CHAPTER 21

THE SHOALING, BREAKING AND RUNUP OF THE SOLITARY WAVE ON IMPERMEABLE ROUGH SLOPES

Tsutomu Kishi
and
Hiroshi Saeki

Department of Civil Engineering, Faculty of Engineering,
Hokkaido University, Sapporo,
Japan

ABSTRACT

Variations in wave characteristics for solitary waves in shoaling water are discussed. The transition of wave character from the solitary wave to the bore is basic to the understanding of the problem.

Experimental curves representing the transformation of wave height prior to breaking as well as the curves giving the breaker conditions are presented. Theories for the transformation of wave height after breaking and the prediction of the plunge point are presented and compared favorably with the laboratory measurements. Runup heights and wave quantities at the shoreline are measured to compare with the theory of a bore on a dry bed.

INTRODUCTION

It has been said that motions of impulsively generated waves are described by the solitary wave theory when waves have values of Ursell's parameter ($\eta_0 L^2/h^3$) between 10 and 40. When values of ($\eta_0 L^2/h^3$) are greater than 40 waves of permanent form cannot exist and inevitably progress to a condition of bore formation or breaking [Wilson, Webb and Hendrickson(1962)] . The above conditions are usually encountered for the nearshore motion of tsunami.

Motion of a solitary wave on a sloping beach is different from that of an oscillatory wave. Therefore, the data for oscillatory waves do not give predictions for the motion of solitary waves. For example, experiments of Ippen and Kulin (1955) led them to the conclusion that the ratio of breaker height-to-initial height obtained from available long-period oscillatory wave data could not be reconciled with solitary wave experimental results.

The authors, in the present paper, describe the results of investigations for the motion of solitary wave in shoaling water to give basic data for tsunami protection.

EXPERIMENTAL APPARATUS

Experiments were carried out in a wave tank of 20m length and 0.8m width. Solitary waves were generated by a pneumatic wave generator at one end of the tank and were propagated through uniform water depth to a sloping beach at the other end of the tank.

Four slopes of 1/10, 1/15, 1/20 and 1/30 were tested in the present experiments. The slopes were roughened by pasting the uniform sand grains of 0.6mm dia on their surfaces.

Details of wave motion were recorded by a 16mm cinecamera which was moved along the side of the wave tank together with the wave. Measurements of length were made by grid lines inscribed on the side glass of the wave tank at 10cm spacing. Measurements of time were made by frame numbers of camera operation (32 and 64 frames per sec) and an electric timer of 1 cycle per sec which was also photographed.

WAVES IN THE APPROACHING CHANNEL OF UNIFORM DEPTH

A solitary wave was generated by sudden release of the air valve of the vacuum tank which held a prescribed volume of water. Wave profiles in the channel of uniform depth were first measured. A typical example of test results is shown in Fig.1.

Profiles of experimental waves were nearly consistent with the theoretical profile of Boussinesq and generally more sharp than the second approximation of Laitone (1960). For large values of the height-to-depth ratio, wave profiles slightly deviated from the Boussinesq theory and became more sharp near the crest and more flat near the tails. The above features of the wave profile agree with the experimental results of Daily and Stephan (1952) and Perroud (1957). It was found from the measurements of wave profile that the wave generator as well as other experimental apparatus had sufficient accuracy for the experiments on solitary waves.

WAVE VELOCITY IN SHOALING WATER

Solitary waves in shoaling water deform more or less and cannot be of permanent form. Wave velocity at any depth on the slope tends to exceed the theoretical value for the solitary wave and approaches that of the bore.

Typical examples of velocity measurements are shown in Fig.2. Wave velocities at the toe of the slope are consistent with the second approximation theory of Laitone (1960) for solitary waves in uniform water. Thus, in Fig.2, the experimental points at the toe of the slope lie on the curve representing the solitary wave theory. The wave velocities then deviate from the solitary wave theory and approach the bore theory as waves advance in shoaling water. The transition of the wave velocity from the solitary wave theory to the bore theory conforms with the observations of the wave deformation.

It should be recognized that the transition from the solitary wave to the bore has an essential importance in considering the motion of solitary waves in shoaling water.

TRANSFORMATION OF WAVE HEIGHT IN SHOALING WATER

It was difficult to find a simple expression for the transformation of wave height in shoaling water.

For waves of small amplitude relation (1) --- Green's formula --- is applied.

$$H / H_0 \propto (h / h_0)^{-1/4} \tag{1}$$

The shoaling effect will be approximately given by relation (2) when waves on a slope are approximated by solitary waves.

$$H / H_0 \propto (h / h_0)^{-4/3} \tag{2}$$

According to relation (2) the shoaling effect for such waves is greater than that for waves of small amplitude.

When waves in shoaling water are not of permanent form and are approximated to the bore, the shoaling effect declines as the relative wave height increases [Keller et al (1960)] . The similar character of the shoaling effect has been pointed out by Kishi (1962) for long waves of finite amplitude.

The transformation of wave heights for solitary waves in shoaling water is made complex by the above contradictions.

Experimental results for 1/20 slope are shown in Fig.3. The values of the ratio H/H_0 for the same shoaling ratio h/h_0 become large as the initial height-to-depth ratio H_0/h_0 increases. This expresses that waves are in transition from the solitary wave to the bore. However, even for large values of H_0/h_0, values of H/H_0 did not exceed relation (1) since waves on the slope were subjected to considerable deformation and relation (2) did not hold. Especially, the fact that the shoaling effect remarkably weakens for values of H_0/h_0 smaller than 0.1 was noticed.

Experimental results for 1/10 slope are shown in Fig.4. In this case waves deform remarkably after passing the toe of the slope. Thus, waves on the slope would be approximated to the bore. Consequently, the effect of the initial height-to-depth ratio on the wave transformation is not significant as in the case of 1/20 slope. The tendency that the values of H/H_0 become small as the initial height-to-depth ratio increases should be observed.

A theoretical relation for the transformation of wave of finite amplitude would have the following form:

$$h/h_* = f(H/h)/f(H_*/h_*) \qquad (3)$$

where H_*, h_* : wave height and water depth at some reference point.

Thus the wave transformation should be represented as a relation of H/h to h/h_* instead of Figs.3 and 4. Especially, the values of H_b/h_b for gentle slopes become independent of H_0/h_0 and seem to be nearly constant, as will be described in the next section. Consequently, the breaking point was taken as the reference point.

The results of experiments are shown in Figs.5 a, b and c. It is found, as expected, that the effect of the initial height-to-depth ratio on the wave transformation is nearly eliminated for 1/30 slope. The effect of the initial height-to-depth ratio is prominent on 1/15 slope for values of the initial height-to-depth ratio smaller than 0.1.

BREAKER CHARACTERISTICS

The breaker heights, breaking points, and plunge points were measured from the photographs taken by the 16mm cinecamera. In the present experiments all waves tended to plunge. Though the classification of breaker types depends somewhat on personal judgement the complete spilling breaker was not observed.

The relation of H_b/H_0 to H_0/h_0 for various slopes together with the data of Ippen and Kulin (1955) is shown in Fig.6. The fact that the curves have the maxima at some small values of H_0/h_0 was noticed in the present experiments. The characteristics of wave transformation shown in Figs.3 and 4 should be clearly understood when the features of breaker height shown in Fig.6 are taken into account.

The relation of h_b/h_0 to H_0/h_0 for various slopes are shown in Fig.7.

The relation of H_b/h_b to H_0/h_0 for various slopes together with the data of Ippen and Kulin (1955) is shown in Fig.8. As is shown in the figure, the effect of H_0/h_0 on H_b/h_b is comparatively small for all slopes for values of H_0/h_0 larger than 0.2. And, in the rough approximation, the values of H_b/h_b for values of H_0/h_0 larger than 0.2 tend to be independent of H_0/h_0 as slopes become gentle. The values of H_b/h_b in shoaling water are generally larger than the theoretical value of $H_b/h_b \approx 0.7 \sim 0.8$ for the solitary wave in uniform water. However, as shown in Fig.8, values of H_b/h_b tend to decrease and approach the above theoretical value as slopes become gentle.

The relation of H_b/h_b to S is investigated in Fig.9. For the measurements of H_b/h_b on gentle slopes a straight line was fitted to estimate the beach slope on which the solitary wave theory in uniform water is approximately applied. The straight line set in Fig.6 gives relation (4):

$$H_b/h_b = 5.68\, S^{0.40} \tag{4}$$

The above relation (4) leads to the prediction that the slope on which the relation of $H_b/h_b = 0.8$ approximately holds would be more gentle than 1/140. It has been said that an oscillatory wave in shoaling water just prior to breaking would be approximated by a solitary wave. However, Ippen and Kulin (1955) had some doubt about this approximation. As stated above, this approximation would hold for only gentle slopes.

DEVELOPMENT OF BREAKERS IN SHOALING WATER

A small bore appearing on the wave crest at the breaking point develops as the wave advances.

Freeman & Le Méhauté (1964) presented a numerical method of calculating the bore development on the basis of a characteristic curve method. However, the procedures of numerical calculation are rather tedious. Kishi (1965) extended the bore theory of Keller et al (1960) to give an analytical representation for the development of plunging and spilling breakers. According to that theory given in the Appendix, the transformation of wave height between the breaking point and the shoreline is expressed by the differential equation (5):

$$\frac{h\,dM}{dh} = -\frac{F_1(M,\alpha)\,F_2(M)}{G_1(M,\alpha)\,G_2(M)} = A(M,\alpha) \qquad (5)$$

where $M = W_1/(g d)^{1/2} = (1+H/2h)^{1/2}$
α : correction factor
h : local water depth
H : wave height
$d = h + H$
W_1 : bore velocity

Functions F_1, F_2, G_1, G_2 and A are defined in the Appendix. By putting $\alpha = 0$ in (5), one obtains the equation for the fully developed bore in shoaling water given by Keller et al (1960) and Amein (1964,1966).

Values of the correction factor α in (5) were determined by comparing the theoretical wave heights with measurements. An example of the calculations is illustrated in Fig.10. Experimentally determined relationships among α, H_0/h_0 and S are shown in Fig.11. In Fig.11 it is noted that values of α for slopes of $S < 1/20$ are nearly independent of S and H_0/h_0, and α seems to be a constant (≈ -2.2) for that condition.

When equation (5) is taken into account, the bore development in shoaling water would be represented by the relation of H/h to h/h_b. In fact, the theoretical limiting value of H/h at the shoreline converges to zero if the friction effect is neglected. However, in practice the wave heights at the shoreline never vanish because of the friction effect, so that values of H/h at the shoreline diverge to infinity.

Therefore, relations of H/h_b to h/h_b for various slopes were investigated, as shown in Fig.12. As shown in Fig.11, values of the correction factor α in (5) are nearly constant for 1/30 and 1/20 slopes but for 1/15 slope they vary with values of H_0/h_0, and consequently with H_b/h_b.

Values of the height-to-depth ratio at the breaking point H_b/h_b for 1/20 slope are not completely independent of values of H_0/h_0. Values of H_b/h_b determine the boundary conditions for (5). Due to the above facts, the effects of H_0/h_0 on wave transformation between the breaking point and the shoreline would not be eliminated for 1/15 and 1/20 slopes.

Theoretical graphs of (5) are compared with measurements in Fig.12. With the exception of the vicinity of the shoreline, theoretical curves are favorably consistent with the measurements. As previously stated, equation (5) can not be applied to the vicinity of the shoreline because of the friction effect. However, it is found from the measurements that the variation in wave height in the vicinity of the shoreline is not remarkable but nearly constant in the rough approximation. Therefore, the transformation of wave height between the breaking point and the shoreline can be approximated, as shown in Fig.12, by combining equation (5) and the experimentally determined wave heights at the shoreline. Wave heights at the shoreline are investigated later.

DETERMINATION OF PLUNGE POINTS

Measurements of plunge point were compared with a theory. Plunge point was defined, in the present experiment, as the point at which the nappe of water jet issued from the wave crest falls to the still water surface.

Kishi (1962) calculated the point at which a wave front stands vertically on a uniformly sloping beach by the characteristic curve method. He discussed the effect of beach slope on breaking with his theory. His theory, however, is more favorably applied to the prediction of plunge point than to the breaking point, since the breaking of a wave develops to the still water level at the plunge point. In the onshore region of the plunge point waves are considered as fully developed bores.

The theoretical relation between the plunge point X_p and the initial surface slope at the wave front m given by Kishi is shown in Fig.13. Definitions of X_p and m are as follows:

$$X_p = x_p / \ell_0 \qquad (6)$$

where x_p : horizontal distance from the toe of the slope to the plunge point.

ℓ_0 : horizontal distance from the toe of the slope to the shoreline.

$$m = \frac{l_o}{h_o(gh_o)^{1/2}} \left(\frac{\partial \eta}{\partial t}\right)_{\substack{x=0 \\ t=0}} \qquad (7)$$

where h_o : water depth at the toe of the slope
η : height of the wave surface above the still water level.
t : time
g : acceleration of gravity

In the theory a wave which has a non-zero slope at the wave front and which propagates shoreward into quiescent water is considered. Therefore, values of m for solitary waves should be calculated at points near the wave front, where $\eta = \eta_1 \neq 0$. The theoretical profile of a solitary wave according to Boussinesq is given by (8).

$$\eta = H_0 \operatorname{sech}^2 \left[\sqrt{\frac{3}{4}\frac{H_0}{h_0^3}}\,(x - ct)\right] \qquad (8)$$

where H_0 : wave height in water of uniform depth
c : wave velocity in water of uniform depth

Equation (8) yields (9).

$$\left(\frac{\partial \eta}{\partial t}\right)_{\eta=\eta_1} = c\sqrt{3}\,(H_0/h_0)^{3/2}(\eta_1/H_0)(1-\eta_1/H_0)^{1/2} \qquad (9)$$

The celerity of a shallow water wave of finite height is given by (10) [Keulegan and Patterson (1940)] .

$$c^2 = gh_0\left(1 + \frac{3}{2}\frac{\eta}{h_0} + \frac{h_0^2}{3\eta}\frac{\partial^2 \eta}{\partial x^2}\right) \qquad (10)$$

Relation (11) is applied to solitary waves.

$$\lim_{\eta \to 0} \frac{h_0^2}{3\eta}\frac{\partial^2 \eta}{\partial x^2} = H_0/h_0 \qquad (11)$$

Substitution of eqs.(9),(10), and (11) into (7) yields (12).

$$m = (\sqrt{3}/S)(1+H_0/h_0)^{1/2}(H_0/h_0)^{3/2}(\eta_1/H_0)(1-\eta_1/H_0)^{1/2} \qquad (12)$$

Equation (12) is applied to waves of nearly permanent form on gentle slopes. However, waves on steep slopes, such as in the present experiments, deform considerably and wave characters transform from solitary waves to long waves or bores. Under such conditions relation (11) will be favorably replaced by (13).

$$\lim_{\eta \to 0} c^2 = g h_0 \tag{13}$$

Substitution of (9),(10), and (13) into (7) gives (14).

$$m = (\sqrt{3}/s)(H_0/h_0)^{3/2}(\eta_1/H_0)(1 - \eta_1/H_0)^{1/2} \tag{14}$$

The relations of X_p to H_0/h_0 for four slopes are shown in Fig.14. In the figure experimental points for 1/10 slope were measured as the middle points between breaking points and plunge points on the dry bed, since the breaking of wave occured very close to the shoreline and, consequently, the falling nappes tended to plunge on the dry bed beyond the shoreline. Theoretical curves which were obtained by inserting values of η_1/H_0 in (14) are entered in the figure and they favorably compare with the measurements.

WAVE RUNUP AND WAVE MOTION AT THE SHORELINE

Details of the wave motion beyond the initial shoreline have been studied by Amein (1964, 1966), Freeman and Le Méhauté (1964), Iwagaki et al (1966) and many other investigators.

Theory for the case of frictionless motion has shown that when the bore reaches the shore line the bore height vanishes. The bore is then replaced by a jet of water, the tip of which is called the leading edge. When friction is considered, the leading edge is replaced by a water front of parabolic form which is called the leading wave element. Freeman and Le Méhauté (1964) determined the motion of the leading wave element by assuming relation (15) at the wave front.

$$c = (g h)^{1/2} = a u \tag{15}$$

where h : depth of the leading wave element
u : fluid velocity in the leading wave element.
a : coefficient

According to their theory, runup height is given by (16),

$$R/h_0 = (u_s^2/2 g h_0)\frac{(1+a)(1+2a)}{1+(f/a^2 s)} \tag{16}$$

where u_s : fluid velocity in the leading wave element at the shore line.
f : friction factor defined by $\tau = \rho f u^2$

A comparison of (15) with measurements was made. At the shoreline fluid velocity u_s is equal to the propagation velocity of the leading wave element W_s by the condition of continuity. Coefficient a in (14) can be determined from the measurements of the depths and propagation velocities of the leading wave elements at the shoreline. Relations between $(h_s/h_o)^{1/2}$ and $W_s/(gh_o)^{1/2}$ for 1/15, 1/20 and 1/30 slopes as obtained experimentally are shown in Fig.15. From the measurements it is found that values of a at the shoreline are nearly constant for a given slope, and are independent of H_o/h_o. The relation of a to s is shown in Fig.16. In the range of the present experiments a tends to increase with s.

Consequently, the relation that R/h_o is proportional to $u_s^2/gh_o\,(= W_s^2/gh_o)$ will be found if flow conditions are hydraulically rough and values of friction factor f are constant and independent of Reynolds number. Results of experiments for the relation of R/h_o to $W_s/\sqrt{gh_o}$, (or $u_s/\sqrt{gh_o}$) are shown in Fig.17. Theoretical relation (16) favorably compares with the measurements.

Application of (16) to the experimental results shown in Fig.17 together with the values of a given in Fig.16 determines the values of f.

Since a is constant for a slope regardless of wave conditions, the value of f is also a constant for the slope. For the purpose of comparison, values of f_1 for the steady flow condition were calculated from Nikuradse's data. The relation between f and the friction factor f_1 of Nikuradse is $f = f_1/8$. Flow conditions of experimental runs used in the calculation are summarized in Table 1:

Table 1. Summary of flow conditions for experiments.

S	H_o/h_o	$W_s \approx u_s$ (cm/sec)	h_s (cm)
1:30	0.473	116.4	1.21
1:20	0.325	106.7	2.10
1:15	0.400	139.8	3.03

Values of f calculated by two methods are compared in Table 2.

Table 2. Comparison of values of f calculated from runup data and Nikuradse's experimental curve

slope	1:30	1:20	1:15
Runup Data	0.0027	0.0040	0.0070
Steady Flow	0.0050	0.0042	0.0037

Since the theory for the leading wave element assumes that $(s + u_t)$ and u_x vanish in the element, values of f are to be consistent with those for steady flow. However, so far as the present experiments are concerned, a definite conclusion was not obtained from Table 2.

Relation of R/h_0 to H_0/h_0 for various slopes were investigated to compare with the experimental results of other investigators. The results of the present experiments as well as the results of Hall & Watts (1953) and Kaplan (1955) gave the following relation (17):

$$R/h_0 = K(H_0/h_0)^{\delta} \tag{17}$$

where K, δ : empirical constants depending on the beach slope.

Relations of K and δ to S are shown in Fig.18. The following conclusions are obtained from the figure:

1) For steep slopes $s \geq 1/6$, K and δ are almost independent of S.
2) For slopes $S < 1/6$, the relation of $K \propto \delta$ approximately holds. The exponent δ decreases with S and approaches 0.5 for slopes of $S < 1/60$. When the value of δ is 0.5 the relation $R \propto E_0^{1/3}$, where E_0 is the energy of the solitary wave at the toe of the slope, is obtained through simple calculations. Thus, for gentle slopes the runup height will be proportional to the one-third power of the wave energy at the toe of the slope.

Finally, in connection with the measurements of runup height, wave heights at the shoreline H_s were measured. The results of laboratory measurements are shown in Fig.15. The maximum wave height at the shoreline H_s is higher than the height of the leading wave element h_s.

It should be stated that the times of maximum velocity and depth at the shoreline are different, since the runup height is determined by the motion of the leading wave element. The interesting feature that values of the ratio of H_s to h_s are independent of H_0/h_0 for all slopes is found in Fig.15. The relation of $\sqrt{H_s/h_s}$ to s is given in Fig.16.

APPENDIX --- THEORY FOR THE MOTION OF A PARTIALLY DEVELOPED BORE IN SHOALING WATER

Tsutomu Kishi

Numerical methods for analyzing the motion of a partially developed bore in shoaling water have been presented by Amein (1964, 1965) and Freeman & Le Méhauté(1964). However, the procedures of calculation are rather tedious. The author extended the theory for a fully developed bore given by Keller et al (1960) to analyze the motion of a partially developed bore.

As is illustrated in Fig.A-1, a partially developed bore is treated as a fully developed bore in the first approximation. Then the bore velocity and the particle velocity behind the bore are given by (A-1) and (A-2).

$$W = (gd)^{1/2}(1+H/2h)^{1/2} + u \qquad \text{(A-1)}$$

$$U = (H/d)W + (h/d)u \qquad \text{(A-2)}$$

where
- d : water depth just behind a bore
- h : water depth just in front of a bore
- H : bore height $(=d-h)$
- u : fluid velocity just in front of a bore

A new variable W_1 defined by (A-3) is introduced.

$$W_1 = (gd)^{1/2}(1+H/2h)^{1/2} \qquad \text{(A-3)}$$

where W_1 : the bore velocity in quiescent water, i.e. $u=0$

Then, (A-1) and (A-2) are simplified to become (A-4) and (A-5):

$$W = W_1 + u \qquad \text{(A-4)}$$

$$U = (H/d)W_1 + u \qquad \text{(A-5)}$$

When a fully developed bore is considered the fluid velocity in front of the bore u may be taken to be zero. However, the wave under consideration is not a fully developed bore and some corrections are necessary to eq.(A-3). In this meaning the fluid velocity u in (A-4) and (A-5) can not be taken to be zero. In fact the velocity of a partially developed bore is small in comparison with that of a fully developed bore which has the same crest height.

The velocity of a partially developed bore approaches that of a fully developed bore as it advances and develops in shoaling water. This implies that the correction velocity u in (A-4) and (A-5) should decrease as the water depth decreases. From dimensional considerations it is reasonable to assume the expression (A-6) for the correction velocity,

$$u = \alpha \, (g h)^{1/2} \qquad \text{(A-6)}$$

where α : a coefficient to be of negative sign

Moreover, the author observed the phenomenon that a plunging water jet forms a vortex just in front of a bore to induce a return flow against the bore [Kishi(1965)] . This phenomenon leads the author to the conclusion that the basic equations should be given by (A-4),(A-5) and (A-6) even for a fully developed bore.

In analyzing the motion of a bore the author works with the Froude number defined by $M = W_1/(gd)^{1/2} = (1+H/2h)^{1/2}$ after Keller et al (1960). In terms of M we have

$$\begin{aligned} W/(gd)^{1/2} &= M(2M^2-1)^{1/2} + \alpha \\ U/(gd)^{1/2} &= 2M(M^2-1)/(2M^2-1)^{1/2} + \alpha \\ d/h &= 2M^2-1 \\ H/h &= d/h - 1 = 2(M^2-1) \end{aligned} \qquad \text{(A-7)}$$

Whitham's rule implies that an approximate formula is derived by applying the characteristic equation (A-8) to the flow quantity immediately behind the bore.

$$dU + 2\,dc - \frac{g\,dh}{U+c} = 0 \qquad \text{(A-8)}$$

where $c = (g d)^{1/2}$

Substitution of the relation (A-7) into (A-8) yields the differential equation (A-9).

$$\frac{h\,dM}{dh} = - \frac{F_1(M,\alpha)\;F_2(M)}{G_1(M,\alpha)\;G_2(M)} = A(M,\alpha) \qquad \text{(A-9)}$$

where

$$F_1(M,\alpha) = (2M^6+6M^5-9M^3-4M^2+3M+2)$$
$$+\alpha(2M^2-1)^{1/2}(2M^3+3M^2-2M-3/2)$$
$$+\alpha^2(2M^2-1)/2$$

$$F_2(M) = (2M^2-1) \qquad \text{(A-10)}$$

$$G_1(M,\alpha) = (2M^3+2M^2-2M-1)$$
$$+\alpha(2M^2-1)^{1/2}$$

$$G_2(M) = 2(4M^4+4M^3-3M^2-2M+1)$$

From (A-9) the values of M at any water depth between the breaking point and the shoreline are readily calculated when values of α are determined experimentally.

Finally, the limiting value of H/h at the shoreline is considered. When the value of α is taken to be zero, (A-9) is reduced to the bore equation of Keller et al and the limiting value of H/h at the shoreline converges to zero. Since (A-9) was derived by adding a correction velocity u defined by (A-6) to the equation of Keller et al, u converges to zero as $h \to 0$. Consequently the limiting value of H/h at the shoreline is zero for equation (A-9). In practice, (A-9) can not be applied in the vicinity of the shoreline, since the friction effect which is neglected in (A-9) is prominent.

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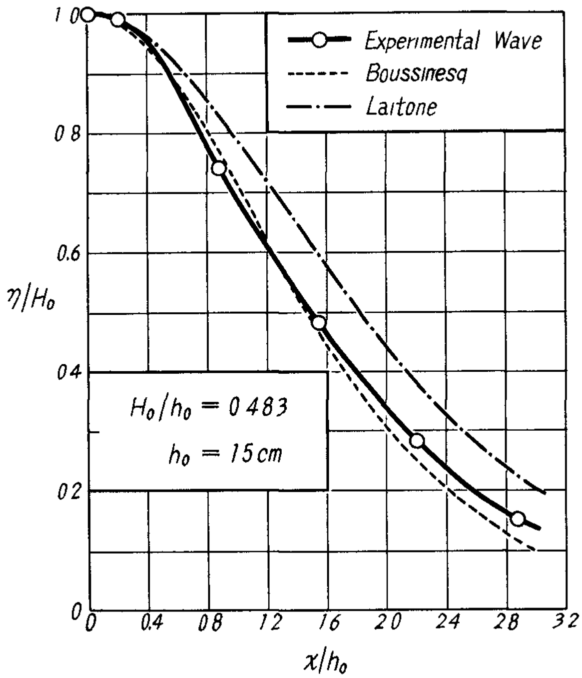


Fig. 1.

Comparison of laboratory measurements of wave profile with theories.

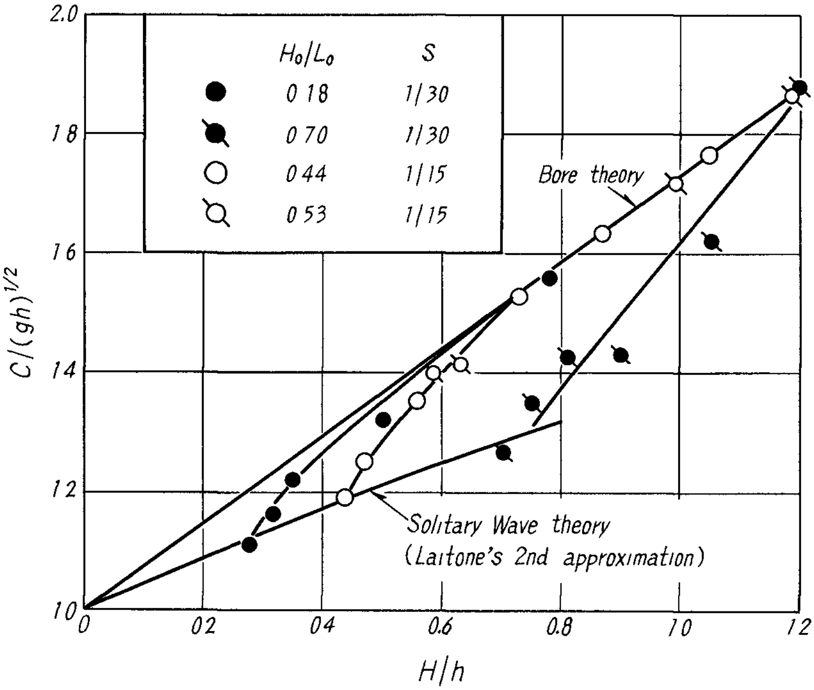


Fig. 2. Laboratory measurements of wave velocities for solitary waves in shoaling water.

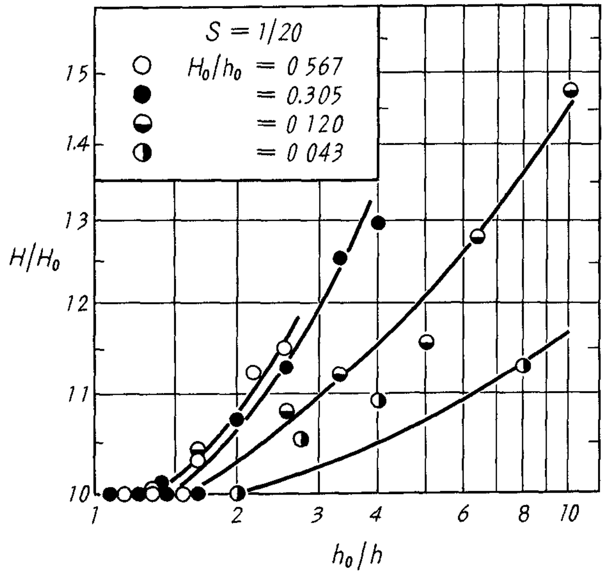


Fig. 3. Relation of H/H_0 to h/h_0 on 1/20 slope for four values of H_0/h_0.

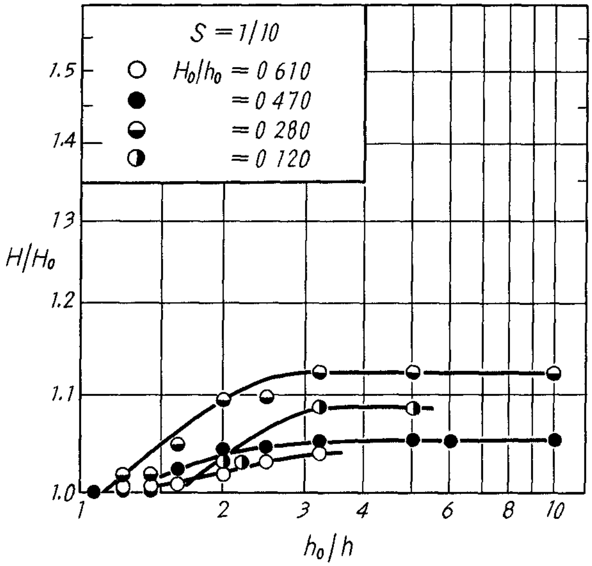


Fig. 4. Relation of H/H_0 to h/h_0 on 1/10 slope for four values of H_0/h_0.

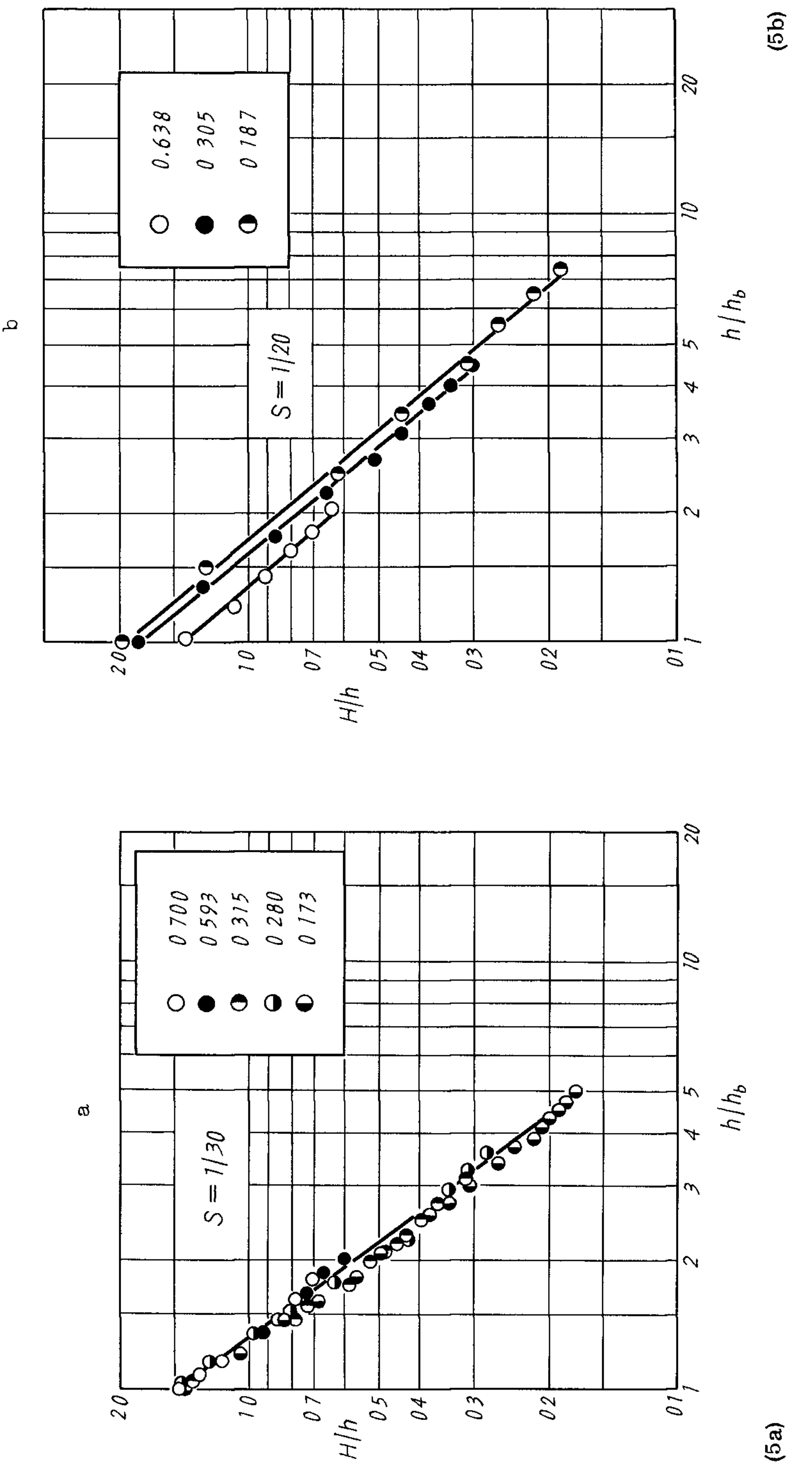


Fig. 5. Relation of H/h to h/h_b for three values of slope.

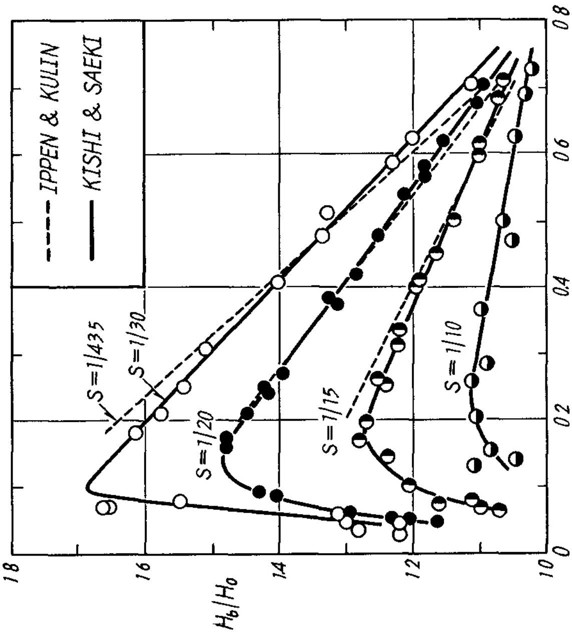
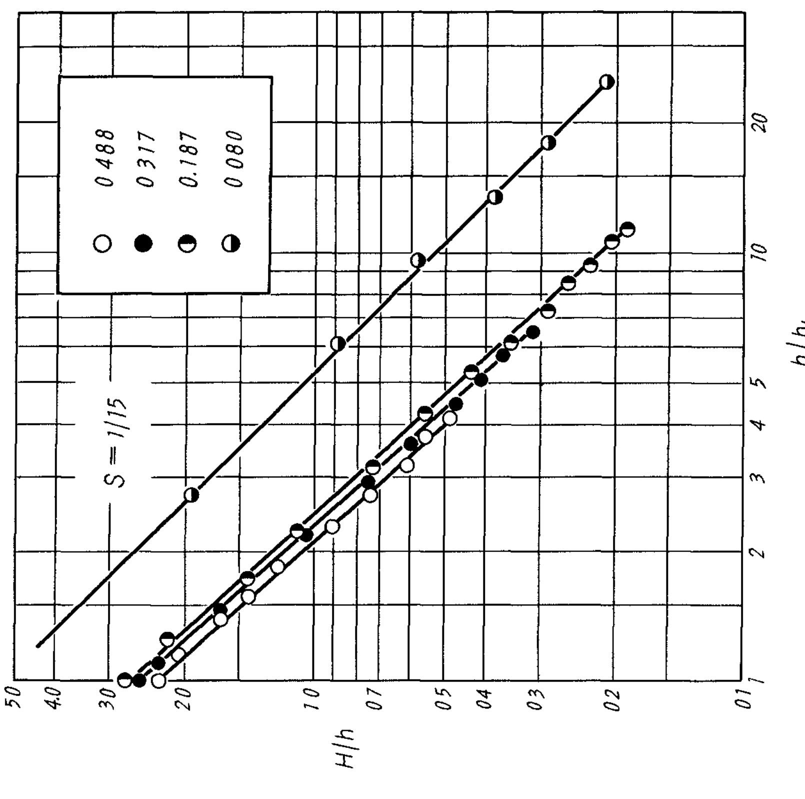


Fig. 6. Relation of H_b/H_o to H_o/h_o on various slopes.



(5c)

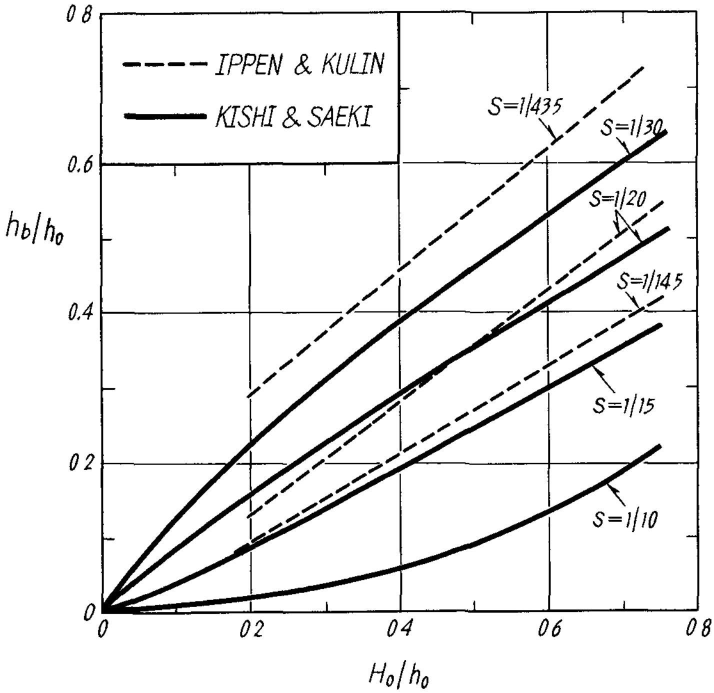


Fig. 7. Relation of h_b/h_o to H_o/h_o on various slopes.

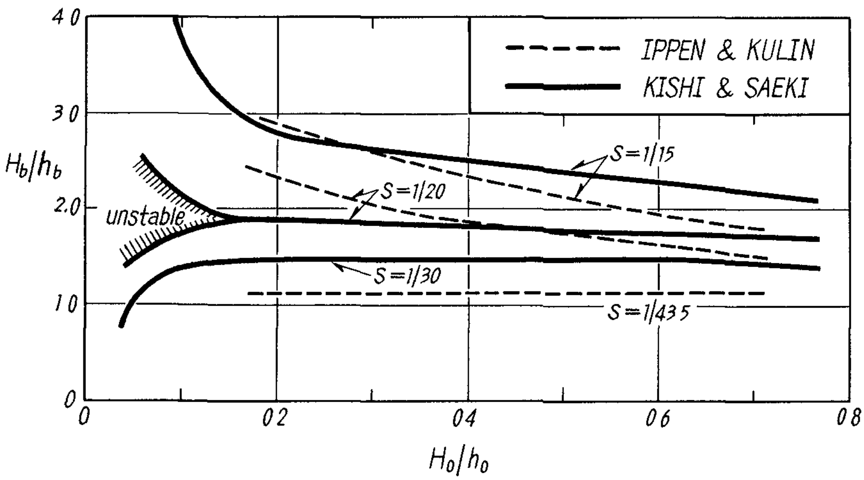


Fig. 8. Relation of H_b/h_b to H_o/h_o on various slopes.

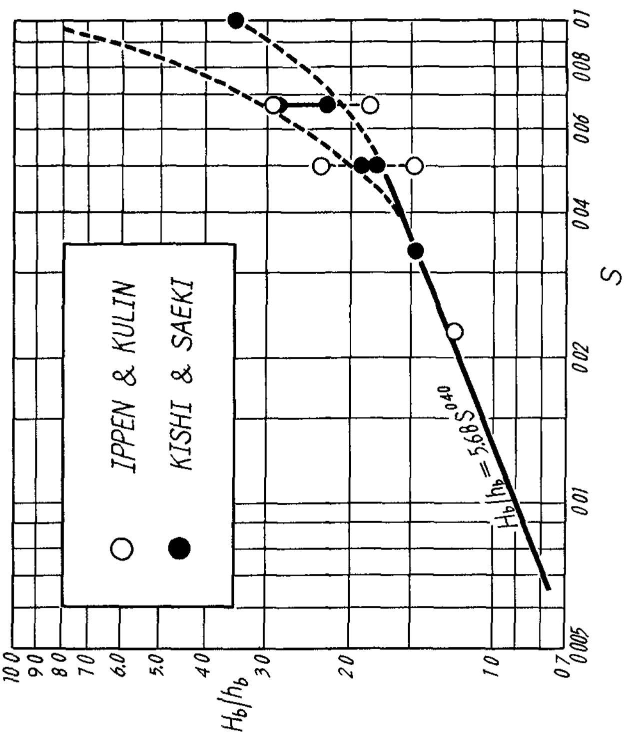


Fig. 9. Relation of H_b/h_b to S.

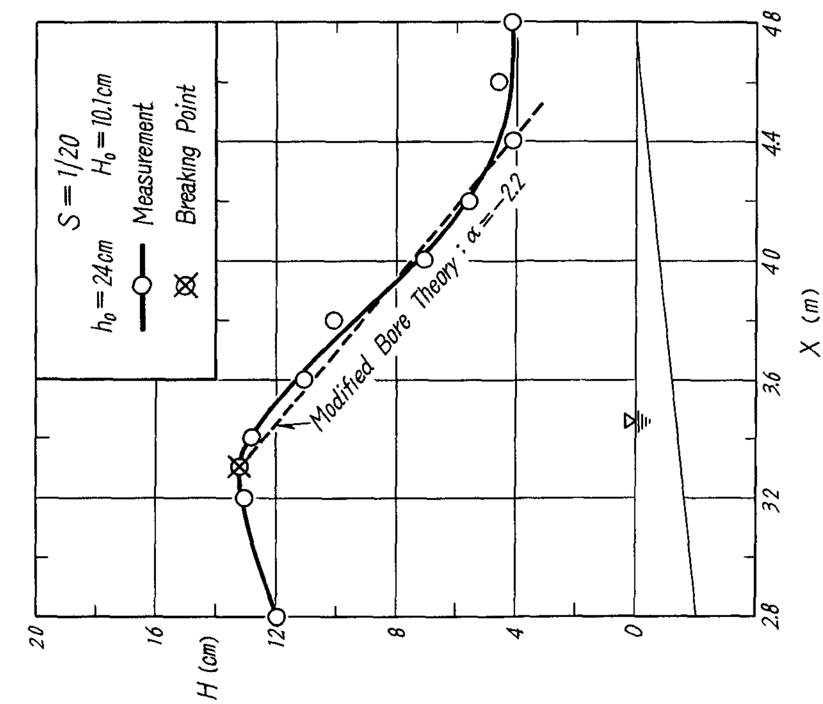


Fig. 10. Comparison of measured wave heights with theory in the onshore region of breaking point.

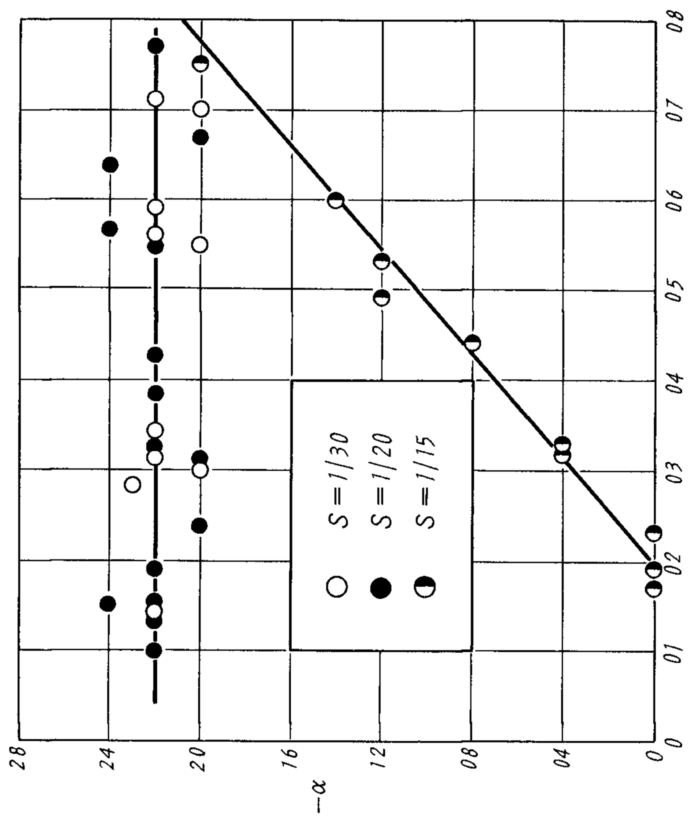
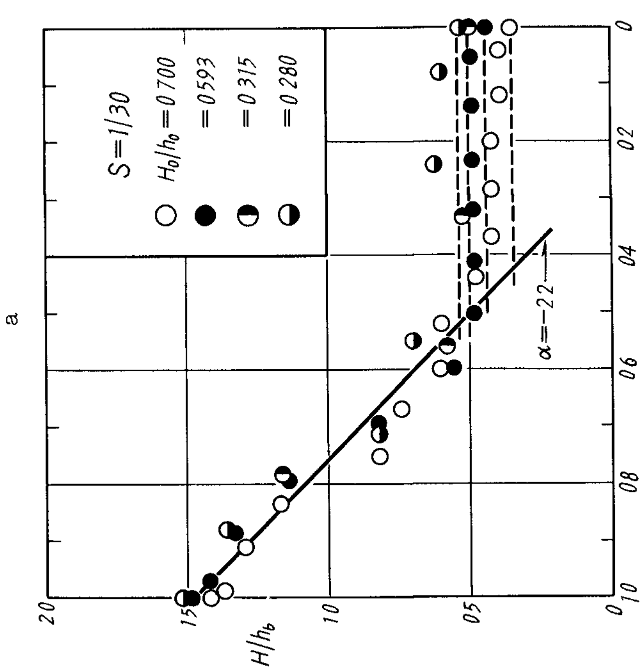
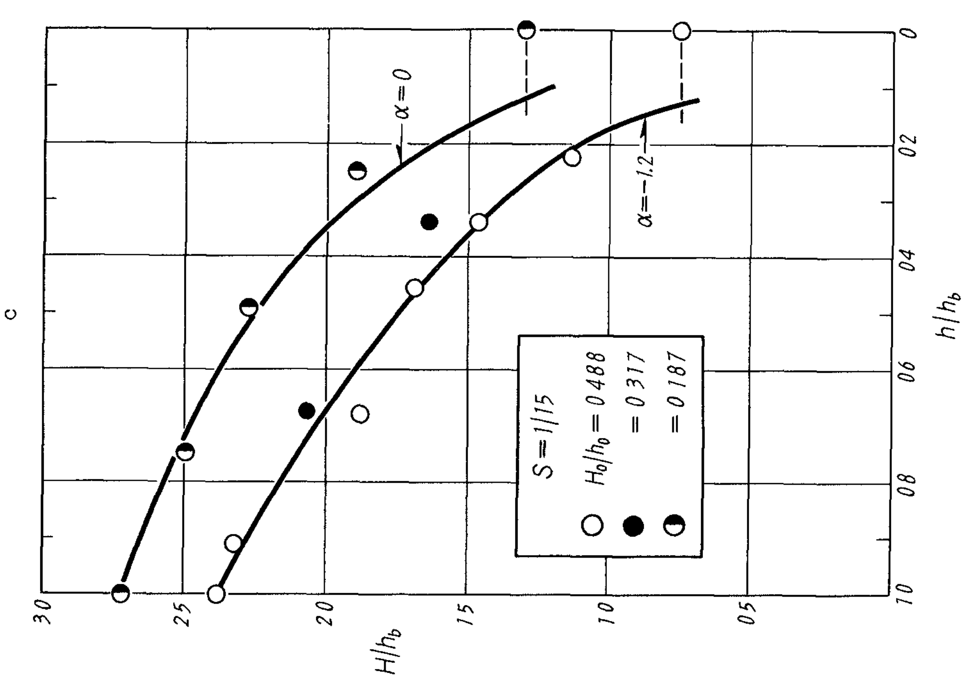


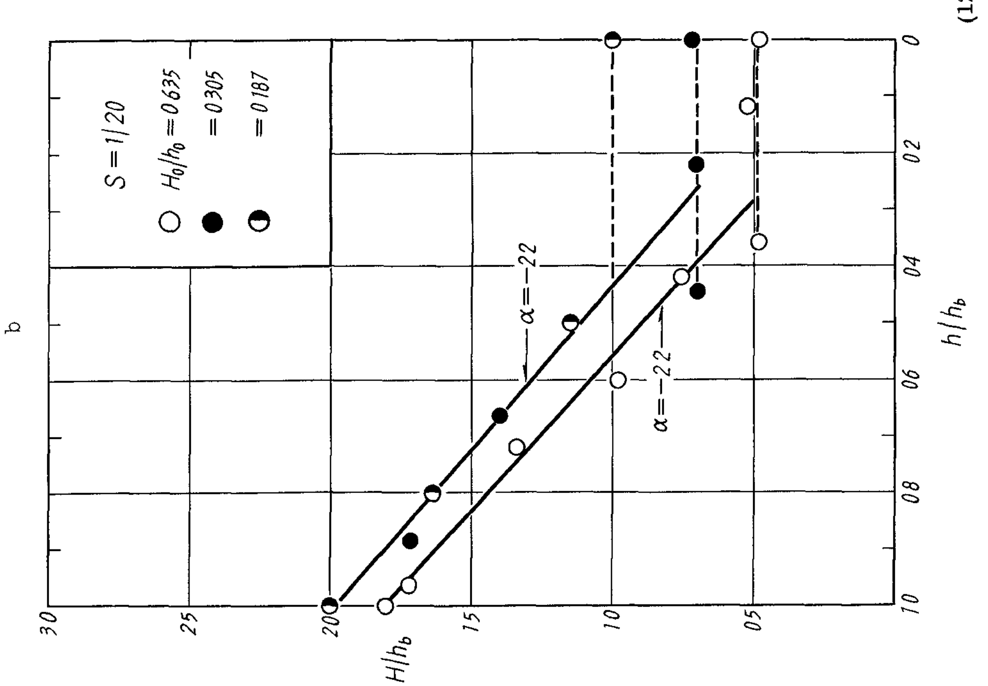
Fig. 11. Relationships among α, H_0/h_0 and S.



(12a)

Fig. 12. Relation of H/h_b to h/h_b in the onshore region of the breaking point for three values of slope.


c
S = 1/15
H0/h0 = 0 488
= 0 317
= 0 187
α = 0
α = −1.2
H/hb
h/hb


b
S = 1/20
H0/h0 = 0 635
= 0 305
= 0 187
α = −22
α = −22
H/hb
h/hb

(12b) (12c)

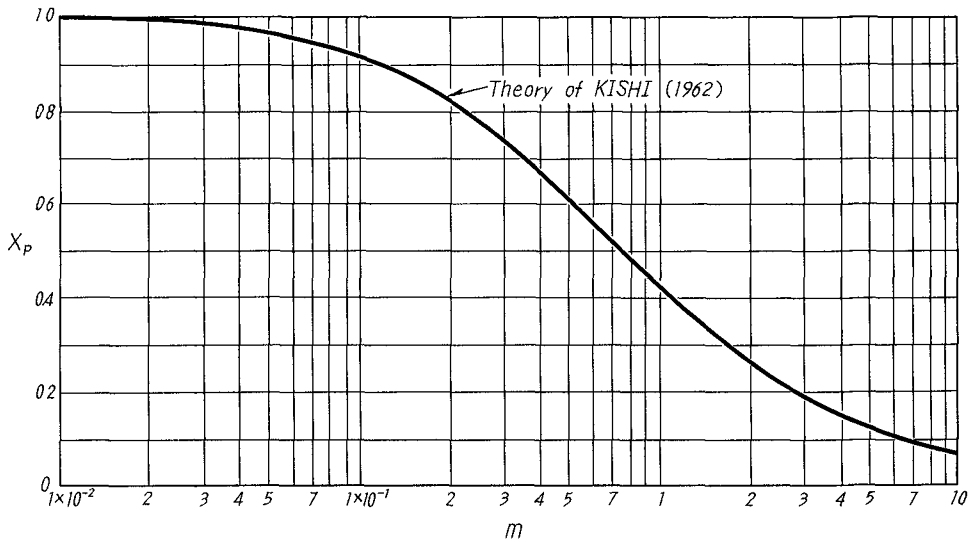


Fig. 13. Relation of X_p to m.

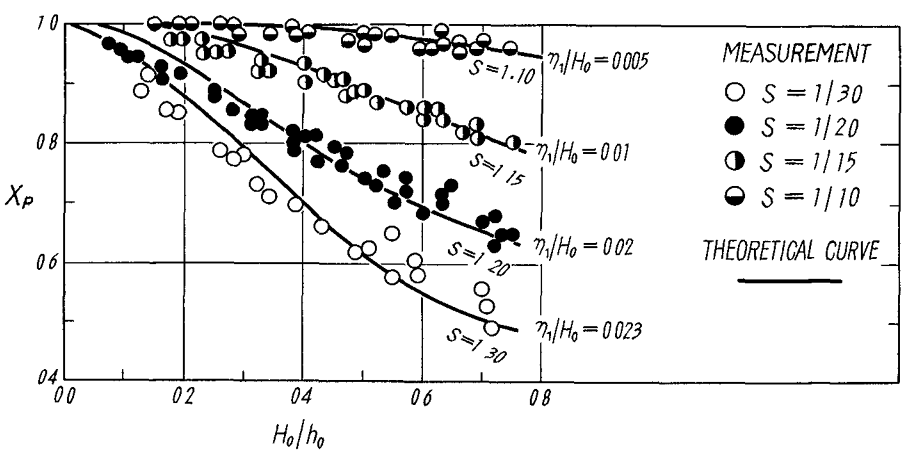
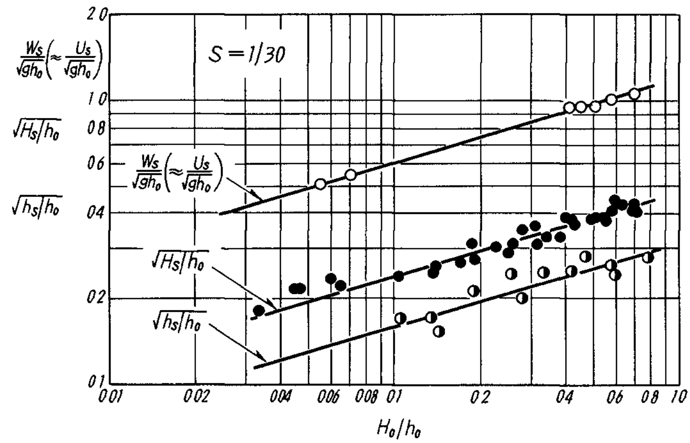
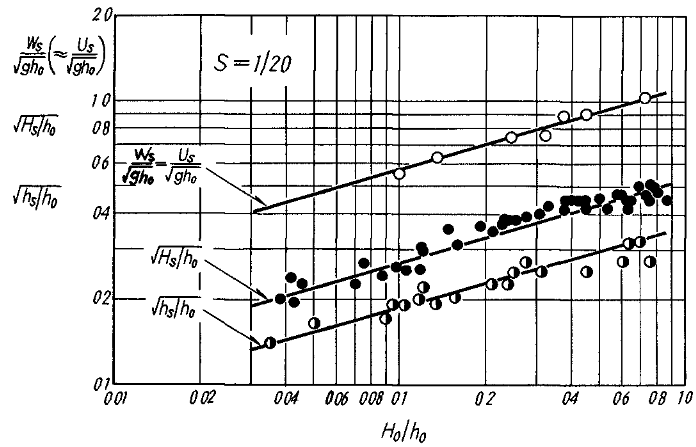


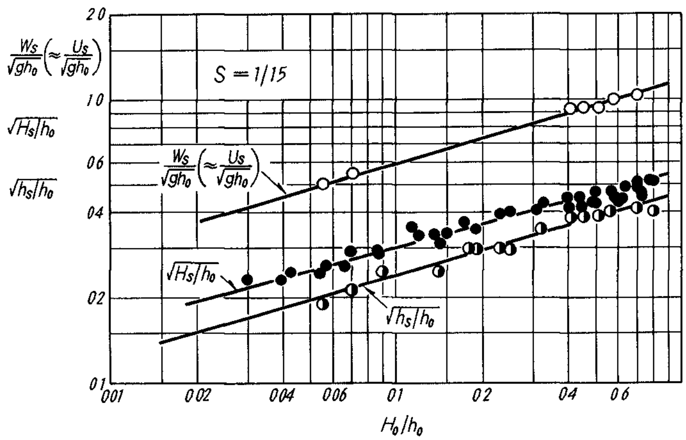
Fig. 14. Comparison of theoretical relationship among X_p and H_0/h_0 with measurements.



a



b



c

Fig. 15. Relations of $W_s/\sqrt{gh_0}$, $\sqrt{H_s/h_s}$, and $\sqrt{h_s/h_0}$ to H_0/h_0 for three values of slope.

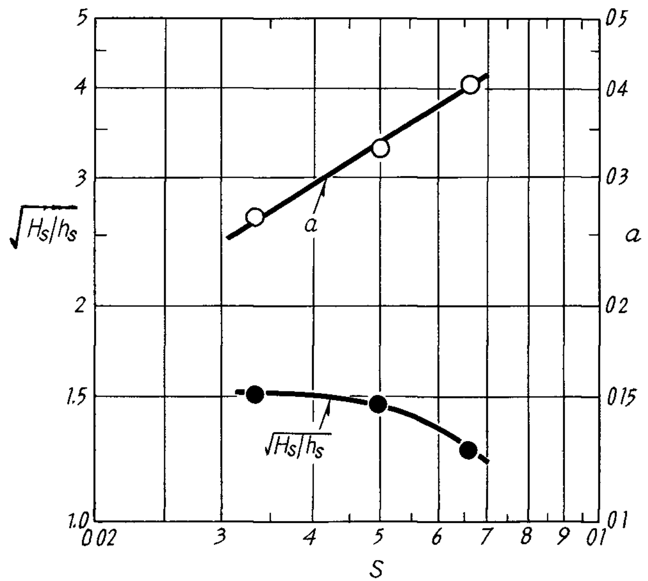


Fig. 16. Relations a and $\sqrt{H_S/h_S}$ to S.

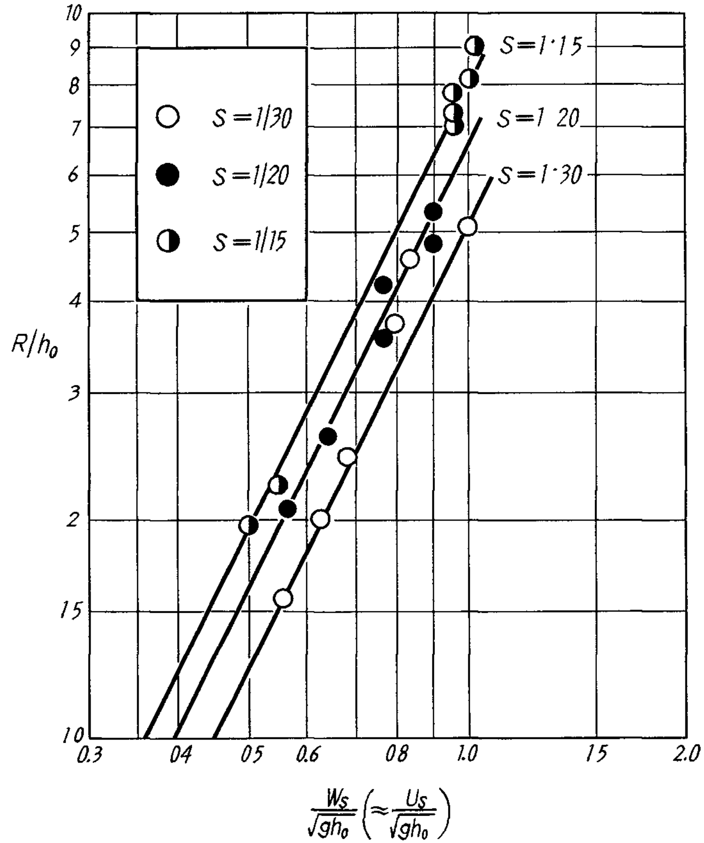


Fig. 17. Relation of R/h_O to $U_S/\sqrt{gho}$ for three values of slope.

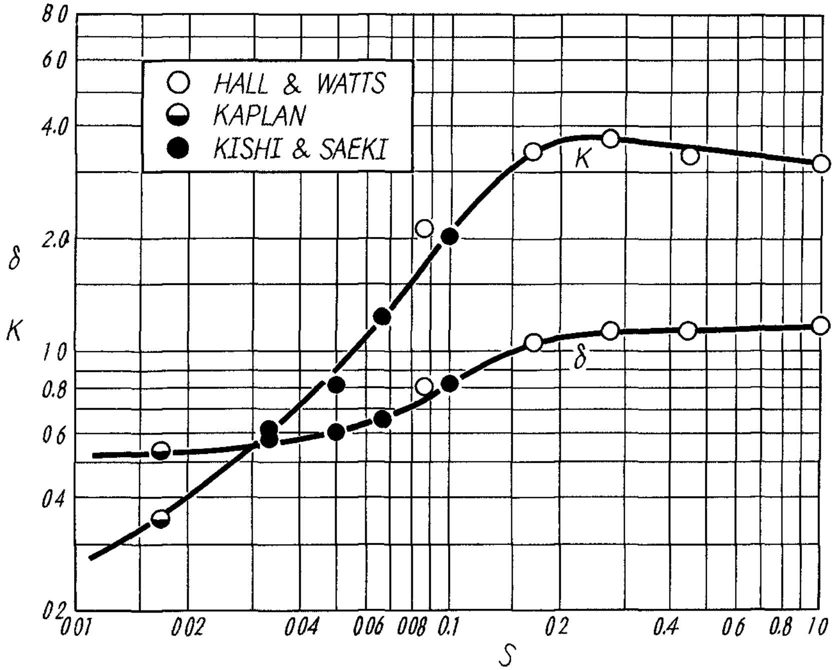


Fig. 18. Relations of K and δ to S.

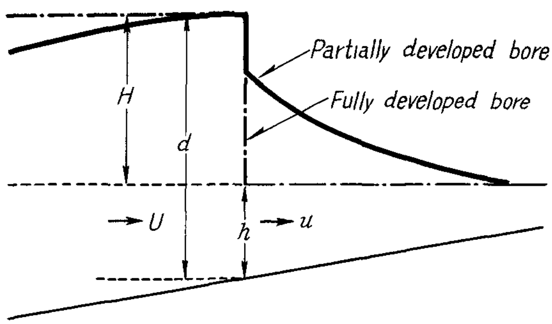


Fig. A-1. Definition sketch of partially and fully developed bores.