CHAPTER 10

EFFECT OF BEACH SLOPE AND SHOALING ON WAVE ASYMMETRY

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SYNOPSIS

This paper is concerned with the quantitative study of the geometrical asymmetry associated with shallow water oscillatory waves in the breaker zone. Three descriptions of wave asymmetry are defined and examined :

(1) Wave vertical asymmetry
(2) Wave slope asymmetry
and (3) Wave horizontal asymmetry

The effects of shoaling, produced by beaches of different slope, on the wave asymmetry are examined. Six beach slopes in the range 1:4 to 1:18 were employed, and a quantitative correlation was found to exist between the wave slope asymmetry, wave horizontal asymmetry and the wave vertical asymmetry.

An expression is given for the wave horizontal asymmetry based on the expression for the wave vertical asymmetry from the cnoidal wave theory. The theoretical study of wave slope asymmetry made by Biesel (1) and the results of the experimental work on the wave slope asymmetry in the present work are compared and gave a good agreement.

INTRODUCTION

The asymmetry of the wave in shallow water induces the asymmetrical forces which influence the near shore sediments and coastal structures to a very great extent. Thus the understanding of the shallow water wave processes is of extreme importance in the design of coastal engineering projects.

As waves approach shallow water over bed slopes similar to those reproduced in the model, they change from the deep water to the shallow water category and become markedly asymmetric before breaking. The shoreward side of the wave becomes nearly vertical whilst the seaward face is appreciably inclined. The actual configuration is affected by the bed slope.

Owing to the continually varying geometry of waves in shallow water and in order to have a detailed understanding of wave transformation in shoaling water, three main types of wave asymmetry are defined in the present work. Their quantitative definitions being as follows :-

$$\text{Wave vertical asymmetry} = \frac{\text{Vertical distance from crest to S.W.L.}}{\text{Total wave height}}$$

$$\text{Wave slope asymmetry} = \tfrac{1}{2}(\text{Front face slope at S.W.L.} + \text{Back face slope at S.W.L.})$$

Both the front face slope and the back face slope were measured in radians. The front face slope was taken as negative and the back face slope as positive. Thus, a negative mean slope would indicate that the wave front is steeper than the rear face slope.

Two types of wave horizontal asymmetry are defined and given the symbols H_A and HA'.

$$H_A = \frac{\text{Horizontal dist. from crest to front face at S.W.L.}}{\text{Horizontal dist. from crest to back face at S.W.L.}}$$

$$H_A' = \frac{\text{Horizontal dist. from crest to preceding wave trough}}{\text{Horizontal dist. from crest to following wave trough}}$$

Diagrams showing the definitions of the wave asymmetry are shown in the appropriate graphs discussed later in the paper.

Some mention of the wave vertical asymmetry has been made in the literature. For instance, the trochoidal wave theory gives the positions of the wave crest and trough relative to the still water level in the shallow water as :-

$$\text{Height of crest} = \frac{H}{2} + \frac{\pi H^2}{4L} \coth \frac{2\pi d}{L} \qquad \text{———(1)}$$

$$\text{and depth of trough} = \frac{H}{2} - \frac{\pi H^2}{4L} \coth \frac{2\pi d}{L} \qquad \text{———(2)}$$

Stokes' finite amplitude wave theory shows that the wave crest lies above the still water level by the amount

$$\frac{\pi H^2}{4L} \left\{ 1 + \frac{3}{2 \sinh^2 \frac{2\pi d}{L}} \right\} \coth \frac{2\pi d}{L}$$

which is more than the value given by the trochoidal wave theory by the factor

$$\left(1 + \frac{3}{2 \sinh^2 \frac{2\pi d}{L}} \right)$$

The experimental results of the wave vertical asymmetry in this work were compared with the expression from the cnoidal wave theory corresponding to the wave vertical asymmetry, and a good agreement was obtained.

It is important to note that whereas the cnoidal wave theory predicts wave vertical asymmetry, it does not predict wave horizontal asymmetry and wave slope asymmetry.

An expression for the wave horizontal asymmetry is provided later in the work reported in this paper based on the expression for the wave vertical asymmetry from the cnoidal wave theory.

A brief summary of both the cnoidal wave theory and the theory of Biesel are presented below :

SUMMARY OF THE CNOIDAL WAVE THEORY

Korteweg and De Vries (10) developed the cnoidal wave theory which is generally considered very useful for describing the propagation of periodic waves in shallow water where the depth is less than one-tenth of the wave length. The cnoidal theory also provides a link between sinusoidal waves and solitary waves.

Korteweg and De Vries, (10) Keulegan and Patterson (9) and Keller (7) used different symbols, but the formulae obtained by them are essentially the same. The equations below are obtained by Wiegel (17) based mainly on the theory of Korteweg and De Vries.

The notations used in the equations below are as follows :

L	Wave length
d	still water depth
K(k)	complete elliptic integral of the first kind
E(k)	complete elliptic integral of the second kind
k	modulus of elliptic integral
H	Wave height (trough to crest)
T	wave period
$\bar{u}$	incomplete elliptic integral of the first kind
cn$\bar{u}$, sn $\bar{u}$	elliptic functions
x	horizontal co-ordinate
Y	vertical co-ordinate measured from the ocean bottom
Ys	vertical distance from ocean bottom to wave surface
Yt	vertical distance from bottom to wave trough
Yc	vertical distance from bottom to wave crest
S.W.L.	still water level

The wave length is given by

$$\frac{L}{d} = \frac{4}{\sqrt{3}} K(k) \left(2\bar{L} + 1 - \frac{Yt}{d}\right)^{-\frac{1}{2}} \qquad \text{---------(3)}$$

where L and k are defined by the following two equations

$$k^2 = \frac{\frac{Yc}{d} - \frac{Yt}{d}}{2\bar{L} + 1 - \frac{Yt}{d}} \qquad \text{---------(4)}$$

$$\left(2\bar{L} + 1 - \frac{Yt}{d}\right) E(k) = \left(2\bar{L} + 2 - \frac{Yc}{d} - \frac{Yt}{d}\right) K(k) \qquad \text{---------(5)}$$

For k to be real in equation (4), the following equations must also hold

$$2\bar{L} + 1 > \frac{Yc}{d} > \frac{Yt}{d} \text{ and } o < k^2 \leq 1 \qquad \text{------------(6)}$$

Equation (4) can be re-arranged in the form

$$\left(2\bar{L} + 1 - \frac{Yt}{d}\right) = \frac{\frac{Yc}{d} - \frac{Yt}{d}}{K^2} = \frac{\frac{H}{d}}{k^2} \qquad \text{------------(7)}$$

Substituting eq. (7) into equation (3) and squaring gives

$$\frac{HL^2}{d^3} = \frac{16}{3}\left[k\,K(k)\right]^2 \qquad \text{------------(8)}$$

$$\text{i.e.} \quad L = \sqrt{\frac{16d^3}{3H}}\; k\,K(k) \qquad \text{------------(9)}$$

Equation (5) can be re-arranged to give

$$E(k) - K(k) = \frac{\left[1 - \frac{Yc}{d}\right] K(k)}{2\bar{L} + 1 - \frac{Yt}{d}} \qquad \text{----------(10)}$$

Substituting equation (3) into equation (10) and re-arranging terms we have

$$\frac{Yc}{d} = \frac{16d^2}{3L^2}\left[K(k)\left((K(k) - E(k)\right)\right] + 1 \qquad \text{---------(11)}$$

multiplying equation (11) by $\frac{d}{H}$ we get

$$\frac{Yc - d}{H} = \frac{16d^3}{3L^2H}\left[K(k)\left(K(k) - E(k)\right)\right] \qquad \text{---------(12)}$$

Equation (12) is important as it expresses the WAVE VERTICAL ASYMMETRY.

The elliptic function cn is in general doubly periodic, it will be singly period (period 4K) providing k is a real number and $o \leq k \leq 1$. It is important to note that when $k = o$; $sn\bar{u} = \sin\bar{u}$, $cn\bar{u} = \cos\bar{u}$ and $K = \pi/2$ i.e. the period $4K = 2\pi$, thus when $k = o$ the elliptic cosine reduces to the circular cosine and the wave profile is given by the trigonometric functions. On the other hand when $k = 1$, $cn\bar{u} = \text{sech}\,\bar{u}$, and we have the hyperbolic function with $K(k) = \infty$ hence the period becomes infinite and we have the solitary wave expression. Thus the enoidal wave theory gives the solitary wave and the sinusoidal wave as its two limiting cases.

The properties of the cnoidal waves are given in terms of the complete elliptic integrals of the first and second kind, and the Jacobian elliptic functions hence the term 'cnoidal' analogous to sinusoidal.

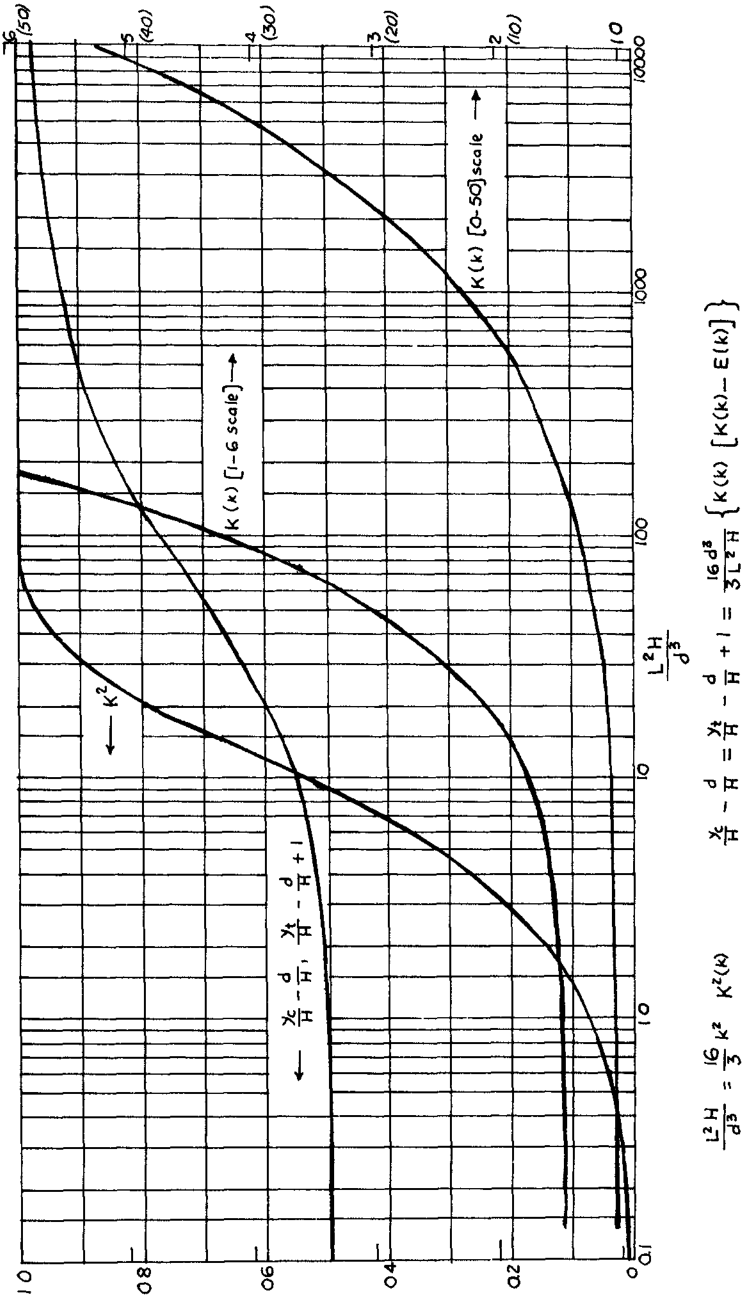

Fig. 1 Relationship among $\frac{L^2H}{d^3}$ and the square of the elliptic modulus (k^2), $\frac{Yc}{H}$, $\frac{Yt}{H}$, and K(k) (after Wiegel,[17] 1960)

Masch and Wiegel (11) computed several of the cnoidal wave characteristics from the equations obtained by Wiegel (17) based on the work of Korteweg and De Vries and they presented the results together with the elliptic integrals and the Jacobian elliptic functions in tabular form. The graph of equation (12) is shown in fig. 1.

WAVE SLOPE ASYMMETRY

In studying the progression of periodic waves in water of variable depth Biesel (1) produced a second-order theory for the wave slope asymmetry at still water level. Let OX and OY be rectangular co-ordinate axes located at mean sea level, with x positive in the direction of wave propagation and OY being directed vertically upward.

The other notations are defined as follows :-

T	Wave period
ω	angular frequency
L	wave length
u, v	OX and OY velocity components
m	$2\pi/L$
a	wave amplitude
d	water depth
$\gamma = -\left(\frac{\partial d}{\partial x}\right)$	= slope of the bed

suffix 0 denotes deep water conditions

ϕ velocity potential

Under conditions of irrotational and incompressible flow, the continuity equation gives

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \qquad \text{------------(13)}$$

The velocity components are given by

$$u = \frac{\partial\phi}{\partial x}, \quad v = \frac{\partial\phi}{\partial y} \qquad \text{------------(14)}$$

In order to satisfy the boundary condition at the bed we have

$$\frac{\partial\phi}{\partial y} - \gamma\frac{\partial\phi}{\partial x} = 0 \quad \text{for } y = -d \qquad \text{------------(15)}$$

Assuming constant surface pressure we have

$$\frac{1}{g}\frac{\partial^2\phi}{\partial t^2} + \frac{\partial\phi}{\partial y} = 0 \quad \text{for } y = 0 \qquad \text{------------(16)}$$

From Biesel the above four conditions are satisfied by the function.

$$\phi(x,y,t) = \frac{-a\omega}{m}\,\frac{1}{\sinh md}\Bigg[\cosh m(y+d)\sin\left(\omega t - \int m\,dx\right) + \gamma\Bigg\{\frac{m(y+d)\sinh m(y+d)}{D^2\tanh md} - m(y+d)\cosh m(y+d) + \frac{m^2(y+d)^2\cosh m(y+d)}{D\sinh md\cosh md}\Bigg\}\cos\left(\omega t - \int m\,dx\right)\Bigg]$$

. . . . (17)

where $D = 1 + \dfrac{md}{\sinh md \cosh md}$ ——————(18)

m, ω and d are related by

$$gm \tanh md = \omega^2 \qquad \text{——————(19)}$$

and

$$a = \frac{ao}{(D \tanh md)^{\frac{1}{2}}} \qquad \text{——————(20}$$

The free surface equation oan be expressed parametrically by the equations

$$\eta = x + a_0 A \sin\left(\omega t - \int m dx\right) + a_0 \gamma A_1 \cos\left(\omega t - \int m dx\right) \qquad \text{——————(21)}$$

$$\lambda = a_0 B \cos\left(\omega t - \int m dx\right) + a_0 \gamma B_1 \sin\left(\omega t - \int m dx\right) \qquad \text{——————(22)}$$

where A, A_1, B and B_1 are functions of md with η and λ being the co-ordinate of a point on the free surface.
The free surface slope of the wave is given by

$$S = \frac{\frac{\partial \lambda}{\partial x}}{\frac{\partial \eta}{\partial x}} = \frac{m a_0 B \sin\left(\omega t - \int m dx\right) + \left(-m\gamma a_0 B_1 + a_0 B\right) \cos\left(\omega t - \int m dx\right)}{1 - m a_0 A \cos\left(\omega t - \int m dx\right) + \left(m\gamma a_0 A_1 + a_0 A\right) \sin\left(\omega t - \int m dx\right)} \qquad \text{——————(23)}$$

For the wave slope in the neighbourhood of a particle with the initial co-ordinates (x, o) at the instant when the particle orosses the plane y = 0 we then have

y = 0 or

$$B \cos(\omega t - mdx) + \gamma B_1 \sin(\omega t - mdx) = 0$$

Hence there are two sets of possible values for cos(ωt - mdx) and sin(ωt - mdx)

i.e.

$$\cos(\omega t - mdx) = \frac{\gamma B_1}{B}; \quad \sin(\omega t - mdx) = -1 \qquad \text{——————(24)}$$

or

$$\cos(\omega t - mdx) = -\frac{\gamma B_1}{B}; \quad \sin(\omega t - mdx) = 1 \qquad \text{——————(25)}$$

Equation (24) corresponds to the passage of the front face of the wave, and equation (25) to the passage of the back face of the wave.

Combining equations (24) and (25) with (23) and noting that terms of γ^2 have been neglected we have :-

$$S_F = -ma_oB - m^2 a_o^2 \gamma (AB_1 + A_1B) - ma_o^2 AB \qquad (26)$$

and for the back of the wave :

$$S_B = ma_oB - m^2 a_o^2 \gamma (AB_1 + A_1B) - ma_o^2 AB \qquad (27)$$

The mean slope $S = \frac{1}{2}(S_F + S_B)$ is a measure of asymmetry of the wave. Thus from equations (26) and (27)

$$S = -m^2 a_o^2 \gamma (AB_1 + A_1B) - ma_o^2 AB \qquad (28)$$

Replacing A, B, A_1 and B_1 by their values we have

$$S = m^2 a_o^2 \gamma \left\{ \frac{3 + \frac{md}{\tanh md} - 3md \tanh md}{D^2 (\sinh md)^2 \tanh md} \right\} \qquad (29)$$

$$\text{where } D = 1 + \frac{md}{\sinh md \cosh md} \qquad (18)$$

As pointed out by Biesel (1) the analysis above assumes that the slope of the bottom is small.

This theory had not previously been tested experimentally, and its verification formed part of the present work. A satisfactory agreement was obtained.

WAVE HORIZONTAL ASYMMETRY

In order to completely define the asymmetry of the wave in shoaling water, note has to be taken of the asymmetry of the wave in the horizontal sense too. For this purpose, two types of wave horizontal asymmetry are noted and are given the symbols H_A and HA'. They are already defined above.

As far as the author is aware, there is no theory describing wave horizontal asymmetry for shoaling waves. As will be shown later in this work it was found that the two types of wave horizontal asymmetry i.e. HA and HA' follow the same trend. It was found possible therefore, to produce an empirical relationship expressing one in terms of the other.

Later in this work an expression is given for the wave horizontal asymmetry HA based on the theoretical expression for the wave vertical asymmetry from the cnoidal wave theory.

APPARATUS

The study described in this paper involved the use of an hydraulic model with an impermeable beach. The impermeable beach was chosen to permit beach slope variation and to enable a detailed study of the effect of the backwash to be made. The study on the effect of backwash is reported in another paper in this same journal (ref.4).

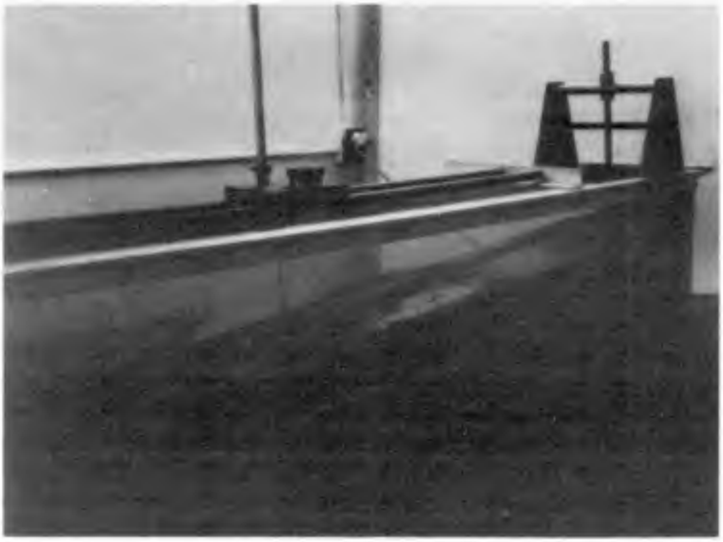

Plate 1

Plate 2

The Beach arrangement and the Wave Generator

In order to overcome the scale limitations imposed by the model size, the measurements were based on parameters used in certain aspects of wave theory, so that the behaviour of the model could be assessed.

The choice of variables was made in the light of pilot experiment conducted by the authur and recent experiments by Plinston (14) and Kemp (8) of wave aotion on granular beaches. These examined amongst other things the variation of beach slope with wave characteristics, and the phase relationship between two successive waves breaking on the beach. The change of beach slope using a plane beach and constant wave characteristics is equivalent to a change in beach material size, although the effect of permeability is not allowed for. Nevertheless, the reduction in phase relationship between uprush and backwash with increasing beach slope, and higher backwash velocities are both produced and are important factors in the movement of beach material.

The overall size of the apparatus was limited by the space available. The wave flume was 24 feet long, 9 inches wide and 15 inches deep. At the wave generator end, the bed of the channel sloped down to a depth of 21 inches. The channel was made of marine plywood, with the exception of long perspex windows which formed the sides of the channel at the beach end.

A satisfactory convex shaped slatted wave absorber was built behind the paddle to damp out reflections from the generator end. The beach was made of marine plywood and stiffened at the back with an I section to prevent vibration of the beach. The toe of the beach was hinged to the beach plate. A false toe was used for the feat beach slope in order to contain the beach in the tank. The upper end of the artificial beach was supported by a vertical screwed rod by which the beach slope was raised. Plate 1 shows the beach arrangement and plate 2 the wave generator.

A special feature of the design was that the motor and reduction gears were mounted on a very strong support, and were quite separate from the channel . Thus there would not be any vibration of the flume arising from the machinery.

Several sets of waves were generated by varying the wave period and the deep water wave height with the intention of selecting suitable wave characteristics, and also finding the position on the horizontal portion of the channel at which the wave had become steady. This enabled a wave measuring station to be chosen in the horizontal section of the channel.

The wave characteristics selected were :- Period T = 0.8 seos Ho = 1.45 ins and horizontal section still water depth = 9.5 ins. These wave characteristics were held constant. The beach slopes used were $\frac{1}{4}$, 1/6, 1/9, 1/12, 1/15 and 1/18.

The filters used consisted of $\frac{1}{2}$ ins. chicken mesh folded many times so as to suitably occupy the width of the channel for a length of $6\frac{1}{2}$ feet. The filters were placed at a suitable distance in front of the paddle. An experiment was conducted to determine the amount of wave reflection in the wave tank for the six beach slopes.

The lowest value of the coefficient of reflection was found to be about 3% for the flattest beach slopes and the highest value was about 8% for the steepest slopes of $\frac{1}{4}$ and 1/6. These values correspond with typical values from Miche (12).

MEASURING TECHNIQUES

General Procedure

The wave lengths were measured by using two capacitance - wire probes suspended separately on a carriage with the ends inserted in water in the tank to the mean operating depth. The output wave form from the wave meter was displayed on a D.C. oscilloscope. The arrangement allowed visual observations to be made on D.C. oscilloscope.

The wave height and the wave shape at the different positions along the beach were found by using the capacitance - wire probe system in conjunction with a 'visicorder' - ultra violet type instrument giving a permanent trace on paper. This recorder offers 8 paper speeds (6 to 800 mm/sec) and also 0.1 and 1 second timing intervals.

Two graduated scales were made on the side of the wave channel for the purpose of reading the still water depth. The scales were graduated in 1/10 of an inch divisions.

The first step for every beach slope was to locate the wave break-point by placing the probe at several marked places very close to each other in the neighbourhood of the position where the wave was visually observed to be breaking and noting the deflection registered on the oscilloscope. The position at which the wave height was a maximum and shoreward of which the wave lost its wave form was selected as the wave break-point.

The location of the break-point was followed by the marking out of the several positions along the beach where the detailed study of the wave asymmetry was to be made. The capacitance-wire probe was positioned at each of the places marked out, and while the wave generator was switched off a line corresponding to the S.W.L. was defined on the recording paper. This was done at the start and end of every reading.

After marking out the S.W.L. line the wave generator was then switched on, with the probe still in position the visicorder was then also switched on, and the wave profile for the particular position of the wire probe was recorded. The probe position was then changed and by repeating the processes again for each probe position the wave trace and wave length measurements were obtained for the different positions marked out along the beach, up to the break-point. At the end of such a series of experiments a calibration test was carried out for the particular setting of the visicorder.

EVALUATION OF WAVE VERTICAL ASYMMETRY

The S.W.L. mark lines on the wave trace paper, one at the start and the other at the end of every reading for every position of the probe were joined, thus providing the S.W.L. line from which measurements were made. Both the measurements of the vertical distance from the wave crest to the S.W.L. and the total wave height were scaled off.

EVALUATION OF WAVE HORIZONTAL ASYMMETRY

Two alternative definitions of wave horizontal asymmetry are made. The evaluation of these two values HA and HA' required the measurements of the horizontal distances from crest to the front and rear faces of the wave at S.W.L. and the horizontal distances from crest to the preceding and the following wave trough. The two types of wave horizontal asymmetry HA and HA' have already been defined earlier in this paper.

EVALUATION OF WAVE SLOPE ASYMMETRY

In the evaluation of the wave vertical asymmetry and the wave horizontal asymmetry any scale exaggeration of the output wave trace from the visicorder would be of no importance, as the wave vertical asymmetry and the wave horizontal asymmetry are dimensionless ratios, of which the terms involved are either both horizontal measurements or both vertical measurements, in which case any scale exaggeration of the output wave trace would cancel out.

On the other hand, the wave slope asymmetry is measured in radians, and because of the scale exaggeration of the wave trace, the profile has to be reproduced to the natural scale from the wave trace.

To evaluate the wave slope asymmetry, the angles made by the front and rear faces of the wave with the still water level line were measured in degrees and then converted into radians. The wave slope asymmetry is defined as :

$\frac{1}{2}$ (Front face slope at S.W.L. + Back face slope at S.W.L.) radians

The front face slope was taken as negative and the back face slope as positive.

REVIEW OF PREVIOUS WORK

Wiegel (16) conducted a series of experiments to compare the wave profile with the trochoidal wave theory. He commented that the surface profiles were similar to that given by the trochoidal wave theory when the wave was not in relatively shallow water (say $d/L > 0.15$) but that near the breakers the experimental results were considerably higher than those given by the theory.

In a study of breakers and beach erosion Hamada (5) remarked that at breaking, the displacement of the orbital centre above the still water level was 0.204 Hb on a 1/10 slope and 0.228 Hb on a slope of 1/15. He noted the observation that when a wave is about to break on a shallow sloping beach the trough becomes wide and flat; and the crest narrow and steep. He considered that the forward leaning of the wave crest might be an important factor in wave breaking.

Eagleson (2) compared experimental values of wave height steepness and wave length with stokes' theory. The commented that stokes' third order theory applied only in the early stages of transformation, the divergencies being too large as the wave begins to deform.

Ursell (15) theoretically predicted in a qualitative sense the effect of shoaling on wave asymmetry by consideration of parameter., $\frac{HL^2}{d^3}$. He concluded that large values of the parameter $\frac{HL^2}{d^3}$ would predict an increased tendency towards wave asymmetry.

Ippen and Kulin (6) studied the shoaling and breaking of the solitary wave. They classified breakers as "symmetric," "asymmetric" and "intermediate." They described waves which retained much of their original symmetry during shoaling and which deformed by what they termed "peaking up" of the crest, as symmetric breakers. On the other hand, they classified as asymmetric breakers, waves which showed a marked steepening of the front face. They did not define what they termed as the intermediate breaker, but presumably they intended something between what they called the symmetric and the asymmetric breakers. When they tried to translate their results into a graph, they found that there was some scatter and they rightly noted that their classification depended very much on personal judgement. From the studies made in the work reported in this paper, and within the experimental limits, it is evident that nothing like a symmetric breaker was observed.

Miller and Zeigler (13) selected about 200 breaker traces from the very many recorded, and plotted each breaker on a dimensionless graph. By comparing the breaker profiles, Miller and Zeigler considered that the breaker forms fell into three major categories, which they referred to as "symmetric", 'asymmetric' and very 'asymmetric' breakers. They then averaged the individual breaker traces for each class to obtain a single trace. Comparing the breaker they classed as asymmetric breaker with the "near breaking wave" i.e. the mean profile just seaward of the break-point they commented that the profile of the asymmetric breaker was more peaked.

The author would like to comment that from the study made in the work reported in this paper, the breaker Miller and Zeigler referred to as a symmetric breaker in fact possessed a distinct vertical asymmetry and some wave horizontal and wave slope asymmetry. Miller and Zeigler themselves remarked that the transition from the profile of the "near breaking wave" which already possessed asymmetry to the symmetric breaker, was difficult to appreciate. On the whole their classification seems to be based on visual judgement of the wave trace with a bias towards what is referred to in the present work as wave slope asymmetry. It was however, very surprising that Miller and Zeigler considered that the breaker type they referred to as the symmetric breaker was possibly analogous to the plunging type and the very asymmetric breaker was similar to the spilling type; the author considers that, if anything, a plunging breaker ought to correspond to a very asymmetric breaker.

EXPERIMENTAL RESULTS ON WAVE VERTICAL ASYMMETRY

On all the beach slopes it was found that as the wave progressed shoreward, the wave vertical asymmetry was continuously increasing and was maximum at the break-point. On the beach slope of $\frac{1}{4}$ it was found that the experimental results gave values very slightly higher than the theory for $d/L > 0.18$. Shoreward of this the experimental results were lower than the theory. The maximum divergence was about 18% of the theoretical value, and that was the greatest divergence found throughout the work including the other beach slopes.

The maximum divergence between the experimental results and the theory for the beach slope of 1/6 was 10% of the theoretical prediction. The experimental result of the wave vertical asymmetry at the wave break-point on the beach slope of 1/6 was found to be 0.707 which was higher than the corresponding value of 0.62 at the break point on the beach slope of $\frac{1}{4}$ but lower than the value of 0.725 at the wave break point on the slope of 1/9.

It was found that whereas the graphs of wave vertical asymmetry against d/L were non-linear for the steeper beach slopes, they became linear for slopes $\angle$ 1/12. The experimental results of the wave vertical asymmetry on the beach slope of 1/15 were quite close to the theoretical predictions. However, the theoretical curve was non-linear.

The graph of wave vertical asymmetry against d/L on the beach slope of 1/18 (see fig. 2) showed the same trend on shoaling as on the slope of 1/15. The maximum divergence between the value of the experimental results and the theory was 5% of the theoretical prediction. The value of the wave vertical asymmetry at the break point on the beach slope of 1/18 was 0.74 which was considerably higher than the corresponding value of 0.62 at the breaker position on the beach slope of $\frac{1}{4}$.

Whereas on the beach slopes of $\frac{1}{4}$, 1/6 and 1/9 the breakers formed were plunging breakers, on the beach slope of 1/12 and for the flatter slopes of 1/15 and 1/18 the breaker in each case was spilling.

The effect of beach slopes on wave vertical asymmetry is shown in figure 3, and it indicated that the wave vertical asymmetry is greater for flat slopes when the wave is in very shallow water of about $d/L \angle 0.10$.

EXPERIMENTAL RESULTS ON WAVE HORIZONTAL ASYMMETRY

The graph of HA and HA' against d/L on the beach slope of $\frac{1}{4}$ is shown in figure 4. The value of HA at the break point was about 0.2 which meant that at the breaker position, the horizontal distance from the wave crest to the wave front face at still water level, was 1/5 of the horizontal distance from the crest to the rear face. At a d/L value of 0.15, the value of HA was 0.80, and so the wave was not really very asymmetric at that stage. The most rapid change in HA and HA' was found to take place at $d/L < 0.15$. The graph of HA' against d/L was quite similar to that of HA against d/L. This was, however, in line with expectation as both HA and HA' are measures of wave horizontal asymmetry.

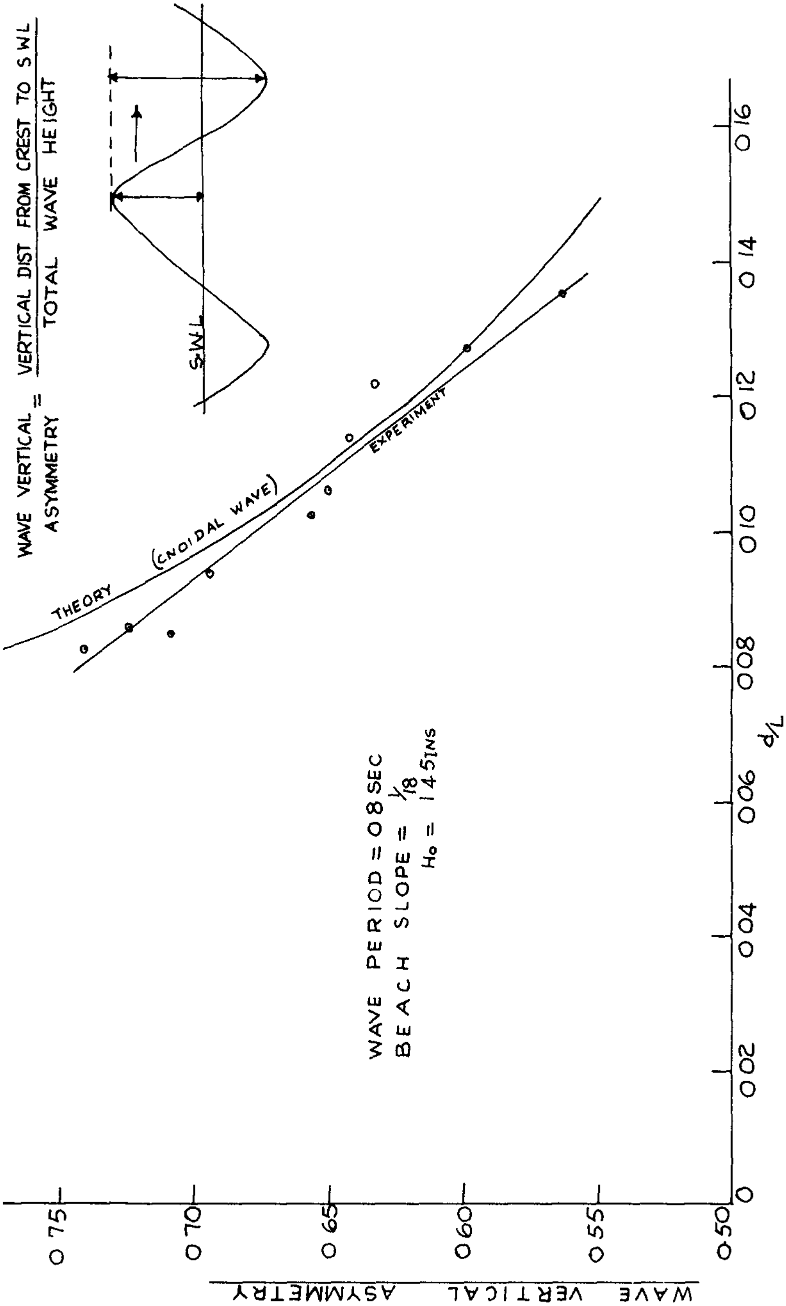

Fig 2. Wave Vertical Asymmetry Experimental and Theoretical Results

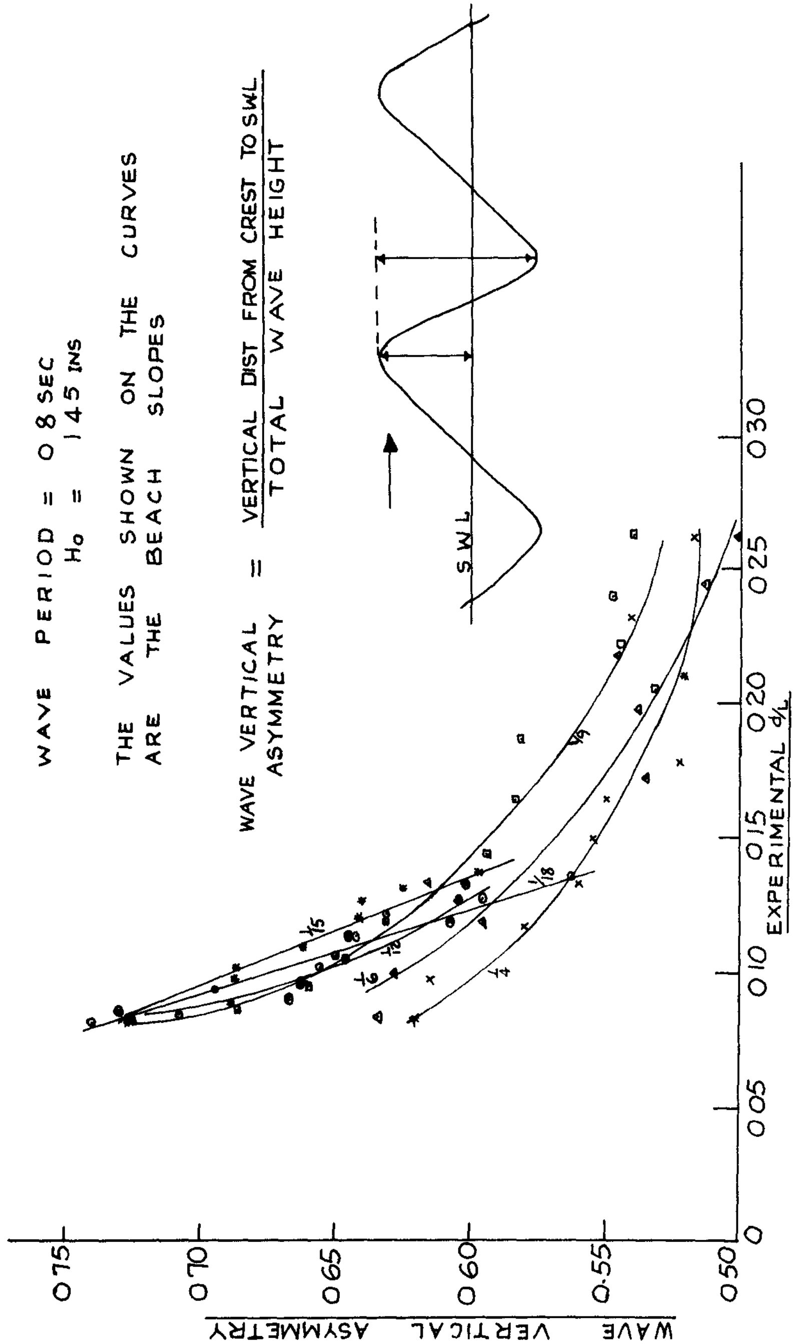

Fig 3 Effect of Beach slope on Wave Vertical Asymmetry

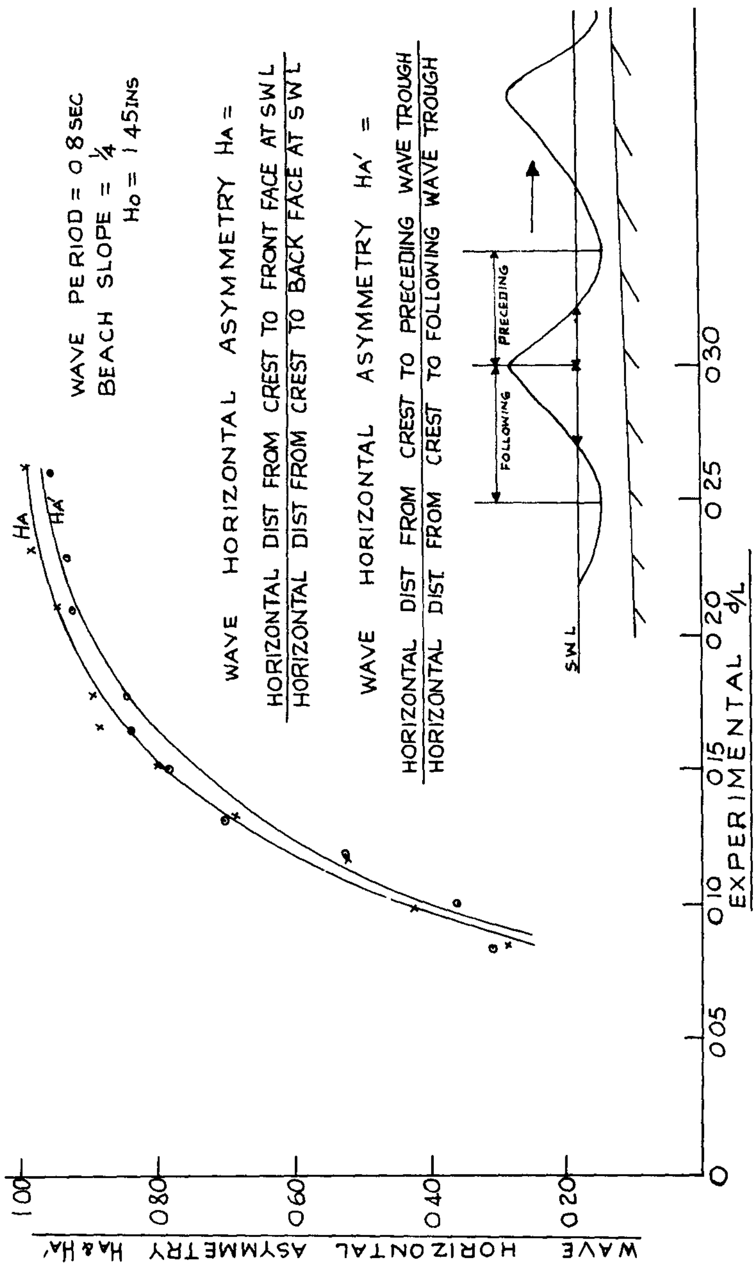

Fig 4. Graph of Wave Horizontal Asymmetry H_A and H_A'

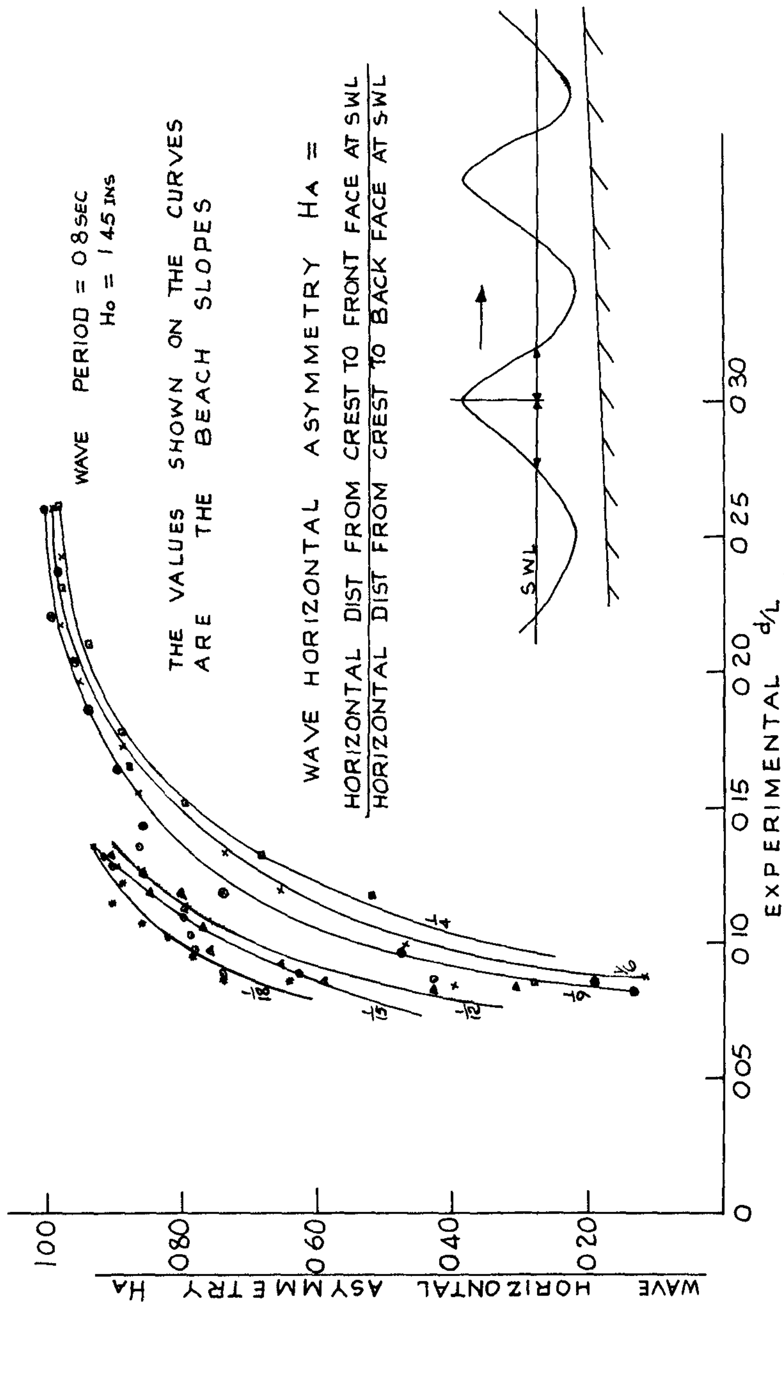

Fig. 5 Effect of Beach slope on Wave Horizontal Asymmetry H_A

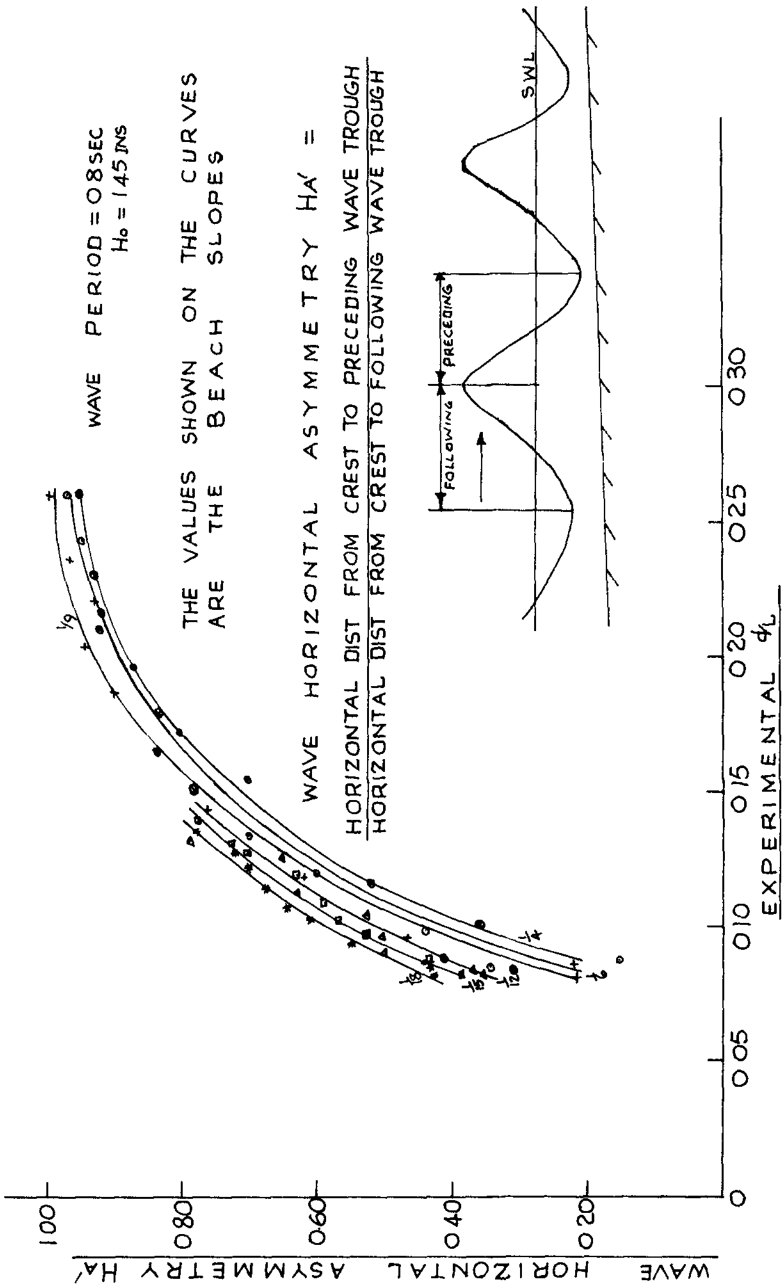

Fig 6 Effect of Beach slope on Wave Horizontal 'Asymmetry H_A'

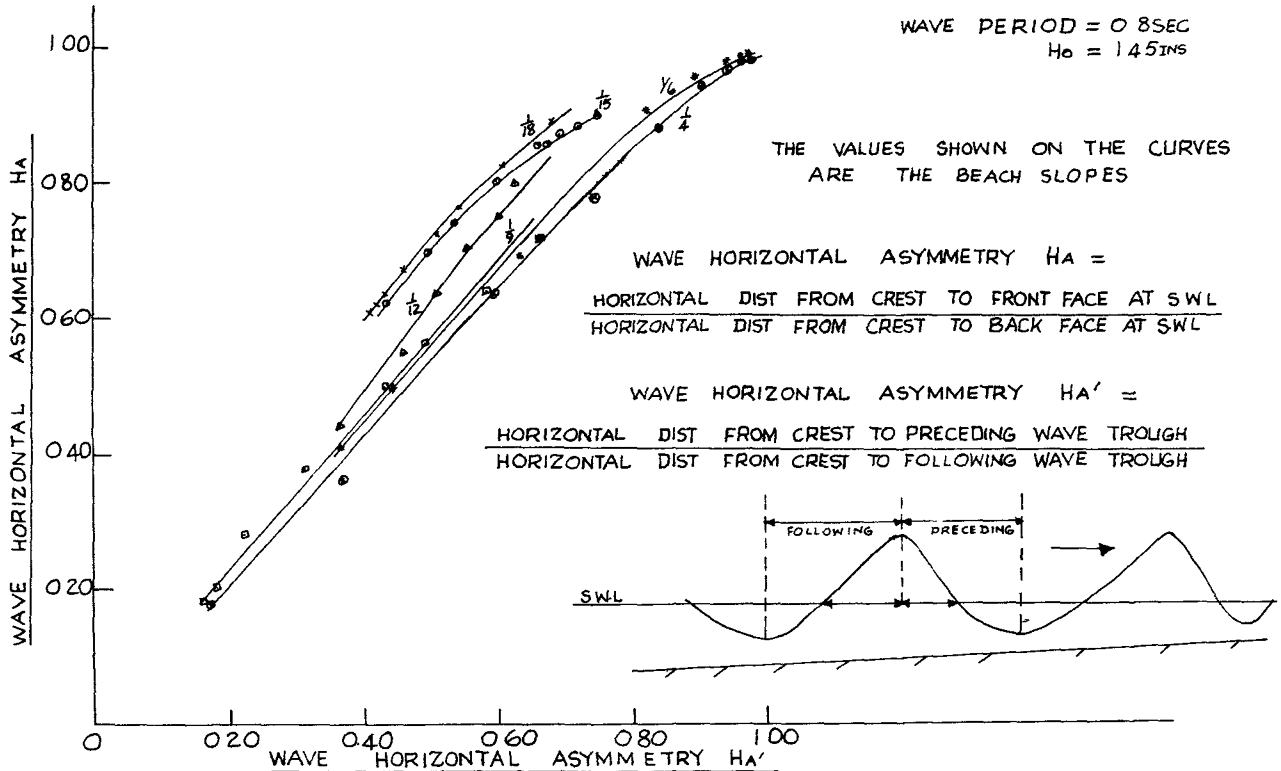

Fig. 7 Graph of Wave Horizontal Asymmetry H_A against H_A'

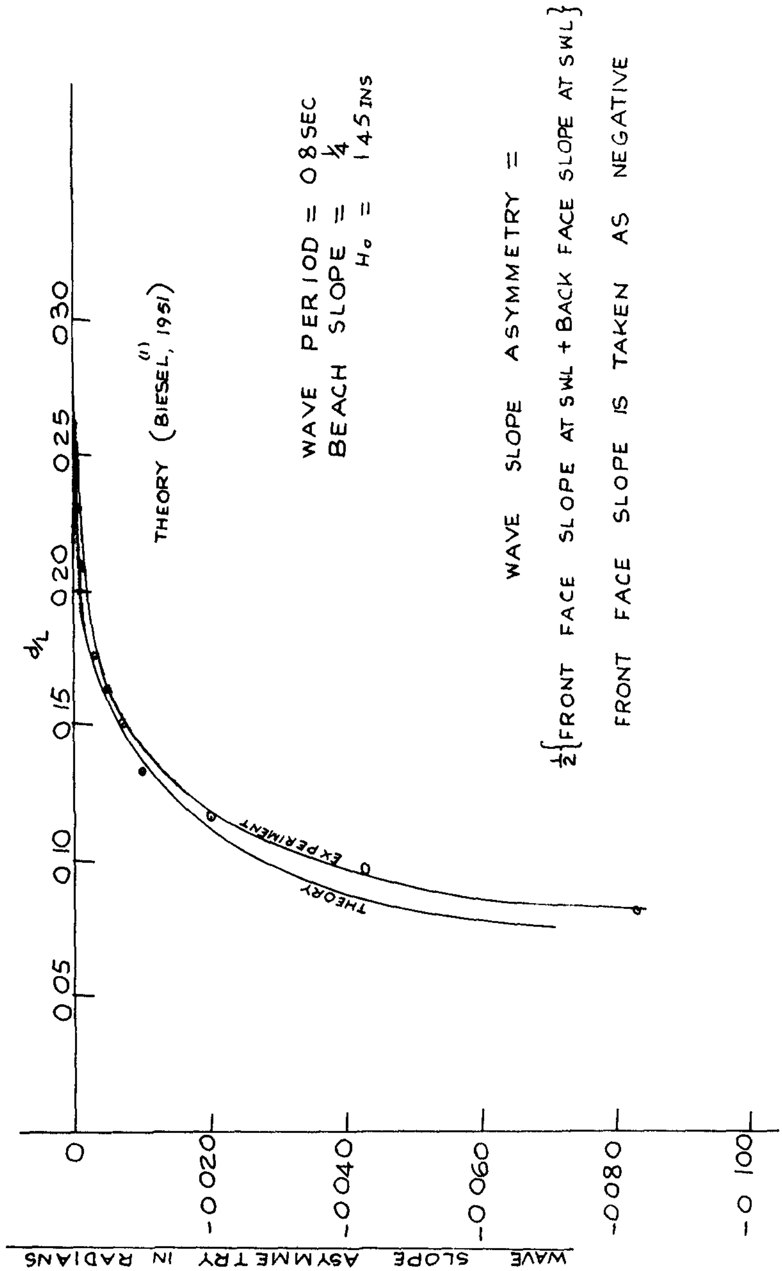

Fig 8. Wave slope Asymmetry Experimental and Theoretical Results

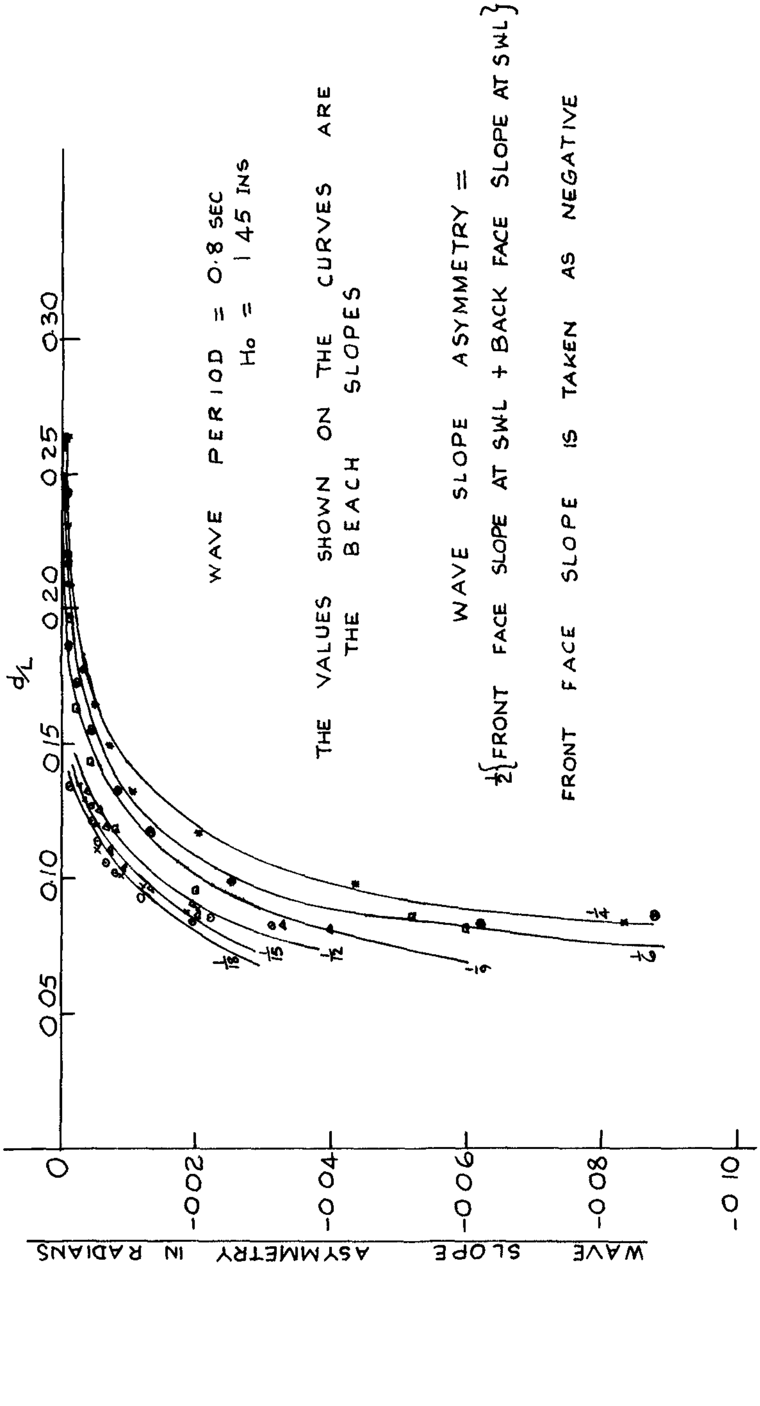

Fig. 9 Effect of Beach slope on Wave Slope Asymmetry

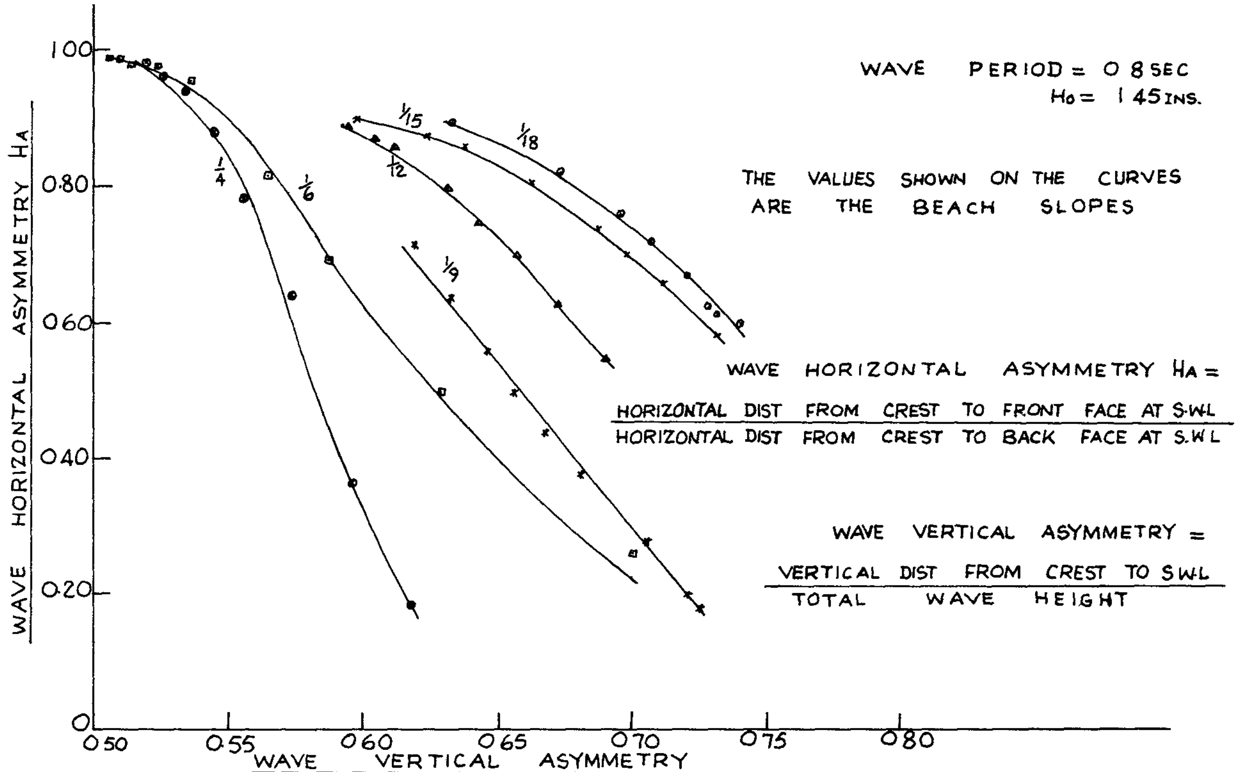

Fig. 10. Graph of Wave Horizontal Asymmetry H_A against Wave Vertical Asymmetry

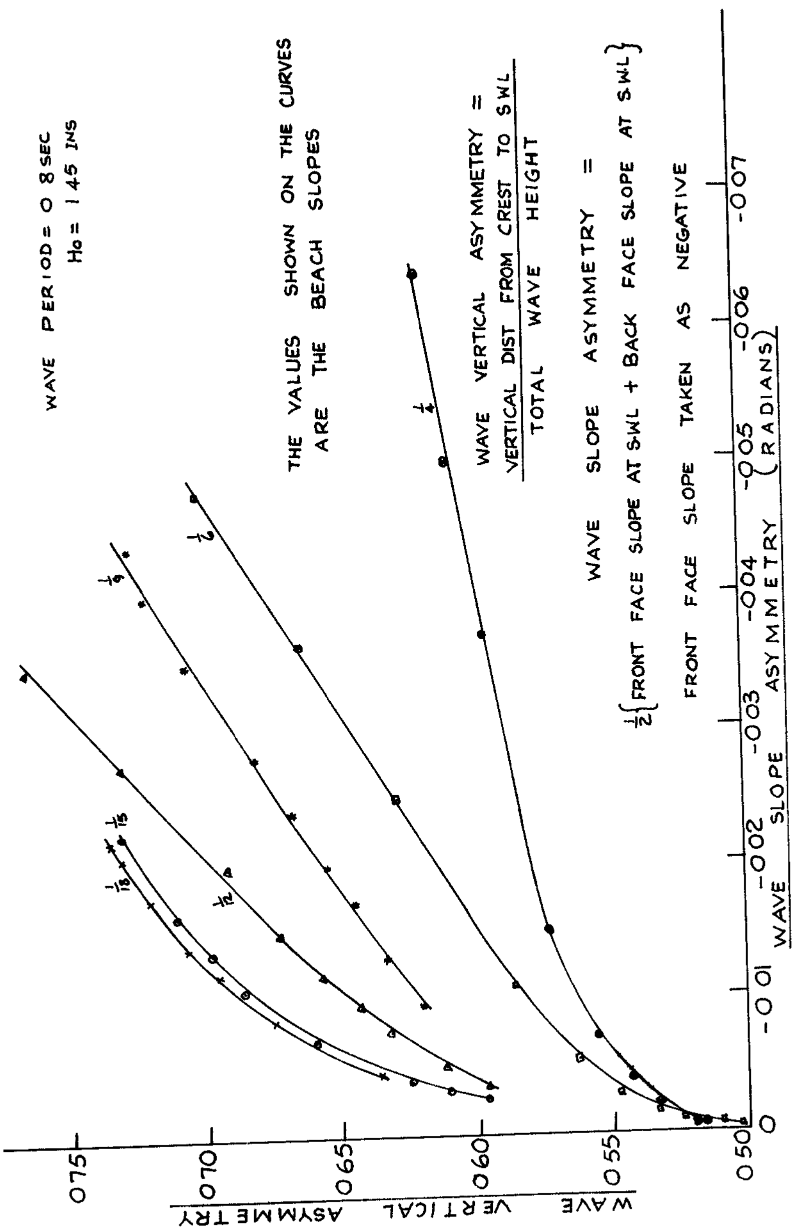

Fig 11 Graph of Wave Vertical Asymmetry against Wave Slope Asymmetry

It was found however that HA' indicated a higher asymmetry than HA.

The graphs of HA and HA' against d/L for the beach slopes of 1/6, 1/9, 1/12, 1/15 and 1/18 follow the same trend as that on the beach slope of $\frac{1}{4}$ discussed above.

Figures 5 and 6 show the effect of beach slope on the wave horizontal asymmetry HA and HA'. It is evident from the figures that the horizontal asymmetry of the wave increases as the beach slope gets steeper, whereas it will be remembered that in very shallow water in the case of wave vertical asymmetry, the vertical asymmetry decreases as the beach slope gets steeper.

As can be seen that figure 7 which shows the graph of HA and HA' for the different beach slopes, establishes a correlation between the two types of wave horizontal asymmetry HA and HA'. This is not unexpected, but it is quite useful, in that it does mean that observations or calculations in wave studies can be made in terms of either parameter.

EXPERIMENTAL RESULTS ON WAVE SLOPE ASYMMETRY

On the beach slope of $\frac{1}{4}$, the graph of wave slope asymmetry against d/L (see figure 8) showed that the wave slope asymmetry increased as the wave advanced into shallower water. The wave slope asymmetry was negligible at d/L value of 0.26 whereas at d/L value of 0.15 the value of the wave slope asymmetry was - 0.007. The most rapid change in wave slope asymmetry took place at $d/L < 0.15$. For instance, at d/L value of 0.10 the value of the wave slope asymmetry became - 0.037. At the breaker position where the wave was most asymmetric the value of the wave slope asymmetry was - 0.083. The theoretical curve gave results rather lower than the experimental results. On the beach slope of 1/6, it was again found that the most rapid change in wave slope asymmetry took place in the region $d/L < 0.15$. The maximum divergence between the theory and the experimental results was about 7% of the theoretical prediction. The results for beach slopes of 1/9, 1/12, 1/15 and 1/18 indicated a similar trend to those discussed above.

As can be seen from fig. 9 showing the effect of beach slope on wave slope asymmetry, the wave slope asymmetry increases as the beach slope gets steeper. This is in marked contrast with the wave vertical asymmetry which decreases as the beach slope gets steeper. On the other hand the wave horizontal asymmetry increases with increasing beach slope.

Figures 10 and 11 show, however, that all of the asymmetries, 'slope', 'vertical', and 'horizontal', are correlated. As a result of the studies, the following relationships were obtained :

Let Av = wave vertical asymmetry
S = wave slope asymmetry
$|s|$ = modulus of s
γ = beach slope

HA & HA' = wave horizontal asymmetry

$$HA = \frac{\text{Horizontal distance from crest to front face at S.W.L}}{\text{Horizontal distance from crest to back face at S.W.L.}}$$

$$HA' = \frac{\text{Horizontal distance from crest to preceding wave trough}}{\text{Horizontal distance from crest to following wave trough}}$$

The empirical relationships between Av, S, γ, HA and HA' are:

$$HA = \frac{1.31}{e^{\gamma/2}} \tanh HA' \quad \text{--------(30)}$$

$$HA = \frac{1.52}{e^{\gamma}} (1.18 - \sinh Av) \quad \text{--------(31)}$$

$$Av = \frac{8.8}{e^{\gamma}} \tanh |s| + 0.5 \quad \text{--------(32)}$$

From the cnoidal wave theory as developed by Korteweg and de Vries (10), the wave vertical asymmetry Av is given by

$$Av = \frac{Y_c - d}{H} = \frac{16 d^3}{3L^2 H}\left\{K(k)\left[K(k) - E(k)\right]\right\} \quad \text{--------(12)}$$

where Yc = distance from the ocean bottom to the wave crest
d = still water depth
H = wave height (trough to crest)
L = wave length
K(k) = complete elliptic integral of the first kind
E (k) = complete elliptic integral of the second kind
k = modulus of the elliptic integral.

From equations (31) and (12)

$$HA = \frac{1.52}{e^{\gamma}}\left\{1.18 - \sinh\left(\frac{16 d^3}{3L^2 H}\left[K(k)\left\{K(k) - E(k)\right\}\right]\right)\right\} \quad \text{-------(33)}$$

Thus equation (33) gives a general expression for wave horizontal asymetrry HA. Values of K(k) and E(k) are tabulated in Masch and Wiegel (11) (1961).

CONCLUSIONS

The wave asymmetry defined in all of the three ways increases as the wave advances into shallower water and the wave is most asymmetric at the wave break point. The most rapid change in wave asymmetry takes place in the region $d/L < 0.15$.

The cnoidal wave theory is adequate for predicting wave vertical asymmetry. However, whereas, the cnoidal theory indicates a non-linear curve throughout, the experimental curves of the wave vertical asymmetry against d/L became linear for slopes $< 1/12$. The studies on the effect of backwash showed that the linearity was due to the backwash and the flat beach slope.

The work on the wave slope asymmetry showed good agreement with the theory of Biesel (1). The results of the experiments for the beach slopes ¼, 1/6 and 1/9 indicated that the wave slope asymmetry ceases at $d/L = 0.26$. The theory of Biesel (1) predicted that the wave slope asymmetry ceases at $d/L = 0.27$. This would appear to be a valuable verification of a theory which had not previously been checked experimentally.

The wave horizontal asymmetry (both HA and HA') and the wave slope asymmetry increase as the beach slope gets steeper, whereas the wave vertical asymmetry is greater for flat slopes when the wave is in very shallow water. The rule of thumb that the trough is a quarter of the breaker height below the undisturbed water level, whatever the beach slope is not quite correct.

A correlation was found to exist between the two types of wave horizontal asymmetry HA and HA'. Further they both show the same trend, thus any studies connected with the wave horizontal asymmetry could be made using either HA or HA'. As shallow water waves tend towards the characteristics of long waves, it is evident that measurement of HA would be more precise than HA'.

Apart from the correlation between the two types of wave horizontal asymmetry HA and HA' a correlation was also found to exist between the wave slope asymmetry, wave horizontal asymmetry and wave vertical asymmetry, this thus make it possible to obtain a general expression for the wave horizontal asymmetry HA.

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