CHAPTER 13

DIFFRACTION OF WIND GENERATED WATER WAVES

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SYNOPSIS

In designing a harbor an engineer must consider the diffraction of waves. In most studies, only uniform periodic waves coming from a single direction are treated. However, wind generated waves in the ocean are two dimensional, and the diffraction of waves due to a breakwater should be treated with this in mind. As methods of measuring the two-dimensional spectra of waves were developed recently, it was decided to determine whether or not diffraction theory could be applied with sufficient accuracy for two-dimensional wave spectra. The results of a laboratory study presented herein show that a knowledge of the two-dimensional spectra can be used together with diffraction theory to predict the energy spectra of waves in the lee of a breakwater within an accuracy that is probably acceptable for many engineering problems.

INTRODUCTION

In designing a harbor, an engineer must insure that certain regions within the harbor will be subject to waves which are less than some specified maximum height in order that the harbor will be useful. A knowledge of water wave diffraction is necessary in planning for the location of breakwaters and other harbor structures to meet this criterion.

Penny and Price (1952) showed that the Sommerfeld solution of the diffraction of light (polarized in a plane parallel to the edge of a semi-infinite screen) was also a solution to the water wave diffraction phenomenon. Putnam and Arthur (1948) performed a laboratory experiment to check the applicability of the Penny and Price solution for the case of a semi-infinite breakwater in deep water. They found that the measured wave heights agreed approximately with the heights predicted by theory in the sheltered region, but were less than the theoretical heights in the unsheltered region. Blue and Johnson (1949) performed an experiment in the laboratory with a gap in a breakwater which was oriented normal to the direction of travel of the incident waves. Tests were made using both deep water and shallow water waves. They compared the results of these experiments with their theoretical analysis, and concluded that the only modification required to use the theory for shallow water waves was that shallow water wave lengths had to be used in the equations instead of deep water wave lengths. The ratios of wave heights at any point on the lee-side of a breakwater to the incident wave heights were computed and put into graphical form by Blue (1948) and by Wiegel (1962). A laboratory study of the effect of bottom configurations within a harbor was made by Mobarek (1962); in this study there was evidence that the bottom slope had little effect on the diffraction phenomenon.

The studies cited above were made using uniform periodic waves coming from a single direction. Although wind generated waves are not regular and consist of a spectrum of short crested waves going in different directions, engineers working on design problems have treated the waves as a train of uniform periodic waves travelling in the predominant wind direction. It has also been assumed that these waves had as characteristics the significant wave height, period and length. It is now possible to replace this simplified approach by a more realistic one using the energy spectra concept (Blackman and Tuckey, 1958; Lee, 1960; Munk et al., 1959; Pierson, 1955; and Putz, 1954 among others). The original simple concepts of one-dimensional wave spectra have been extended to the concept of two-dimensional spectra. The two-dimensional spectra are obtained through the use of the co- and the quadrature spectra which are calculated from the outputs of several wave recorders (Longuet-Higgins, 1961; Chase et al., 1957; Nagata, 1964; and Mobarek, 1965).

The purpose of this paper is to describe the method by which a knowledge of two-dimensional wave spectra can be used to obtain the one-dimensional energy spectra of waves at several positions in the lee of a breakwater, and to test its validity by comparing the results of laboratory measurements with theory. An investigation was made of a two-dimensional spectrum of wind generated waves in the laboratory for a specific wind speed and fetch. In addition, the waves were measured at several locations in the lee of a breakwater, and one-dimensional spectra were calculated from these data. The linear theory of diffraction was applied to the measured two-dimensional spectrum to calculate the one-dimensional energy spectra for these locations. The two sets of one-dimensional energy spectra were compared.

It is possible to calculate the two-dimensional energy spectra at any point in the lee of a breakwater from a knowledge of the two-dimensional spectrum seaward of the breakwater, together with diffraction theory. The authors did not do this, however, as they did not know how to measure the two-dimensional spectra at different points in the lee of the breakwater as the water surface time histories are not ergodic in this region.

THEORETICAL CONSIDERATIONS

DIFFRACTION THEORY

The main assumptions in the Penny and Price solution (see Wiegel, 1966, 1964 for a summary of the theory and its verification) are:

(i) The motion is irrotational.
(ii) The wave amplitude is infinitely small.

(iii) At a fixed boundary the normal component of the orbital velocity is zero.
(iv) At the free surface, the pressure is constant.

These assumptions are the main assumptions of the linear theory of wave motion. The water surface elevation, y_s, can be expressed as:

$$y_s = \frac{Aikc}{g} e^{ikct} \cosh kd \cdot F(x,z) \tag{1}$$

where A is a constant, i is $\sqrt{-1}$, k is $2\pi/L$, L is the wave length, c is the wave speed, g is the acceleration due to gravity, d is the water depth, and x and z are the horizontal coordinates. For an infinitely thin, vertical, rigid, impermeable, semi-infinite breakwater, the Sommerfeld solution would be

$$F(x,z) = \frac{1+i}{2}\left\{e^{-ikx}\int_{-\infty}^{\sigma} e^{-\pi i u^2/2}\,du + e^{iku}\int_{-\infty}^{-\sigma'} e^{-\pi i u^2/2}\,du\right\} \tag{2a}$$

where

$$\sigma^2 = \frac{4}{L}(r-x), \quad \sigma'^2 = \frac{4}{L}(r+x), \quad r^2 = x^2 + z^2$$

and u is a dummy variable. The diffraction coefficient, K', is given by the modulus of F(x,z) for the diffracted wave:

$$K' = |F(x,z)| \tag{2b}$$

The origin of the coordinate axes is at the tip of the barrier, while x is the coordinate normal to the barrier and z is the coordinate along the barrier (Fig. 1). Eq. (2a) can be transformed into an equation expressed in terms of Fresnel integrals. A computer program (called WDIFFR) has been written by J. D. Cumming in FAP language for the alternate form of the equation, to compute diffraction coefficients (Cumming and Fan, 1966). Another computer program was written to utilize the WDIFFR program to calculate the one-dimensional energy spectrum at any specified location for a measured or assumed two-dimensional spectrum input. This program is shown in Table 1.

SPECTRA

Mobarek (1965) has shown that the covariance function R(x,y,t) is given by:

$$R(Z,X,T) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(\ell,m,f)\, e^{i2\pi(\ell Z+mX-fT)}\, d\ell\, dm\, df \tag{3}$$

where $E(\ell,m,f)$ is the two-dimensional energy spectrum, ℓ and m are the two space frequencies, f is the time frequency and X,Z are spacings between pairs of wave recorders along the x and y axes and T is an increment of time. The autocovariance function can be defined as:

TABLE 1

DIFFRACTED TWO-DIMENSIONAL SPECTRUM

```
      DIMENSION E(5,9),ANG(5),FREQ(9),AL(9),X(4),Y(4),WA(4,5,9),
     1 WASQ(4,5,9),SUM(4,9),DIFEN(4,5,9)
      READ 10,((E(I,J),I=1,5),J=1,9)
      PRINT 10,((E(I,J),I=1,5),J=1,9)
      ANG(1)=140.
      ANG(2)=110.
      ANG(3)=80.
      ANG(4)=50.
      ANG(5)=20.
   10 FORMAT(5F6.3)
      DO 20 K=1,9
      FREQ(K)=1.6+((FLOAIF(K-1))/10.)
   20 AL(K)=(32.2/6.283)*(1./(FREQ(K)**2))
      X(1)=1.5
      X(2)=0.5
      X(3)=0.5
      X(4)=1.5
      Y(1)=2.5
      Y(2)=2.5
      Y(3)=1.0
      Y(4)=1.0
      DO 50 I=1,4
      DO 40 K=1,9
      SUM(I,K)=0.0
      DO 30 J=1,5
      CALL WDIFFR(AL(K),ANG(J),X(I),Y(I),WA(I,J,K))
      PRINT11,AL(K),ANG(J),X(I),Y(I),FREQ(K),WA(I,J,K)
   11 FORMAT(6F15.5)
      WASQ(I,J,K)=WA(I,J,K)**2
      DIFEN(I,J,K)=WASQ(I,J,K)*E(J,K)
   30 SUM(I,K)=SUM(I,K)+DIFEN(I,J,K)
   40 CONTINUE
   50 CONTINUE
      PRINT 60,((SUM(I,K),K=1,9),I=1,4)
   60 FORMAT(1HO,14HDIFFRAC.WAVES,9E12.4/(12X9E12.4))
      DO 90 K=1,9
      DO 80 J=1,5
      PRINT 70,(WA(I,J,K),I=1,4)
   70 FORMAT(1HO,12HDIFFRAC.COEF,4E20.8/(12X4E20.8))
   80 CONTINUE
   90 CONTINUE
      CALL EXIT
      END(1,1,0,0,0,0,1,1,0,0,0,0,0,0,0)
```

$$R(0,0,T) = R(T) = \int_{-\infty}^{\infty} E(f)\, e^{-i2\pi fT}\, df \tag{4}$$

where

$$E(f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\ell,m,f)\, d\ell\, dm \tag{5}$$

Eq. 3 can be written as

$$R(Z,X,T) = \int_{-\infty}^{\infty} \left[C(Z,X,f) + i\, Q(Z,X,f)\right] e^{-i2\pi fT}\, df \tag{6}$$

C and Q are the co-spectrum and the quadrature spectra, respectively; they are given by

$$C(Z,X,f) + i\, Q(Z,X,f) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(\ell,m,f)\, e^{i2\pi(\ell Z + mX)}\, d\ell\, dm \tag{7}$$

In the solution described herein, it is assumed that the energy $E(f,\theta)$, in polar coordinates, can be treated as if it were concentrated in a finite number of directions $\theta_1, \theta_2, \ldots\ldots \theta_{\bar{r}}$. The amount of energy in the directions $\theta_1, \theta_2, \ldots\ldots \theta_{\bar{r}}$ is designated as $a_1, a_2, \ldots\ldots a_{\bar{r}}$, respectively. Eq. 7 can be rewritten as

$$C_n(f) + iQ_n(f) = \sum_{h=1}^{\bar{r}} a_h \exp\left[i2\pi\bar{k}D_n \cos(\theta_n - \theta_h)\right] \tag{8}$$

$$= \sum_{h=1}^{\bar{r}} a_h\, S_{n,h} \tag{9}$$

where $S_{n,h} = \exp\left[i2\pi\bar{k}D_n \cos(\theta_n - \theta_h)\right]$

- h = a particular direction of the wave
- n = a particular probe spacing
- $\bar{k}$ = $1/L$
- k = the wave number, $2\pi/L$
- D = the spacing between any two probes
- $\bar{r}$ = the number of wave directions considered

It is almost impossible to get an accurate estimate of $\bar{r}$ unknowns from $\bar{r}$ pieces of experimental information. Therefore, if j is the number of wave directions, less than $\bar{r}$, then the $\bar{r}$ equations can be solved by the linear least squares technique to calculate the values of the j unknowns. This has been discussed by Mobarek (1965) in detail, and will not be repeated herein. The following equation was given by Mobarek:

$$\sum_{h=1}^{j} \left(a_h \sum_{h=1}^{n'} S_{n,s} \cdot S_{n,h}\right) = \sum_{n=1}^{n'} R_n \cdot S_{n,s} \tag{10}$$

where $R_n = C_n + i\, Q_n$

n' = the number of probe spacings

A computer program was written to utilize the Bell Laboratories Subroutine BE-GI-TISR in order to calculate E(f) from Eq. 4. Also, a computer program was written to solve Eq. 10 and to determine the values of $E(f,\theta)$ for the two-dimensional spectrum (Mobarek, 1965). Then, the main program, mentioned before, utilizing the Subroutine, WDIFFR was applied to the two-dimensional spectrum to determine the energy spectra at several locations in the lee of the breakwater.

EXPERIMENTAL SET-UP

Experiments were performed at the Hydraulic Engineering Laboratory of the University of California, Berkeley. Wind waves were generated on the water surface in a wind-wave tunnel about 60 feet long and 12 feet wide with the water 1.1 feet deep (Fig. 2). The wind for generating the waves was produced by five blowers mounted in parallel at one end of the tunnel. A plenum chamber was installed between the blowers and the tunnel so that there would not be five jets of air blowing over the water surface. The clearance between the water surface and the top of the tunnel was 4.0 feet. The wind wave tunnel was located in one corner of a model basin 64 feet wide by 150 feet long by 2-1/2 feet deep. This permitted the waves to travel into a fairly large area after they had left the generating area. The measurements were made in the area in which the waves were "free" waves, rather than in the generating area where they were being forced by the wind. The waves finally ran onto a beach where their energy was dissipated.

An array of four wave gages arranged in a star shape was used (Fig. 2) to measure the two-dimensional spectrum. Each gage consisted of two half round shaped stainless steel wires glued together by an epoxy which had a very high electrical resistance (greater than 5.0 meg-ohms). The two halves, joined together, formed a "parallel-wire type" wave gage about an eighth of an inch in diameter. Thus, although the gages were of the parallel type, each gage measured the water surface time history at essentially a point. These gages were designed specifically for this facility by J. D. Cumming and R. L. Wiegel.

Run No. 1

Fig. 3 is a sketch of the experimental set-up used for Run No. 1. The wave gages placed at points 1, 2, 3 and 4, shown in the figure, were of the same type as were used for measuring the two-dimensional spectrum. The "semi-infinite" breakwater consisted of a sheet of plywood 2.0 feet high by 6.0 feet long by 1/2 inch thick.

The blowers were turned on, and after the waves reached their equilibrium condition for this particular fetch length, the recorder was started. Waves were recorded at eight locations (the four "star" array plus the four that would be in the lee of the breakwater when it was installed) without the breakwater in place. After recording the waves for a sufficient length of time, the recorder was stopped. The blowers were left running so that the wind continued to blow over the water surface. The breakwater was then placed in the desired location, and after the disturbance due to the barrier placement had been dissipated, the oscillograph was started again and the outputs of the four gages in the lee of the barrier were recorded. The measurements of the diffracted waves were made for an interval of about four minutes. Three thousand ordinates were read from the wave record at a 0.05 second interval (that is, for a 2-1/2 minute interval of the 4-minute record).

Run No. 2

Owing to reasons that will be discussed in a later section of this paper, it was throught best to change the location of the barrier. Fig. 4 shows a sketch of the experimental set-up for Run No. 2. The barrier was transferred to the opposite side of the tank and placed at the end of the tunnel side. The four gages were rearranged so that their positions would be similar to those used in Run No. 1 with respect to the breakwater. The experimental procedure used in Run No. 1 was also used in Run No. 2.

RESULTS AND DISCUSSION

As an example of the estimation of the two-dimensional spectrum, which was generated by the wind, some of the results of the earlier work of Mobarek (1965) are shown in Figs. 5-8. One of the main features of this two-dimensional spectrum is the deviation of the direction of the peak energy from the main wind direction. This deviation varies between 10^o and 20^o, depending on the frequency being considered. It was concluded by Mobarek that this deviation was due to artificial factors pertaining to the local conditions of the model basin, which was the same one as used in this study.

The two-dimensional spectrum was treated as if the waves were traveling in only five directions, 40^o, 70^o, 100^o, 130^o, and 160^o. The energy associated with the waves treated as if they were moving in each of these discrete directions was the sum of the energy of the waves moving in a continuous increment of directions on either side of the direction being considered. The calculations were repeated for each frequency band. The problem was reduced to a multi-calculation procedure. Diffraction theory was then applied to each of these "wave trains" in order to calculate the energy level at each of the locations in the lee of the breakwater for which measurements had been made. This was done for each wave frequency and direction, as if there was only this particular wave train present. Then, the linear sum of all wave energies pertaining to each frequency was calculated for each location, forming one point of the one-dimensional energy spectrum for this location. This process was repeated for all frequencies and the frequency spectrum, referred to herein as the predicted frequency spectrum, was obtained for each of the four locations in the lee of the barrier (Figs. 3 and 4). A two-dimensional spectrum cannot be obtained from measurements made in the lee of the barrier as the process is not ergodic there; that is, the time averages of records taken at different locations (different x and z) are different owing to the diffraction phenomenon.

The main assumptions that were made in the development of the process described above were that the wave system and the diffraction were linear.

Fig. 9 shows the diffracted wave energy for each frequency as calculated using diffraction theory, together with the measured values obtained from Run No. 1 (see also Table 2). Three observations can be made from the data shown in these figures. First, the total amount of energy contained in the wave system in the lee of the breakwater is about the same for the calculated as for the measured diffracted waves. Second, the peak energy is about the same for both the calculated and measured waves. Third, the peak energy occurs at a lower frequency for the spectra calculated from the measurements made in the lee of the breakwater than occurs for the spectra predicted from the two-dimensional spectrum using diffraction theory. It appears that an amount of energy has been transferred to the lower frequencies by some mechanism.

It was thought that the shift in energy might be due to the vibration of the wooden plate used in the experiments to simulate a breakwater. Measurements showed that its natural frequency was about 1.8 to 2.0 cycles/second. This, as well as the statistical reliability, may also account for some of the slightly increased peak energy measured at the gages closest to the breakwater (Figs. 9c and 9d). Reflections from the side walls of the wind-wave tunnel near the breakwater may also have affected the redistribution of energy with respect to frequency.

Fig. 10 shows both the measured and calculated diffracted wave energy for each frequency, for Run No. 2 (see also Table 3). The data in these figures show an increase in the total amount of diffracted wave energy in the measured spectra compared with the predicted spectra.

This might be explained to some extent by reference to Figs. 4 and 5. From these figures one can see that the two-dimensional spectrum is shifted to the right of the wind "direction," rather than being symmetrical about the main wind direction as discussed before. Taking this into consideration, together with the experimental set-up for Run No. 2, it can be seen that a certain amount of energy will be reflected from the right wall (with one's face to the wind) of the wind-wave tunnel, which might cause some of the increase. In addition to this, some energy is produced by the vibrations of the wooden plate (breakwater), as described for Run 1. Finally, the confidence limits for the measured spectra were between 0.7 and 1.5 times the measured value (Mobarek, 1965).

Another possibility is that, due to multiple reflections, more energy may be ultimately directed into the lee of the breakwater.

Finally, there may be another explanation. Considering the "infant" state in regard to calculating two-dimensional wave spectra from a star array, the difficulty may be that the two-dimensional spectra is not of sufficient accuracy.

TABLE 2a

MEASURED DIFFRACTED WAVE SPECTRAL ENERGY x 10^5 (ft^2)

RUN NO. 1

Freq. c/sec.	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
gage 1	3.3	4.5	4.25	3.3	2.5	1.38	0.75	0.42	0.35
gage 2	1.25	2.6	3.4	2.6	1.4	1.4	0.91	0.85	0.50
gage 3	1.5	2.2	1.5	0.6	0.5	0.35	0.25	0.30	0.10
gage 4	3.2	4.22	3.48	2.12	1.32	0.99	0.75	0.51	0.37

TABLE 2b

THEORETICALLY CALCULATED DIFFRACTED WAVE SPECTRAL ENERGY x 10^5 (ft^2)

RUN NO. 1

Freq. c/sec.	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
gage 1	0.16	1.32	1.93	3.65	5.67	4.74	2.97	1.64	1.50
gage 2	0.07	0.52	0.72	1.3	2.4	1.92	1.22	0.742	0.64
gage 3	0.06	0.39	0.52	0.84	1.15	0.78	0.44	0.20	0.18
gage 4	0.14	1.04	1.46	2.45	3.39	2.57	1.53	0.77	0.73

TABLE 3a

MEASURED DIFFRACTED WAVE SPECTRAL ENERGY x 10^5 (ft^2)

RUN NO. 2

Freq. c/sec.	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
gage 1	2.08	3.5	4.8	4.3	3.12	1.98	1.3	1.0	0.67
gage 2	1.6	3.0	4.15	3.7	2.65	1.86	1.28	1.13	0.96
gage 3	0.75	1.15	1.42	1.25	0.82	0.48	0.34	0.30	0.26
gage 4	2.3	3.75	4.5	3.75	2.9	2.25	1.7	1.19	0.75

TABLE 3b

THEORETICALLY CALCULATED DIFFRACTED WAVE SPECTRAL ENERGY x 10^5 (ft^2)

RUN NO. 2

Freq. c/sec.	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
gage 1	0.41	1.52	2.29	3.79	3.39	1.82	1.16	0.61	0.35
gage 2	0.22	0.77	1.18	2.07	1.62	0.61	0.49	0.36	0.10
gage 3	0.11	0.46	0.64	1.0	0.85	0.43	0.25	0.12	0.06
gage 4	0.27	1.15	1.67	2.68	2.41	1.29	0.78	0.41	0.22

CONCLUSIONS

1. There is a strong evidence supporting the assumption of linearity in the theory of diffraction - as far as many practical considerations are concerned.

2. The diffraction theory can be applied, for some practical purposes, to the two-dimensional spectrum at a harbor entrance to calculate the energy level at the various points inside the harbor.

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REFERENCES

Blackman, R. B., and Tuckey, J. W., "The Measurement of Power Spectra," Dover Publications, New York, 1958, 190 pp.

Blue, F. L., Jr., "Diffraction of Water Waves Passing Through a Breakwater Gap," Ph.D. Thesis, Univ. of California, Berkeley, 1948.

Blue, F. L., Jr., and Johnson, J. W., "Diffraction of Water Waves Passing Through a Breakwater Gap," Trans. Amer. Geophys. Union, Vol. 30, No. 5, October 1949, pp. 705-718.

Chase, J., et al., "The Directional Spectrum of a Wind Generated Sea as Determined from the Data Obtained by the Stereo Wave Observation Project," N. Y. Univ., Dept. of Meteorology and Oceanography, July 1957.

Cumming, J. D., and Fan, Shou-Shan, "Computer Solution of Wave Diffraction by Semi-infinite Breakwater," Univ. of California, Hydraulic Eng. Lab., Tech. Rept. HEL 1-7, July 1966.

Lee, Y. W., "Statistical Theory of Communications," J. Wiley & Sons, Inc., 1960.

Longuet-Higgins, M. S., et al., "Observations of the Directional Spectrum of Sea Waves Using the Motions of a Floating Buoy," Conf. on Ocean Wave Spectra, Easton, Maryland, 1961, pp. 111-136.

Mobarek, I. E., "Effect of Bottom Slope on Wave Diffraction," Univ. of California, Berkeley, IER Tech. Rept. HEL 1-1, 1962.

Mobarek, I. E., "Directional Spectra of Laboratory Wind Waves," Jour. Waterways and Harbors Div., Proc. ASCE, Vol. 91, No. WW3, August 1965, pp. 91-116.

Munk, W. H., et al., "Spectra of Low Frequency Ocean Waves," Bull. Scripps Inst. of Oceanography, 7:283, 1959.

Nagata, Y., "The Statistical Properties of Orbital Velocity and Their Application for the Measurement of Directional Wave Spectra," Jour. Coastal Eng. of Japan, 1964.

Penny, W. G., and Price, A. T., "The Diffraction Theory of Sea Waves and the Shelter Afforded by Breakwaters," Phil. Trans., Royal Soc. of London, Ser. A, Vol. 244, March 1952, pp. 236-253.

Pierson, W. J., Jr., "Wind Generated Gravity Waves," Advances in Geophysics, Vol. 2, 1955, pp. 93-179.

Putnam, J. A., and Arthur, R. S., "Diffraction of Water Waves by Breakwater," Trans. Amer. Geophys. Union, Vol. 29, No. 4, August 1948, pp. 481-490.

Putz, R. R., "Statistical Analysis of Wave Records," Proc. Fourth Conf. Coastal Eng.," Council on Wave Research, The Engineering Foundation, 1954, pp. 13-24.

Wiegel, R. L., "Diffraction of Waves by Semi-infinite Breakwater," Jour. Hyd. Div., Proc. ASCE, 88, HY1, January 1962.

Wiegel, R. L., "Oceanographical Engineering," Prentice-Hall, Inc., 1964, pp. 180-194.

NOTATION

The following symbols have been adopted for use in this paper:

A	=	constant
C, Q	=	the co-spectrum and quadrature spectrum, respectively
c	=	the phase velocity, ft/sec
D	=	probe spacings, ft
d	=	water depth, ft
E	=	the wave energy
F(x,z)	=	a function
f	=	the frequency in cycles/sec
g	=	gravity, ft^2/sec
H	=	diffracted wave height, ft
H_i	=	incident wave height, ft
h	=	a particular direction of wave advance
i	=	$\sqrt{-1}$
k	=	the wave number, $2\pi/L$
$\bar{k}$	=	1/L
K'	=	diffraction coefficient, $\lvert F(x,z) \rvert$, the ratio of the diffracted wave height to the incident wave height, H/H_i
L	=	wave length, ft
l, m	=	the x and y components of the wave number
n	=	a particular wave probe spacing
n'	=	the number of probe spacings

R	=	the covariance function
r	=	$\sqrt{x^2 + z^2}$, ft
$\bar{r}$	=	the number of wave directions considered
T	=	the time increment, seconds
t	=	time, seconds
u	=	a dummy variable
x	=	horizontal coordinate in the direction normal to the breakwater, ft
y_s	=	the water surface elevation, ft
Z, X	=	the components of the spacings between the gages of the array of wave detectors in the x direction and y direction, respectively
z	=	horizontal coordinate in the direction of the breakwater, ft
θ	=	direction, degree
σ, σ'	=	$4(r-x)/L$, $4(r+x)/L$

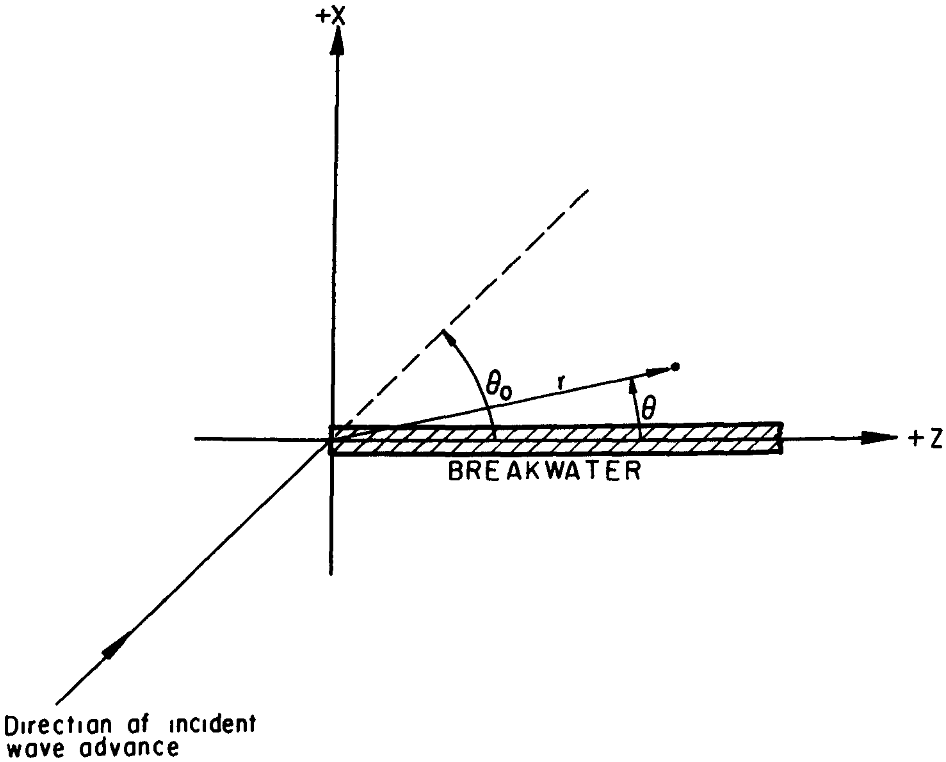

Fig. 1. Coordinate system.

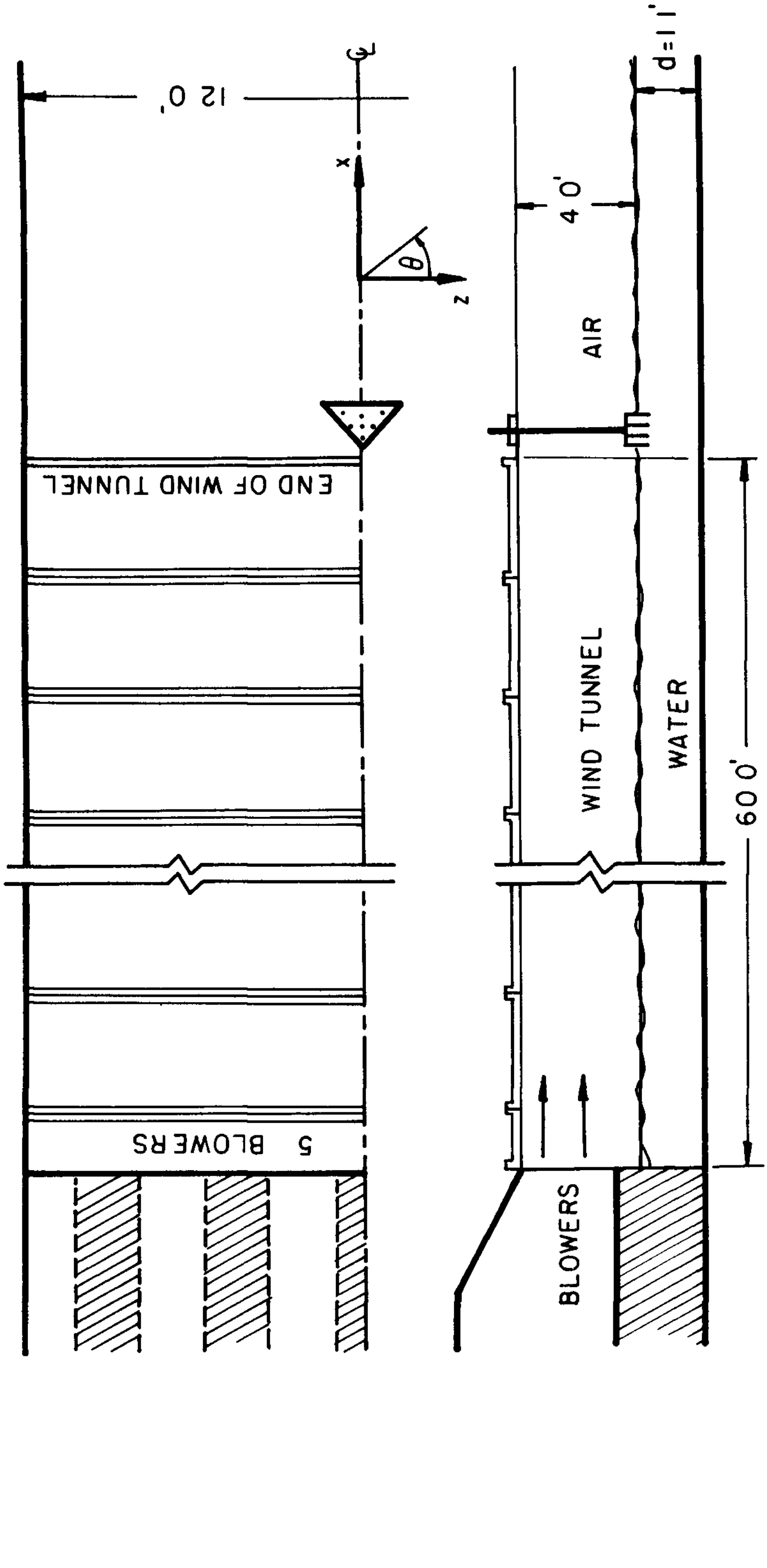

Fig. 2. Schematic drawing of wind tunnel layout and location of measuring instruments for the two dimensional wind wave spectrum study.

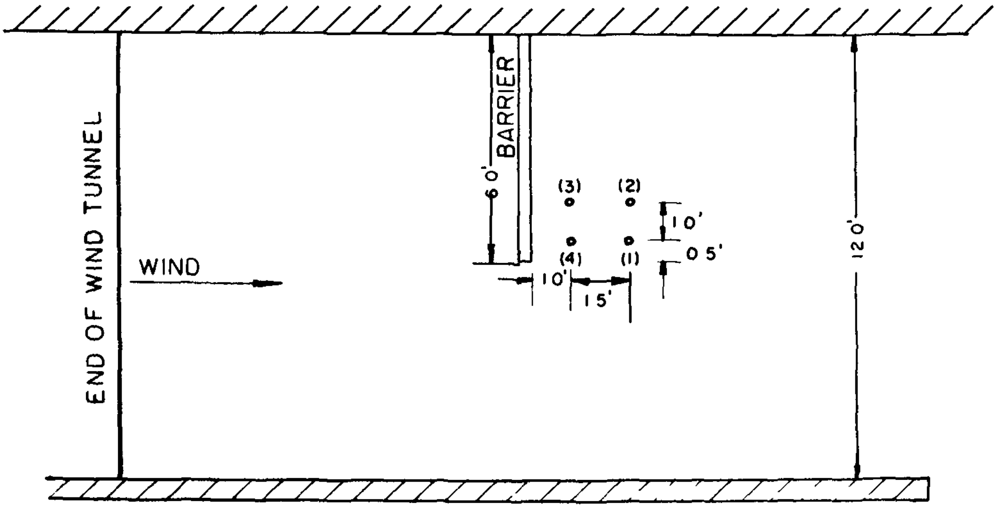

Fig. 3. Sketch of set-up for run no. 1.

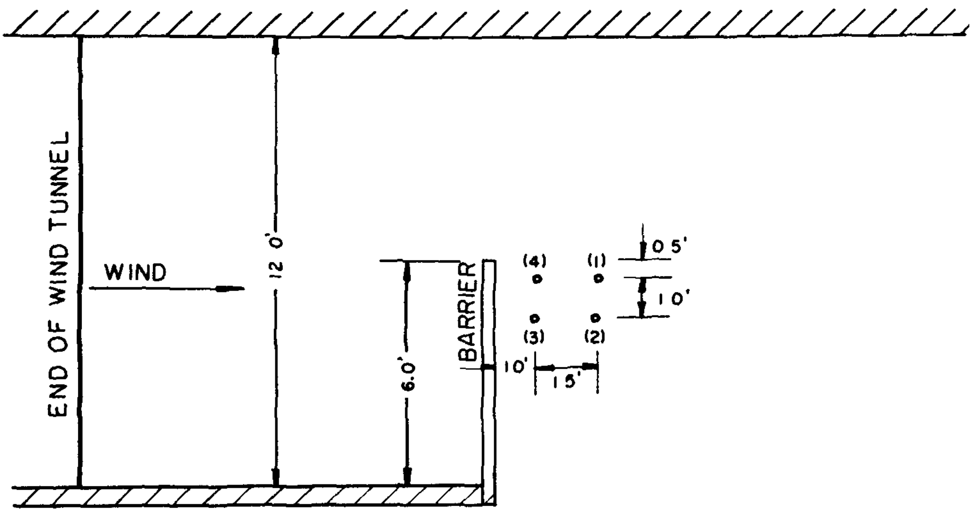

Fig. 4. Sketch of set-up for run no. 2.

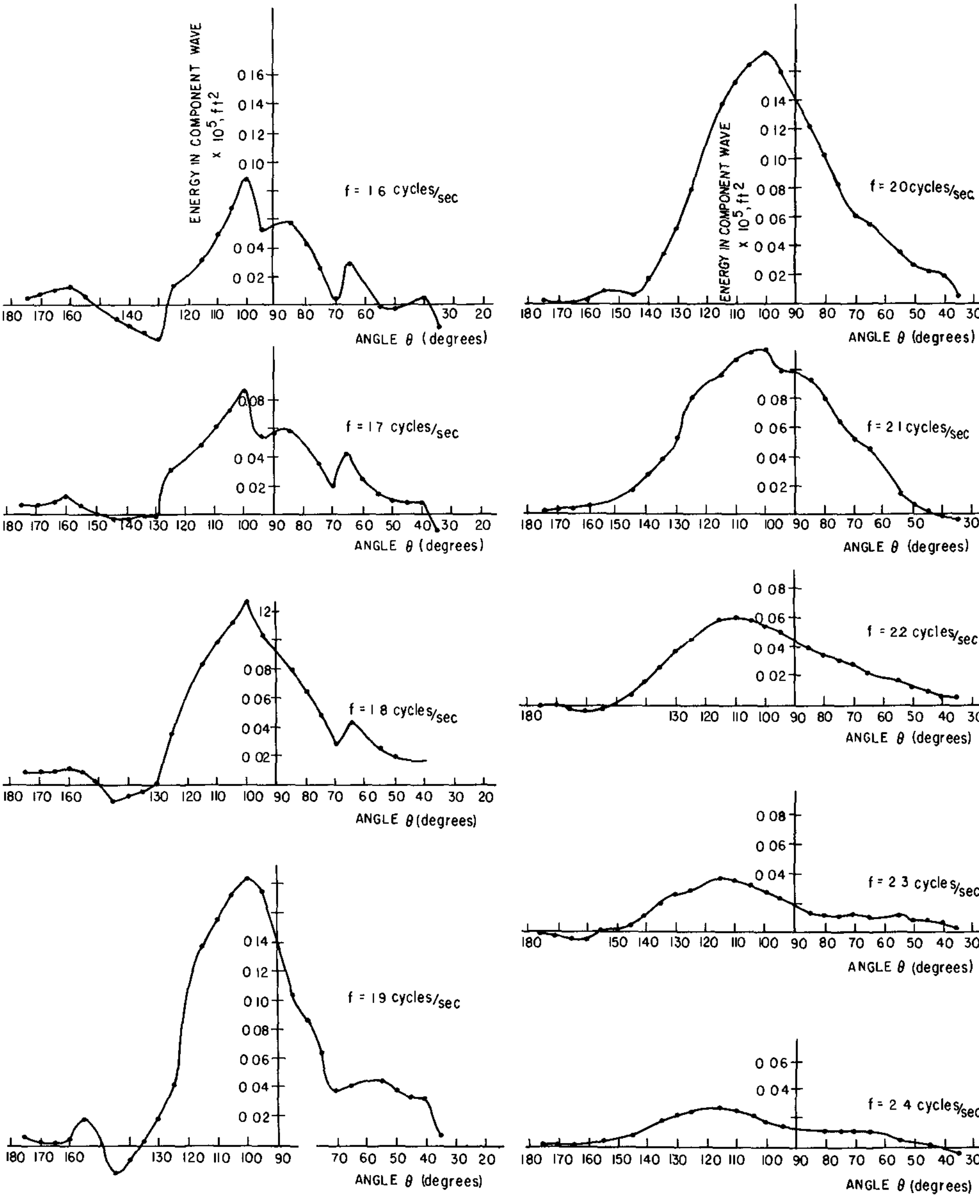

Fig. 5. Sample two dimensional energy-frequency spectrum (from Mobarek, 1965).

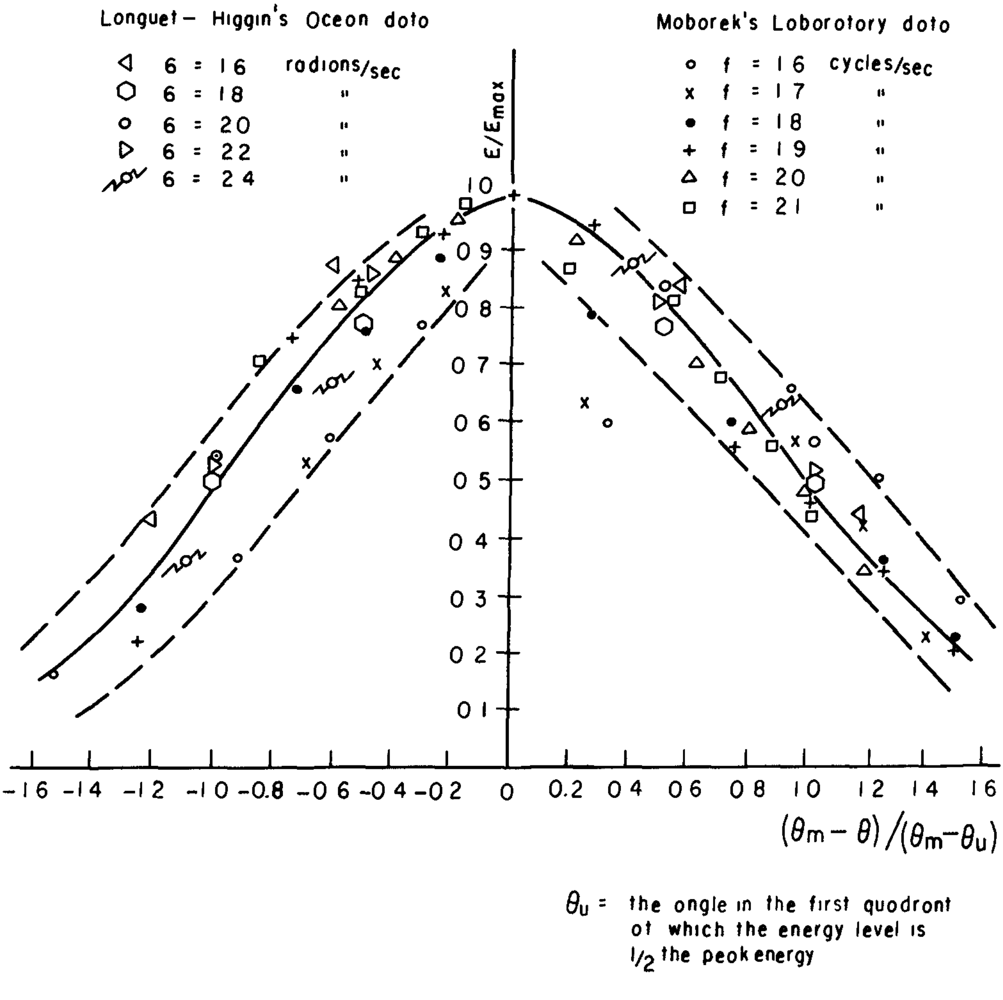

Fig. 6. Dimensionless plot of the two dimensional spectrum (from Mobarek, 1965).

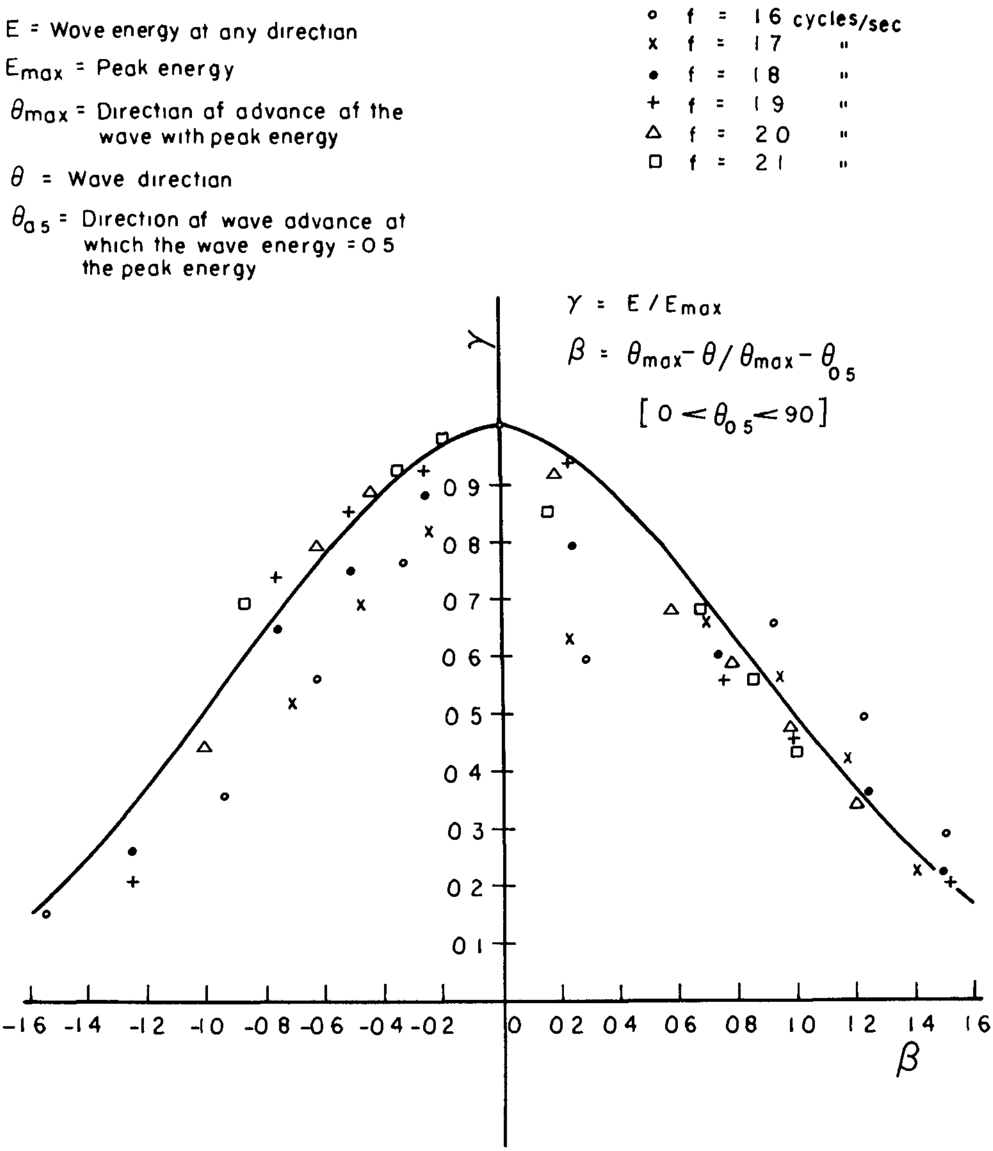

Fig. 7. The circular normal distribution (from Mobarek, 1965).

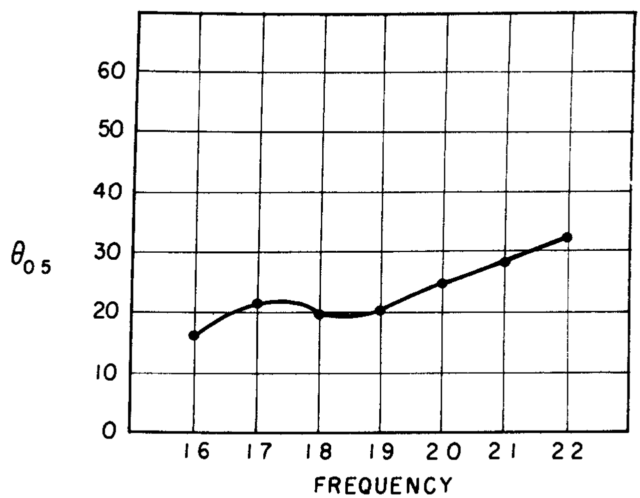

Fig. 8. Sketch of $\theta_{0.5}$ vs. frequency.

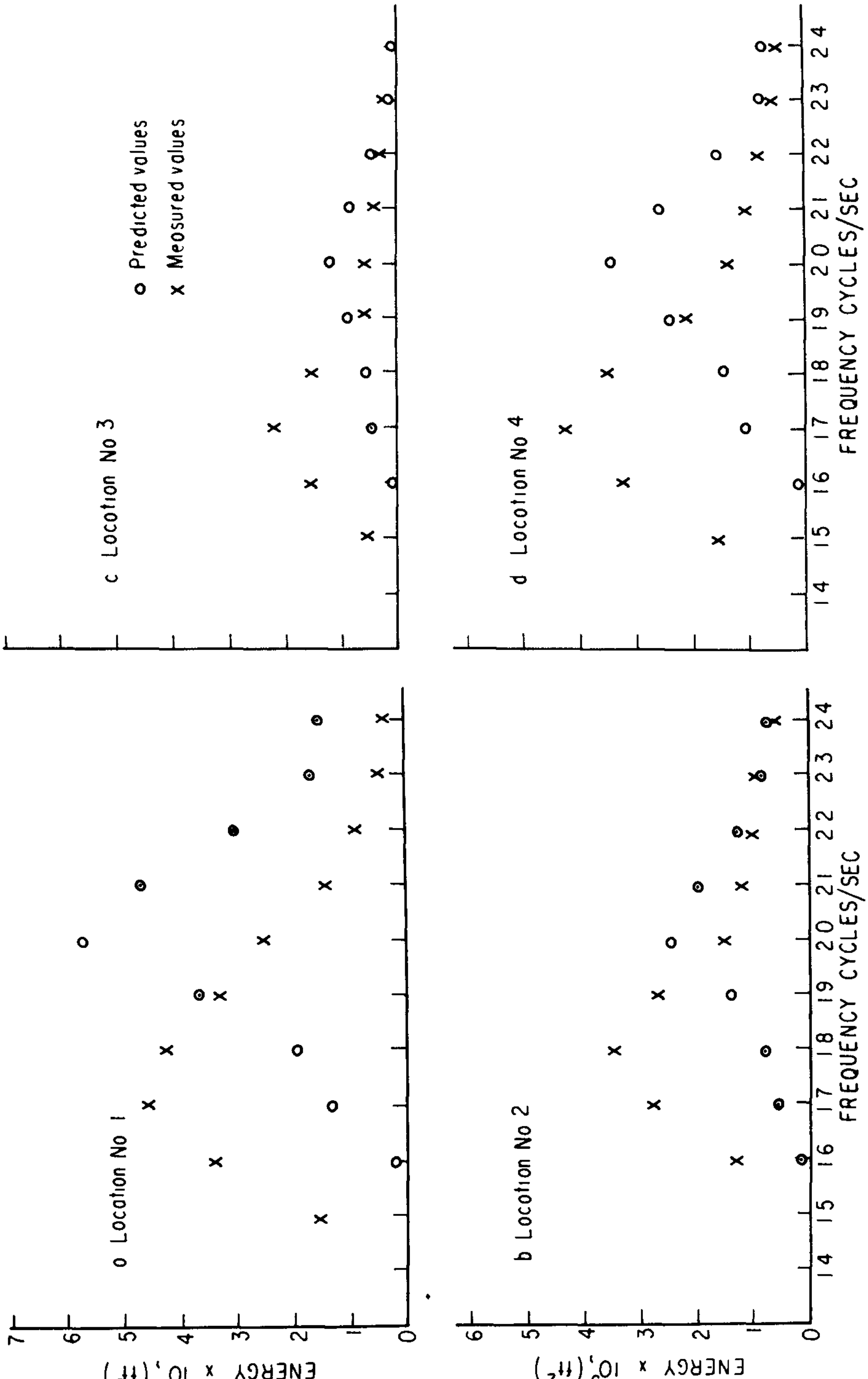

Fig. 9. Predicted and measured one dimensional energy-frequency spectra run no. 1.

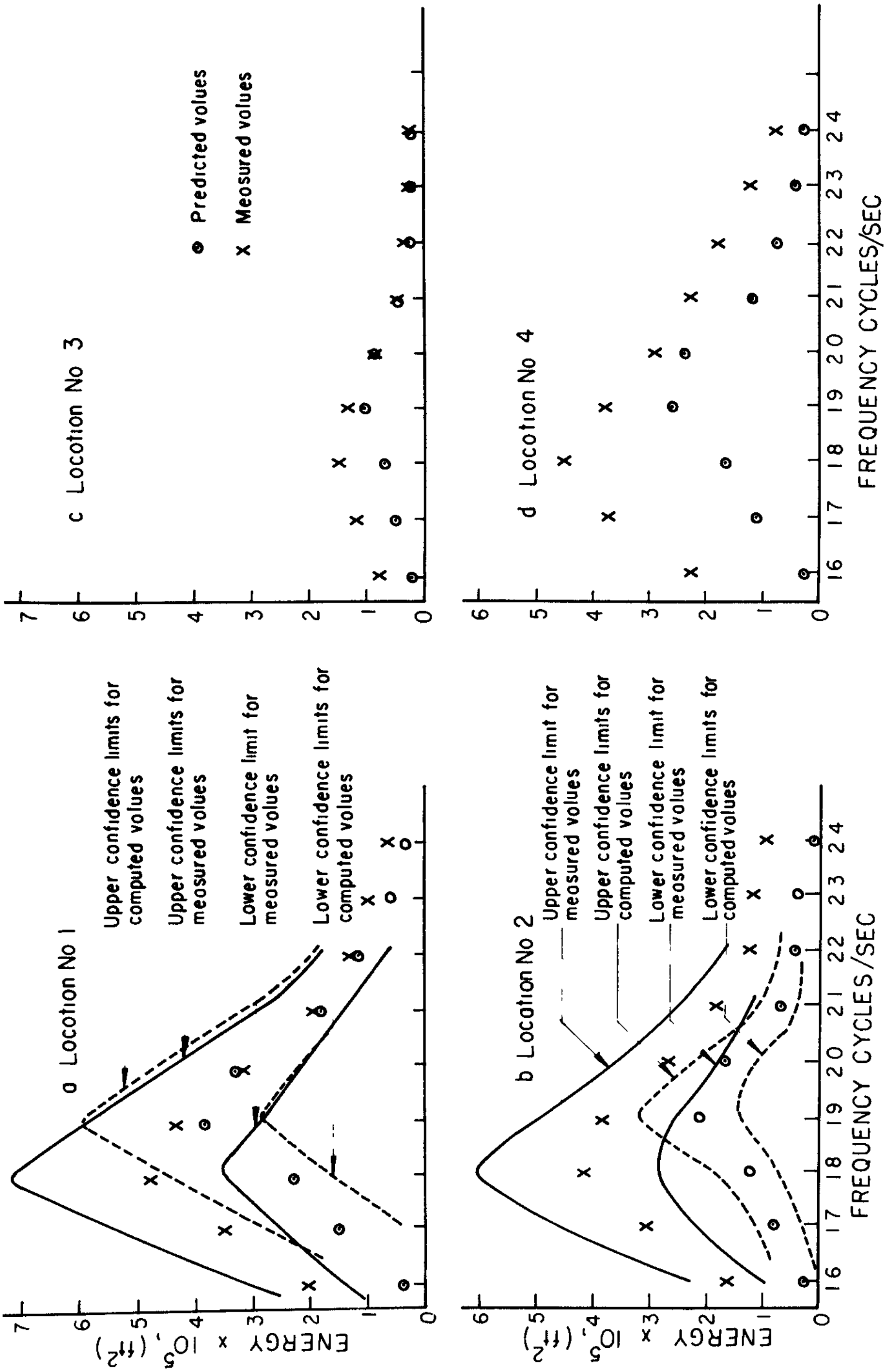

Fig. 10. Predicted and measured one dimensional energy-frequency spectra run no. 2.