Chapter 47

SEA TESTS OF A SPREAD-MOORED LANDING CRAFT

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ABSTRACT

Sea tests of motion and mooring force were conducted on an LST (Landing Ship Tank) of about 4400 long tons displacement. The LST was spread-moored by six 2-1/16 inch and one 1-1/4 inch (port breast) stud-link chains in simple catenary configuration in about 45 feet of water in the open Gulf of Mexico about 65 air miles south of New Orleans, Louisiana. Water-level variations at a single location, ship rotations and accelerations, mooring force, and wind were measured in sea states of 2 and 4. Three recordings of 38, 62, 67 minutes duration were analyzed, using time-series techniques to provide apparent amplitude-response operators for all of the ship's motions and seven mooring chains. Theoretical prediction of the operators using long crested regular waves was made also. In longitudinal plane, theory predicts motions 1/3 to 4 times and chain tensions 1/4 to 9 times those measured. The most probable maximum-motion amplitude responses in sea state 4 are found to be 1.7, 1.1, and 1.7 feet, respectively, in surge, sway and heave, and 3.4 and 0.5 degrees, respectively in pitch and yaw. Roll was measured only in sea-state 2 with a corresponding maximum of 2.1 degrees. Maximum wave-induced chain tensions in kips were: 85.1 and 48.0 in port and starboard bow chains respectively; 10.6 (sea state 2) and 19.7 in port and starboard breast chains; 13.9 and 4.3 in port and starboard quarter chains (sea state 2) and 9.7 in stem chain. Total tension in port bow chain was 116.1 kips (85.1 plus initial tension of 31.0 kips). Chain response operators vary directly with initial tension, whicl complicates design.

It is concluded that: (i) moor was unbalanced, i.e., port bow chain took most of load; (ii) chains loaded lightly, e.g., maximum wave induced tension was 116 kips compared to new proof load of 300 kips for the particular chain, the port bow; (iii) water level should be measured at more than one point; (iv) discouragement over differences is balanced by encouragement over agreements between measurements and theoretical prediction of motion and chain tension; (v) toward improvement: Theory needs extension to include short crested waves and barge types; (vi) initial tension unique to problem of mooring design; (vii) propulsion devices may be needed toward maintaining design initial tension, especially in storm; (viii) if directional spectra had been measured and if theory involving short crested waves had been available and used, then discrepancies between observation and theory likely would have been less.

INTRODUCTION

As the need to operate from fixed platforms floating at the sea surface at points beyond the continental shelf boundaries increases due to dictates of National defense, economic forces and scientific requirements, ways to hold these platforms in position when subjected to the forces of wind, wave, and current have to be developed.

One way to hold such a platform in position is to power it, continually sense its position, and continually apply power to maintain it on station.

A second way is to moor the platform by means of anchor and lines. Systems using conventional and even stake pile anchors and heavy anchor chains are very common. They have been employed in water of varying depth to moor a number of different kinds of platforms.

In high wave and strong winds, platforms moored in this way are subject to forces and motions of a complex nature. In turn, the mooring chains experience tensions that are much greater than those experienced under calm conditions. Such platforms have, in fact, broken their mooring chains and been lost or damaged.

The failure of the anchor chains can be due to the imposition of forces beyond their design capability or to the gradual deterioration of the chain in a marine environment. In either case, the nature of the forces applied to the chain needs to be investigated theoretically and compared with measurements.

Also the nature of the motions of the moored platform due to the forces of wind and wave needs to be investigated as a measure of the kinds of work that can be done on such a platform.

A platform restrained by mooring chains in the two horizontal directions will seek an equilibrium position in which the chains, hung from the platform in the form of a catenary, when displaced from this equilibrium position by the force of the wind on the "sail" area of the platform and by the effects of waves and currents, exert forces (of the nature of spring constants) in surge, sway, and yaw. These tend to restore the platform to its equilibrium position. These restoring forces have to be added to the equations that describe the motions of the platform.

As a simple case, the restoring forces can be treated as linear, but, especially for extreme motions that may occur, these forces may be non-linear depending on terms of the form $(k_1 + k_2 (x)^2)x$ where x is the displacement in surge, and k_1 and k_2 are constants reflecting particular chain properties. Due to coupling with the other motion components, the analysis of the non-linear problem appears to be difficult. Some analytical and theoretical success has been achieved with the linear model.

The other complicating effect in this problem is the nature of the applied excitations. The wind, being turbulent, exerts a mean force on the platform and a fluctuating component with periods that are more than 30-minutes on down through 10-minutes, 5-minutes, 1-minute, 30-seconds, and so on. The waves, in turn, contain periods ranging from 20 seconds on down.

Both the wind and the waves can be represented by stationary random processes that oscillate about certain mean values and that grow and shrink from oscillation to oscillation. The application of random process theory to the problem of the moored platform is essential toward understanding its behavior.

However, and this is the rub, the extreme values that occur in random process theory are the least well-documented aspect of the theory and depend on probability concepts that are the most subject to criticism. In particular, the non-linear and not well-understood model of the moored system needs to be analyzed when the motions are extreme.

Despite these difficulties, some progress has been made in gathering and analyzing the data pertinent to the behavior of moored systems and in developing a theory to explain and interpret the data. It is the purpose of this paper to present the highlights of the results of tests of a particular system and to point out where theoretical and observational improvements can be obtained. It is based on the report to the U.S. Naval Civil Engineering Laboratory by O'Brien and Muga (1964).

THE SHIP AND ITS INSTRUMENTATION

THE SHIP

The craft studied is a landing ship tank (LST CLASS 542) modified to serve as a tender for an offshore drilling platform about 30 feet forward of her bow. The owners, the California Company, designate the craft as S-23. It has a length of 319 feet at the waterline, an extreme moulded breadth of 50 feet, a draft of 12 feet, and a displacement of 4420 long tons. Other characteristics are given in Table I.

The craft (Figure 1) was spread-moored in 45 feet of water in the open Gulf of Mexico, at about N 29-01-42 W 90-09-18, approximately 65 air miles south of New Orleans, Louisiana, by six 2-1/16 and one 1-1/4 inch stud-link chains with proof breaking loads when new of 300 and 185 kips, respectively. Their length varied from approximately 440 feet at the port bow to 1, 140 feet at the quarter (Figure 2) as shown in Table II, which lists the results of on-site measurements. These were difficult and required about three weeks of work by a large crew, including a three man team of divers.

The port bow chain (No. 2) was anchored to a 10,000 pound conventional anchor. The remaining six chains were anchored to stake piles. In addition, a 10,000 pound conventional anchor helped to hold the stern chain (No. 6).

The craft was displaced analytically in the longitudinal and transverse directions, i.e., in surge and sway respectively. The resulting change in chain tensions were calculated and resolved in proper components to provide values for restoring force of 20.0 kips per foot in surge (k_x), 12.7 kips per foot in sway, (k_y) and 5.62 kip-feet per degree of the yaw (k_ψ), as shown in Table I. The chains were considered ineffective against heave, pitch, and roll.

DATA PICKUPS

A total of seventeen measurements were made simultaneously as a function of time on 3 September, namely: water level variation (wave) at one point; wind speed and direction; ship acceleration in surge, sway, and heave at bow and sway and heave at stem; roll and pitch; tension in all seven chains (Figure 1). On 23 March, roll, pitch and tension, in three of the chains (#1, 5, and 7) were not measured due to malfunctioning of the pickups.

Water level variation (wave) was measured at a point approximately 30 feet forward from the bow of the craft by means of a vertical staff of the resistance type. Electrical type water level sensors, similar to sparkplugs, were fastened to the staff at one half foot intervals in the vertical from about -10 to +20 feet mean water level; their shorting on contact with water was sensed through a proper electrical circuit.

Table I. Characteristics of LST (S-23), Class 542

Length (ft):	
overall	327.8
at water-line	318.6
between perpendiculars	316.0
Breadth, B, extreme molded, (ft)	50.0
Draft, D, (ft)	12
Depth, h, molded, at midships, (ft)	25.16
Water Depth, d, (ft)	45
Displacement, long tons	4,420
Mass, M, $lbs\text{-}sec^2/ft^4$ (slug)	307,478
A_s surface area at water-line (3rd deck), (ft^2)	14,887
Free period of oscillation (seconds/cycle): *surge = 24.6; *sway = 31.1	
roll	6.8
*pitch	3.3 to 4.4
*heave	5.1
BG, Vertical distance between CG and CB (ft)	5.9
OG, Vertical distance from free surface to CG (ft)	0.2
GM, Metaccentric height (ft)	11.8
CG, Center of gravity (ft):	
aft fwd perpendicular	171.6
above keel	12.2
I_{x_T}, virtual moment of inertia about x-axis (including added moment of inertia) ($slug\text{-}ft^2$)	1.38×10^8
I_y, ship's moment of inertia about y-axis ($slug\text{-}ft^2$)	1.97×10^9
I_z, ship's moment of inertia about z-axis ($slug\text{-}ft^2$)	2.020×10^9
ξ_s, coordinate of aft-most point at water-line (center-of-gravity coordinate system), (ft)	-155.3
ξ_b, coordinate of fore-most point at water-line (center-of-gravity coordinate system), (ft)	163.3
Restoring force:	
surge, k_x, kips/foot	20.0
sway, k_y, kips/foot	12.7
yaw, k_ψ, kip-feet/degree	5.62

*calculated.

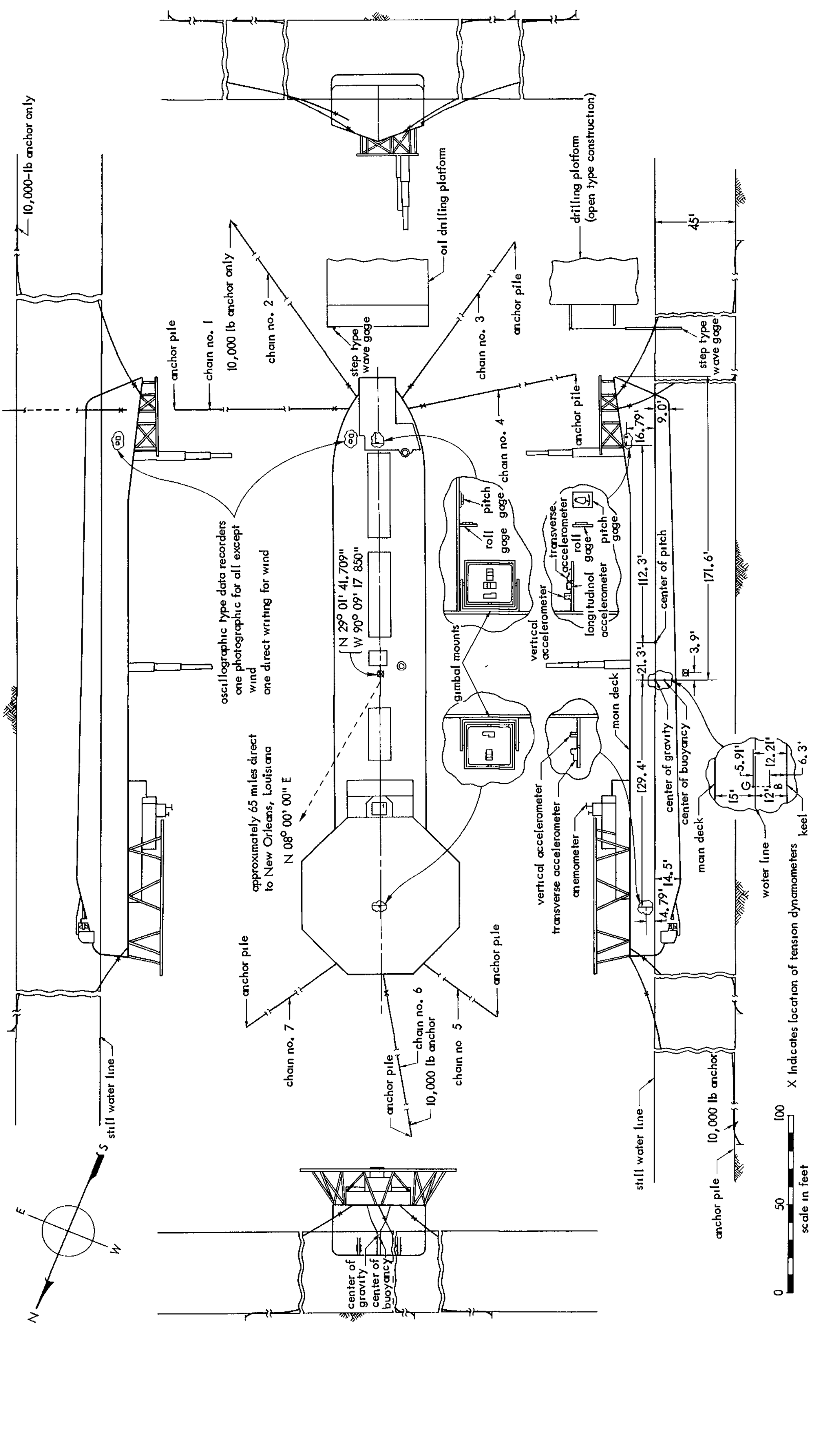

Figure 1 LST as moored and instrumented.

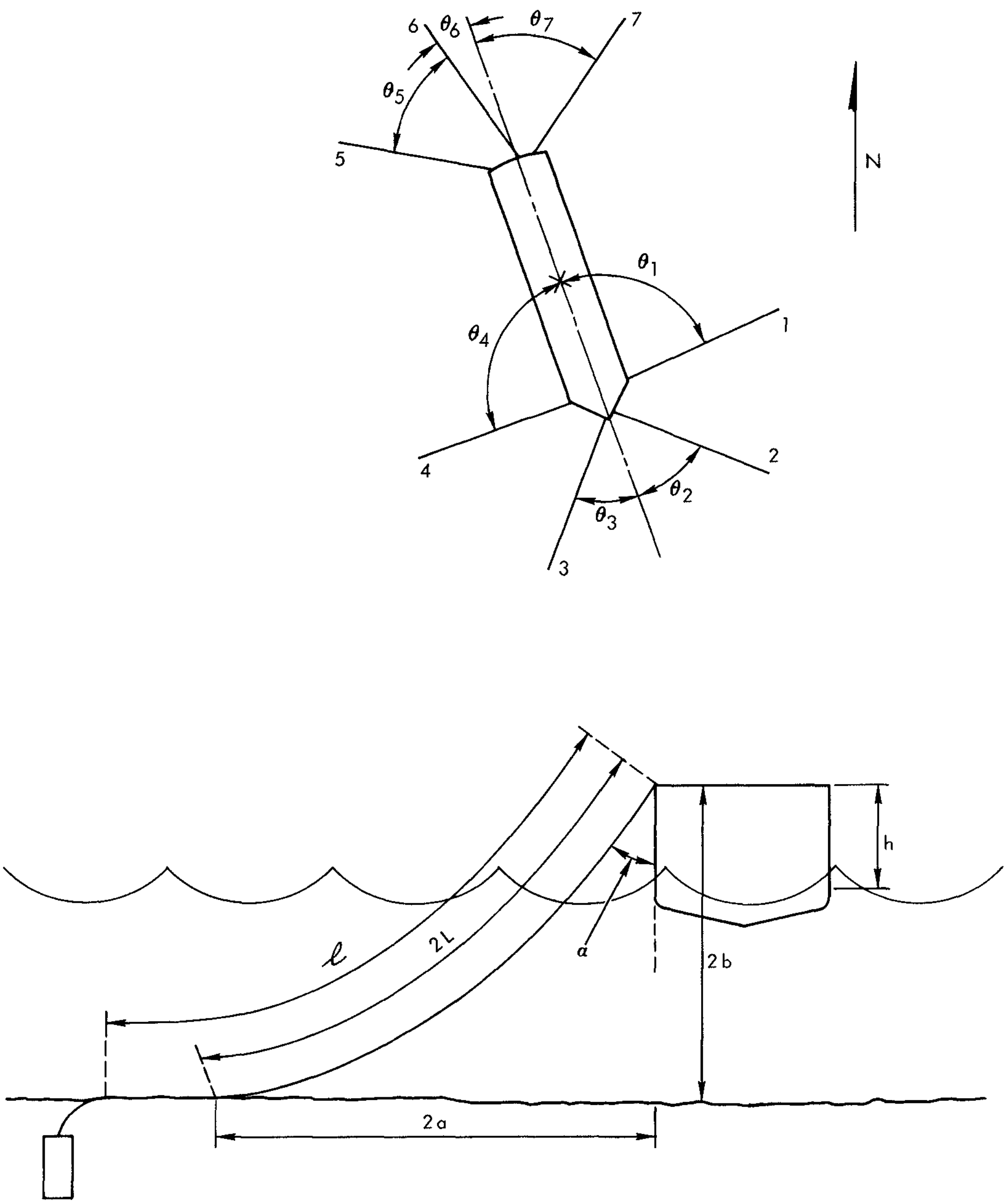

Figure 2 Convention used to describe mooring chain geometry. (See Table II for actual dimensions and tensions.)

Table II. Characteristics of LST Moorings
(See Figure 3 for explanation of symbols)

Chain No.	Chain Position	2b (feet)	h (feet)	s (feet)	ℓ (feet)	2L (feet)	2a (feet)	α (deg.)	θ (deg.)	Equilibrium (kips)	Initial Tension 23 March AM (kips)	Initial Tension 23 March PM (kips)	Initial Tension 3 Sept (kips)
1	Port Breast	35.7	10.0	910.5	915.4	174.8	169.9	24.7	87.25	6.0	---	---	10.8
2	Port Bow	63.0	21.0	430.7	438.8	328.1	320.0	20.6	35.00	31.1	31.0	30.6	34.6
3	Starboard Bow	71.6	23.0	598.3	609.1	321.0	310.2	23.9	34.78	26.4	30.2	30.7	28.3
4	Starboard Breast	66.0	20.5	868.4	884.0	191.0	175.4	37.2	87.53	10.8	17.7	17.8	6.6
5	Starboard Quarter	51.0	17.5	971.4	981.6	173.7	163.5	32.6	51.92	11.2	---	---	12.2
6	Stern	57.7	18.0	817.1	824.3	312.7	305.5	22.2	11.0	30.6	7.4	7.6	24.1
7	Port Quarter	54.0	17.0	1132.4	1140.8	233.5	224.7	26.6	53.97	18.6	---	---	21.5

Note: (1) Values for 2b through θ are for the equilibrium condition; they are assumed to reflect those for 23 March and 3 September to a significant accuracy.

(2) All chains are stud-link. No. 1 is 1-1/4 inch; all others are 2-1/16 inch.

A conventional torque type recording selsyn-type anemograph was used to measure wind speed and direction. Both the direction vane and the speed cups were located on the bridge of the ship, approximately 40 feet above the water surface or 25 feet above the main deck.

Ship acceleration was measured by gimbal mounted linear accelerometers with sensitivity of about 0.001 G as installed on the longitudinal axis of the ship; three 112.3 feet forward of the C.G. for measurements in respectively surge, sway, and heave directions; two about 129.4 feet aft of the C.G. for measurements in respectively sway and heave directions.

Separate conventional pendulum type inclinometers were used to measure roll and pitch.

Chain tension was measured by means of a four wire strain gage bridge, cemented to a standard link in each of the seven chains. The vertical inclination of the link was not recorded; however, it was measured manually periodically.

All pickups were part of separate energized electrical circuits such that pickup response was sensed by the deflection of a galvanometer, and this deflection suitably recorded by means of a photographic type oscillograph at relatively low paper speeds, i.e., 13.2 inches per minute on 23 March and 2.75 inches per minute on 3 September. The exception was the wind velocity pickup, whose output was recorded by a direct writing oscillograph, which was part of the anemograph.

The velocity of the surface currents was measured periodically with a propeller type meter and also with floats. In general, these measurements were difficult and not always successful.

MEASUREMENTS MADE

GENERAL

Figure 3 is a facsimile of four minutes duration, taken from an oscillogram on 3 September. It is typical of the total of about 100 hours of measurements made in 1958. The highly embroidered profile for water level is reflected in that of the responses. Water level was recorded as a series of equivalent one-half foot steps on the original oscillogram; they have been rendered as a continuous line on the facsimile.

Note the low frequency component of about 39 seconds period (0.17 radians per second) in the chain response, particularly for numbers 2, 3, and 7, as excited by a beam-on wave of this period.

Wind velocity was measured continuously with all other quantities and recorded on a conventional type anemogram.

MEASUREMENTS SELECTED FOR ANALYSIS

Three continuous records of measurement, during which a stationary random process was assumed to exist, were selected for analysis: 38 minutes in the morning of 23 March (1146 to 1223 CST), 62 minutes in the afternoon of 23 March (1225 to 1326 CST), and 67 minutes on 3 September (1200 to 1306 CST).

As noted under "Data Pickups," the 3 September data contains output from seventeen different pickups; that for 23 March is less data from the inoperative roll, pitch and three chain tension pickups (#1, 5, 7).

The measurements of wind velocity, although complete, were only scanned since, although the wind displaced the craft from its equilibrium position, it was not high enough to interfere significantly with the response of the craft to waves, which was the main concern of the study.

To permit machine analysis, the height of each pickup trace above a base line was read at intervals of one-second and recorded as punches (data points) on standard machine type cards; specifically: 2281 and 3721 points for 23 March and 4021 points for 3 September.

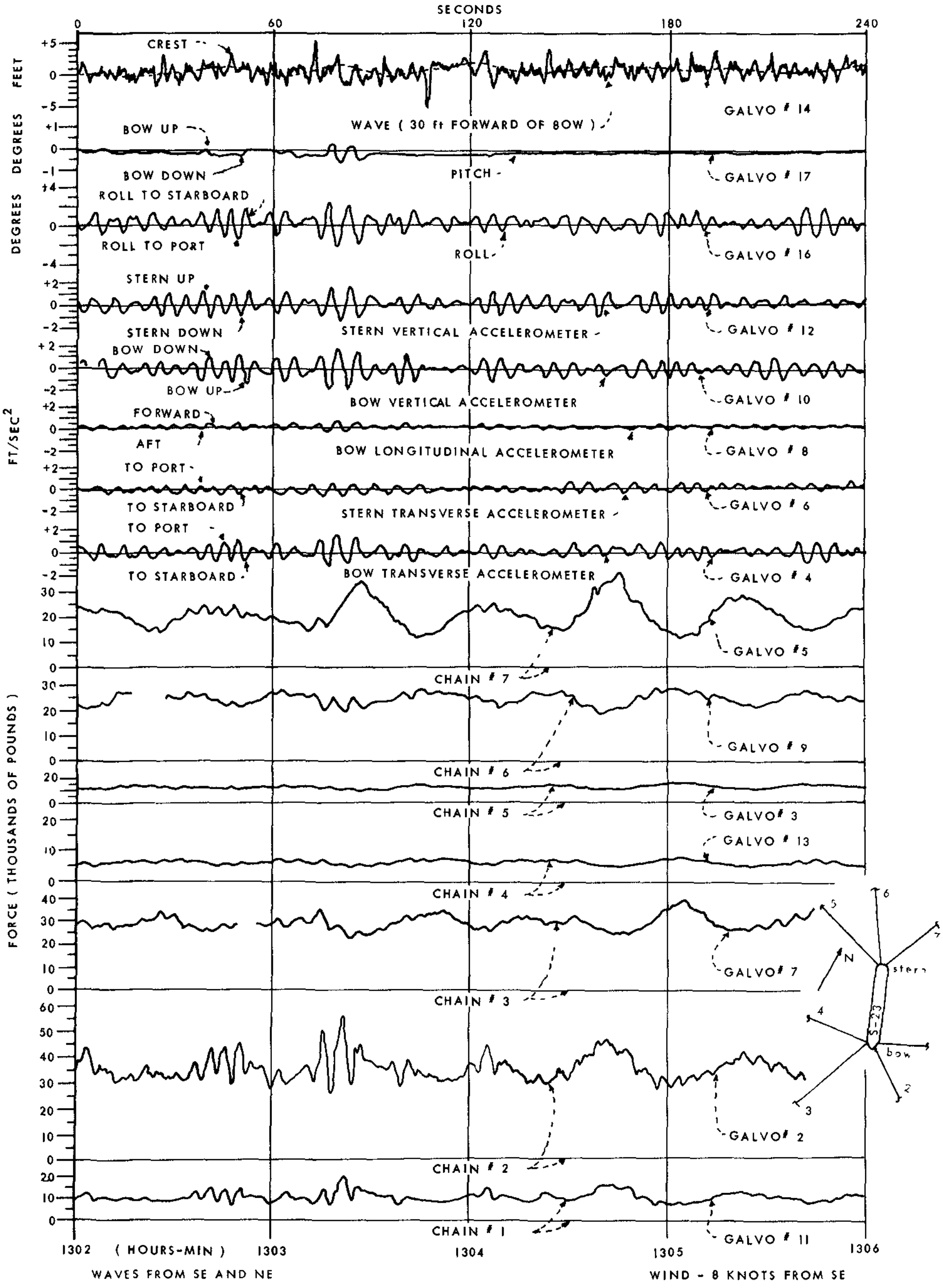

Figure 3 Facsimile of oscillogram for 3 September, 1302 to 1306 CST.

METHODS OF ANALYSIS

The analysis of measurements as complex as those obtained in this study (Figure 2) would have been improbable if not impossible a few years ago. However, due to the pioneer work of Pierson (1952), based on the earlier work of Blackman and Tukey (1958) in electrical engineering, and applied by Canham, et. al. (1963), among others, to seakeeping trials, the analysis technique now is nearly routine, albeit tedious. Therefore, only a very general explanation of it is given here.

The time-series technique forms the basis for the analysis. A stationary random process is assumed over a period of sensible duration. All answers obtained are statistical in nature with the attendant variability. The principle of linear superposition is used as a device for breaking down the complex records of excitation (sea state), mooring force, and ship response into simple elements. The irregular excitations and responses are regarded as the approximate sum of a number of regular sinusoidal components in a random phase relationship.

By use of correlation functions and transforms, the energy in the irregular wave record is isolated into particular magnitudes (square of the amplitude (A) of water level, motion or force) in particular frequency (ω) bands. These two quantities are used to plot a diagram known as the "energy spectrum" specifically (A^2/ω) versus ω.

For a particular frequency, the ratio of the response ordinate to the excitation ordinate gives an ordinate on a diagram known as the "response amplitude operator." For the sea tests such as these, the response operator tends to provide a check on the rigor of the sea measurements since it can also be obtained experimentally in the laboratory with a reduced scale model and also theoretically where, in both cases, regular long-crested waves are used.

The phase of one motion relative to another is obtained by comparing the recorded amplitude of one motion to that of the other by use of cross spectral analysis technique.

The quantity, which indicates the departure of the field measurements from those predicted by experiments and theories based on long-crested waves, is termed "coherency." This measures the ratio of that part of two signals having a definite phase relationship to the total power in the two signals.

For spectra and cross spectra, the coherency is related to the correlation coefficient of the records after passing them through the equivalent of a band-pass filter to study a particular narrow filter range.

A value of coherency equal to, or near, unity provides a confirmation of the linearity of the relationship between two particular motions within the frequency range analyzed, and of essentially long-crested wave action on the system. If the data are believed to be accurate, and digitization and computations are carefully checked, then coherencies less than one are a measure of the short crestedness of the forcing waves.

Finally, the degree to which the true value is approximated by an estimate is defined by "confidence limits" as computed from familiar probability theory.

By use of the area (E) under the energy spectrum, termed the variance, important statistical predictions can be made on the basis of a Rayleigh distribution and information contained in Longuet-Higgins (1952). For example, $\sqrt{E}$ multiplied by the following factors gives the following amplitudes: 0.707 for the most frequent; 0.886 for the average; 1.416 for average of highest one-third; 1.800 for average of highest one-tenth. The factor for the most probable maximum is not so straightforward. It is necessary to first determine the average number of oscillations in a particular record and thereby to determine the average period. The number of oscillations in the record is then computed and this number applied to the proper curve in Longuet-Higgins (1952) to obtain the factor by which $\sqrt{E}$ is multiplied in order to obtain the value of the most probable maximum. This factor differed for each of the three test records, namely: 2.60 and 2.67 on AM and PM 23 March respectively and 2.72 on 3 September. The values in Table 5 are based on all of these factors, using the values of the variances as given.

Sorting indicates that the excitation and response amplitudes tend to have a Rayleigh distribution. This is more the case for wave amplitudes than for the chain tensions as is apparent from comparing the results in Tables 4 and 5 for sorting and variance type prediction, respectively.

Even if it had been possible to measure the directional spread of the wave energy, it would not have been possible to obtain the response operators from response measurements since particular motions could not be related to waves from a particular direction. However, since some insight into these important operators was desired, a uni-directional wave system was assumed so that by use of measurements of water level at a single point and of the analytical techniques outlined previously, it was possible to obtain a quantity termed "apparent response amplitude operator." The adjective "apparent" is dropped hereinafter. Yawing of the vessel as well as the directional effects are contained in this somewhat crude operator.

Heave, surge, sway and yaw were not measured directly. Rather, they were deduced along with pitch from acceleration measurements using the technique described by Cartwright (1957) and also O'Brien and Muga (1963). Pitch was also measured directly along with roll on one day: 3 September. Acceleration measurements pertinent to roll were not made, unfortunately.

Simple harmonic motion was assumed for both excitation and response, e.g., acceleration was divided by the square of the pertinent frequency to obtain displacement.

Table III. Frequency of Peak-Response Energy and Corresponding Amplitude (To obtain corresponding chain tension add the initial tensions given in Table II)

	Frequency (ω) (raps)			Period (T) (spc)			Amplitude			
	23 March		3 Sept	23 March		3 Sept	23 March		3 Sept	
	AM	PM		AM	PM		AM	PM		Units
Surge	0.60	0.66	0.75	10.5	9.5	8.4	1.22	1.45	0.25	feet
Heave	0.60	0.57	1.07	10.5	11.0	5.9	0.89	1.03	0.31	feet
Pitch	0.63	0.63	1.01	10.0	10.0	6.2	2.2	2.3	0.31	degrees
Pitch Measurement			0.97			6.5			0.31	degrees
Sway	0.88	0.85	0.94	7.1	7.4	6.7	0.84	0.78	0.53	feet
Yaw	0.60	0.63	0.63	10.5	10.0	10.0	0.35	0.31	0.24	degrees
Roll			0.94			6.7			0.92	degrees
Water Level	0.66	0.66	1.04	9.5	9.5	6.0	3.43	3.75	1.41	feet
Chain Nos:										
1			0.16			39.3			1.3	kips
			1.20			5.7			not definable	
2	0.16	0.16	0.16	39.3	39.3	36.0	5.3	5.4	2.6	kips
	0.63	0.66	0.97	10.0	9.5	6.5	7.2	8.9	3.1	kips
3	0.22	0.19	0.16	28.6	33.1	39.2	4.45	3.5	2.0	kips
	0.63	0.66	1.04	10.0	9.5	6.0	7.95	8.15	1.45	kips
4	0.16	0.16	0.16	39.3	39.3	39.3	1.37	1.37	0.45	kips
	0.63	0.63	1.01	10.0	10.0	6.2	4.60	4.43	1.35	kips
5			0.16			36.0			1.0	kips
			1.20			5.7			not definable	
6	0.19	0.19	0.25	33.1	33.1	25.1	1.73	1.65	1.73	kips
	0.63	0.66	1.01	10.0	9.5	6.2	2.10	2.15	1.2	kips
7			0.16			39.3			3.9	kips
			1.20			5.7			not definable	

Table IV. Amplitudes From Sorting

1	2 Most Frequent	3 Average	4 Average Of Highest 1/3	5 Average Of Highest 1/10	6 1/2 of Maximum Double Amplitude	7 Maximum Single Amplitude
Chain 1 (kips):						
3 Sept	1.5	1.6	2.9	3.8	6.5	10.6
Chain 2 (kips):						
23 Mar AM	9.5	8.8	14.8	23.5	54.0	85.1
23 Mar PM	10.5	9.0	15.0	21.1	32.5	58.4
3 Sept	2.5	3.2	5.7	8.0	16.5	22.9
Chain 3 (kips):						
23 Mar AM	3.8	6.2	11.4	17.4	29.5	44.5
23 Mar PM	3.8	6.6	11.3	15.1	23.5	29.8
3 Sept	1.5	1.7	3.2	4.3	7.0	10.6
Chain 4 (kips):						
23 Mar AM	2.5	3.3	5.5	7.9	13.5	19.7
23 Mar PM	2.8	3.3	5.5	7.0	10.0	13.8
3 Sept	0.3	0.3	0.6	1.1	2.5	4.7
Chain 5 (kips):						
3 Sept	0.5	0.5	1.1	1.7	2.5	4.3
Chain 6 (kips):						
23 Mar AM	1.8	1.9	3.2	4.0	6.5	9.9
23 Mar PM	1.8	2.0	3.1	3.5	5.5	9.8
3 Sept	1.3	1.4	2.3	3.1	4.5	5.0
Chain 7 (kips):						
3 Sept	1.8	2.4	5.0	6.8	11.0	13.9
Water Level (feet):						
23 Mar AM	1.8	2.6	4.4	5.8	8.0	9.5
23 Mar PM	1.8	2.6	4.3	5.8	7.8	9.0
3 Sept	1.3	1.1	1.7	2.0	4.5	5.0
Roll (degrees):						
3 Sept	0.6	0.7	1.1	1.5	2.1	2.1
Pitch (degrees):						

Table V. Amplitudes Obtained from Spectral Variance

1	2		3	4	5	6	7	8	9
	Limits Lower	Upper (raps)	E	$\sqrt{E}$	Most Frequent	Average	Average Of Highest 1/3	Average Of Highest 1/10	Most Probable Value of Maximum
Water Wave (feet):									
23 Mar AM	0.44	1.88	5.50	2.35	1.66	2.08	3.32	4.22	6.11
23 Mar PM	0.44	1.80	5.57	2.36	1.66	2.09	3.34	4.25	6.30
3 Sept	0.44	1.80	1.10	1.05	0.74	0.93	1.49	1.89	2.86
Surge (feet):									
23 Mar AM	0.50	1.57	0.44	0.66	0.47	0.59	0.93	1.19	1.72
23 Mar PM	0.50	1.57	0.59	0.77	0.54	0.68	1.09	1.38	1.58
3 Sept	0.50	1.57	0.02	0.15	0.11	0.14	0.22	0.28	0.41
Heave (feet):									
23 Mar AM	0.44	1.57	0.34	0.58	0.41	0.51	0.82	1.04	1.51
23 Mar PM	0.44	1.57	0.45	0.67	0.47	0.59	0.94	1.20	1.79
3 Sept	0.70	1.57	0.04	0.19	0.13	0.17	0.26	0.34	0.52
Pitch, measured (degrees):									
3 Sept	0.44	1.57	0.03	0.18	0.13	0.16	0.26	0.33	0.49
Pitch, derived (degrees):									
23 Mar AM	0.44	1.57	1.37	1.17	0.83	1.04	1.66	2.11	3.04
23 Mar PM	0.44	1.57	1.64	1.28	0.91	1.13	1.81	2.31	3.42
3 Sept	0.44	1.57	0.03	0.18	0.13	0.16	0.26	0.33	0.49
Sway (feet):									
23 Mar AM	0.69	1.57	0.19	0.43	0.31	0.38	0.61	0.78	1.12
23 Mar PM	0.69	1.57	0.17	0.41	0.29	0.37	0.59	0.74	1.09
3 Sept	0.82	1.57	0.10	0.32	0.23	0.28	0.45	0.58	0.87
Yaw (degrees):									
23 Mar AM	0.44	1.88	0.03	0.18	0.13	0.16	0.26	0.33	0.47
23 Mar PM	0.44	1.88	0.05	0.23	0.16	0.20	0.32	0.41	0.61
3 Sept	0.44	1.88	0.02	0.13	0.09	0.12	0.18	0.23	0.35
Roll (degrees):									
3 Sept	0.63	2.51	0.62	0.79	0.56	0.70	1.11	1.42	2.15

Units: (a) Column 3 is square of underscored unit in Column 1.
(b) Columns 4 through 9 are the same as underscored unit in Column 1.
For example, "feet" in Column 1 gives "feet^2" in Column 3, but "feet" in columns 4 through 9.

Table V. Amplitudes Obtained from Spectral Variance (Cont)

1	2		3	4	5	6	7	8	9	
	Limits (raps) Lower	Upper	E (kips squared)	√E (kips)	Most Frequent (kips)	Average (kips)	Average Of Highest 1/3 (kips)	Average Of Highest 1/10 (kips)	Most Probable Value of Maximum By Frequency (kips)	Total (kips)
Chain 1:										
3 Sept	0.00	0.63	1.57	1.25	0.89	1.11	1.77	2.26	3.25	
3 Sept	0.63	1.57	1.10	1.05	0.74	0.93	1.49	1.89	2.80	
Chain 2:										
23 Mar AM	0.00	0.41	27.30	5.22	3.69	4.63	7.40	9.40	13.57	21.11
23 Mar PM	0.00	0.44	28.30	5.32	3.76	4.71	7.53	9.58	14.20	21.76
3 Sept	0.00	0.63	6.45	2.54	1.80	2.25	3.60	4.57	6.91	9.36
Chain 2:										
23 Mar AM	0.41	1.88	38.60	6.21	4.39	5.50	8.80	11.18	16.15	
23 Mar PM	0.44	1.88	38.10	6.17	4.36	5.47	8.74	11.11	16.48	
3 Sept	0.63	2.20	5.35	2.31	1.64	2.05	3.28	4.16	6.28	
Chain 3:										
23 Mar AM	0.00	0.47	12.55	3.54	2.50	3.14	5.02	6.38	9.20	15.44
23 Mar PM	0.00	0.47	10.50	3.24	2.29	2.87	4.59	5.83	8.65	15.86
3 Sept	0.00	0.63	3.93	1.98	1.40	1.76	2.81	3.57	5.39	5.82
Chain 3:										
23 Mar AM	0.47	1.88	22.70	4.76	3.37	4.22	6.75	8.58	12.38	
23 Mar PM	0.47	1.88	24.80	4.98	3.52	4.41	7.05	8.96	13.30	
3 Sept	0.63	2.20	0.63	0.79	0.56	0.70	1.12	1.43	2.15	
Chain 4:										
23 Mar AM	0.00	0.41	1.96	1.40	0.99	1.24	1.98	2.52	3.64	7.67
23 Mar PM	0.00	0.41	1.96	1.40	0.99	1.24	1.98	2.52	3.74	7.88
3 Sept	0.00	0.63	0.31	0.56	0.39	0.49	0.79	1.00	1.52	3.40
Chain 4:										
23 Mar AM	0.41	1.88	6.75	2.60	1.84	2.30	3.68	4.68	6.76	
23 Mar PM	0.41	1.88	6.75	2.60	1.84	2.30	3.68	4.68	6.94	
3 Sept	0.63	1.88	1.26	1.12	0.79	0.99	1.59	2.02	3.05	
Chain 5:										
3 Sept	0.00	0.63	0.70	0.84	0.59	0.74	1.18	1.51	2.28	
		[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.60	2.42	

Table V. Amplitudes Obtained from Spectral Variance (Cont)

1	2		3	4	5	6	7	8	9	
	Limits Lower (raps)	Limits Upper (raps)	E (kips squared)	$\sqrt{E}$ (kips)	Most Frequent (kips)	Average (kips)	Average Of Highest 1/3 (kips)	Average Of Highest 1/10 (kips)	Most Probable Value of Maximum By Frequency (kips)	Most Probable Value of Maximum Total (kips)
Chain 6:										
23 Mar AM	0.00	0.41	2.20	1.48	1.05	1.31	2.10	2.67	3.85	5.25
23 Mar PM	0.00	0.41	1.89	1.37	0.97	1.22	1.95	2.47	3.66	5.18
3 Sept	0.00	0.63	2.20	1.48	1.05	1.31	2.10	2.67	4.03	4.41
Chain 6:										
23 Mar AM	0.41	1.88	1.89	1.37	0.97	1.22	1.95	2.47	3.56	
23 Mar PM	0.41	1.88	1.89	1.37	0.97	1.22	1.95	2.47	3.66	
3 Sept	0.63	1.88	0.42	0.65	0.46	0.57	0.92	1.17	1.77	
Chain 7.										
3 Sept	0.00	0.63	8.95	2.99	2.12	2.65	4.24	5.38	8.13	
3 Sept	0.63	1.88	0.94	0.97	0.69	0.86	1.37	1.75	2.64	

Frequency (ω) is expressed in radians per second abbriviated as raps; period (T) is expressed in seconds per cycle abbreviated as spc; $\omega = 2\pi/T$.

The digitized measurements were manipulated in a high-speed computer to provide a seemingly endless stream of answers, of which a few of the more important are presented herein.

EXCITATION

HIGH FREQUENCY WAVES

These were wind generated with peak frequency of 0.66 raps on 23 March and 1.04 raps on 3 September (Figure 4 and Table 3). Both spectra seem conventional with variance (area under the spectrum) of 5.6 $ft.^2$ on 23 March and 1.1 $ft.^2$ on 3 September. These indicate to sea states of 4 and 2 respectively.

Distribution of amplitudes was obtained by sorting and by predicting from the spectral variance as outlined in Method of Analysis. Results (Table 4 and 5) indicate that amplitudes from sorting are 1.1 to 1.5 times those predicted. This of course detracts from the neatness of the value of the variance type prediction. It could be due to: Improper pickup performance; lack of Rayleigh amplitude distribution; and errors in data analysis.

LOW FREQUENCY WAVES

Although a beam-on seiche of about 39 seconds period was present, as deduced from chain response, the water level pickup with sensors at 1/2 foot intervals in the vertical did not sense it, at least consistently. Hence, its amplitude is assumed to have been less than 1/4 foot.

WIND AND CURRENTS

Winds on 23 March were generally from 135 - 146 degrees azimuth at 20 to 24 knots with gusts to 28 knots and from 113 - 135 degrees on 3 September with gusts to 24 knots where bow on winds are from 159 degrees. Currents on both days are considered to have had negligible dynamic effect. Along with winds they tended only to displace the ship from its position of static equilibrium and hence to alter the initial tension in the mooring chains.

RESPONSE

GENERAL

Response (Figures 4, 5 and 6) like excitation, was in two frequency bands; a

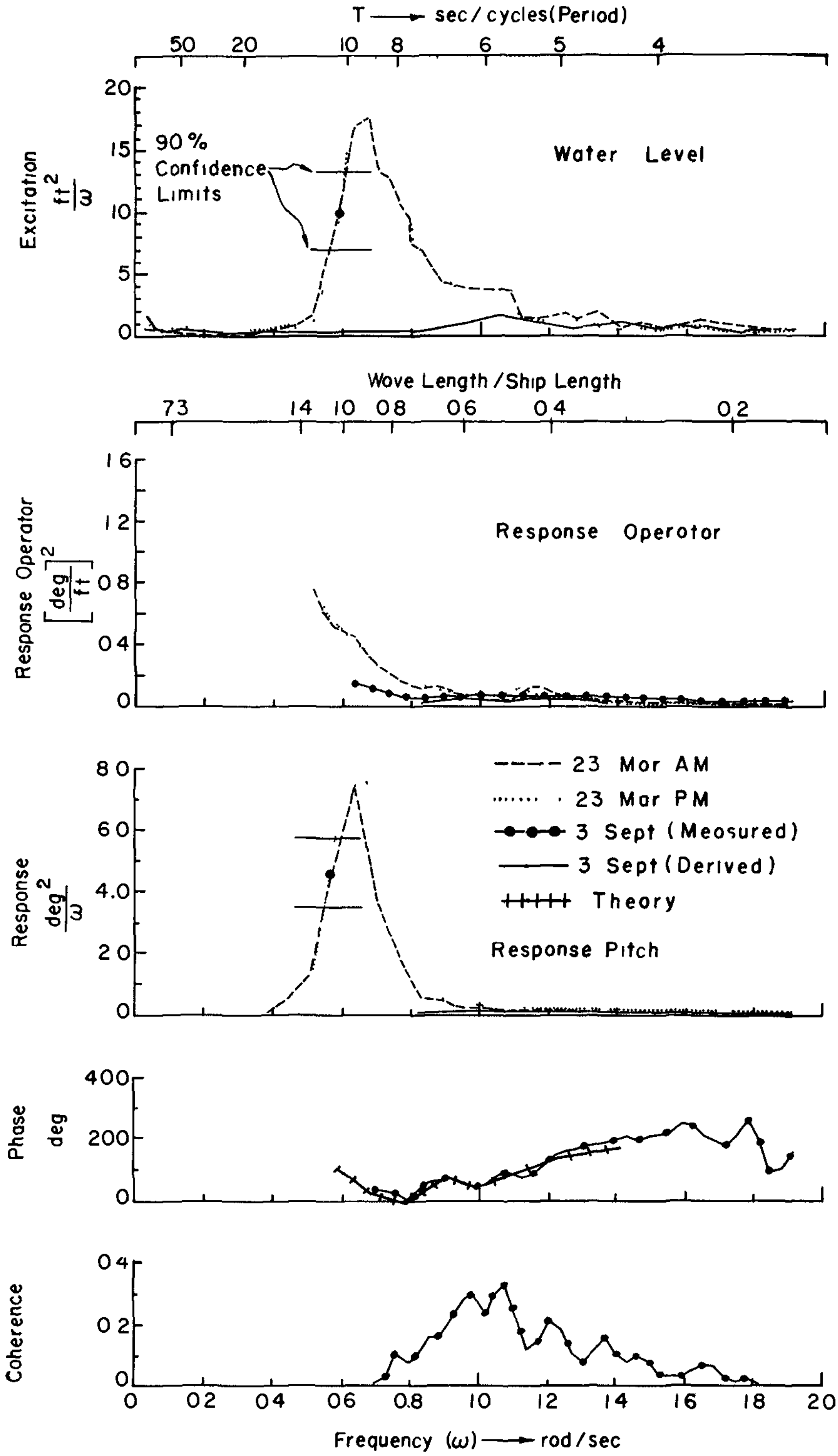

Figure 4 Water level versus pitch. In the range 1.0 to 2.0 radians per sec the response as measured direct and as derived from acceleration measurements are sensibly the same; therefore, the former is not shown. The phase convention is: Pitch bow up lags water level (wave) crest up.

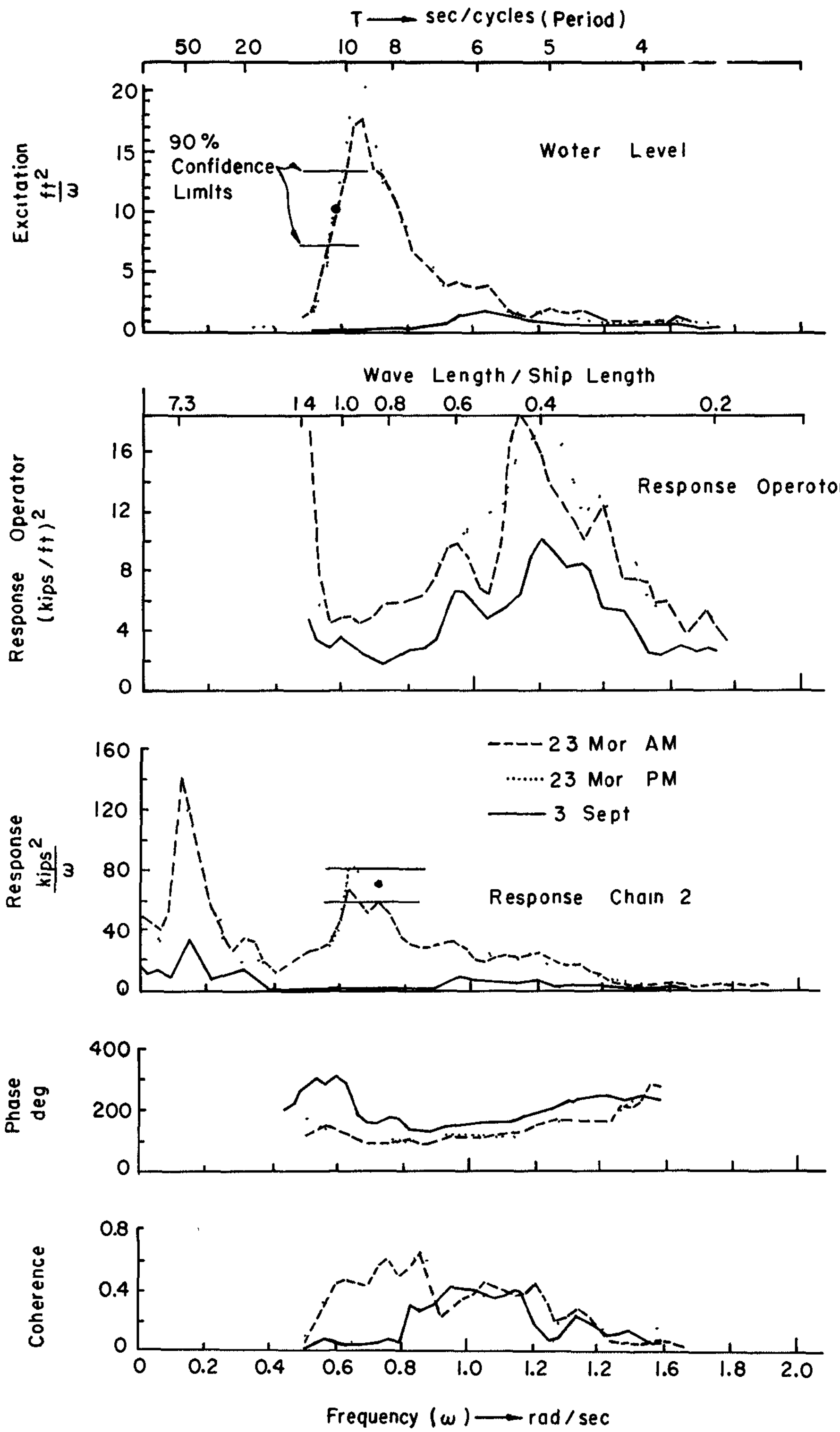

Figure 5 Water level versus tension in the port bow chain (No. 2). The phase convention is: Tension increase lags water level (wave) crest up.

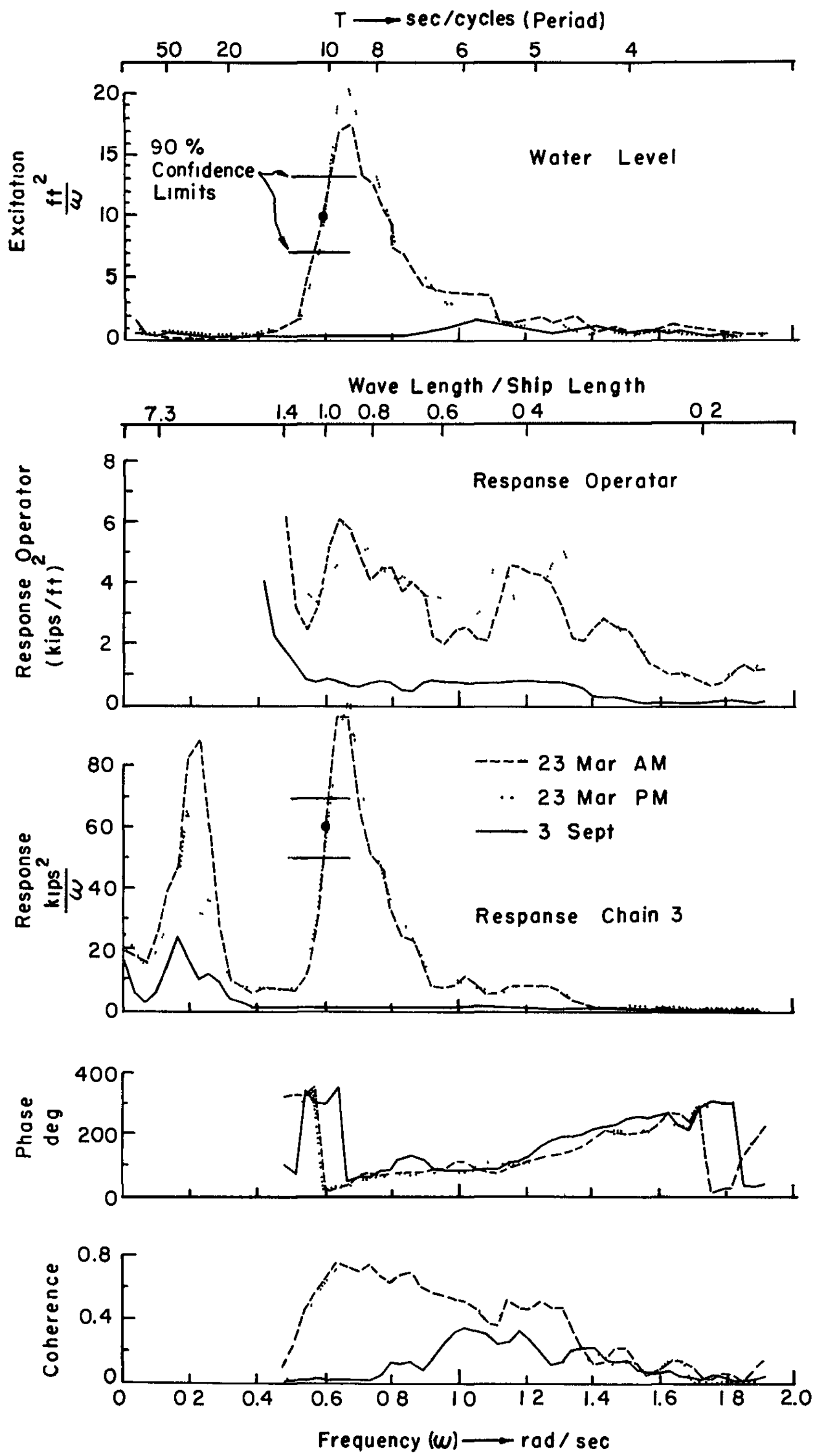

Figure 6 Water level versus tension in starboard bow-chain (No. 3). The phase convention is: Tension increase lags water level (wave) crest up.

high one due to wind wave excitation of from 0.50 to 1.28 raps frequency (12.6 to 4.9 spc period) as sensed by all pickups; a low one, due to unexpected seiche action of from 0.16 to 0.19 raps (39.1 to 32.1 spc) as sensed only by the chain tension pick-ups. The latter was a happy accident.

It likely is feasible to calculate motion of a moored ship from measurements of tension induced in its moorings. However, in the case of these tests, such calculations were considered much too tedious. Thus, low frequency ship motion, although likely considerable, is not accounted for.

There was ship response in all six degrees of freedom; that in the longitudinal plane was more evident.

Phase angle, relative to the excitation (wave) was obtained for all responses over a considerable frequency range; however, only those for pitch and the bow chains are presented (Figures 4, 5, and 6).

LONGITUDINAL MOTIONS

Values for surge, heave and pitch as predicted from the spectral variance are tabulated in Table 5. Note that the most probable maximum amplitude for the three periods was: 1.7 feet surge and heave and 3.4 degree pitch. These are slight motions but sufficient to induce noticeable changes in chain tensions.

LATERAL MOTIONS

In the high frequency range, the lateral motions were much less than the longitudinal. The most probable maximum was 1.1 feet sway and 0.5 degrees yaw (Table 5). The equivalent for roll on 3 September was 2.1 degrees; however, this is not likely a maximum for the three periods since the roll on 23 March was likely higher.

The high frequency lateral motions are the result of the directional nature of the short-crested nearly bow on waves. For example, in commenting on the roll of a ship in head-seas and symmetrically oriented relative to the encounter spectrum of short-crested waves, Pierson (1957) writes:

> " . . . . The oncoming apparent waves will at one time be high on the port side and at another be high on the starboard side causing the vessel to roll first one way and then the other for the same apparent wave form . . . . "

He implies that the coherencies will be near zero in head seas between the following, water level variation and roll; heave and pitch; and pitch and roll. This was found to be the case in these tests; for example, the coherency between water level and roll was found to be 0 to 0.3 on 3 September.

As mentioned, the ship motions excited by the low frequency beam on wave was not sensed. They were likely high especially in surge. Some idea of their magnitude could be obtained from an analysis of the changes in chain tension at this frequency; however, this would be tedious and very time-consuming.

CHAIN TENSIONS

Note in Table 5 that the significant and most probable maximum change in tension, as predicted using the spectral variance, are 8.8 and 21.8 kips respectively. Both were in the port-bow chain (#2). Responses at high and low frequencies were of comparable magnitudes.

However, these do not agree with those obtained by sorting (Table 4) where comparable values of 14.8 and 54.0 kips were found (The maximum single amplitude was 85.1 kips and that in the port-bow chain). Generally, it appears that sorting gives amplitudes 1.5 to 2.5 times those predicted from the spectral variance. The reasons are the same as those given for similar disagreements noted for the high frequency waves.

Sorting within the two frequency bands, rather than overall, as in Table 4, is of course feasible and might provide a better basis for checking variance type prediction. It would not of course change the value of the overall maximum.

The wave induced tensions do not seem alarming when considered relative to the new chain proof load. This is 185 kips for the single light chain (#3) and 300 kips for the rest, compared to the maximum total tension of 116.1 kips, i.e., 85.1 kips maximum wave induced amplitude plus 31.0 kips initial tension, as measured in the heavy port chain (#2) on 23 March A.M. (Table 4).

Of course, it is not obvious that all chains could have withstood even their new proof load, no less their ultimate, due to deterioration by, for example, corrosion and fatigue and without parting due to poor connections of the chain to anchor and ship. Therefore, during storm, new proof load could be small consolation to the captain charged with the safety of the ship. He might understandably tend to drop the moorings and take to the sea unfettered when his seaman's eye seemed to sense critical chain tensions. This tendency existed and one of the objectives of these tests was to quantify the seaman's eye toward keeping the ship on station longer in higher seas and thereby, to reduce the down time on the bow-on oil drilling rig which the ship served.

RESPONSE AMPLITUDE OPERATORS FROM MEASUREMENTS

GENERAL

This operator was developed from the measured data by operations at successive frequencies (ω) over the full spectral range as follows: The amplitude of the response spectrum (ft^2 or deg^2 or kips$^2/\omega$) was divided by amplitude of the excitation spectrum (ft$^2/\omega$) to obtain the corresponding amplitude of the response amplitude operator (ft^2 or deg^2 or kips^2/ft^2). Linearity between excitation and response and long-crested waves were assumed in all cases. That these assumptions were not realized fully is discussed in the concluding part of this chapter.

When linearity exists, the operator is a powerful design tool for it is apparent that it can be used generally to predict response from excitation and of course, excitation from response, e.g., wave spectrum from the response of a buoy.

Because of their importance, the analysis of the test results was directed mainly to obtaining these operators so as hopefully to predict ship response at sea states higher than those encountered in the tests and also to compare the operators with those predicted by theory since of course this is the ultimate design tool. Operations derived from model tests were not available.

LONGITUDINAL MOTION OPERATORS

These are shown in Figures 7 and 8 where it is apparent that the amplitude of surge and heave decreases with increased frequency as would be expected. The pitch operators tend to peak at about 0.7 raps frequency or about where wave length equals ship length. Thereafter, they decrease with increased frequency. Operators derived from the three separate sets of measurements agree fairly well, although less so at the lower frequencies, especially for heave (Figure 7). A curve eye-balled through them is likely adequate for engineering design.

LATERAL MOTION OPERATORS

These were determined but are not presented here. As mentioned under "Response," they are the result of the directional nature of the short-crested nearly bow-on waves. Roll and yaw amplitudes did not exceed 1.0 and 0.1 degrees per foot of wave amplitude respectively over the most of the significant frequency range of 0.5 and 1.6 raps. Sway showed a notable peak of about 0.4 feet/foot at about 0.9 raps (7.0 spc) on the 23 March operator. That from the 3 September measurements was generally 1-1/2 to 3 times higher.

CHAIN TENSION OPERATORS

All operators have a peak at about 1.25 and 0.16 raps frequency. The equivalent periods are 5.0 and 39.3 seconds per cycle and wave length/ship length ratios

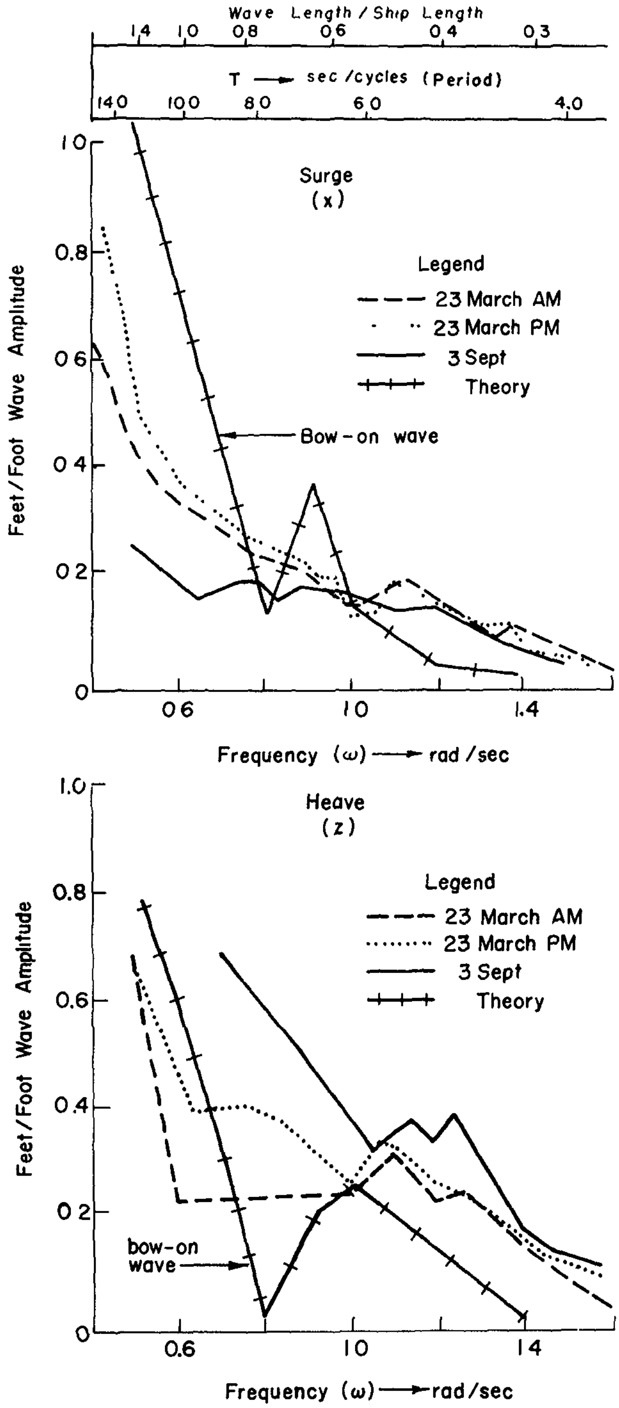

Figure 7. Response amplitude operator for surge and heave.

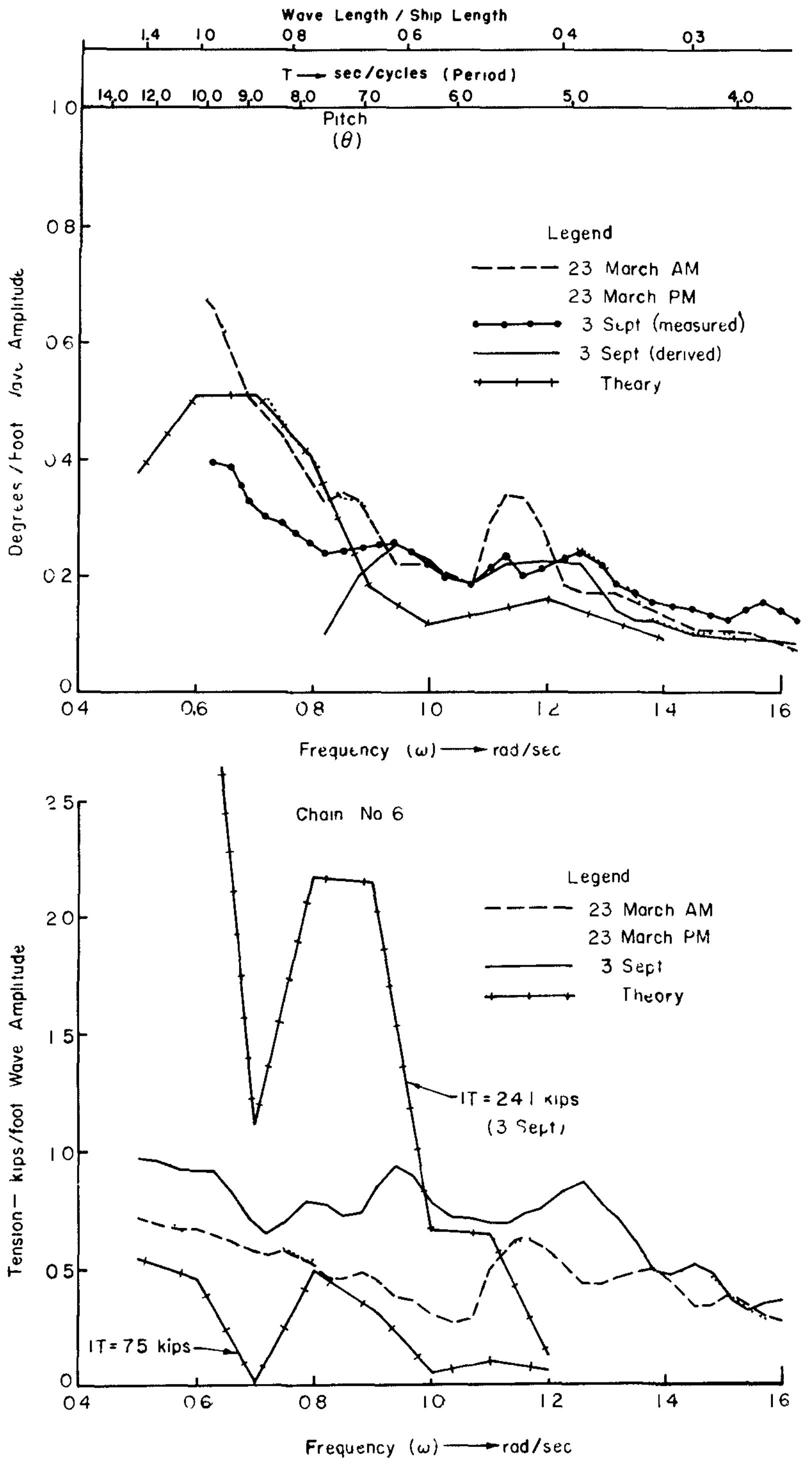

Figure 8. Response amplitude operator for pitch and stern chain (#6).

0.4 and 4.6 respectively.

The high frequency peak is that predicted for the surge of the unmoored ship by Wilson (1959 Equation 21). The low frequency peak is close to the calculated resonant frequency in surge of 0.25 raps.

The two bow chains dominated with that on the port the greater; the other five chains served mainly to keep the ship from fishtailing significantly. For example, at 1.25 raps, the change of tension in kips per foot of wave amplitude is: 4.5 and 2.2 for the port and starboard bow chains respectively (#2 and #3); 1.4 and 1.5 for port and starboard breast chains (#1 and #4); 1.0 and 0.3 for the port and starboard quarter chains (#7 and #5) and 0.7 for the stem chain (#6).

The non-linear nature of the spring restoring force, i.e., the tension-displacement relation for the mooring chain-in-catenary configuration, is apparent in the operators for those chains for which measurements were made on the two days, i.e., the stem cabin (#6); the starboard breast chain (#4) and the two port chains (#2 and #3) in Figures 8, 9, and 10, respectively. That is, generally for a given chain and wave the change in tension varies directly with the initial tension. The exception is the important port bow chain (#2) where, to spoil an apparently sound generality, the reverse is true, e.g., the greater changes in tension occur on 23 March when the chain had a lower initial tension, 31.1 kips, than on 3 September with 34.6 kips, although the difference is obviously not great (Figure 10).

It is satisfying to note that the pairs of operators derived from measurements made on both AM and PM on 23 March for chains #2, 3, 4 and 6 are significantly the same since the respective initial tensions are sensibly equal (Table 2 and Figures 8, 9, 10).

RESPONSE OPERATORS FROM THEORY

GENERAL

The model used in these theoretical predictions consisted of the ship with zero forward speed, spread-moored by four mooring lines. Two lines were attached forward and two aft, representing the seven actual chains. The model was situated in a windless, currentless sea, 45 feet deep, and exposed to regular long crested sinusoidal waves of unit amplitude with fixed frequency and heading of either bow-on or beam-on. Waves with circular frequencies of from 0.5 to 1.4 radians per second (raps) were considered. These are equivalent to waves with periods of 12.5 to 4.5 seconds per cycle (spc) or 530 feet to 100 feet in length, in water 45 feet deep on the ship 319 feet long at the water line.

The theory used is a modification of the deep water theory used by Kaplan and Putz (1962). Specifically, the proper shallow water wavelength is imposed on the deep water theory, and the change in the orbital velocity pattern from circular, as in deep water, to elliptical, as in shallow water, is neglected. The rigorous shallow water wave theory was not used since the amount of computation required was considered excessive.

The craft had the usual 6 degrees of freedom; 3 translational (surge, sway and heave) and 3 rotational (roll, pitch and yaw).

BASIC ASSUMPTION

The basic assumption is that of linearity. Specifically, it is assumed that in the absence of excitation, the ship motion can be described in terms of homogeneous, second-order, linear, differential equations with time as the independent variable. An excitation term is added to the homogeneous equations as a "right-hand-side term" which, in the present case, is sinusoidal. In accordance with linear theory, it is assumed that there is no coupling between the variables in the two planes of motion; that is, those longitudinally in heave, pitch and surge, and those laterally in sway, yaw, and roll. However, the longitudinal motions are coupled with each other and, similarly, the lateral motions are coupled with each other.

A fundamental analytical tool in carrying out the prediction is the slender-body theory. Essentially, this theory makes the assumption that, for an elongated body where a transverse dimension is small compared to its length, the fluid flow at any cross-section is independent of the flow at any other section; therefore, the flow problem is reduced to a two-dimensional problem in the transverse plane. The force is found by integrating the pressure over the length of the body.

Equations are formulated by the balance of inertial, damping, restoring, exciting, and coupling forces and moments. Hydrodynamic and hydrostatic fluid effects, together with body inertia and mooring influences, are included in the analysis.

INERTIAL FORCES

The forces exerted by the ship in accelerating the surrounding water give rise to equal and opposite forces by the water on the ship. These are termed inertial forces and, correspondingly, inertial moments. Since they are proportional to acceleration, they are usually expressed in terms of a fictitious added mass. The total inertia force has components in all three directions of translation and rotation.

DAMPING FORCES

Damping forces involve the dissipation of energy and are due to wave generation, viscosity and eddy-making. Except in the case of roll, damping due to wave generation only is considered in this study. Total damping force, like inertia force, has components in all three directions of translation and rotation.

HYDROSTATIC RESTORING FORCES

These forces are due to the buoyancy effect arising from static displacements. Total hydrostatic restoring force has a component only in the vertical, or heave direction. The hydrostatic restoring moment has components only in the roll and pitch directions.

MOORING RESTORING FORCES

This additional restoring force has been added, in the case of the landing craft under analysis, in addition to the hydrostatic restoring force which is always present in the case of a craft in water. The mooring restoring force is effective only in surge, sway and yaw. While it is present in the three remaining modes (heave, pitch, and roll), it is never significant in relation to the hydrostatic restoring forces. Mooring restoring force is assumed to be a linear function of displacement in surge and sway as well as yaw. The assumption of linearity is a proper one for the small displacements encountered during the sea tests; however, for large displacements, linearity is not assumed.

EXCITATION FORCES

Excitation forces could also be called wave forces, since they represent the waves which excite the ship motion and which give rise to all the preceding forces. They are sinusoidal in nature, and have components in all three directions of translation and rotation, although some of these components vanish at certain headings of the craft.

EQUATIONS OF MOTION

The following five forces, as explained above, are considered in the analysis, namely; inertial, damping, excitation and restoring force. In the latter, the hydrostatic and mooring forces are combined. On the basis of these five items, six equations of motion, one for each degree of freedom, are written as follows:

$$m\ddot{x} = F_x^i + F_x^d + F_x^m + F_x^w \quad \text{(surge)} \qquad (1)$$

$$m\ddot{y} = F_y^i + F_y^d + F_y^m + F_y^w \quad \text{(sway)} \qquad (2)$$

$$m\ddot{z} = F_z^i + F_z^d + F_z^h + F_z^w \quad \text{(heave)} \qquad (3)$$

$$I_x \ddot{\varphi} = M_\varphi^i + M_\varphi^d + M_\varphi^h + M_\varphi^w \quad \text{(roll)} \qquad (4)$$

$$I_y \ddot{\theta} = M_\theta^i + M_\theta^d + M_\theta^h + M_\theta^w \quad \text{(pitch)} \qquad (5)$$

$$I_z \ddot{\psi} = M_\psi^i + M_\psi^d + M_\psi^m + M_\psi^w \quad \text{(yaw)} \qquad (6)$$

where m = mass of the ship
F = force
M = moment

with superscripts on the F's and M's indicating components, and subscripts indicating the type of force or moment, according to the following notation:

i = inertial
d = damping
h = hydrostatic restoring
m = mooring restoring
w = wave
x = surge
y = sway
z = heave
φ = roll angle
θ = pitch angle
ψ = yaw angle

Acceleration is indicated by the superscript (¨) above the motion symbol; for example $\ddot{x}$, etc.

Equations of motions consist of linear combinations of terms which vary in time and are proportional to acceleration, velocity and displacements; the latter included those which vary sinusoidally and are connected with the wave term which excites the system. Each of these contains certain coefficients which must be evaluated.

SHIP MOTION OPERATORS

By considerable effort, mainly due to the difficulty of determining the coefficients simultaneous equations for the longitudinal motion were evolved in the form of a 3 by 5 matrix. By inverting this matrix, solutions for motions in three degrees of freedom are obtained. The same procedure is followed in the case of the lateral motions. By varying the value of the exciting frequency, responses are calculated in the form of amplitude of motion for a unit amplitude excitation. When these responses are expressed as a function of frequency they are known as "response operators." They were computed for both the lateral and longitudinal motions with the latter shown in Figures 7 and 8. The phase relationship of the motion to the wave was calculated for all motions but is presented only in the case of pitch (Figure 4).

There is surge and pitch only in head seas (Figure 7 - 8); also, at frequencies greater than about 1.2 raps (5.24 seconds period), the motion in both modes is relatively small. Both beam-on and head-on waves produce noticeable heave. In head seas, heave becomes relatively small (Figure 7) at frequencies greater than 1.2 raps (5.24 seconds period). In beam seas, t heave slightly exceeds the amplitude of the incident wave, up to about 0.8 raps; after this, it decreases relatively slowly to about 0.3 of the amplitude of the incident wave at 1.4 raps. The magnification of the heave is puzzling, but seems to be explainable by the equations used, if not physically.

Consistent with linear theory, there is sway, yaw and roll, only in beam-on waves. The sway up to about 0.9 raps is slightly greater than the amplitude of the incident wave. After 0.9 raps, the sway decreases slowly to about 0.4, the amplitude of the incident wave at 1.4 raps. The roll in beam-on waves peaks at about 0.9 raps raps (5.2 degrees per foot) and then decreases to about 1.90 degrees at 1.2 raps. The yaw in beam seas attains 0.15 radians at about 1.2 raps.

RESPONSE AMPLITUDE OPERATORS FROM THEORY

CHAIN TENSION OPERATORS

Motion of ship end of chain in longitudinal plane. This point is designated by subscript "c" and located at a distance "ℓ" forward or aft of the C.G. as measured along the center line. The previously calculated phase angle (ϵ) and maximum amplitude of surge (X) and heave (Z) of the C.G. and pitch (θ) about the C.G. were used in the equation for simple harmonic motion, e.g., surge of point "c" is

$$X_c = X \cos(\omega t + \epsilon_1) \quad \ldots (7)$$

The heave of the point is a combination of the effect of heave and pitch at the C.G., i.e.,

$$Z_c = Z \cos(\omega t \;\; \epsilon_2) + \ell \theta \cos(\omega t + \epsilon_3) \quad \ldots (8)$$

By suitable trigonometric manipulation, Equation (8) can be written as:

$$Z_c = (A + B)^{\frac{1}{2}} \cos(\omega t + \tan^{-1}(B/A) \quad \ldots (9)$$

where $A = Z \cos \epsilon_2 + \ell \theta \cos \epsilon_3$

$B = Z \sin \epsilon_2 + \ell \theta \sin \epsilon_3$

At a given frequency, previously calculated maximum values of surge, heave and pitch due to waves of unit amplitude with their respective phase angles are substituted into Equations (7) and (9) at various times (t) from 0 to T (wave period) sufficient to plot the closed path of the point in the X - Z plane. It will be elliptical, i.e., a Lissajou figure with the point of initial tension at the center of the figure.

Change of tension due to the movement of the ship end of the chain, as it oscillated in the Lissajou figure in the longitudinal plane, was calculated using the Table 2 chain geometry and the equations of the catenary given by O'Brien and Kuchenreuther (1958). The maximum change in tension was determined thereby and plotted as a function of frequency to form the response amplitude operators given in Figures 8 through 11.

As with the motion operators, the change in tension is low at frequencies greater than about 1.2 raps.

Note, particularly in Figures 8 and 9, that the amplitude of the operator varies directly with the magnitude of the initial chain tension, as discussed under the Operators derived from measurements. This is due to the non-linear relationship between tension and geometry for a chain in a catenary configuration.

Thus, a chain tension operator derived for one initial tension cannot be expected to agree with one derived for another initial tension. This, of course, limits seriously the usefulness of chain tension operators in design.

However, for small ship motions and chains with normal sag, of the order of those

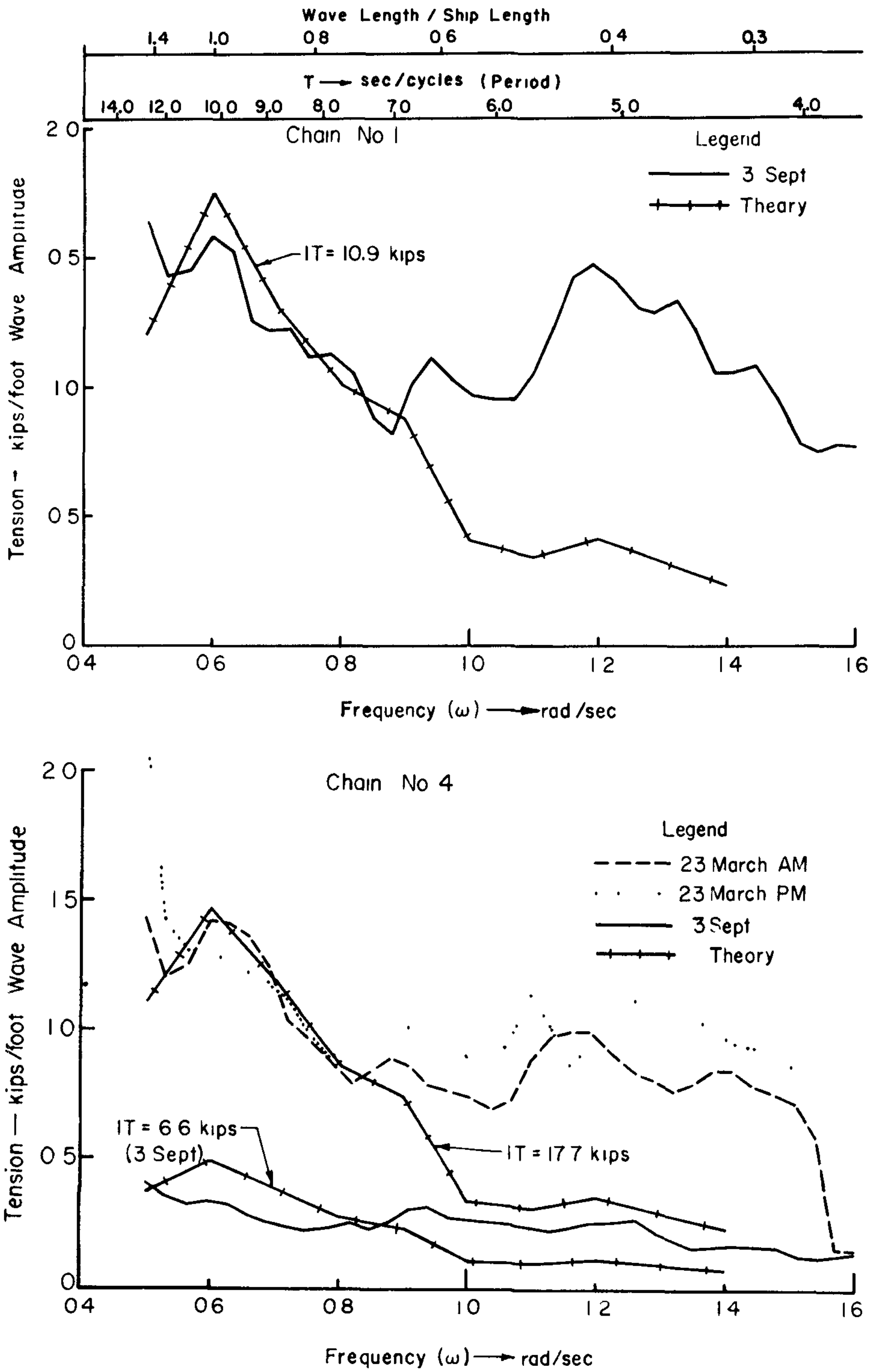

Figure 9. Response amplitude operator for port and starboard breast chains (#1 and #4).

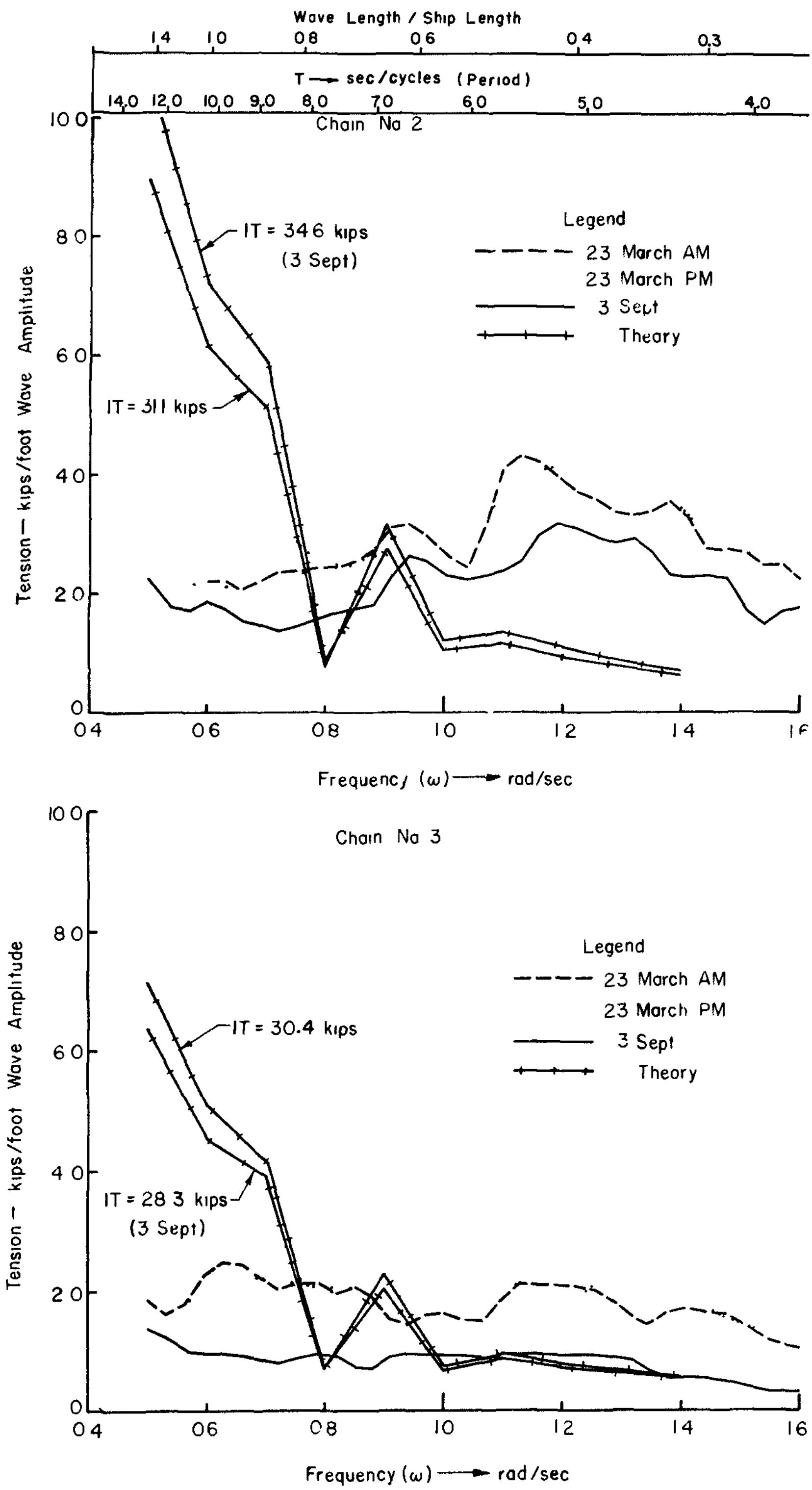

Figure 10. Response amplitude operator for port and starboard bow chains (#2 and #3).

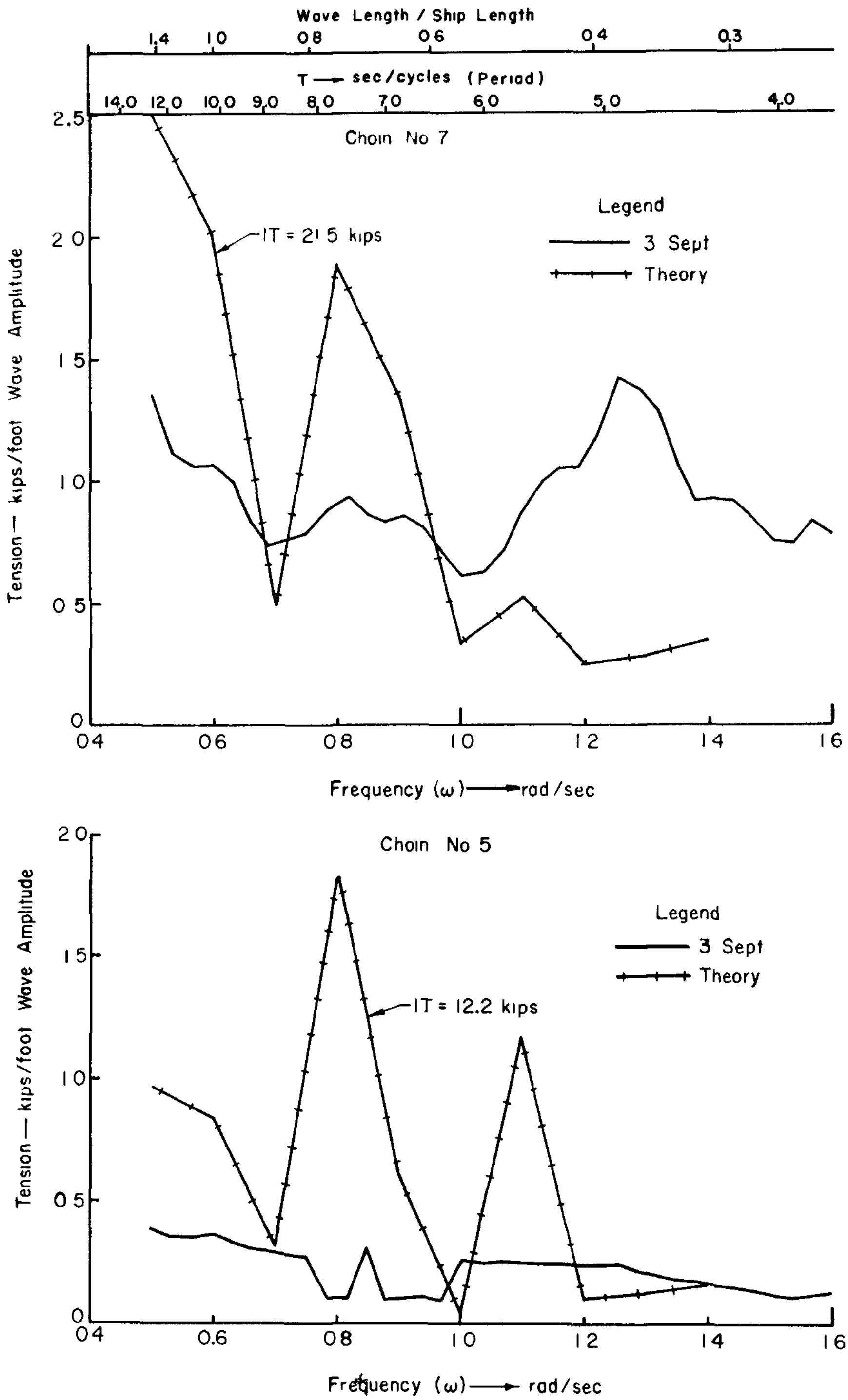

Figure 11. Response amplitude operator for port and starboard quarter chains (#7 and #5).

encountered in these tests, the tension-displacement relationship (restoring force) for a given initial tension may be considered to be sensibly linear for design purposes. Hence, ship motion calculations made on this basis, as in the preceding paragraphs, may be considered meaningful, although strictly the pertinent motion operators so derived, i.e., surge, sway and yaw, are subject to the same initial tension limitation as the chain tension operators.

COMPARISON OF EXPERIMENTALLY AND THEORETICALLY DERIVED OPERATORS

LONGITUDINAL SHIP MOTIONS OPERATORS

In the frequency range of 0.5 to 1.4 raps, the agreement is worst at the lower frequencies where theory predicts 3/4 to 4 times measurement. It is best near 1.0 raps where this factor is 3/4 to 1 and somewhat poorer at 1.4 raps in that theory predicts 4/3 to 3/4 or measurement (Figures 7 and 8).

The agreement in the case of pitch is relatively good with theory predicting about 1/2 to 4/3 of measurement. It is not nearly as good in surge and heave where theory predicts 1/3 to 4 times measurement and in one exceptional case 12 times measurement (heave of 0.8 raps, Figure 7).

LATERAL SHIP MOTION OPERATORS

There is no basis for comparison here since the measurements reflect response due to short crested bow-on waves while the theory used for bow-on waves assumes zero response in the lateral plane.

CHAIN TENSION OPERATORS

Generally the agreement is best in the middle frequencies, i.e., 0.8 to 1.0 raps with the glaring exception of the quarter chains (Figures 9, 10, and 11).

Overall, there is better agreement in the breast chains (#1 and #4) with prediction about 1/2 measurement and to a lesser extent in the bow chains (#2 and #3) with prediction about 1/4 to 3 times measurement than for the quarter chains (#7 and #5) with prediction 1/4 to a fantastic 9 times measurement (at 0.8 raps in the starboard quarter).

The stem chain (#6) deserves special comment since its initial tension was markedly different on the two days, i.e., 7.5 kips on 23 March and 24.1 kips on 3 September. Here the agreement was very poor on both days at the lower frequencies, e.g., at 0.5 raps theory predicted 3/2 to 4 times measurement. However, in the middle and high frequencies the agreement was better with these factors varying from 1/4 to 1.

PHASE AND COHERENCY

LONGITUDINAL SHIP MOTION

Pitch bow-up lagged water level crest from about 0 degrees at 0.7 raps to about 150 degrees at 1.2 raps based on the only available direct measurements in this plane, e.g., those on 3 September. Theoretical predictions agree rather well (Figure 4).

Phase, for surge and heave, were not determined from the pertinent acceleration measurement; the task would have been tedious, although not difficult. Theory indicates that surge-aft lagged water level crest by about 180 degrees at less than 0.7 raps and by 150 degrees at greater than 0.9 raps; in between the phase lag dropped to 0 degrees at 0.8 raps. Heave-up lagged water level crest by about 80 degrees at 0.9 raps then gradually increased to 150 degrees at 1.2 raps.

Heave and pitch had similar phase patterns over 0.5 to 1.2 raps with the pitch lags lower by about 40 degrees. At about 0.6 and 1.2 raps, they were in phase at about 80 and 150 degrees lag, respectively.

Coherency between excitation (wave) and response, is an experimentally derived quantity, and hence available only for pitch. It is low generally being 0 at 0.7 and 1.8 raps with peak of 0.3 at 1.1 raps (Figure 4).

LATERAL SHIP MOTIONS

The only pertinent phase and coherency information is for roll, since that was the only lateral motion measured directly. Roll port-up lagged water level crest in degrees as follows: 200 at 0.6 raps; 100 at 0.7 raps; 300 at 1.4 raps. As mentioned under Response, the roll motion in this frequency range is due to the short-crested nature of the bow-on waves; hence, the phase calculated for beam-on waves is not pertinent.

Coherency between water level and roll was low, being 0 at about 0.6 and 1.8 raps and with peak of 0.3 at about 1.1 raps.

CHAIN TENSION

In the range 0.6 to 1.4 raps an increase in chain tension lagged water level crest by the following degrees: 100 to 180 for bow chains (#2 and #3); 50 to 250 for the breast chains (#1 and #4); 180 to 450 or 90 lead for the quarter chains (#7 and #5) and 100 to 360 for stem chain (#6). All in all, there was an exasperating lack of harmony in the phase behavior of the chains, particularly since they were fastened to a sensibly rigid body, although the patterns for surge and the starboard bow (#3) and stem (#6) chains appear to be sensibly similar.

Coherency of chain tension with water level was rather low at about 0 to 0.75 for all chains. The peak on 23 March was at about 0.70 raps and on 3 September at 1.0 raps.

SUMMARY

GENERAL

Sorting generally gave amplitudes of excitation and response which were 1.1 to 2.5 times those predicted from the spectral variance.

EXCITATION

1. Sea state was 4 on 23 March and 2 on 3 September.

2. Two wave systems were operative. One was wind-generated from about bow-on with significant amplitude of about 4.4 feet and peak response at 9.5 seconds period on 23 March and 1.7 feet and peak response at 6.0 seconds period on 3 September. The other was a seiche, apparently, beam-on, with significant amplitude of about 1/4 foot and period of about 39.3 seconds.

3. Maximum single wave amplitude measured was 9.5 feet on 23 March and 5.0 feet on 3 September.

4. Winds were from about bow-on at 14 to 24 knots on both days.

5. Currents were negligible.

RESPONSE

1. Response was greater on 23 March.

2. Significant amplitude (average of highest one-third) of ship motion did not exceed: 1.1 feet surge; 0.9 feet heave; 1.8 degrees pitch; 0.6 feet sway; 0.3 degrees yaw; and 1.1 degrees roll (3 September only).

3. Significant amplitude of wave-induced chain tension in kips did not exceed 15.0 for bow chains (#2 and #3); 5.5 for breast chains (#1 and #4); 5.0 for quarter chains (#7 and #5) with measurements for 3 September only; and 3.2 for stern chain (#6).

4. The maximum wave-induced tension in kips measured in each of the seven chains was: 85.1 and 48.0 in port and starboard bow chains respectively; 10.6

(3 September) and 19.7 in port and starboard breast chains; 13.9 and 4.3 in port and starboard quarter chains both on 3 September; and 9.7 in the stern chain. To obtain total tension add the initial tensions in Table 2 to the above, e.g., the maximum total tension in the port bow chain was 85.1 plus 31.0 or 116.1 kips. New proof load in the port breast chain is 185 kips; it is 300 kips for all the others.

RESPONSE AMPLITUDE OPERATORS FOR SHIP MOTION

1. Those for surge, and heave decreased with increased frequency; pitch tends to peak at about 0.7 raps(about 9.0 spc period) or where wave length equals ship length.

2. In the range 0.6 to 1.6 raps motions per foot of wave amplitude as derived from acceleration measurements did not exceed 0.4 feet surge; 0.5 feet heave; 0.7 degrees pitch; 1.0 degrees sway; 0.1 degrees yaw; 1.0 degrees roll (direct measurement). Sway shows peak of 0.4 feet/foot, 0.9 raps, 23 March.

3. In the longitudinal plane, linear bow-on wave theory predicts motions 1/3 to 4 of those obtained from measurements. In one exceptional case this factor is 12 (heave at 0.8 raps).

4. Ship motion in lateral plane, i.e., sway, yaw, roll, in the high frequency range, is the result of the directional nature of the short crested nearly bow-on waves. The ship motion theory used with those waves assumes no response in the lateral plane. Thus, there is no basis for comparing measurements with theoretical prediction.

RESPONSE AMPLITUDE OPERATORS FOR CHAIN TENSION

1. All operators show a peak at 1.25 radians per second (raps) frequency or 5.0 seconds per cycle (spc) period and 0.16 raps (39.3 spc).

2. The two bow chains dominate, e.g., change of tension in kips/foot wave amplitude at 1.25 raps is: 4.5 and 2.2 for port and starboard bow chains, respectively; about 1.5 for both breast chains; 1.0 and 0.3 for port and starboard quarter chains, respectively; and 0.7 for stern chain.

3. Theoretical prediction (made only in high frequency range) generally is not good, i.e., prediction 1/4 to 3 times measurement in the bow chains; 1/2 in breast; 1/4 to 9 in the quarter, and 1/4 to 3/2 in the stern chain.

4. The amplitude of the operators varies directly with the magnitude of the initial chain tension due to the non-linear relationship between tension and geometry for the chain-in-catenary configuration used in the tests. Thus, operators derived for one initial tension do not agree with those derived for another.

PHASE AND COHERENCY

Longitudinal ship motions

1. In the range 0.7 to 1.2 raps, the following lagged water level crest in degrees by: pitch bow-up, 150; surge-aft, 150 - 180; and heave, 80 - 150. All three were in phase at about 1.2 raps at about 150 degrees lag.

2. Coherency between water level and motion was low, being 0 at both 0.7 and 1.8 raps with peak of 0.3 at 1.1 raps.

Lateral ship motion

1. Roll information only available. Roll-port-up lagged water level crest by 100 - 300 degrees in the range 0.6 to 1.4 raps.

2. Coherency was low being 0 at both 0.6 and 1.8 raps with peak of 0.3 at 1.1 raps.

Chain tension

1. In range 0.6 to 1.4 raps increased tension lagged water level crest in degrees by: 100 - 150 bow chains; 50 - 250 breast chains; 180 - 450 (90 lead) quarter chains; 100 - 360 stern chain. Disharmony is noted.

2. Coherency was low at 0 to peak of 0.75 on 23 March.

CONCLUSIONS

1. The moor was unbalanced in that the bow chains, particularly the port, tended to take the bulk of the load.

2. Even in sea-state 4, the maximum total tension in the lines, 116 kips in the port bow chain, is considered low relative to the new proof load of 300 kips for the chain concerned. It is realized that this margin needs to be large toward combating corrosion and fatigue excitations in addition to those of wind, wave, and current.

3. Water level variations should be measured at more than one point in ocean tests such as these, so as to permit definition of the directional wave spectrum. Measurements at some three to six points about the platform would be required; a single instrumented buoy of the type described by Cartwright (1964) could be used instead.

4. Discouragement over the extreme differences between theoretical prediction and measurements found in these tests is balanced by encouragement over agreements.

5. Toward improvements, the fairly well-established theory for motion of unmoored ships as used herein, needs to be extended to short crested waves and to moored platforms of relatively broad beam, including those of odd shape, such as barges. It needs to include dealing with short crested waves and the difficult problem of providing properly for damping and initial tension. The latter is unique to the problem. It is dependent on the whims of the wind and current which tend to alter the attitude and position of the platform and hence the tension in the lines fastened to it. The prediction of initial tension during storm is most demanding. However, unless this can be done, it will not be feasible to predict the motion and mooring tensions properly, either by theory or model tests.

6. Propulsion devices controlled by read-out of tension in key moorings may be necessary toward realization of the design initial tensions in the prototype. Also non-catenary line configurations may be necessary, e.g., lines so buoyed as to hang in straight lines, so as to provide a linear type spring of constant initial tension.

7. If the directional properties of the waves had been measured (Directional spectra) and if a theory involving short crested waves had been available and used, then the discrepancies between observations and theory likely would have been less.

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