Chapter 8

STATISTICAL DISTRIBUTION OF WAVE HEIGHTS IN CORRELATION WITH ENERGY SPECTRUM AND WATER DEPTH

by

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Wave motion has been studied at various places in the south western part of the Netherlands (the Delta area) as part of a study on coastal morphology and the design of coastal structures.

The main part of this study deals with the statistical distribution of wave heights in relation to:

a. the height of the sea surface at fixed points, as a function of time;
b. the water depth;
c. the energy spectrum of the wave motion.

The results of theoretical studies by D.E.Cartwright and M.S.Longuet Higgins (lit. 1) and data on wave measurements obtained by Rijkswaterstaat in the North Sea were used. These measurements were taken by means of an electrical step-capacity gauge, with wireless transmission of the data to shore . Three gauges, fastened to fixed poles (named E, K and T) were placed at points at depths of 5, 10 and 15 meters respectively. (fig. 1).

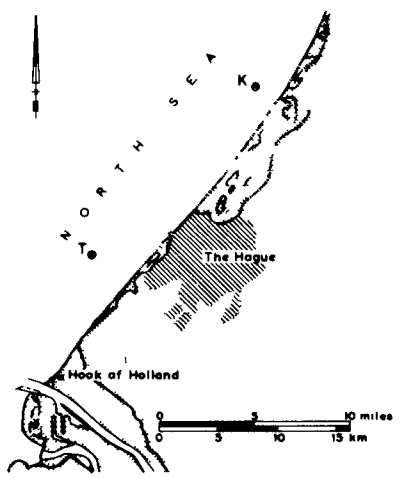

Fig 1 Situation of measuring stations

The theoretical frequency distributions of the wave heights, derived by Cartwright and Longuet Higgins for the "lineair model", are based on the assumption that the distribution of the ordinates

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of the sea surface as a function of time is Gaussian. From the data of wave records, it appeared that in relatively shallow water (depths varying from 3.5 to 17.0 meters) Gaussian as well as non-Gaussian distributions occur.

In § 1 a brief description is given of the "lineair model" and its limitations. The deviations of frequency distributions of the sea surface from those of the lineair model are studied in § 2.

Four parameters are used to describe the wave heights. In § 3 the distributions of these four wave height parameters are considered as a function of three variables:
the standard deviation $\sqrt{m_o}$, the skewness α_3 of the sea surface distribution and ε , the relative width of the energy spectrum.
From the frequency distributions obtained from observations the quotients of the significant wave heights $H_{1/10}$, $H_{1/3}$ and H_m are determined as a function of ε and α_3 in § 4 . In § 5 the wave height parameters are further discussed.

§ 1. The lineair model of the wave motion and its limitations

Let $z=f(t)$ denote a continuous function of the time t, representing the ordinate of the sea surface above the mean water level; z is measured at a fixed point and it is assumed that $f(t)$ is a statistically stationary function during a limited time interval T_o.

Any function $f(t)$ thus defined can be represented by the sum of a large number of sine-waves:

$$z = \sum_n a_n \sin(\omega_n t + \varepsilon_n) \tag{1}$$

a_n = amplitude, ω_n = angle-frequency and ε_n = phase-angle.

In the "lineair model" the ordinates z measured from the mean sea level, z=o, are assumed to be Gaussian distributed during the time interval T_o.

Let the function for which this is valid be represented by:

$$z_a = f_a(t) \tag{2}$$

Analogous to (1) we thus obtain:

$$z_* = \sum_n a_n \sin(\omega_n t + \varepsilon_n) \tag{3}$$

But now the phases ε_n are random and distributed uniformly between 0 and 2π and the frequencies ω_n are distributed densely in the interval $(0, \infty)$; (lit.2).

Furthermore it is assumed that for every sine-wave, according to the classic wave theory:

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{2\pi g}{L} \operatorname{tgh} \frac{2\pi D}{L}} \tag{4}$$

where T and L represent the wave period and length respectively, D is the water depth and g the gravitational acceleration.

Function (4) is valid when (lit.3):

$$\frac{2\pi a}{L} \ll 1 \text{ and } \frac{2\pi a}{L} \ll \left(\frac{2\pi D}{L}\right)^5 \tag{5}$$

Furthermore for every very small interval $d\omega$ the sum of the squares of the amplitudes is a continuous function of the frequency ω or:

$$E(\omega)\, d\omega = \Sigma \tfrac{1}{2} a_n^2 \tag{6}$$

The function $E(\omega)$ is called the energy spectrum. The moments of $E(\omega)$ about the origin are defined as:

$$m_p = \int_0^\infty E(\omega)\, \omega^p\, d\omega \tag{7}$$

The relations (3), (4) and (5) define the "lineair model" of the wave motion and it has been found that it is possible to compute the distribution of the maxima and minima of $f_*(t)$. (lit.1).

The energy-spectrum $E(\omega)$ of a continuous, statistically stationary function f (t) can be determined by means of: (lit.4)

$$E(\omega) = \frac{1}{\pi} \int_0^\infty R(\tau) \cos \omega\tau\, d\tau \tag{8}$$

where $R(\tau)$ is the auto-correlation function of f(t), defined as:

$$R(\tau) = \frac{1}{T_0} \int_0^{T_0} f(t+\tau) f(t)\, dt \tag{9}$$

From (8) and (9) it follows that if $\tau = 0$,

$$R(0) = \overline{z^2} = \int_0^\infty E(\omega)\, d\omega \tag{10}$$

in which $\overline{z^2}$ is the variance of f(t) and $\int_0^\infty E(\omega) d\omega$ is the area m_0 of the energy spectrum.

Thus: $\overline{z^2} = m_o$ (11)

Let E_p denote the mean potential energy per unit of surface $(E_p = \frac{1}{2}\rho g z^2)$**, then:

$$m_o = \int_0^\infty E(\omega)\, d\omega = 2E_p \tag{12}$$

For the "lineair model", in which E_p is half of the total energy E_{tot}, it follows from (12) that: $m_o = E_{tot}$ (13)

Equation (10) may also be computed by integration of (1) and (6).

To check in how far agreement between the "lineair model" and the measured wave motion could be expected, the distribution of the ordinates of the sea surface was examined.

§2. Distribution of the ordinates of the sea surface.

In the following pages the probability distribution of the ordinates z of the sea surface will be indicated by the symbol p(z). The ordinates, measured from the mean sea level, are computed from a wave record obtained at a fixed point in the sea; $\sqrt{m_o}$ is the standard deviation of p(z).

The distribution p(z) has been determined by a computer from 35 wave records using a continuous measuring period of 10 minutes from each record. The intervals ΔZ were kept constant for a specific record, but they varied for the different records from 10 to 25 cm., depending upon the wave height.

To obtain sufficient variations in the water depths, wave heights and wave lengths, a selection has been made from the available wave records. The relation between wind force and wave height has not been considered.

It appeared that in the relatively shallow water, Gaussian as well as non-Gaussian distributions occur. Some examples are given in fig. 2.

**Introducing specific units, ρg can always be made 1.

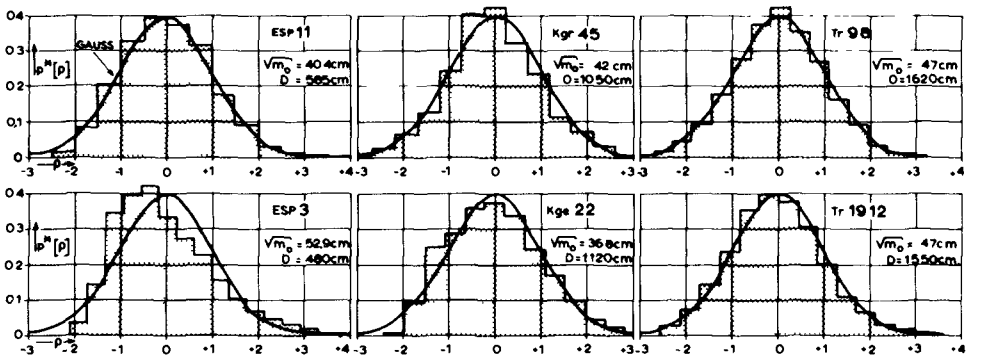

Fig 2 Probability distributions p[ρ] obtained from measurements, compared with the Gauss distribution

To facilitate comparison these distributions have been expressed in terms of $p^*\left(\frac{z}{\sqrt{m_o}}\right)$ where:

$$p^*\left(\frac{z}{\sqrt{m_o}}\right) = \sqrt{m_o} \cdot p(z) \tag{14}$$

Every distribution has been compared with the normal distribution. The deviation of a distribution $p^*\left(\frac{z}{\sqrt{m_o}}\right)$ from the Gaussian distribution, may be defined by the coefficient of skewness $\alpha = M_3 / M_2^{3/2}$.

The factors M_2 and M_3 represent the second and third moment about the mean value of the distribution p^*.

The deviation between both distributions was also defined by an emperical parameter:

$$\alpha_* = \frac{q(\rho=0) - q(\rho=1)}{q_1(\rho=0) - q_1(\rho=1)} \tag{15}$$

where: $\rho = \frac{z}{\sqrt{m_o}}$,

$q(\rho=0)$ and $q(\rho=1)$ are the values of the cumulative distribution $q(\rho)$ for the values $\rho = 0$ and $\rho = 1$,

$q_1(\rho=0)$ and $q_1(\rho=1)$ are the corresponding values for the Gaussian distribution.

Consequently in the parameter α_* the areas between the values $\rho = 0$ and $\rho = 1$ are compared for the distribution $p(\rho)$ and the Gaussian distribution.

It also follows that for a Gaussian distribution: $q(\rho=0) = 0{,}5$ and $q(\rho=1) = 0{,}159$, so that the considered area is $0{,}341$. If the distribution $p^*\left(\frac{z}{\sqrt{m_o}}\right)$ is Gaussian, then $\alpha_* = 1$ and $\alpha = 0$.

The coefficients α and α_* computed from the records appeared to be correlated with the factor: $L_m^2\sqrt{m_o}/D^3$ (fig.3), where D is the water depth; L_m is a value for the wave length, computed by means of equation (4) using the mean period T_m. The value T_m is defined as twice the average time interval between two successive zero-crossings of z=f(t).

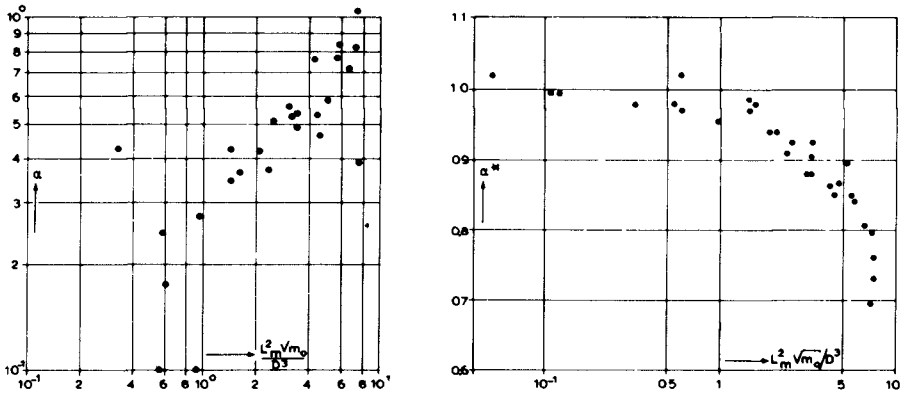

Fig 3 Relation between $L_m^2\sqrt{m_o}/D^3$ and α c q α*

The parameters obtained from the records studied, varied as follows:

$\sqrt{m_o}$ from 0,15 m up to 0,80 m. ($H_{1/3}$ from 0,60 m up to 3,50 m).

L_m from 18,0 m up to 60,0 m.

D from 3,50 m up to 17,0 m.

The scattering of the points in figure 3 may be due to the fact that an interval of only 10 minutes was taken from each record in order to obtain a statistically stationary process; a longer time interval may result in a significant change in water depth (tidal motion) or wind force. Furthermore, the smaller the wave energy spectrum, the better L_m will represent the average wave length.

For most p(z) distributions, the high values of z show a significant departure from the Gaussian curve. It is not clear wether these deviations can be fully explained by statistical variations. This phenomen has not been studied in detail.

From the correlations of fig.3 it appears that α_* shows a better correlation than the description of the skewness by means of α. The scatter in the results of α may also be due to the fact

that M_3 does not give an unambigious interpretation of the skewness.

In the following pages only the values of α_* will be considered; the results, however, are based on separate computations with α as well as α_*

§ 3. Distribution of the wave heights.

Four parameters (fig.4) have been used to describe the wave heights as fully as possible.

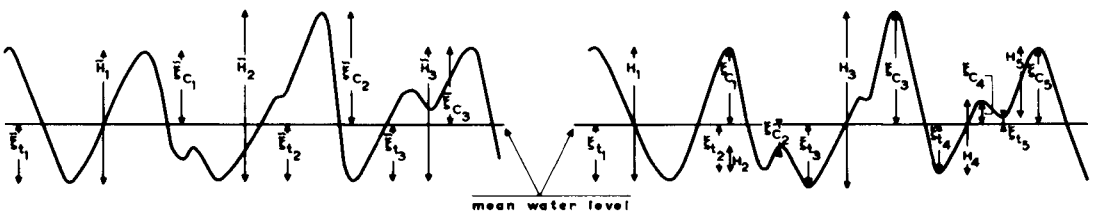

Fig 4 Wave height parameters

The wave heights $\tilde{\tilde{H}}$, measured from trough to crest and $\tilde{\tilde{\zeta}}$ measured from mean level to crest or trough, are defined for those waves which have exactly one zero crossing between crest and the predeeding trough. Between two successive zero crossings the highest crest and the lowest trough are considered.

The wave heights H and ζ are defined for all waves irrespective of the zero crossings.

When determing the wave heights, H and ζ, the sensitivity of the measuring instrument is important. The more sensitive the instrument, the more detailed the surface movement recorded. Waves less than 5 cm high and those between 5 and 10 cm with a period less than 1 second, were not included in this study. The measuring steps of the gauge were 4 cm. It was evident from the energy spectra of the records that waves with a period less than 1 sec contribute very little to the total wave energy.

Furthermore the distribution of ζ_c (measured from mean level to crest) and the distribution of ζ_T (measured from mean level to trough) will be considered separately.

The statistical distributions of the heights H, $\tilde{\tilde{H}}$, ζ_c, $\tilde{\tilde{\zeta}}_c$, ζ_T, and $\tilde{\tilde{\zeta}}_T$ show remarkable differences. The cumulative distribution of the

several parameters obtained from a 10 minute record are shown in fig.5 (wave height interval 10 cm.).

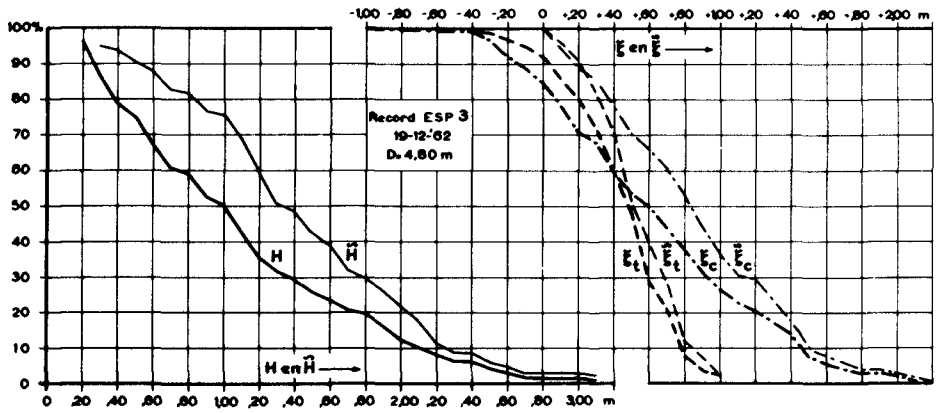

Fig 5 Example of a cumulative probability distribution q[H],q[H̃],q[ξ] and q[ξ̃]

The factors which influence these distributions will be discussed first.

In the lineair case (§§ 1, 2 and lit.1), probability distributions of ξ are derived by Cartwright and Longuet Higgins (assuming that the distribution of the ordinates of the sea surface should be Gaussian.) Then the probability distribution $p(\xi)$ is a function of $\sqrt{m_o}$ and ε ; $\sqrt{m_o}$ is the area of the energy spectrum and ε defines the relative width of the energy spectrum :

$$\varepsilon^2 = \frac{m_o m_4 - m_2^2}{m_o m_4} \tag{16}$$

where m_n is the n^{th} moment about the origin of the energy spectrum (see equation (7))

A general assumption of the distributions of H, $\tilde{H}$, ξ and $\tilde{\xi}$ may be

$$p(H;\xi;\tilde{H} \text{ or } \tilde{\xi}) = f\left[p(z), \sqrt{m_o}, \varepsilon\right] \tag{17}$$

or

$$p(H;\xi;\tilde{H} \text{ or } \tilde{\xi}) = f\left[\alpha_4 , \sqrt{m_o}, \varepsilon\right] \tag{18}$$

By means of (18) the wave height distributions for general cases $\alpha_4 = 1$ and $\alpha_4 \neq 1$ are studied in comparison with those derived from the lineair model.

a. Distributions of the values ξ_c and ξ_T

When $\alpha_4 \approx 1$ (distribution of sea surface is nearly Gaussian), the

distributions derived from the wave records correspond to a great extent with the theoretical curves given by Cartwright and Longuet Higgins. After application of the χ^2 test, no significant deviation was found. Three examples are given in table 1. The records with $\alpha_* < 1$ have been divided into three groups in which α_* is 0,9 - 0,8 and 0,7 respectively.

For each group a comparison has been made between the cumulative distributions of ξ_c and ξ_T obtained from observations and the corresponding theoretical distributions with the same values of $\sqrt{m_o}$ and ε. Correction factors, by which the theoretical values must be multiplied to obtain the measured ones, have been derived from this comparison. It was found that these factors are not influenced by ε. From the result in fig. 6 it is seen that ξ_c increases and ξ_T decreases when α_* decreases.

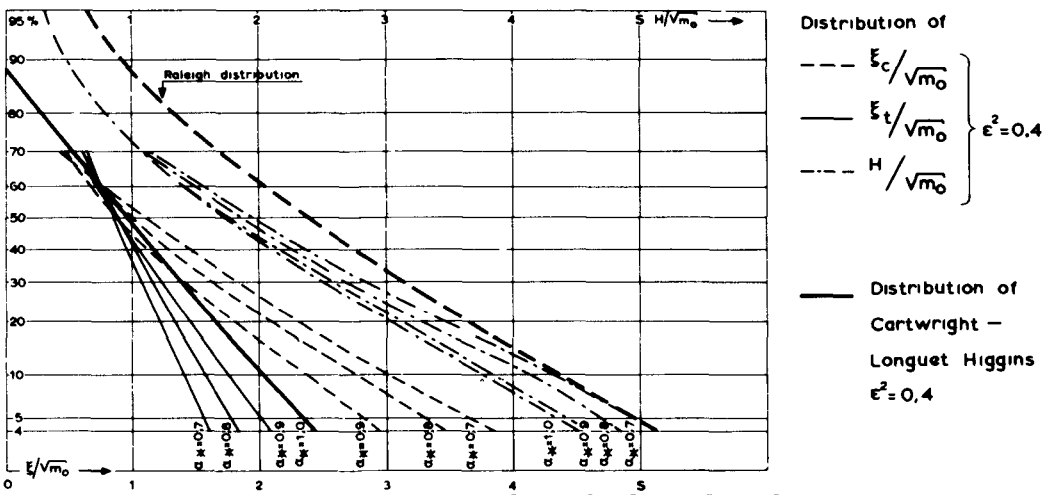

Fig 6 The cumulative probability distributions $q\left[\xi_c/\sqrt{m_o}\right]$, $q\left[\xi_t/\sqrt{m_o}\right]$ and $q\left[H/\sqrt{m_o}\right]$ for different values of α_*

b. Distribution of the wave height H. (measured from trough to crest).

When $\alpha_* \simeq 1$, the probability distribution of H can be represented by the formula:

$$p(\varphi) = \tfrac{1}{4} r^{1/2} \cdot \varphi \cdot e^{-\frac{1}{8}\varphi^2/r^{1/4}} + a^2\left(1 - r^{3/4}\right)\varphi \cdot e^{-a\varphi} \qquad (19)$$

where $\varphi = H/\sqrt{m_o}$, $r = 1 - \varepsilon^2$ and $a = 2\left(1 + r^{3/8}\right)/(2\pi r)^{1/2}$

The first term in the right member of (19) represents a Raleigh distribution, multiplied by a factor; the additional term is

derived empirically.

Formula (19) has been verified for about 100 wave records, for which $\alpha_* \approx 1$, while different values of $\sqrt{m_o}$ and ε were considered. Furthermore ε has been computed from:

$$1 - \varepsilon^2 = (a/2n)^2 \qquad \text{(lit.1)} \qquad (20)$$

where a is the number of zero crossings for a specific record and n is the number of waves occuring in the record; $\sqrt{m_o}$ is the area of the energy spectrum.

These spectra have been computed by a computer.

The limiting case of (19), $\varepsilon \to 0$, corresponds with the distribution for a very narrow spectrum: then

$$p(\varphi) \to \tfrac{1}{4}\varphi e^{-\frac{1}{8}\varphi^2} \qquad \text{(Raleigh distribution)} \quad (21)$$

The limiting case $\varepsilon \to 1$ gives the distribution for a very broad spectrum, then $p(\varphi) \to 0$ This case can occur when a wave of high frequency and small amplitude is superimposed on another wave of lower frequency (lit.1). The high frequency wave forms a ripple in the base wave and the distribution of the maxima $p(\xi/\sqrt{m_o})$ of the composed wave tends to the assumed Gaussian distribution of the ordenates of the sea surface $p(z/\sqrt{m_o})$

The cumulative probability $q(\varphi)$ is defined as the probability of φ exceeding a given value:

$$q(\varphi) = \int_{\varphi}^{\infty} p(\varphi)\, d\varphi \qquad (22)$$

Substituting from (19) it is found:

$$q(\varphi) = r^{3/4} \cdot e^{-\frac{1}{8}\varphi^2/r^{1/4}} + (1-r)^{3/4}(1+a\varphi)\cdot e^{-a\varphi} \qquad (23)$$

Graphs of $p(\varphi)$ and $q(\varphi)$ for various values of ε^2 are shown in fig.7.

In those wave records where $\alpha_* < 1$, correction factors have been computed analogous to those for ξ. The corrected graphs $q(\varphi)$ for various values of α_* and for $\varepsilon^2 = 0.4$ are given in fig.6. Examples of the application of the χ^2 test are given in table 1.

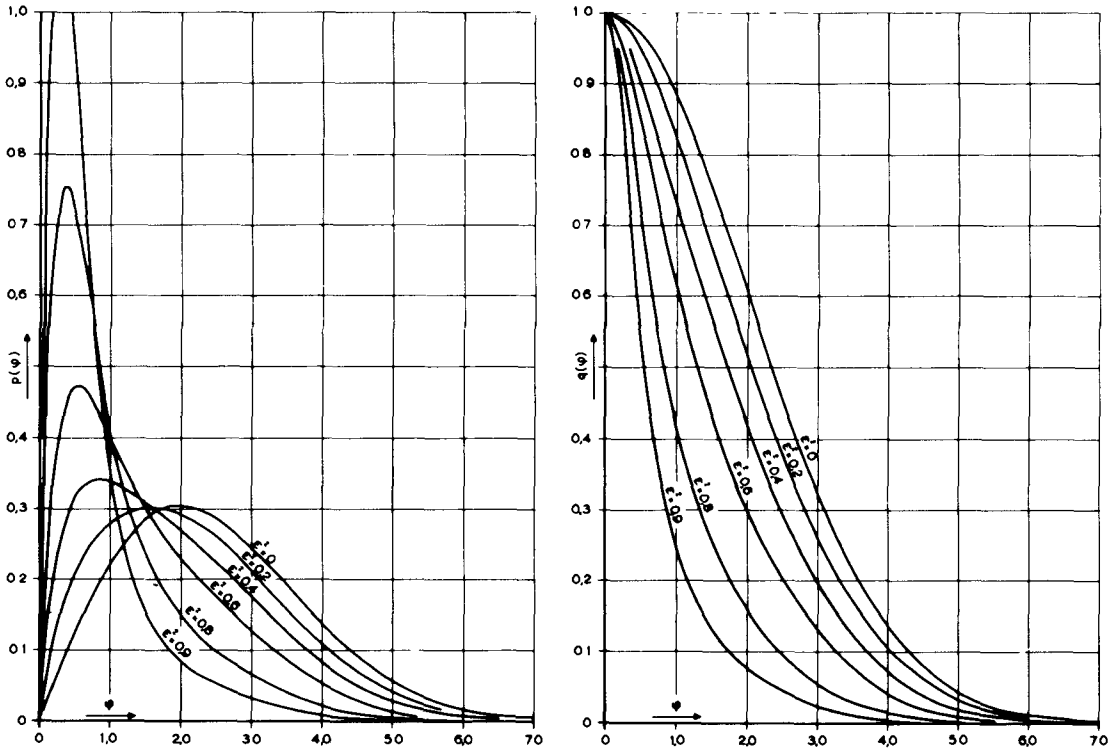

Fig 7 Probability distribution $p[\varphi]$ and cumulative distribution $q[\varphi]$, for $\alpha_x = 1$ and different values of ε^2

c. Distribution of $\tilde{\xi}$ and $\tilde{H}$.

The distributions of $\tilde{\xi}_c$ and $\tilde{\xi}_t$ obtained from the records, correspond sufficiently with the Raleigh distribution when $\alpha_x \approx 1$ The distribution of $\tilde{H}$, where $\alpha_x \approx 1$, deviates from the Raleigh distribution. The graphs for the cumulative probabilities

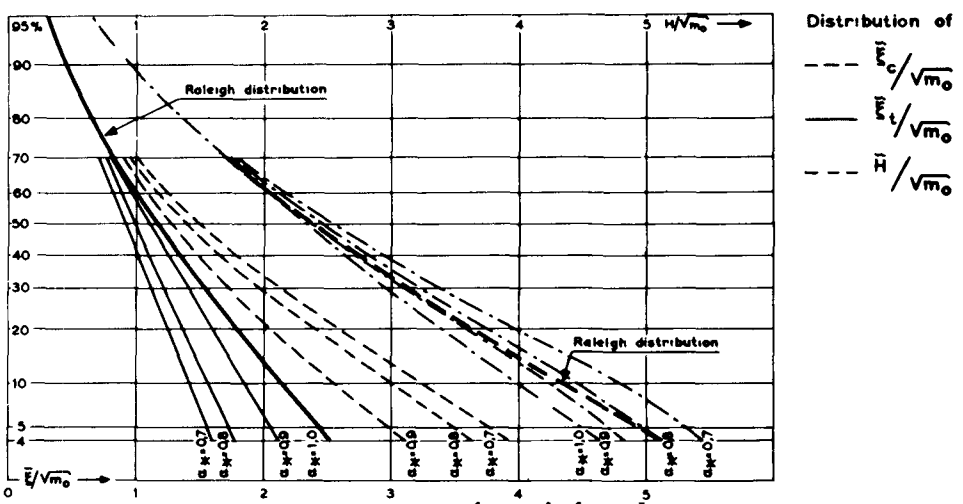

Fig 8 The cumulative probability distributions $q\left[\tilde{\xi}_c/\sqrt{m_0}\right]$, $q\left[\tilde{\xi}_t/\sqrt{m_0}\right]$ and $q\left[\tilde{H}/\sqrt{m_0}\right]$ for different values of α_x

record	$\sqrt{m_o}$ [cm]	ε^2	α_*	$p(\zeta_c)$		$p(\zeta_t)$		$p(H)$		$p(\tilde{\zeta}_c)$		$p(\tilde{\zeta}_t)$		$p(\tilde{H})$	
				χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$	χ^2	$p(\chi^2)$
Tr 98a	41	0.441	1.00	5.51	40%	9.96	8%	11.99	10%	7.28	20%	11.0	5%	9.60	20%
Tr 24.7	28	0.202	1.02	8.54	28	13.23	7	6.95	79	13.97	6	12.70	8	21.05	3
Tr 9.8	47	0.370	0.998	8.86	29	4.52	73	14.22	15	7.82	18	7.75	17	4.72	85
ESP 2	57	0.412	0.88	16.64	4	12.00	15	18.27	5	32.29	< 1	15.39	4	19.61	5
ESP 15	56	0.391	0.765	46.03	< 1	57.66	< 1	15.59	21	65.30	< 1	54.45	< 1	12.12	21
ESP 3	53	0.412	0.82	49.16	< 1	24.21	1	15.03	15	71.29	< 1	20.82	< 1	9.20	55
						I		II				III			

I. comparison with frequency distributions of Cartwright and Longuet Higgins ($\alpha_* = 1$).

II. comparison with frequency distributions according to equation (19); ($\alpha_* = 1$).

III. comparison with Raleigh distribution.

table 1

$q(\tilde{H}/\sqrt{m_o})$ and $q(\tilde{\zeta}/\sqrt{m_o})$ for $\alpha_* \approx 1$ and the corrected graphs for various values of α_* are shown in fig. 8. See also table 1. and § 3a.

The results shown in fig. 8, correspond on the whole with those obtained from the measurements by Goodknight and Russel (lit.6). These measurements were taken during hurricanes in the Gulf of Mexico and off the Pacific Coast of the U.S.A.

§ 4. Significant wave heights.

Irregular wave motion can be conveniently described by taking the highest $1/n^{th}$ waves of the total number in a sample. In practice the average value of the highest waves $H_{1/3}$ or $H_{1/10}$ is often used, as is the mean wave height H_m. For practical purposes the relations between these quantities have been studied by several investigators using theoretical formula (lit.5) as well as measurements taken in nature and in a laboratory. The relations can be computed from the formula given in § 3. This has been done only for the wave heights H and $\tilde{H}$; (for § see lit.1).

The $1/n^{th}$ highest waves correspond to the values of $\varphi(=H/\sqrt{m_o})$ greater than φ', which is defined by

$$q(\varphi') = \int_{\varphi'}^{\infty} p(\varphi)\, d\varphi = 1/n \qquad (24)$$

The average value of φ for these maxima, denoted by $\varphi_{1/n}$ is:

$$\varphi_{1/n} = n \int_{\varphi'}^{\infty} p(\varphi) \cdot \varphi \, d\varphi \qquad (25)$$

The mean wave height φ_m is found from:

$$\varphi_m = \varphi_1 = \int_{0}^{\infty} p(\varphi) \cdot \varphi \cdot d\varphi \qquad (26)$$

For the lineair case ($\alpha_* \approx 1$), $\varphi_{1/n}$ can be computed from (19) and (25). The values of $\varphi_{1/n}$ (n=1, 3 and 10) have been computed for different values of ε, as have the ratios $H_{1/10}/H_{1/3}$, $H_{1/10}/H_m$ and $H_{1/3}/H_m$; they are given in table 2 (see also fig.9).

A formula of $p(\varphi)$ for the non-lineair cases ($\alpha_* < 1$) has not yet been derived. In order to dertermine the relation between $H_{1/10}$, $H_{1/3}$ and H_m for such cases, use has been made of the fact that $H_{1/3}$ almost always corresponds to $H_{13,5\%}$ and $H_{1/10}$ with $H_{4\%}$. The values of $H_{13,5\%}$ and $H_{4\%}$ are exceeded by resp. 13,5 and 4% of the total number of wave heights in the sample. From the 100 records with different values of ε and α_* it was found that on the average $H_{1/3} = 0,996\ H_{13,5\%}$ and $H_{1/10} = 1,027\ H_{4\%}$; only a slight scattering of the points in the correlation occurs. By means of these relations the ratios $H_{1/10}/H_{1/3}$ $H_{1/10}/H_m$ and $H_{1/3}/H_m$ for $\alpha_* = 0,9 - 0,8$ and 0,7 have been determined (table 2) from the distributions $p(H)$ and $p(\tilde{H})$, given in §3.

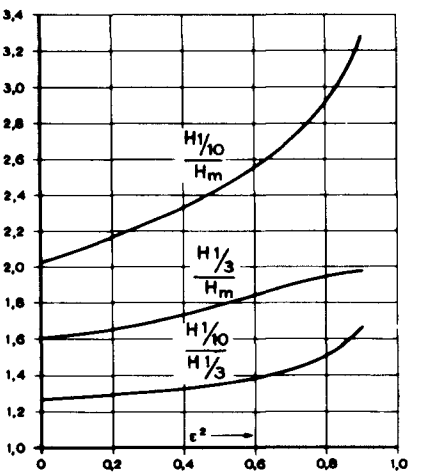

Fig 9 The quotients of $H_{1/10}$, $H_{1/3}$ and H_m [for $\alpha_* = 1$] as a function of ε^2

	$\alpha_* = 1$			$\alpha_* = 0,9$			$\alpha_* = 0,8$			$\alpha_* = 0,7$		
	$\varepsilon^2=0$	0.4	0.6	$\varepsilon^2=0$	0.4	0.6	$\varepsilon^2=0$	0.4	0.6	$\varepsilon^2=0$	0.4	0.6
$H_{1/10}$ / H_m	2.03	2.53	2.56	2.03	2.29	2.55	2.03	2.31	2.56	2.05	2.30	2.58
$H_{1/3}$ / H_m	1.60	1.74	1.85	1.61	1.77	1.86	1.62	1.79	1.87	1.62	1.79	1.88
$H_{1/10}$ / $H_{1/3}$	1.27	1.33	1.38	1.26	1.31	1.37	1.26	1.29	1.37	1.26	1.30	1.37

table 2

When the distribution $p(\bar{\varphi})$ approximates to a Raleigh distribution, $\bar{\varphi}_{1/10}$, $\bar{\varphi}_{1/3}$ and $\bar{\varphi}_m$ can also be computed from (25). From these values it was found that:

$$\bar{H}_{1/10}/\bar{H}_m = 2{,}00, \quad \bar{H}_{1/3}/\bar{H}_m = 1{,}60 \text{ and } \bar{H}_{1/10}/\bar{H}_{1/3} = 1{,}27.$$

It has been found that average values of ε and of the various ratios $H_{1/3}/H_m$ etc. may be applied along the Dutch North Sea Coast. The value of ε^2 is almost always found in the interval 0,4 -0,6.

The average ratios have been determined for different values of α_* from 100 records. It was found that the deviation of α_* from unity is not important (table 3).

	$H_{1/10}/H_m$	$H_{1/3}/H_m$	$H_{1/10}/H_{1/3}$	$\bar{H}_{1/10}/\bar{H}_m$	$\bar{H}_{1/3}/\bar{H}_m$	$\bar{H}_{1/10}/\bar{H}_{1/3}$
$\alpha_* \sim 1$	2,45	1 72	1 54			
$\alpha_* < 1$	2,34	1 75	1,30	2 04	1,58	1 24

table 3

However the value of ε greatly affects the distribution of the wave height H. The value of ε is largely determined by the sensitivity of the measuring instrument. When an instrument is used which damps out high frequency waves (for example a pressure meter) the energy spectrum is narrowed artificially and the values of ε will decrease. Thus the kind of instrument with which the records have been made must be taken into account. Wiegel (lit.7) reviewed the ratios between certain values of $H_{1/n}$ measured in various seas. Most of these measurements were taken with a pressure meter. When ε is zero, the values correspond on the whole with the values given in table 1.

Some results of measurements by Piest and Walden (lit.8) in the German Bight are given, which also deals with the relation between $\bar{H}_{1/10}$, $\bar{H}_{1/3}$ and $\bar{H}_m$. These ratios correspond well enough with the above mentioned values.

§5. Choice of a significant wave height.

In the foregoing the parameters H, ζ, $\overset{\frown}{H}$ or $\overset{\frown}{\zeta}$ were used to describe the wave motion for practical purposes. It appears however, that the parameters ζ and $\overset{\frown}{\zeta}$ are preferable to H and $\overset{\frown}{H}$ when the difference between deep water and shallow water waves is important. The distribution of H or $\overset{\frown}{H}$ is not greatly affected by the difference between deep and shallow water wave motion but the distribution of ζ, (and especially ζ_c and ζ_T) is. This can be expressed in terms of α_* which depends on the factor $L_m^2 \sqrt{m_o}/D^3$

In the preceeding pages it has been demonstrated that high waves in shallow water show an increasing eccentricity relative to the mean level if the water depth decreases. It is evident that the movement of the water particles in shallow water waves differs considerably from that in deep water waves. Therefore, when a wave motion is described by means of the parameters H or $\overset{\frown}{H}$, the factor $L_m^2 \sqrt{m_o}/D^3$ must also be taken into account.

In the second place we have seen that the distributions of $\overset{\frown}{H}$ and $\overset{\frown}{\zeta}$ are independent of the width of the energy spectrum (expressed in terms of ε). The distributions $p(\overset{\frown}{H})$ and $p(\overset{\frown}{\zeta})$ correspond to a great extent with the formula of the Raleigh distribution, which occurs when the energy spectrum of a wave motion is very narrow. Thus, by using the parameters $\overset{\frown}{H}$ or $\overset{\frown}{\zeta}$ (by which usually only the higher waves are described), a rather rough picture of the actual wave motion can be given. The cumulative distribution of $\overset{\frown}{H}/\sqrt{m_o}$ and $\overset{\frown}{\zeta}/\sqrt{m_o}$ have the shape of a Raleigh distribution with the same m_o as the energy spectrum of the actual wave motion.

Finally, it should be noted that for shallow water wave research in the laboratory, the influence of α_* and thus of $L_m^2 \sqrt{m_o}/D^3$ cannot be neglected.

Conclusions and discussion.

In the case of wave motion in relatively shallow water, the distribution of the "trough to crest" wave heights H or $\tilde{H}$, do not differ much from those in deep water. However, the heights from mean level to crest ζ_c or $\tilde{\zeta}_c$, and to trough, ζ_T or $\tilde{\zeta}_T$, may differ greatly. It appeared that the statistical distribution of wave heights in relatively shallow water depends on:

a. The standard deviation $\sqrt{m_o}$ of the distribution of the sea surface ordinates z(m_o is also the area of the energy spectrum)
b. The relative width of the energy spectrum (denoted by ε);
c. The relation between the wave length L_m, the mean energy m_o and the water depth. This relation can be expressed by $L_m^2 \sqrt{m_o}/D$ Transition from deep water into shallow water wave distributio also results in a skewness of the distribution of the ordinate of the sea surface. This skewness is defined by α or α_*; especially α_* is closely correlated with the factor $L_m^2 \sqrt{m_o}/D^3$

In this study the statistical variation of m_o has not been examined, nor has the problem of how far the wave motion can be considered as a statistically stationary process. This condition has been satisfied as well as possible by taking at least one hundred waves during the measuring time under which conditions depths and wind forces did not change significantly during the measurement. The maximum time interval of records was 30 minutes; the minimum was 10 minutes.

In some cases very broad energy spectra were found; it appeared that in those cases the wave motion was composed of two or more separate wave trains from different directions. Such spectra have not been used for this study, as a mean wave length cannot be defined.

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