CHAPTER 58

THE EFFECT OF UNIT WEIGHTS OF ROCK AND FLUID ON THE STABILITY OF RUBBLE MOUND BREAKWATERS

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A. ABSTRACT

To study the effect of the specific weights of armour block material and fluid on the stability of rubble mound breakwaters a total of 110 model tests were made, with varying specific weights of armour and fluid, sizes of blocks and slopes of the breakwater face. The tests indicate that in cases where the specific weights deviate much from usual values, the current design formula (Eq. (1)) should be modified by entering a variable quantity, φ, instead of the figure "1" in the denominator. Within the scope of these tests, values varying from $\varphi = 0,37$ to $\varphi = 1,05$ were indicated. Theoretical considerations seem to show that also higher values of φ may be expected, in particular on account of the effect of the sloping water surface on the buoyancy of the armour blocks. As neither tests nor analysis have given conclusive evidence as to under what conditions higher respectively lower values of φ should be applied, at present only model tests can give the answer in any particular case.

B. INTRODUCTION

In locations where suitable rock material is easily available, rubble mound breakwaters with armour blocks of blasted rock, more or less arbitrarily placed, are often economically preferable. This applies to most breakwaters in Norway, and the stability problems relating to such structures therefore are of particular interest to us.

The specific weights of rock and fluid are important factors in the conditions of stability of such breakwaters. Practically all current design formulae have this form:

$$Q = \frac{H^3}{c^3} \frac{\gamma_r}{(\gamma_r/\gamma_f - 1)^3} = \frac{H^3}{c^3} R(\gamma) \qquad (1)$$

where Q is the weight of individual blocks necessary for stability, H is the wave height, γ_r and γ_f are the specific weights of rock and fluid respectively and c is a factor representing all other variables. (In Hudson's well known formula (1), $c^3 = (K_\Delta \cot\alpha)$ where α is the slope angle). The above form of the function $R(\gamma)$ has been derived by calculating the buoyancy of the blocks as it would be if the downrushing water were at rest with a horizontal surface. On the whole the experimental evidence in support of the above form of $R(\gamma)$ so far published seems to be somewhat incomplete.

As in most cases the specific weights do not deviate much from those for which the factor c in Eq. (1) was originally determined, the point is mostly unimportant. But in special cases it may be economically preferable, or even necessary, to use materials with unusual specific weights, in which case the resulting influence on stability becomes a matter of considerable interest.

The purpose of this paper is to present the results of rssearch regarding this problem, carried out during the years 1961-1965 at the River and Harbour Research Laboratory at the Technical University of Norway, in Trondheim. *) In short, our tests indicated that the figure "1" of Eq. (1) should be replaced by a variable quantity, φ, which may vary from values of less than one-half to considerably more than one.

The investigation comprises two fairly comprehensive series of model tests, and an attempt at analytical treatment of the process of failure. This attempt has not given any full sxplanation of the above indications. In certain respects, however, the analysis has yielded results so far in agreement with experimental data, that its basic concepts may, it would seem, derive some support therefrom.

C. THE TESTS

Two series of tests were made, Seriss 1, the most comprehensive one, by Mr. Olaf Kydland, in 1961 and 1962, as part of his work for the degree of Licentiatus Technicae, and Series 2 by Mr. Alf T. Sodefjed, in 1965, as part of his work for the degree of Civil Engineer (2), (3) and (4), supervised by the writer.

The scope of the two series of tests is shown in Tables I and II. In Series 1 only one slops of the breakwater model front was ussd, the slope of 1 in 1,5, most commonly used in actual construction in Norway. Broken natural rock with three different specific weights were used, and in addition blocks broken from a cast of cement and plaster of Paris with sand. For each specific weight parallel tests were made with three different sizes of blocks. An intermediate size was introduced for the 4,52 rock (pyrite), because armour of 105 cm^3 blocks of such heavy material could not be broken down in our wave channel.

The blocks were broken manually. Great care was taken to have the blocks of each set of tests as uniform as possible, and at the same time to avoid any consistent diffsrencs in shape between blocks of different materials. The weights of individual blocks were kept within 10 % of the average for each group, and the ratio of the greatest to the smallest of the linear dimensions of sach block was kept below 2,5.

Also the specific weights of the fluid were varied by using, besides fresh water, solutions of NaCl with specific weights of 1,065 and 1,13.

In Series 2 three different slopes of the breakwater front were used: 1 in 1,25, 1 in 1,5 and 1 in 2. The same types of block material as in Series 1 were used, but in this series all blocks weighed about the same,which made the volume of any block smaller, the heavier its material. The specific

*) Later referred to as the RHRL.

weights varied slightly from those in Series 1 for the four different types. The greatest individual deviation from the average was about ± 2 % for the two lighter materials and about ± 5 % for the two heavier types. The dimensions of the blocks were kept within the same limits as mentioned for Series 1. Only fresh water was used in this Series.

The model of the breakwater front was built on a wooden slab, Fig.1, to eliminate variation in permeability. In Series 1 the slab was covered with two layers of secondary stones, the mean linear dimension of which varied with the size of cover blocks from about 1 cm to 3 cm. On top of this sublayer, two layers of the cover blocks, as described above, were placed. In Series 2, a similar arrangement was used, with the sublayer about 5 cm thick.

All tests were made in an ordinary wave channel, 60 cm wide with depth of water 70 cm, (14)*). Each test was started with a wave height well below that causing damage. The height was then raised in decreasing increments as the range of damage was reached.

In Series 1 the periods were chosen so as to have in all cases as nearly as possible the same steepness of wave at breakdown of the model. In Series 2 the period was 1,8 s in all tests recorded here.

The wave generator was run continously for 20 min in Series 1 and 15 min in Series 2 at each wave height. Secondary reflexion from the paddle could not be entirely avoided, but the model was built on wheels as done by Hedar (5), and was moved in each case to a position where the uprush with and without this secondary reflexion were practically equal.

In all tests the degree of damage was noted, as the wave height was increased. The extent of damage was given as the percentage of the total number of cover blocks within a certain specified region, which had rolled down the slope. The wave height corresponding to a given percentage was determined by linear interpolation.

A major problem was how to build all the models sufficiently alike. It proved difficult to avoid a certain improvement of the stability of the model as the routine of the operators improved with time. In fact the first set of tests of Series 1 had to be discarded, because the date on which the model had been built, appeared as a dominant variable in the results.

This difficulty was, it is believed, fairly well overcome in the subsequent tests, in the first place by adopting a strictly standardized method of building the model, and in the second place by making up the test programme so that a possible effect of improved routine on the average results should be about the same for all combinations tested.

In Series 2 this plan could not be followed throughout, because the tests with slope 1 in 1,5 were decided on only after the other tests were completed. This may partly explain why models with slope 1 in 1,5 apparently were more stable than those with slope 1 in 2, as seen in Table VI.

The standard method adopted for placing the cover blocks was not quite the same in the two series.

*) See Fig. 7 of References (14). Figure 2, therefore, is omitted from this paper

In Series 1 the blocks of the first layer were dropped on to the sublayer at a point about twice the expected wave height for "no damage" above the SWL, and from there rolled down the slope till it stopped against the blocks already placed. If it stopped earlier, the rolling was started again by touching with a finger. The eecond layer of cover blocke was placed in the eame way, but here the finger assistance was more frequently needed, becauee of the greater roughnese of the elope on which the blocks must roll.

In Series 2 each block was dropped ae directly as possible into its intended place. Blocks belonging below the SWL were dropped from the water surface and those belonging higher up from a height of some 5 - 10 cm above the breakwater face. The upper edge of the part of a cover layer already placed was kept sloping from one side of the wave channel to the other, at an angle of about 45° against the axis of the wave channel when seen normally against the face of the breakwater. Thereby each individual block was guided sideways into a position where it mostly came to rest against two of the blocks previously placed, instead of just one. It was found that this method gave greater stability than that used in Series 1. Thie difference should be more pronounced the steeper the breakwater front is. That may be part of the reason for the relatively low stability found with a slope of 1 in 2 in this series.

In both Series, at least three identical tests were made with each slope, each size of blocks and each combination of specific weights.

In Series 1, if any one of these three tests gave results deviating more than ± 10 % from the average of the three, that test was discarded and a new one made. With 3 x 20 = 60 programmed tests, only two individual resulte were discarded due to this 10 %-rule, while three more were diecarded due to other irregularities discovered during the tests, although their results were within the 10 % limit.

In Series 2, a somewhat stricter rule was used, requiring that the total difference between the maximum and minimum results of identical teets should not exceed 10 % of the average. Here 3 x 12 = 36 tests were programmed, but several tests were repeated more than twice, so that in total 55 tests were made. Of theee 45 gave results within the adopted limit, while 10 fell outside, mostly for obvious reasons.

The wave data pertaining to the tests of the various combinations of specific weights, sizes of blocks and slopee of the breakwater face summarized in Table I and II, may be seen from Tables III and IV.

D. PRESENTATION OF RESULTS

The results of the tests are most easily preeented by bringing Eq.(1) on linear form, and introducing a possibly variable φ instead of the fixed quantity, 1. Eq.(1) may be written:

$$H/k = \lambda = D(\gamma_r/\gamma_f - \varphi) \qquad (2)$$

Here Q is replaced by $\gamma_r \cdot V = \gamma_r \cdot C_V \cdot k^3$, where V is the volume and k is a characteristic linear dimension of the cover block in question, C_V is a "coefficient of volume" and $D = C_V^{1/3} \cdot c$.

If the general form of the widely accepted Eq.(1) is reasonably correct, aside from the value of φ, then the observed values of $\lambda = H/k$ should, when plotted against γ_r/γ_f as abscissa, group themselves about a straight line, which line will define the values of φ and D.

In Fig. 3 the average values of H/k from each set of three parallel tests for each one of the 20 combinations indicated in Table I for Series 1 have been plotted, as observed with 1 % of damage. It is seen that the data are all reasonably close to the straight line drawn in full, which corresponds to φ = 0,44 and D = 0,99. The data for the heaviest rock material tested, pyrite, fall somewhat below the line, which is more or less evident throughout both series of tests. Naturally the drawing of the best fitting straight line may be disputed, but it would hardly seem reasonable to draw the line so as to bring the value φ closer to 1 than indicated in Fig.3.

Actually the line has been drawn after study of similar diagrams for each of the five groups of combinations tested in Series 1, shown in Figures 4 to 8. In these diagrams have been plotted the maximum and minimum and the mean value of λ found in each of the three individual tests made for each combination, at 1 % of damage. In the same figures the straight lines corresponding to higher percentages of damage have been shown. For the sake of clarity the data themselves have not been included, but the agreement with the straight lines is as good as for 1 %, or better.

For each of the five groups of combinations, values of φ and D corresponding to 1 % and to 4 % of damage have been taken off the diagrams and tabulated in Table V.

The results of the tests of Series 2 for 1 % of damage have been similarly plotted in Fig. 8. As practically the same block weight was used throughout this series, there is just one group of combinations for each value of the angle of slope, α. Corresponding values of φ and D have been taken off these diagrams and entered in Table VI. Similar diagrams for 10 % of damage have been plotted (not shown) and values of φ and D shown in Table VI.

It is seen from the diagrams and tables that higher values of φ are consistently found for higher percentages of damage, that is for higher stability of the remaining blocks on the breakwater front. Similarly Series 2 gave higher values of φ than Series 1, as well as higher stability.

During the tests notes were carefully taken of the locations on the slope from which blocks were successively washed away. In Fig.10 is shown how the damage was distributed over the slope, relatively to the wave heights, for all tests of both series.

E. CONDITION OF STABILITY

A full theoretical explanation of the variation of φ indicated by the tests would be most desirable, but the problem is very complicated, and no full solution, however approximate, has as yet been found. Nevertheless, a rational study of the conditions of stability of the armour blocks on a rubble mound breakwater slope, based on fairly reasonable assumptions, may be of some value in clarifying part of the problem.

Any such study must be based on a certain concept of the mode of failure of an irregular block of stone forming part of the cover layer on a rubble mound breakwater, as it is being washed away by the downrushing water. *) The main question is: What will, in most cases, be the initial movement of such a block ?

Some investigators, among them Svee (7), have assumed that at certain moments some block may become entirely free of restraint from neighbouring blocks and be thrown right out into the downrushing stream. I have no doubt that this may, and occasionally does occur. It was expressly noted by Kydland, who performed with acute observation the tests of Series 1, that very often there seemed to be a lockering of the cover layer around the SWL, before real damage started. Probably some few blocks may then have become entirely free of restraint.

Nevertheless, the writer is inclined to believe that the mode of failure assumed by Hedar (5), whereby the moving block rolls away, initially in contact with its downstream neighbour, corresponds more nearly to what usually happens. It is hard to see how a block, once it starts to lift from its base, can avoid being pressed by the downrushing stream against its neighbour below.

On this basis, and referring to the force diagram in Fig.11, we shall study the condition of stability of a block "n" against rotation about its point of support, A_n, on block "n + 1" below. Block "n" may, or may not, be steadied by contact with the block "n - 1" above. **) In this Section we shall assume that it is not, see Section H .

If block "n" is free of contact with block "n - 1", its stability depends largely on the angle θ. Blocks who happen to have the smallest angle θ will, other conditions being equal, roll away first.

The forces to be considered are the weight of the block, Q, its buoyancy, B, (which is not directed vertically and is not equal to $\gamma_f V$) a drag force, F_{DP} and an inertial force, F_{MP} both expected to act parallel to the slope at some distance $\varepsilon\, k/2$ above the center of gravity of block "n", and a lift force due to the parallel velocity, F_{LP} . Finally there is introduced an hypothetical normal force, F_h, directed downwards and proportional to the volume of block n, not to its projected area. This hypothetical force will be discussed later.

In the "detailed summary" previously printed, also a normal drag force due to a supposed current directed out of the breakwater body was included. Subsequent study has indicated that within the region close to the SWL any normal velocity may be quite small and may possibly even be directed into, not out of the breakwater body. The assumption of an outward normal drag force of any consequence has therefore been dropped.

*) With the slopes of breakwater front here considered, and aside from occasional "shock forces" from uprushing waves, failure is regularly caused only by the downrush, as shown by Hedar (5).

**) Of course, the real configuration of blocks is not two-dimensional, as in Fig.11, and a block may be held by more than one downstream and one upstream neighbour. This, however, can not materially alter our reasoning.

The forces on block "n", Fig. 11, may be written:

$$\left.\begin{aligned} Q &= C_{\forall}\,\gamma_r\,k^3 \\ B_N &= C_{\forall}\,\gamma_f\,k^3\cos\alpha \\ B_P &= C_{\forall}\,\gamma_f\,k^3\cos\alpha\,\tan\beta \\ F_{DP} &= C_{AP}\,k^2\,C_{DP}\,\gamma_f\,C_{VP}\,H \\ F_{MP} &= C_{\forall}\,k^3\,\gamma_f\,C_{MP}\,a_P/g \\ F_{LP} &= C_{AN}\cdot k^2\,C_{LP}\,\gamma_f\,C_{VP}\,H \\ F_h &= C_h\,\gamma_f\,C_{\forall}\,k^3 \end{aligned}\right\} \quad (3)$$

As stated before, k is a characteristic linear dimension of block "n". H is the height of the regular waves in the wave channel, and a_P is the acceleration of the downrushing stream at block "n". The various coefficients, C, will be discussed in Chapter F.

Block "n" will be stable against rotation about point A_n, Fig.11, if

$$Q\,k/2\,\sin(\theta-\alpha) + B_P\,k/2\,\cos\theta + F_h\,k/2\,\sin\theta \geq$$

$$F_{DP}\,k/2\,(\varepsilon+\cos\theta) + F_{DM}\,k/2\,(\varepsilon+\cos\theta) + F_{LP}\,k/2\,\sin\theta \quad (4)$$

By entering equations (3) in Eq. (4) and arranging the terms we arrive at the following condition of stability of block "n" : *)

$$\frac{k}{H} = \frac{1}{\lambda} = \frac{A_1\mu_1 + A_2\mu_2}{\gamma_r/\gamma_f - [\psi + C_M\,\mu_1\,a_P/g - \mu_2\,C_h]} \quad (5)$$

where:

$$A_1 = \frac{C_{AP}\,C_{DP}\,C_{VP}}{C_{\forall}} \qquad A_2 = \frac{C_{AN}\,C_{DN}\,C_{VN}}{C_{\forall}} \quad (6)$$

$$\mu_1 = \frac{\varepsilon+\cos\theta}{\sin(\theta-\alpha)} \qquad \mu_2 = \frac{\sin\theta}{\sin(\theta-\alpha)} \quad (7)$$

$$\psi = \frac{\tan\theta-\tan\beta}{\tan\theta-\tan\alpha} \quad (8)$$

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*) It is interesting to note that in principle the "Initial Motion Condition" given by Kamphuis,1966 (12) is identical with Eq.(5) as far as the hydraulic situations treated are alike.

F. DISCUSSION OF COEFFICIENTS

1. Shape coefficients: The projected area of block "n" in parallel and normal direction is $A_P = C_{AP}\ k^2$ and $A_N = C_{AN}\ k^2$, respectively. The volume of the block, as defined earlier, is $C_V\ k^3$. The values $C_{AP} = C_{AN} = 1{,}0$ and $C_V = 0{,}5$ represent fairly well the actual shape of the blocks.

2. Drag coefficients in parallel flow, C_{DP}: Reynold's Number in our cases is mostly between 10^4 and 10^5 at critical stages and the corresponding value of C_D for a smooth sphere, given in current literature, is about 0,4 to 0,5. From the ordinary Prandtl friction formula Hedar (5) deducted for the waves on a break-water front a boundary resistance corresponding to

$$\frac{1}{16(\log_{10} \frac{14{,}8}{k} z)^2},$$

where z is the depth of water above the armour blocks, normally to the slope. Subsequent tests by Andersson (9) have indicated that with very rough slopes the figure 14,8 should be replaced by about 5. Using this figure, and assuming values of z as found in an earlier study (8) at the critical stage of downrush, values of C_{DP} of 0,3 to 0,4 are found. The value $C_{DP} = 0{,}35$ is chosen for use.

3. Lift coefficient in parallel flow, C_{LP}: The greater parallel velocity above than below a block will create a lift force. Little is known on which to base an assumption as to the size of this force. From tests on pipes placed on the bottom, the Hydraulic Research Station at Wallingford (10), 1961, reported lift forces from about 3/4 to about 1/2 of the corresponding drag forces,(pp 2 and 3), while Johansson, ((11, p. 32), reports lift forces up to twice the drag force. Here is assumed $C_{LP} = C_{DP} = 0{,}35$.

4. Coefficient of parallel velocity, C_{VP}: Velocities up and down a breakwater slope are generally taken to be related to the wave height by the equation: $v_P = \sqrt{C_{VP} \cdot 2gH}$. Here our concern is with the maximum velocities around SWL during downrush. From the mathematical model previously presented, (8), it is found that $C_{VP} = 0{,}35$ seems to be a reasonable assumption.

5. Coefficient of mass, C_{MP}: For reasons stated in (6), $C_{MP} = 1{,}5$ has been assumed.

Entering values of coefficients 1 to 4 in Eq. (6) we obtain

$$A_1 = \frac{1{,}0\ \ 0{,}35\ \ 0{,}35}{0{,}5} = 0{,}245$$

$$A_2 = \frac{1{,}0\ \ 0{,}35\ \ 0{,}35}{0{,}5} = 0{,}245$$

In the stability condition, Eq.(5), A_1 and A_2 represent the general conditione, as determined by the general shape of blocke and by the hydraulic relations involved. On the other hand, the factors μ_1 and μ_2 represent the geometrical stability conditions of those individual blocks which, at the moment considered, are just about to be carried off.

The values assumed for the coefficients can all be disputed, but it is believed that none of them should be considered directly unreasonable. If fair agreement with test result can be shown by applying the same values of A_1 and A_2 to all combinations of specific weights, block sizes and slope angles, that might be taken to indicate that our condition of stability may not be too unrealistic.

The geometric stability factore, μ_1 and μ_2 depend, when the angle of slope, α is given, only on the fraction, ε , and the angle, θ. The former is, of course, unknown, but does not play an important part in the calculations. A value, $\varepsilon = 0{,}15$ has been used here. Using other values, like 0,10 or 0,20 does not ohange the following argument, it just leads to slightly different "best fit values" of θ.

The same percentage of damage should represent the same stability condition and therefore the same value of θ, irrespective of specific weights, sizes of blocks or angels of slope, as long as we are dealing with armour layers that have been constructed alike.

G. CALCULATION OF φ

Eq. (5) gives λ as a linear function of γ_r/γ_f, like Eq. (2), provided the other members in the equation are independent of γ_r/γ_f:

$$\lambda = \frac{\gamma_r/\gamma_f - [\psi + C_M \mu_1 \, a_P/g - \mu_2 C_h]}{A_1\mu_1 + A_2\mu_2} \tag{9}$$

The two equations (2) and (9) then must be identical:

$$\varphi = \psi + C_M \mu_1 \, a_P/g - \mu_2 C_h \tag{10}$$

$$\lambda = \frac{\gamma_r/\gamma_f - \varphi}{A_1\mu_1 + A_2\mu_2} \tag{11}$$

$$\varphi = \gamma_r/\gamma_f - \lambda(A_1\mu_1 + A_2\mu_2) \tag{12}$$

$$D = \frac{1}{A_1\mu_1 + A_2\mu_2} \tag{13}$$

If the angle θ is known, φ can be calculated from Eq. (11), with the values of coefficients stated in Section F. By trial calculation it is possible to determine those values of θ which will agree most closely with the experimental values of φ, taken off the diagrams in Figures 3 to 9. Such "best fit values" of θ have been calculated for each series (each method of construction) and for two percentages of damage within each series. The following "best fit values" were found:

For Series 1, with 1 % of damage: θ = 56°
" " 1, " 4 % " " : θ = 62°
" " 2, " 1 % " " : θ = 66°
" " 2, " 10 % " " : θ = 73°

The values of φ and D, thus calculated from Equations (11) and (12), have been entered in Tables V and VI for comparison with the experimental values. It is seen that while there are some differences, these are mostly quite small, especially in view of the fact that Eq. (11) gives φ as the difference between two numbers, the smaller of which is at least twice the difference.

The general requirement stated at the end of Section F thus is fairly well well satisfied. Also a higher "best fit value" of θ is found for the higher percentages of damage, and higher values for Series 2 than for the less stable models of Series 1, all of which agrees with what must be expected.

While this agreement certainly is no proof of the correctness of the experimental results and of the condition of stability arrived at, it may possibly be taken as an indication that the results may deserve a certain degree of confidence.

H. THE ANGLE θ AS A PARAMETER OF STABILITY

So far we have assumed that our armour block "n", Fig.11, is not steadied by any contact with its upstream neighbour, "n - 1". If, however, it is so steadied, a certain force, P_{n-1}, acting from block "n - 1" on block "n" must be included in our stability relations.

It seems reasonable to assume that the set of forces acting on block "n - 1" at the moment of critical forces on block "n" near the SWL, will not be very different from the set acting on block "n". If this is so, the force P_{n-1} may be considered as composed of a certain fraction, p, of the same forces as those already discussed for block "n", including weight and buoyancy. Based on this assumption, calculations have been made, assuming different values of the fraction, p. It has been found that entering such a force P_{n-1} does not materially alter the calculations, the only effect being that the "best fit values" of the angle θ are lowered somewhat. For instance, p = 0,2 leads to 3° to 5° lower values of θ than p = 0.

This means that the stabilizing effect of a force P_{n-1} is roughly equivalent to a certain increase in that value of θ which is necessary for stability. It appears, therefore that the angle, θ, may usefully be considered as a <u>general parameter of stability.</u>

I. THE "HYPOTHETICAL FORCE", F_h.

While fair agreement between the experimental values of φ and those calculated from Eq. (11) is easily obtainable, the matter with regard to the other equation for φ, Eq. (10), stands quite differently. The first member, ψ, must always be greater than 1. The variation of ψ with θ and α is shown in Fig. 11, for $\tan\beta = 0{,}40$, and it is seen that in particular with the smaller values of θ, ψ may easily reach values of 1,4 or more. The second member on the right hand side of Eq. 10 must also be positive, and is not negligible. It seems reasonable to use for the acceleration down the slope the values estimated in (8) for the time when the boundary forces at the SWL pass their maximal value ((8), Table IV, p. 459). Assuming for a_p a value of about 0,1 g, with $C_{MP} = 1{,}5$, the second member amounts to about 0,20 for the case of 4 % of damage in Series 1.

Consequently, unless there is a third, negative member, due to our "hypothetical force", F_h, or other causes, only values of φ greater than 1 can satisfy Eq. (10).

It may be of some interest to see, if a force like F_h should exist, what must be the value of the "hypothetical coefficient", C_h, to make Eq. (10) agree with the experimental values of φ. Therefore, values of C_h have been calculated from Eq. (10) for each of the combinations of specific weights, block sizes and slope angles included in the tests, using in each case the experimental value of φ. In the calculation of ψ, $\tan\beta$ has been determined from Eq. (7) of reference (6).

The values of C_h thus determined have been entered in Tables V and VI. It is seen they do not vary much. The mean values of C_h and the corresponding standard deviations, σ, are

For Series 1, with 1 % of damage: $\bar{C}_h = 0{,}525$, $\sigma = 5{,}7$ %
" " 1, " 4 % " " : $\bar{C}_h = 0{,}472$, $\sigma = 11{,}0$ %
" " 2, " 1 % " " : $\bar{C}_h = 0{,}380$, $\sigma = 12{,}9$ %
" " 2, " 10 % " " : $\bar{C}_h = 0{,}306$, $\sigma = 8{,}3$ %

(in the last figure, the values for $\cot\alpha = 2{,}0$ have been left out)

Considering the wide variety of conditions included in the tests, the moderate variation in C_h seems remarkable, considering that the individual experimental values of φ were used in the calculation.

Still, it is possible, although hardly very probable, that the agreement found may be accidental, as it has not been shown that a force like F_h does actually exist. To enter into Eq. (10) F_h must be proportional to the volume of the block. It seems reasonable, then, to look for a regular inertial force, due to an accelerated stream into the breakwater body, or a retarded stream out of it. Attempts at showing the exixtance of such accelerations so far have not succeeded.

It may seem difficult to accept the notion of a force like F_h, in view of the fact that important normal forces directed out of the breakwater have been observed in several investigations, most clearly, perhaps, by Sigurdsson (13).

It should be noted, however, that we are concerned here with the situation slightly below, but quite close to the SWL, where the bulk of the damage took place in our tests (see Fig. 10), while the great upward normal forces have mainly been observed at points further down the slope, close to the trough between downrushing and oncoming wave. While at the SWL or slightly below, the water surface is at its steepest, further down it flattens out and the slope is even reversed. The great effect of surface slope on the pressure distribution in the fluid (see (8), Eq. (3), p. 448) may well be one cause of a force like F_h. In fact, while numerical evaluation is difficult, there are indications in several of Sigurdssons diagrams of negative (upward) normal forces close to the SWL at certain stages of the wave cyclus.

Finally, in the highly turbulent and most complicated stream of downrushing water around and over the armour blocks there seems to be ample opportunity for development of forces like F_h, proportional to the volume of the blocks, although the demonstration of such forces, either by experiment or by theory may be most difficult.

It is concluded, therefore, that the possibility of a force like F_h should not be excluded, as far as the case of armour layers of irregular blocks of blasted rock irregularly placed is concerned. In the case of regularly shaped blocks, regularly placed and even bonded, with an all over more smooth breakwater face, the situation may well be quite different.

J. PRACTICAL CONSEQUENCES

If the indications of the present study should be proved in the main correct, if it has to be accepted that φ may assume values as different from 1 as, say 0,5 and 1,2, not to go to extremes, such values will have to be taken into consideration in the design of rubble mound breakwaters where the use of material with very unusual specific weights are contemplated.

If any of the current design formulae are employed, the correct value of φ should be entered, instead of 1. At the same time, of course, the coefficients of the formulae must be changed so as to give correct block weights at some usual value of γ_r.

In Table VII an example has been shown, based on Hudson's formula (1) with $K_\Delta = 3{,}2$ at $\gamma_r = 2{,}65$. It is seen that with values of γ_r close to normal, the difference is not great, but with value like 3,5 or 2,3 the difference should be taken into account, and with still higher or lower values the difference may be decisive.

There remains, however, the big question, what will be the correct value of φ in any particular case. While certain indications can be had from the study here presented, a prediction would be hazardous. Therefore, with unusual specific weights, the only safe procedure at present seems to be to base the design on direct model tests with the materials in question, and with all conditions, including those of building the breakwater, as close to reality as possible.

It is to be hoped that further study of the problem will make safe design recommendations possible.

K. CONCLUSIONS

1. The tests indicate that it may be advisable to replace the term $(\gamma_r/\gamma_f - 1)$ in current design formulae for rubble mound breakwaters (Eq.(1)) by $(\gamma_r/\gamma_f - \varphi)$, where φ is a variable quantity. Within the scope of these tests values of φ ranging from 0,37 to 1,05 were found.

2. The tests are believed to be representative, ae great care was taken to eliminate irrelevant variables and the agreement between the various test results seems quite satisfactory.

3. While no full theoretical explanation of the results is given, an analysis of the stability condition of an armour block on a breakwater slope has yielded results in good agreement with the experimental onee.

4. The assumption of a normal force directed into the breakwater and proportional to the volume of the block leads to quite consistent resulte as regards the magnitude of such a force which would be required for stability under the various test condition.

5. The analysis indicated that values of φ exceeding those found in these experiments may well occur.

6. Experiments and analysis both indicate that greater values of φ are to be expected, the more stable the placing of the armour blocks has been. Also, φ increaeed with increase in $\cot\alpha$, within the range of $\cot\alpha$ = 1,25 to 2,0.

7. The present inveetigation is insufficient to permit definite predictions as to what value of φ to expect in particular cases. Therefore, where quite unueual specific weighte occur, it is recommended to resort to model tests in each case.

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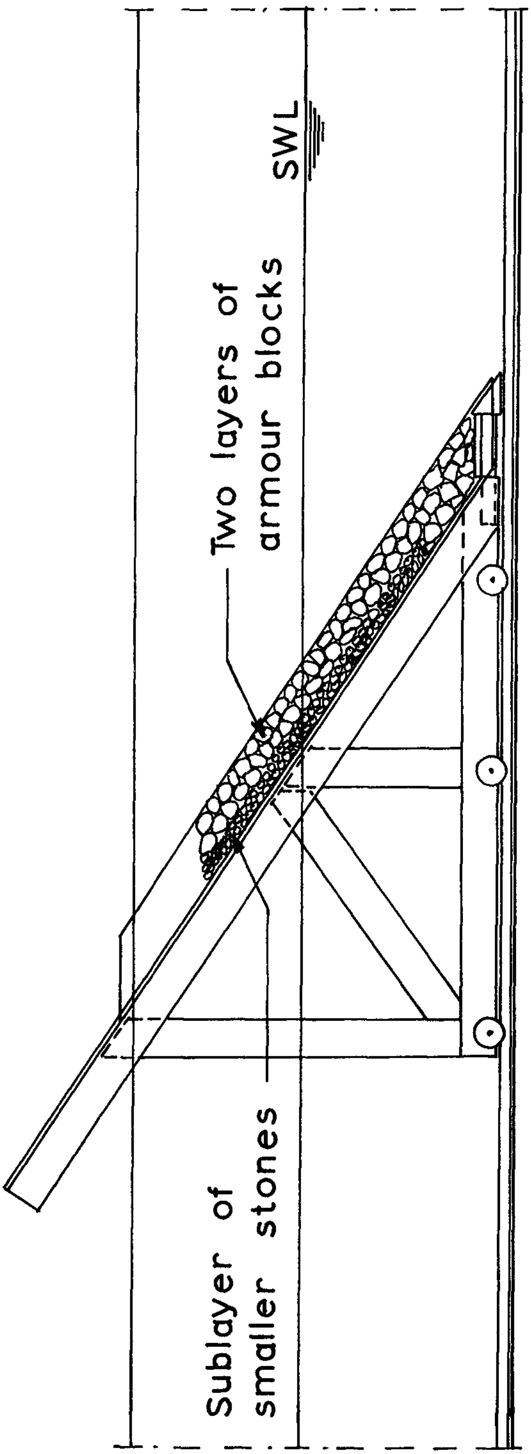

Fig. 1. The breakwater model.

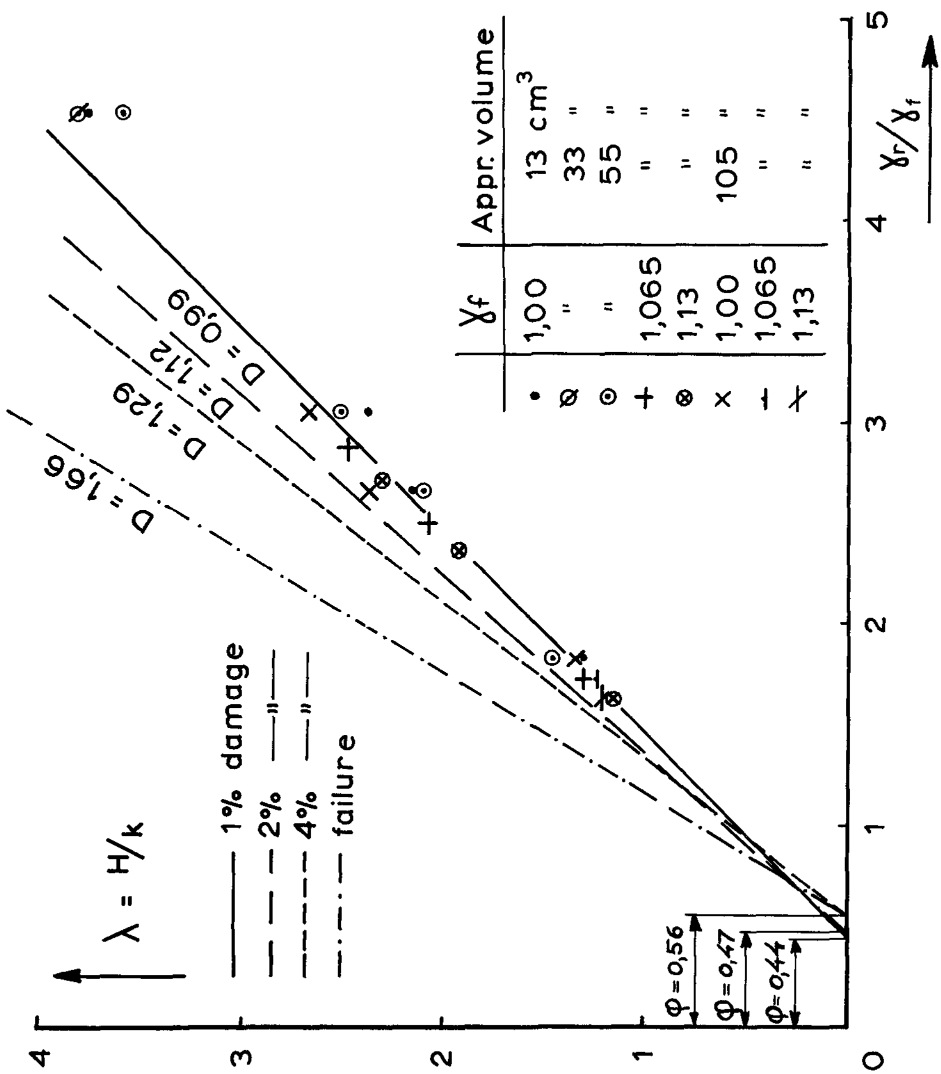

Fig. 3. Results of all Tests of Series 1. Data from Olaf Kydland (2)

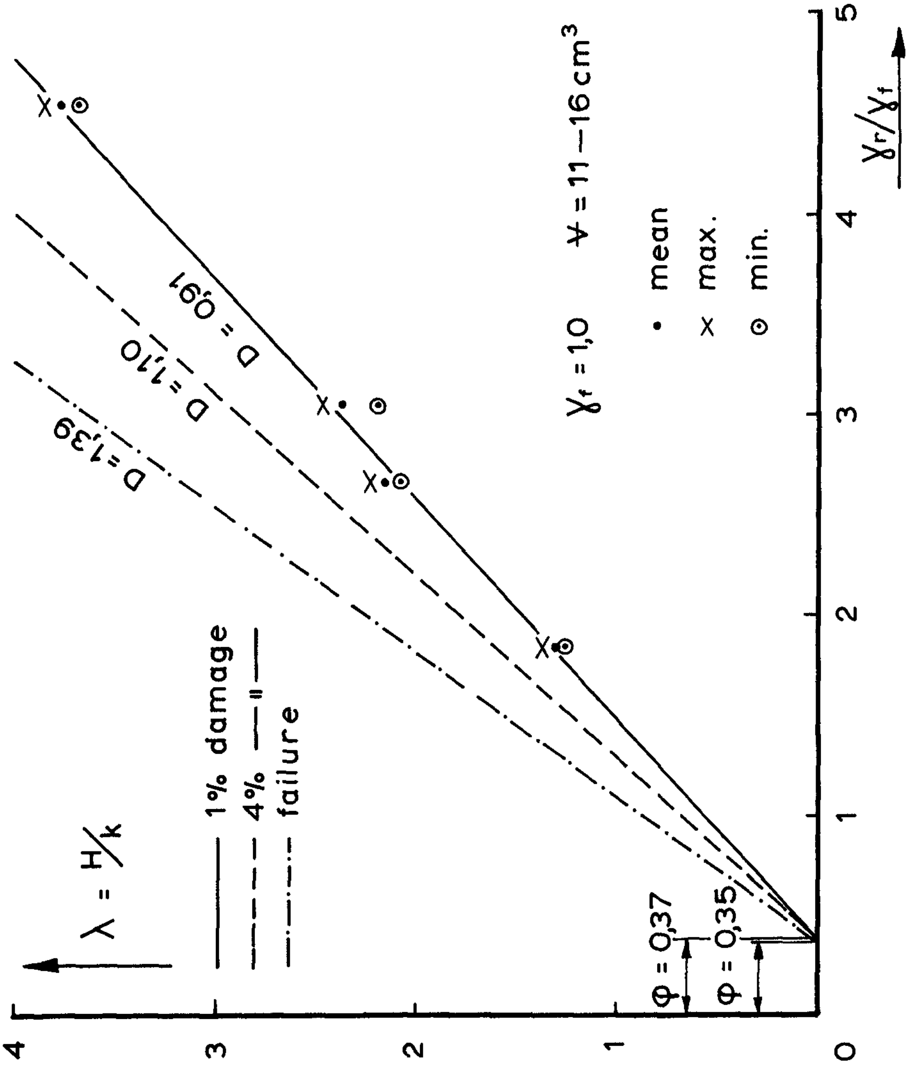

Fig. 4. Results of Tests, Series 1, Group I. (Table I) Data from Olaf Kydland (2)

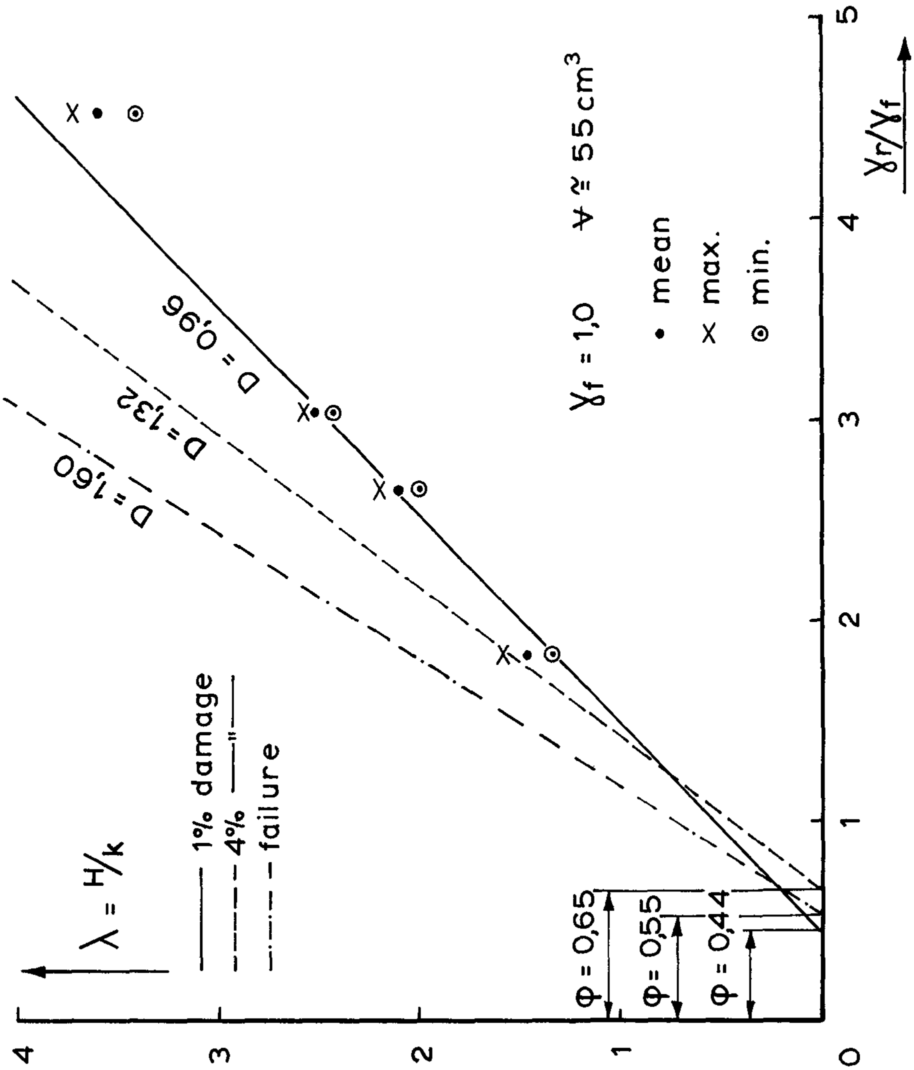

Fig. 5. Results of Tests, Series 1, Group II. (Table I) Data from Olaf Kydland (2)

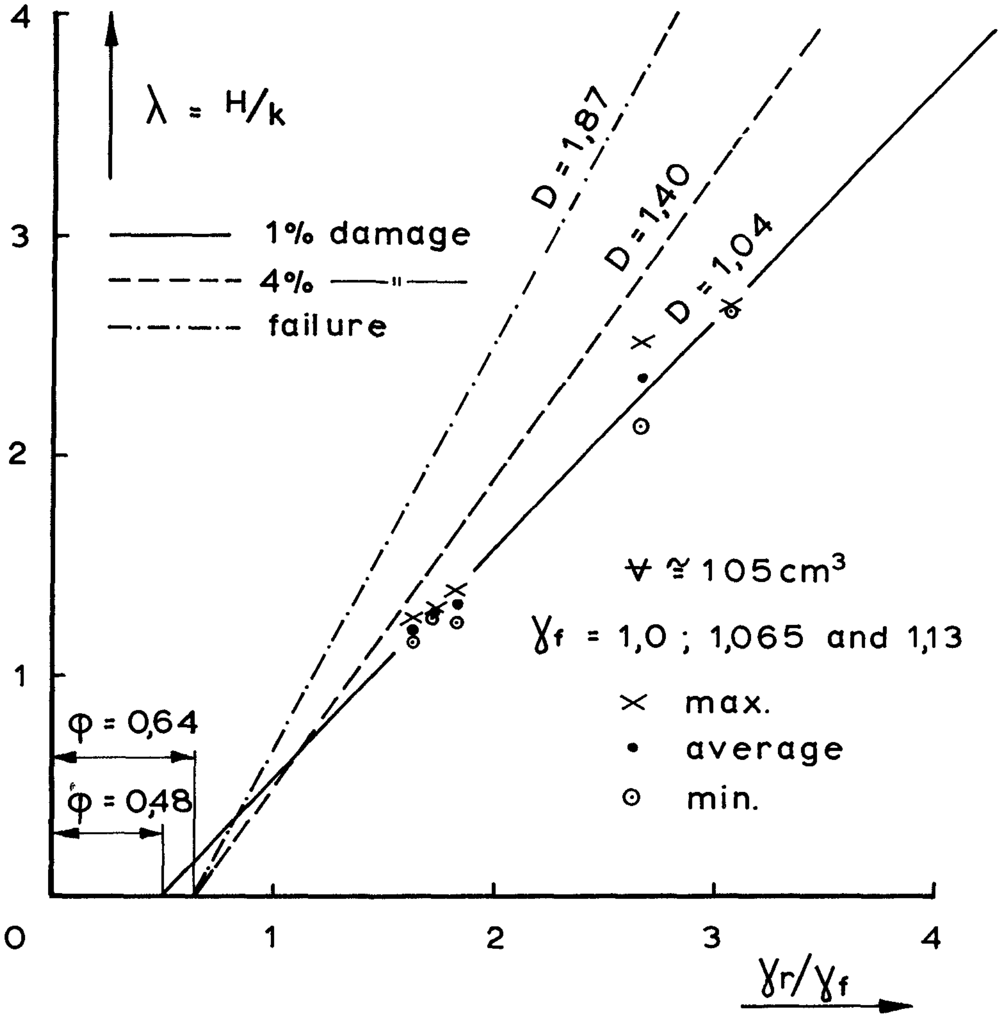

Fig. 6. Results of Tests, Series 1, Group III.
(Table I) Data from Olaf Kydland (2)

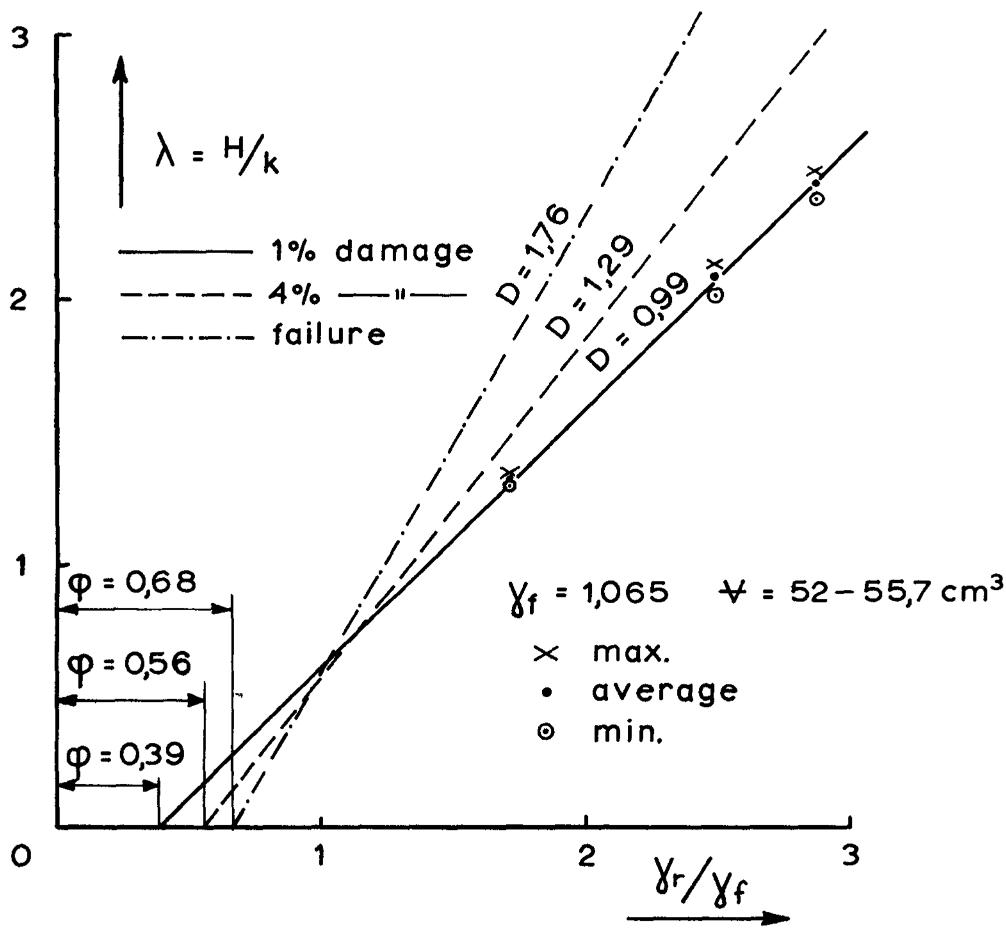

Fig. 7. Results of Tests, Series 1, Group IV. (Table I) Data from Olaf Kydland (2)

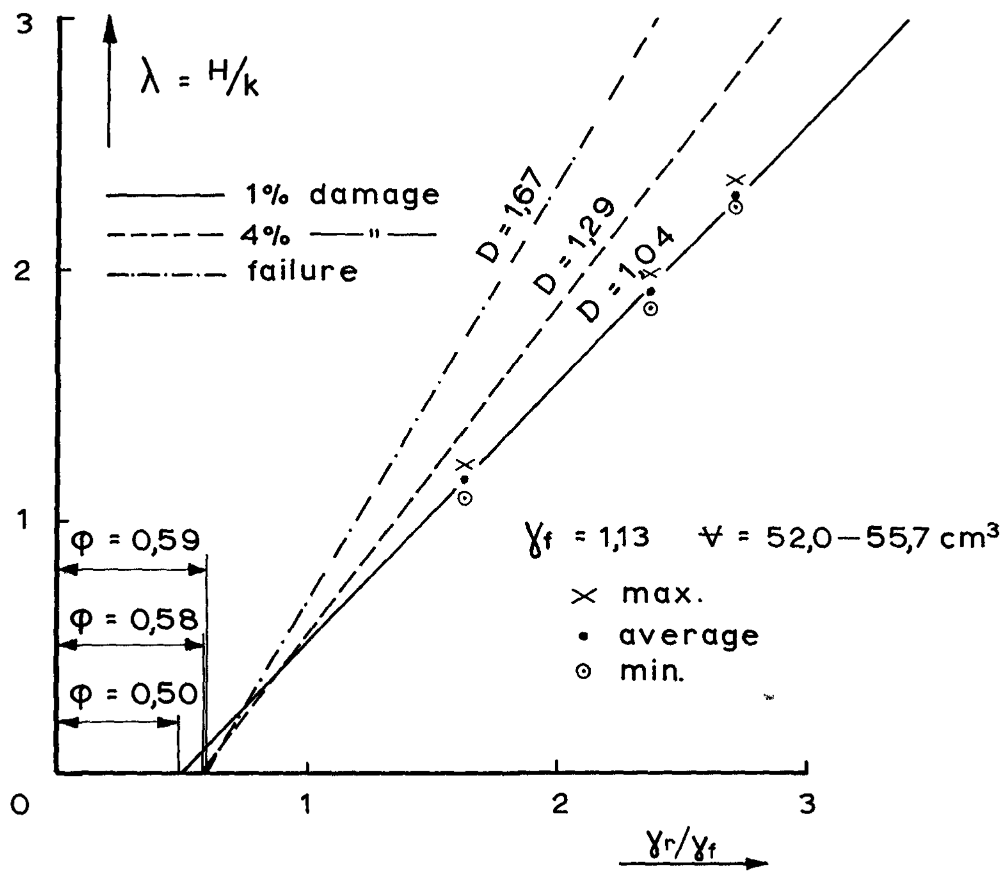

Fig. 8. Results of Tests, Series 1, Group V. (Table I) Data from Olaf Kydland (2)

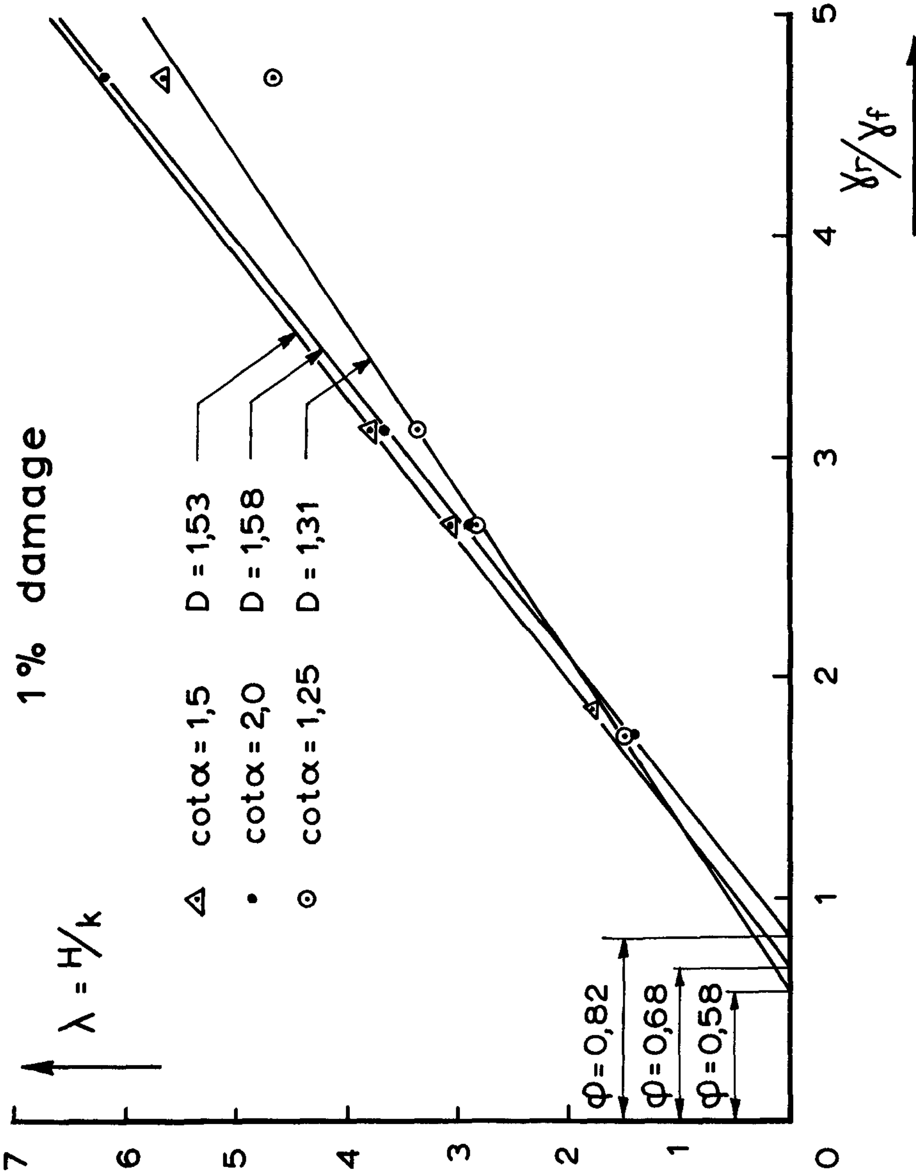

Fig. 9. All Results from Series 2 (Table II). Data from Alf T. Sodefjed (3) and (4)

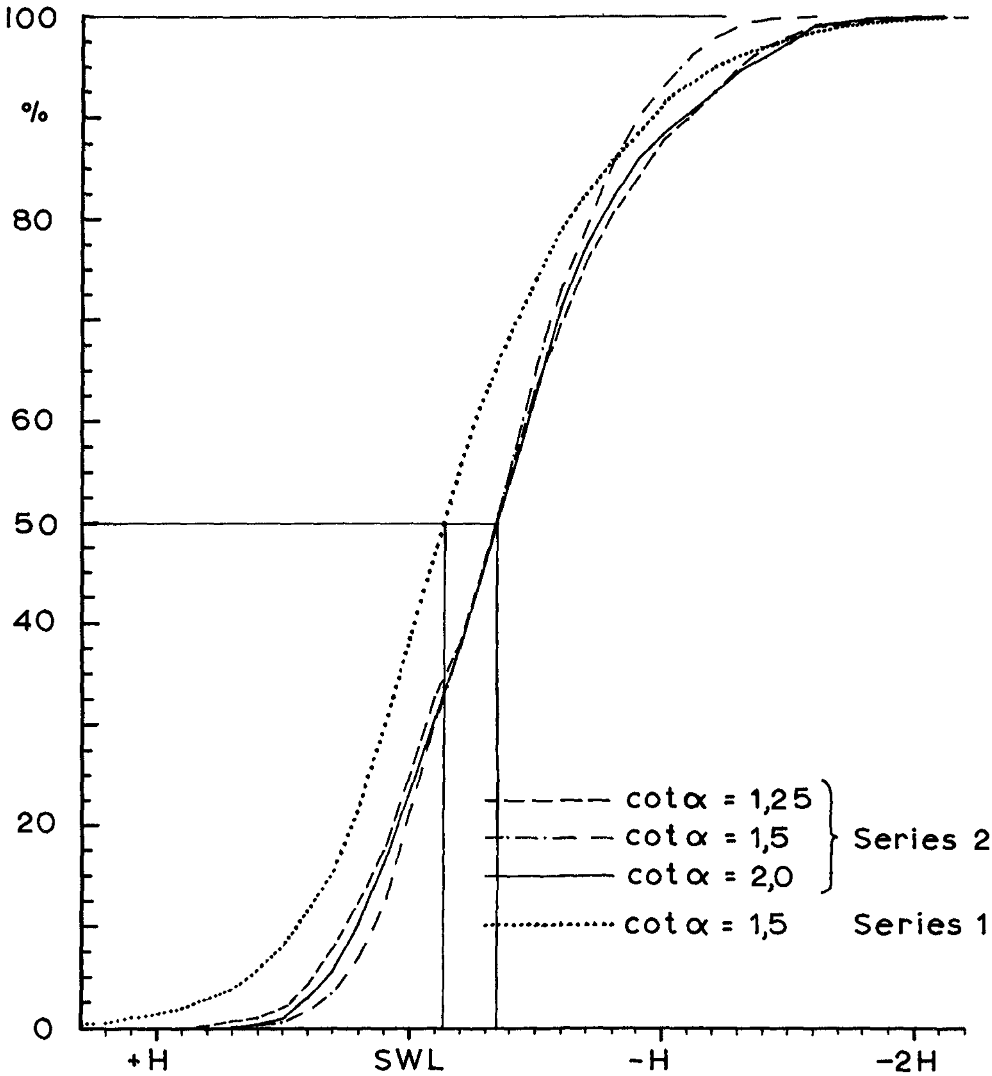

Fig. 10. Cumulative Distribution of Damage along the Breakwater Face, Series 1 and 2.

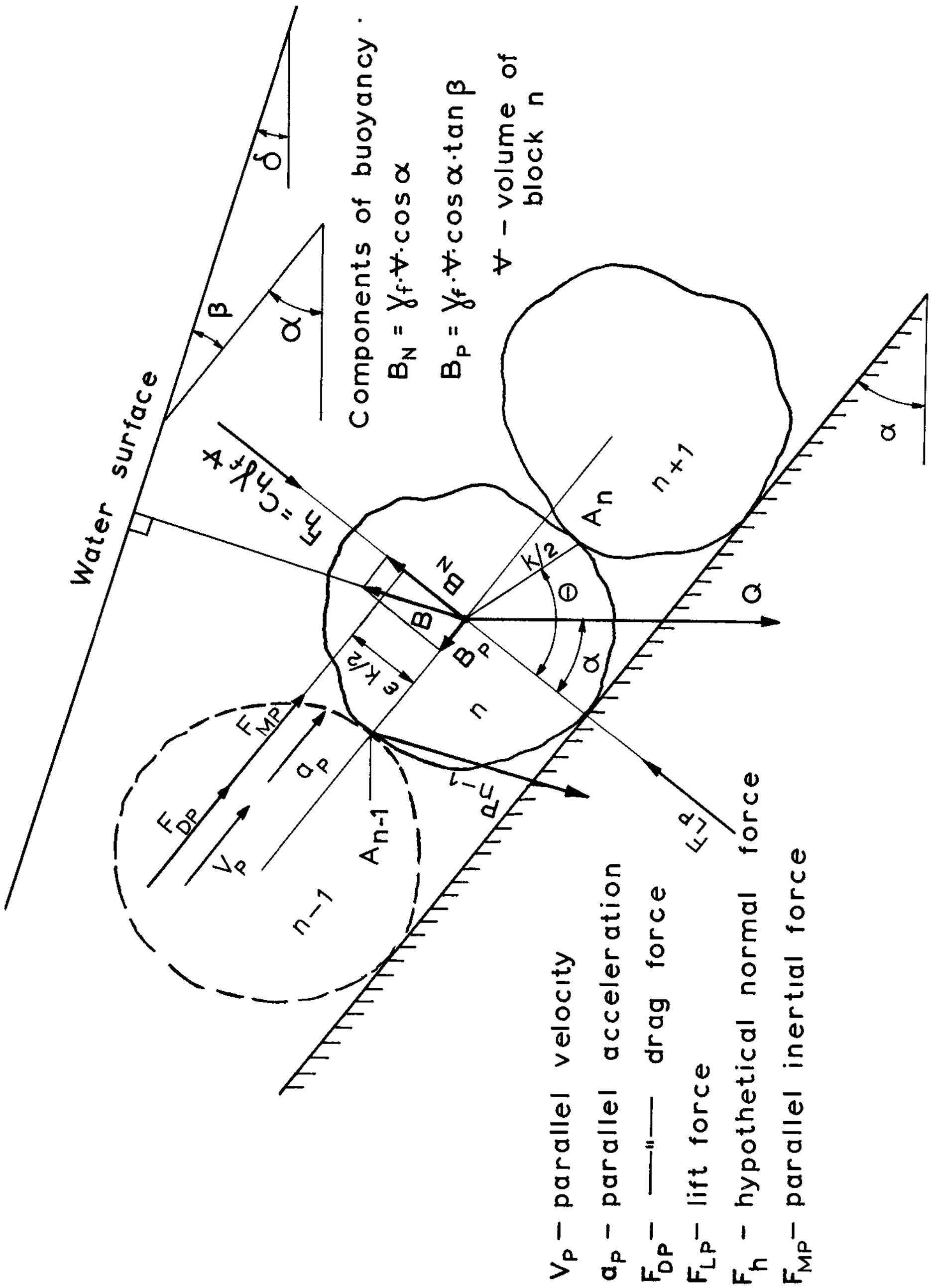

Fig. 11. Force Diagram for one Armour Block.

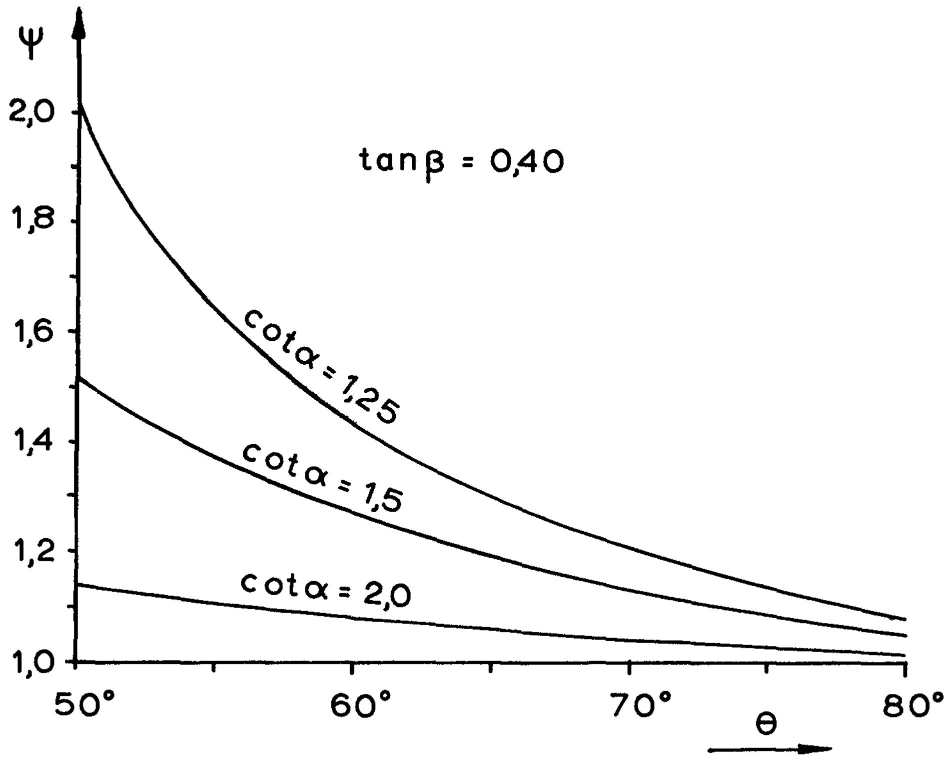

Fig. 12. Variation of ψ with θ and α.

TABLE I — Scope of tests · Series 1 cot α = 1,5						
Size of Armour Blocks	Combinations of Specific Weights					Group of combinations
	γ_f	Types of Armour Blocks A γ_r = 1,83	B γ_r = 2,66	C γ_r = 3,05	D γ_r = 4,52	
1 ∀ = 11—16 cm^3	1,00	▭▭▭	▭▭▭	▭▭▭	▭▭▭	I
2 ∀ = 32,7 cm^3	1,00				▭▭▭	
3 ∀ = 52,0 – 55,7 cm^3	1,00	▭▭▭	▭▭▭	▭▭▭	▭▭▭	II
	1,065	▭▭▭	▭▭▭	▭▭▭		IV
	1,13	▭▭▭	▭▭▭	▭▭▭		V
4 ∀ = 104 - 106,5 cm^3	1,00	▭▭▭	▭▭▭	▭▭▭		III
	1,065	▭▭▭				III
	1,13	▭▭▭				III

In total 20 combinations

TABLE II

Scope of tests: Series 2 γ_f = 1,00 throughout

Types of Armour Blocks

Specific Weights and Sizes		A	B	C	D
γ_r	g/cm³	1,725 ± 2% [1)]	2,70 ± 2,2 %	3,13 ± 2%	4,72 ± 5 %
Q_{av}	g	140,1	136	143	140
$\forall_{av}$	cm³	81	50	46	30
Slope of break-water face	1:1,25				
	1:1,5				
	1:2,0				

In total 3 x 4 = 12 combinations

1) For cot α = 1,5, γ_r = 1,86 g/cm³

TABLE III

Wave Data for Tests of Series 1

Combination of spec. weight and size (TableI)	Group of combinations (TableI)	γ_r/γ_f	k cm	At 1% of damage			At 4% of damage		
				H cm	λ	tanβ 1)	H cm	λ	tanβ 1)
A4	III	I,83	5,97	8,0	1,34	0,462	10,0	1,67	0,419
A3	II	1,83	4,81	7,0	1,46	0,425	7,4	1,54	0,413
A1	I	1,83	3,16	4,1	1,30	0,439	5,o	1,58	0,395
A3 - S1	IV	1,72	4,81	6,3	1,31	0,445	7,2	1,50	0,419
A3 - S2	V	1,62	4,81	5,6	1,16	0,468	6,6	1,37	0,438
A4 - S2	III	1,62	5,97	7,2	1,20	0,481	8,2	1,37	0,460
A4 - S1	III	1,72	5,97	7,7	1,29	0,471	9,1	1,52	0,439
B4	III	2,66	5,96	14,1	2,37	0,420	16,8	2,82	0,381
B3	II	2,66	4,76	10,0	2,10	0,416	12,7	2,67	0,360
B1	I	2,66	2,83	6,1	2,16	0,452	7,2	2,54	0,419
B3 - S1	IV	2,50	4,76	9,9	2,08	0,418	11,9	2,50	0,376
B3 - S2	V	2,36	4,76	9,1	1,92	0,437	10,8	2,27	0,401
C4	III	3,05	5,92	15,8	2,67	0,396	18,8	3,17	0,351
C3	II	3,05	4,78	12,0	2,51	0,372	15,1	3,15	0,309
C3 - S2	V	2,70	4,78	11,0	2,30	0,396	13,2	2,77	0,351
C1	I	3,05	2,87	6,8	2,37	0,431	8,6	2,99	0,349
C3 - S1	IV	2,87	4,78	11,7	2,47	0,378	14,2	2,97	0,327
D3	II	4,52	4,70	16,9	3,60	0,380	22,1	4,70	0,302
D2		4,52	4,03	15,4	3,82	0,400	19,7	4,88	0,337
D1	I	4,52	2,83	10,7	3,78	0,402	13,0	4,60	0,397

1) Calculated from (6), Eq.7

TABLE IV

Wave Data for Tests of Series 2

Slope of break-water face (TableII)	Types of armour blocks (TableII)	k	At 1% of damage			At 10% of damage		
			H	λ	$\tan\beta$ 1)	H	λ	$\tan\beta$ 1)
		cm	cm			cm		
1 : 1,25	A	5,47	8,16	1,49	0,601	8,95	1,64	0,582
	B	4,66	13,15	2,82	0,494	14,83	3,18	0,460
	C	4,51	15,20	3,37	0,454	17,80	3,94	0,404
	D	3,90	18,20	4,67	0,396	20,60	5,28	0,350
1 : 1,5	A	5,33	9,68	1,82	0,458	10,57	1,98	0,439
	B	4,66	14,27	3,06	0,368	15,86	3,41	0,340
	C	4,51	17,05	3,79	0,317	20,08	4,45	0,264
	D	3,90	22,06	5,65	0,229	24,24	6,22	0,193
1 : 2,0	A	5,47	7,68	1,40	0,350	8,13	1,49	0,339
	B	4,66	13,50	2,90	0,248	16,10	3,46	0,202
	C	4,51	16,60	3,69	0,194	21,35	4,74	0,114
	D	3,90	24,15	6,18	0,073	27,27	7,01	0,018

1) Calculated from (6), Eq.7.

TABLE V, SERIES 1

VALUES OF φ, D AND C_h

$A_1 = A_2 = 0,245$ $C_{MP}\ {}^{a}p/g = 0,15$ $\cot\alpha = 1,5$

γ_r/γ_f	At 1% of damage					At 4% of damage				
	From test diagrammes		Calculatd with p=0, θ=56°, ε=0,15			From test diagrammes		Calculated with p=0, θ=62°, ε=0,15		
	φ	D	φ	D	C_h	φ	D	φ	D	C_h
1,83	0,48	1,04	0,50		0,480	0,64	1,40	0,54		0,407
1,83	0,44	0,96	0,38		0,519	0,65	1,32	0,64		0,404
1,83	0,37	0,91	0,54	1,01	0,543	0,37	1,10	0,61	1,29	0,563
1,72	0,39	0,99	0,42		0,530	0,56	1,29	0,56		0,450
1,62	0,50	1,04	0,47		0,468	0,58	1,29	0,56		0,431
2,66	0,48	1,04	0,31		0,503	0,64	1,40	0,48		0,424
2,66	0,44	0,96	0,58		0,524	0,65	1,32	0,59		0,427
2,66	0,37	0,91	0,54	1,01	0,536	0,37	1,10	0,70	1,29	0,552
2,50	0,39	0,99	0,44		0,546	0,56	1,29	0,54		0,469
2,36	0,50	1,04	0,46		0,484	0,58	1,29	0,60		0,447
3,05	0,48	1,04	0,40		0,516	0,64	1,40	0,60		0,436
3,05	0,44	0,96	0,56		0,548	0,65	1,32	0,61		0,450
2,70	0,50	1,04	0,42	1,01	0,508	0,58	1,29	0,55	1,29	0,469
3,05	0,37	0,91	0,70		0,548	0,37	1,10	0,74		0,583
2,87	0,39	0,99	0,43		0,568	0,56	1,29	0,57		0,490
4,52	0,44	0,96	0,96	1,01	0,544	0,65	1,32	0,88	1,29	0,453
4,52	0,37	0,91	0,78		0,564	0,37	1,10	0,96		0,563

$\bar{C}_h = 0,525$ $\sigma = \pm 0,0275$ (at 1% of damage)

$\bar{C}_h = 0,472$ $\sigma = \pm 0,0518$ (at 4% of damage)

TABLE VI, SERIES 2

VALUES OF φ, D AND C_h

$A_1 = A_2 = 0{,}245$ C_{MP} $^a p/g = 0{,}15$ T = 1,8 sec.

cotα	γ_r g/cm³	At 1% of damage					At 10% of damage				
		From test diagrammes		Calculated with p=0, θ=66°, ε=0,15			From test diagrammes		Calculated with p=0, θ=73°, ε=0,15		
		φ	D	φ	D	C_h	φ	D	φ	D	C_h
1,25	1,725			0,56		0,372			0,73		0,286
	2,70	0,58	1,31	0,50	1,28	0,410	0,72	1,60	0,78	1,65	0,315
	3,13			0,50		0,423			0,74		0,328
	4,72			1,08		0,444			1,52		0,342
1,50	1,86			0,63		0,357			0,79		0,260
	2,70	0,68	1,53	0,63	1,47	0,390	0,80	1,84	0,86	1,85	0,285
	3,13			0,56		0,409			0,73		0,305
	4,72			0,89		0,441			1,37		0,323
2,00	1,725			0,93		0,276			1,02		0,076
	2,70	0,82	1,58	1,07	1,77	0,316	1,05	2,16	1,07	2,12	0,113
	3,13			1,05		0,338			0,90		0,137
	4,72			1,23		0,386			1,40		0,163

$\overline{C}_h = 0{,}380$ $\sigma = \pm 0{,}049$

$\overline{C}_h = 0{,}306$ $\sigma = \pm 0{,}0253$

TABLE VII

Block Weight Required for H = 6,0 m and cot α = 1,5 with φ = 0,5; 1,0 and 1,2 and varying γ_r, Based on Hudson's Formula :

$$Q = \frac{\gamma_r H^3}{K_{\Delta 1} \cot \alpha \, (\gamma_r/\gamma_f - 1)} \quad \text{and} \quad K_{\Delta\varphi} = K_{\Delta 1}\left[\frac{\gamma_r/\gamma_f - 1}{\gamma_r/\gamma_f - \varphi}\right]^3 \quad ; \quad K_{\Delta 1} = 3,2$$

γ_r	Q_1^t at φ = 1,0 $K_{\Delta 1}$ = 3,2	$Q_{0,5}^t$ at φ = 0,5		$Q_{1,2}^t$ at φ = 1,2	
		$Q_{0,5}^t$ $K_{\Delta\varphi}$=1,41	$Q_{0,5}^t/Q_1^t$	$Q_{1,2}^t$ $K_{\Delta\varphi}$ = 4,79	$Q_{1,2}^t/Q_1^t$
2,0	105,o	67,o	0,64	143,o	1,37
2,3	54,3	45,o	0,83	61,5	1,13
2,65	29,7	29,7	1,oo	29,7	1,oo
3,o	19,1	21,7	1,14	17,8	0,93
3,5	11,5	14,6	1,27	9,9	0,86
4,0	7,3	10,4	1,4o	6,1	0,82