PART 3

COASTAL STRUCTURES

CHAPTER 48

MODELING OF STRUCTURES SUBJECTED TO WIND GENERATED WAVES

by

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ABSTRACT

The difficulties inherent in the direct determination of loads on off-shore structures which are exposed simultaneously to wind and waves make it desirable to model each situation in the laboratory It is shown here that scaling of the loads and the waves is possible by using waves which are generated by blowing air over the surface of a laboratory channel, and by choosing a model material with an appropriate modulus of elasticity Wind-generated waves such as those measured in the wind water tunnel of Colorado State University have a dimensionless spectrum (Hidy and Plate (1965)) that is identical in shape to that found off the coast of Florida under hurricane conditions (Collins (1966)) Furthermore, it has been shown that hydro-elastic modeling is quite feasible (LeMehaute (1966)) These two results are combined to give modeling criteria for off-shore structures if direct wind forces are disregarded

INTRODUCTION

The increased use of off-shore structures for the exploration and exploitation of the oceans has created a demand for accurate design information on the load conditions to which the structures are subjected. In contrast to most land based structures, the critical load conditions for an off-shore structure may be dynamic in nature and induced by the water surface waves, so that analytical design procedures may become very complicated except for simply shaped structural elements

Present analytical procedures for determining the response even of simple structures to periodic waves are not exact For example, it is customary to use the Morrison equation to determine the wave forces acting on a vertical cylinder. In the Morrison equation, the inertia and drag coefficients must be determined experimentally In some cases these coefficients can only be described statistically Furthermore, they may vary with depth, as has recently been demonstrated by Pierson and Holmes (1965)

Under these circumstances it becomes desirable to study the dynamic behavior of a structure on a laboratory scale, which is quite feasible as will be shown in this paper. LeMehaute (1966) has shown that hydro-elastic modeling can be accomplished conveniently by using modern plastics for model construction If it can be ascertained that the response of the structure is linear, then the modeling problem consists of requiring identical shapes of transfer functions in model and prototype obtained by the modeling transformation The transfer function is then the quantity which needs to be determined from the experiment, and it can be found conveniently by exciting the model structure with a sequence of sine waves, and finding the response to each of them

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Most structural responses are nonlinear (although many are nearly linear), so that their transfer functions cannot be constructed by superposition. In such cases laboratory experiments are the only alternatives to potentially extremely complex calculations, for which the model structure must be excited with a forcing function which is dynamically similar to the prototype forcing function. In particular, for structures excited by wind generated waves, the model wave spectrum must be dynamically similar to that of the prototype Dynamic similarity of the spectrum does imply similarity of the shape of the spectra, as well as similarity of the energy contained in the spectra.

The need for similarity in spectral shape has been realized for some time Thus, Nath and Harleman (1967) produced a spectrum whose shape was similar to that of spectra found in parts of the Atlantic Ocean by Pierson and Moskowitz (1964). They used a wave generator for exciting model structures which was programmed to generate a quasi-random wave train with a spectrum of the desired shape.

Wave trains produced by a wave generator have several features which differ from that of a wind generated wave pattern. A wave generator can only produce wave components which agree with the free modes of the water surface. That is, each of the wave components is sinusoidal in shape and travels independently of all other wave components Thus, when a large wave is formed by the superposition of component sinusoidal waves that are momentarily in phase, although of different wave lengths, one finds that the life of the wave is fairly short. The large wave forms when the components are in phase, and then disappears as the components become out of phase due to the different component celerities. In contrast to this, one finds that large wind generated waves at high wind speed are quite long-lived and are, therefore, not composed of independently traveling sinusoidal waves. Nor is their total shape sinusoidal In Fig. 1 an example is shown of a significant wave which is found at a wind speed of approximately 10 m/sec This wave has been obtained by averaging the highest 20 waves from a record of about 300 waves measured in a laboratory channel. The waves were superimposed in such a way that their highest points coincided on the time axis and the ordinate values were averaged. A confidence band given by the local standard deviations of the coordinate values about the mean is also shown. It is apparent that even though the wave travels as a whole, its shape is definitely not sinusoidal Consequently, even though the spectrum of generator produced random waves might match that of wind generated waves, the shapes of the resulting individual waves are not exactly the same. Results obtained by this procedure are therefore subject to question, particularly if the dynamic response of the structure is nonlinear.

A second, and possibly more important difference between a generator produced spectrum and a fully developed wind wave spectrum lies in the inability of the former to properly model the magnitudes of the spectral densities. Complete dynamic modeling requires that both wave lengths and amplitudes are modeled by the same scale factor As shall be shown later, this condition requires that the dominant wave, which corresponds to the frequency at the peak in the spectrum for fully developed conditions, reaches its maximum possible amplitude. It is doubtful that this condition can be met while maintaining the required spectral shape, because the variance of a wave spectrum driven by strong winds increases with fetch, while that of the paddle generated wave spectrum decreases, partly due to viscosity, and partly due to breaking when superposition of component waves increases wave amplitudes beyond their stability limit

More suitable test conditions are obtained when wave trains are used, whose spectra, as well as shape and amplitudes of individual waves, resemble those of wind generated waves, but on a smaller scale. Such wave trains are found in the laboratory

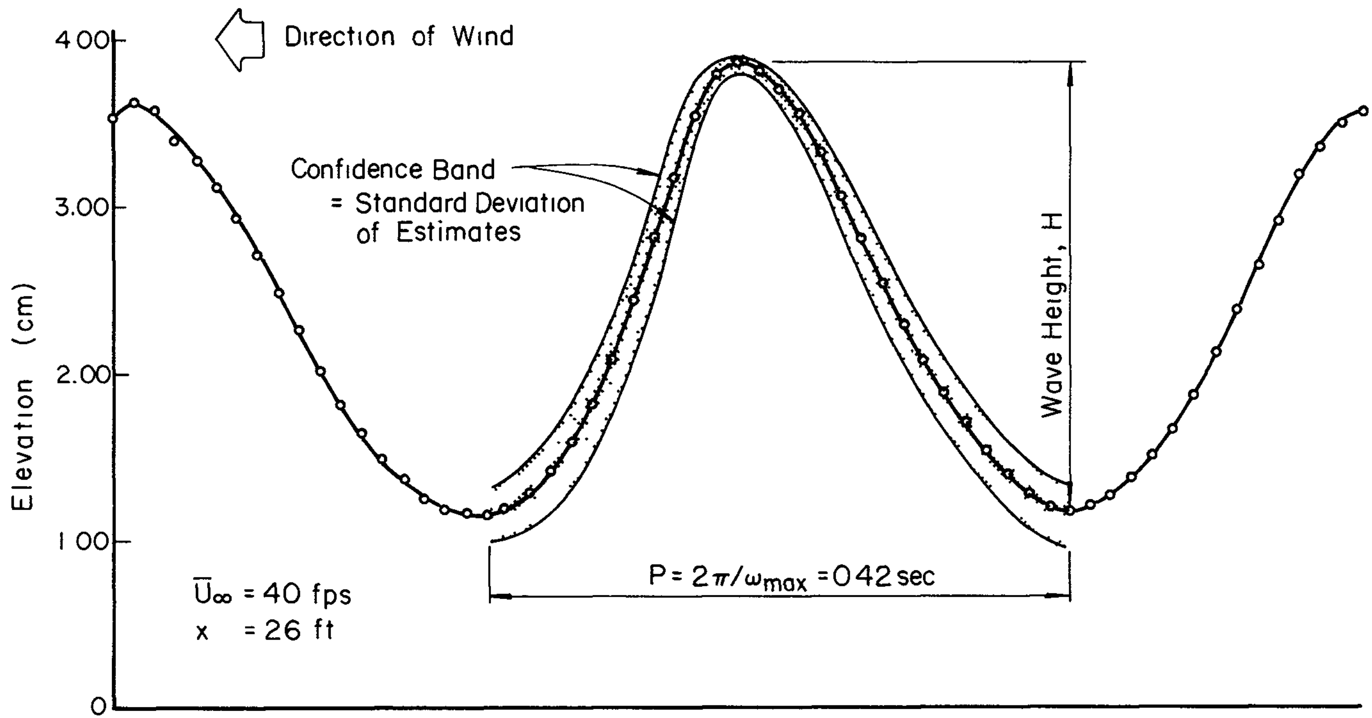

Fig. 1 Average Wave Form of wave at high wind velocity (from Po-Chang).

if air is blown over the surface of water standing in a channel. In this paper, the application of wind generated waves to modeling of wave forces is discussed. Since the wind generated laboratory waves have fewer drawbacks than paddle generated waves, it would be advisable to use them to obtain the load conditions on model structures By this method one also obtains a means to evaluate the wind loads on the part of the structure above the water surface

As shall be discussed in the first part of this paper, the spectrum of fully developed wind generated waves in a laboratory is similar in shape to that found for ocean waves, and it also yields a relationship between peak frequency and variance of the spectrum which is scaled by the Froude number in such a way that no change in scale between vertical and horizontal dimensions becomes necessary In the second part, considerations will be given to the modeling requirements of the structure This will be discussed on the basis of simple linear structures, and it is shown that Cauchy number similarity is feasible together with Froude number similarity.

The Similarity Spectrum of Wind Generated Waves

The driving forces acting on the submerged portion of a dynamically loaded offshore structure result from the wave field generated by the wind. They act in addition to the wind force on the superstructure Even though this wave field usually appears random both in space and time, it is nevertheless possible to distinguish, especially in the neighborhood of coasts, crests of waves which give the wave field an appearance of local two-dimensionality, with a predominant direction of progression perpendicular, or almost perpendicular, to the crest For such a wave pattern, a laboratory analogue exists in the wind generated waves which are obtained when air is blown over the surface of the channel in which water is standing It seems possible that the majority of all wind driven ocean waves outside of a storm center consists basically of a dominant pattern of this kind This would offer a logical explanation for the observation that one-dimensional wave spectra both in the laboratory and in oceans have an approximately equal shape. An illustration of this phenomenon is given in Fig 2a where a typical laboratory spectrum, obtained by Hidy and Plate (1965), is compared with a set of ocean wave spectra generated by strong off shore winds off the coast of Florida shortly after the passage of Hurricane Dora, in September 1964. The spectra were calculated by Collins (1966) The peak density of the larger ocean wave spectrum is 10,000 times larger than the peak density of the laboratory spectrum Yet, the shapes do not differ significantly.

The identical shape of the dimensionless spectra shall be explored here A non-dimensional form of the spectral shape that is suitable for the purpose of this paper has been suggested by Hidy and Plate (1965) Hidy and Plate (1965) recommended to non-dimensionalize the spectra by dividing the frequency axis by ω_{max}, which is the angular frequency at which the spectral peak $\phi(\omega_{max})$ is observed The spectral density $\phi(\omega)$ was reduced so that the area under the non-dimensional spectrum $S(\omega/\omega_{max})$ is equal to 1 This leads immediately to the requirement

$$S\left(\frac{\omega}{\omega_{max}}\right) = \frac{\omega_{max}}{\sigma^2} \phi(\omega) \tag{1}$$

where σ^2 is the variance of the water surface elevation. This non-dimensionalizing procedure is equally valid for ocean wave data, as was shown by Colonell (1966). In Fig. 2b, the average curve of Hidy and Plate (1965) is given, which is representative for many different laboratory spectra. The similarity of this shape with that of Fig. 2a is noted Also, it is seen that the high frequency end of the spectrum follows

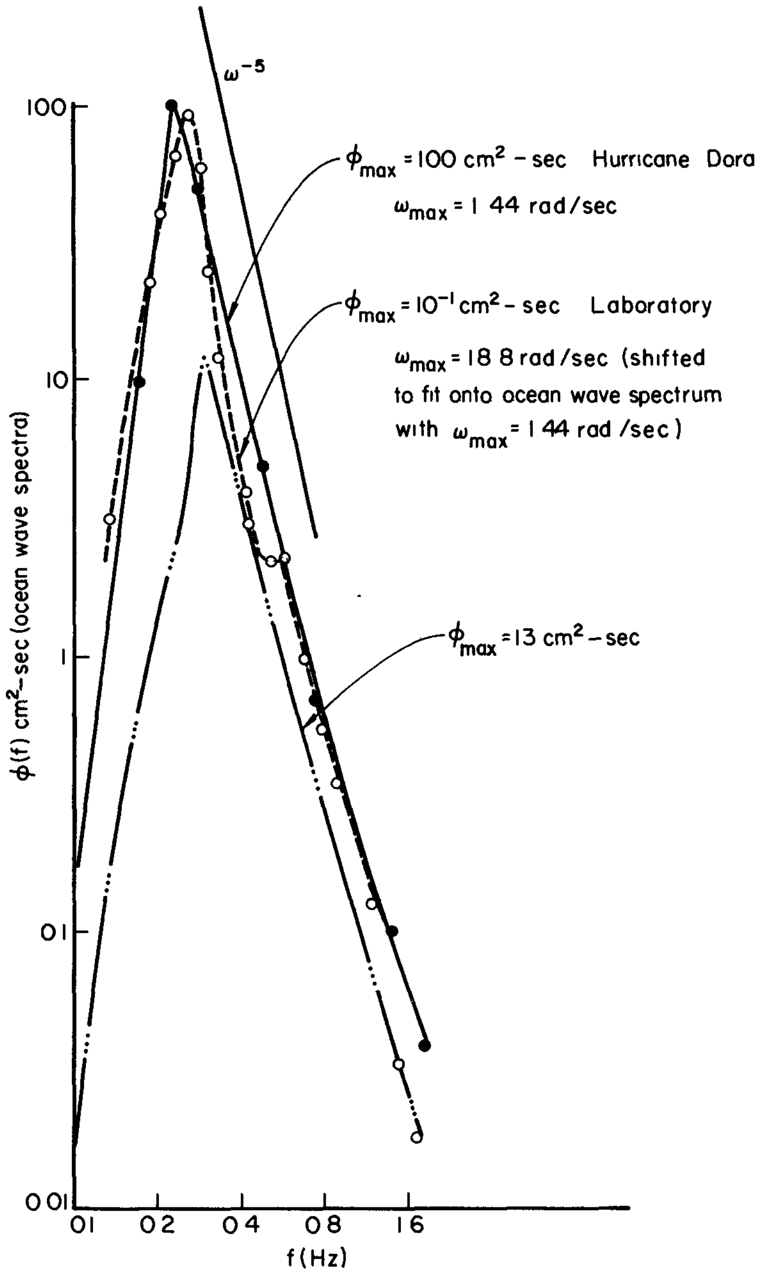

Fig 2a Examples of Ocean and Laboratory Spectra

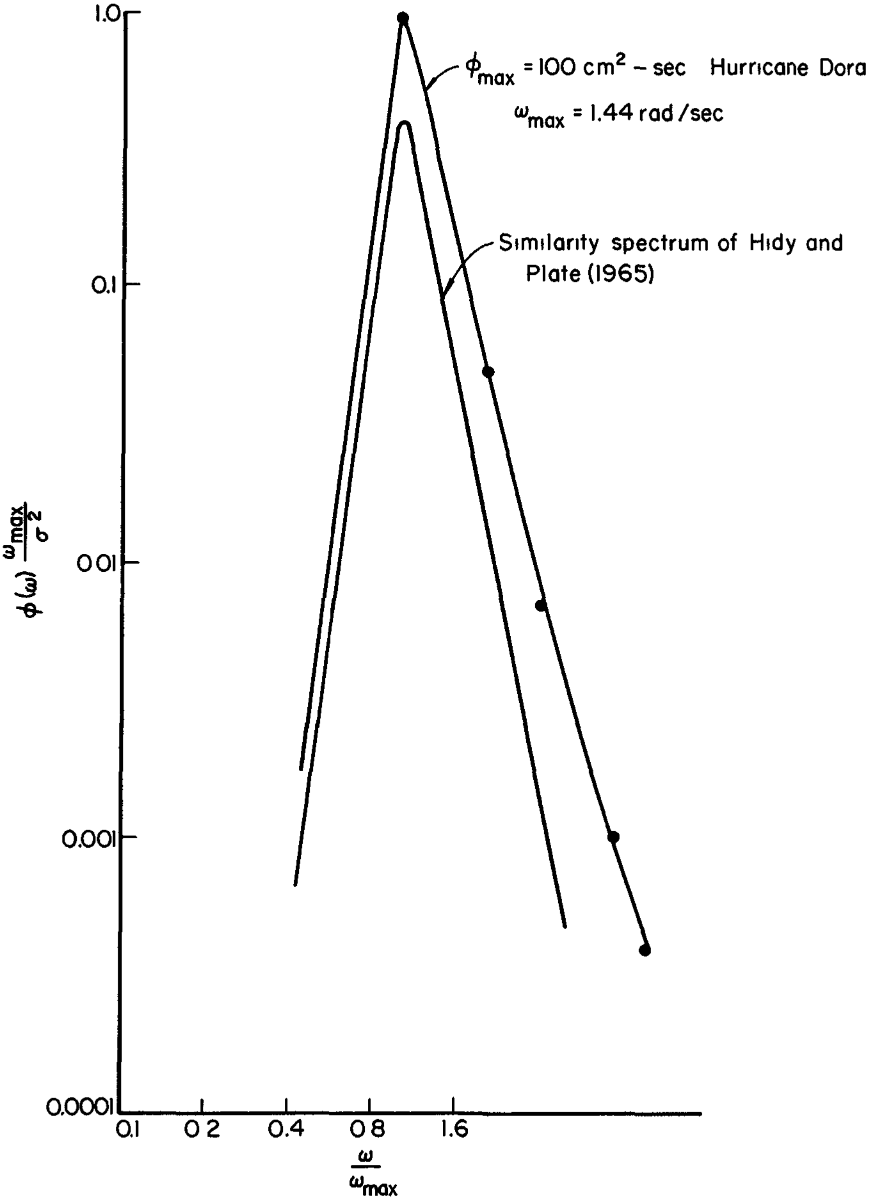

Fig 2b Examples of Ocean and Laboratory Spectra

approximately a ω^{-5} law, as predicted by Phillips (1958) However, the -5 law seems to be only an approximation to the spectra at high frequencies There is evidence that the exponent in the power law varies from about -7 near the spectral peak to -4 at higher frequencies.

More significant than the high frequency behavior of the individual spectrum is the fact that all spectra obtained from ocean or laboratory are bounded at the high frequency end by a universal curve given by the equation of Phillips

$$\phi(\omega \gtrsim \omega_{max}) = 1\ 05 \quad 10^{-2} g^{+2} \omega^{-5} \tag{2}$$

where g is the constant of gravity This is shown in Fig. 3, which has been reproduced from Hess (1968). Phillips (1958) derived Eq. 2 by using dimensional analysis. Recently Plate, Chang and Hidy (1968) have provided arguments which deduce the -5 power law as an upper limit of spectral growth, independent of the shape of the individual spectrum, provided only that all spectra are similar The -5 power law then becomes a law which relates peak spectral density $\phi(\omega_{max})$ to ω_{max} Since this law plays a key role in the modeling criteria to be developed, the derivation shall be outlined here

The basic assumptions are

a. The spectrum has a sharp peak near ω_{max} , and can be described by the similarity shape of Fig 2b. According to Longuet-Higgins (1952) this implies that the water surface undulations consist mainly of a train of waves of frequency ω_{max} whose amplitudes are subject to a random modulation. This model agrees well with observations both of water surface elevation recordings and of wave spectra The wave of frequency ω_{max} shall be called the dominant wave, whose height is denoted by H .

b The maximum growth of the dominant wave component is limited by the acceleration of gravity such that

$$a_{max} = \alpha g \tag{3}$$

where a_{max} is the maximum acceleration of the surface of the dominant wave, and α is a number which is smaller than 1 It will be shown that $\alpha \approx 0.3$ leads to an estimate which is consistent with the numerical factor of 0.0105 in Eq. 2.

Longuet-Higgins (1952) has shown that a Gaussian wave record which satisfies condition (a) above has wave heights H which are Rayleigh distributed. Consequently, the mean value, $H^{(p)}$, of the pN highest of N waves is a constant multiple of the mean wave height $\bar{H}$, or

$$\frac{H^{(p)}}{\bar{H}} = m \tag{4}$$

where m is a constant. Typical values of m are $m = 1\ 42$ for $p = 1/3$ and $m = 1.26$ for $p = 1/2$. Thus, if each wave is basically sinusoidal, then the average vertical acceleration of the water particles at the peak of the highest pN dominant waves is found to be equal to

$$a_m = \frac{1}{2} \omega_{max}^2 \quad H^{(p)} \quad . \tag{5}$$

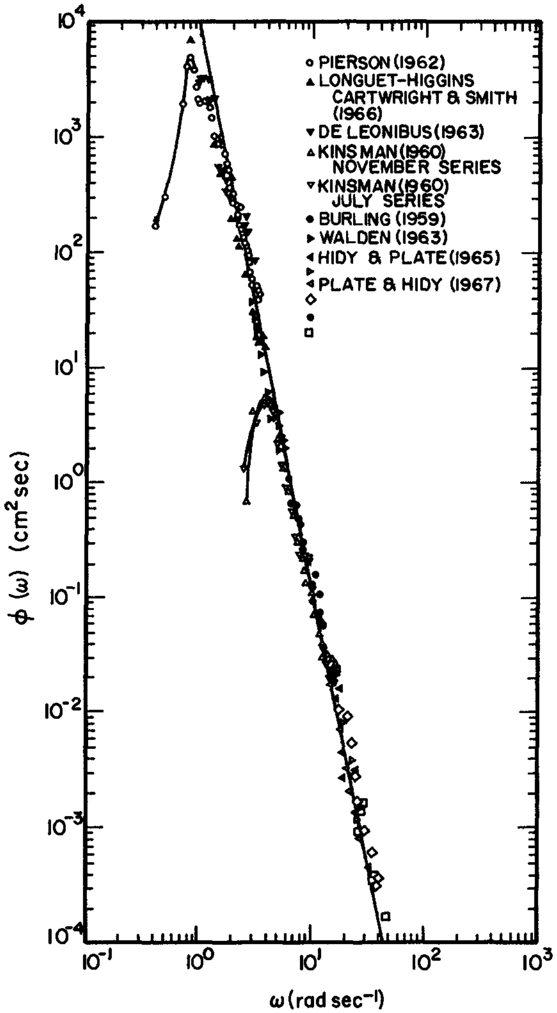

Fig. 3 A Comparison of Observed Wave Spectra with Geophysical Wave Spectra (from Hess, 1968)

Somewhat arbitrarily, it is assumed that the significant acceleration is that of the significant wave $H^{(1/3)}$. For a wave record whose water surface elevation is Gaussian (which is approximately true both for the laboratory and the field), there exists the following relation between mean wave height and variance of the water surface elevation (Collins, 1966)

$$8\ \sigma^2 = \overline{H}^2 \tag{6}$$

with m = 1.42, the relationship between the acceleration of the significant wave $H^{(1/3)}$ and the variance σ^2 of the wave record is found as

$$a_m^2 = 4\ \omega_{max}^4\ \sigma^2 \tag{7}$$

But according to assumption Eq. 3, this is also equal to $\alpha^2 g^2$, so that

$$\sigma^2 = \frac{\alpha^2 g^2}{4\ \omega_{max}^4} \tag{8}$$

We can eliminate σ^2 in Eq. 1 and let $\omega = \omega_{max}$ It is then seen from Fig 2 that $S(\omega_{max}/\omega_{max}) = S(1) \approx 0\ 5$ for a fully developed sea, and consequently

$$\phi(\omega_{max}) = \frac{\sigma^2}{\omega_{max}}\ 0\ 5 = \frac{\alpha^2 g^2}{8\ \omega_{max}^5} \tag{9}$$

which is independent of the spectral shape except for the requirement of similarity. This result is now compared with Eq 2 where ω is replaced with ω_{max}. One obtains

$$\phi(\omega_{max}) = 1\ 05\ 10^{-2}\ g^2\ \omega_{max}^{-5} \tag{10}$$

Consequently, one finds

$$\alpha = 0\ 29 \tag{11}$$

which is somewhat lower than the limiting vertical acceleration of the Stokes wave, where $\alpha = 0.5$.

The reasoning leading to Eq. 10 is of consequence for the purpose of modeling. When conducting a model study, it is naturally desirable to model the maximum forces that can occur Waves with amplitudes exceeding that of the significant wave would presumably break, because their accelerations exceed the critical value αg Therefore, waves at frequencies $\omega > \omega_{max}$ of larger amplitudes than that at ω_{max}, cannot occur in an equilibrium spectrum described by the similarity shape and by the relation between peak spectral density and the corresponding frequency expressed through Eq 10 If larger waves are to occur they must therefore be of lower frequencies.

Equation 10 also implies that the waves that reach the limit of growth are subjected to continuous addition of energy through work done by the wind, so that the dominant wave remains at its maximum height. A spectrum of waves with a dominant wave consisting of swell from a far away, and perhaps long subsided storm, might still have the similarity shape of wind generated waves, but its maximum spectral density will be below that of Eq. 10. Consequently, the maximum possible spectral density for waves of frequency ω_{max} is given by Eq. 10, which therefore describes the envelope for all fully developed wave spectra.

Fully developed wave spectra are found in particular when wind of long duration is blowing, such as during hurricanes An example is given by results of Collins (1966) which were obtained at two different times at two different off-shore fetches during an off-shore blowing wind. In Table 1 the peaks of the spectra calculated from Eq. 10 are compared with the peaks obtained by Collins. It is remarkable that the long fetch spectra are in exact agreement with Eq 10, while the lower fetch data are below the saturation value given by Eq 10 Many other data show the same behavior, as is evident from the data of Fig 3.

TABLE 1. PEAK AMPLITUDES OF HURRICANE DORA DATA (from Collins, 1966)

Case	f_{max} (Hz)	$\phi(f_{max})$ (observed) m^2-sec	$\phi(f_{max}) = 2\pi\phi(\omega_{max})$ (from Eq 10) m^2-sec	Remarks
AI	0.35	0.130	0 124	Sept. 9, 64 Fetch 11 miles
AII	0 45	0 018	0 034	Sept. 9, 64 Fetch 1.7 "
BI	0.23	1 05	1.00	Sept 10, 64 Fetch 11 "
BII	0 27	0.2	0 45	Sept.10, 64 Fetch 1.7 "

We notice that the results of Eqs 9 and 11 yield the important relationship between variance, σ^2 , and ω_{max}

$$\sigma^2 = 2.1 \quad 10^{-2} g^2 \omega_{max}^{-4} \quad . \tag{12}$$

Equation 12 shows that the scale of the amplitudes of the wave which must also scale the square root of the variance of the spectrum fixes the scale of the frequencies, so that it is not possible to adjust the two scales independently. If it is desired to model wave heights so that

$$(\sigma/L)_m = (\sigma/L)_p \tag{13}$$

then it follows that the frequencies must be related like

$$\frac{\sigma_m}{\sigma_p} = \frac{L_m}{L_p} = \frac{\omega^2_{max\ p}}{\omega^2_{max\ m}} \tag{14}$$

Here, L is any characteristic length, the subscript m refers to the model, and the subscript p to the prototype. This condition is in accord with the requirements imposed on the wave length λ . If it is desired to have identical non-dimensional wave lengths, λ/L , for model and prototype, then it would be necessary to set

$$\frac{L_m}{L_p} = \frac{k_p}{k_m} = \frac{\lambda_m}{\lambda_p} \quad . \tag{15}$$

However, for gravity water waves, the frequency is related to the wave number, $k = 2\pi/\lambda$, by.

$$\omega^2 = g\ k \tanh kh \tag{16}$$

where h is the depth, as defined in Fig 4, and thus

$$\frac{\omega_p^2}{\omega_m^2} = \frac{k_p}{k_m} = \frac{L_m}{L_p} = \frac{h_m}{h_p} \tag{17}$$

in agreement with Eq 14. Consequently, the use of a fully developed wave spectrum in the laboratory for simulating a fully developed wave spectrum for prototype conditions results in identical scale ratios of both wave heights and wave lengths, if the spectra are related by Eq 14. The use of Eq. 14 thus leads to an undistorted geometrical scaling of the whole wave field This result establishes the advantage of using wind generated waves in the laboratory for modeling wind generated waves in the field

It is interesting to note that modeling according to Eq. 14 implies Froude number scaling, i.e.,

$$Fr = \frac{u_{wave}}{\sqrt{gL}}_m = \frac{u_{wave}}{\sqrt{gL}}_p \tag{18a}$$

where u_{wave} is a wave related velocity such as c_o, or the wave induced at some reference depth. For the latter, u_{wave} is proportional to $\sigma\omega$ for any frequency component and one obtains

$$\frac{\omega_m \sigma_m}{\sqrt{gL}_m} = \frac{\omega_p \sigma_p}{\sqrt{gL}_p} \quad . \tag{19}$$

If σ is eliminated through Eq. 14, then Eq 17 follows Consequently, Froude number modeling according to Eq 19, in conjunction with Eqs. 9 and 10, results in fully developed wind generated wave spectra in model and prototype which are geometrically similar with equal vertical and horizontal scales Therefore, the model structures can be built to scale, and the complications which arise from distorted scales can be avoided. It should be mentioned that the definition of a modeling Froude number according to Eq 19 is somewhat more stringent than is required for linear response of structures under the effect of a wave force. For such a system, the amplitude appears only in the load function and therefore need not be scaled properly because its effect can be included into the conversion factor which is used for calculating prototype response data from model data. Then the Froude number is more suitably defined by

$$Fr = \frac{c_o}{\sqrt{gL}} \tag{20}$$

Since c_o is independent of the vertical scale of the wave motion, this modeling criterion only suffices to satisfy Eq 15, but not Eq. 13.

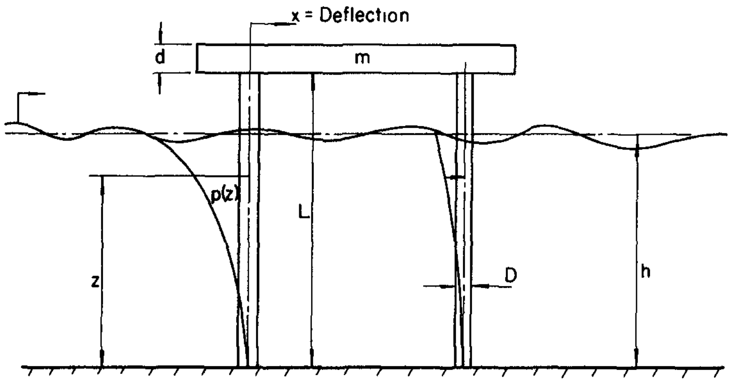

Fig 4 Definition Sketch for Off-Shore Structure

In applying Eq. 10 to laboratory modeling, it is required that some value of the height of the significant wave is known which can be expected under prototype conditions at the position where the structure is to be constructed. How this wave height can be found shall not be discussed here If it is known, then the variance of the water surface follows from Eqs. 4 and 6, the peak frequency, ω_{max} , from Eqs. 8 and 11, and the spectral peak from Eqs 9 and 11. The spectrum can then be constructed with variance σ^2 , ω_{max} , and the similarity spectrum of Fig. 2. After choosing a suitable length scale, the model spectrum can be obtained by reducing the variances of the water surface and the frequencies according to Eq. 14. It remains to show under what conditions it will be possible to obtain a scaled dynamic response of the structure.

Modeling of Structures Subjected to Wave Forces

When a linear structure is excited by a random dynamic load whose stationary spectrum is $\phi_L(\omega)$, it is well known that the deflection spectrum $\phi_x(\omega)$ of a characteristic point on the structure can be expressed by

$$K^2 \phi_x(\omega) = | H(\omega) |^2 \phi_L(\omega) \quad . \qquad (21)$$

where K is the spring constant of the support legs and H is a transfer function. Multiplication of the deflection spectrum by the square of the spring constant K of the support legs signifies that we assume the structure to be so stiff that deflections are within the range of validity of Hooke's law. Then the spring constant is a multiplier whose magnitude must be known. It is to be chosen according to dynamic similarity of the structural response

The function $H(\omega)$ of Eq. 20 is the transfer function of the structure which establishes the dynamic response of the structure under the effect of the load spectrum. In our notation, it is a dimensionless function whose value at zero frequency must be one, to correspond to the case of static loading. The simplest structure, such as shown in Fig 4, consists of one or more cylindrical supports which are clamped to some degree into the ground and into the working platform. The transfer function of each of these cylinders is then given by that of a simple second order system

$$| H(\omega) |^2 = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2} \tag{22}$$

which is valid for the dominant first vibrational mode, whose natural frequency is equal to ω_n The coefficient ζ is the relative damping factor It is determined almost completely by the internal structural damping, while the natural frequency is given to

$$\omega_n = \sqrt{\frac{K}{m}} \tag{23}$$

where m is the mass of the load on the single cylinder, and K is the spring constant

$$K = \gamma \frac{3\ EI}{L^3} (1-\beta) \tag{24}$$

In this equation, E is the modulus of elasticity, I the moment of inertia, and L the length of the cylinders, as indicated in Fig 4 The clamping coefficient γ corrects for the possibility of rotation of cylinder top and bottom When the cylinder is clamped into the ground, and into the platform (i e., the cylinder is one leg of a multilegged structure) so that the joints cannot rotate then $\gamma = 1$ A more flexible structure, resulting from only partial clamping in the ground or at the platform yields a smaller γ, while increased stiffness, as obtained for example by braces across the legs results in a larger value of γ Evidently, for more complex structures a simple correction of the single cylinder spring constant does not suffice, and a suitable elastic model may be the only feasible alternative The coefficient β is the reduction factor due to vertical loads, i e.,

$$\beta = \frac{4WL^2}{\pi^2 EI} \tag{25}$$

where W is the vertical load applied to the cylinder top As long as β is significantly smaller than 1, a vertical load can be used to tune the structure, so that its natural frequency assumes the desired value.

For the multilegged structure as a whole, the transfer function is the same as that for the individual legs except for a modification factor which results from the time lag between the forces on different cylindrical legs. The phase shift may either lead to adding total forces on the structure or subtracting, and might even cancel depending on the distance between legs as related to wave length, and on the orientation of the structure This effect has been discussed by Nath and Harleman (1968)

In a model study, the effects of structural complications and intramodal coupling on the transfer function may possibly be determined experimentally We notice that for the transfer function to be modeled in the laboratory, it shall have to meet the following requirements

1 Model and prototype must be geometrically similar, and must be scaled such that the length scales of the wave motion are identical to the scale of the geometry of the structure. In this manner, interference effects due to loads on different individual legs and other structural elements are accounted for.

2. Dimensionless (or relative) damping coefficients of model and prototype associated with each mode must be identical. This requirement is not met easily. It is probably suitable to use a damping coefficient based on experience and correct the damping of the model structure by an external arrangement of dashpots

3. The natural frequencies of model and prototype must be chosen such that ω/ω_n is identical in model and prototype This must be interpreted as requiring that the ratio of the ω_{max} of the peak of the load spectrum to the natural frequency of the structure, ω_n , is the same, or

$$\left(\frac{\omega_{max}}{\omega_n}\right)_{model} = \left(\frac{\omega_{max}}{\omega_n}\right)_{prototype} \tag{26}$$

Evidently, the preceding requirements imply that the output spectrum, as well as the input spectrum, is similar in model and prototype. The modeling requirement expressed by Eq. 26 implies simultaneous equality of both Froude number and Cauchy number in model and prototype. Generally speaking, the Cauchy number is defined as

$$C_a = \frac{u_W}{u_E} \tag{27}$$

where a wave reference velocity might be $u_W = \sigma\omega_{max}$, while the reference elastic velocity u_E usually is the speed of sound E/ρ However, in the case of bending deformation of the structure, it appears to be more rational to use an elastic velocity $u_E = L\omega_n$, so that

$$C_a = \frac{\omega_{max}\,\sigma}{\omega_n\,L} \tag{28}$$

for similar geometrics, Cauchy number similarity for model and prototype requires that Eq. 26 be satisfied This equality together with Froude number equality can be met, however, only if the elastic properties are suitably adjusted By using the definition of the natural frequency, Eq 23, in conjunction with Eqs. 24, 14 and 27, it is seen that

$$\alpha \frac{E_m\,\rho_p}{E_p\,\rho_m} = \frac{L_m}{L_p} \tag{29}$$

when ρ_p and ρ_m are the bulk densities of the loads on the cylinder for prototype and model, respectively. The factor α , given by

$$\alpha = \frac{\gamma_m (1 - \beta_m) I_m L_p^4}{\gamma_p (1 - \beta_p) I_p L_m^4} \tag{30}$$

reduces to one if exact geometric similarity of the model and prototype support structures exist, and if the damping conditions are identical. A factor $\alpha = 1$ is, of course, not necessary. For example, instead of the thin-walled cylinders used for the legs of prototype structures, we might prefer to use solid legs in the model.

The result given by Eq 28 can be derived more formally by the methods of inspectional analysis, as was evidently done by LeMehaute (1966) It gives a means of obtaining, by suitably adjusting α , E_m and ρ_m , the dynamically correct response of the structure to wave forces, provided that the load spectrum is also modeled properly.

Acknowledgments

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