CHAPTER 6

THE ANALYSIS OF WAVE RECORDS

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ABSTRACT

Data obtained from two surface profile wave gages and two pressure wave gages at the Steel Pier in Atlantic City, New Jersey, are used to check the consistency of the analysis variables obtained from a given set of records by several commonly used analysis procedures

All estimates of the characteristic height tested are found to be correlated better than 86 The estimates of characteristic period are not so satisfactory and in some cases are below 25 Consideration of several proposed definitions of the characteristic period indicates that they are based more on convenience in data processing than on application of the derived data Consideration of the use of wave data in engineering design shows that no one definition of the period can be satisfactory for all applications The best definition of the characteristic wave period for a given engineering problem can be specified only when the dynamic aspects of the problem have been identified

INTRODUCTION

The concept of a "significant wave height" and a "significant wave period" which can be used to characterize a wave field is appealingly simple It suggests a simple transition from the experimental results in a laboratory wave tank and the theoretical results obtained with monochromatic wave theory to the phenomena that occur in the real ocean

This concept was first introduced when sailors were asked to report the height and the period of " the larger, well formed waves, and omit entirely the low and poorly formed waves " as part of the synoptic weather reports from ships Comparison of early wave gage records, with visual observations, led to the opinion that the wave height "H " given by visual observers was the average height of the one-third highest individual waves, "$H_{1/3}$" Figure 1, taken from Ross (1966) and based on an earlier figure by Cartwright (1962) provides some perspective on the reliability of this approximation Figure 1 is based on 905 pairs of visual and instrument observations from a weather ship equipped with a shipboard wave recorder For the data included in this figure $H_{1/3}$ = 1 1 H_V Comparisons of shipboard observations by two or more observers are given by Hogben and Lumb (1964, 1967) and likewise show considerable scatter

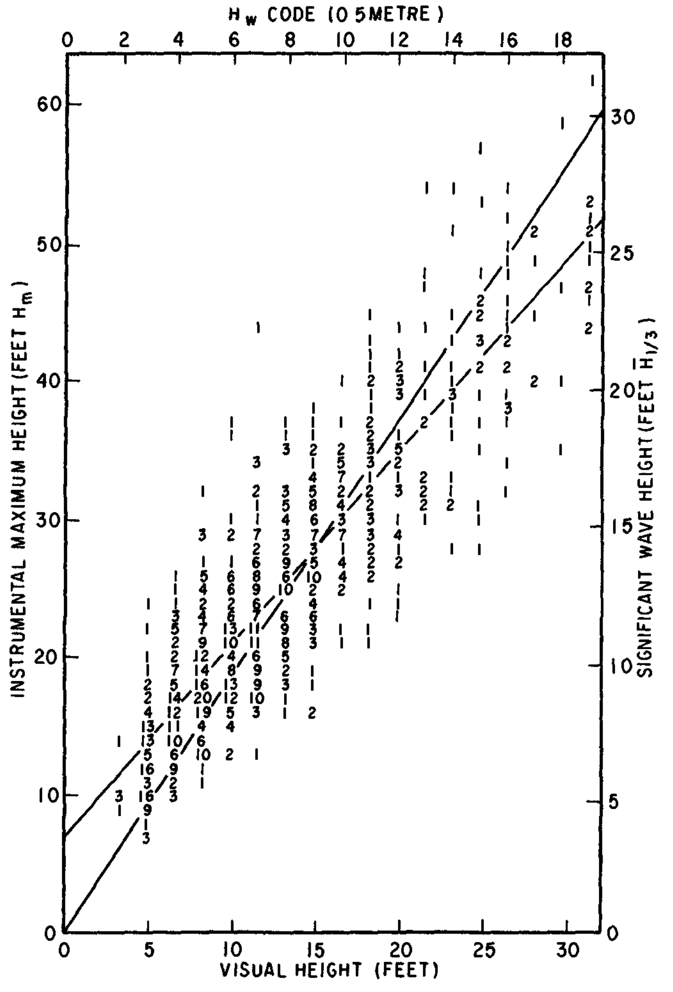

Figure 1 Comparison of instrument and visually observed heights (after Ross)
For the data in the above figure $H_{1/3} = 1\ 1\ H_V$

Many wave records have been analyzed by listing the heights and periods of individual waves for the determination of $H_{1/3}$ and the corresponding period $T_{1/3}$ This process is rather tedious and not truly objective, for there is no purely objective way of making a clear distinction of which perturbations on the record are waves that should be counted and which are ripples that should be omitted

This problem is illustrated by Figure 2, where simultaneous records from four wave gages, located within a 12-foot circle on the Steel Pier in Atlantic City, are shown Note that some of the waves appearing on the surface gage records do not appear on the pressure records, and that almost any procedure for determining which waves should be counted would accept some waves on one surface record that would be rejected on the other

OTHER DEFINITIONS OF WAVE PERIOD

To obtain a more objective measure of the wave period, various writers (Draper, 1966, Tucker, 1961) have suggested using the average period of all zero up-crossings as the characteristic period Both of these depend somewhat on the resolution of the wave recording system and Tucker's depends on a practical method for determining the zero line Draper suggests that it can be estimated by eye Thus two different estimates may lead to two different values If interest is centered on the wind-generated waves, it would be more appropriate to consider crossing of a trend line The difference would not be important with large waves and small tides, but it could be significant with large tide ranges and small waves This latter combination can be important in sedimentation problems

Since 1965, it has been customary at CERC to identify the most prominent period in a 7-minute wave record as the significant wave period

To clarify the meaning of wave period estimates, it is useful to note that according to the linear theory for monochromatic progressive waves,

$$h(t) = \bar{h} + A\cos(kx-\sigma t-\phi) \tag{1}$$

$$w = \sigma A \frac{\sinh k(z+D)}{\sinh kD} \sin(kx-\sigma t-\phi) \tag{2}$$

$$u = \sigma A \frac{\cosh k(z+D)}{\sinh kD} \cos(kx-\sigma t-\phi) \tag{3}$$

$$p = \rho A C^2 k \frac{\cosh k(z+D)}{\sinh kD} \cos(kx-\sigma t-\phi) \tag{4}$$

$$k^2C^2 = \sigma^2 = gk\tanh kD = (4\pi^2)/T^2 \tag{5}$$

where w, u are the vertical and horizontal components of velocity, p the

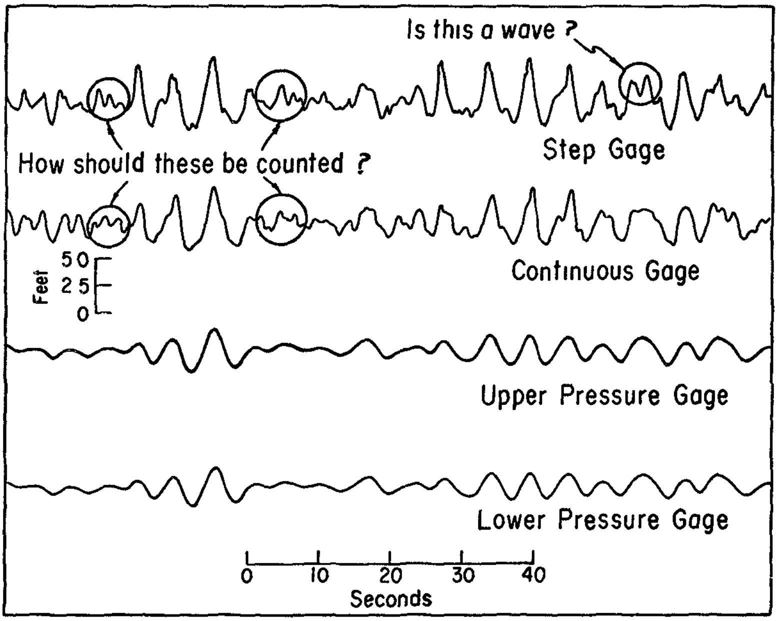

Figure 2 A sample of the simultaneous record from four wave gages at the Steel Pier in Atlantic City, N J , illustrating the difficulty in deciding which perturbations are to be considered as waves in determining $H_{1/3}$ Records (a) and (b) are for pressure gages, (c) and (d) are pressure gages with (d) about 6 feet below (e)

pressure, k the wave number, σ the frequency, ϕ the phase at $kx = \sigma t$, C the phase speed, T the period, ρ the density and g the acceleration of gravity The origin of z is taken at the water surface and the upward direction is taken as positive Since, for any given value of D, there is a one-to-one relationship between period, frequency, wave number, wave length and phase velocity, functions which logically depend on any one of these parameters may be expressed unambiguously as functions of the period

In the more general case when several wave trains are present, it is more useful to consider

$$h(t) = \bar{h} + \sum_{m=1} A_m \cos(k_m x - \sigma_m t - \phi_m) \quad (6)$$

Similar expressions may be readily constructed for u, w, and p In general k_m and x should be regarded as vectors with $k_m x$ as the scalar product of the vector wave number, k_m and the position vector x This detail is not important here, however, for if the gage position is fixed, $k_m x$ is constant for each m, and may be absorbed into the phase angle ϕ_m This procedure will be followed in the remainder of the paper If the sequence of σ_m, important in a given application is known in advance, as in the case of astronomical tides, the most expedient analysis procedure for the calculation of the amplitudes A would be a least squares evaluation of the coefficient s, a_m, b_m, in the expression

$$h(t) = \bar{h} + \sum_{m=1} [\, a_m \cos\sigma_m t + b_m \sin\sigma_m t] \quad (7)$$

$$\begin{aligned} A_m^2 &= a_m^2 + b_m^2 \\ \phi_m &= \tan^{-1} b_m/a_m \end{aligned} \quad (8)$$

If the sequence of σ_m is not known in advance, and this is the usual case, the amount of arithmetic involved in the solution can be greatly reduced by choosing $\sigma_m = 2m\pi/T_a$, where T_a is the period of record selected for analysis

It can be shown by Parseval's relation that the sum of all the A_m^2 is equal to the variance of h(t) It can also be shown that the variance of h(t) is proportional to the average potential energy of the wave (Kinsman, 1965, p 145ff) According to Taylor (1937), this is the idea Rayleigh had in mind when he first introduced the concept of an energy spectrum

If one value of A_m^2 say A_s^2 is much larger than all of the others, it seems natural to select the corresponding period, $T = 2\pi s/T_a$ as the significant period If there are many A_m^2's with nearly the same magnitude it may be desirable to regard the true energy as a continuous function of the frequency and the computed A_m^2 as estimates of this function, integrated over a small frequency interval centered on σ_m. When this procedure is followed, it seems natural to define the significant period as the period corresponding to the peak energy density per unit frequency. If the wave energy is known, or estimated, as a continuous function of the frequency, a simple transformation of variables provides the energy as a function of the wave period. Thus, the significant wave period could be defined as the period of maximum energy density per unit period.

Short definitions of six proposed measures of the significant wave period are listed below.

1. The average period of the one-third highest waves, $T_{1/3}$
2. The period most prominent in the record, T_{CERC}
3. The average period of all waves, T_{all}
4. The average period of all waves that cross the mean water level, T_{ZUC}
5. The period of maximum energy density, T_{PM}
6. The period corresponding to frequency of maximum energy density, T_{FM}

The last of these seems most suitable for a study of wave dynamics, and is suggested as a standard. The first two definitions have been proposed for convenience in collecting data, and the third and fourth to make the determinations more objective and reproducible.

OTHER DEFINITIONS OF WAVE HEIGHT

Since the energy of a simple wave is proportional to the square of the wave height, it is natural to define a measure of the wave height in terms of the square root of the average energy. This estimate, called the root-mean-square wave height by Tucker (1961) is equivalent to the standard deviation of the wave record and is defined by

$$H_{RMS} = \left[\frac{1}{N} \sum_{n=1} [h(n\Delta t) - \bar{h}]^2 \right]^{1/2} \tag{9}$$

where $h(n\Delta t)$ is the water surface elevation at time $t=n\Delta t$, and $\bar{h}$ is the mean water level for the analysis interval. Thus H_{RMS}, unlike $H_{1/3}$, has a clear physical definition and can be easily determined by either digital or analog computers. Both theoretical and empirical evidence suggest that the average value of the ratio $H_{1/3}/H_{RMS}$ is about 4. The actual value obtained from a given observation depends on the full wave spectrum.

Since 1965, it has been customary at CERC to estimate the significant wave height as the N'th highest wave in a 7-minute wave record where N is a function of the selected period and is approximately 1/6 of 420 seconds divided by the "significant period " This procedure can be performed very rapidly by making use of suitable transparent nomograms, but the determination of the "most prominent period in the record" is too subjective to be readily programed for a computer

Draper (1966), making use of some work by Tucker (1961), proposed a more objective system which can be reaily programed The standard deviation of the wave record called H_{RMS} by Tucker is estimated from the highest and lowest water elevations in the record, and the significant wave height is estimated as the product of the sum of the highest and lowest departures of the surface elevations from its mean position and a factor, which depends on the period of zero up-crossings

A COMPARISON OF THE PARAMETERS AS EVALUATED BY SEVERAL DEFINITIONS

Since 1966, CERC has been making a digital record from the step resistance wave gage at the Steel Pier in Atlantic City, New Jersey A computer program has been developed for calculating each of the measures of wave height and period discussed above with the exception of T_{CERC} and H_{CERC} which must be obtained manually This program has been used to analyze the records from November and December of 1966

It has been found, for the records analyzed, that all of the measures for wave height are highly correlated The correlation matrices for the principal measures are shown in Tables I and II Consequently, it appears to make relatively little difference how a record is analyzed to obtain wave height, since very nearly the same answer is obtained for any method The Fast Fourier Transform algorithm of Cooley and Tukey (1965), often called the "FFT" was used to analyze records 1024 seconds (17 minutes, 4 seconds) long for the computation of energy spectra This permits a detailed definition of the spectrum with a frequency resolution slightly better than 10^{-3} Hertz

Spectra with resolution per unit frequency similar to that obtained with the auto-correlation technique (Blackman and Tukey, 1958), were obtained by averaging across frequency bands of constant width Spectra in terms of energy density per unit period were computed by averaging across frequency bands of variable width

The correlation matrices for the various measures of the wave period are given in Tables III and IV, and a comparison of a few of the individual estimates in Table V It can be seen that two estimating procedures which may agree to within 1 second in some cases, may differ by as much as 10 seconds in other cases The correlation appears to be better for the higher waves This is shown in Figure 3, in which the ratio of T_{FM} to T_{CERC} is shown as a function of H_{RMS} H_{RMS} is a measure of wave height defined on page 6

TABLE I

CORRELATION MATRIX FOR WAVE HEIGHT ESTIMATES

Atlantic City, N J , December 1966 (84 records)

	H_{RMS}	H_{CERC}	H_{Tucker}	$H_{1/3}$
H_{RMS}	1 000	958	987	970
H_{CERC}	958	1 000	908	917
H_{Tucker}	987	908	1 000	948
$H_{1/3}$	970	917	948	1 000

H_{RMS} is the Standard Deviation of the Record

H_{CERC} is the Value Obtained from the Analysis System Used at CERC Since 1965

H_{Tucker} is the Estimate of H_{RMS} Based on the Highest Crest and Lowest Trough as Recommended by Tucker (1961)

$H_{1/3}$ is the Average Value of the One-Third Highest Waves as Obtained by a Digital Computer

H_{CERC} is based on a seven-minute record Other estimates are based on records of 1024 seconds The correlation between records of 420 seconds and 1024 seconds duration is approximately 0 98

TABLE II

CORRELATION MATRIX FOR WAVE HEIGHT ESTIMATES

Atlantic City, N J , November 1966 (129 records)

	H_{RMS}	H_{CERC}	H_{Tucker}	$H_{1/3}$
H_{RMS}	1 000	949	982	938
H_{CERC}	949	1 000	945	868
H_{Tucker}	982	945	1 000	918
$H_{1/3}$	938	868	918	1 000

See Table I for explanation

TABLE 11I

CORRELATION MATRIX FOR WAVE PERIOD ESTIMATES

Atlantic City, N J , December 1966 (84 records)

	T_{FM}	T_{CERC}	T_{PM}	T_{ZUC}	T_{all}	$T_{1/3}$
T_{FM}	1 000	564	402	264	- 037	028
T_{CERC}	564	1 000	510	390	038	135
T_{PM}	402	510	1 000	583	236	365
T_{ZUC}	264	390	583	1 000	751	867
T_{all}	- 037	038	236	751	1 000	951
$T_{1/3}$	028	135	365	867	951	1 000

T_{FM} is the period corresponding to the frequency of maximum energy density per unit frequency

T_{CERC} is the most prominent period as determined by the CERC method (see text)

T_{PM} is the period of maximum energy density per unit period

T_{ZUC} is the average period of zero up-crossings

T_{all} is the average period between maxima in the record

$T_{1/3}$ is the average period of the one-third highest waves

T_{CERC} is based on a seven-minute record All other measures are based on a record of 1024 seconds

TABLE IV

CORRELATION MATRIX FOR WAVE PERIOD ESTIMATES

Atlantic City, N J , November 1966 (129 records)

	T_{FM}	T_{CERC}	T_{PM}	T_{ZUC}	T_{all}
T_{FM}	1 000	725	581	493	131
T_{CERC}	725	1 000	615	552	,236
T_{PM}	581	615	1 000	648	260
T_{ZUC}	493	552	648	1 000	710
T_{all}	131	236	260	710	1 000

See Table III for explanation

TABLE V

ESTIMATES OF THE "CHARACTERISTIC WAVE PERIOD"
ESTIMATES USING VARIOUS DEFINITIONS
CERC STEP RESISTANCE WAVE GAGE, STEEL PIER, ATLANTIC CITY, N J
DECEMBER 1966

Day	Hour	T_{FM}	T_{CERC}	T_{PM}	T_{ZUC}	T_{all}	$T_{1/3}$
2	1600	9 1	7 0	8 0	4 2	2 3	3 3
2	2000	10 6	12 0	8 0	4 2	2 2	3 3
3	0000	10 6	11 0	10 0	4 5	2 5	3 6
3	0400	12 7	13 0	11 0	5 4	2 9	4 4
3	0800	12 7	12 0	11 0	4 7	1 9	2 4
3	1200	12 7	13 0	11 0	4 4	2 0	2 6
3	1600	12 7	11 0	11 0	4 5	2 2	3 3
3	2000	12 7	11 0	2 0	3 5	2 3	2 8
4	0000	10 6	5 0	2 0	3 4	2 3	3 0
4	0400	12 7	10 0	2 0	3 6	2 3	3 0
4	0800	5 8	6 0	5 0	4 6	2 7	3 8
4	1200	6 4	7 0	5 0	4 5	3 0	4 2
4	1600	9 1	10 0	8 0	5 0	2 7	4 3
4	2000	8 0	8 0	7 0	5 4	3 2	4 7
5	0000	9 1	8 0	8 0	5 3	3 3	4 8
5	0400	10 6	11 0	5 0	6 2	4 2	5 9
5	0800	12 7	9 0	12 0	6 2	3 1	4 7
5	1200	12 7	14 0	11 0	6 5	3 7	5 6
5	1600	12 7	12 0	11 0	5 8	3 1	5 1
5	2000	15 9	12 0	6 0	6 2	3 5	5 2

See Table III for explanation

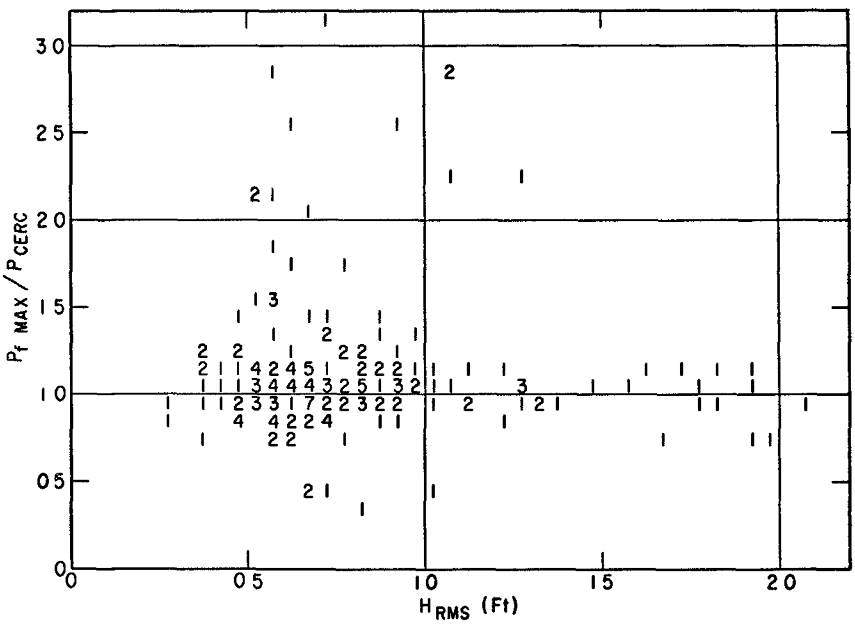

Figure 3 The ratio of $T_{f\ max}$ to T_{CERC} as a function of H_{RMS} The numbers represent the number of cases in which the observation fell in 0 1 x 0 05 foot square

THE INTERPRETATION OF A CHARACTERISTIC WAVE PERIOD

For linear wave theory there is a one-to-one correspondence between period, frequency, wave length, wave number and phase velocity Because of this, experimental results obtained with monochromatic waves are often tabulated or graphed as a function of period A careful analysis, however, may show that one of the other variables is more fundamental to the problem Thus in coastal engineering design, the wave period may be used to specify either the characteristic time scale or the characteristic length scale of some process due to waves A little reflection on the physical processes involved will show that no one variable can provide the best estimate of the time and space scales for all processes due to a given sea state This can be seen most clearly by considering a Fourier Transform of the wave record

The energy spectrum for the wave height can be expressed in the form

$$E_h(\sigma_m)\,d\sigma = A_m^2 \tag{10}$$

The corresponding expressions for the spectra of the velocity components are obtained from (2) and (3) in the form

$$E_w(\sigma_m)\,d\sigma = A_m^2\,\sigma_m^2\,\left[\frac{\cosh k(z+D)}{\sinh kD}\right]^2 \tag{11}$$

$$E_u(\sigma_m)\,d\sigma = A_m^2\,\sigma_m^2\,\left[\frac{\cosh k(z+D)}{\sinh kD}\right]^2 \tag{12}$$

The term σ_m^2 produces an amplification of the height spectrum with increasing frequency Thus if the height spectrum is flat or contains two or more nodes of nearly the same value, the peak of the velocity spectrum, on the surface, is likely to occur at a higher frequency than the peak of the height spectrum The terms in square brackets decrease with depth and decrease more rapidly with increasing frequency Thus the peak of the velocity spectrum will have a tendency to shift toward lower frequencies with increasing depth

Equation (5) can be used to eliminate C^2 from (4) to obtain

$$p = \rho g\,A\,\frac{\cosh k(z+D)}{\cosh kD}\,\cos(\sigma t-\phi)$$

The horizontal displacement, X, may be obtained from (3) by integration with respect to time to obtain

$$X = A\,\frac{\cosh k(z+D)}{\cosh kD}\,\sin(\sigma t-\phi)$$

The corresponding spectra are

$$E_p(\sigma)\ d\sigma = \rho^2 g^2\ A^2 \left[\frac{\cosh k(z+D)}{\cosh kD}\right]^2$$

$$E_X(\sigma)\ d\sigma = A^2 \left[\frac{\cosh k(z+D)}{\cosh kD}\right]^2$$

At the surface the peak of these spectra must agree with the peak of the height spectra, but the high-frequency components are attenuated with depth more than the low frequency components Thus with relatively flat or bimodal spectra, there is a tendency for the peak of the spectrum to shift toward low frequencies

Other transformations, which will produce other changes in the spectra, and in the period which seems to be most important will be appropriate to some engineering problems

It seems that no one definition of the "significant period" for a wave field in which waves of several frequencies are present, can provide the best value for use in all engineering calculation

SUMMARY AND CONCLUSIONS

The wave-height estimates obtained from any particular wave gage by any of the analysis procedures tested are consistent, in the sense that the ranking of estimates obtained by one procedure will be nearly the same as that obtained by any other analysis procedure tested But a scale correction may be necessary to obtain the best fit between data analyzed by two different procedures

The period data obtained by different analysis procedures are not consistent and the estimates of a characteristic wave period have little value unless the procedure used in obtaining the period estimate is known Comparisons between wave periods may be more satisfactory when the data are stratified in some way which makes the data sample more homogenous Restricting attention to waves more than 3 feet high is one such stratification which improves the consistency of the estimates Other forms of stratification may also be useful

It appears that the best procedure for engineering design will be to disregard the tabulated periods, and to consider every period that might reasonably occur, along with the given height estimate, to determine the critical conditions or the design wave

The best choice for a design wave for a particular environment depends on the problem to be considered No single value can be sufficient for all problems

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