Chapter 50

ON THE PROCESS OF CHANGE FROM SALT WATER TO FRESH WATER BY EFFECTIVE CONTROL OF OUTLET GATES FOR A LAKE OR RIVER DISCHARGING TO THE SEA

Isao Minami[1)]

1. SUMMARY

In previous studies, the mechanisms for changing salt water lake to fresh water were analysed by Dr. Jansen[2)] and Okuda[1)]. But in these theories, the effectiveness of the structure of the outlet gate and its control for salinity had not been considered. For this reason, the above studies are not precise enough to estimate the necessary time required for changing salt water to fresh water in the actual sea-reclamation planing. The author has proposed a new method to calculate the necessary time. In this paper the mechanism of transport of salinity in water is also considered.

2. INTRODUCTION

In general the lake with a narrow opening at the sea side has vertically stratified salinity distribution. The upper water contains little salinity, but the lower water near the bed has much salinity and high density. The mechanism of transport of salinity in stratified turbulent flow must be applied to the water exploitation plan for the change from salt water lake to fresh.

Fig. 1 shows an example of the lake that is separated by the gate from the outer sea. At the gate, the outflow velocity must have greater value than the flow in velocity which is produced by the water density difference between seaside and lake.

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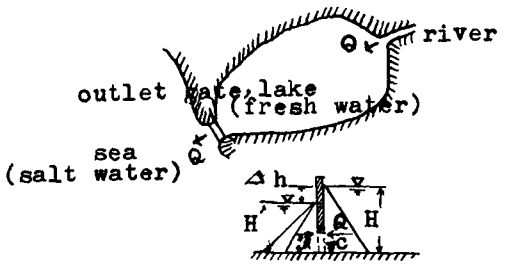


Fig.1

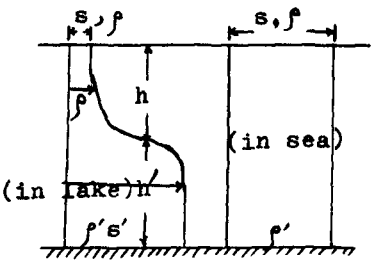


Fig.2

The progress of change from salt water to fresh water could be classified as follows;

1st step: The salt water goes out to the sea through the lower part of the gate. In this case, the salt water exiting near the gate plate in the lake will be almost entirely diminished. But the salt water below the elevation of the sill of gate cannot diminish so fast because of its vertical density difference.

2nd step; The salinity will go out with discharge by vertical mixing from lower salt water and this is strengthened by wind force or flowing upper water. But in these types of mixing, the salt content mixed with upper fresh water would be negligiblly small. These phenomena are seen in the lake having deeper parts than the elevation of the sill of outlet gate.

3rd step; The salinity in the soil will go into upper water from its void.

We must take into consideration the above three hydraulic steps in planning, and precise analysis of diffusion or movement of salt water must be effectively used for reclamation planning.

3. GATE EQUATIONS

In Fig.1, to flush out the salinity from the lake by the gate management, the critical difference of water elevation must exist between the outer sea and the inner lake as follows:

$$\Delta h > \frac{\rho' - \rho}{\rho} H' \qquad (1)$$

At the sluice in Fig.2, the lost salinity mixed with discharge from sluice are supplied by the movement of salinity in the lake. At the upper open gate, as Fig.3a, the salinity out flow is smaller than that lower the open gate. Therefore, the time necessary to change the salty lake to fresh is surely dependent on the structure of

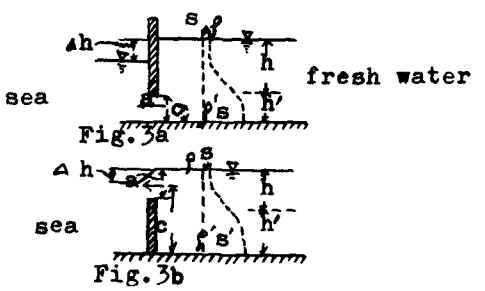


Fig.3a

Fig.3b

the gate and its management. At the upper open gate, the mechanism of vertical transport of salinity in the lake is an important control factor. At the lower open gate, the mechanism of horizontal transport of salinity is most important. In small discharge the horizontal transport of salinity by the density action is larger than the vertical transport.

The discharge from the outlet gate is calculated by the next equation by Dr. Kuwano.

$$Q = C_1 B a \sqrt{2g\Delta h} \qquad (2)$$

$$C_1 = \frac{2}{3} C \frac{\Delta h}{2a} \left\{ \left(1 - \frac{\varepsilon h - \Delta h}{\Delta h}\right)^{3/2} - \left[1 - \frac{\varepsilon(a + h' + \Delta h)}{\Delta h}\right]^{3/2} \right\}, \quad \varepsilon = \frac{\rho' - \rho}{\rho}$$

The salinity mixed in the discharge can be written as follows due to dimensional theory:

$$\rho_g = \rho + K C^{\alpha_1} h^{\alpha_2} h'^{\alpha_3} a^{\alpha_4} g^{\alpha_5} g^{\alpha_6} (\rho' - \rho)^{\alpha_7}$$

To determine the index. $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7$, the author performed the experiment in the laboratory. Fig.4 shows the model of the outlet gate.

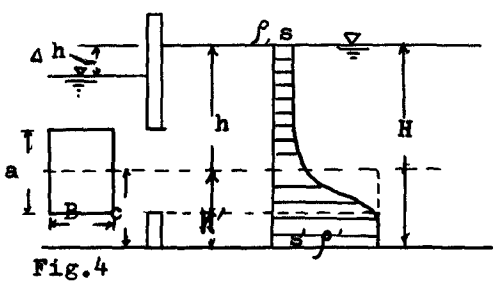


Fig.4

The dimensions of all hydraulic factors are as follows:

$[\rho] = ML^{-3}$
$[C] = L$
$[h] = L$
$[h'] = L$
$[a] = L$
$[g] = L^2T^{-1}$
$[\rho' - \rho] = ML^{-3}$
$[g] = LT^{-2}$

The dimension equation is

$$ML^{-3} = K[L]^{\alpha_1}[L]^{\alpha_2}[L]^{\alpha_3}[L]^{\alpha_4}[L^2T^{-1}]^{\alpha_5}[ML^{-3}]^{\alpha_6}[LT^{-2}]^{\alpha_7} \qquad (3)$$

about the dimension M $\quad 1 = \alpha_7$

about the dimension L $\quad -3 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + 2\alpha_5 - 3\alpha_6 + \alpha_7$

about the dimension T $\quad 0 = -\alpha_5 - 2\alpha_7$

By dimensional analysis, the next equation is given:

$$\rho_g = \rho + K(\rho'-\rho)\left(\frac{h}{C}\right)^{\alpha_1}\left(\frac{\rho'}{C}\right)^{\alpha_2}\left(\frac{q}{C}\right)^{\alpha_3}\left(\frac{H^2g}{q^2}\right)^{\alpha_4} \tag{4}$$

By the data from experiment, we can decide the value of index as follows:

$$\rho_g = \rho + 0.2793(\rho'-\rho)\left(\frac{h}{C}\right)^{-0.00101}\left(\frac{\rho'}{C}\right)^{0.0361}\left(\frac{q}{C}\right)^{-0.00304}\left(\frac{H^2g}{q^2}\right)^{+0.000231} \tag{5}$$

By the equation (5), we can calculate the salinity of discharge flowing through the gate.

In the above equation (5), the relation between density and salinity was given as follows:[3]

$$\rho = 1 + \frac{1}{1000}(-0.069 + 1.4768\,Cl - 0.00157\,Cl^2 + 0.0000398\,Cl^3) \tag{6}$$

4. STABILITY OF BOUNDARY LAYER IN THE ESTUARY

The vertical distribution of salinity of lake water in the estuary has the following characteristics, namely upper fresh water and lower salty water exist as Fig.5, and in boundary of two layers the spring layer of salinity is developed. To know the stability of the boundary layer or diffusion from the lower water, we used Richardson's number. But Richardson's number is not so convenient to calculate because it demands a gradient of density and velocity that is difficult to know.

Therefore, we should modify Richardson's number as follows:

$$R_i = \frac{\frac{1}{\rho}\frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2} \tag{7}$$

$$R_{im} = \frac{g(\rho'-\rho)h}{\rho U^2}$$

$$\Delta\rho = \rho'-\rho, \quad \Delta z = h, \quad \Delta U = u'-u$$

This number is useful because the shape of vertical distribution of salinity has always a typical style, as Fig. 5, and the depth of the upper layer is generally about 3m to 5m, and the value of density is 1,000--1,025.

When the modified Richardson's number has the relation,

$$R_{im} > 1 \tag{8}$$

the boundary layer is stable, but when thenumber has the relation,

$$R_{im} < 1 \qquad (9)$$

the boundary layer is not so stable.

In stable conditions, the vertical diffusion of salinity from lower salt water is very small, but in unstable conditions, the diffusion is actively developed by the wind or flowing water.

1. Stability of boundary layer against the wind.

The modified Richardson's number is deduced by the following equation:

$$R_{im} = \frac{C_f g(\rho' - \rho)h}{\tau_a} \qquad (10)$$

$$\tau_a = 2.6\times10^{-3}\rho_a W^2, \quad \rho_a = 1.25\times10^{-3}$$

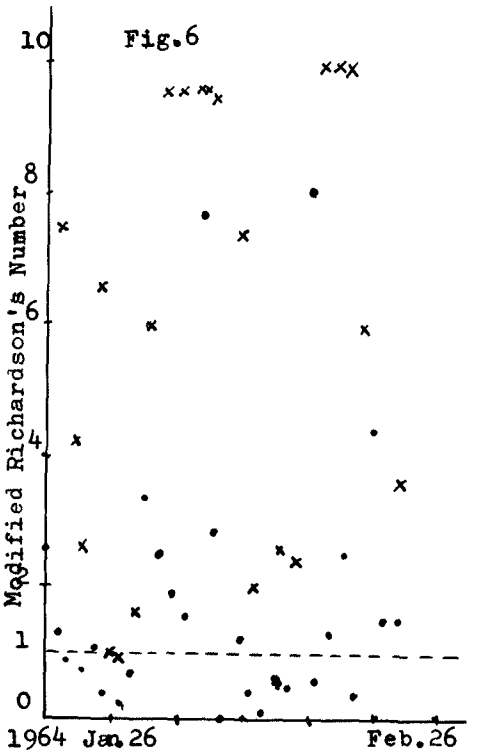
In equation (10), resistant coefficient c_f is taken as follows:

$$C_f \fallingdotseq 0.001 \qquad (11)$$

These data were given by the observation of Nakauma[7] in Simane Prefecture in Japan. Fig. 6 shows the modified Richardson's number of the boundary layer during February 1964.

(The depth of boundary layer was during 5m or 3m from water suface)

Fig.6



In Fig. 6, about few days per month, the modified Richardson's number was smaller than 1, and the other day it was larger than 1. This shows that the vertical diffusion exists only in strong windy days. In calm days, the vertical diffusion is very small.

Now we can show the value of c_f against the difference of density.

Table 1.

$\rho' - \rho$	C_f
0.001	0.00375
0.002	0.00106
0.003	0.00072
0.004	0.00088
0.005	0.00086
0.013	0.00068

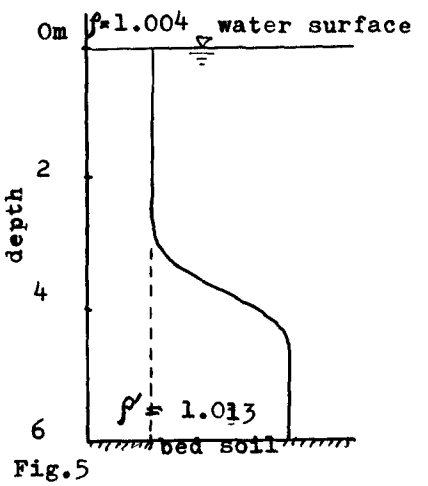


Fig.5

2. Stability of boundary layer due to flow of upper fresh water.

The stability of boundary layer is given by modified Richardson's number as follows:

$$R_{im} = \frac{\frac{1}{\rho}\frac{\Delta\rho}{\Delta z}}{\left(\frac{\Delta U}{\Delta z}\right)^2} = \frac{1}{\rho}\frac{g(\rho'-\rho)h}{U^2} \qquad (12)$$

If the Richardson's number is larger than 1, the boundary layer is stable and in the case of $R_{im} < 1$, the boundary layer is unstable and mixing between upper water and lower water is actively progressed.

5. HORIZONTAL TRANSPORT OF SALINITY IN STRATIFIED TURBULENT FLOW

The horizontal transport of salinity in the stratified flow is shown in Fig. 7. If the loss of salinity is produced in a part lower than the elevation of the boundary layer near the gate, the lost salinity will be supplied from the neighbour by the velocity of the internal wave. In general, the sudden loss of salinity makes the local deformation of equi-salinity line on the horizontal plane as Fig. 7. The discontinuous deformation of the equi-salinity line will propagate as internal wave and by this movement the transport of salinity is completed in horizontal direction. Then the value of the horizontal transport of salinity is calculated by tne next equation:

$$S = (S'-S)(h'-h_2')\sqrt{\frac{gh_2'h'}{h+h'}\frac{\rho'-\rho}{\rho}} \qquad (13)$$

If the out flow discharge is very large, the internal equi-salinity line would be much deformed near the gate surface, but the water surface would not be so deformed. In this mechanism of horizontal transport of salinity, fluctuation velocity is not important and the transport energy is supplied by density difference in the water. The critical transport velocity of salinity is given by the next equation:

$$S = (S'-S)h'\sqrt{\frac{gh'}{\rho}\frac{\rho'-\rho}{\rho}} \qquad (14)$$

But these phenomena are only seen when there is small discharge. In the case of large discharge, the upper layer would be mixed with lower water.

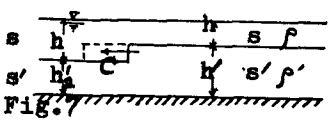


Fig. 7

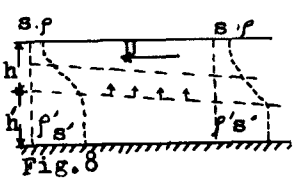


Fig. 8

6. VERTICAL TRANSPORT OF MOMENTUM IN STRATIFIED TURBULENT FLOW

The vertical eddy viscosity in stratified turbulent flow is a function of Richardson's number. η_0 is eddy viscosity when there is no stratified turbulent flow, but the eddy viscosity η would be changed by salinity distribution as the next equation[8] shows:

$$\eta = \eta_0 (1 + e Ri)^{-1/2} \qquad (15)$$

Namely the vertical transport of momentum is given as follows in unit time and unit area:

$$\tau = \eta \frac{\partial u}{\partial z} = \eta_0 (1 + e Ri)^{-1/2} \frac{\partial u}{\partial z} \qquad (16)$$

Fig. 9 shows the above relation coincides with experimental results. The value of index$(-\frac{1}{2})$ and e are decided by experiment. The turbulent diffusion coefficient in no salinity flow is shown as follows:

$$\eta_0 = 1.8 \times 10^4 \text{ cm}^2/\text{s}$$
$$\eta_0 = 1.2 \cdot 10^4 \text{ cm}^2/\text{s} \quad \text{(Okuda)} \qquad (17)$$

At the vertical section;

$$= 0.01 L^{4/3} \text{ cm}^2/\text{s} \quad \text{(H.Stommel)} \qquad (18)$$

But we can estimate the eddy viscosity from the **folllow equations;**

$$\eta_0 = \frac{K h \sqrt{g h i}}{6} \qquad (19) \quad \text{(by water flow)}$$

$$\eta_0 = 1.02 W^3 \qquad W < 6 \text{ m/s} \quad \text{(Thordde, 1914)}$$

$$\eta_0 = 4.3 W^2 \qquad W > 6 \text{ m/s} \qquad (20) \quad \text{(by wind velocity)}$$
(Ekman, 1905)

By Munk's ,by other's and by the author's experiments, the following value is deduced.

$$e = 8 - 13 \qquad (21)$$

But in this paper, the author used the modified Richardson's number expressed as equation (7) and (10). If the modified Richardson's number is used, the transport of momentum is given as follows:

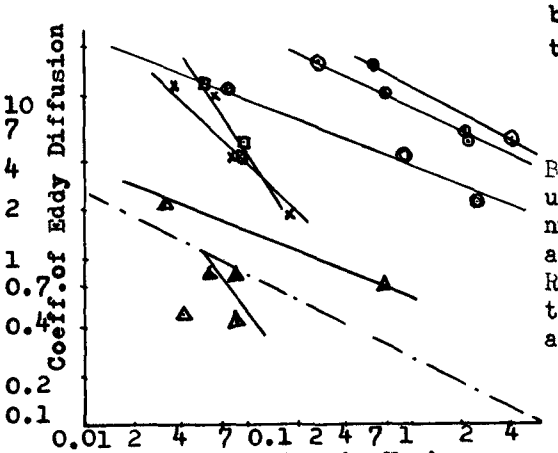


Fig.9

$$\tau = \eta_0 (1 + e' R_{im})^{-\frac{1}{2}} \frac{\partial u}{\partial z} \quad [8] \qquad (22)$$

7. VERTICAL TRANSPORT OF SALINITY IN THE TURBULENT FLOW

The coefficient of vertical eddy diffusion and diffused salinity is also a function of Richardson's number and Fig. 10 shows that the tendency of equation (23) coincides with experimental results.

$$\eta_s = \eta_0 (1 + b R_i)^{-3/2} \quad [8] \qquad (23)$$

(i) by flowing water

If we use the modified Richardson's number, the vertical transport of salinity in stratified turbulent flow is shown as follows:

$$S = \eta_s \frac{dS}{dz} = \eta_0 (1 + b_m R_{im})^{-3/2} \frac{\partial S}{\partial z} \quad [8] \qquad (24)$$

In upper equations, the writer used the modified Richardson's number instead of Ri and the constant bm is decided by the experiment.

$$b_m \fallingdotseq 10 \qquad (25)$$

(ii) by wind action

$$S = \eta_0 (1 + b_{mw} R_{im})^{-3/2} \frac{\partial S}{\partial z} \qquad (26)$$

In equation (26), the modified Richardson's number must be used.

$b_{mw} \fallingdotseq 10000$

This is deduced from field survey of Nakaumi.[7] Then the value of η_0 was calculated by Thordde[3] equation.

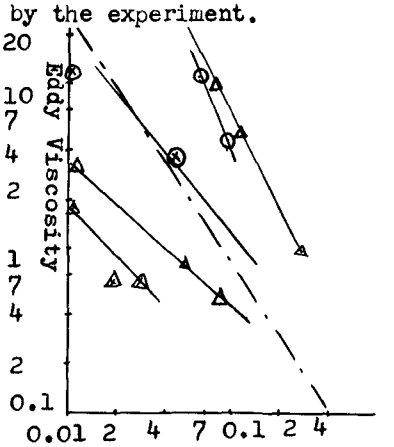


Fig.10 Modified Richardson'

8. THE CALCULATION OF LONGITUDINAL ELEVATION OF THE BOUNDARY LAYER

To know the vertical salinity distribution, the elevation of the boundary must be calculated for all discharge. In hydraulics, the shape of that boundary is called the salt wedge. The elevation of the salt wedge is a very important factor in understanding the progress of fresh water from salt water. The salt wedge may be calculated by following equations: The following equations are deduced by Prof. Dr. Schönfeld.[5]

The momentum equation are

$$-i+\frac{\partial h}{\partial x}+\frac{\partial h'}{\partial x}+\alpha\frac{\partial}{\partial x}\left(\frac{u^2}{2g}\right)+\frac{1}{g}\frac{\partial u}{\partial t}+i_f=0$$
$$-i'+(1-\varepsilon)\frac{\partial h}{\partial x}+\frac{\partial h'}{\partial x}+\alpha\frac{\partial}{\partial x}\left(\frac{u^2}{2g}\right)+\frac{1}{g}\frac{\partial u'}{\partial t}+i_f'=0 \quad (27)$$

The continious equations of discharge are

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+h\frac{\partial u}{\partial x}=0$$
$$\frac{\partial h'}{\partial t}+u'\frac{\partial h'}{\partial x}+h'\frac{\partial u'}{\partial x}=0 \quad (28)$$

These equations are modified as follows:

$$\frac{dh}{dx}=\frac{1}{\phi(\omega)}\left[\left(1-\frac{u'|u'|}{gh'}\right)\left\{i-\frac{f_i}{2gh}(u-u')|u-u'|\right\}-\left\{i-\frac{f_i'}{2gh'}u'|u'|-\frac{f_2'}{2gh'}(u-u')|u-u'|\right\}\right]$$
$$\frac{dh'}{dx}=\frac{1}{\phi(\omega)}\left[\left(1-\frac{u|u|}{gh'}\right)\left\{i-\frac{f_2'}{2gh'}u|u|+\frac{f_i'}{2gh'}(u-u')|u-u'|\right\}-(1-\varepsilon)\left\{i-\frac{f_i}{2gh}(u-u')|u-u'|\right\}\right] \quad (29)$$

$$\phi(\omega)=\frac{u^2}{gh}\frac{u'^2}{gh'}-\frac{u^2}{gh}-\frac{u'^2}{gh'}+\varepsilon \quad (30)$$

If we assume the values for salt wedge as follows:

$$u'=0, \qquad f_i=0.01$$
$$\varepsilon=\frac{\rho'-\rho}{\rho}=0.025 \quad (31)$$

We can write equation (29) as follows.

$$\frac{dh}{dx}=f$$
$$\frac{dh}{dx}=g \quad (32)$$

In the equations (32) the values of f and g are calculated for many values of depth of fresh water, depth of salt water and velocity of upper fresh water. Table 2. shows the value of f and g. By the Table 2, we can calculate the salt wedge and the elevation of water surface as follows.

(i) The simplest calculation method

$$x=x_0+\Delta x$$
$$h=h_0+\Delta h=h_0+f\cdot\Delta x$$
$$h'=h_0'+\Delta h'=h_0'+g\cdot\Delta x \quad (33)$$

Table 2 The values of f and g .

h	h	u	i=0.00005		i=0.0001		i=0.000133		i=0.0002	
			f_i=0.01		f_i=0.01		f_i=0.01		f_i=0.01	
m	m	m/s	f	g	f	g	f	g	f	g
2.5	6	1.5	+000973	-0.00393	+0 00973	-0 00387	+0.00973	-0.00392	+0 00973	-0.00391
		1.1	+0.01434	-0 00576	+0 01434	-0.00581	T0 01434	-0 00576	+0 01434	-0.00576
		0.8	+0 16485	-0.06675	+0.16485	-0.07053	+0.16485	-0.06675	+0.16485	-0.06675
		0.5	-0 00489	+0 00207	-0 00489	+0 00249	-0 00489	+0.00212	-0 00487	+0.00221
		0.3	-0 00122	+0 00059	-0.00122	+0.00092	-0 00122	+0.00059	-0 00122	+0 00059
		0.05	-0.000029	+0.00011	-0.000029	+0.00041	-0.000029	+0 00014	-0.000029	+0.00021
2.5	5	1.5	+001031	-0.00338	+0.01031	-0 00329	+0 01031	-0.00338	+0.01031	-0.00336
		1.1	+001519	-0.00494	+0 01519	-0 00494	+0 01519	-0 00494	+0 01518	-0 00499
		0.8	+0 17455	-0.05710	+0 17455	-0.06029	+0 17454	-0 05709	+0 17454	-0 05790
		0.5	-0 00517	+0 00178	-0 00517	+0.00218	-0 00519	+0 00183	-0 00517	+0.00191
		0.3	-0.00129	-0 00052	-0 00129	+0 00084	-0.00129	+0.00052	-0.00127	+0 00052
		0.05	-0.000031	+0.00011	-0.000031	+0.00041	-0.000031	+0 00014	-0 000031	+0.00021
2.5	4	1.5	+0.01116	-0 00260	+0.01116	-0 00296	+0 01116	-0 00259	+0.01116	-0 00255
		1.1	+001645	-0 00372	+0.01645	-0 00364	+0.01645	-0.00372	+0 01645	-0.00372
		0.8	+0 18909	-0.04269	+0.18909	-0 04500	+0.18909	-0.04269	+0.18909	-0 04268
		0.5	-0 00560	+0 00135	-0.00560	+0 00172	-0 00560	+0.00139	-0 05600	+0.00147
		0.3	-0.00139	+0 00041	-0 00139	+0.000 73	-0.00140	+0.00041	-0.00139	+0.00041
		0.05	-0 000033	+0.000011	-0.000033	+0.00041	-0.000033	+0.00014	-0.000033	+0.00021
2.5	3	1.5	+0 01259	-0.00137	+0 01259	-0 00133	+0.01257	-0.00131	+0.01250	-0 00126
		1.1	+0 01856	-0.00172	+0 01856	-0 00152	+0.01856	-0.00172	+0 01856	-0.00172
		0.8	+0 21333	-0.01888	+0.21333	-0.01974	+0.21333	-0.01888	+0.21333	-0 01888
		0.5	-0.00632	+0.00057	-0 00632	+0 00062	-0.00632	+0.00066	-0.00632	+0.00074
		0.3	-0 00158	+0.00023	-0 00157	+0.00054	-0.00158	+0.00023	-0.00157	+0.00023
		0.05	-0.000038	+0.000053	-0.000038	+0.00010	-0.000038	+0.00014	-0.000037	+0.00020
2.5	2	1.5	+0.01546	+0.00097	+0.01546	+0.00104	+0.01546	+0.00108	+0.01546	+0.00117
		1.1	+0 02278	+0 00215	+0.02278	+0 00261	+0 02278	+0.00215	+0.02278	+0.00215
		0.8	+0 26182	+0.02793	+0.26182	+0 02998	+0.26182	+0.02793	+0 26182	+0.02990
		0.5	-0.00776	-0.00085	-0 00776	-0 00081	-0.00776	-0.00078	-0.00776	-0.00093
		0.3	-0 00194	-0 00019	-0.00194	+0 00015	-0.00194	-0.00014	-0.00194	-0.00014
		0.05	-0.000046	+0.000054	-0.000046	+0.00009	-0.000046	+0.00013	-0.000046	+0.00019
2.5	1	1.5	+0.02405	+0.00669	+0.02405	+0.00689	+0.02405	+0.00694	+0.02405	+0.00705
		1.1	+0.03544	+0.01272	+0.03544	+0.01393	+0.03544	+0.01272	+0.03544	+0.01272
		0.8	+0.40727	+0.16205	+0 40727	+0.17282	+0.40727	+0.16205	+0.40727	+0.16205
		0.5	-0.01207	-0.00504	-0.01207	-0.00504	-0.01207	-0.00504	-0.01207	-0.00505
		0.3	-0.00301	-0.00122	-0.00301	-0 00099	-0.00301	-0.00122	-0.00301	-0.00122
		0.05	-0.000072	+0.000018	-0.000072	+0.000068	-0.000072	+0.00010	-0.000072	+0.00017

(ii) By Runge - Kutter's method

$$x = x_0 + \Delta x, \qquad h = h_0 + k, \qquad h' = h_0' + l \tag{34}$$

$$k = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$\begin{cases} k_1 = f(x_0, h_0, h_0')\Delta x \\ k_2 = f(x_0 + \frac{\Delta x}{2}, h_0 + \frac{k_1}{2}, h_0' + \frac{l_1}{2})\Delta x \\ k_3 = f(x_0 + \frac{\Delta x}{2}, h_0 + \frac{k_2}{2}, h_0' + \frac{l_2}{2})\Delta x \\ k_4 = f(x_0 + \Delta x, h_0 + k_3, h_0' + l_3)\Delta x \end{cases}$$

$$l = \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

$$\begin{cases} l_1 = g(x_0, h_0, h_0')\Delta x \\ l_2 = g(x_0 + \frac{\Delta x}{2}, h_0 + \frac{k_1}{2}, h_0' + \frac{l_1}{2})\Delta x \\ l_3 = g(x_0 + \frac{\Delta x}{2}, h_0 + \frac{k_2}{2}, h_0' + \frac{l_2}{2})\Delta x \\ l_4 = g(x_0 + \Delta x, h_0 + k_3, h_0' + l_3)\Delta x \end{cases}$$

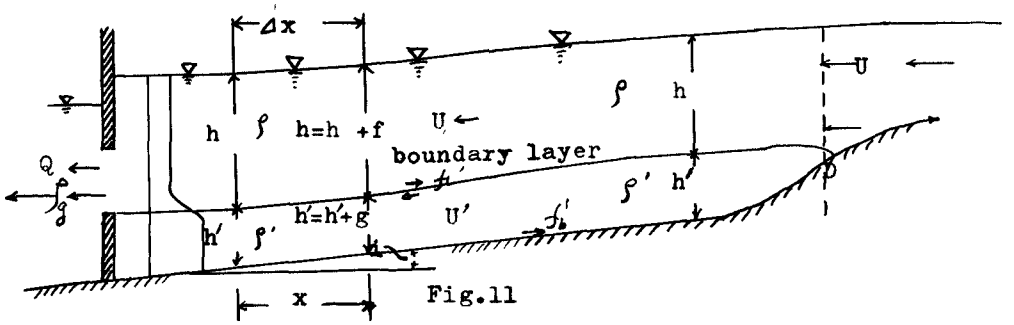


Fig.11

In the place furtner from the salt wedge front D in Fig. 11, the flowing water will directly contact the soil surface and the salinity has vertically constant value. The point D is given by the front of the salt wedge. But in the case of small water velocity of the upper water, the elevation of point D is nearly equal to the elevation of the sill surface of the gate. But in actually, the wind velocity is also an important factor in deciding the elevation of the boundary layers.

9. THE EQUATION OF THE CHANGE OF SALINITY OF THE LAKE IN THE ESTUARY

The fundamental differential equation is given by Prof. Jansen.[2] But in this equation the value of the ratio r between outflow salinity and original salinity is not exactly considered.

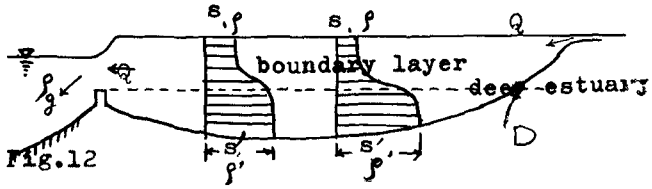


Fig.12

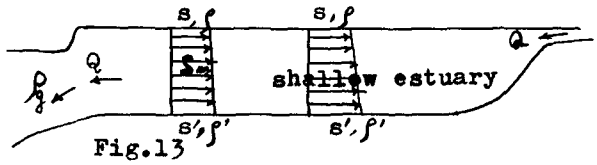


Fig.13

In the case of r=const., the solution is given by Prof. Jansen's equation as follows:

$$S = \frac{a}{r} + (S_0 - \frac{a}{r})\, e^{-\frac{rQ}{U}t} \qquad (35)$$

$$r = \frac{S}{Sm}.$$

After these studies Prof. Okuda[1] assumed the next equation for the value of r, $r = 1 - k\,(Sm - a)$

(36)

and the solution of the salinity in the estuary is given as follows:

$$S = \frac{\frac{c}{P} - a\, e^{\frac{PQ}{U}t}}{c - e^{\frac{PQ}{U}t}} \qquad (37)$$

where

$$P = 1 - ka$$

$$c = \frac{kS_0 - ka}{kS_0 - 1} \qquad (38)$$

In Fig. 12 and Fig. 13, we can understand that these equations are only effective for the shallow lake as Fig. 13. When the elevation of the sill of the gate is higher than the bed of lake, the equations (35) and (37) are not so effective.

The above two equations are not so precise because of following reasons:

(i) There are no equations to calculate the value of r.

(ii) They neglect the density effect against the eddy diffusion coefficient. The vertical salinity distribution changes the modified Richardson's number and this also changes the diffusion of salinity.

(iii) They neglect the vertical diffusion from the lower salty water.

(iv) They don't consider the effectiveness of the structure of the outlet gate and its control.

10. LONGITUDINAL SALINITY DISTRIBUTION IN THE UPPER WATER BEING ON THE BOUNDARY LAYERS

The salinity mixed the upper fresh water may be calculated by the equations below: In this case the coefficient of vertical turbulent diffusion is calculated by the next equations.

$$\eta = \eta_0 (1 + a_m R_{im})^{-1/2} \tag{39}$$

$$\eta_s = \eta_0 (1 + b_m R_{im})^{-3/2} \tag{40}$$

where

$$R_{im} = \frac{g(\rho' - \rho)h}{u^2} \quad \text{(for water flow)} \tag{41}$$

$$R_{im} = \frac{C_f(\rho' - \rho)gh}{\tau_a} \quad \text{(for wind action)} \tag{42}.$$

Because the shape of vertical salinity distribution has the same form below the water surface in the lake, the above equations would have practical use. Namely, the fresh water is sitting on the boundary layer between fresh water and salt water. In the part below the boundary layer, there is salt water and its salinity is an almost constant value.

(1) AT THE SALINITY MIXED IN FRESH WATER FROM LOWER SALT WATER

If there is a flow of upper fresh water, salinity is diffused from the lower salt part only very slightly. Then the salinity of the upper water is changed from the mouth of the river to the outlet gate as Fig. 14,

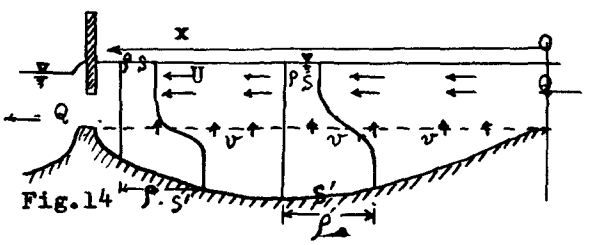


Fig.14

The differencial equations of the change of salinity are led as follows:

$$u\frac{ds}{dx} = \eta_s \frac{d^2S}{dx^2} + \frac{\eta_s}{h}\left(\frac{ds}{dz}\right) \qquad (43)$$

$$u\frac{ds}{dx} = \eta_0 (1 + b_m R_{im})^{-3/2} \frac{d^2S}{dx^2} + \frac{\eta_s}{h^2}(S'-S) \qquad (44)$$

The boundary equations are:

$$\begin{aligned} x &= 0, \quad S = 0 \\ x &= l, \quad S = S_0 \end{aligned} \qquad (45)$$

From the upper boundary condition, we can deduce the next equation of salinity.

$$S = S_0\left\{1 - \frac{e^{\alpha x}\sin h\beta(l-x)}{\sin h\beta l}\right\} \qquad (46)$$

where,

$$\alpha = \frac{u}{2\eta_s}, \qquad \beta = \frac{\sqrt{u^2 + 4\eta_s^2/h}}{2\eta_s} \qquad (47)$$

(2) FUNDAMENTAL DIFFUSION EQUATION FROM SOIL

The fundamental differencial equation is led as follows:

If these conditions occur, that is, the mean velocity U is const., mean water depth h is const., and turbulent coefficient of diffusion η is constant, then, the equation of diffusion of salinity is given as follows by Okuda[1]

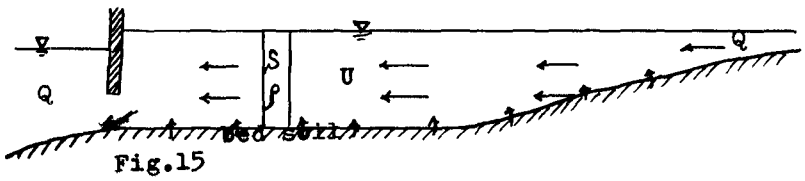


Fig.15

$$u\frac{ds}{dx} = \eta_s \frac{d^2s}{dx^2} + \frac{k}{h^2}(s'-s) \qquad (48)$$

The boundary condition of differential equations are shown as follows.

$$\begin{aligned} x &= 0, \quad s = 0 \\ x &= \ell, \quad s = s_0 \end{aligned} \qquad (49)$$

By the boundary condition, we can deduce the solution as follows:

$$s = s_0\left\{1 - \frac{e^{\alpha x}\sinh\beta(\ell-x)}{\sinh\beta\ell}\right\} \qquad (50)$$

where

$$\alpha = \frac{u}{2\eta_s} \qquad (51) \qquad \beta = \frac{\sqrt{u^2+4\eta_s k/h}}{2\eta_s} \qquad (52)$$

11. THE CHANGE OF THE SALINITY IN DEEP ESTUARY NEAR THE OUTLET GATE

If the outlet gate is open at the lower part, and the modified Richardson's Number in the lake is larger than 1, the required time to replace old salt water with fresh water is calculated as follows.

1st step

The outflow salinity through the gate is calculated by equation (5).

$$\Delta s \sim \rho_g Q = \Delta V \cdot \Delta\rho$$
$$\Delta V = h \cdot A_m \qquad (53)$$

We can assume the following important facts by the field survey and experiments of salinity distribution:

(i) The boundary layer exists on the same level as the sill of gate. (at the start of calculation)

(ii) The decrease of salinity happens only in the upper region of the boundary layer in the deep estuary.

If the vertical salinity distribution and discharge from the outlet of the gate are given, the outflow salinity mixed in the discharge is calculated by the equation (5).

The decreased salinity of the lake is given as follows:

$$\rho - \Delta\rho = \frac{\rho_g Q}{h \cdot A_m} \qquad (54)$$

The sunk depth of the boundary layer is calculated by the next equation:

$$\Delta h' = \frac{\Delta \rho \cdot h}{\rho'} \qquad (55)$$

2nd step

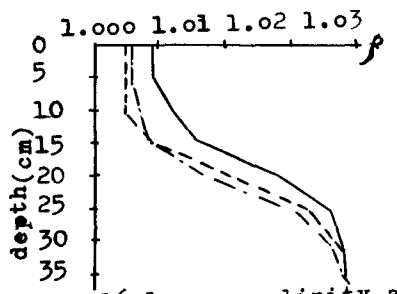
We must modify the salinity distribution of the upper layer by the following method:

$\rho = \rho + \Delta\rho$, $\rho' = \rho'$

But if the elevation of the boundary layer is lower than the elevation of the sill surface, the escape of salinity from the estuary is very difficult. This calculation method was verified by the following experiment as Fig. 16:

3rd step

We must modify the salinity distribution by wind action by equation (26).



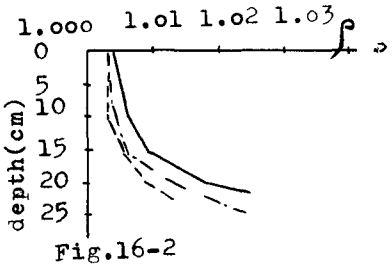
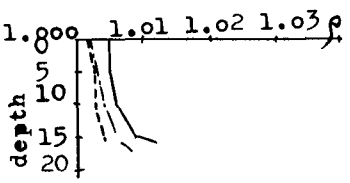


Fig.16-2

Fig.16-1 -----salinity after 60 min.
-.-.-.salinity after 30 min.
——— original salinity distribution



4th step

We must calculate the salinity in the discharge by equation (5) and return to the 1st step.

12. MODEL TEST OF ISAHAYA SEA RECLAMATION PLAN

The author made the model of Isahaya bay in Fig. 21. The bay opens to Nagasaki bay through the gate. The hydraulics factors of the model are as follows:

Mean sea water elevation	± 0 cm
Amplitude of water surface	± 3.5 cm
Salinity in Nagasaki Bay	$\rho' = 1.025$
The elevation of sill of the outlet gate	-18 cm
The effective total width of the outlet gate	10 cm

The structure of the outlet gate and V-H, A-H curve of Nagasaki bay are given by Fig. 18 and Fig. 21. Fig. 16 shows the changing process of vertical salinity distribution in the pool inner the embankment. In this experiment, the discharge from the outlet gate was 200 cm^3/s. Before the reclamation works in Isahaya bay, the distribution of salinity had the same value in vertical directions. This shows that there was intense mixing by wind friction on the sea surface and tidal actions of total amplitude of 3 m. But after the reclamation works, the wave in the pool would be very small and the vertical distribution of salinity would abruptly change at the same elevation as the sill. The upper fresh water would have a density about $\rho = 1.002$ and the salt water would have a density about $\rho' = 1.020$.

To chase out the salinity by the outlet gate, we must constantly operate the gate on the condition that the difference of water surface between outer sea and inner pool has the value of equation (1). In this case the salinity distribution in the pool will change as Fig. 16 and has stratified salinity distribution. In this case the boundary line between fresh water and salt water has the elevation nearly equal to the height of the sill of the outlet gate. This is first step for making the change in salinity. In the 2nd step, the salinity lower than the spring layer or in the bed soil would be transported by mixing with fresh water flow. If the strong wind blows on the sea, the salinity of the fresh water will increase and the depth of the spring layer will be deep.

In the sea reclamation, the most important factor is to make the elevation of the boundary layer deep. The method to make the boundary layer deep is shown as follows:

(a) The gate must work to flow out the flooding discharge.

(b) The gate must work to flow out the salinity from the deepest part of the gate on the embankment. To accomplish the above, we must open the lowest part of the gate. As an example in Fig. 17, it is best to make the one deep gate for chase out the high salinity water.

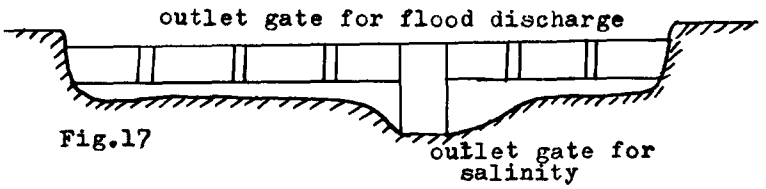


Fig.17

If the elevation of the boundary layer is sufficiently deep, at the strong wind or flooding flow, we can constantly gain fresh water from the lake.

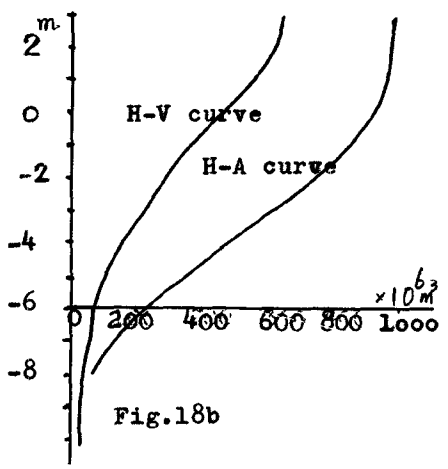


Fig.18b

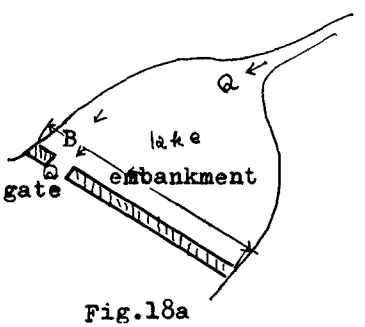


Fig.18a

13. EXAMPLE

Initial conditions

$h = 8\ m$, $\rho = 1.025$, $B = 300\ m$, $\Delta t = 0.432 \times 10^6\ sec$,
$h' = 4\ m$, $\rho' = 1.025$, $C = 0.8$, $Q = 580.8\ m^3/s$,
$c = 4.25\ m$, $\rho_g = 1.025$, $2g = 19.6\ m/s$, $B_l = 5000\ m$,
$a = 0.5\ m$, $\Delta h = 0.3\ m$,

1st step in calculation,

elevation of boundary layer ($\fallingdotseq$ elevation of sill) $-8.0\ m$

salinity out flow through the gate

$S = Q(\rho_g - 1) = 5808(1.025 - 1) = 14.52\ ton$

water volume existing on the boundary layer(in Fig.18)

$\Delta V = (655 - 90)\,10^6 = 565 \times 10^6\ m^3$

lost salinity from the upper region

$\Delta\rho = \frac{14.52}{565 \times 10^6}\Delta t = 0.01105$

modification of vertical salinity distribution

$\rho = 1.025 - 0.01105 = 1.01395$

$\rho' = 1.025$

water velocity of upper region

$u = Q/A = 580.8/8 \times 5000 = 0.014452\ m/s$

modified Richardson's Number

$R_{im} = \frac{(\rho' - \rho)hg}{u^2} = \frac{(1.025 - 1.01395)\,8 \times 9.8}{(0.014452)^2} = 4196$

salinity mixed into upper fresh water

$\Delta\rho = \eta_0 (1 + 10 R_{im})^{-\frac{3}{2}} \frac{(\rho' - \rho)}{h}$

$\eta_0 = \frac{\kappa h \sqrt{ghi}}{6}$

gradient of water surface(by Manning equation)

$i = \left(\frac{un}{h^{2/3}}\right)^2 = \left(\frac{0.014452 \times 0.01}{8^{2/3}}\right)^2 = 0.363 \times 10^{-8}$

$\eta_0 = \frac{0.4 \times 8 \times \sqrt{9.8 \times 8 \times 0.363 \times 10^{-8}}}{6} = 2.843 \times 10^{-4}$

$\rho = 0.000\,2843\,(1 + 10 * 4196)^{-\frac{3}{2}} \frac{0.01105}{8} \times 0.432 \times 10^6 \div 0$

$\rho = 1.01395$

$\rho' = 1.025$

mixing by the wind velocity

$\tau_a = 2.6 \times 10^3 \times 1.25 \times 10^{-3} W^2 = 11.7 \times 10^{-6}\ ton/m$

modified Richardson's Number

$R_{im} = \frac{C_4(\rho' - \rho)hg}{\tau_a} = \frac{0.002(0.01105) \times 9.8 \times 8}{11.7 \times 10^{-6}} = 14.808$

$\eta_0 = 1.02 \times 6^3 = 220.32$

$\Delta\rho = \eta_0 (1 + 10000 R_{im})^{-\frac{3}{2}} \frac{\rho' - \rho}{\rho} = 220.32\,(1 + 10000 \times 14.808)^{-\frac{3}{2}} \frac{0.01105}{\rho} \times 0.432 \times 10^6 = 0.0028$

$\rho = 1.01395 + 0.0023 = 1.01625$

$\rho' = 1.025$

$\delta h_l = \frac{0.0023 \times 8}{0.025} = 0.736\ m$

$h' = 8 + 0.736 = 8.736\ m$ (elevation of boundary layer)

2nd step in calculation

The outgoing salinity through the gate is calculated through the next equation.

$$\rho_g = \rho + 0.2793(\rho' - \rho)\left(\frac{h}{C}\right)^{-0.00101}\left(\frac{h'}{C}\right)^{0.0361}\left(\frac{a}{C}\right)^{-0.00304}\left(\frac{H^2 g}{g^2}\right)^{0.000231}$$

$H = 8+4 = 12\,m$, $C = 4.25\,m$, $h' = 3.264\,m$

$\rho' = 1.025$, $a = 0.5\,m$, $g = \frac{580.8}{300} \approx 1.936\ m^3/s/m$

$\rho = 1.01625$, $h = 8.736\,m$

$$\rho_g = 1.01625 + 0.2793(1.025 - 1.01625)\left(\frac{8.736}{4.25}\right)^{-0.00101}\left(\frac{3.264}{4.25}\right)^{0.0361}\left(\frac{0.5}{4.25}\right)^{-0.00304}\left(\frac{12^2 \times 9.8}{1.936^2}\right)^{0.000231}$$

$$= 1.01899$$

Return to the calculation of the 1st step.

14. SYMBOLS

Δh; critical difference of water surface between lake and sea,
ρ; water density of the upper region,
ρ'; water density of the lower region,
s; salinity of the upper region,
s; salinity of the lower region,
h; water depth of the upper region,
h; water depth of the lower region,
k; experimental constant,
Q; total discharge through the gate,
q; discharge through the gate of unit width,
g; acceleration of the gravity,
a; open height on the sill,
s; salinity mixed in discharge outgoing through the gate,
ρ_g; density of the water of discharge outgoing through the gate,
u; water velocity in horizontal direction,
z; vertical coordinate,
x; horizontal coordinate,
t; time,
η; eddy viscosity,
η_0; eddy viscosity which is no salinity,
η_s; eddy coefficient of diffusion,
κ; Karman's constant,
i; gradient of the water surface,
c; velocity of the internal wave,
α; constant of momentum distribution,
f_i'; resistant coefficient between upper fresh water and lower salt water,
f_b; resistant coefficient between salt water and soil,
ΔV; water volume of the lake,

r;ratio of salinity in discharge against lake,
H;total water depth of a lake,
H';the water depth of the sea,
C;discharge constant,
B;width of the outlet gate,
B_1;width of the lake,
c;height of the center of the open part ,
Cl;content of the chlorinity,
Ri;Richardson's Number,
Rim;Modified Richardson's Number,
u';velocity of salt water,
c_a;coefficient of resistance by wind velocity,
τ_a;shesring force acting on the sea surface by the wind,
ρ_a;density of the air,
L;water depth,
w;wind velocity,
e;experimental constant,
e';experimental constant (Modified Richardson's Number),
b;experimental constant,
bm;experimental const.(water flow),
bmw;experimental const.(wind),
i_f ,i'_f ;constant for resistant,
f_b ,f_b ;resistant constant,
a ;salinity ratio in flow in discharge,
S. ;original salinity in the lake,
k;molecular diffusion coefficient,
Am;area of the lake at the optional elevation,
$\Delta h'$;sunk depth of boundary layer by the vertical diffusion of salinity

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