Chapter 48

RESONANCE CONDITIONS IN Nº.1 DOCK OF LUANDA HARBOUR

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1. OBJECT OF THE PAPER

About 900 m of berth length arranged around a single pier are at present available in the port of Luanda.

Due to the constant increase of traffic through the port, the "Brigada de Estudos do Porto de Luanda" (Brigade of Studies of the Luanda Harbour) decided to prepare a general plan of extension of the port facilities. The extension planned comprises the construction of a new pier, which together with the existing one will delimit a triangular dock - dock nº.1 —, and of a series of rectangular basins between the new dock and the area of S.Pedro da Barra fortress (fig.1).

The danger of possible resonance phenomena and the fact that this problem can be dealt with before construction begins, led the Brigade to undertake a detailed analysis of it before taking decisions on the extension works of the port. This analysis comprised three stages:

a) Observation in nature of long-period waves in Luanda bay.

b) Analytic study of the behaviour of the planned docks under the action of possible long-period waves.

c) Experimental study in model of the same phenomenon.

The author, as consulting engineer of the Brigade of Studies, programmed observations a) and analysed their results, performed the analytic study b) [1] , [2] , [3] and, as head of the Hydraulic Department of Laboratório Nacional de Engenharia Civil, planned, supervised and interpreted the model studies (1). These concerned dock nº 1 alone because, on one hand, this is the structure whose construction will begin first and, on the

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(1) - These model tests were directed by Mr. J. Pires Castanho

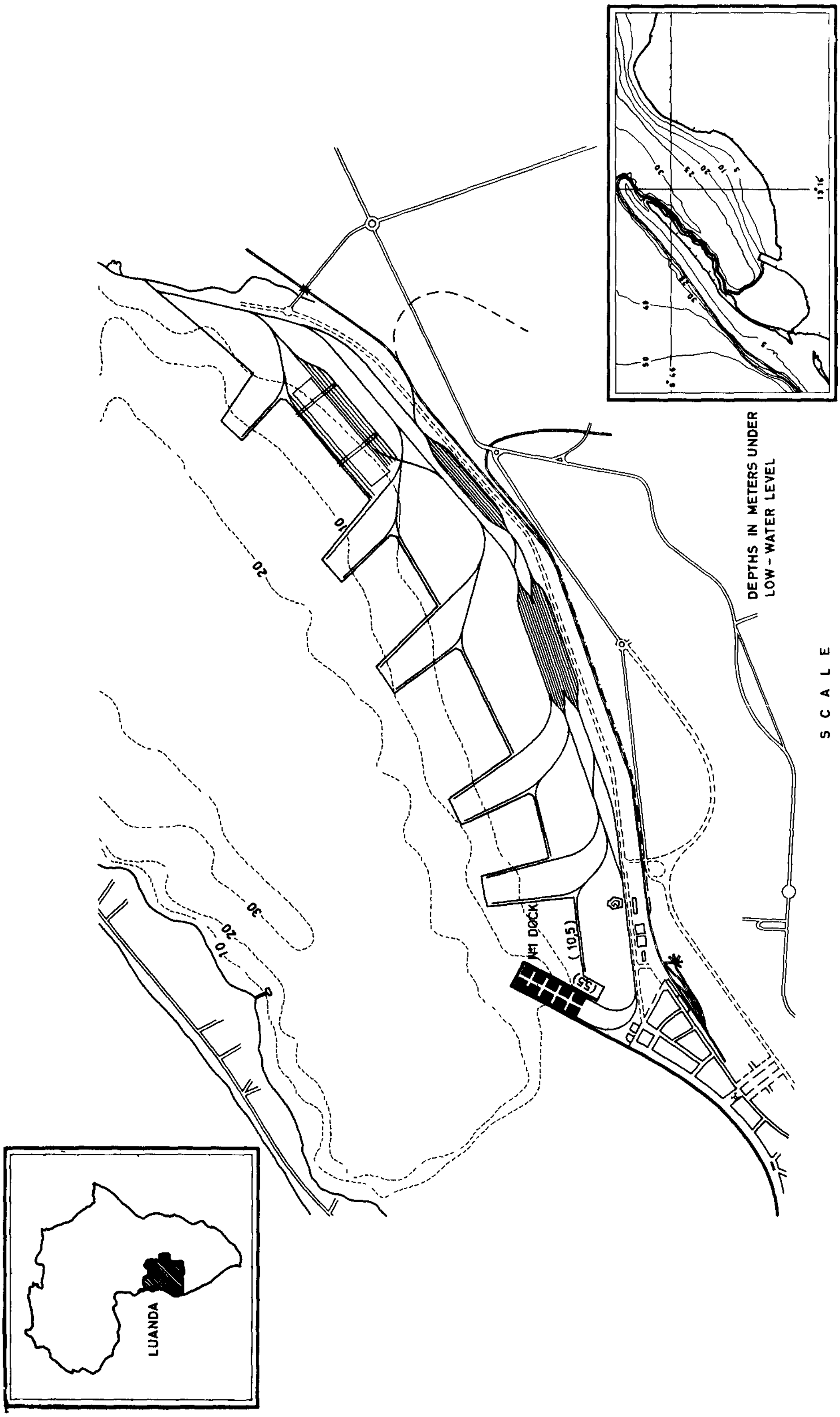

FIG 1 --LOCATION MAP AND PLAN OF ENLARGEMENT OF LUANDA HARBOR other, due to its shape, the conclusions of the analytic study about this dock are to be viewed with great reservations.

In the present paper the author presents the results of the studies in reference on dock nº 1, seeking in special to compare the results of the analytic study and of the model tests, stressing agreements and differences, trying to explain the latter and drawing conclusions applicable in practical cases.

2 - OBSERVATIONS IN NATURE

2.1 - Observations carried out in 1958 - Quick rotation tide recorders were installed after 1959, so that their records were not available for the first analytic studies. This lack was made good by means of a standard rotation tide recorder installed in the pier of Departamento Marítimo (fig.2). The supplied data concern the period between 4th December 1957 and 1st January 1959.

These tide records, to a scale of time of 1 mm≃2 min and of heights of 1:20, were analysed by direct observation methods as follows: whenever a certain regularity was observed in the distances between peaks of recorded curves, the time during which this regularity lasted was determined and by dividing this time by the number of intervals between peaks, the wave period was obtained. The amplitude was not taken into account as it remained fairly uniform between 6 and 9 cm. The last figure was rarely exceeded, the maximum recorded value being about 20 cm.

The results of this analysis are indicated in Table 1 which lists the times during which the waves, classified according to their periods in classes of 10 s, were observed; the cumulated times during which waves with periods inside a certain interval were observed are then determined. Then the percentages of durations in each class and of cumulated durations are calculated, both for the total time of observation (565 920 minutes) and the time during which regular long-period waves lasted (28 844 minutes).These data enable two histograms and two cumulated frequency curves to be plotted for the two total durations indicated above (fig. 3 and 4).

According to the histograms, the long-period waves most frequent in Luanda harbour range from 130 to 140 seconds (30 per cent of the recorded values), followed by those between 140 and 150 seconds and between 110 and 120 seconds. The first cu

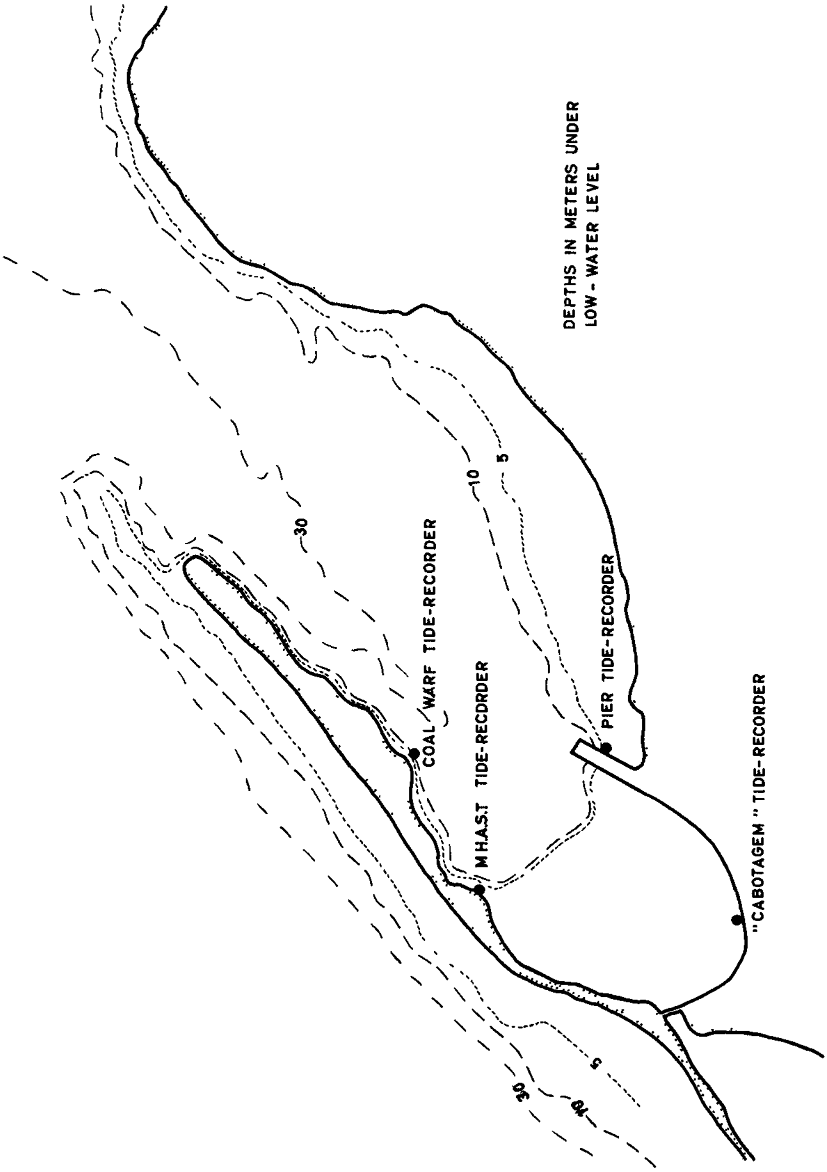

FIG. 2.--LOCATION OF TIDE-RECORDERS

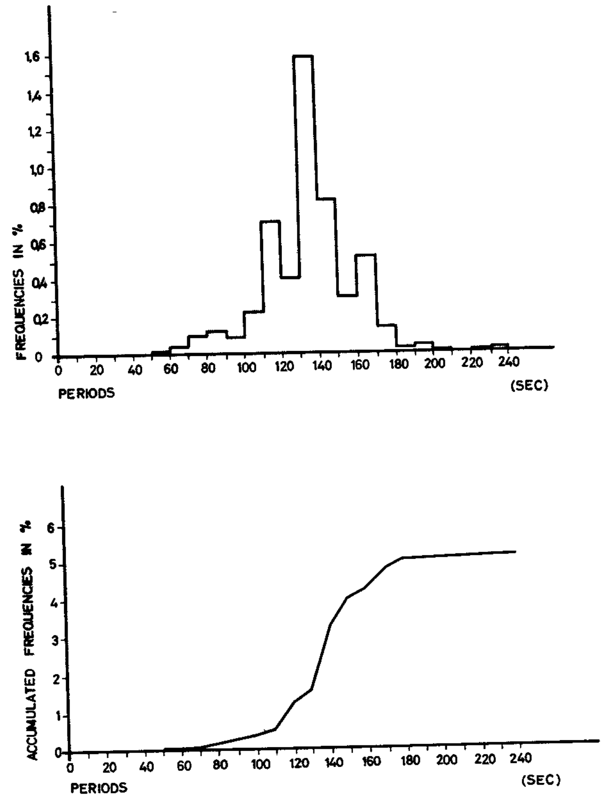

FIG. 3 --OBSERVATIONS OF 1958 HISTOGRAM AND CUMULATED CURVE OF OBSERVED PERIODS IN % OF TOTAL OBSERVATION TIME

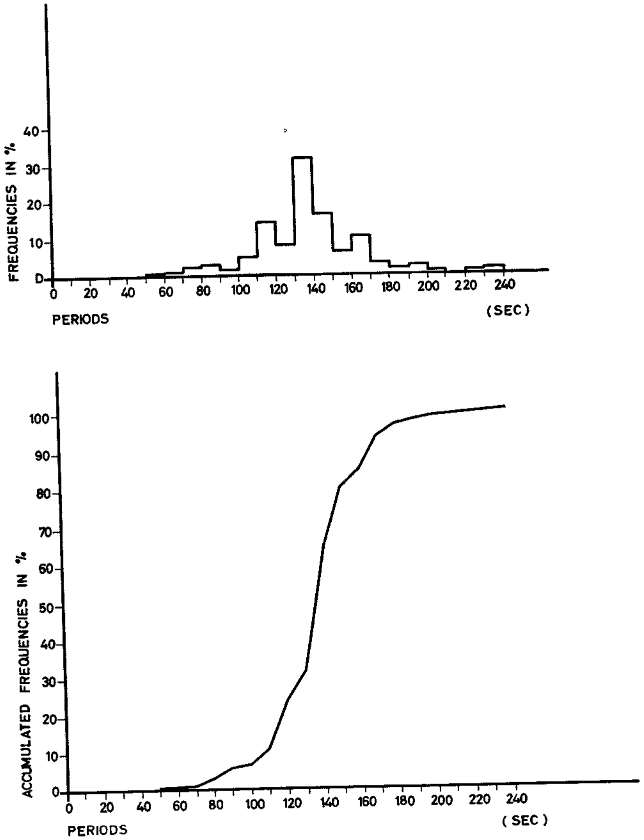

FIG 4 --OBSERVATIONS OF 1958 HISTOGRAM AND CUMULATED CURVE OF OBSERVED PERIODS IN % OF TIME OF POSSIBILITY OF ANALYSIS OF LONG PERIOD WAVES IN RECORDS

TABLE 1

10-sec period ranges	Obser-vation times (min)	Cumu-lated times (min)	Percentage with respect to		Percentage of cumulated times with respect to	
			28 844m	565 920m	28 844m	565 920m
41 to 50 s						
51 to 60 s	76	76	0.26	0.013	0.26	0.013
61 to 70 s	164	240	0.57	0.028	0.83	0.041
71 to 80 s	660	900	2.29	0.116	3.12	0.157
81 to 90 s	692	1 592	2.40	0.122	5.52	0.279
91 to 100 s	418	2 010	1.45	0.073	6.97	0.352
101 to 110 s	1 282	3 292	4.44	0.226	11.41	0.578
111 to 120 s	3 946	7 238	13.68	0.697	25.09	1.275
121 to 130 s	2 262	9 500	7.84	0.399	32.93	1.674
131 to 140 s	8 934	18 434	30.97	1.578	63.90	3.252
141 to 150 s	4 588	23 022	15.90	0.810	79.80	4.062
151 to 160 s	1 650	24 672	5.72	0.291	85.50	4.353
161 to 170 s	2 756	27 428	9.55	0.486	95.07	4.839
171 to 180 s	802	28 230	2.78	0.141	97.85	4.980
181 to 190 s	198	28 428	0.69	0.034	98.54	5.014
191 to 200 s	278	28 706	0.96	0.049	99.50	5.063
201 to 210 s	28	28 734	0.10	0.004	99.60	5.067
211 to 220 s	-	28 734	-	-	99.60	5.067
221 to 230 s	42	28 776	0.16	0.007	99.76	5.074
231 to 240 s	68	28 844	0.24	0.012	100.00	5.086

mulated frequency curve shows that 90 per cent of the observed periods are concentrated in the interval 100-170 sec.

It should be noted however that:

a) It is very likely, from the general look of the tide records, that waves of long period in Luanda bay have been more frequent than the preceding results seem to show but, due to the scale of the records and the apparenthy irregular distribution of peaks, their presence could not be analysed.

b) As said, the amplitude of the oscillations recorded by the tide-recorders do not exceed a few centimeters, as a rule, but in three exceptional cases amplitudes of 15-20 cm (with periods of 130-160 sec) were recorded. It is noteworthy that long-period waves with periods of the same order of magnitude were observed on the same occasions at Lobito, 400 km to the south, and also that the external regular normal-period waves of the swell type had a small amplitude (0.40 to 0.60 m).

2.2 - The observations of 1960 - Two quick-rotation tide recorders installed in Luanda bay were observed during several months: one in the cabotage quay (to be called hereafter "cabotage-tide recorder") and another in the eastern portion of the pier (to be called "pier tide recorder"). A standard rotation tide recorder was installed in the coal wharf (fig.2). The location of the two quick rotation tide recorders was chosen bearing in mind two basic requirements: first to avoid disturbances due to waves generated by local winds or by the frequent passage of ships nearly; second to avoid sites likely to lie in nodal zones of possible long-period stationary oscillating systems which, in that case, would not be recorded.

The results obtained show that the position of the "cabotage tide-recorder" was not very favourable as regards the former requirement which rendered more difficult the analysis of the records; on the other hand the location of the pier tide-recorder proved excellent in this connexion.

The pier tide-recorder started operating on the 19th March 1960, the cabotage tide recorder on the 5th April 1960 and the coal wharf tide recorder on the 31st December 1959. The tide records used in the study concern a period of time from the dates above up to 28th June 1960.

Experience showed that due to the time scale chosen, the coal wharf tide records were not worthwhile using as they could yield reliable data on the amplitudes alone.

The other two tide records were on the contrary excellent: the scale of time was 1 mm ≃ 17.15 seconds and of height 1.5. Yet, due no doubt to the preceding reasons, the values of the pier tide recorder were much clearer than those of the cabotage tide-recorder (fig.5), and so they alone were used in the analysis of long-period waves in the harbour.

The cabotage tide-records were used merely for comparison of periods and amplitudes notably in the case of marked oscillations in the pier tide-records. The results obtained in the coal wharf served only for comparison of amplitudes.

2.3 - Analysis of the tide records - As precedingly explained, long -period waves in Luanda bay and their frequency were studied on basis of the values of the pier tide-record alone, between 19th March and 28th June 1960, i.e. about 100 days.

The records were carefully analysed all the time save for some short periods in which this proved impossible due to gaps or superpositions in the records.

The indicated period was deemed sufficiently significant as neither trends for concentration of long-period waves in certain epochs nor deviations of the results of the analysis with respect to the results of 2.1 were observed.

The records were studied according to the rules indicated above.

The results obtained are presented in Table 2, as a list of the times during which were observed long-period waves classified in 10-s intervals.

The same table contains the cumulated times during which waves with periods inside certain intervals were observed and the percentages of duration of each class and of the cumulated durations with respect to both the total time of observation (about 2,400 hours = 8 640 000 seconds) and the time during which regular long period waves were observed (3 976 528 seconds).

Two histograms and two cumulated frequency curves with

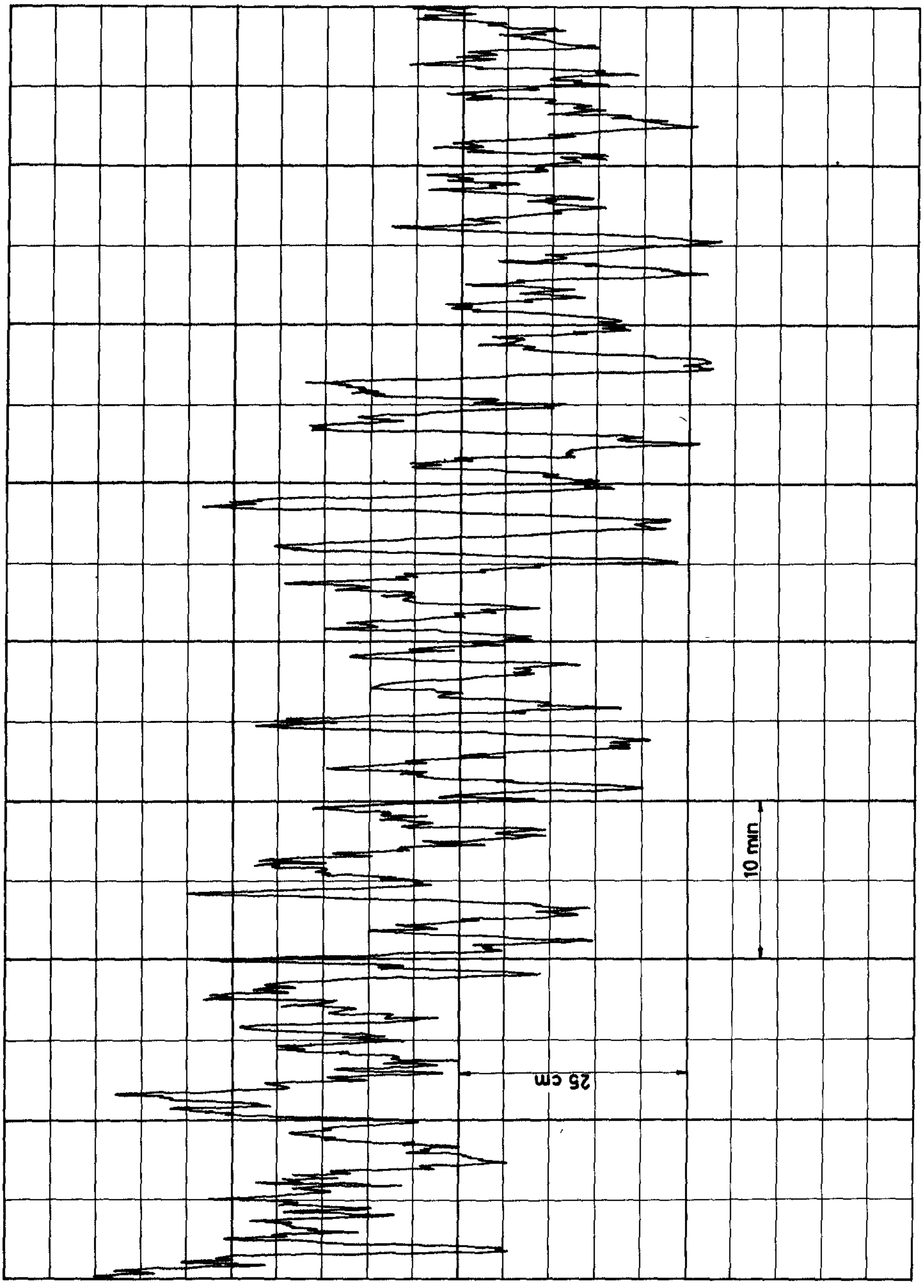

FIG. 5 --EXAMPLE OF RECORDS OBTAINED IN PIER QUICK-ROTATION TIDE-RECORDER

TABLE 2

10-sec period ranges	Observation times (sec)	Cumulated times (sec)	Percentage with respect to		Percentage of cumulated times with respect to	
			3 976 528 sec	8 640 000sec	3 976 528 sec	8 640 000 sec
31- 40	720	720	0.02	0.008	0.02	0.008
41- 50	7 013	7 733	0.18	0.081	0.19	0.089
51- 60	52 547	60 280	1.32	0.609	1.52	0.698
61- 70	86 559	146 839	2.18	1.002	3.69	1.700
71- 80	172 003	318 842	4.32	1.992	8.02	3.692
81- 90	209 743	528 585	5.27	2.429	13.29	6.121
91-100	289 692	818 277	7.28	3.356	20.60	9.477
101-110	379 242	1 197 519	9.54	4.393	30.11	13.870
111-120	496 036	1 693 555	12.47	5.745	42.59	19.615
121-130	639 672	2 333 227	16.09	7.409	58.67	27.024
131-140	587 064	2 920 291	14.80	6.800	73.44	33.824
141-150	551 777	3 472 068	13.90	6.391	87.31	40.215
151-160	311 420	3 783 488	7.83	3.607	95.14	43.822
161-170	84 485	3 867 973	2.12	0.979	97.27	44.801
171-180	31 930	3 899 903	0.80	0.370	98.07	45.171
181-190	17 184	3 917 087	0.43	0.199	98.50	45.370
191-200	7 666	3 924 753	0.19	0.089	98.70	45.459
201-210	806	3 925 559	0.02	0.009	98.72	45.468
211-220	4 255	3 929 814	0.11	0.049	98.82	45.517
221-230	2 022	3 931 836	0.05	0.023	98.87	45.540
231-240	25 761	3 957 597	0.65	0.299	99.52	45.839
321-330	2 948	3 960 545	0.07	0.034	99.60	45.873
361-370	1 097	3 961 642	0.03	0.013	99.62	45.886
381-390	1 560	3 963 202	0.04	0.018	99.66	45.904
441-450	2 211	3 965 413	0.06	0.025	99.72	45.929
451-460	9 041	3 974 454	0.20	0.105	99.95	46.054
511-520	2 074	3 976 528	0.05	0.024	100.00	46.058

respect to the above-mentioned total durations were plotted on basis of these data (figs.6 to 9).The curve of observed values in 1958 (see 2.1) was superposed to the last cumulated curve.

A comparison of the records of the two rapid rotation tide recorders shows that:

a) As said, the pier tide records are much smoother and clearer than the cabotage tide records; the location of the former recorder seems excellent.

b) The wave-periods recorded in both instruments are approximately the same, although occasionally somewhat imprecise in the cabotage tide-recorder.

c) When absent in one tide-recorder,long-period waves are also absent in the other.

d) As a rule, amplitudes are larger in the pier tide-recorder. Thus, for instance, amplitudes were maximum (0.60m) in the pier tide-recorder on the 19th June 1960, whereas they did not exceed 0,35m in the cabotage tide-recorder that same day.The simultaneous maximum amplitude value in the coal wharf tide-recorder was 0.40m. These facts show that amplitudes increase in the area of the pier tide-recorder. At first sight this could apparently be ascribed to resonance but a careful analysis of the phenomenon shows it to be due to a mere concentration of energy in the funnel-shaped zone where the pier tide-recorder was installed, as selectivity for certain periods is not observed.

A comparison of the values observed in 1958 and in 1960 shows that:

a) Long-period waves in Luanda bay are much more frequent than the analysis of the 1958 records would indicate.

Thus, whereas long-period waves could be clearly detected in the former in no more than 5% of the total observation time due to the scales used, these same waves were observed during about 46% of the time in the 1960 records.

b) A remarkable agreement is observed between the results of 1958 and 1960, in spite of a slight deviation towards the shortest periods. The most frequent periods ranged from 120 to 130 seconds (from 130 to 140 seconds in the 1958 results) , fol

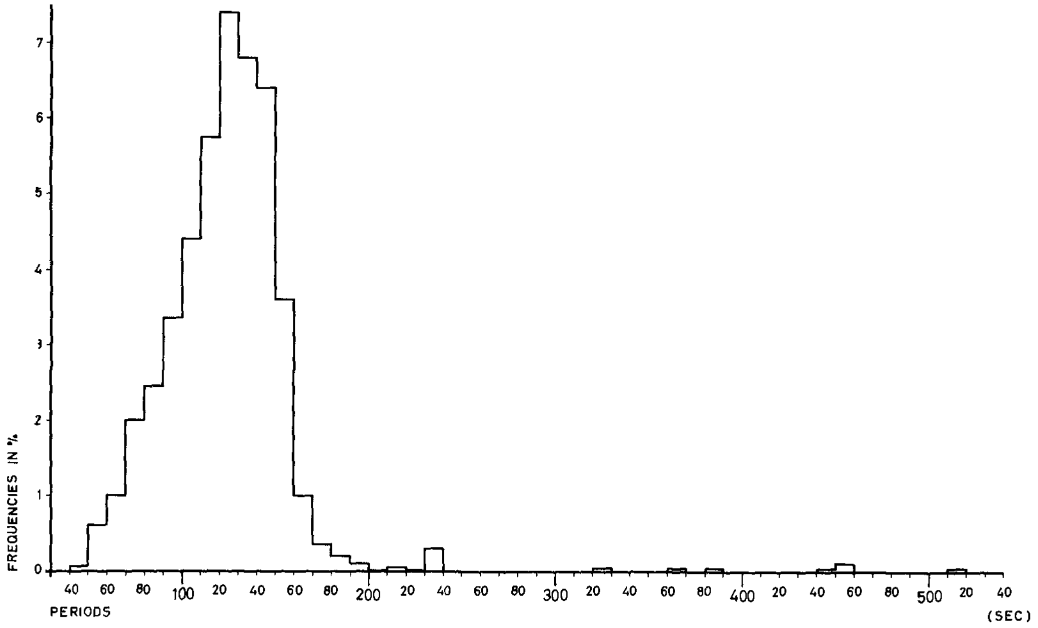

FIG 6 --OBSERVATIONS OF 1960 HISTOGRAM OF OBSERVED PERIODS IN % OF TOTAL OBSERVED TIME

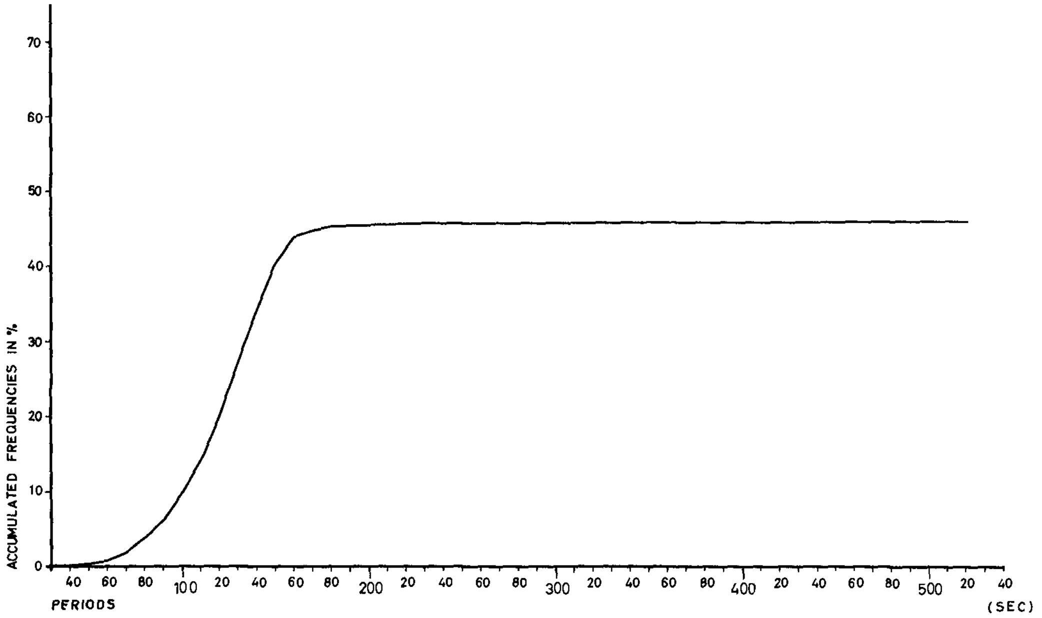

FIG 7 --OBSERVATIONS OF 1960 CUMULATED CURVE OF OBSERVED PERIODS IN % OF TOTAL OBSERVATION TIME

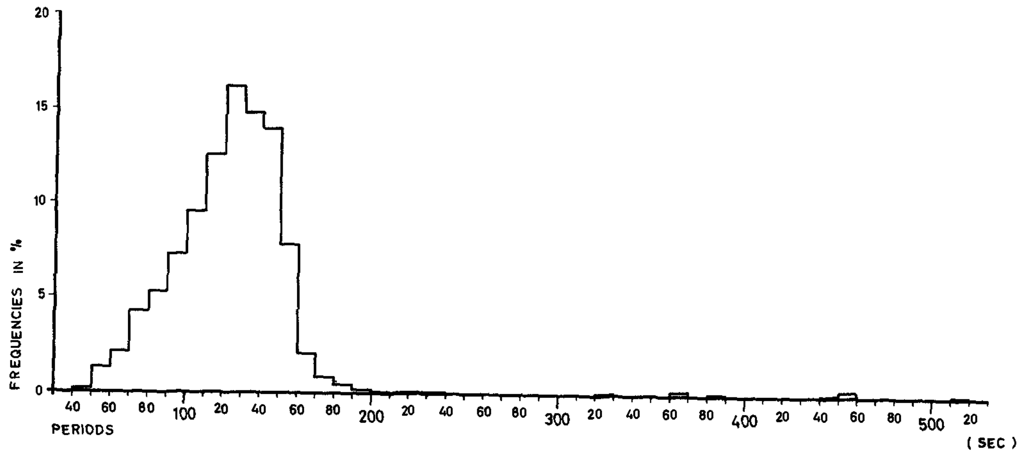

FIG 8. --OBSERVATIONS OF 1960 HISTOGRAM OF OBSERVED PERIODS IN % OF TIME OF POSSIBILITY OF ANALYSIS OF LONG PERIOD WAVES IN RECORDS

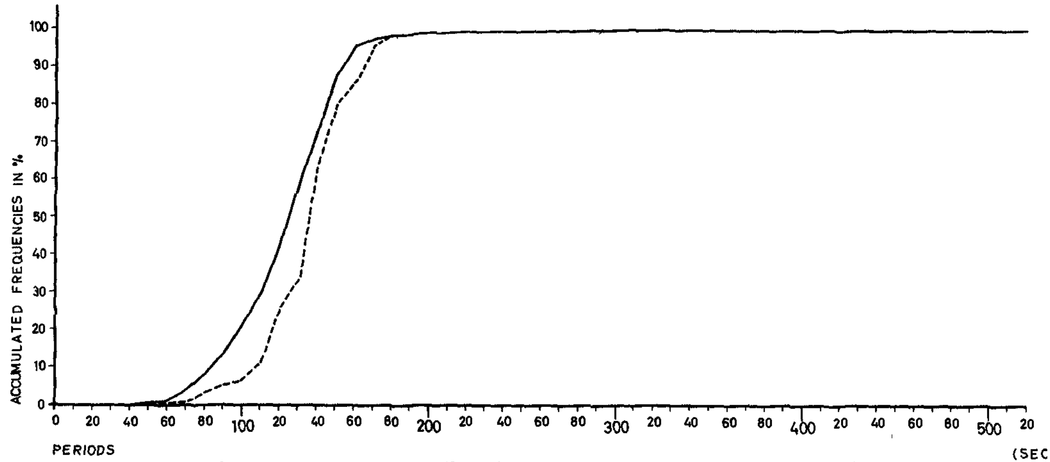

FIG. 9 --OBSERVATIONS OF 1960· CUMULATED CURVE OF OBSERVED PERIODS IN % OF TIME OF POSSIBILITY OF ANALYSIS OF LONG PERIOD WAVES IN RECORDS

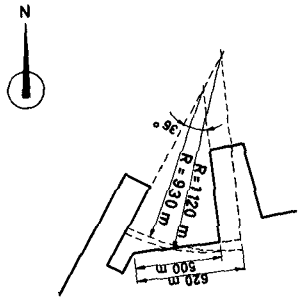

FIG 10. --SOLUTIONS O and I

lowed by those between 130 and 140 seconds and between 140 and 150 seconds. According to the cumulated frequency curve, 90% of the periods (between the 5% and the 95% ordinates) range from 74 to 160 seconds.

3 - ANALYTIC PREDICTIONS

3.1 - Solution 0 - As indicated above, although analytic studies were performed for all the docks, the results presented concern nº 1 dock alone, because on one hand only this one was tested in model so far and, on the other, the remaining docks being rectangular, no difficulties arose in connexion with the analytic calculation of their natural periods of oscillation.

The solution proposed by "Brigada de Estudos do Porto de Luanda", for nº 1 dock, presented in fig.10, will be called solution 0. When the analytic study presented below was carried out, only the 1958 observations in nature were available .

Given the irregular shape of the dock, an accurate analytic study of its resonant conditions was not feasible. Nevertheless, the dock could be roughly assimilated to a constant-depth solid, sector-shaped in plan. The natural period of oscillation of basins with this shape is given by the expression (see [4], [5]):

$$T = \frac{2\pi}{\sqrt{gK \text{ th } Kh}} ,$$

where h is the depth of the basin and K is a coefficient depending on the shape of the basin that can be obtained as follows.

Let us denote by θ the angle at the center of the sector and by R its radius.

Let us consider

$$n = \frac{\pi}{\theta}$$

The values of the coefficient K in the expression above are the results of the division by R of the roots of the derivatives of the Bessel functions of 0, n, 2n, 3n, ... order.

In the present case, $\theta = \frac{\pi}{5}$, $R = 920$ m and $h = 11,5$[x].

Consequently

$$n = \frac{\pi}{\theta} = 5$$

and the roots of the derivatives of Bessel functions J_o, J_5, J_{10}... equal to KR were the unknowns from which the values of K could be obtained.

The zeroes of functions $J'_o(x)$ are:

$x_1 = 3.8317$
$x_2 = 7.0156$
$x_3 = 10.1735$
$x_4 = 13.3237$
$x_5 = 16.4706$

Those of function $J'_5(x)$ are:

$x_1 = 6.415$
$x_2 = 10.520$
$x_3 = 13.985$
$x_4 = 17.310$

For lack of tabulated values, only the order of magnitude of the zeroes of $J'_{10}(x)$ could be determined

$x_1 = 12$

$x_2 = 16.5$

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[x] - The bottom of the dock lying at a depth (-10.5m) and the maximum range of the tide being about 1.5m, the study was performed for a constant depth of 11.5m, roughly corresponding to the mean level depth.

The values of K corresponding to the zeroes of $J'_o(x)$ are:

$K_1 = 0.004120$
$K_2 = 0.007544$
$K_3 = 0.010939$
$K_4 = 0.014327$
$K_5 = 0.017710$

The corresponding periods of resonance are:

$T_1 = \underline{143}$ seconds
$T_2 = \underline{79}$ "
$T_3 = \underline{54}$ "
$T_4 = 41$ "
$T_5 = 34$ "

To the values of $x = \underline{KR}$ of about 13 there correspond periods of resonance of about 40 seconds, of no interest in the case in reference, and so only the roots x_1 and x_2 of $J'_5(x)$ were taken into account. The corresponding natural periods of oscillation are:

$T_1 = \underline{86}$ seconds
$T_2 = 47$ seconds

The fundamental period of oscillation of the basin (143 seconds) falls within a range of periods somewhat frequent in Luanda bay, so that the dimensions of the basin were considered unsuitable.

3.2 - Solution I - In order to prepare recomendations on how to change the characteristics of the dock, the following data were considered according to the directions of "Brigada de Estudos do Porto de Luanda":

- The bottom of the dock should remain at a depth of (-10.5) ;

- the location of the end quay should remain unchanged although its length could be altered;

- even if displaced, the western quay in the second pier should remain perpendicular to the above mentioned end quay;

- it was desirable that neither the length of the end quay (500m) nor the free width of the entrance to the dock were diminished.

Thus the only possible change was an eastward displacement of the second pier, which amounted to maintaining the angle at the center (and therefore n), increasing the radius of the sector and consequently the fundamental period of resonance of the basin. Another consequence to be taken into account, nevertheless, was that the period of oscillation corresponding to the first root of J'_5 (x) was also increased and so it was advisable to avoid for it values falling in the range of frequently observed wave periods in Luanda bay.

In order to determine dimensions of the basin enabling this disadvantage to be avoided, the radius of the sector was chosen so that

$$T = \frac{2\pi}{\sqrt{g\ K\ \text{th}\ K\ h}} \geqslant 175 \text{ seconds},$$

as oscillation periods exceeding this value are very seldom observed in Luanda.

The equation above, apparently difficult to solve due to the presence of the transcendent function th Kh, can be simplified taking into account that the values of Kh of interest in this case are very small, and consequently it is possible to take

$$\text{th}\ Kh = Kh$$

up to the fourth decimal place. The equation then becomes

$$\frac{2\pi}{K\sqrt{g\ h}} \geqslant 175,$$

from which

$$K \leqslant \frac{6.28}{175 \times 10.60}$$

Now the value x=KR corresponding to the fundamental oscillation period is, as indicated

$$K\ R = 3.8317$$

whence

$$R \geqslant 1{,}120\ m$$

Thus a radius of 1,120 m would be required for the sector, in order to ensure a fundamental period of oscillation of 175 sec.

It was first advisable to determine the period of oscillation corresponding to this radius for the first root of $J'_5\ (x)$ which, as is known, is x = 6.415 = KR.

The value of K is:

$$K = \frac{6.405}{1{,}120} = 0.005728$$

and the corresponding period:

$$T = \frac{2\pi}{\sqrt{g\ K\ th\ Kh}} = 103 \text{ seconds}$$

falls within a range of infrequent periods in Luanda bay and so is aceptable, above all bearing in mind that this is not the fundamental oscillation.

A radius of 1,120 m for the sector implied for the southern quay of nº 1 dock a length of 620 m, i.e. more 120 m than the planned value. These characteristics, recommended for nº 1 dock, gave rise to solution I.

3.3 - Solution III - Meanwhile "Brigada de Estudos do Porto de Luanda", on basis of the first model tests of solution I, prepared solution III (presented in fig.11) that, while maintaining the general shape of solution I, eliminated the small rectangular dock which, according to the experimental studies, proved very harmful.

Before testing this solution in model, its resonance conditions were investigated in an analytic study briefly presented below.

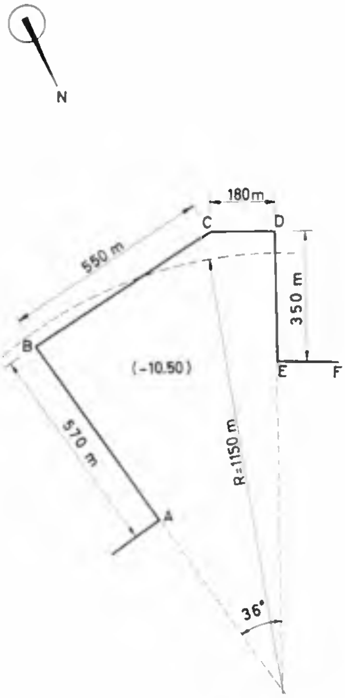

FIG. 11. --SOLUTION III

FIG. 12. VIEW OF MODEL

The basic data of solution III were:

$$R = 1{,}150 \text{ m} \qquad \theta = 36^{\circ}$$

The periods of natural oscillation corresponding to the zeroes of the derivatives of the Bessel function are:

	$x = KR$	$T = \dfrac{2\pi}{\sqrt{gK \text{ th } Kh}}$
$J'_0(x)$	3.8317	177 seg
	7.0156	97 "
	10.1735	67 "
	13.3237	51 "
	16.4706	41 "
$J'_5(x)$	6.415	106 "
	10.520	65 "
	13.985	49 "

The result of the analytic study was deemed satisfactory as no period of natural oscillation of solution III was contained in the range of most frequent periods in Luanda bay - 105 to 175 sec.

4 - MODEL TESTS: 1st stage

4.1 - General - A detailed description or discussion of the experimental conditions in which these tests were carried out being outside the scope of the present paper, the only indication given about them is that the scales of the model were 1/400 in plan and 1/200 in height (fig.12).

The coeficient of amplification of the response curves obtained in the tests is the ratio of the maximum amplitude observed inside the dock to the mean amplitude in the neighbourhood of the entrance.

The tide records of 1960 were already available when the model tests were carried out. So, although the analytic study was based on the observations of 1958, the response curves obtained in the tests are superposed to the histograms of the periods determined in the observations of 1960.

4.2 - Model test of solution I - The response curve obtained in the model test of solution I is presented in fig.13, superposed on the histogram of the periods of the tide records of

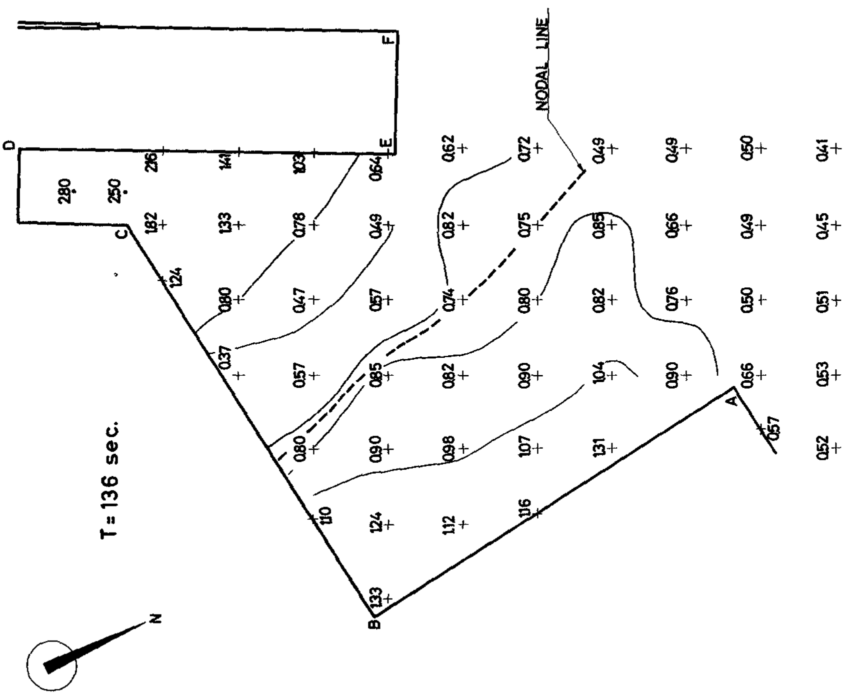

FIG. 14 --RESONANT OSCILLATION OF SOLUTION I T = 136 S EC

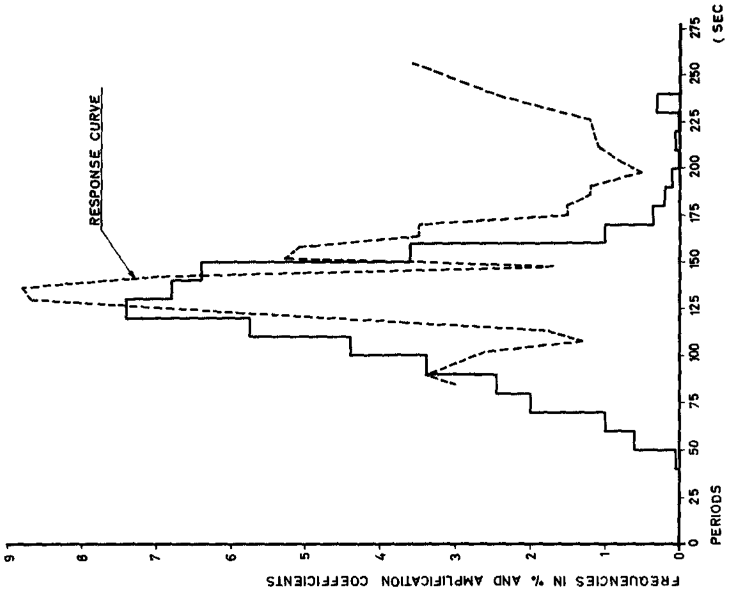

FIG 13 --RESPONSE CURVE OF SOLUTION I

1960. As shown, contrary to the predictions of the analytic study, according to which no period of oscillation between 103 and 170 sec. would be present in the harbour, the maximum amplification occurred for a period of 136 seconds (with a band of periods above and below where amplification remained considerable). According to fig.14, in which the amplitudes at the different points and the lines of equal amplitude are indicated, this resonance corresponded to a one-node oscillation between the area of the small rectangular dock and quay AB.

The tests also showed that, independently of resonance, the rectangular dock had the worst possible effects on the water movement in nº 1 dock. In fact the entrance to the dock lay in an area where the existant pier (DE) would meet quay BC. The increased amplitudes thus generated together with the strong inflow due to the rectangular dock gave rise to extremely strong currents which, on being reflected inside the dock, originated alternate water movements, not only near the entrance to the rectangular dock but also everywhere in nº 1 dock, in special along quay BC. Fig.14 shows the extremely rough conditions inside the rectangular dock. It is noteworthy that this same phenomenon took place even for non-resonant oscillations in nº 1 dock. On the other hand, when the rectangular dock was closed in the model, the alternate water movements along quay BC were considerably reduced.

A first and obvious conclusion of these tests is the need to eliminate the rectangular dock and that was why "Brigada de Estudos do Porto de Luanda", as indicated above, presented solution III (fig.11).

4.3 - <u>Model test of solution III</u> - The response curve obtained in this test (fig.15) presented 4 more or less marked peaks with periods of 56, 96, 108 and 136 sec. in the period range of interest, each corresponding no doubt to a different resonance mode. The three latter periods fall in a range of very frequent periods in Luanda bay.

Curves of equal amplitude for periods of 96, 108 and 136 seconds are presented in figs. 16, 17 and 18.

Solution III proved much better than solution I. In addition to the elimination of the above-mentioned disturbances due to the small rectangular dock, the coefficients of amplification were appreciably reduced, assuming acceptable values (the "coefficient

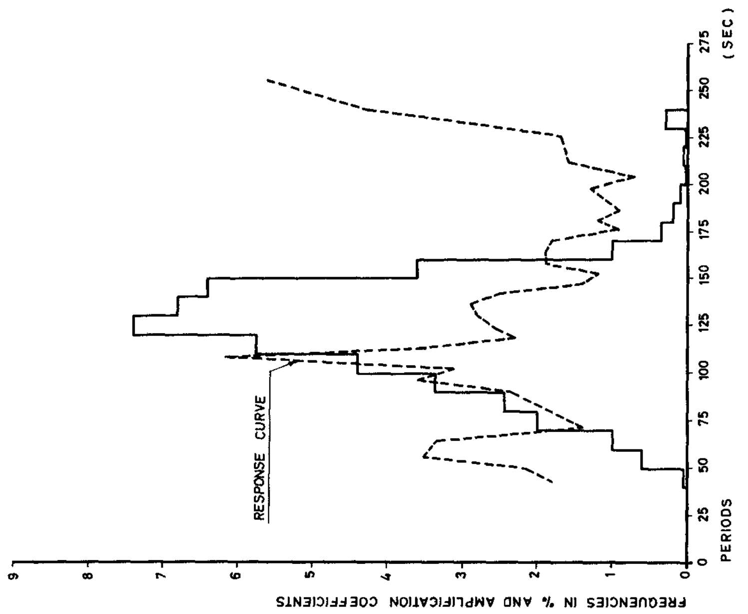

FIG 15 --RESPONSE CURVE OF SOLUTION III

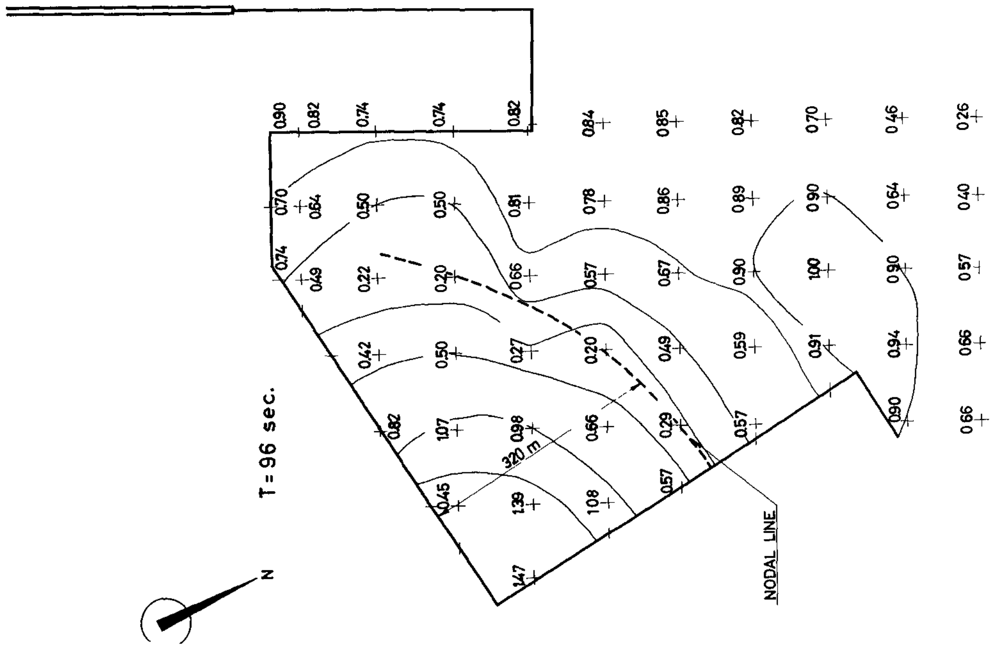

FIG 16 --RESONANT OSCILLATION OF SOLUTION III T = 96 SEC

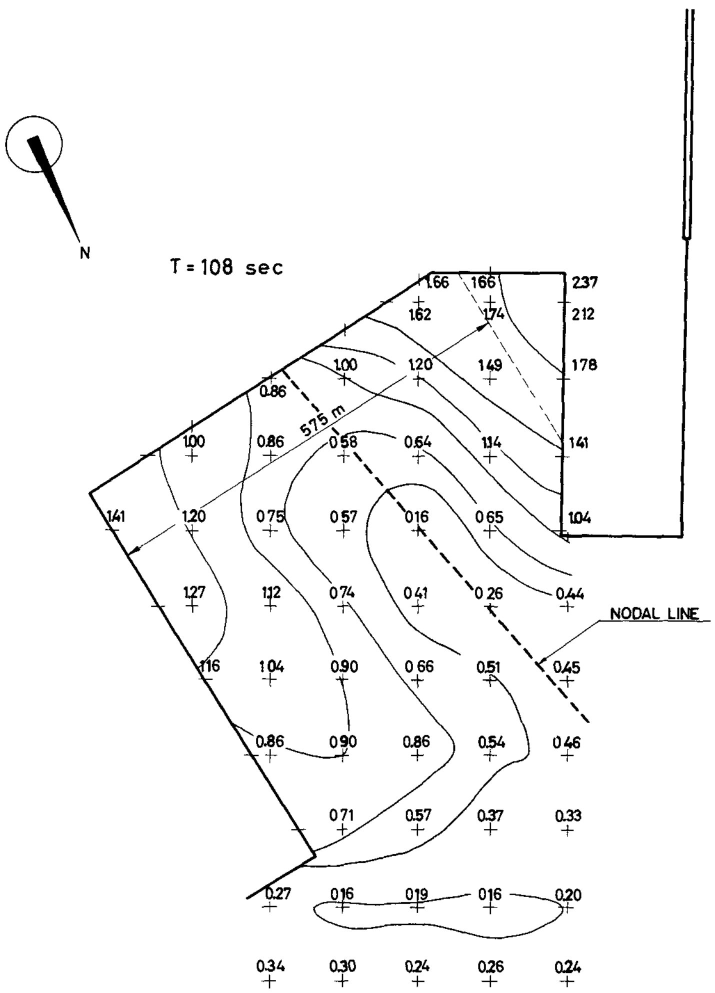

FIG 17 --RESONANT OSCILLATION OF SOLUTION III T = 108 SEC

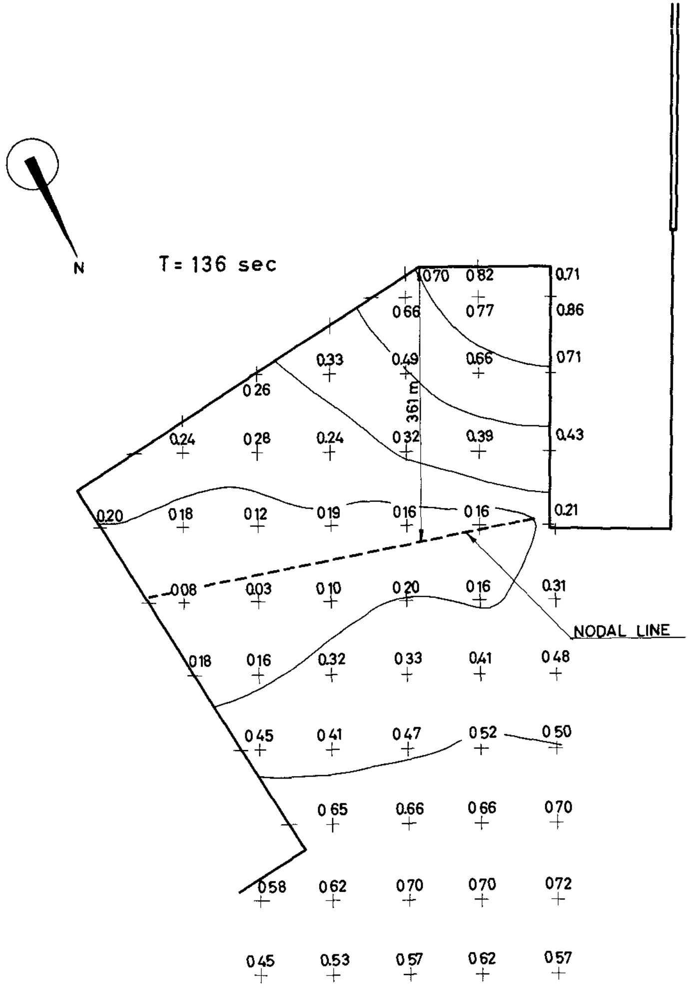

FIG 18 --RESONANT OSCILLATION OF SOLUTION III T = 136 SEC of amplification" was defined as the ratio of the maximum internal amplitude to the mean external amplitude). Nevertheless, as some periods of resonance corresponded to waves frequently occurring in Luanda bay, the shape of nº 1 dock had to be changed.

5 - COMPARISON AND ANALYSIS OF THE RESULTS OBTAINED

5.1 - Comparison of analytic and experimental results-As shown in figs. 13 and 15, the analytic predictions were not confirmed by the tests, notably as regards solution I, possibly on account of the disturbances due to the small rectangular dock . On one hand the response curve displayed peaks for periods non-predicted in the analytic study, whereas for some predicted resonance periods moderate amplification coefficients were obtained. Only for solution III, in which deviations are slighter, peaks of the response curve for periods approaching the predicted values could be obtained. Thus, resonance had been predicted for periods of 97 and 106 seconds and the response curve displayed peaks for 96 and 108 seconds; in the zone of 49, 51, 65, 67 seconds, where resonance phenomenon were anticipated, considerably high amplification coefficients and a peak for 56 seconds were obtained. On the contrary, for the period of 77 seconds, where resonance had been predicted, the response curve presented a minimum.

The deviations just described should be ascribed to the non-validity of assimilating the dock to a sector.

5.2 - Interpretation of the experimental results - A careful analysis of the equal amplitude curves (figs. 16, 17, 18) is extremely useful for the interpretation of the experimental results.

Thus, according to fig.16, the oscillation with a period of 96 seconds corresponds to a quarter-wave-length oscillation in an open basin with respect to the larger quay at the end of the dock the nodal line is removed slightly more than a quarter-wave-lengtl from this quay. In fact the wave length for a period of 96 seconds is

$$L = CT = \sqrt{gh} \times T = \sqrt{9.8 \times 11.5} \times 96 = 1{,}020 \text{ m}$$

and

$$\frac{L}{4} = 255 \text{ m}$$

The oscillation of 108 seconds (fig.17) corresponds to a half-wave-length oscillation in a closed basin between the new pier and a line near the angle of the present pier with the new quay. In fact the wave length is now

$$L = CT = \sqrt{g\ h} \times T = \sqrt{9.8 \times 11{,}5} \times 108 = 1{,}150 \text{ m}$$

and

$$\frac{L}{2} = 575 \text{ m},$$

that is the distance between the two lines.

The oscillation of 136 sec.(fig.18) corresponds to a quarter-wave-length oscillation in an open basin with respect to the smaller quay at the end of the dock, a line parallel to the quay through the end of the present pier behaving roughly as the entrance to the dock. In fact, the wave length for a period of 136 seconds is

$$L = CT = \sqrt{9.8 \times 11.5} \times 136 = 1{,}446 \text{ m}$$

and

$$\frac{L}{4} = 361 \text{ m},$$

which is sensibly the distance between the quay and the above-mentioned line.

Finally, the 56 sec oscillation, whose equal-amplitude curves are not presented due to their minor interest, proved to be a binodal oscillation, with nodal lines roughly parallel to the large quay at the end of the dock. This oscillation was neglected for the following reasons:

a) It was a binodal oscillation, which in itself at once made it of minor importance as it tended to be damped much faster than uninodal oscillations.

b) its amplification coefficient was not high;

c) its period fell in an interval of very rare periods in Luanda bay.

6 - SOLUTION IV AND ITS EXPERIMENTAL STUDY

6.1 - Solution IV - From the foregoing and from an analysis of the shape of nº 1 dock (fig.19) it is clear that distances d_1, d_2, d_3 conditioned resonances corresponding to the periods of 136, 108 and 96 seconds respectively. So it was decided to investigate how to change these distances so as to obtain a satisfactory solution.

It was proved that some advantages could be derived from a change in d_1. In fact, its value gives rise to a quarter-wave-length resonance which, although slight, corresponds to a very frequent period (136 seconds) in Luanda bay. The problem nevertheless was difficult to solve. In fact, for this resonance to fall in the period range of 170 seconds, comparatively rare in the bay, distance d_1 should obey the condition

$$L = 4\ d_1 = \sqrt{g\ h} \times T = \sqrt{9.8 \times 11{,}5} \times 170 = 1{,}800\ \text{m}$$

and

$$d_1 = 450\ \text{m}$$

This value had the disadvantage of extending the present pier, eliminating its end quay and giving rise to a resonance with a period of 96 seconds (with a higher amplification coefficient), therefore in the range of the most frequent wave periods in Luanda bay.

In order to solve the problem by means of a resonance in the low periods (about 60 seconds), which are infrequent, it would be necessary to have

$$L = 4\ d_1 = \sqrt{gh} \times T = \sqrt{9.8 \times 11{,}5} \times 60 = 638\ \text{m}$$

and

$$d_1 = 159\ \text{m},$$

which is obviously unacceptable.

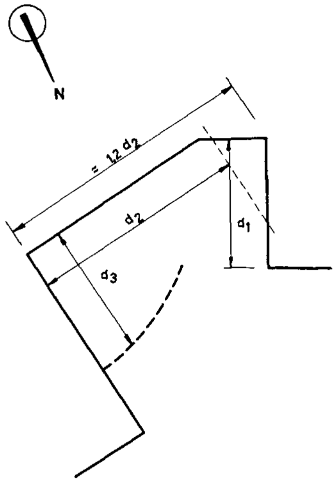

FIG 19 -DISTANCES COMMANDING RESONANT OSCILLATIONS

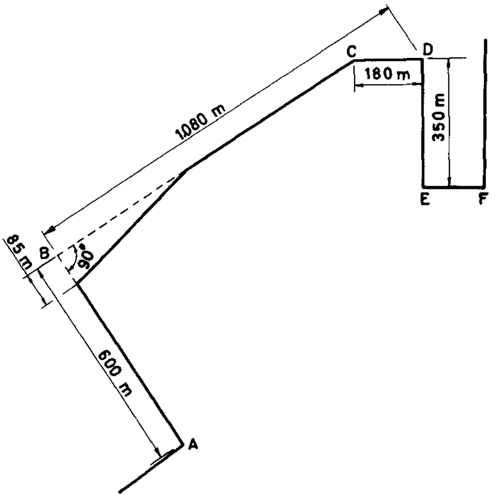

FIG 2- --SOLUTION IV

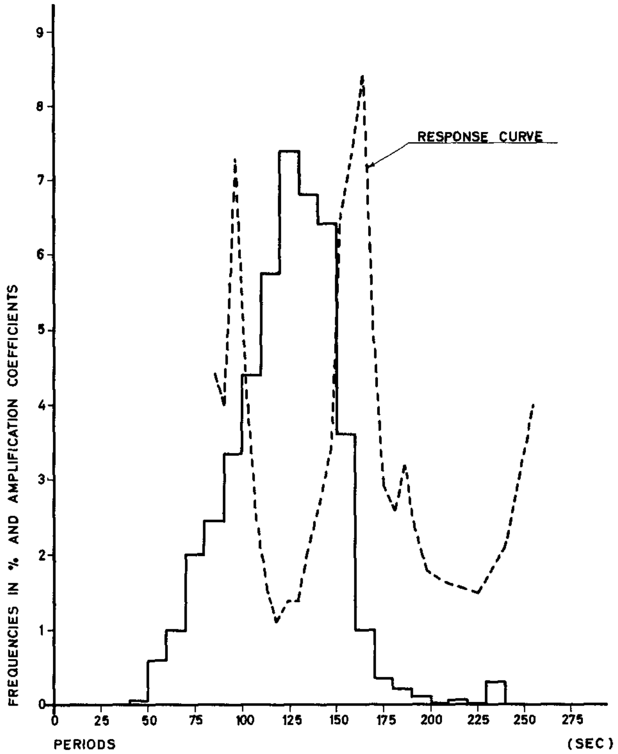

FIG 21 --RESPONSE CURVE OF SOLUTION IV

Nevertheless, given its small amplification coefficient that probably could be further reduced by the increase of distance d_2 (which, as will be seen below, was necessary for other reasons), this resonance did not seem very dangerous.

Distance d_2 had to be changed so that the corresponding half-wave-length resonance, that was marked, would fall in an infrequent period. This could be done by decreasing or increasing d_2 and consequently the corresponding resonance period. In the former case, reducing the resonance period to 70 seconds, which is an infrequent period, d_2 should be such that:

$$L = 2\,d_2 = \sqrt{gh} \times T = \sqrt{9.8 \times 11{,}5} \times 70 = 742 \text{ m}$$

and

$$d_2 = 371 \text{ m}$$

This value was deemed unacceptable as it would reduce too much both the length of the end quay and the entrance width. It was thus necessary to increase d_2, so as to obtain a resonance period of about 170 seconds. Consequently:

$$L = 2\,d_2 = \sqrt{gh} \times T = \sqrt{9.8 \times 11{,}5} \times 170 = 1{,}800 \text{ m}$$

and

$$d_2 = 900 \text{ m}; \quad 1.2\,d_2 = 1{,}080 \text{ m}$$

Therefore d_2 had to be increased by about 320 m.

As for distance d_3, it gave rise to a not very marked quarter-wave-length resonance with a period of 96 seconds which is not very frequent. Nevertheless only a short reduction of d_3, and consequently of d_1 connected with it, was advantageous.

The foregoing reasons led to solution IV presented in fig. 20. The angle of the end quay was suggested by "Brigada de Estudos do Porto de Luanda".

6.2 - Model test of solution IV - The model test of solution IV yielded the response curve presented in fig. 21 that displays two comparatively marked peaks. One, for a period of 165 seconds, is due to the half-wave-length resonance between quays AB and DE that had been predicted for a period of about 170 seconds; the other, with a period of 96 seconds, corresponds to the first harmonic of the former. In fact, the equal-amplitude curves (fig.22, 23) show that in the former case two lines of maxima (loop lines) are generated near the quay walls and a node line roughly in the middle dock; in the latter case the oscillation is binodal with two loop lines near the quays and another roughly in the middle dock.

The quarter-wave-length oscillation of solution III were not conspicuous in solution IV, probably due to the increased width of the dock entrance and the reduced ratio of its cross dimension (distances d_1 or d_3) to its length (distance d_2).

The superposition of the response curve on the histogram of long-period waves observed in Luanda (fig.21) shows a shift between the peaks of both graphs, which means that the probabilities of resonance are minimum. The peak in the uninodal oscillation corresponds to a very small frequency of the long-period waves; as for the binodal oscillation, its frequency is slightly higher but anyhow this is always less dangerous than the fundamental oscillation. For all these reasons, solution IV was considered as satisfactory.

7 - CONCLUSIONS

In addition to enabling the determination of a suitable shape for nº 1 dock of Luanda port, we believe that from the studies presented the following general conclusions of practical interest can be drawn:

a) Special care is required when choosing the location of quick rotation tide recorders for long-period waves. It is necessary not only to prefer locations in or near loop zones for all the periods but also to avoid at all costs the disturbing influences of normal waves coming from the open sea or locally generated by the wind or ships passing nearby. Without these precautions the interpretation of the obtained results will be very difficult.

b) In the analytic study of oscillations in an irregular shaped basin, an extreme prudence is recommended in the assimilation of the basin to another with simpler geometric shapes making an analytic study possible.

c) The consideration of half-wave length and one-quarter

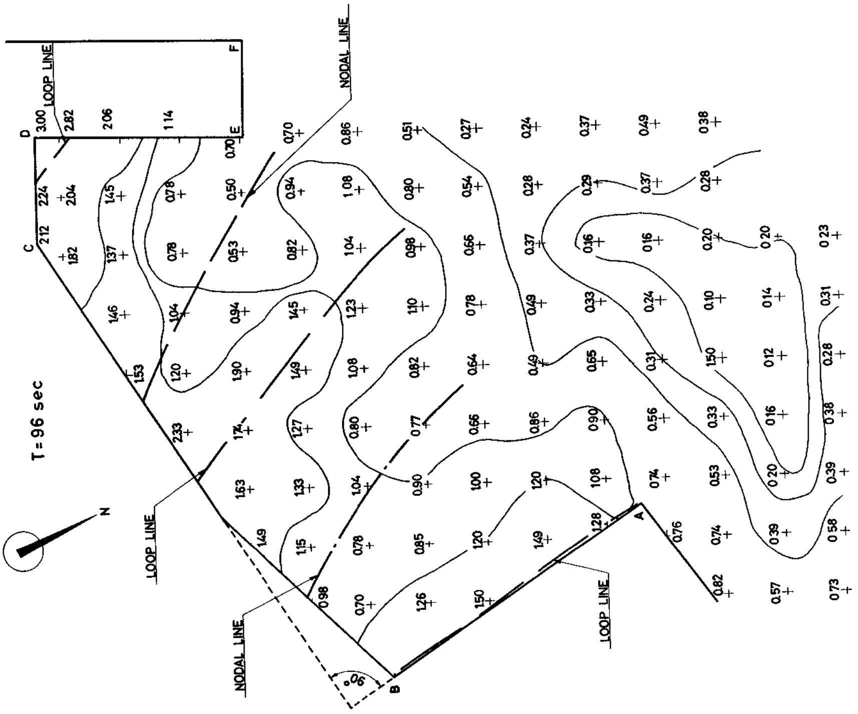

FIG 23 --RESONANT OSCILLATION OF SOLUTION IV T - 96 SEC

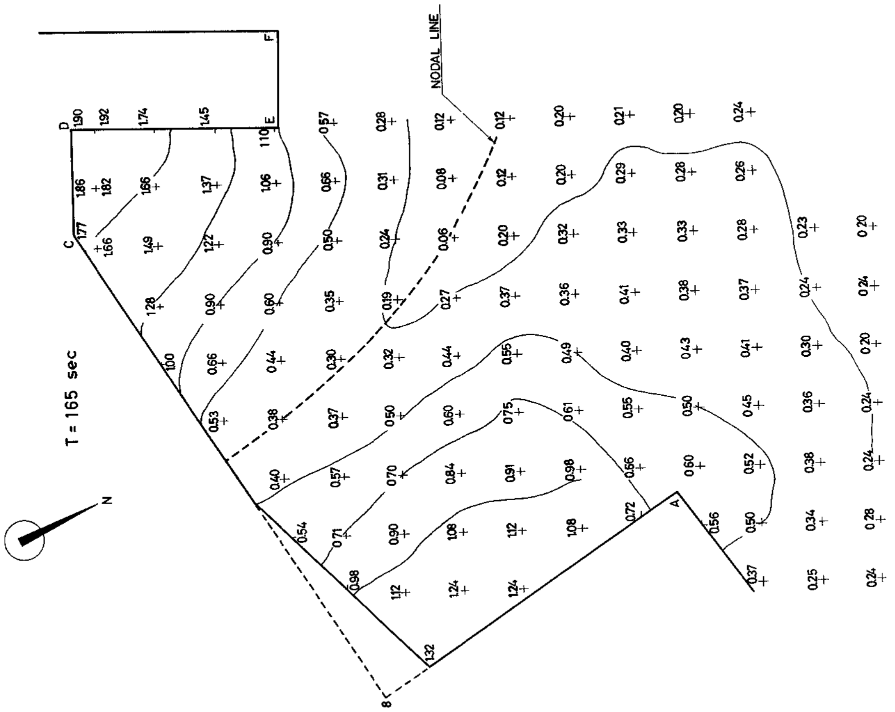

FIG 22 --RESONANT OSCILLATION OF SOLUTION IV T - 165 SEC wave length oscillations and their harmonics between roughly regular reflecting surfaces can be very useful when predicting oscillation conditions in an irregular shaped basin.

d) The presence of small docks near loop zones is extremely harmful as regards oscillations in a wider space.

e) The intensity of one-quarter wave length oscillations is substantially reduced when the opening increases. The latter conclusion, in fact, agrees with others obtained by analytic or experimental means [6], [7] .

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