.64	1.32	2.09	1.48		5 mos., 12-20 min. interval every 12 hrs., water depth 46' MLLW (WIEGEL & KUKK, 1957)
2. N. Atlantic Ocean NIO ship-borne recorder			2.40			1.60		138 records, 26-124 waves per record (mean of 69 waves) every 3 hrs. (DARLINGTON, 1954)
3. Pt.Arguello, Calif. bottom-pressure recorder	1.42	1.85		1.30				3 mos., 20 min. every 8 hrs.; figures refer to daily ave. & max., recorder in 75' of water MLLW (WIEGEL, 1949)
4. Pt. Sur, Calif. bottom-pressure recorder	1.46	1.85		1.27				14 mos., 20 min. interval every 8 hrs.; figures refer to daily ave. & max.; recorder in 68' of water MLLW (WIEGEL, 1949)
5. Heceta Head, Ore. bottom-pressure recorder	1.47	1.91		1.30				14 mos., 20 min. interval every 8 hrs.; figures refer to daily ave. & max.; recorder in 50' of water MLLW (WIEGEL, 1949)
6. Cuttyhunk, Mass. bottom-pressure recorder						1.57		10 mos., 20 min. interval every 6 hrs., wave recorder in 75' of water (SEIWELL, 1948)
7. Bermuda bottom-pressure recorder						1.57		4 mos., 20 min. interval every 2 hrs., recorder in 120' of water (SEIWELL, 1948)
8. Cape Cod, Mass.							1.56	1 record, 14 waves; 1 record, 28 waves, waves only 2-3 sec. period (GIBSON, 1944)
9. LaJolla, Calif. bottom-pressure recorder		1.63				1.49		46 waves (MUNK & ARTHUR, 1951)
10. North Sea pressure recorder hanging on cable below float						1.85		688 waves (every bump counted as a wave); wind waves (EHRING, 1940)
		1.5				(1.58)		517 waves (neglecting all waves less than 14.8 cm high in above sample) (HARNEY, SAUR & ROBINSON, 1949)
11. Pacific Coast, U.S.A. & Guam, M.I., bottom-pressure recorder						1.63		25 records, 20 min. interval (PUTZ, 1952)
12. Greymouth, N. Z. bottom pressure recorder				1.24	1.94	1.58		109 records, 17 min. interval, every 2 hrs. (WATTERS, 1953)
13. Hachijo I. Japan (surface ?)			2.0			1.5		11 records, 10 min. interval, various locations (YOSHIDA, KAJIURA & HIDAKA, 1953)
14. Long Branch, N.J. BEB surface recorder		1.29	1.93			1.50		1 record of 102 waves (PIERSON, NEUMANN & JAMES, 1955)
15. Bermuda, NIO ship-borne recorder				1.24		1.61		6 records, 12-18 min. interval (FARMER, 1956)
16. Oceanside, Calif. bottom-pressure recorder				1.25				1 mo., 20 min. interval every 8 hrs. (WIEGEL & KUKK, 1957)

Putz, 1952; Pierson and Marks, 1952; Watters, 1953; Yoshida, Kajuira, and Hidaka, 1953; and Darlington, 1954). These papers advanced the concept of the "statistical nature" of ocean waves. Many of the data were summarized by Longuet-Higgins (1952) in his paper on the statistical distribution of heights of ocean waves.

Almost all of the data were obtained from bottom pressure-type recorders, with their inherent practical and theoretical limitations (Folsom, 1949; Pierson and Marks, 1952; Fuchs, 1955; Gerhardt, Jehn and Katz, 1955; Neuman, 1955). Similar studies were made with both surface recorders and bottom pressure recorders by Wiegel and Kukk (1957), the results of which are shown in Fig. 5. It was found that the statistical ratios obtained by measuring the waves at the surface differed from the ratios obtained from the sub-surface pressure recorders (Table 1). It is believed that the difference, and the sign of the difference, can be explained by the method of analyzing the records of the sub-surface pressure recorders combined with the fact that the sub-surface dynamic pressures decrease more rapidly with depth for the lower period waves than for the longer period waves. Of particular importance is the fact that the surface measurements in the ocean of wind waves and swell of Wiegel and Kukk (1957) resulted in nearly the same ratios as those of the surface measurements by Sibul (1955) of wind-waves generated in a wind-wave tunnel. Although many of the measurements of Sibul were for waves in relatively shallow water the author states that no observable shallow water effect was noticed in the wave height ratios. It should be noted that the wave height ratios obtained by Wiegel and Kukk were for swell with wind waves often superimposed, while the ratios obtained by Sibul were only for wind waves. It appears then that the wave height distribution about a mean value is about the same for swell as for wind waves.

Several investigators were able to fit existing mathematical curves to the empirical wave height data, curves of the "random distribution" type (Putz, 1952; Longuet-Higgins, 1952; and Walters, 1953). The work of Longuet-Higgins is in most general use (it predicts nearly the same values as do the curves of Putz). It allows the prediction of the most probable maximum wave height for a given number of waves, providing the mean wave height (or some other measure of wave height) is known. These values are presented in Table 2. In this table N is the number of consecutive waves considered and $\bar{\mu}(a_{max})$ is the most probable maximum wave amplitude (half the wave height) that will occur in N consecutive waves if the sample has a root mean square wave height of $2\,\bar{a}$. In order to find the most probable maximum wave to expect in N waves if the significant height is known, rather than the root mean square wave height, it is only necessary to divide the value $\mu(amax)$ by 1.416 (Table 3). For example, if N = 500 waves, from

Table 2

Value of E $(a_{max})/\bar{a}$ and μ $(a_{max})/\bar{a}$ for different values of N, for a narrow spectrum (after Longuet-Higgins, 1952)

N	$(\log N)^{\frac{1}{2}}$	E $(a_{max})/$ a exact expression	E $(a_{max})/$ a asymptotic expression	$\mu(a_{max})/\bar{a}$
1	0.000	0.886		0.707
2	0.833	1.146		1.030
5	1.269	1.462		1.366
10	1.517	1.676	1.708	1.583
20	1.731	1.870	1.898	1.778
50	1.978	--	2.124	2.010
100	2.146	--	2.280	2.172
200	2.302	--	2.426	2.323
500	2.493	--	2.609	2.509
1,000	2.628	--	2.738	2.642
2,000	2.757	--	2.862	2.769
5,000	2.918	--	3.017	2.929
10,000	3.035	--	3.130	3.044
20,000	3.147	--	3.239	3.155
50,000	3.289	--	3.377	3.296
100,000	3.393	--	3.478	3.400

Table 2, μ (amax) / $\bar{a}$ = 2.509, from Table 3 $a^{(0.333)}/\bar{a}$ = 1.416, and the most probable maximum wave height/significant wave height = 2.509/1.416 = 1.77. If the significant wave was 10.0 ft. then the most probable maximum wave height would have been 17.7 ft.

The entire spectrum of wave heights can be obtained by use of Table 3.

Table 3

Representative values of $a^{(p)}/\bar{a}$ in the case of a narrow wave spectrum (after Longuet-Higgins, 1952).

p	$a^{(p)}/\bar{a}$	p	$a^{(p)}/\bar{a}$
0.01	2.359	0.4	1.347
0.05	1.986	0.5	1.256
0.1	1.800	0.6	1.176
0.2	1.591	0.7	1.102
0.25	1.517	0.8	1.031
0.3	1.454	0.9	0.961
0.3333	1.416	1.0	0.886

In Table 3 p = 0.1 refers to the average of the highest one tenth of the waves, p = 0.333 refers to the average of the highest one third of the waves p = 1.0 refers to the mean of all of the waves, etc. The ratio $H_{1/10}/H_{1/3}$ can be obtained from Table 3 as 1.800/1.416 = 1.28; and the ratio $H_{1/3}/H_{mean}$ as 1.416/0.886 = 1.60. It is evident from a comparison of these values with those shown in Table 1 that the wave height distribution function of Longuet-Higgins is of practical importance.

The wave height distribution function of Longuet-Higgins is for the case of a narrow wave spectrum. For a wide spectrum the work of Cartwright and Longuet-Higgins (1956; also, Williams and Cartwright, should be considered.

Another important set of statistical data deals with the wave period (or frequency) spectrum. Putz (1952) made measurements of twenty-five wave records (bottom pressure type), each record of approximately twenty-minute duration. He found the relationship between the significant wave period to the mean wave period shown in Fig. 6. The complete period distribution function of Putz is shown in Fig. 7. As an example of its use, suppose the significant period was 11.5 sec., then from Fig. 6 it would be found that the mean wave period would be 11.0 sec. From Fig. 7 it would be found that 99.5 percent of the wave periods would be less than 18 seconds and that only 0.5 percent would be less than 4.5 sec. Of more importance in the consideration of waves in the generating area is the work of Darbyshire (1959), the data on wave frequency distribution being given in Fig. 14.

Darlington (1954) has made a similar study of wave records obtained with the NIO shipborne wave recorder (Tucker, 1956) in the North Atlantic. Most of the recordings reported were made with the ship stopped. As can be seen in Table 1 his results on wave height distribution agree well with those of Putz, and the data extended the results to a mean wave height up to 28 ft. with a maximum wave height of 42 ft. The results of the relationship between the period of the highest one third of the waves and the mean period are shown in Fig. 6. The results are not the same as those obtained by Putz, but are not too different. Part of the difference must be due to the fact that Putz's records were obtained from a bottom pressure recorder. In addition, Darlington's measurements were made in storm areas (with background swell in many cases), while Putz's measurements were mostly of swell. It is interesting to note that both results show that for lower period waves T_m should be greater than $T_{H_{1/3}}$. Darlington found that by interpolating the best fit straight line that $T_m > T_{H_{1/3}}$ for $T_m < 6$ sec, while Putz found that $T_m > T_{H_{1/3}}$ for $T_m < 9$ sec. No actual measurements of this condition were made however, Both of these findings are in conflict with Sibul's (1955) measurements of short period waves generated in a wind-wave tunnel, Fig. 8. It would appear then that the relationship

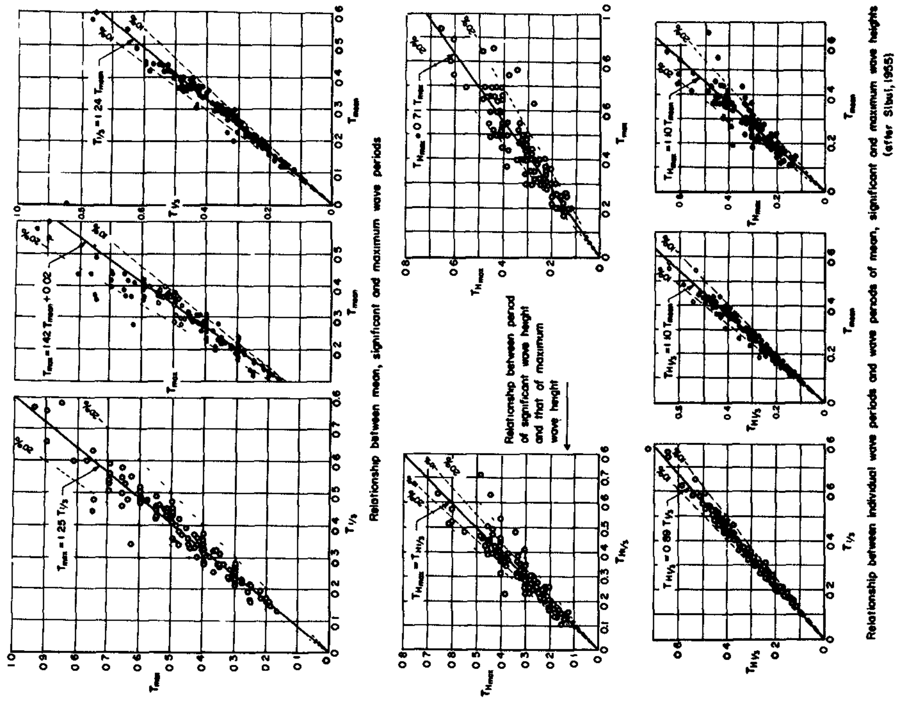


Fig. 8

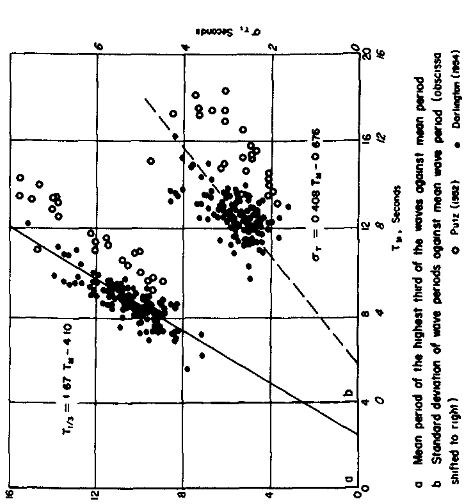


Fig. 6

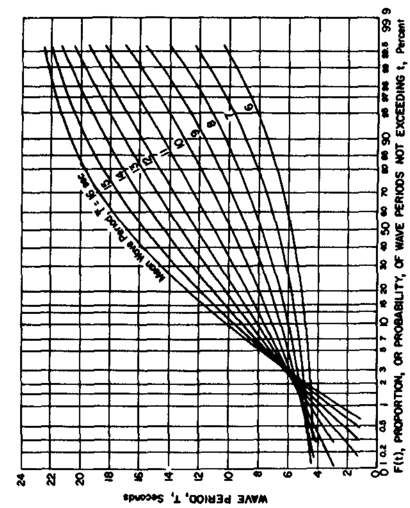


Fig. 7. Prediction curves for wave period distributions for various mean periods (after Putz, 1952).

between $T_{H_{1/3}}$ and T_m is a non-linear one. Part of the difference may be that both Putz's and Darlington's wave measurements are either wholly or partially dependent upon the prediction of surface waves from sub-surface pressure measurements with some inherent difficulties in measuring the true mean and significant wave periods.

What is the relationship between the heights and periods of the wind waves? In Fig. 9 are shown the relationship between T_{mean} (the length of the wave record divided by the number of waves in the record) and $H_{1/3}$. These two parameters were chosen as they were the most generally available ones. The data shown are for wind waves rather than swell, although there may be some swell present in the records of Bretschneider (1954) and Darlington (1954). A line was drawn through the wind-wave tunnel data of Sibul (1955), neglecting the smallest waves as these probably were affected considerably by surface tension. This line was extended through several cycles on the log-log paper. This line is expressed by

$$H_{1/3} = 0.45\ T_{mean}^2 \tag{1}$$

If we assume that for non-periodic waves $L = gT^2/2\pi$

$$\frac{H_{1/3}}{(L_{mean\ o})} = \frac{1}{11.4} \tag{2}$$

If we assume that the relationship derived by Pierson, Neumann and James (1955) for a gaussian sea surface is correct, that is the apparent wave length L* is about 2/3 the value of a period wave, then

$$\frac{H}{(L^*_{mean\ o})} = \frac{1}{11.4} \times \frac{3}{2} = \frac{1}{7.6} \tag{3}$$

which is very close to the theoretical maximum wave steepness, and nearly identical to the maximum steepness of mechanically generated waves. This appears to be the physical reason for the limit of the maximum wave heights; for a given mean wave period this is the maximum that can exist without breaking.

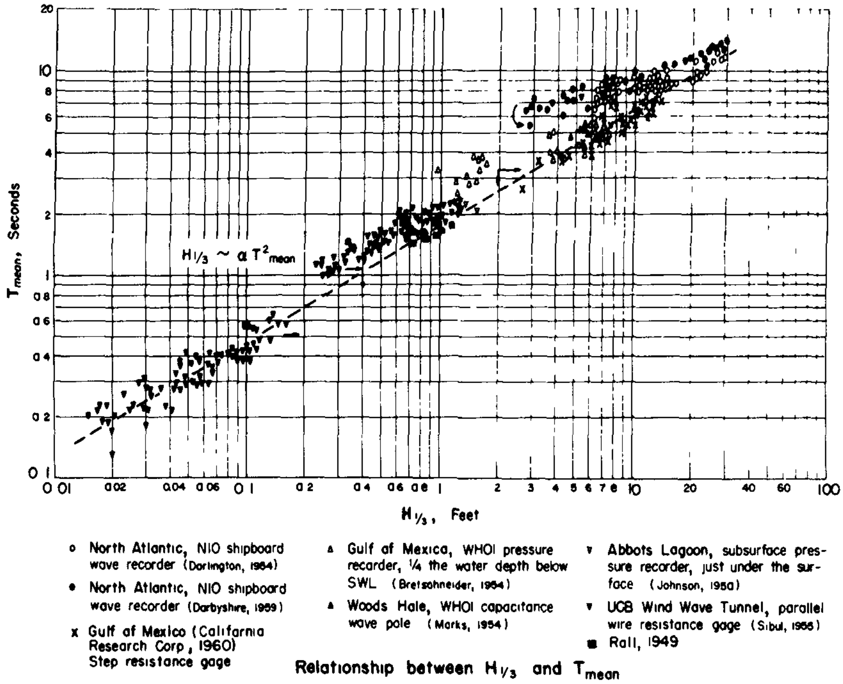
The California Research Corporation (1960) has measured waves from several oil platforms in the Gulf of Mexico using a step resistance gage that is essentially self-calibrating in that as each electrode is shorted by the sea water passing over it a "step" occurs on the record. The data shown in Fig. 9 are for four hurricanes in the Gulf of Mexico, with the measurements being made in water 30 feet deep. Each point shown was obtained from a sample of between 100 and 200 waves. The fact that the water was not deep should be considered as the maximum wave steepness in transitional water is not as great as it is in deep water On the other hand, some of the data were for essentially deep water waves, and these data show the same trend with respect to the dashed line as do transitional waves.

The extension of the line through Sibul's data does not go through all of the data taken in the field. Many of the field data were obtained with instruments either entirely or partially dependent upon subsurface pressure measurements. It is known that these pressure measurements projected to surface values by use of the first order theory relationship between wave height and sub-surface pressures, when using an average pressure factor associated with a mean period, underpredicts the surface wave by an average of about 25 percent (Folsom, 1959; Gerhardt, John and Katz, 1955). This same technique utilizes a subjective method of determining the number of waves in a record which when combined with rapid attenuation with distance below the water surface of the short period wave components, results in a mean wave period that is longer than would be the case in analyzing a surface wave record. The field data, then, should be shifted in the manner indicated by the arrows in Fig. 9. The data of Darlington (1954) and Darbyshire (1959) should probably be rotated in the manner shown in the figure as the recorder used in obtaining the data is very insensitive to waves with periods less than about 6 sec (Marks, 1955; Marks, 1956; Williams and Cartwright, 1957). In addition, the NIO shipborne recorder utilized a vertical acceleration sensing device, the output of which must be integrated twice to give a reference for the pressure cells. The reliability of the results of such a technique are not yet fully understood. This would tend to make them conform to the other data.

The data shown in Fig. 9 due to Sibul (1955), Johnson (1950) and the California Research Corporation (1960) are for only wind waves while these of Bretschneider (1954) and Darlington (1954) probably include swell as well as wind waves. Studies made of data largely of swell do not show this trend, as can be seen in Fig. 10 (Putz, 1951).

Is there any clear relationship between the heights and periods of individual waves? For wind waves generated in the laboratory the answer is a qualified yes. The results for this case can be seen in Fig. 11 (Johnson and Rice, 1952). In general, the longer the wave the higher the wave. For swell, or a mixture of swell and wind waves the joint frequency distributions are shown in Fig. 12 (Putz, 1951, 1952). There is no evidence of a relationship between these quantities although there is a tendency for highest wave heights to occur with near average periods.

Ocean waves can be considered to be the combination of a series



Relationship between $H_{1/3}$ and T_{mean}

Fig. 9

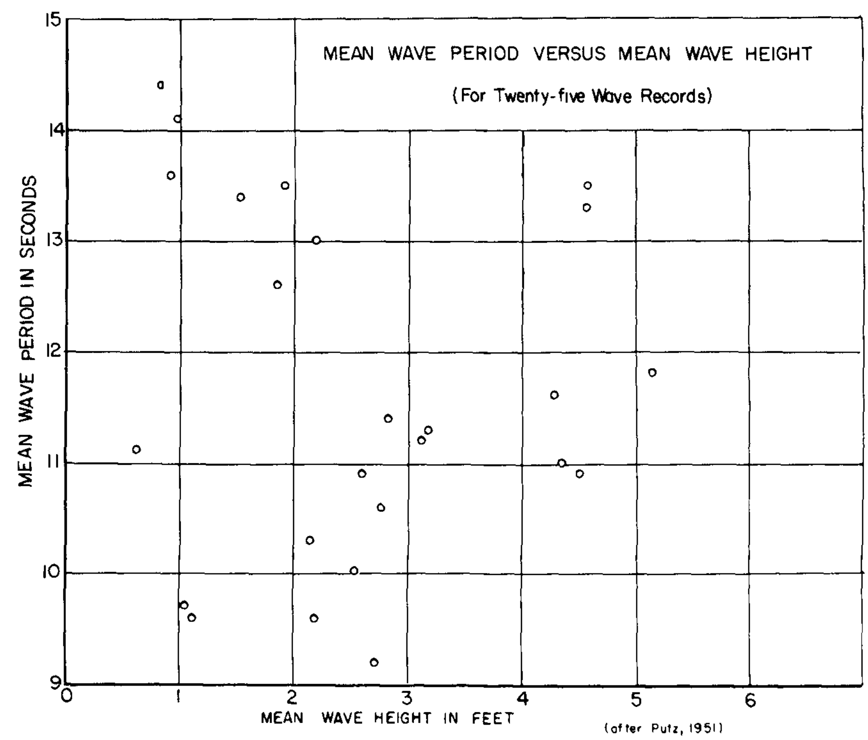
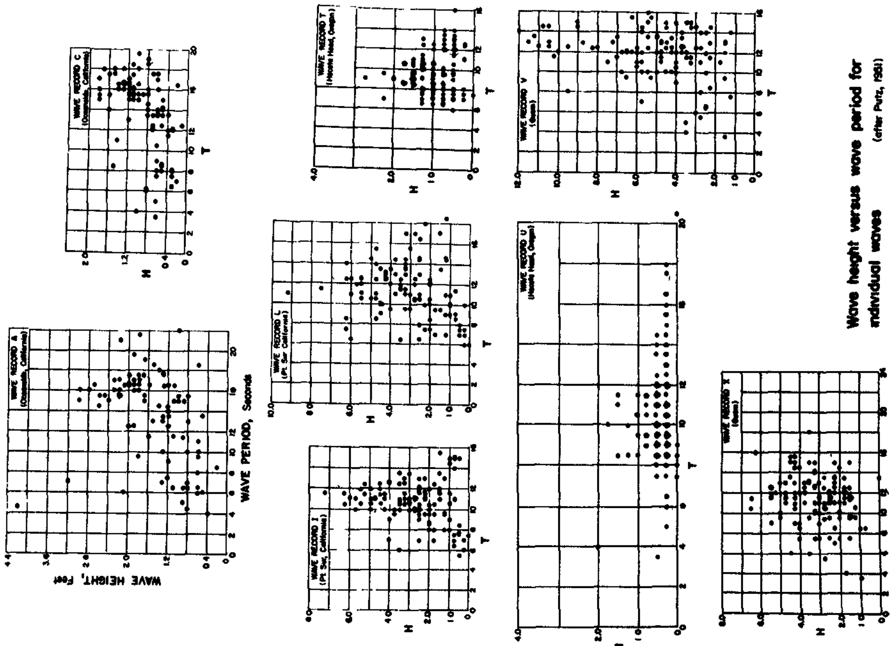
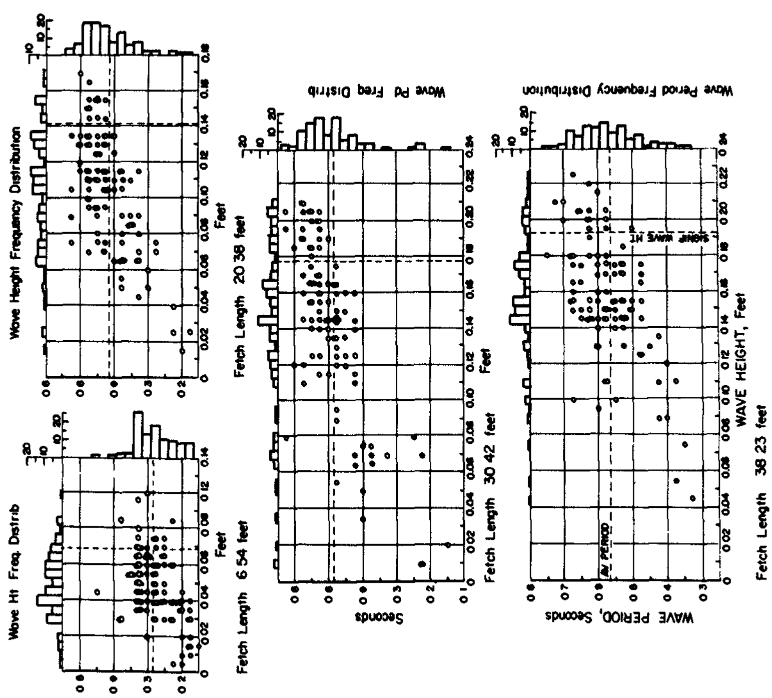


Fig. 10



Wave height versus wave period for individual waves (after Putz, 1951)

Fig. 12



Joint frequency distribution of wave period and wave height (U = 42 7 ft/sec) (after Johnson and Rice, 1952)

Fig. 11

of components of different periods, or frequencies, with a certain amount of power being transmitted by each component. In describing waves using this concept the term wave spectrum is used. The "spectrum" describes in some manner the distribution of the energy density present in a wave system with respect to the wave period, or frequency. An example of one type of wave spectrum is shown in Fig. 3.

Darbyshire (1959) has obtained the wave spectra for recorded waves of a large number of storms in the North Atlantic using the NIO shipborne wave recorder. The records were analyzed first to obtain values of $T_{H1/3}$, $H_{1/3}$, etc., using a wave recorder calibration curve based upon the wave component percent associated with f_o. Then the records were analyzed by an approximate Fourier method to obtain certain information on the wave spectra, namely, the square of the wave height components, H_f, within each frequency interval $\Delta f = 0.007$ sec.$^{-1}$. f_o was defined as the frequency of the class having the largest value of H_f^2 on the spectrum. The heights H_f were corrected using a calibration curve based upon the component period associated with each frequency f. The wave period associated with this, T_{fo}, was found to be closely related to the significant wave period (T_{fo} = $1.14\ T_{H1/3}$) as is shown in Fig. 13. It appears from this, and from the relationships shown by Putz and Sibul, that the choice of the significant wave to describe waves was a good one.

Darbyshire (1959) has found a consistent relationship between H_f^2/H^2 and $f - f_o$ (Fig. 14), where H is defined as the height of a hypothetical single sine wave train which has the same energy density as the actual wave system. The close relationship between this hypothetical wave and other surface wave characteristics can be seen in Fig. 15, where $H_{max} = 2.40$ H and $H = 0.604\ H_{1/3}$ with very little scatter of the data.

The scatter of data of H_f^2/H^2 vs $f - f_o$ is considerable when compared with the scatter of data for $H_{1/3}$ vs H_m, $T_{H1/3}$ vs T_m. For example, if H were 10 ft., H_f could be between 41 and 56 ft. at its maximum.

As pointed out by Darbyshire this empirically determined spectrum is inconsistent with the previous spectrum of Darbyshire (1952; 1955), the spectrum of Neumann (1953) and the spectrum of Roll and Fisher (1956).

Burling (1955) computed the spectra associated with various fetches and meteorological conditions on a reservoir (fetches from 1200 to 4000 feet and wind speeds from about 15 to 25 ft/sec. The results are shown in Fig. 16a, where ω is the circular wave frequency component $(2\pi/T)$ and

$$\Phi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \xi(X, t + \tau)\, \xi(X, t + \tau)^{-i\omega\tau} d\tau \tag{4}$$

Eq. 4 is given in the form of Phillips (1958b) where ξ (x,t) is the surface displacement at fixed point, and τ is a time displacement. $\Phi(\omega)$ has the dimensions of ft^2 - sec, (or cm^2 - sec) and the area under the curve $\Phi(\omega)$ versus ω is related to the energy per unit area being transmitted by the wave system. It is important to note that Burling's data seem to lie along a single curve for circular frequencies greater than about 6 radians/sec (wave periods smaller than about 1 sec). The curve in this region has been developed theoretically by Phillips (1958b).

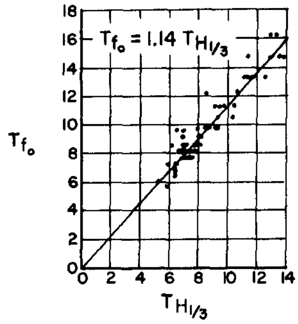
$$\Phi(\omega) \approx \alpha g^2 \omega^{-5} = \text{constant}/\omega^5 \tag{5}$$

The solid curves in Fig. 16 are $\Phi(\omega) = 7.4 \times 10^{-3} g^2 \omega^{-5}$, in cgs units. In Fig. 16b the results of Project SWOP (Chase, et al, 1957) are shown compared with Eq. 5 (Phillips, 1958c). This extends the relationship shown in Fig. 16a to ocean conditions, and it appears that the "fully developed sea" extends only to wave components down to $\omega \approx 2$; i.e., wave period components up to about 3 seconds. The physical significance of this limiting curve is that a wave of a given frequency can increase in energy only up to a certain limit ($H/L = 1/7$ in deep water for a uniform periodic wave). Hence, if the fetch and duration are long enough $\Phi(\omega)$ must have a unique value which depends only upon the frequency.

Some of the differences between the spectra of different investigators have been explained by Pierson (1959a) based upon a random nonlinear model of waves (Tick, 1958). An additional spectrum plus a discussion of the above cited spectra have been presented by Bretschneider (1959).

Directional spectrum have been measured by Chase et al (1957), but their results are difficult to interpret. Phillips (1958c) and Cox (1958) offer conflicting interpretations of the results. The major difficulty, aside from experimental errors is that the measurements were made of ocean waves, and it is not possible to be sure of the level of wave action with respect to the local winds.

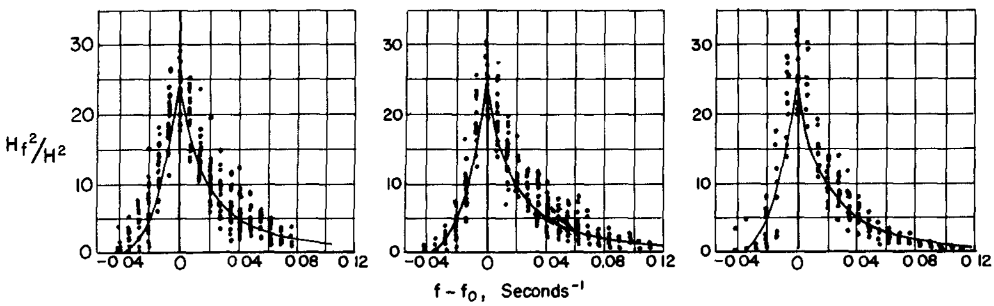
Wave are short-crested: that is, they have a dimension in the horizontal direction normal to the direction of wave advance. This dimension has been termed the crest length of a wave. There are very few measurements of this wave dimension. The average ratio of crest length to wave length (L'/L) of some waves measured on aerial photographs was from 2 to 3. Johnson (1948) made some measurements on waves in a lake and found (L'/L) = 3. Yampol'ski (1955) found that the distribution curves for wave lengths and crest lengths were nearly



Relationship between T_{f_o} and $T_{H_{1/3}}$

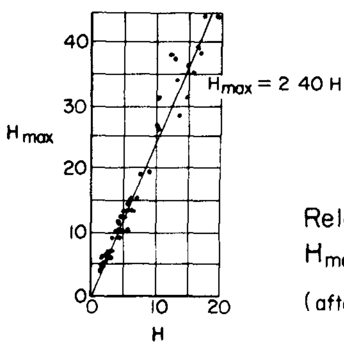
(after Darbyshire, 1959)

Fig. 13



Relationship between $H_f^{\,2}/H^2$ and $f - f_0$

Fig. 14



Relationship between H_{max} and H

(after Darbyshire, 1959)

Fig. 15

identical in appearance, the measurements being made from aerial photo graphs of the sea surface.

A three-dimensional laboratory study was made by Ralls and Wiegel (1956) of wind-generated waves in which the crest length was meas ured as shown in Fig. 17 (notice surface tension waves also in this figure). The results of measurements of crest lengths and wave lengths are shown in Fig. 18. In deep water the ratio is approximately 3.

2. Relationships among Wave Dimensions, Winds, and Fetches

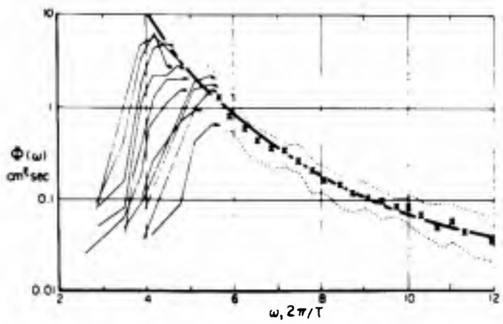
The relationships among the wave dimensions, wind speeds and direction, atmospheric stability, fetch length, and wind and fetch variability are not well established. It is not surprising, considering the extreme difficulty in obtaining reliable measurements in the open ocean combined with the near impossibility of obtaining sea conditions consisting of only wind waves, with no swell present. To these difficulties must be added the problems of determining the wind fetches and durations. Because of this the data obtained will be presented, but ideas from studying data obtained in the laboratory and in lakes will be used to interpret the ocean data.

First, let us consider waves from the standpoint of wave spectra. The data of Burling (1955) has already been covered. The remaining data will pertain to ocean waves.

In deep water the average power, averaged over a complete cycle, transmitted per unit area of surface is proportional to the square of the wave height and inversely proportional to the wave period. The energy per unit area, the energy density, is proportional to the square of the wave height. Darbyshire (1952; 1955) used the equation

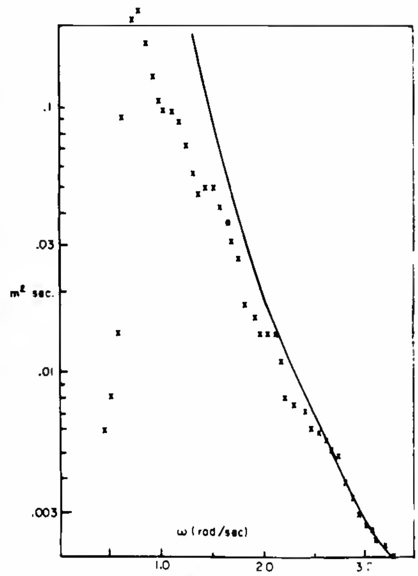
$$E_T = \frac{1}{8} \rho g \sum_{T-\frac{1}{2}}^{T+\frac{1}{2}} h_n^2 = \frac{1}{8} \rho g H_T^2$$

where E_T is the energy per unit area of horizontal sea surface in a wave-period interval from $T-\frac{1}{2}$ to $T+\frac{1}{2}$, and H_T is defined as the equivalent wave height for waves of period between $T - \frac{1}{2}$ and $T + \frac{1}{2}$ (that is, in a one-second interval) as obtained by a Fourier analysis and taking the square root of the sum of the squares of the peaks within each one-second interval of wave period. The results, plotted in terms of H_T/T versus T/U, where U is the gradient wind speed, are shown in Fig. 19. The solid line curve shown in the figures is the gaussian function that Darbyshire states expresses the relationship between H_T/T and T/U. It is evident that the scatter is extreme, and that the relationship between the



Spectra of wind-generated waves measured by Burling (1955). The cluster of lines on the left are representative of the spectra at low frequencies for which equilibrium has not been attained. On the right the curves merge over the equilibrium range, and the dotted lines indicate the extreme measured values of $\Phi(\omega)$ at each frequency ω. The x's represent the mean observed value at each ω, and the heavy line the relation $\Phi(\omega) = \alpha g^2 \omega^{-6}$ with $\alpha = 7.4 \times 10^{-5}$. (after Phillips, 1958 b)

(a)



The frequency spectrum $\Phi(\omega)$ from the S.W.O.P. wave pole data. The solid curve represents the equilibrium range spectrum $\Phi(\omega) \sim 7.4 \times 10^{-5} g^2 \omega^{-5}$. (after Phillips, 1958c)

(b)

Fig. 16

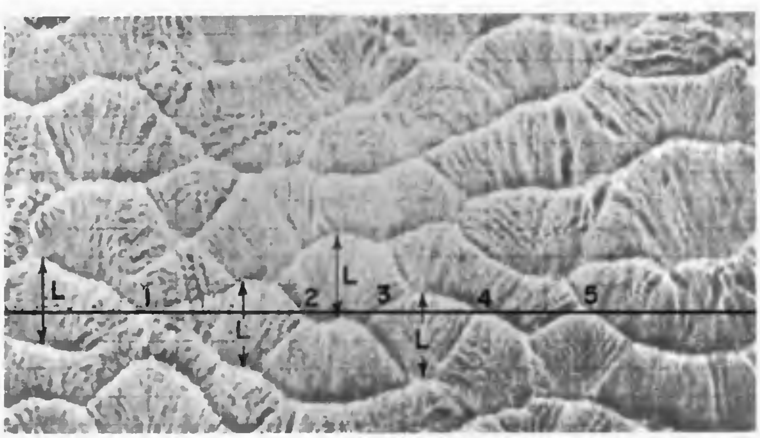


Fig. 17. Distribution method of measuring crest lengths (after Ralls and Wiegel, 1956).

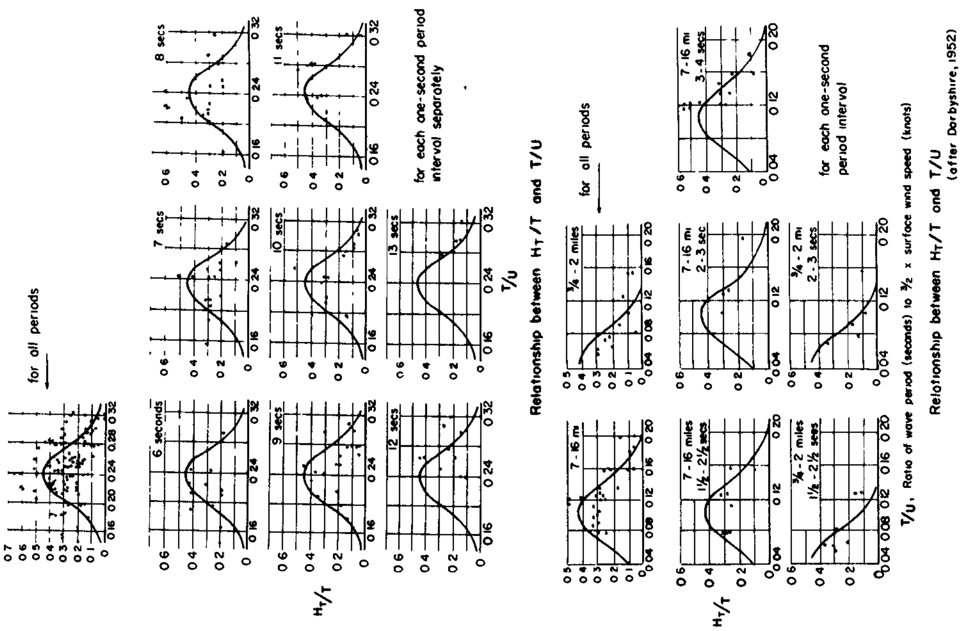

for all periods
for each one-second period interval separately
Relationship between H_T/T and T/U
for all periods
for each one-second period interval
T/U, Ratio of wave period (seconds) to 3/2 x surface wind speed (knots)
Relationship between H_T/T and T/U
(after Darbyshire, 1952)

Fig. 19

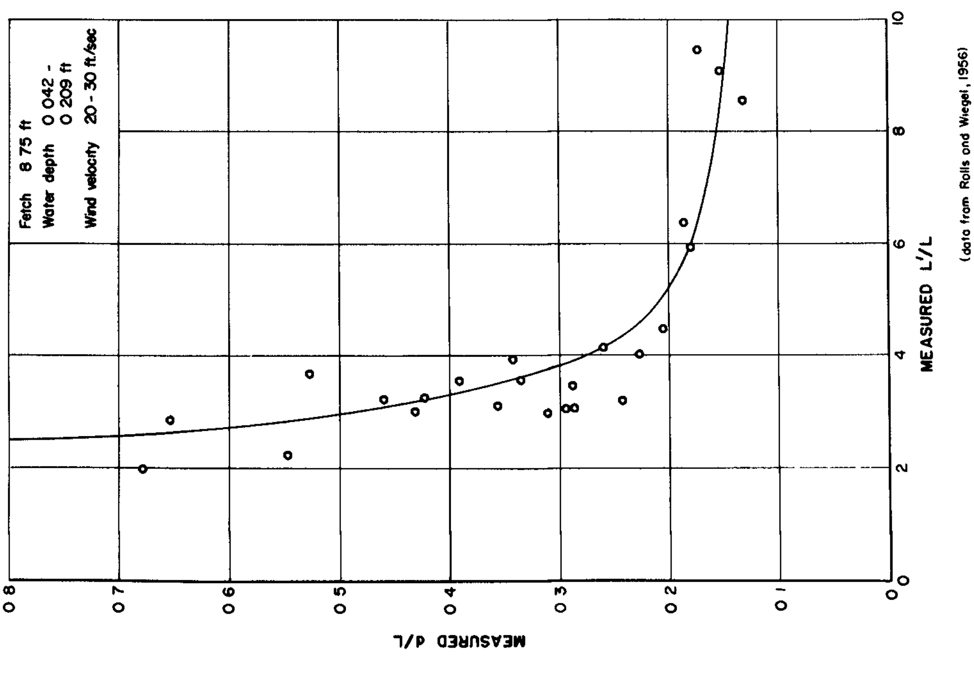

Fetch 8 75 ft
Water depth 0 042 - 0 209 ft
Wind velocity 20 - 30 ft/sec
MEASURED d/L
MEASURED L'/L
(data from Rolls and Wiegel, 1956)

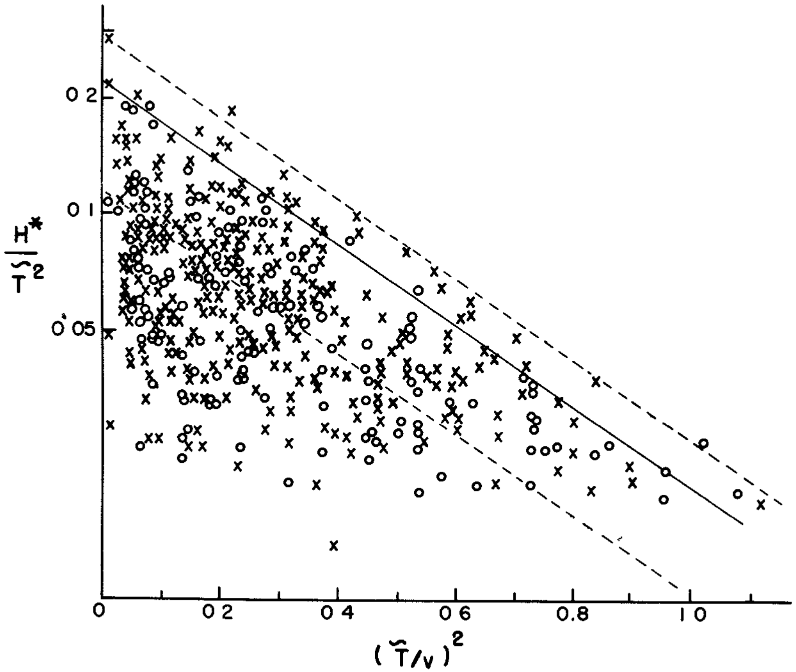
Fig. 18

data and the curves practically non-existent. Darbyshire has found a much more reliable spectrum, leaving out the relationship of the spectrum to the wind (Fig. 14).

Other data for which a spectrum equation has been developed are shown in Fig. 20 (Neumann, 1953). The relationships among wave height, wave period and wind speed, as given by Neumann, is stated to be an exponential curve, and that this curve forms the upper envelope of the data in Fig. 20. The values of H and T used in this figure refer to the height and period of surface waves, many of them visual observations from a ship. They do not refer to data obtained by a Fourier analysis or similar method. This upper envelope is considered by Neumann to represent the "fully arisen sea," where the term refers to the case where no more energy can be added to the wave system regardless of the length of time the wind blows or the length of the fetch (Pierson, Neumann and James, 1955). This means that the waves, at all frequencies, are dissipating energy and radiating it from the storm area at the same rate that wind is transferring wave energy to the sea surface. It is doubtful that this condition exists in the open ocean for all frequencies although it does exist for the higher frequency components (Fig. 16). It is, in fact, contradicted by the tendency for wave steepness to decrease with increasing values of T/U.

Some actual measurements of ocean wave spectra have been made by Ijima (1957), but these are for cases so complicated that it is difficult to compare anything but their gross characteristics with the simple expressions of wave spectra (Fig. 21). The spectrum for a relatively simple case is shown in Fig. 16b (Chase, et al, 1957).

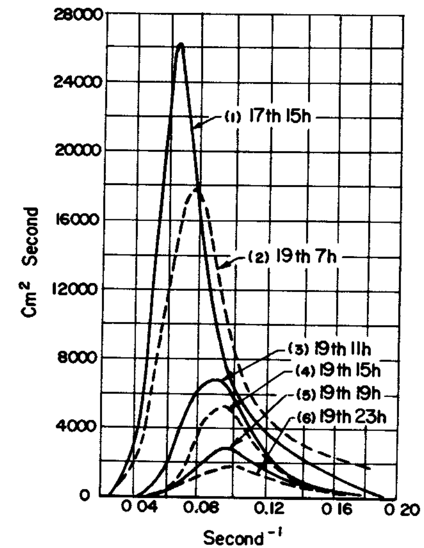
It is difficult to obtain certain data for wind waves in the ocean, such as the wave height and period as a function of wind speed for a constant fetch, the wave height and period as a function of fetch for a constant wind speed. In fact it is difficult even to obtain data for either a constant wind speed or fetch, or for even a stationary storm. It is possible to obtain these data in the laboratory or in relatively small lakes or reservoirs. These data are presented in Figs. 22 and 23, together with some ocean data where the fetches were not constant, nor were the fetch limits even stated. The wind velocities were often measured at different elevations above the water surface. However, the elevation where the winds are measured, as long as it is fairly close to the surface, should not affect the power relationship between H and U and T and U, although it will affect the constant of proportionality. That this is true can be seen from a study of the wind-speed data of Thornthwaite and Kaser (1943) for winds at 1/2 and at 15 ft. above the ground, the data of Deacon (1949) for winds at 1/2 and 4 inches above the ground, and the data of Deacon, Sheppard and Webb (1956) for winds at 6.4 and 13 meters above the sea surface. These data show that the winds are linearly related (Fig. 24), the



Ratio of the wave heights to the square of the apparent wave periods plotted against the square of the ratio of the apparent wave period to the wind velocity

(after Neumann, 1953)

Fig. 20



Time changes of wave energy spectrum in the decrease of swell of a typhoon

(after Ijima, 1957)

Fig. 21

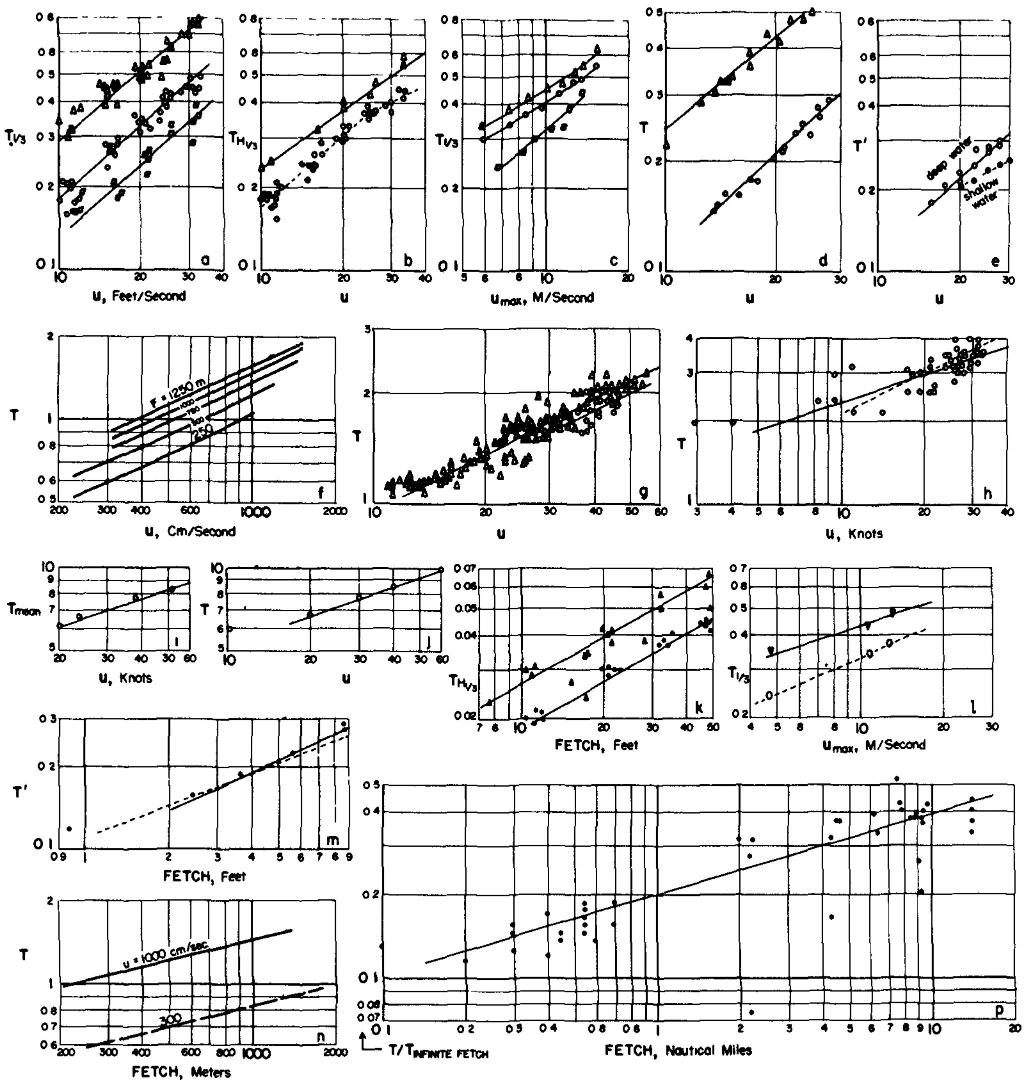
slope of the line on the log-log plot being unity.

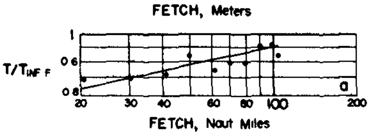
There is scatter in the data. Some data were taken for neutral stability winds (all of Sibul's laboratory data, for example), some of the data for unstable winds and some of the data for stable winds. A good deal of the scatter is probably due to the different wind types. That this is so has been shown by Roll (1952), Burling (1955) and Brown (1955). The data of Brown are presented in Fig. 25. It can be seen that for a given wind speed the wave heights are higher for both stable and unstable winds for neutral winds (essentially $T_s - T_a = O$, where T_s is the temperature of the sea surface and T_a is the air temperature). Examples of the three types of wind profiles over the ocean are shown in Fig. 26.

Figures 22 and 23 show that in the laboratory $T_{1/3} \propto U^{0.8}$ and $H_{1/3} \propto U^{1.1}$, approximately (Stanton, Marshall and Houghton, 1932; Hamada, Mitsuyasu, and Hase, 1953; Flinsch, 1946; Sibul, 1955, Ralls and Wiegel, 1956). Studies in a lagoon and in a reservoir show that $T_{mean} \propto U^{0.5}$, $T_{1/3} \propto U^{0.4}$ to $U^{0.5}$ and $H_{1/3} \propto U^{1.1}$ to $U^{1.4}$ (Johnson, 1950; Burling, 1955). Studies in a lake show that $T \propto U^{0.35}$ and $H_{max} \propto U^{1.2}$ to $U^{1.5}$ (Darbyshire, 1956). Most of the data available for ocean waves do not include information even as to the range of fetches for which the measurements were made. The relationships among T, H, and U are for a variety of fetches. The ocean data show $T \propto U^{0.3}$ to $U^{0.4}$ and $H \propto U^{0.5}$ to $U^{1.1}$ (Brown, 1953; Roll, 1953).

The relationships between wave period and fetch and wave height and fetch are not as clear as the relationships among H, T. and U. The laboratory data for constant wind velocities show approximately that $T_{1/3} \propto F^{0.3}$ to $F^{0.5}$, $T' \propto F^{0.5}$, and $H_{1/3} \propto F^{0.5}$ (Hamada,, Mitsuyasu and Hase, 1953; Sibul, 1955; Ralls and Wiegel, 1956). The reservoir measurements, for constant wind velocity (Burling, 1955) show $T_{1/3} \propto F^{0.25}$ and $H_{1/3} \propto F^{0.4}$ (Darbyshire, 1956) measurements on a lough give $T/T_{infinity} \propto F^{0.3}$ and $H_{max}/H_{infinity} \propto F^{0.3}$, for varying wind speeds, where $T_{infinity}$ and $H_{infinity}$ refer to values of T and H for infinite fetches. It is interesting to note that Stevenson (1886) found the wave height to be proportional to the square root of the fetch. Because the wave height is proportional to $F^{0.3}$ to $F^{0.5}$ an increase in fetch from 400 nautical miles, say, to 600 nautical miles would cause an increase in wave height of from 12-1/2 to 22-1/2 percent which might not be noticed in the scatter of the data.

The physical reason for the increase in height and period with wind velocity and fetch is clear considering the way wave records are analyzed. Waves in deep water can have a maximum steepness (H/L) of 1/7. Thus at the end of a short fetch enough energy has been trasnferred from the wind to the water to make only the shortest waves of any significant height. As the fetch is increased more energy can only be added to the

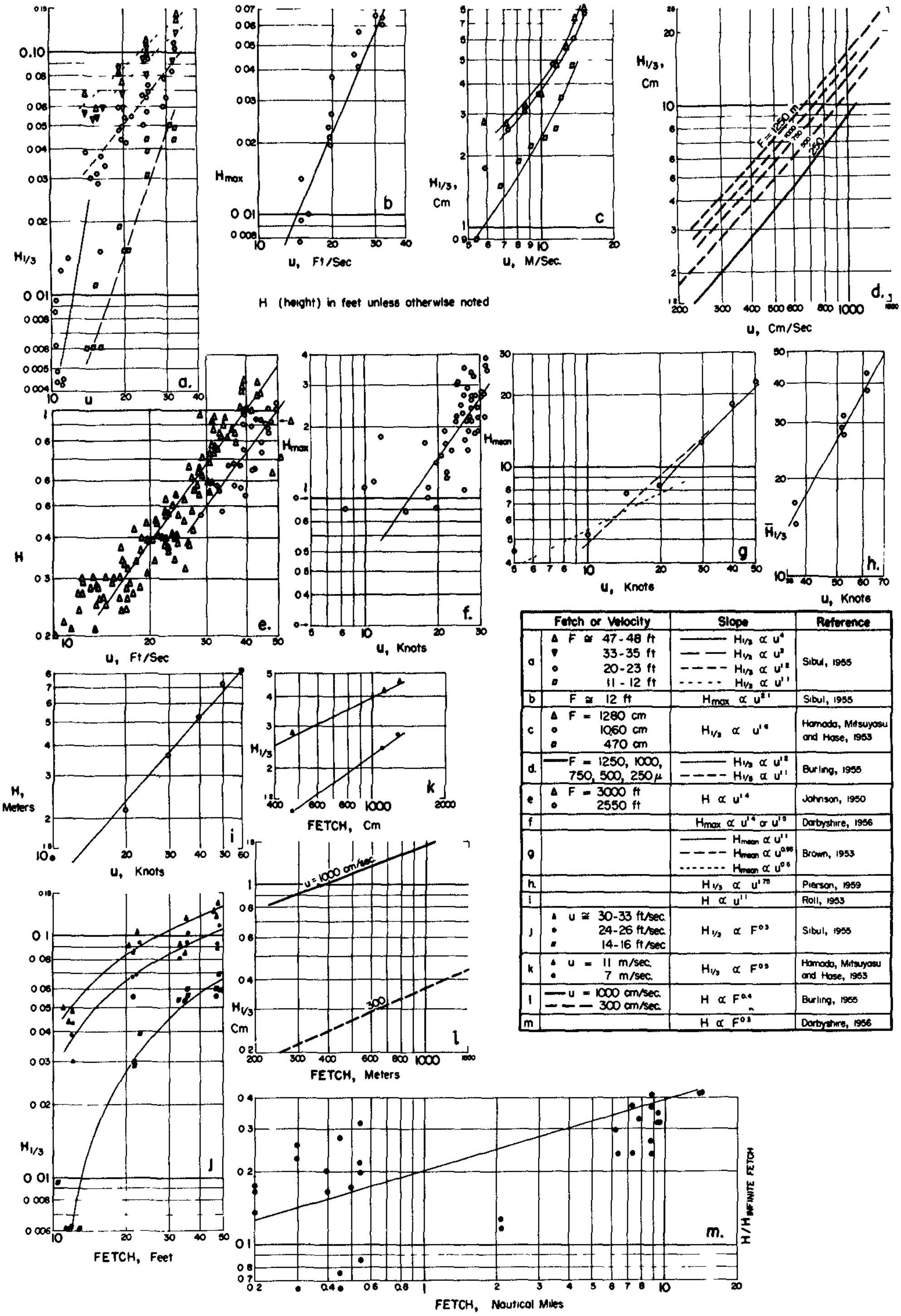




	Fetch or Velocity	Slope	Reference
a	Δ F ≅ 48 feet • 22 feet # 12 feet	$T_{1/3} \propto u^{0.89}$	Sibul, 1955
b	Δ F ≅ 35 feet • 22 feet	—— $T_{H_{1/3}} \propto u^{0.8}$ - - - - $T_{H_{1/3}} \propto u^{0.85}$	Sibul, 1955
c	Δ F = 1280 cm • 1060 cm # 470 cm	$T_{1/3} \propto u^{0.5}$ to $T_{1/3} \propto u^{0.7}$	Hamada, Mitsuyasu and Hose, 1953
d	Δ F = 41 feet • 15 feet	$T \propto u^{0.94}$	Stanton, Marshall & Houghton, 1932
e	F = constant	—— $T' \propto u^{0.85}$ - - - $T' \propto u^{0.6}$	Rolls and Wiegel, 1956
f	—— F = 1250, 1000, 750, 500, 250 meters	$T \propto u^{0.8}$	Burling, 1955
g	Δ F = 3000 feet • 2550 feet	$T \propto u^{0.5}$	Johnson, 1950
h	(for higher values of u)	—— $T \propto u^{0.38}$ - - - - - $T \propto u^{0.8}$	Darbyshire, 1956
i		$T_{mean} \propto u^{0.4}$	Brown, 1953
j		$T \propto u^{0.5}$	Roll, 1953
k	• u = 30 ft/sec • 20 ft/sec	$T_{H_{1/3}} \propto F^{0.5}$	Sibul, 1955
l	▼ u = 12 m/sec. o 7 m/sec.	—— $T_{1/3} \propto F^{0.3}$ - - - - $T_{1/3} \propto F^{0.4}$	Hamada, Mitsuyasu & Hose, 1953
m	u_{max} = 25 ft/sec. $T' = \frac{L_{measured}}{C_{measured}}$	—— $T' \propto F^{0.5}$ - - - - $T' \propto F^{0.4}$	Rolls and Wiegel, 1956
n	—— u = 1000 cm/sec — — — 300 cm/sec	$T \propto F^{0.35}$	Burling, 1955
o		$T \propto F^{0.2}$	Darbyshire, 1956
p		$T \propto F^{0.3}$	Darbyshire, 1956

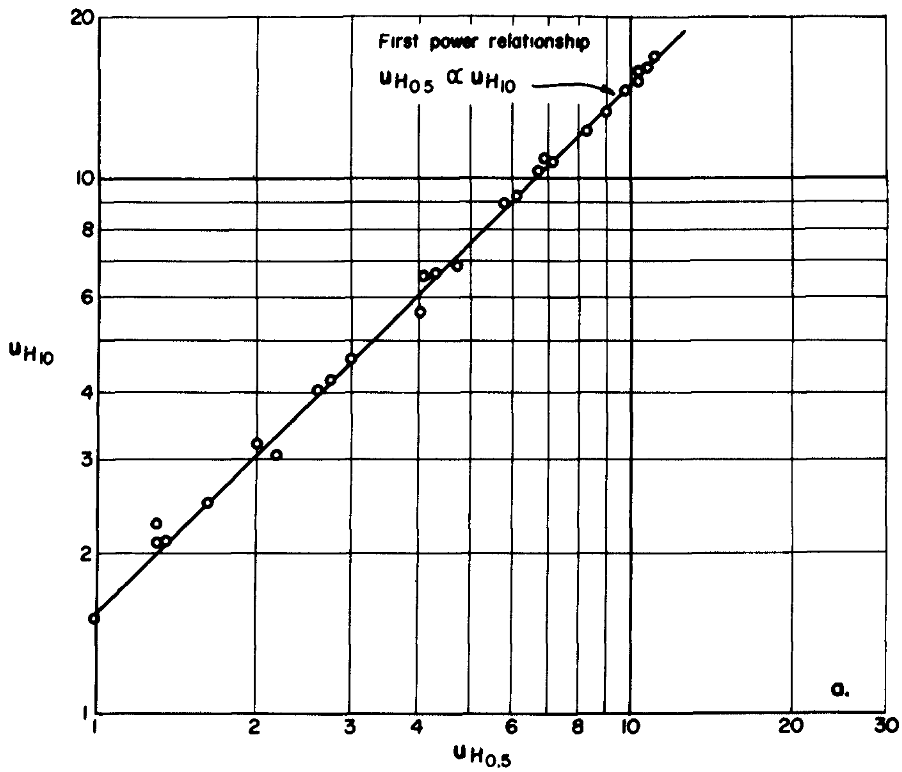
u (velocity) in feet/second, unless otherwise noted
T (period) in seconds

Fig. 22. Wave period as a function of fetch and wind speed.

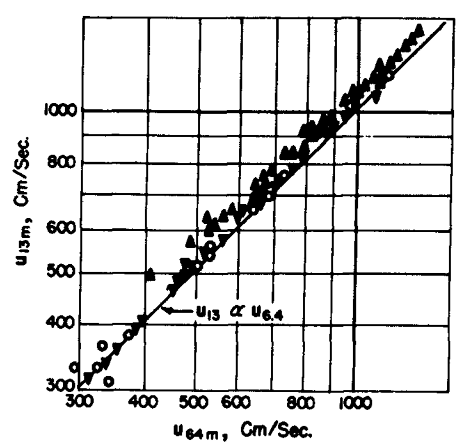


	Fetch or Velocity	Slope	Reference
a	▲ $F \cong$ 47-48 ft ▼ 33-35 ft ○ 20-23 ft ▱ 11-12 ft	——— $H_{1/3} \propto u^{4}$ — — $H_{1/3} \propto u^{3}$ - - - $H_{1/3} \propto u^{1.2}$ ----- $H_{1/3} \propto u^{1.1}$	Sibul, 1955
b	$F \cong$ 12 ft	$H_{max} \propto u^{2.1}$	Sibul, 1955
c	▲ $F =$ 1280 cm ○ 1060 cm ▱ 470 cm	$H_{1/3} \propto u^{1.6}$	Hamada, Mitsuyasu and Hase, 1953
d.	——— $F =$ 1250, 1000, 750, 500, 250 μ	——— $H_{1/3} \propto u^{1.2}$ - - - $H_{1/3} \propto u^{1.1}$	Burling, 1955
e	▲ $F =$ 3000 ft ○ 2550 ft	$H \propto u^{1.4}$	Johnson, 1950
f		$H_{max} \propto u^{1.4}$ or $u^{1.9}$	Darbyshire, 1956
g		——— $H_{mean} \propto u^{1.1}$ - - - $H_{mean} \propto u^{0.96}$ ------- $H_{mean} \propto u^{0.6}$	Brown, 1953
h.		$H_{1/3} \propto u^{1.75}$	Pierson, 1959
i		$H \propto u^{1.1}$	Roll, 1953
j	▲ $u \cong$ 30-33 ft/sec. ● 24-26 ft/sec. ▱ 14-16 ft/sec	$H_{1/3} \propto F^{0.3}$	Sibul, 1955
k	▲ $u =$ 11 m/sec. ● 7 m/sec.	$H_{1/3} \propto F^{0.5}$	Hamada, Mitsuyasu and Hase, 1953
l	——— $u =$ 1000 cm/sec. — — — 300 cm/sec.	$H \propto F^{0.4}$	Burling, 1955
m		$H \propto F^{0.3}$	Darbyshire, 1956

Fig. 23. Wave height as a function of fetch and wind speed.



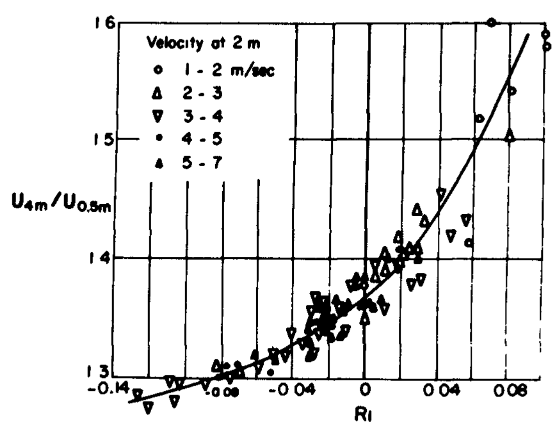
Winds over land (data from Thornthwaite and Kaser, 1943)



Winds over the sea

▼ $T_a - T_s < -1°F$ ▲ $T_a - T_s > +1°F$
o $T_a - T_s$ from $-1°F$ to $+1°F$

(data from Deacon, Sheppard and Webb, 1956)



4:0.5 m wind ratio related to Richardson number; short grass surface

(after Deacon, 1949)

Fig. 24

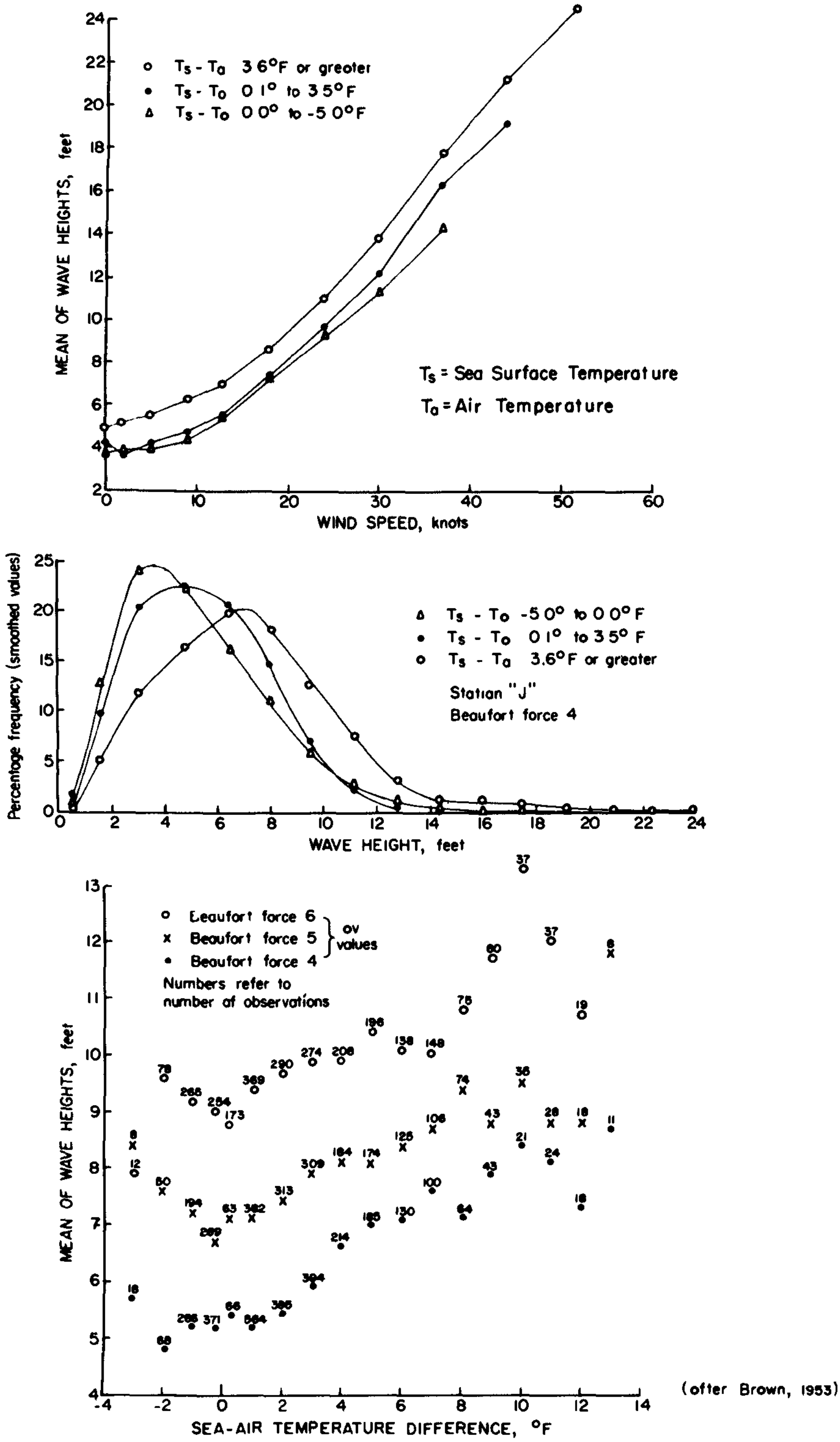

MEAN OF WAVE HEIGHTS, feet
Ts - Ta 3 6°F or greater
Ts - To 0 1° to 3 5°F
Ts - To 0 0° to -5 0°F
Ts = Sea Surface Temperature
Ta = Air Temperature
WIND SPEED, knots
Percentage frequency (smoothed values)
Ts - To -5 0° to 0 0°F
Ts - To 0 1° to 3 5° F
Ts - Ta 3.6°F or greater
Station "J"
Beaufort force 4
WAVE HEIGHT, feet
Beaufort force 6
Beaufort force 5
Beaufort force 4
av values
Numbers refer to number of observations
SEA-AIR TEMPERATURE DIFFERENCE, °F
(after Brown, 1953)

Fig. 25

longer waves as the short ones have reached their maximum steepness; the longer waves then dominate the wave system as far as the eye is concerned.

If the wave height and wave period depend upon wind velocity, fetch, duration of wind, air temperature and sea surface temperature, then by dimensional analysis

$$\frac{gT}{U} = f_1\left(\frac{gF}{U}, \frac{gt}{U}, \frac{Ta}{Ts}\right) \tag{7a}$$

$$\frac{gH}{U^2} = f_2\left(\frac{gF}{U^2}, \frac{gt}{U}, \frac{Ta}{Ts}\right) \tag{7b}$$

The functions f_1 and f_2 cannot be determined by dimensional analysis; they must be determined either empirically or theoretically. Effectively then, after quasi-steady-state conditions are reached (gt/U large), the wave height and period depend upon the stability of the wind blowing over the water surface and a Froude number based upon the linear dimension of the storm, as long as the waves are in "deep water." The wave height and period should depend critically upon the turbulence of the air flow, and a logical criteria for this would be a modified Froude number based upon the mean horizontal length of the eddies in the air, such a criteria would be useful in studying coupled wave effects. A more complete analysis would show that in deep water the wave height and period would also depend upon Reynolds number (damping), Webers number (surface tension effects), Richardson number rather than simply T_a/T_s, the ratio of fetch length to fetch width, and a boundary roughness parameter H/L (which of course would be a function of the wind speed, etc., again). The air-sea boundary process is one of fluid flow and thus the dimensionless numbers controlling all other fluid flow processes should be the ones used in describing it, neglecting the ones which obviously are not important in this such as Mach number and the cavitation number. In Fig. 27 are shown the empirical relationships between gT/U and gF/U^2 and gF/U^2 with the effect of T_a/T_s being lost in the scatter of data. The empirical relation ship between gT/U and gt/U and gH/U^2 and gt/U are shown in Fig. 28. In some references curves have been drawn through the data in such a manner that they leveled off in the region of gt/U and gF/U^2 greater than about 10^5. This is because the lines are drawn through the averages of all of the data. A close inspection of the data show that this leveling off did not occur for any of the individual sets of measurements, with the exception of Darbyshire's (1959) data for $gH_{1/3}/U^2$. The best curves through these individual sets of measurements giving approximately $gT/2\pi U \propto (gF/U^2)^{0.3}$ to $(gF/U^2)^{0.4}$, or $T \propto U^{0.2}$ to $U^{0.4}$, $T \propto F^{0.3}$ to $F^{0.4}$, and $gH/U^2 \propto (gF/U^2)^{0.25}$ to $(gF/U^2)^{0.35}$ or $H \propto U^{1.3}$ to $U^{1.5}$, $H \propto F^{0.25}$ to $F^{0.35}$. These data seem to indicate that the "fully developed sea" does not occur in the open ocean for winds of any importance. Fig. 28 can be treated in a similar manner. Here, $gT/2\pi U \propto (gt/U^{0.3}$, or $T \propto U^{0.7}$, $T \propto t^{0.3}$, and $gH/U^2 \propto (gt/U)^{0.4}$ to $(gt/U)^{0.5}$, or $H \propto U^{1.5}$ to $U^{1.6}$, $H \propto t^{0.4}$ to $t^{0.5}$.

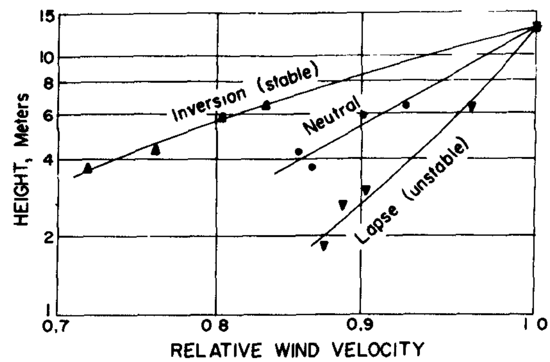
It is clear that these data are in conflict with the original empirical spectrum of Neumann, where $H \propto U^{2.5}$ (Neumann and Pierson, 1957). In order for $H \propto U^{2.5}$ the curve relating gH/U^2 and gF/U^2 would have to have a negative slope, which would also mean that $H \propto F^{-0.25}$ which is in obvious conflict with the physical situation. However, the original spectrum is apparently in the process of being modified to include recent findings (Pierson, 1959b).

There is evidently still a pressing need for a large number of reliable measurements of waves in the open oceans.

The data shown in Fig. 27 are for the case described by Sverdrup and Munk (1947) as "fetch-limited", the other condition being "duration-limited",(Fig. 28). Phillips (1957; 1958) developed a mathematical model that predicted that the mean square wave height is proportional to the square root of time for the duration limited case and to the square root of fetch for the fetch limited case. What is meant by these two terms? Consider an infinite fetch, with the wind suddenly starting with a given velocity and then remaining at this velocity. At any point in the fetch the significant wave height and period will increase with time. The significant waves moving past this point will have travelled a distance equal to the product of the group velocity of the significant waves and the length of time the wind has been blowing. Power will have been added to the wave system by the wind during the entire time. Now real fetches are finite, so eventually the significant waves reaching a given point will be associated with the component wave periods that originated at the beginning of the fetch. The time for this to occur will be the distance divided by the group velocity (Sverdrup and Munk, 1947; Phillips, 1958a). After this duration has been reached the waves are said to be fetch-limited.

It is possible for a third case to exist, the fully developed sea. This case would require a storm duration and fetch both long enough that energy is being dissipated and radiated at the same rate that it is being transferred from the wind to the water in the form of waves.

Some measurements in the laboratory of Sibul (1955) show the



Sample wind profiles

▼ Ri (13 4) = -0.07 • Ri (13 4) = 0.01
▲ Ri (13 4) = 0.18

(after Deacon, Sheppard and Webb, 1956)

Fig. 26

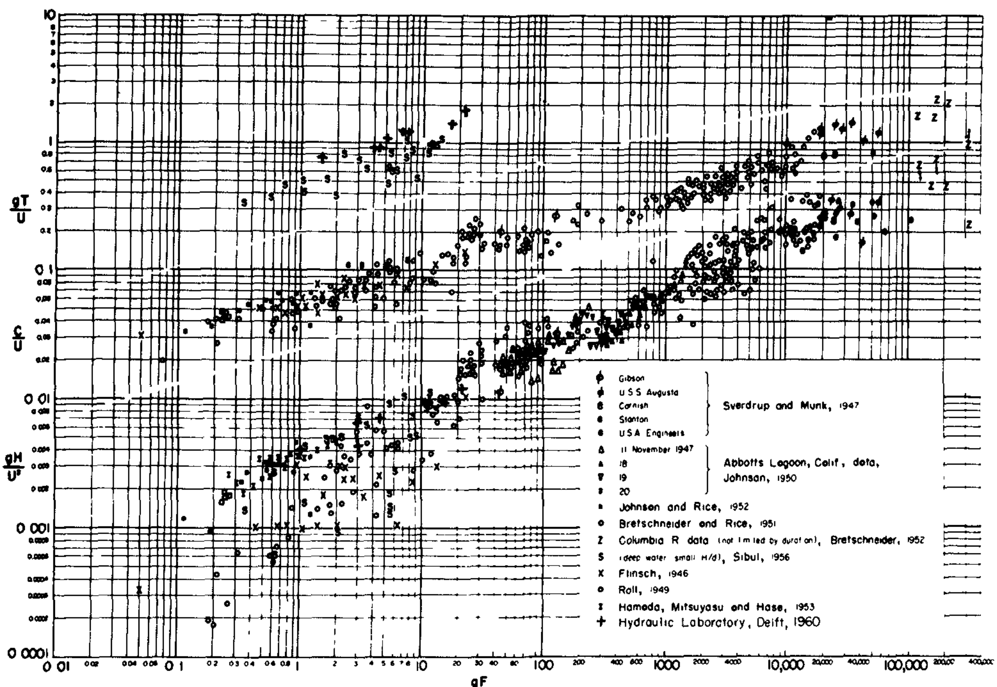


Fig. 27. Relationship among fetch, wind velocity and wave height, period and velocity.

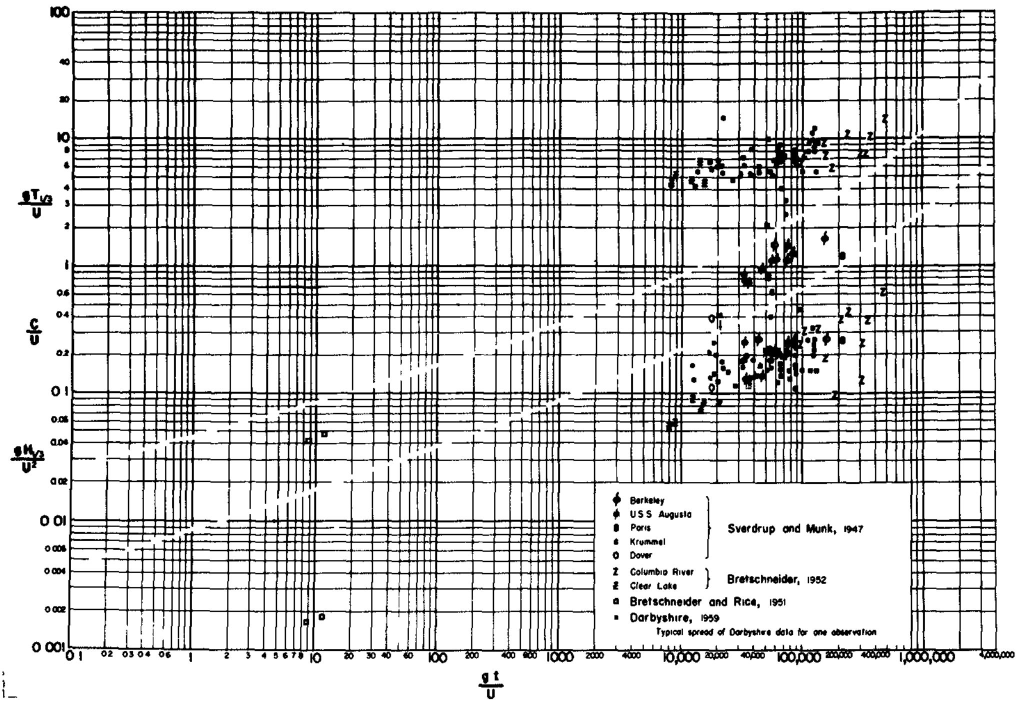


Fig. 28. Relationship among duration wind velocity and wave height, period and velocity.

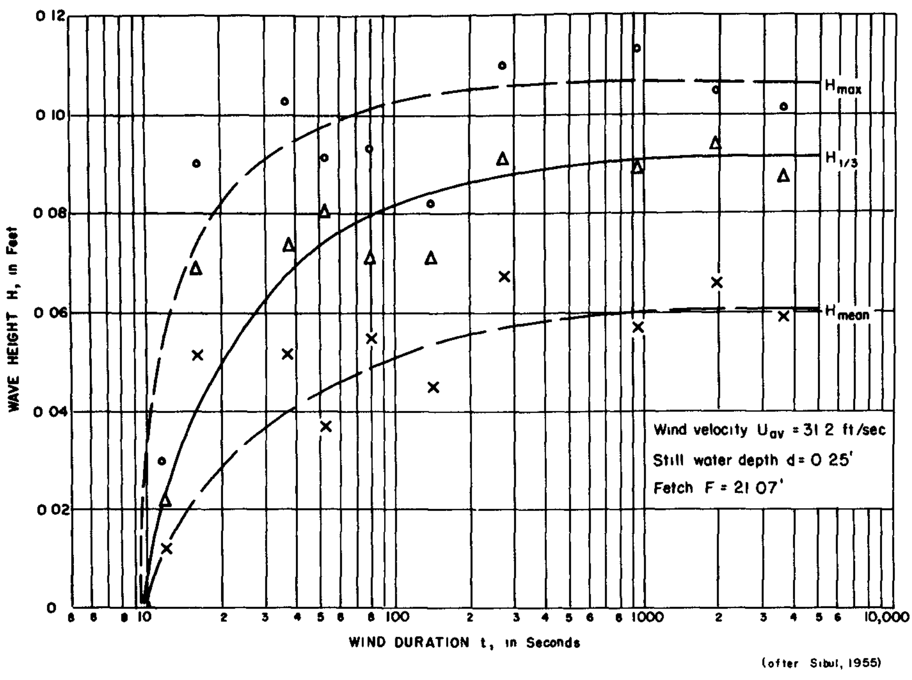


Fig. 29. Wave height as a function of wind duration.

effect of duration-limited and fetch-limited conditions on the wave heighl as can be seen in Fig. 29.

In Fig. 27 are shown the data for C/U vs gF/U^2. There are a cc siderable number of measurements that show C/U in excess of unity. Tl is not surprising, nor is it a paradox. The flow of air over water is a boundary layer phenomenon and the phase speeds of the waves should be related to the free stream wind speed (the geostrophic wind speed) which is in considerable excess of the wind speed at anemometer height, which is the wind speed given for the ocean data of Figs. 27 and 28.

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