CHAPTER 17

BOTTOM BOUNDARY SHEAR STRESSES ON A MODEL BEACH

by

P G Teleki
Department of Geology, Louisiana State University, Baton Rouge, and
U S Army Coastal Engineering Research Center, Washington, D C

and

M W Anderson, A M , ASCE
Department of Structures, Materials and Fluids
University of South Florida, Tampa

SYNOPSIS

The maximum amplitude of shear stress in the bottom boundary layer of water waves was evaluated with a Preston probe inclined on a 1 12 5 slope beach Near bottom velocity profiles were obtained in laminar and developing turbulent flow conditions from which the experimental boundary layer thicknesses were evaluated Agreement between experimental bottom velocities and those calculated from Airy theory deteriorate with decreasing depth on the beach resulting in lower shear stress values than predicted by linear theory The measured boundary layer thickness on the slope exceeds the predicted for horizontal bottom, increasing shoreward to some critical depth outside the breaker zone from where it decreases shoreward The influence of roughness on the shear stress distribution is considerable in the "offshore" region, but becomes negligible near the breaker zone On a smooth bottom the coefficient of friction agrees with Kajiura's expression

INTRODUCTION

In order to quantitatively evaluate nearshore sediment transport, it must be recognized that substantially more is required to be known about oscillatory boundary layers, the magnitude of energy dissipation due to bottom friction and fluid turbulence and about the effect of permeability in various sediments, in other words about the physical phenomena near the fluid-solid interface Our present knowledge on the flow of fluid near the bottom boundary is negligible because analytical solutions of mass transport exist only for laminar flow and for horizontal impermeable boundary Nature prescribes, in contrast, sloping beaches, loose boundaries and waves undergoing transformation and breaking Furthermore, at this time we cannot yet solve the equations for turbulent boundary layer in unidirectional steady flow, let alone when waves are present One of the early efforts of investigating the nature of the oscillating laminar boundary layer was Eagleson's (1959) Solutions for the horizontal boundary were given by Longuet-Higgins (1958), Grosch (1962) and Iwagaki, et al (1967) Turbulent boundary layers were investigated by Jonsson (1963) and Horikawa and Watanabe (1968) The authors of the latter reference employed Kajiura's (1968) theory for turbulent boundary layers with some success Direct measurement of boundary shear stress and evaluation of the friction coefficient have been made by Eagleson (1959) and Iwagaki et al (1965) The indirect method of velocity measurements has been applied by Jonsson (1963), Horikawa and Watanabe (1968) and Sleath (1968) Sleath's results are particularly interesting because he used a permeable bottom boundary One of his conclusions was that increase in the permeability of a sandy bottom brings ab out an increase in the near bottom mass transport velocity Experiments by Horikawa and Watanabe, employing the hydrogen bubble technique, hold out promise toward understanding not only the measurement of instantaneous velocities and boundary layer thickness but a very critical aspect of boundary layer research, namely phase differences between the velocities within and outside of the boundary layer and between the local velocities and the boundary shear stress

The use of the Preston probe for evaluating boundary shear stress from measurements of dynamic pressure is well known from literature for unidirectional turbulent flows Its theoretical development is due to Preston (1954) who used the probe on smooth boundaries Evidence for its applicability to rough boundaries was presented by Hwang and Laursen (1963) and Ghosh and Roy (1970) Nece and Smith (1970) used an enlarged version of the probe on loose boundary of a tidal estuary with partial success The Preston probe, as presented in this report, has not been applied to oscillating flows nor to a sloping bottom In presence of waves, pressure gradients will necessarily be present in the direction of flow Hsu (1955) and Patel (1965) have given proof of the probe's use in pressure gradients In addition, Hsu (1955) extended its use to laminar flow

The aim of this paper is to report on some aspects of the oscillating flow near the bottom, in particular about resistance on a sloping beach and its manifestation in velocity and shear stress distributions near a solid boundary The theoretical developments of Kajiura (1968) for the oscillating boundary layer were followed Velocity and shear stress measurements were obtained for various wave conditions, a fixed slope, smooth and a rough boundary in a wavetank, using the indirect method of a Preston probe

THEORETICAL CONSIDERATIONS

For the case of unsteady flow of a viscous, incompressible fluid flow the Navier-Stokes equations of motion, two-dimensional case, is given as

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial z}\right) = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial z^2} \qquad (1\ 1)$$

where the x-axis is positive in the direction of wave propagation, z is the vertical coordinate measured positive upward, ρ is the fluid density, μ is the viscosity, p is pressure and u, v are the local velocity terms in the horizontal and vertical directions respectively, defined

$$u = -\frac{\partial\phi}{\partial x} \tag{1 2}$$

and

$$v = -\frac{\partial\phi}{\partial z} \tag{1 3}$$

so that the condition of continuity is met by

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \tag{1 4}$$

given ϕ as the velocity potential Defining d as the local water depth positive upward from the free surface, U as the free stream velocity, and δ the boundary layer, the boundary conditions are

$z = -d,\ u = 0,\ v = 0$	at the bottom
$z = \delta\ ,\ u = U(x,t)$	at the outer edge of the boundary layer
$z \to \infty\ ,\ u = U(x,t)$	at the free surface

Introducing harmonic velocity components

$$\begin{aligned} U(x,t) &= \overline{U}(x) + U'(x,t) \\ U'(x,t) &= U(x)e^{i\sigma t}\ ,\ \sigma = 2\pi/T \end{aligned} \tag{1 5}$$

$$\overline{U}'(x,t) = 0 \tag{1 6}$$

$$\begin{aligned} u(x,y,t) &= \overline{u}(x,y) + u'(x,y,t) \\ v(x,y,t) &= \overline{v}(x,y) + v'(x,y,t) \\ p(x,t) &= \overline{p}(x) + p'(x,t) \end{aligned} \tag{1 7}$$

we integrate Eq 1 1 over one wave period and obtain

$$\rho\left(\overline{\frac{\partial u}{\partial t}} + \overline{u}\,\overline{\frac{\partial u}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}} + \overline{v}\,\frac{\partial \overline{u}}{\partial z} + \overline{v'\frac{\partial u}{\partial z}}\right) + \frac{\partial \overline{p}}{\partial x} - \nu\,\frac{\partial^2 \overline{u}}{\partial z^2} = 0 \tag{1 8}$$

for the flow in the boundary layer Using the same procedure, the averaged expression for the flow outside the boundary layer will be

$$\rho\left(\overline{\frac{\partial U}{\partial t}} + \overline{U}\,\frac{\partial \overline{U}}{\partial x} + \overline{U'\frac{\partial U'}{\partial x}} + \overline{v}\frac{\partial \overline{U}}{\partial z} + \overline{v'\,\frac{\partial U'}{\partial z}}\right) + \frac{\partial \overline{p}}{\partial x} - \nu\,\frac{\partial^2 \overline{U}}{\partial z^2} = 0 \tag{1 9}$$

Because its effect is negligible we can omit the viscous term in Eq 1 9 and extract the pressure term

$$-\frac{\partial \bar{p}}{\partial x} = \rho\left(\frac{\partial \bar{U}}{\partial t} + \bar{U}\frac{\partial \bar{U}}{\partial x} + \overline{U'\frac{\partial U'}{\partial x}}\right) \tag{1 10}$$

having also neglected the terms containing v, assuming that the vertical velocity is small Substituting Eq 1 10 into Eq 1 8, gives

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial u}{\partial x}} - \left[\frac{\partial \bar{U}}{\partial t} + \bar{U}\frac{\partial \bar{U}}{\partial x} + \overline{U'\frac{\partial U'}{\partial x}}\right] = \mu\frac{\partial^2 u}{\partial z^2} \tag{1 11}$$

which describes the total velocity distribution

Based on the parameter $\delta/L \ll 1$ (where L is the wavelength), we make the assumption that the contributions represented by the nonlinear convective terms (except turbulence) are numerically not significant Therefore we rearrange Eq 1 11 to show the defect velocity relationship

$$\frac{\partial}{\partial t}(\bar{u} - \bar{U}) = \frac{\partial}{\partial z}\left(\frac{\tau}{\rho}\right) \tag{1 12}$$

where

$$\tau = \mu\frac{\partial u}{\partial z} \tag{1 13}$$

and

$$\tau = K_{\angle}\frac{\partial u}{\partial z} \tag{1 14}$$

are the laminar and turbulent horizontal shear stress relationships respectively, and the dynamic viscosity $\mu = \rho\nu$ Eq 1 12 is the expression for oscillatory mean motion in the boundary layer based on potential theory According to Schlichting (1960), the validity of Eq 1 12 can be established if the laminar boundary layer thickness

$$\delta_L = (2\nu/\sigma)^{1/2} \tag{1 15}$$

The Laminar Case

Recalling that u is the horizontal velocity in the boundary layer and U just outside the layer, so that

$$\lim_{z \to \delta} u(x,y,t) = U(x,t) = a/d \quad C \sin(kx - \sigma t) \tag{2 1}$$

where C is the wave celerity, "a" the wave amplitude and $k = 2\pi/L$ is the wave number Grosch (1962) has shown that for $a/d \ll 1$ or a/d near $(kx-\sigma t) = 0$ the linearized theory in the laminar case provides an adequate description of the flow because the sum of all the terms $O(a/d)$ in the nonlinear solution for the bottom shear stress equal the linear solution and are hence negligible The solution becomes analogous to the Blasius series for steady flows

The steady state solution of the velocity in the boundary layer can be written as

$$u_o = U[\sin(kx-\sigma t)-e^{-z/\delta_L} \sin(kx-\sigma t-z/\delta_L)] \tag{2 2}$$

where the subscript "o" refers to boundary conditions, δ_L is defined in Eq 1 15 and

$$U = \frac{\pi H}{T} \sinh^{-1} kd \tag{2 3}$$

is the velocity immediately outside the boundary layer

Iwagaki, et al (1967) based on Grosch's solution obtained the approximate expression for the bottom shear stress

$$\tau_o/(\rho u_o{}^2) = \mathbb{R}^{-\frac{1}{2}} \sin (kx-\sigma t - \pi/4) \tag{2 4}$$

where the local Reynolds number is

$$\mathbb{R} = \frac{u_o{}^2 T}{2\pi\nu} \tag{2 5}$$

and the phase difference between U and τ_o is $\pi/4$

The maximum shear stress was given by Iwagaki, et al (1967) as

$$\frac{\tau_{o\ max}}{\rho g H} = \frac{\sqrt{2\nu}}{g \sinh kd} \left(\frac{\pi}{T}\right)^{3/2} \tag{2 6}$$

$$\tau_{o\ max} = \rho \hat{U}\left(\frac{2\nu\pi}{T}\right)^{\frac{1}{2}} \tag{2 7}$$

which shows that the amplitude ^, of the local boundary shear stress is a function of the local free stream velocity amplitude (therefore the water particle excursion distance) and a boundary layer thickness The wave height $H = \frac{1}{2}a$ is applicable to sinusoidal wave profiles

In the laminar case we can describe the flow regime with the appropriate Reynolds number, which should contain the parameters for the boundary layer thickness, local velocity and viscosity Hence, re-arranging Eq 2 5, we get

$$\mathbb{R}^{-\frac{1}{2}} = \frac{1}{U}\left(\frac{2\nu\pi}{T}\right)^{\frac{1}{2}} = \frac{\nu}{U} \frac{\sqrt{2}}{\delta_L}$$

and

$$\mathbb{R}^{\frac{1}{2}} = \frac{U\delta_L'}{\nu} = \mathbb{R}_S \tag{2 8}$$

where $\delta_L' = (\nu/\sigma)^{\frac{1}{2}} = \delta_L/\sqrt{2}$ for smooth bottom The Reynolds number in Eq 2 8 is identical with Kajiura's (1968) and that of Horikawa and Watanabe (1968)

For laminar flow we now nominally define the wave friction coefficient to be of the form

$$C_f \hat{U}U = n(\tau_o/\rho) \tag{2 9}$$

where n must be determined from experiments For the case of smooth bottom laminar flow Kajiura (1968) approximates the amplitude of the friction coefficient by

$$\hat{C}_f = 1/\mathbb{R}_S \quad \text{for} \quad \mathbb{R} < 200 \tag{2 10}$$

for the case of the smooth boundary Introducing now z_o, as the characteristic roughness length, so that when

$$D = 30\ z_o \tag{2 11}$$

we have the equivalent Nikuradse's roughness expressed, we follow Kajiura's notation and write the Reynolds number for the rough boundary as

$$\mathbb{R}_R = \frac{\hat{U}D}{\nu} \tag{2 12}$$

and the friction coefficient can be expressed as

$$\hat{C}_f = 1\ 70\ (\frac{\hat{U}}{\sigma z_o})^{-2/3} \tag{2 13}$$

The Turbulent Case

From experiments of steady turbulent flow (e g , see Clauser, 1954) we know that the turbulent boundary layer structure is threefold, consisting of the inner or laminar sublayer at the wall, the outer or defect layer near its outer edge, the two connected by the overlap layer A similar breakdown for oscillatory layers was suggested by Jonsson (1966)

The general form of the turbulent velocity profile can be written as

$$\frac{u}{u^*} = A \log (\frac{zu^*}{\nu}) + C \tag{3 1}$$

which in the presence of roughness on the wall is modified to read

$$\frac{u}{u^*} = A \log (\frac{zu^*}{\nu}) + C - \frac{\Delta u}{u^*} \tag{3 2}$$

where $\Delta u/u^*$ represents the vertical shift of the logarithmic profile caused by the roughness elements, A and C are constants to be appraised experimentally and

$$(u^*)^2 = \tau/\rho \tag{3 3}$$

Similarly $$(u_o^*)^2 = (\tau_o/\rho) \quad z = 0 \tag{3 4}$$

is the shear velocity

and the shear stress is given by

$$\tau = K_z \, \partial u/\partial z \tag{3 5}$$

K_z is Kajiura's (1968) eddy viscosity (i e , the Boussinesq effective viscosity) of the form

$$K_z = \begin{cases} \nu & \text{in the inner layer} \\ \kappa \; u^*_o \; z & \text{in the overlap layer} \\ K_d & \text{in the outer layer} \end{cases} \tag{3 6}$$

where $K_d = K \hat{U} \delta_L'$, $K = 0.2$ is a universal constant, δ_L' is defined in Eq 1 15 and $\kappa = 0.4$ is Kármán's universal constant Eq 3 6 is in relation to a smooth boundary For the rough case the overlap expression of Eq 3 6 applies for $z_o < z < \delta$, except when $(\delta_L'/D) < 1$ in which case

$$K_z = 5.53 \, \kappa u^*_o \, z_o \tag{3 7}$$

for the laminar sublayer Kajiura's eddy viscosity assumption is based on analogy to steady state flow It presents the possibility, however, of being an improved estimate because it takes the structure of the layer into consideration But as Clauser (1956) pointed out for steady flow, "the turbulent eddies introduce shearing stresses for which no reliable method of calculation exists", and this phenomenon should only be more complex in time periodic flows

Even small changes in bottom roughness will drastically alter the profile of the turbulent boundary layer, which makes the vertical distribution of K_z difficult to establish Liu (1967 , as quoted by Kline, 1969, vol I, p 529) showed that changing z_o can result in a change of K_z by a factor of four in the defect layer Indeed, Horikawa and Watanabe (1968) showed that K_z attenuates with respect to z on both smooth and rippled boundaries, therefore further research is needed before accepting the formulations given in Eq 3 6, 3 7, and 3 8

The friction coefficient for the smooth bottom in turbulent flow is given by Kajiura as

$$\hat{C}_f = (\mathbb{R}_S \, m y_L)^{-2} \quad \text{for} \quad \mathbb{R}_S > 200 \tag{3 9}$$

with

$$m = \frac{1}{2}\sqrt{\frac{\kappa}{N}}, \quad N = \text{Const} \simeq 12 \quad \text{assumed} \tag{3 10}$$

$$y_L = \frac{2D_L}{\sqrt{\kappa N \delta_L}} \qquad (3\ 11)$$

$$D_L = N\nu/\hat{u}^* \qquad (3\ 12)$$

where y_L is the distance between the bottom and the lower limit of the overlap layer, we obtain the the aid of Eq 2 8 the approximate expression

$$\hat{C}_f = \left(\frac{\hat{u}^*_\delta}{\hat{U}}\right)^2 \qquad (3\ 13)$$

For rough boundaries Kajiura gives

$$\hat{C}_f = \left(\frac{60\ z_o\sigma}{\kappa \hat{U} y_R}\right) \qquad (3\ 14)$$

where y_R is the upper limit of the laminar sublayer

For $\hat{U}/\sigma z_o$ < 1000 Eq 3 14 can be approximated by Eq 2 13

The Preston Probe

Preston (1954) assumed that in turbulent flow on a smooth boundary a region must exist close to the wall in which the "law of the wall" of the form

$$\frac{U}{u*} = f_1\left(\frac{zu*}{\nu}\right) \qquad (4\ 1)$$

applies, so that the local shear stress at the boundary τ_o, can be related to the velocity distribution by measuring the differential pressure Δp, with a modified Pitot tube in contact with the wall The inter-relations are

$$\frac{\Delta p}{\tau_o} = f_2\left(\frac{u*d}{\nu}\right) \qquad (4\ 2)$$

and

$$\frac{\tau_o}{\rho \nu^2} = f_3\left(\frac{\Delta p d^2}{\nu^2 \rho}\right) \qquad (4\ 3)$$

where f_2, f_3 denote functional dependence, and d is the outside diameter of the probe The logarithmic expression obtained by Preston (1954)

$$\log_{10}\frac{\tau_o d^2}{4\rho\nu^2} = \bar{2}\ 604 + 7/8\ \log_{10}\frac{\Delta p d^2}{4\rho\nu^2} \qquad (4\ 4)$$

was modified by Hsu (1955) for the laminar sublayer (hence for laminar boundary layers) to read

$$y^* = \tfrac{1}{2} \log_{10} \left(\frac{8}{4+t^2}\right) + \tfrac{1}{2} x^* \quad (4\ 5)$$

and for the turbulent portion

$$y^* = \log_{10} k + \frac{7}{8} x^* \quad (4\ 6)$$

Where $y^* = \log_{10} (\tau_o d / 4\rho\nu^2)$ and $x^* = \log_{10} (\Delta p d^2/4\rho\nu^2)$, t = the ratio of inner to outer diameter of the stagnation tube and should be in the vicinity of 0 6, k = I(t) and I(t) is given by Hsu in tabulated form

Preston also advanced the hypothesis that the relationships presented in Eqs 4 2 and 4 3 are independent of the x-wise pressure gradient in the turbulent boundary layer Patel (1965) indicated, however, that for severe favorable and adverse pressure gradients the Preston probe overestimates skin friction The analytical solution given by Yalin and Russell (1966)

$$\tau_o = \rho(\alpha U^2 + \beta g S \delta) \quad (4\ 7)$$

given g as the gravitational acceleration and α, β as empirical constants, takes into consideration the instantaneous position of the free surface S, thus the pressure gradient It is clear therefore, that to avoid the influence of vertical accelerations on the local static pressure, as well as the influence of wave set-down near the breaker zone, initial evaluation of flow parameters with the Preston probe must be restricted to S=0, i e , to the wave crest and wave trough, or approximately (kx- t) = 0

EXPERIMENTAL APPARATUS AND PROCEDURE

Experiments were carried out in a fixed level, concrete floor, concrete and plexiglas-wall wavetank designed by the authors and constructed in 1968-69 at Louisiana State University Dimensions of the channel were 65x3x3 feet and it is shown in Figure 1 A fixed angle beach was constructed at the downstream end of the tank with a slope of 1 12 5, covered with aluminum sheet to provide a smooth surface For part of the tests sand of uniform size was attached to the beach face in thickness of one grain dimension (z_o = 00123 ft)

Resistance type wave gauges and the Preston probe were suspended from a forward-reverse gear, variable speed carriage and positioned with the aid of point gauges A Sanborn Model 150 oscillograph served as the excitation source and recording unit for the wave and pressure recording Waves with fixed periods of T=1 0, 1 5, 2 0 seconds were generated with a bottom-hinged paddle-type wavemaker The surface configuration of these waves were quite asymmetric for low frequencies and high amplitudes, consequently they were damped using various combinations of baffles following recommendations of Keulegan (1968) An undamped short period wave train is shown in Figure 1

Figure 1. Experimental set-up showing the wavetank, instrument carriage with Preston probe, and pressure transducer. Beach is nearest observer.

Since the Preston probe was to be used in wave boundary layers, the calibration procedure had to conform to the oscillatory motions experienced. For this purpose a variable stroke, variable frequency calibrating apparatus was built whose real time response was recorded by a linearsyn differential transformer. This enabled calculation of both the pressure-velocity relationship for the probe and the associated time lag. Differential pressures were sensed by a Pace 90B pressure transducer.

For the given wave periods the wavemaker stroke was changed to provide two wave amplitudes, noted as "large" and "small" in the graphs.

Water depth was fixed at 2.0 feet in the horizontal floor portion of the tank. Waves were measured in this part of the channel as well as adjacent to the Preston probe, aligned with the sloping bottom (Figure 2). The definition sketch for the probe is presented in Figure 3. Measurements of the differential pressure were made under the wave crests and troughs only by orienting the probe upslope and downslope, keeping other conditions the same. Adjustments for the phase difference between the surface, the free stream velocity and the velocity in the boundary layer were recorded. Measurements were carried out first on a smooth, then on a roughened bottom keeping other conditions the same. Water temperature was regularly recorded during the experiments.

Figure 2. Preston probe aligned with rough boundary. Static probe is in front.

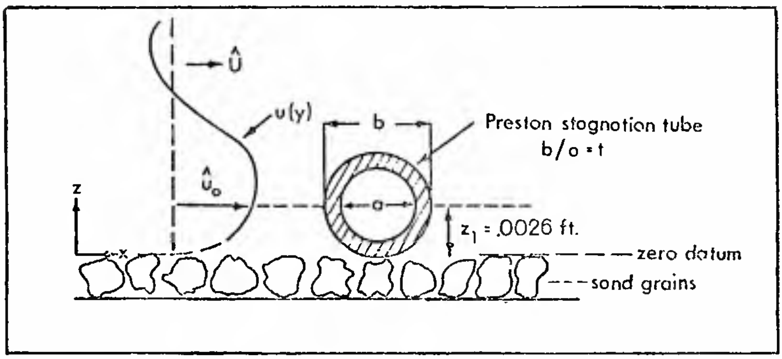


Figure 3. Preston probe resting on rough boundary, definition sketch

EXPERIMENTAL RESULTS

Rigorous evaluation of the velocity distributions on a slope, relative to waves undergoing transformation and without specific knowledge about boundary layer structure, growth and separation during a wave cycle will have to await further experimentation and a solution to the nonlinear phenomena experienced. Results presented on the velocity profiles on a sloping bottom are therefore exploratory only. In Figures 4, 5, 6, a set of profiles are illustrated from one of the experiments conducted on a smooth bottom with "small" amplitude waves.

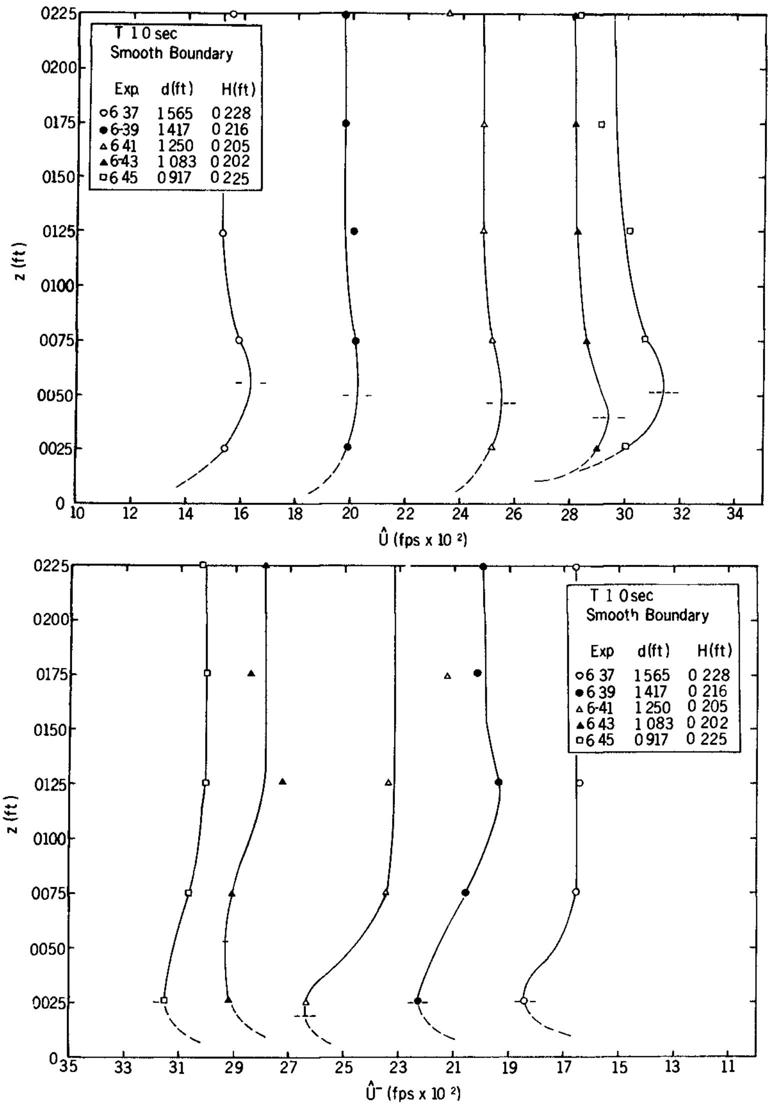
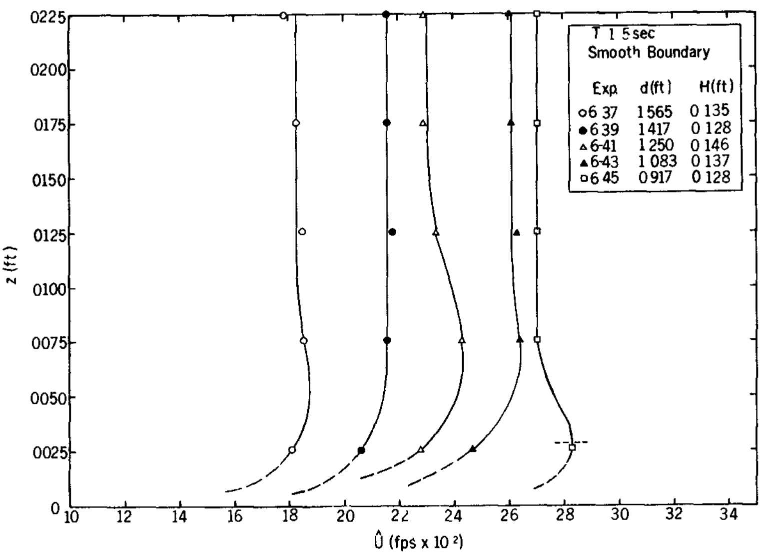


Figure 4 Experimental velocity profiles for T=1 0 second on smooth boundary Upper graph shows measurements under wave crests, lower graph, wave troughs



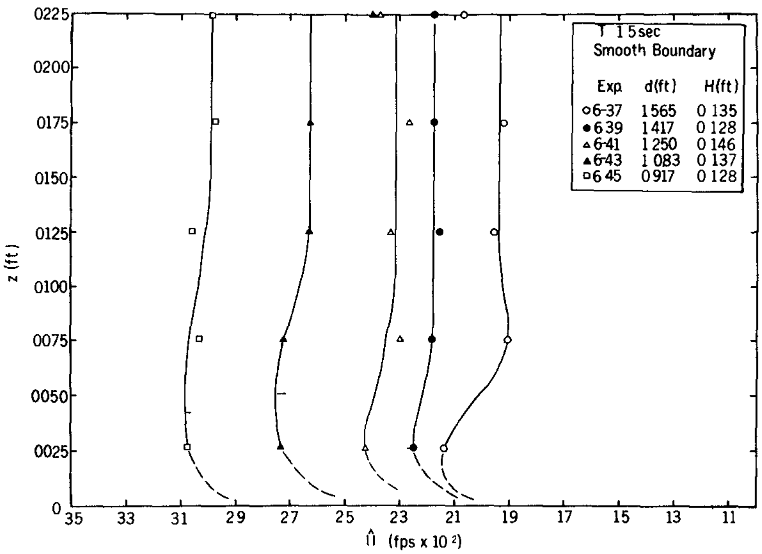


Figure 5 Experimental velocity profiles for T=1 5 seconds on smooth boundary Upper graph refers to wave crests, lower refers to wave troughs

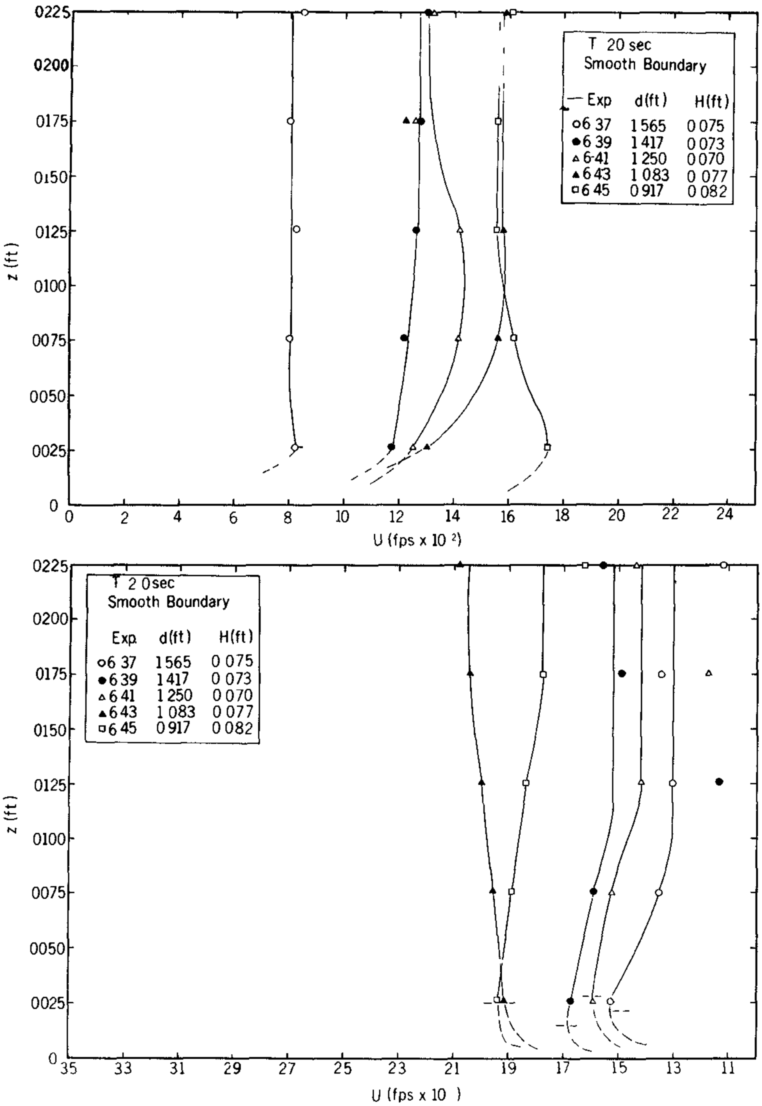


Figure 6 Experimental velocity profiles for T=2 0 seconds on smooth boundary Upper graph measurements with respect to wave crests Lower graph wave troughs

Figure 7. Boundary layer obtained with injected dye on a rough boundary.

Visual confirmation of the presence of a boundary layer is shown in Figure 7, showing displacements of a dye streak upslope, indicative of incremental mass transport with successive wave cycles. The dashed lines of Figures 4 - 6 refer to the elevation of the maximum velocity in the boundary layer $\hat{u}_{o\ max}$ following the procedure of Eagleson, Dean and Peralta (1958). Experimental values of $\hat{U}$, were found to agree very well for "deep" water on the slope, but the agreement declined for decreasing water depth and the linear model was found to overestimate the measured values. It should be noted the maximum reflection (H_r/H_I) was less than 10% for any one test. Grosch's (1962) criterion for neglecting nonlinear terms in the N-S equation was $a/d < .01$; it was exceeded in all cases. The effect of the convective velocity terms should be additive under the crest and subtractive under the wave trough with respect to linear values (Eq. 2.3).

Experimental data on boundary layer thicknesses are presented in Table 1 for various combinations of wave amplitude, water depth and wave period. Values of $\hat{\delta}$ exceed the theoretical δ_L (Eq.1.15), by factors of 3 to 8. Boundary layer thicknesses were found to be greater under wave crests than under wave troughs, this is no doubt influenced by vertical accelerations prior to attainment of $\hat{U}_{max}$ under the crest. Growth of the layer is also affected by the contribution of upslope mass transport which allows a longer excursion distance for its development. This phenomenon occurs on both smooth and rough boundaries. The effect of roughness is to increase $\hat{\delta}$. The contribution is more under wave crests and this is to be expected because the rate of boundary layer growth accelerates with increasing roughness element size. Wave amplitude influence on $\tilde{\delta}$ was difficult to discern, more tests are needed to evaluate this relationship. In some cases, boundary layers were observed to grow to some maximum value offshore of the breaker zone from which point $\hat{\delta}$ diminished shoreward, the reversal usually taking place at $d \approx 1.0$ foot.

Table 1 EXPERIMENTAL BOUNDARY LAYER THICKNESSES (in 10^{-3} feet)

	$\hat{\delta}$								δ_L
Wave Ampl	"Large"				"Small"				
Boundary	Smooth		Rough		Smooth		Rough		
Wave -	Crest	Trough	Crest	Trough	Crest	Trough	Crest	Trough	
d/T^2									
1 56	11 9	6 9	15 3	13 1	12 5	7 6	12 3	7 0	1 79
1 42	16 7	7 3	7 5	10 2	12 5	9 4	12 5	7 1	
1 25	15 8	8 0	7 3	12 5	12 5	10 3	17 2	-	
1 083	22 6	6 2	13 5	7 1	16 3	10 6	17 5	10 4	
0 92	8 1	10 6	7 5	-	17 5	17 5	7 5	-	
0 695	14 5	7 5	11 3	7 6	11 5	6 2	12 0	14 5	2 18
0 629	12 6	8 2	14 6	11 2	15 0	8 4	10 7	-	
0 555	8 7	8 7	16 1	-	16 2	10 5	13 3	18 2	
0 481	12 6	9 0	16 4	5 7	12 0	13 8	10 5	16 5	
0 407	15 0	9 5	11 1	16 2	7 5	14 4	8 2	13 7	
0 391	14 5	12 5	16 8	12 1	6 0	10 1	11 6	5 9	2 54
0 354	-	14 1	12 4	10 2	14 0	11 2	8 5	10 6	
0 312	12 6	14 1	9 5	7 6	13 5	12 5	10 3	8 2	
0 271	7 0	8 9	15 4	10 7	12 0	17 5	12 2	17 4	
0 229	8 7	5 9	13 0	-	12 5	12 5	16 2	12 5	

The classification of boundary conditions in terms of the prevalent flow regime, i e to establish where laminar gives way to turbulent flow is a difficult task Kajiura specified the transition region as

$$25 < \mathbb{R}_S < 650 \text{ for smooth bottom}$$

$$\text{and} \quad 100 < \mathbb{R}_R < 1000 \text{ for rough bottom}$$

Collins' (1963) critical Reynolds number of 113 (by transformation) is in the range for $\mathbb{R}_S$ Both ranges are wide and until a universal velocity distribution for oscillatory boundary layers is established, we do not know when to assume inception of turbulence or when full turbulence appears Considerable data fell into the transition ranges specified above The critical Reynolds numbers of $\mathbb{R}_S = 250$ for the smooth bottom and $\mathbb{R}_R = 500$ for the rough boundary are proposed

Evaluation of boundary shear was based on Eqs 4 5 and 4 6, using the given critical Reynolds numbers Maximum amplitudes of the shear stress were corrected for phase lag The distribution of $\tau_{o\ max}$ upslope is shown in Figure 8 corresponding to wave crests and in Figure 9 for wave troughs

Although the rate of increase varies depending on wave period and amplitude, the trends are linear The dependence on wave amplitude is clearly discernible, higher shear stresses are associated with "large" amplitude waves In "deep" water initial values of $\tau_{o\ max}$ are smaller under wave troughs than under wave crests Convergence was noted, however, for both cases of boundary conditions on nearing the breaker zone, indicating also that the effect of roughness used becomes negligible for very shallow water

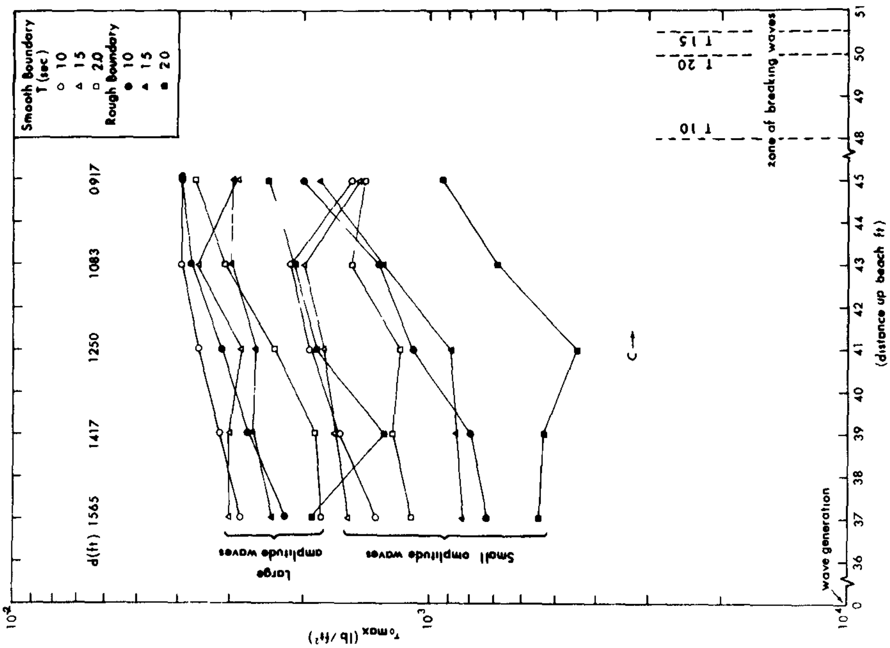


Figure 8 Shear stress distribution upslope corresponding to wave crests

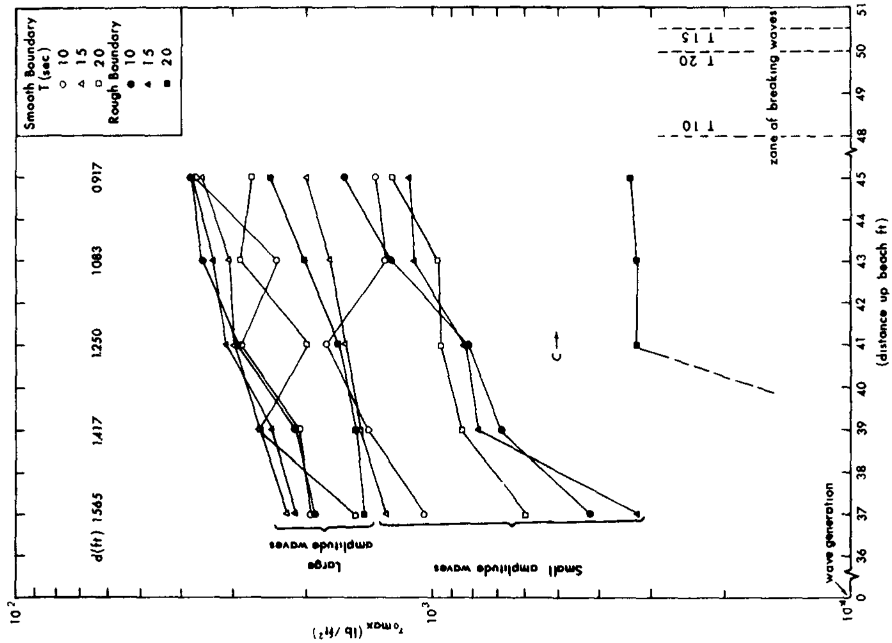


Figure 9 Shear stress distribution upslope corresponding to wave troughs

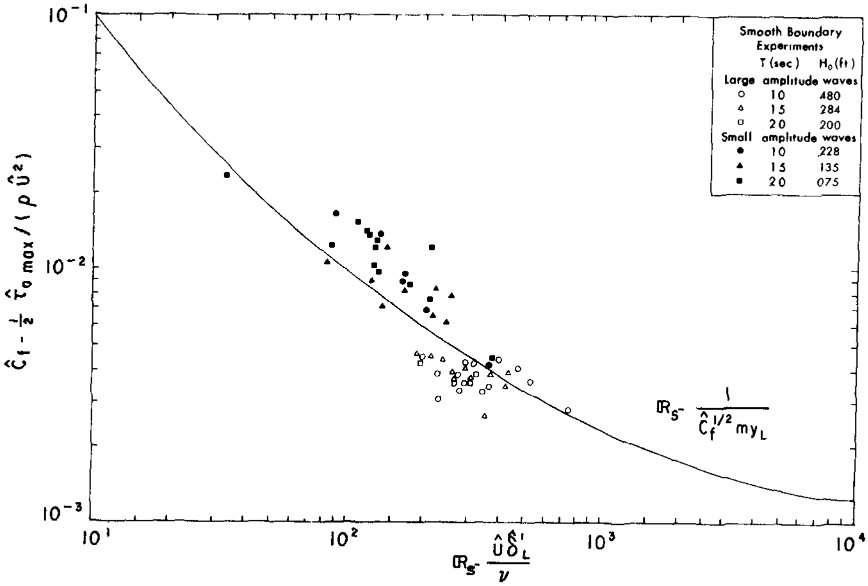


Figure 10 Relation between the coefficient of friction $\hat{C}_f$ and the Reynolds number $\mathbb{R}_S$ for smooth boundary

The relation between the shear stress and the corresponding velocity is dependent upon the coefficient of friction, whose value is a function of boundary conditions and the local flow regime For the smooth bottom Figure 10 is presented where agreement between Kajiura's curve and the computed $\hat{C}_f$ is very good for n = ½ This is in contrast with n=2 for the rather widely used $\tilde{C}_f$ (e g , see Eagleson, 1959) It is shown that the friction coefficient increases for decreasing wave amplitude

In presence of boundary roughness, Eq 2 13 is applicable when $\hat{U}/\sigma z_0 < 1000$, and this condition is validated for the test cases In Figure 11 the linear trends show that the friction coefficient increases for increasing wave frequency, decreasing Reynolds numbers and decreasing wave amplitude

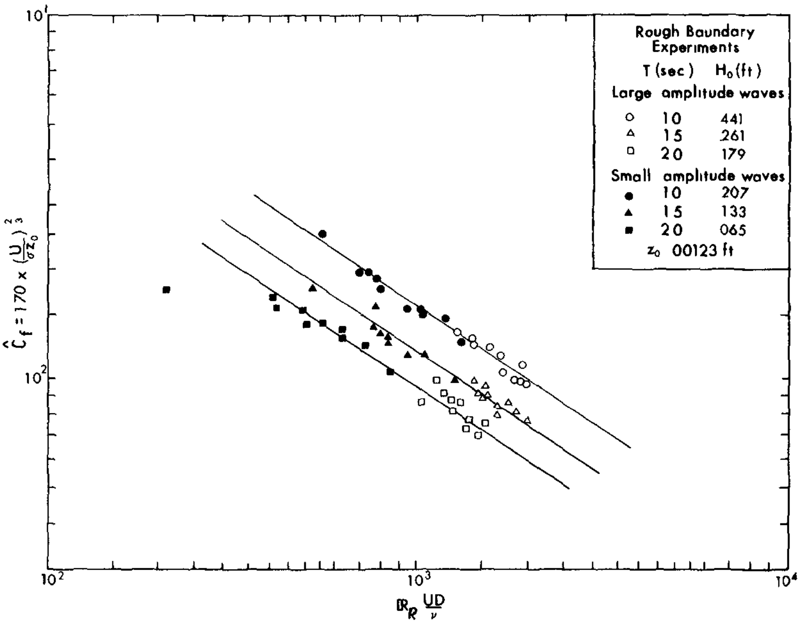


Figure 11 Relation between the coefficient of friction $\hat{C}_f$ and the Reynolds number $\mathbb{R}_R$ for rough boundary

CONCLUSIONS

Laboratory investigations of boundary layer thickness, near bottom velocity profiles and bottom friction were carried out on a slope with the aid of a Preston probe in hopes of initiating greater understanding about littoral sediment transport The use of Preston probe is limited under oscillating flow conditions although its simplicity of construction and applicability to both laminar and turbulent flows is appealing Wave boundary layers are complex phenomena in time and space and their analysis on a sloping bottom presents considerable difficulties The thickness of fully developed layers greatly exceeds the theoretical, and this is probably the result of non-negligible vertical accelerations and the presence of mass transport The nonlinear effects could not be appraised for the velocity distribution and the agreement between the theoretical and experimental reference velocity only occurs for relatively deep water The shear stress distribution appears to have a linear relation to decreasing water depth, with higher amplitudes as wave height is increased

REFERENCES

Clauser, F , 1954, Turbulent boundary layers ın adverse pressure gradients, J Aero Sci , 21 91-108
Clauser, F , 1956, The turbulent boundary layer, Adv in Appl Mech , 4 1-51, Academıc Press, New York
Collıns, J I , 1963, Inceptıon of turbulence at the bed under periodıc gravıty waves, J Geophys Res 68(21) 6007-6014
Eagleson, P S , 1959, The damping of oscillatory waves by lamınar boundary layers, Mass Inst Tech , Hydrodyn Lab Tech Rpt 32, 37 p
Eagleson, P S , Dean, R G and Peralta, L A , 1958, The mechanıcs of the motıon of dıscrete spherical bottom sediment particles due to shoalıng waves, U S Army Corps of Engrs, Beach Erosıon Board Tech Memo 104, 41 p
Ghosh, S N and Roy, N , 1970, Boundary shear in open channel flow, Proc ASCE, J Hydr Dıv , 96 (HY4) 967-994
Grosch, C E , 1962, Lamınar boundary layer under a wave, Phys of Fluids 5 1163-1167
Horıkawa, K and Watanabe, A , 1968, Laboratory study on oscillatory boundary layer flow, Coastal Engıneering ın Japan 11 13-38
Hsu, E Y , 1955, The measurement of local turbulent skın frıctıon by means of surface Pıtot tubes, Davıd Taylor Model Basın Rpt 957
Hwang, L S and Laursen, S M , 1963, Shear measurement technique for rough surfaces, Proc ASCE, J Hydr Dıv 89(HY2) 19-37
Iwagakı, Y , Tsuchiya, Y and Chen, H , 1967, On the mechanısm of lamınar dampıng of oscıllatory waves due to bottom frıction, Bull Dis Prev Research Instıtute, Kyoto Unıversity, 16(3), 49-75
Iwagakı, Y , Tsuchıya, Y and Sakaı, M , 1965, Basıc studies on the wave dampıng due to bottom frıctıon, Coastal Engrg in Japan, 8 37-49
Jonsson, I G , 1963, Measurements ın the turbulent wave boundary layer, Proc IAHR Conf , London, 85-92
Jonsson, I G , 1966, On the exıstence of unıversal velocity dıstrıbutıons ın an oscillatory turbulent boundary layer, Coastal Eng Lab Tech University, Denmark, Basıc Res Prog Rpt 12 2-9
Kajiura, K , 1968, A model of the bottom boundary layer ın water waves, Bull Earthquake Res Inst , Tokyo Unıversity 46 75-123
Keulegan, G H , 1968, Wave dampıng effects of screens, U S Army Corps of Engıneers Waterways Exp Sta Rpt 2-12, 62 p
Klıne, S J , Morkovın, M V , Sovran, G and Cockrell, D J , 1969, Computatıon of turbulent boundary layers - 1968, Proc AFOSR-IFP-Stanford Conf , vol I, p 529
Longuet-Hıggıns, M S , 1958, The mechanıcs of the boundary layer near the bottom ın a progressıve wave, Proc 6th Conf Coastal Engıneerıng, Gaınesvılle, Fla, Ch 10 184-193
Nece, R E and Smıth, J D , 1970, Boundary shear stress ın rivers and estuaries, Proc ASCE, J Waterways & Harbors Dıv 96(WW2) 335-358
Patel, V C , 1965, Calıbratıon of the Preston tube and lımıtations on ıts pressure gradıents, J Fluıd Mech 23(1) 185-208
Preston, J H , 1954, The determınation of turbulent skin frıctıon by means of Pıtot tubes, J Royal Aero Soc 58 109-121
Schlıchting, H , 1960, Boundary layer theory, McGraw-Hill, N Y 647 p
Sleath, J F A , 1968, The effect of waves on the pressure in a bed of sand ın a water channel and on the velocıty dıstrıbutıon above ıt, unpub Ph D Thesıs, St John's College, Cambrıdge, 141 p
Yalın, M S and Russell, R C H , 1966, Shear stresses due to long waves, J Hydr Res , 4(2) 55-98