CHAPTER 40

SCOUR OF SAND BEACHES IN FRONT OF SEAWALLS

by

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ABSTRACT

Many previous studies were confined to problem of beach erosion due to waves breaking on the structure. The investigation reported here involved regular non-breaking, shallow water waves progressing toward a seawall. An analytical solution was developed and compared with laboratory-scale experiments.

The shallow-water wave theory and boundary layer equations were used in theoretical development, which resulted in a mathematical model for the ultimate scour depth in front of a seawall.

The theoretical equation for scour is as follows:

$$S = (D-1/2A) \left[(1-C_r)\ u^* \ \left(3/4\ C_D\ \rho \frac{\cot\theta}{d\ (\gamma s-\gamma)}\right)^{1/2} - 1\right]$$

where D = still water depth, $A = H_I + H_R$, H_I = incident wave height, H_R = reflected wave height, and $C_r = \frac{H_R}{H_I}$ = coefficient of reflection

also u* = horizontal velocity within boundary layer, C_D = coefficient of drag, ρ = fluid density, θ = seawall slope angle, d = effective sand diameter, 50% finer, γs = specific weight of sand, and γ = specific weight of water.

The comparison between theoretically calculated values and experimental results indicates fairly good agreement. The model experiments also indicate that depth of scour depends to a large extend on wave characteristics and that scour length (distance between scour trough or crests) is independent of time, but is a function of incident wave length.

INTRODUCTION

Most of the studies in the past were confined to problems of beach erosion due to "breaking waves."

The investigation discussed here involved regular, water waves progressing toward a seawall. In such a case, waves may or may not break on the seawall, and the objective of the study was to investigate the nature of scour of a flat sand beach in front of a simple seawall. (Figure 1) Earlier part of the study was reported elsewhere(1)*.

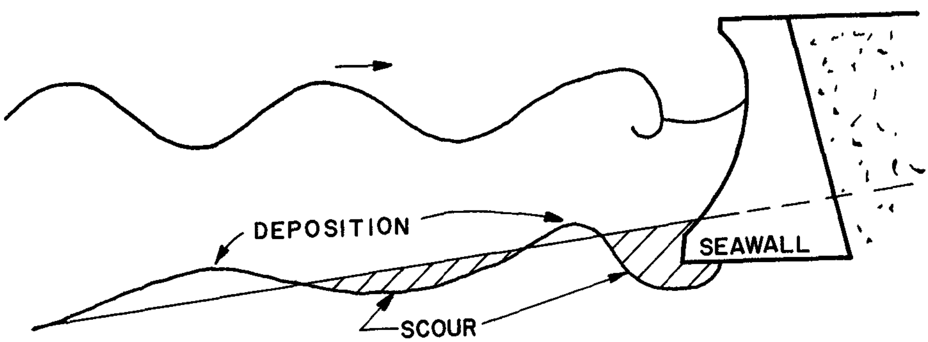

FIG. 1 DEFINITION SKETCH

When a system of oscillatory, incident waves progresses toward the seawall and hits the seawall, another system of waves if formed due to the reflection from the seawall. These two wave systems form a third wave system as the reflected wave is superimposed on the incident wave. The velocity components of the new wave system are given by the summation of the velocity component vectors of the incident and reflected waves. (Figure 2)

The theoretical development is based on the continuity equation and on boundary layer equations. The development resulted in a mathematical model which predicts the "ultimate" scour depth.

An experimental physical laboratory model was constructed and data on the "ultimate" scour depth were obtained.

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* Numbers in parenthesis refer to references on page 20.

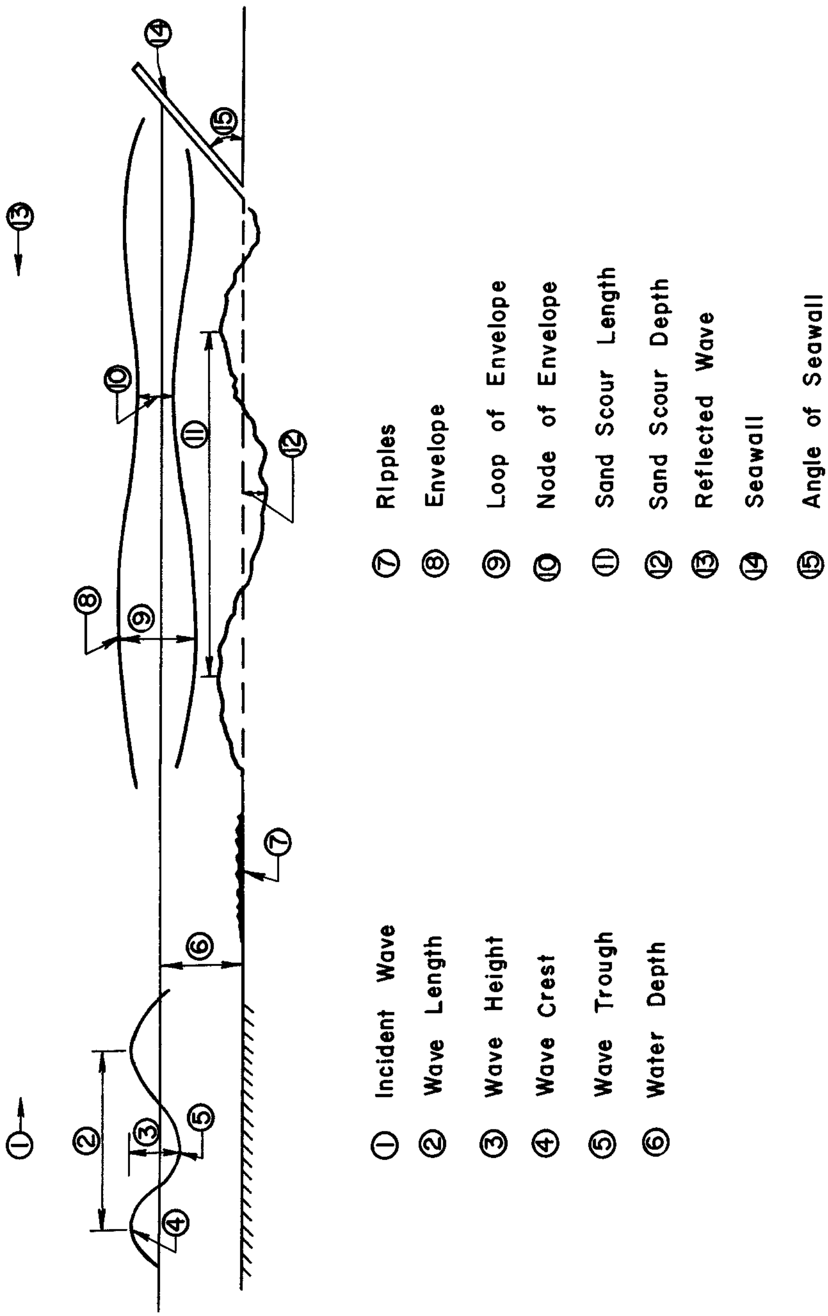

FIGURE 2. SCHEMATIC EXPLANATION OF TERMINOLOGY

Reasonably good agreement was obtained between the mathematical and physical model.

THEORETICAL CONSIDERATIONS

Equation of Motion and Equation of Continuity - Assume that water wave motion is generated from rest by a horizontal force, and that the fluid pattern is irrotational and satisfies the velocity potential requirements. Under these assumptions two equations must be satisfied, namely, Euler's equation of motion and Laplace's equation of continuity.

Euler's equation of motion may be stated as follows:

$$- \frac{1}{\rho} \text{grad } p + \overline{F} = \frac{\overline{DV}}{Dt} \quad (1)$$

and the Laplace Equation may be written as:

$$\nabla^2 \overline{\psi} = \overline{\psi}_x^2 + \overline{\psi}_y^2 + \overline{\psi}_z^2 \quad (2)$$

or in two-dimensional cartesian coordinate form:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \quad (3)$$

In addition, the following boundary conditions must be satisfied:

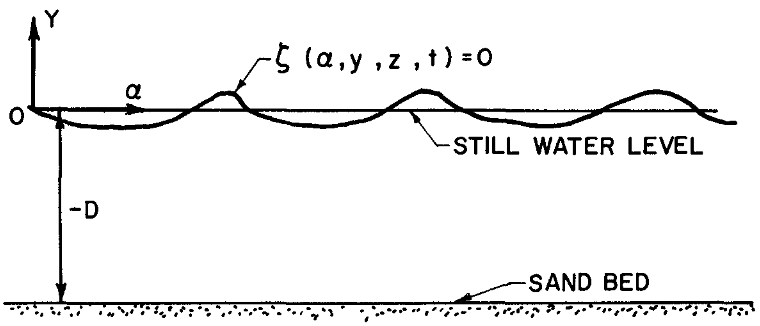

FIG. 3 GEOMETRY OF WAVE MOTION

The free surface of the water in contact with air can be defined by:

$$\xi\ (x,y,z,t) = 0 \tag{4}$$

Along the surface the pressure must be zero, so that,

$$-\frac{1}{\rho}\nabla p = 0 \tag{5}$$

Along the bottom, where y = -D, the water particles must remain in contact with it,

$$\frac{\partial\Phi}{\partial y} = -\frac{\partial\Phi}{\partial x} = 0 \text{ at } y = -D \tag{6}$$

In two-dimensions, Equation (1) can be written as:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{7}$$

or $$\frac{1}{\rho}\frac{\partial p}{\partial x} = -\frac{\partial\Omega}{\partial x} - \frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}$$

$$= -\frac{\partial\Omega}{\partial x} - \frac{\partial^2\Phi}{\partial t\partial x} - \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial x}\right)^2\right] - \frac{\partial\Phi}{\partial y}\frac{\partial^2\Phi}{\partial x\partial y}$$

$$= -\frac{\partial\Omega}{\partial x} - \frac{\partial^2\Phi}{\partial t\partial x} - \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial x}\right)^2 + \frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial y}\right)^2\right] \tag{8}$$

where Ω = gy positive for upward direction. Integrating Equation (8), we have,

$$\frac{P}{\rho} = -gh - \frac{\partial\Phi}{\partial t} - \frac{1}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial y}\right)^2\right] \tag{9}$$

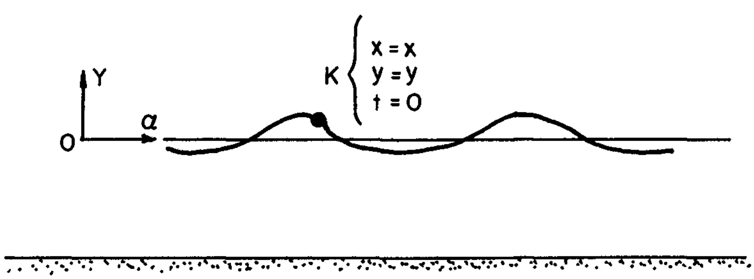

FIG. 4 FREE SURFACE OF WAVE MOTION

From Figure 4, consider a particle K on the surface. After an infinitesimal time δt, the particle will move to $x + \delta x$.

$$u \, \delta t = \delta x$$

$$\frac{\partial \Phi}{\partial x} = \frac{\delta x}{\delta t} \tag{10}$$

The corresponding pressure change will be,

$$\delta P = \frac{\partial p}{\partial t} \delta t + \frac{\partial p}{\partial x} \delta x + \frac{\partial p}{\partial y} \delta y \tag{11}$$

From Equation (10) and (11), we will have the condition of,

$$\frac{\delta p}{\delta t} = \frac{\partial p}{\partial t} + \frac{\partial \Phi}{\partial x}\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y}\frac{\partial y}{\partial t} = 0, \text{ along free surface} \tag{12}$$

From Equation (9) and Equation (12), neglecting the second order of small values, we have,

$$g \frac{\partial \Phi}{\partial y} + \frac{\partial^2 \Phi}{\partial t^2} = 0, \text{ when } y = 0 \tag{13}$$

Equation (5), (6), (9), or (13) represent the boundary conditions for the wave motion.

Stokes' Wave With Finite Amplitude

Shallow water waves were first studies by Stokes (1880), he obtained the solution by expanding the velocity potential about the still water level. In this theory, it is not necessary to assume that amplitude and steepness are small. The final results are presented as non-linear equations.

From Equation (4), the hydrostatic pressure of any water particle is,

$$P = \rho g \, (\xi - y) \tag{14}$$

where ρ is the density of water, g is the gravitational acceleration.

The differentiation of Equation (14) is,

$$\frac{\partial P}{\partial x} = \rho g \frac{\partial \xi}{\partial x} \tag{15}$$

From Equation (15), it is obvious that u, horizontal component of velocity, is independent of y.

The second order differentiation of Equation (15) will yield the horizontal component of acceleration which, again, is independent of y.

Up to this point in using the theoretical hydrodynamic concepts, no approximations have been made. It is obvious, that the important question is how to define the stream function Φ and free surface ξ.

Fourier's theorem, Jacobian elliptical function and complete elliptic integral have been used by many investigators as approximative approaches. In 1952, Biesel developed a second order approximation of potential function as

$$\Phi = \frac{HL \cosh 2\pi(y + D)/L}{2T \sinh (2\pi D/L)} \sin 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) +$$

$$\frac{3\ H^2 \cosh\ 4\ (y + D)/L}{16T\ \sinh^4 \left(\frac{2\pi D}{L}\right)} \sin 4\pi\left(\frac{x}{L} - \frac{t}{T}\right) \qquad (16)$$

By differentiating Equation (16) with respect to x and Y, the velocity components of any particle are obtained

$$u = \frac{\partial \Phi}{\partial x}$$

$$u = \frac{\pi H}{T} \frac{\cosh \frac{2\pi(y + D)}{L}}{\sinh (2\pi D/L)} \cos 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) +$$

$$3/4 \left(\frac{\pi^2 H^2}{TL}\right. \frac{\cosh\ 4\pi(y + D)/L}{\sinh^4 (2\pi D/L)} \cos 4\pi\left(\frac{x}{L} - \frac{t}{T}\right) \qquad (17)$$

and, $v = \frac{\partial \Phi}{\partial y}$

$$v = \frac{\pi H}{T} \frac{\sinh 2\pi(y + D)/L}{\sinh 2\pi D/L} \sin 2\pi\left(\frac{x}{L} - \frac{t}{T}\right) +$$

$$3/4 \left(\frac{\pi^2 H^2}{TL}\right. \frac{\sinh\ 4\pi\ (y + D)/L}{\sinh^4 (2\pi D/L)} \sin 4\pi\left(\frac{x}{L} - \frac{t}{T}\right) \qquad (18)$$

The free surface is

$$\xi = \frac{H}{2} \cos 2\pi \left(\frac{x}{L} - \frac{t}{T}\right) \frac{\pi H^2}{4L} + \frac{\pi H^2}{4L}$$

$$\frac{H^2}{4L} \left(1 + \frac{3}{2 \sinh^2 (2\pi D/L)}\right) \coth \frac{2\pi D}{L} \cos 4\pi \left(\frac{x}{L} - \frac{t}{T}\right) \tag{19}$$

Cnoidal Theory

In 1844, Russell described a different kind of wave called the "solitary wave", which represents a single disturbance, propagated essentially unaltered in form over long distance at a constant velocity. A few years later, Boussinesq (1871), (9) and Rayleigh (1876), (9) developed mathematical equations for its profile and velocity. In 1895, Korteweg and deVries modified Rayleigh's theory in such a way as to obtain waves that are periodic in profile and which tend to the solitary wave in the limiting case of long wave length. (The cnoidal theory).

The velocity components of any water particle in the water can be determined from

$$u = (\sqrt{gd}) \left[\frac{h}{D} - \frac{h^2}{4D^2} + \left(\frac{D}{3} - \frac{y^2}{2D}\right) \frac{\partial^2 h}{\partial x^2}\right] \tag{20}$$

$$v = (\sqrt{gd})\ y \left[\left(\frac{1}{D} - \frac{h}{2D^2}\right) \frac{\partial h}{\partial x} + \frac{1}{3} \left(D - \frac{y^2}{2D}\right) \frac{\partial^3 h}{\partial x^3}\right] \tag{21}$$

where

$$h = -D + y_t + H\, cn^2 \left[2K(k) \left(\frac{x}{L} - \frac{t}{T}\right), k\right] \tag{22}$$

where K(k) is the first kind of complete elliptical integral, defined as

$$K(k) = \int_0^{\pi/2} \frac{d\Phi}{(1 - k^2 - \sin^2 \Phi)^{\frac{1}{2}}} \qquad 0 \le k \le 1$$

The free surface profile is

$$\xi = y_t + H\, cn^2 \left[2K(k) \quad (\frac{x}{L} - \frac{t}{T}),\ k\right] \tag{23}$$

A comparison of horizontal velocity components of water particles among the Linear Theory, Stokes Second Order Theory and Cnoidal Theory had been made by Wiegel (1960). (9)

The Mechanics of Sediment Movement

If the mean diameter of a sediment particle (50 per cent finer by weight) is d, and u_* is the local fluid velocity parallel to the bottom, then the drag force just sufficient to initiate movement of a particle in the bed is

$$F_D = C_D \; \rho \frac{u_*^2}{2} \; \frac{\pi d^2}{4} \tag{24}$$

where C_D is the coefficient of drag, ρ is the density of the fluid.

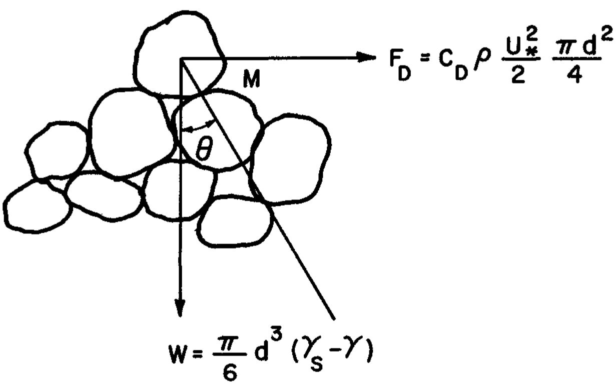

FIG. 5 INITIAL MOVEMENT OF A SAND PARTICLE

Consider a particle P as shown in Figure 5; θ is the angle of repose. The moment which is just sufficient to initiate movement of the particle about M must equal the moment of its own weight about point M.

$$F_D = C_D \rho \frac{u_*^2}{2} \frac{\pi d^2}{4} = \frac{\pi}{6} d^3 (\gamma_s - \gamma) \tan\theta \tag{25}$$

where γ_s is the specific weight of the sediment, γ is the specific weight of the fluid, and C_D is the coefficient of drag, which is a function of Reynolds Number. (Re)

Boundary Layer Along a Flat Sand Bed

The mechanics of sediment movement were discussed in the previous section and u* was defined as the local horizontal velocity parallel to the bottom. However, the sand particle is so small that the boundary layer effect must be taken into consideration. Fortunately the boundary layer along a flat plate is the simplest case of the application of Prandtl's Boundary Layer Theory.

Based upon the cnoidal wave theory and laboratory observations (Figure 4), it is reasonable to assume that the fluid flow pattern between the sand bed and water surface (within the scour wave length) is uniform and steady, or that $\frac{\partial v}{\partial x}$ = constant. Following Prandtl's development the thickness of boundary layer may be expressed as:

$$\delta \approx \left(\frac{\nu x}{U} \right)^{\frac{1}{2}} \tag{26}$$

or $$\eta = y \left(\frac{U}{\nu x} \right)^{\frac{1}{2}} \tag{27}$$

where $\eta = y/\delta$ is defined as a dimensionless term.

From the definition of stream function, we have

$$u_* = \frac{\partial \Psi}{\partial y} = \frac{\partial \Psi}{\partial \eta} \cdot \frac{\partial \eta}{\partial y} = U f'(\eta) \text{ or} \tag{28}$$

$$\frac{u_*}{U} = f'(\eta) \tag{29}$$

The solution of this equation was obtained by Howarth. (10)

A Theoretical Equation For Wave Scour

Wave reflection from a simple seawall (or a flat plate) is a function of its angle of inclination, (i.e. for a vertical wall the reflection is 100 per cent and for sloping beaches partial reflections will occur) (2). The incident and superimposed waves will form an envelope as shown in Figure 6.

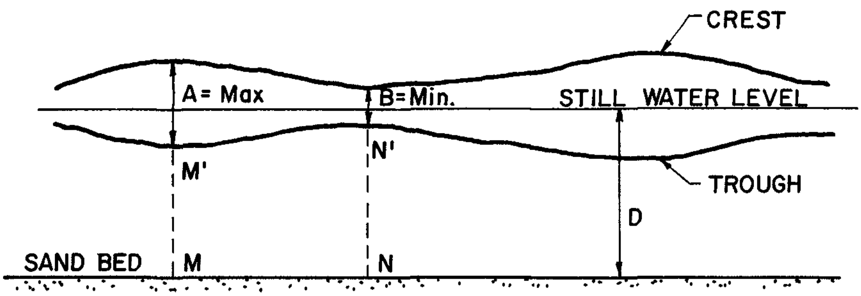

FIG. 6 ENVELOPE OF WAVE MOTION

It was observed that the surface of the sand bed first becomes rippled under the nodes of the envelope. (Point N in Figure 6). A few minutes later the rippled surface extends to cover the entire sand bed. Soon after the formation of these ripples the actual scour formations appear. The crests and troughs of the sand formations correspond to the loops and nodes of the envelope.

The mechanics of the scouring process may be explained as follows: When the experiment is started with a flat sand bed, the horizontal velocity component under the node is affected more than the horizontal velocity component under the loop, so that the primary scour occurs under the nodes of the envelope. A few hours later (usually 1 to 3 hours) the crests of the sand formation move under the nodes of the envelope. This relative position will normally last throughout the duration of the experiment.

From equation (25), for a particular grain size of sand, the most important is u_*. In other words, u_*, the local velocity parallel to the bottom, is the main factor determining the depth of scour. Since the wave is generated from rest, the equation of continuity is valid in this case. It is then logical to say, that when scour depth increases, the local velocity must decrease until a certain point when the ultimate scour depth is reached. This does not imply that scour and sediment transfer come to a halt, it is only a limit which is approached asymptotically.

Figure 7 shows a side elevation of sand bed. Section "a" represents the initial condition (before the scour) and section "b" represents the condition when the ultimate scour is reached.

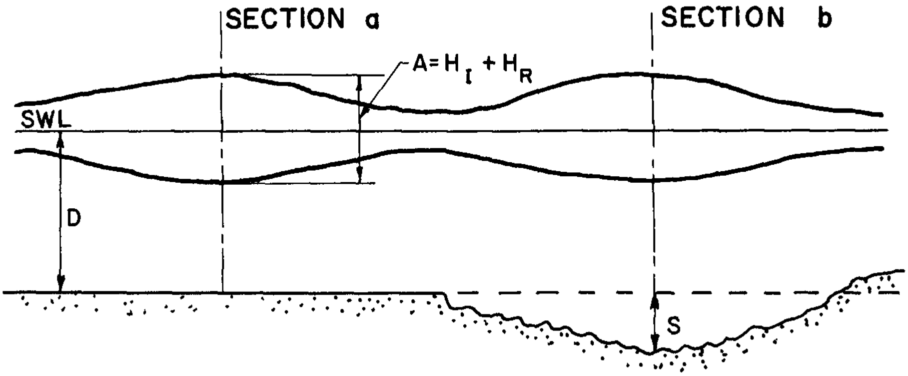

FIG. 7 GEOMETRY OF SAND SCOUR

Continuity equation may be written between section "a" and "b".

$$U_a \quad (D - \tfrac{1}{2}A) = U_b \quad (D - \tfrac{1}{2}A + S) \tag{30}$$

or

$$U_b = U_a \frac{D-\frac{1}{2}A}{D-\frac{1}{2}A + S} \tag{31}$$

This equation must be modified to take account of reflection which is a function of the slope of the seawall. When the two wave systems approach from opposite directions and are superimposed, the velocity components may be added vectorially. Introducing equation (28), we have

$$f'(\eta)\ (1 - C_r)\ U_b = (1 - C_r)\ f'(\eta)\ U_a \left(\frac{D - \frac{1}{2} A}{D - \frac{1}{2} A + S}\right) \qquad (32)$$

where C_r is reflection coefficient, defined as:

$$C_r = \frac{A - B}{A + B} = \frac{H_R}{H_I} \qquad (33)$$

Substituting Equation (32) into Equation (25) and simplifying, the following expression is obtained:

$$S = (D - \tfrac{1}{2} A) \left[(1 - C_r)\ u_* \left(3/4C_D\ \rho \frac{\cot\theta}{d\ (\gamma_s - \gamma)}\right)^{\frac{1}{2}} - 1 \right] \qquad (34)$$

where $A = H_I + H_R$

Dimensional Analysis

The following dimensionless terms may be obtained from the dimensional analysis:

$$\frac{S}{K}, \frac{\lambda}{L}, \frac{T}{t}, R_e, \frac{V^2 \rho}{d(\gamma s - \gamma)}, F_r, C_r$$

where the following functional equation may be written:

$$f\left(\frac{S}{K}, \frac{\lambda}{L}, \frac{T}{t}, \frac{1}{R_e}, \frac{V^2 \rho}{d(\gamma_s - \gamma)}, \frac{1}{F_r}, \theta, \alpha, C_r\right) = 0 \qquad (35)$$

or

$$\frac{S}{K} = f'\left(\frac{\lambda}{L}, \frac{T}{t}, \frac{1}{R_e}, \frac{V^2 \rho}{d(\gamma_s - \gamma)}, \frac{1}{F_r}, \theta, \alpha, C_r\right) \qquad (36)$$

For the sake of comparison Equation (34) may be re-written in the following form:

$$\frac{S}{K} = (1 - C_r)\ (3/4\ C_D)^{\frac{1}{2}} \left(\frac{V^2 \rho}{d(\gamma_s - \gamma)}\right)^{\frac{1}{2}} (\cot\theta)^{\frac{1}{2}} - 1 \qquad (37)$$

It is obvious that equations (36) and (37) are very similar. However, it must be pointed out that equation (34) should be used only for the "ultimate" scour depth.

EXPERIMENTAL STUDY

Facility

The wave channel employed was two feet wide, two feet deep, and sixty seven feet long, glass walled, with absorbers at both ends.

Pendulum-type wave generator was used, its frequency could vary from zero to 2.1 cycles per second. The stroke, period and movement of oscillating plate of the generator were all adjustable. The capacitance-type probe and wave recorder were used to collect the data.

Geometric Configuration

A false bottom of five inches deep and a bulk-head of plywood were placed under the generator and extended toward seawall of a distance of about 15 feet. Sand bed five inches deep extended throughout the remainder of the tank. The seawalls, also made of plywood, were placed in front of the downstream absorber. Wire-mesh filter (about 5 feet thick and 2 feet wide) was placed about 5 feet in front of the generator in an attempt to reduce the reflected-waves hitting the paddle and causing re-reflected waves. Unfortunately, it was very difficult (if not impossible) to eliminate the reflected-waves completely. Grain size distribution curve for the sand is shown in Figure 8.

Procedure

The experimental program was as follows:

Experiment No.	H/L	H/D	L/D	T (sec)	Remarks
A1	0.03314	0.38400	10.500	1.265	15 Degree Seawall
A2	0.04012	0.42125	10.500	1.600	" " "
A3	0.03291	0.34556	10.500	1.695	" " "
A4	0.02343	0.24600	10.500	1.790	" " "
B5	0.03621	0.34625	9.563	1.410	" " "
B6	0.02680	0.33500	12.500	2.080	" " "
B7	0.02741	0.38375	14.000	2.100	" " "
B8	0.01732	0.35959	17.466	1.620	" " "
C9	0.02609	0.33552	12.656	2.000	No Seawall
C10	0.02609	0.33552	12.656	2.000	45 Degree Seawall
C11	0.04057	0.44000	10.844	1.500	" " "
C12	0.03852	0.49000	10.600	1.500	" " "

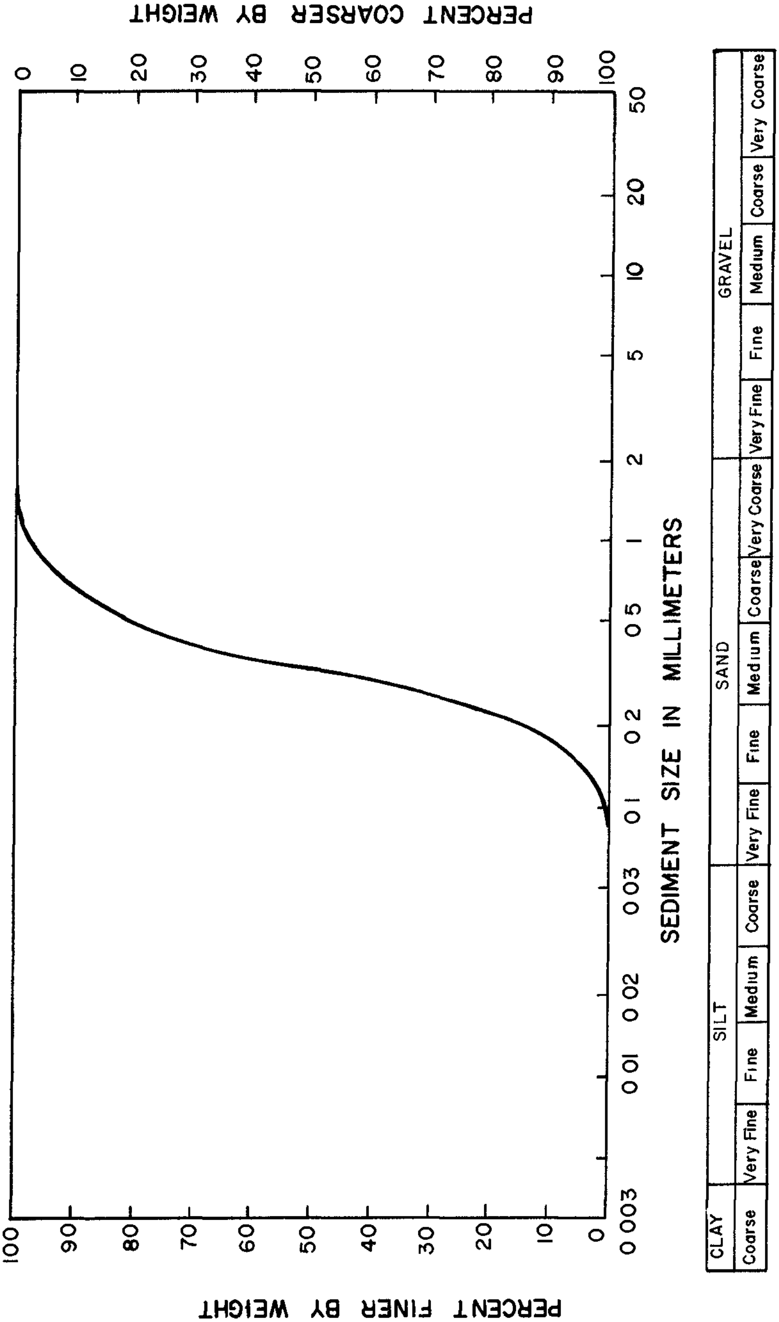

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FIG. 8 GRAIN SIZE DISTRIBUTION

Before each experiment, the sand bed was carefully leveled, wave recorder calibrated, water depth checked, and wave period and wave length adjusted. The reflection coefficients and wave heights were taken at a location between 10 to 20 feet away from the seawall. The scour depths were taken upstream of the seawall over a distance of 20 feet. All experiments were conducted until the scour depths became fairly constant with time.

Results

All experiments performed indicate that there may be a limit of scour depth which is approached asymptotically. The scour depth increases very rapidly during the first few hours and then the erosive process slows down and reaches a state of what other investigators called "ultimate" scour depth. Figure 9 is a plot of $\frac{\bar{S}}{K}$ as a function of $\frac{T}{t}$, where $\bar{S}$ is the average scour depth taken within a range of 15 feet in front of the seawall. The term T/t is actually the number of waves generated.

Figure 10 is a plot of $\frac{\bar{S}}{K}$ as a function of the coefficient of reflection, C_r, for a 15 degree seawall. It appears that the scour depth is only a random function of the reflection coefficient. This is probably due to the fact that the reflection coefficient depends on the wave characteristics, seawall slope and the kinematic behavior upon hitting the seawall. There is a great deal of difference between the reflection coefficients of a non-breaking and breaking wave on the seawall for the same wave characteristics.

Fluctuations in values of measured wave height and wave reflection were observed throughout the test, these are shown in Figure 11. It should be noted that the average wave height was used in all calculations.

Herbich and Murphy (1) concluded previously that the scour length is not influenced by wave height, water depth, seawall slope or the reflection coefficient. The same was observed in the current study and the scour length was approximately equal half the wave length.

The following comparison was made between theoretically calculated values and experimental results.

Experiment	Calculated Value from Equation 34 (inches)		Average Scour Depth $\bar{S}$ (inches)	Maximum Scour Depth S max (inches)
	$y = \frac{1}{2}\delta$	$y = 5d$		
A1	-1.13	0.585	0.457	0.50
A2	1.56	1.664	1.103	1.50
A3	1.22	1.950	1.170	1.55
A5	1.35	1.272	1.300	1.80
A6	1.09	1.170	1.420	1.80
A7	1.21	2.380	1.373	1.45

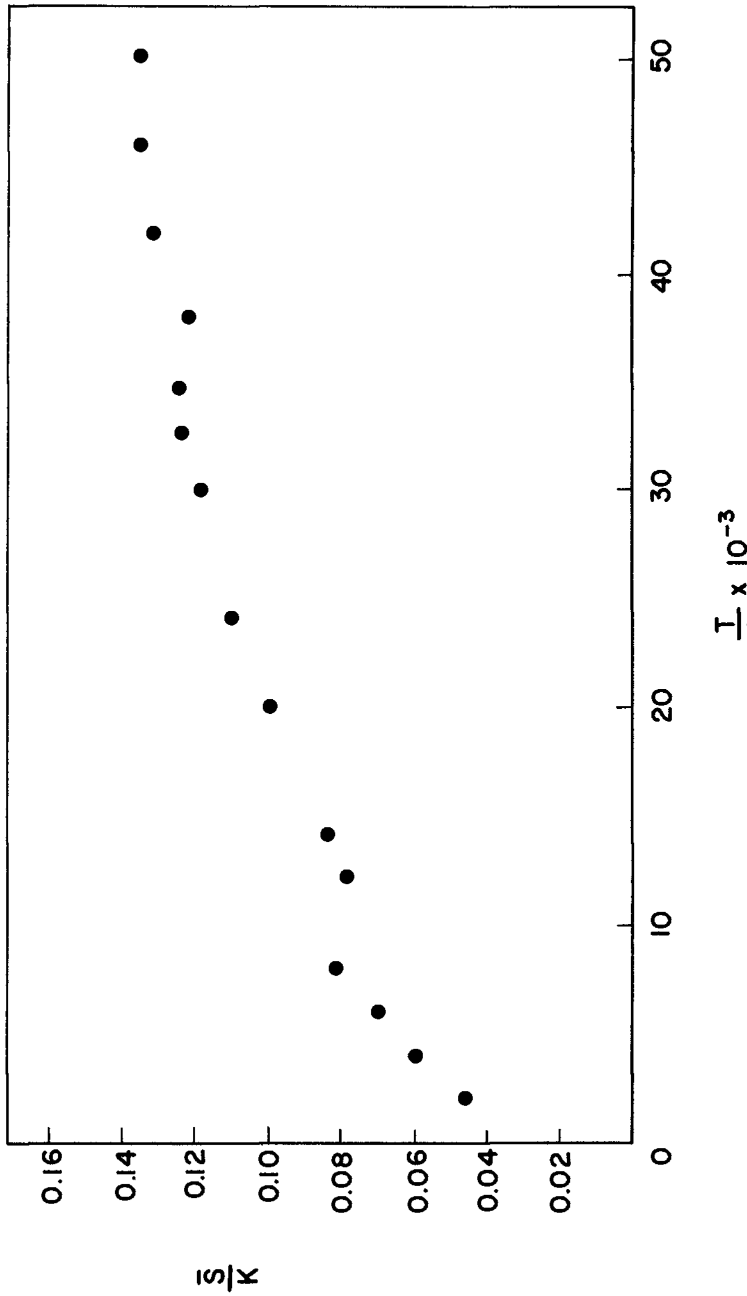

FIGURE 9 RELATIONSHIP BETWEEN $\frac{\bar{S}}{K}$ AND $\frac{T}{t}$

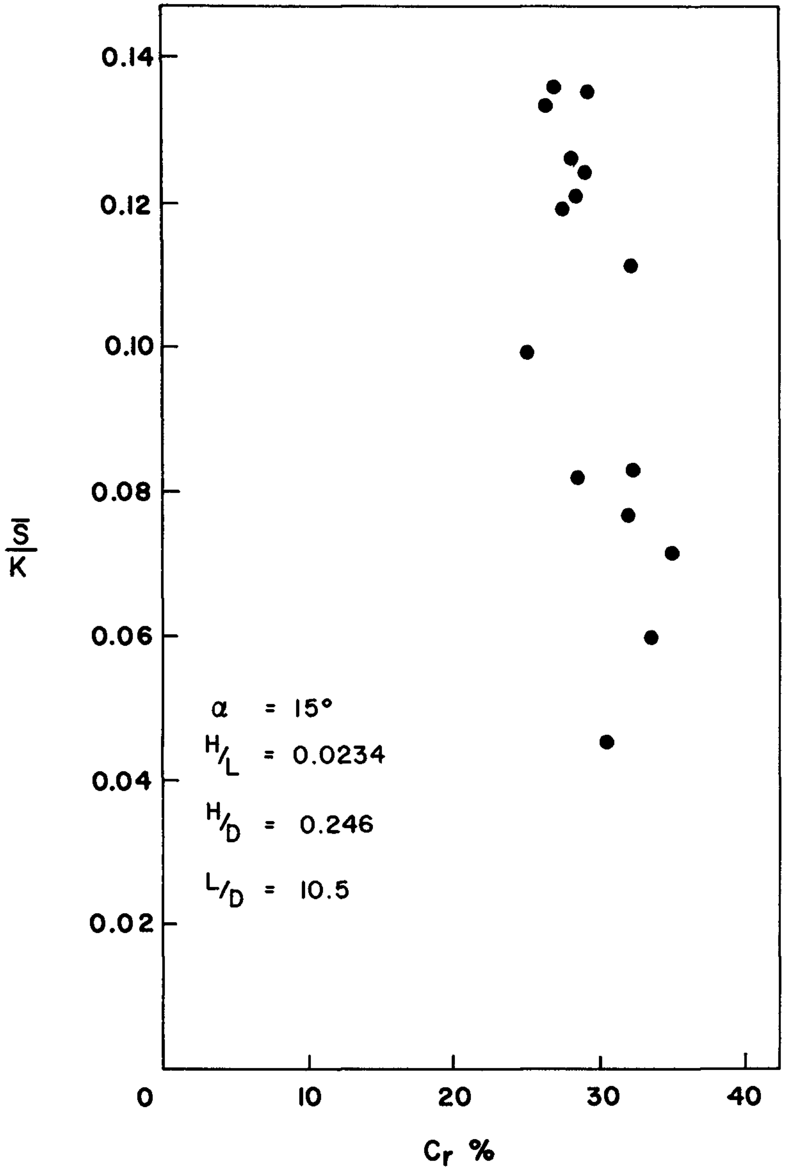

FIG. 10 RELATIONSHIP BETWEEN $\frac{\bar{S}}{K}$ AND C_r

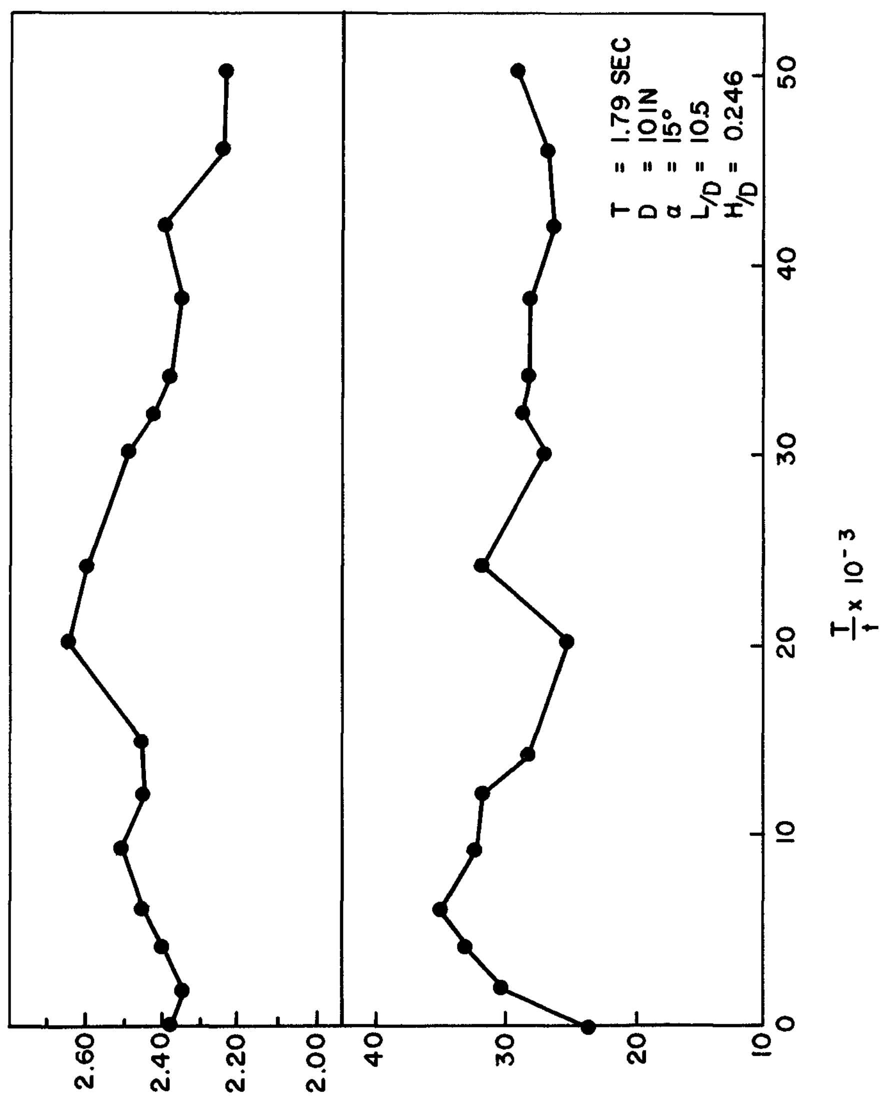

FIG. 11 FLUCTUATION OF H, C_r AS A FUNCTION OF $\frac{T}{t}$

The calculated values were based on two assumptions (a) that $y = \frac{1}{2}$ and that $\delta = 1.72 \left(\frac{ux}{U}\right)^{\frac{1}{2}}$ and $\eta = y \left(\frac{U}{L/2}\right)^{\frac{1}{2}}$ (column 2) and (b) that $y = 5d$ (column 3).

The agreement between theoretically predicted values and experimentally obtained values is considered good.

CONCLUSIONS

1. A mathematical model to describe scour in front of seawalls was developed. In considering the model the most important factors affecting ripple formation are water velocity and sand diameter.
2. The scour length is independent of time and only a function of wave length.
3. The "ultimate" scour limit is approached asymptotically.
4. Agreement between proposed theoretical equation and experimental data is reasonably good.

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DISCUSSION

Mr. J. A. Zwamborn, of Pretoria, South Africa, asked a question regarding the slope of the beach and said that it was not clear to him whether the slope was one of the variables investigated. He also indicated that Sato, Tanaka and Irie have done some work on scour of beaches as reported in paper 116 at this Conference. He also inquired whether similar vertical scour patterns may be found in front of natural coastal structures, such as sand dunes.

Mr. A. Paape, of Delft, The Netherlands, stressed the fact that the study was conducted in the laboratory and that difficulties may arise in predicting the behavior in the prototype due to possible scale effect. He also mentioned that nothing was said about the possible scale effect.

Mr. Thorndike Saville of Coastal Engineering Research Center, Washington D. C., U.S.A., concurred with Mr. Paape and indicated that more field studies are needed.

In closing the author said the following:

(A) In answer to Mr. Zwamborn: The first phase of the study was conducted with a horizontal beach. The next step in this investigation will be to vary the slope of the beach and study the effect of the slope on scour patterns and scour depth. The author was not aware of the study by Sato and others and will be looking forward to reading their complete paper. It is assumed that similar scour patterns will occur in front of other obsticles such as natural dunes.

(B) In answer to Mr. Paape and Mr. Saville: The question of scale effect between the model and prototype has not yet been solved for sediment movement in the rivers and no attempt has been made to predict the depth of scour on beaches under prototype conditions.

Two sizes of sand were used in this investigation and the results were found to be comparable. It is planned to make measurements in the field, some time in the future, along the beaches of the Gulf of Mexico.

Originally it was thought that scour would occur at the toe of the seawall - this however proved not to be the case as the first scour hole developed some distance seaward from the structure.

LIST OF SYMBOLS

A	$H_I + H_R$	L
B	$H_I - H_R$	L
C_D	Drag Coefficient	—
cn	Jacobian Function	—
C_r	Reflection Coefficient	—
D	Still Water Depth	L
$\underline{d}$	Effective Sand Diameter 50 per cent finer	L
F	Drag force, vector	F
F_x, F_y, F_z	Force Component	F
g	Acceleration due to gravity	L/T^2
h	ξ - D, Water particle under free surface	L
H	Wave Height	L
H_I	Incident Wave Height	L
H_R	Reflected Wave Height	L
K	D - ½ A, refer to Figure 2	L
K(k)	Elliptical Integral, first kind	—
L	Wave Length	L
P	Pressure	F/L^2
$\underline{S}$	Scour Depth	L
$\bar{S}$	Average scour depth in front of a seawall	L
t	Wave Period	T
T	Time Elapsed	T
u_*	Horizontal velocity within boundary layer	L/T
U	Horizontal velocity of wave motion	L/T
V	Velocity Vector	L/T
ρ	Water Density	F/L^3
ρ_s	Sand Density	F/L^3
θ	Angle of repose	degrees
Φ	Velocity potential function	—
γ	Specific weight of water	F/L^3
γ_s	Specific weight of sand	F/L^3
μ	Dynamic viscosity	FT/L^2
ν	Kinematic viscosity	$\frac{L^2}{T}$
R_e	Reynolds Number = $\frac{KV}{\nu}$	—
F_r	Froude Number = $\frac{V^2}{gK}$	—
α	Slope of seawall	degrees
λ	Scour wave length	L
ξ	Free water surface	—
η	Dimensionless co-ordinate of boundary layer	—