CHAPTER 9

THE ANALYSIS OF HARBOR AND ESTUARY SYSTEMS

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INTRODUCTION

It is one of the paradoxes of our age that hydraulic engineering is concerned with problems that in many ways exceed in difficulty those encountered in the more glamorous fields of science. One measure of this situation is that classical mathematics, which is a powerful tool when applied to simple systems, has proved to be a rather impotent aid in hydraulic calculations, except when the geometry is simple. Harbor and estuary systems are usually associated with complex geometry, and thus we ordinarily cannot depend on mathematics to give general solutions. Instead of solving the hydraulic equations of flow in the complex geometry, we have sought to reconstruct the geometry of the prototype in a reduced scale, by means of models, and by assuming that the equations governing the full-size and reduced-scale systems are the same, to find specific solutions through direct measurements in the latter.

Because of scale effects, the hydraulic model is not perfect, but it does reproduce the complex geometry and some of the complexities of three dimensional flows in the prototype. These cannot at present be described mathematically, and where they are important a properly constructed and adjusted hydraulic model is our most powerful tool in the investigation of estuarial problems, and is likely to remain so.

However, hydraulic models are very expensive, especially if they are built to a reasonably large scale, so it behooves us to be sure that we have not overlooked other, perhaps less powerful, methods when we are confronted with a particular problem. Within the past decade two methods have been developed that will eventually replace hydraulic models for tidal flow and river flood routing investigations. One is based on the use of digital computers to numerically integrate the differential equations of open channel flow; the realization of this method by a digital computer program for a particular system has been called a "mathematical model". The second is based on the use of analog elements that behave with respect to electrical current in the same way as the prototype behaves with respect to flows of water. When an assembly of such elements is adjusted to duplicate the behavior of a hydraulic system in a way similar to the way a hydraulic model is adjusted during its verification period, the result may be called an analog model.

THE MATHEMATICAL BASIS OF TIDAL FLOW CALCULATIONS

Each of the two methods depends on a mathematical description of the flow in open channels that was already well developed at the turn of the century, and which has formed the basis for many hand calculation methods during the past fifty years. One of the earliest practical applications was by Parsons(1) who published in 1918 a history of the construction of the Cape Cod Canal in Massachusetts and incidentally presented an excellent summary of the theory of tidal hydraulics, a subject that had at that time not been treated in hydraulic texts, together with his careful measurements of the tides and currents in the canal for comparison with theory. He brought to the attention of his American colleagues the classical work of Sir George Biddell Airy of Cambridge, on "Tides and Waves" (2). Parsons recognized the limitations of using linear friction (an assum tion that leads to a simple solution of the equations) and recalled the meth ods being used in the computation of surge tank behavior, where the equations are the same and where a useful concept of linearized friction was being employed. In this concept the friction is described by a term that depends linearly on the velocity, but which has a coefficient that makes the total energy dissipated over a complete tidal cycle the same as would be the case if square-law friction had been assumed. He credited this idea to Prasil(3) and Dubs(4).

When the geometry is simple, as in the case of artificial canal, analytic solutions based on linearized friction are often all that are neede These are well described by Einstein and Fuchs(5) and by Dronkers and Schoenfeld(6). The latter also describe some approximate analytic methods for predicting higher harmonics due to non-linear friction. These methods require both a good mathematical and physical insight on the part of the computor and considerable tedious calculation. In contrast the "brute force" methods of direct integration of the differential equations, either along characteristics or in a properly chosen rectangular grid, can be programmed for digital computers, and it is likely that this will be the future trend. These numerical methods are described in references (5) and (6) and by Stoker(7).

SCHEMATIZATION AND VERIFICATION

As we consider the attractions of turning the tedious work of com putation over to a computer or an analog model, we should anticipate an additional difficulty, that of schematization. This is not a difficulty inher ent in any method, but arises from the complexity of the geometry. In both the mathematical integration methods and the analog model methods the system of waterways must be divided into short reaches within which we can assume that the channel properties can be averaged into a representative set of values. We must assign values of friction factor for each reach; these values must be guesses, for we normally cannot establish steady flow in an estuary and measure the water surface slope due to friction acting alone. This incidentally is a familiar task to operators of hydraulic models. In any computation or analog model approach, however, there are additional assumptions that must be made about the contribution that each part of a given cross section makes to the inertia. The division of a cross section into a main part that contributes inertia and a shoal part that contributes only to storage is illustrated in Figure 1.

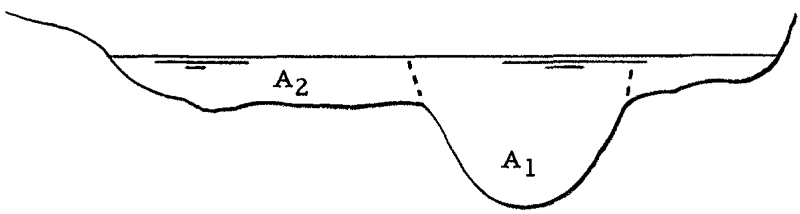

Figure 1. Irregular cross section in an estuary

In a compact but irregular cross section theory tells us that the shallow water wave velocity should be $\sqrt{gA/B}$, where A is the cross sectional area and B is the water surface width. The shoal areas do not contribute completely to the area A, however, but are often the scene of sluggish and sometimes reverse currents. Thus, in an investigation of tide wave motion in San Francisco Bay we found that the active cross section, A_1, could be considerably less than half of the total, and that a workable general rule was that we could assume that areas shallower than ten feet did not contribute to the inertia; however, it was still necessary to check this assumption.

Fortunately, friction had only a small effect on the velocity of propagation and a principal effect on the attenuation, so we could adjust our assumptions of inertia to achieve the correct wave speed, and the friction to achieve the correct attenuation. Thus, in contrast to the verification procedure for hydraulic models, where only the friction ordinarily needs adjusting, both friction and inertia must be adjusted in an analog model. This is, of course, equally true for a computational model.

Figure 2 shows superimposed on a map of the San Francisco Bay-Sacramento-San Joaquin Delta System the network of analog elements employed in a study of tidal flows there. The system contained analog elements, of which twenty were supplied with square-law resistors developed in the Hydraulics Laboratory at the University of Californi Berkeley. Figure 3 shows the result of a verification run. Both the tidal amplitude and phase were obtained for a large number of stations along the hundred-mile course leading from the Golden Gate outlet and the Pacific Ocean to the City of Sacramento. These measurements are shown by circled points. The results of measurements on the analog model are shown as small triangles. At all stations except the last, at Sacramento, the difference between the readings is within the error of measurement. This work was reported on in greater detail in reference (8). It was performed under contract for the California Department of Water Resources.

RELATIVE MERITS OF ANALOG AND MATHEMATICAL MODELS

In estimating the relative advantages of mathematical models and analog models, one can say that the digital computer is inherently capable of the greater precision. It will be found, however, that accurac will depend in each instance on the completeness of the verification process. Here the much higher speed of the analog is an advantage, for it can go through thousands of tidal cycles per second, and any adjustments to the inertial elements or the friction is reflected almost instantaneously in the results (a graphical presentation on an oscilloscope screen). When using a digital computer one must wait for a complete new solution to eva uate the effect of each change in a coefficient, and even on the fastest machines this may take many minutes. When both the friction and the inertia must be adjusted for as many as one hundred elements, and due to interactions this must be repeated many times, the temptation to stop short of the best adjustment must be great. We have found that it takes several weeks of intermittent work to adjust a moderate size analog, ever when the results of a given adjustment are immediately visible to the ope ator.

A limitation of the analog, that may be of importance in some instances, is that it is impractical to include all of the terms in the hydraulic equations. For example, total hydraulic head may be accounted for, but the velocity head may not be separated out. Two-dimensional flows require additional hardware to bring the velocity vector to the same direction as the loss vector, etc. None of these has proved insurmountable, though they may suggest that once the principal values of inertia anc friction are determined on an analog a greater refinement may be possibl using a digital computer, with these as starting values. Some experience is essential here, however, to avoid the beginner's mistake of asking for a far greater accuracy in the computations than is warranted by the variability of the data used in the verification procedure.

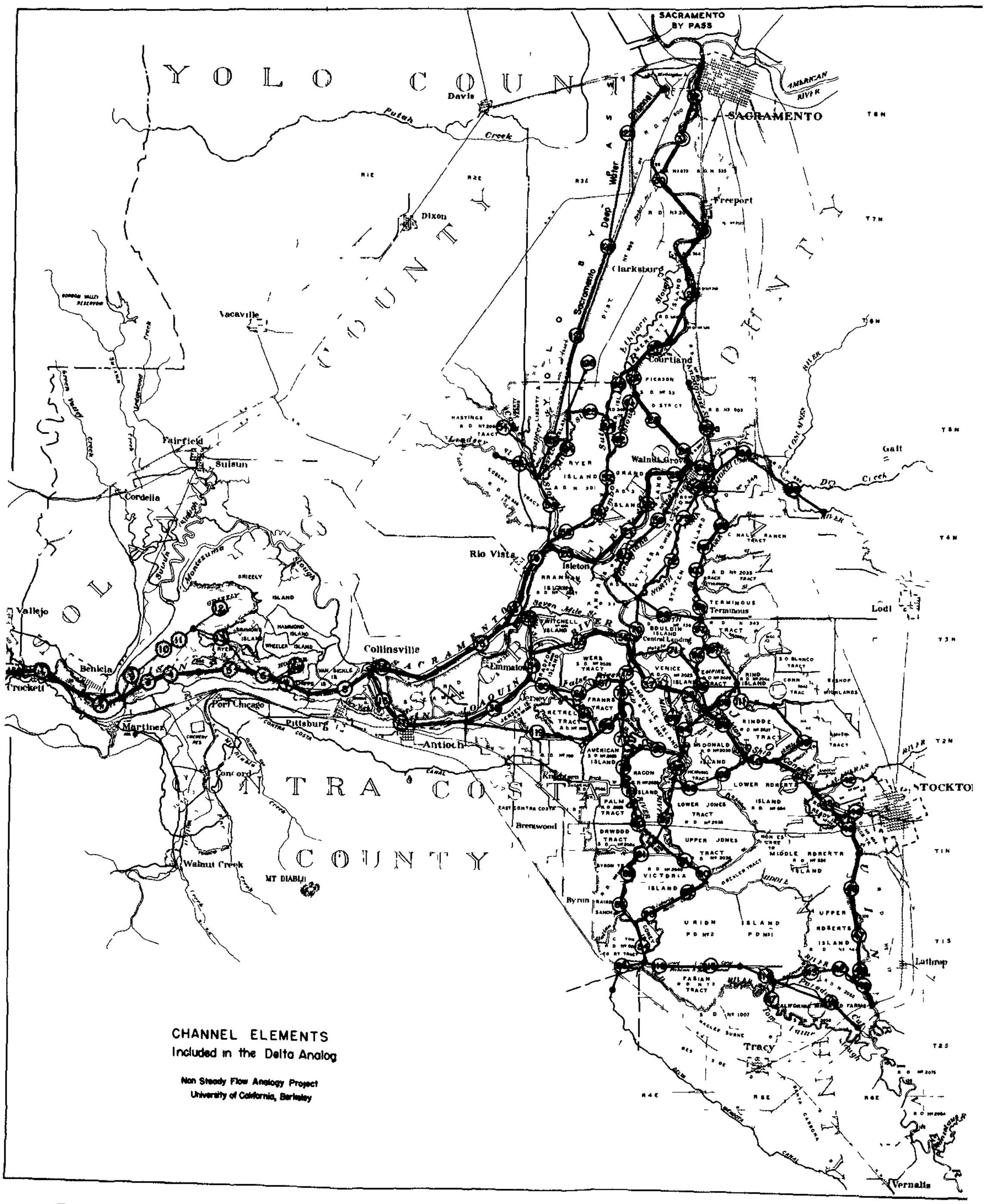

Figure 2. Network of analog elements for the Sacramento-San Joaquin Delta Analog

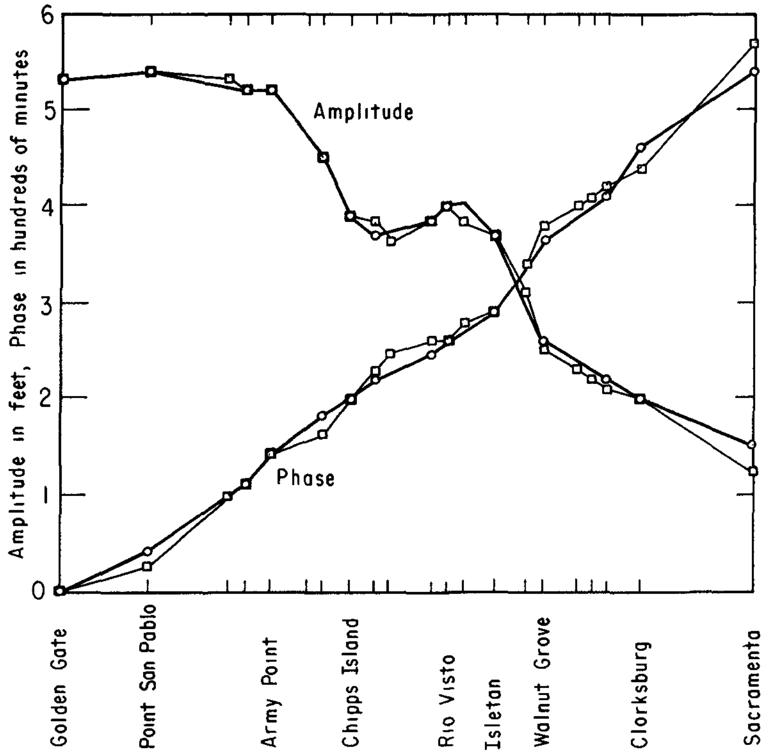

Figure 3. Prototype vs. analog measurements in the Delta

PRESENT AND FUTURE TRENDS

Salinity intrusions into estuaries would seem to be an appropriate subject for a digital computer application. The approach might be to try to solve the convected diffusion equation by numerical integration. The operative equation is:

$$\frac{\partial \phi}{\partial t} = K \frac{\partial^2 \phi}{\partial x^2} - u \frac{\partial \phi}{\partial x}$$

where ϕ is the salinity, K is the diffusivity coefficient, and u is the velocity with which the salinity would be convected downstream in the absen of diffusion. Practically, this would have to be solved with the boundary conditions of u as a function of time at the upper end and ϕ as a constant at the ocean outlet. The production of a set of K's for the reaches that would lead to a duplication of the time history and profile of salinity, as between the prototype during a particular period and the mathematical model, would constitute a partial verification. It would only be partial because there is a strong possibility that K is not only a function of distance along the estuary, but of u and possibly ϕ.

Thus, if the diffusivity depends on the salinity and the local velocity, or velocity gradients, we are no longer very sure that a duplication in one case would lead to a duplication for another. Furthermore, we are really not even sure that the equation itself is a good description of the process of salinity intrusion. What then is possible? To gain a little perspective on this problem, let us step back and ask what information is available. If a considerable body of historical data is available for values of ϕ and u, we may be able to form a connection between them that will serve as a predictor. If the relationship is a linear one, even though there is a time lag the solution is reasonably well assured. That the connection can be made if the relationship is non-linear is also possible under some circumstances as shown by Jacoby(9). The proof is involved, but an idea of the power of the method is apparent in the following example drawn with slight modification from Jacoby's doctoral dissertation.

Assume that we have a basin connected to the sea by a canal and that a long series of measurements are available of both the tidal fluctuations in the sea and in the basin. We do not know the size of the basin, the length of the canal, or the friction, or anything at all about the physica system. However, we know that it is non-linear because the friction depends on the square of the velocity.

We built such a system out of electronic components in our laboratory, and set about trying to make enough measurements to enable us to fully predict the behavior of the system, without knowing how it was put together. Since we were assuming a natural system, we did not allow ourselves the luxury of applying steady flows, etc., to the system, as this would be impossible in most natural systems. We did apply a random signal in the place of the ocean tide, and made simultaneous measurements of the tide inside the simulated basin. A typical section of the record obtained is given in Figure 4.

From approximately 10,000 data points (more than was necessary in all probability) we were able to derive 28 predicting coefficients. More coefficients would have improved the predictions, but would also have increased the amount of the computations, which already required about 45 minutes on a 704 computer.

With the predictor equation developed, we applied a pure sine wave to the input of the simulated system and measured it plus the output. A comparison between the predicted and the observed output is given in Figure 5. Notice that the correct amplitude and time delay are predicted. More remarkable is the fact that the correct degree of flattening of the wave crests is predicted. This flattening of the crests might be expected on the basis of square law friction in the connecting channel.

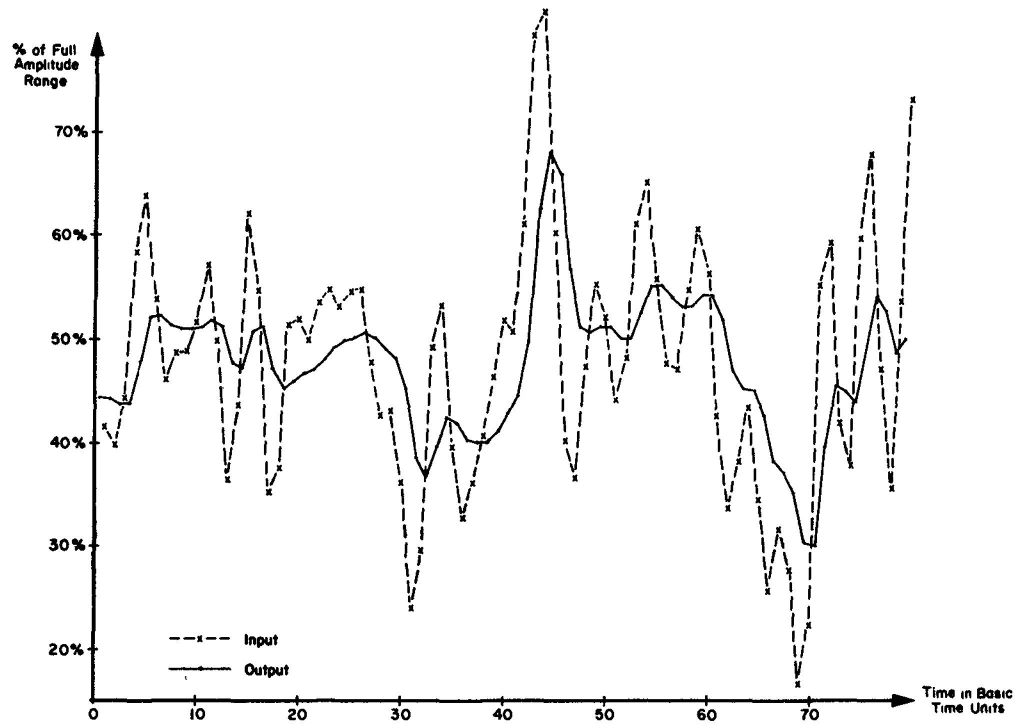

Figure 4. Typical section of input and output records

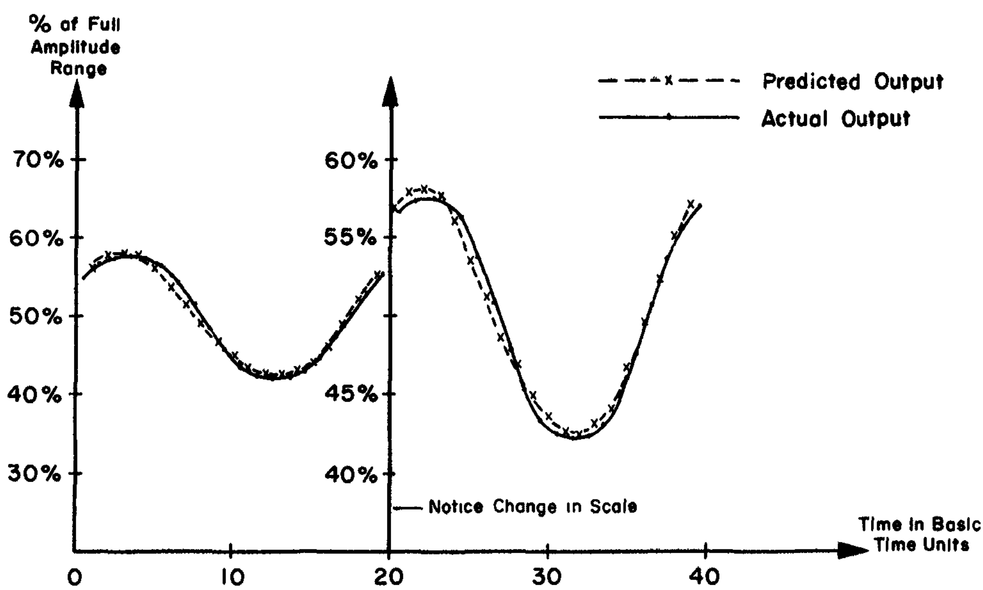

Figure 5. Comparison between predicted and measured output from the experimental system

This example should not be taken too literally -- there is little likelihood that this method would be the best one to use in the problem as given, for the simple reason that we would ordinarily be able to start with a knowledge of the basin and channel dimensions and be able to make predictions on the basis of far less data. In a more abstract sense, though, this example illustrates how other problems can be attacked. The method needs further improvement, in the direction of making the most efficient use of the data available in constructing the predictor, but it constitutes an important advance in our analysis of engineering systems. It also illustrates the powerful methods that have been made feasible by the digital computer, and that we need not be forever limited to using the computer merely to mechanize methods already developed.

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