CHAPTER 26

CHARACTERISTICS OF WAVES BROKEN BY A LONGSHORE BAR

By

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ABSTRACT

A two-dimensional model submerged offshore bar was installed in a Texas A&M Hydrodynamics Laboratory wave tank Monochromatic waves with a range of heights and periods were generated at this bar for three different depths of water over the bar For each wave, water surface time-histories were measured at points before and after the bar and spectral analyses of these measurements were performed

The analysis of each wave record yielded an equivalent wave height which is proportional to the square root of the wave energy per unit surface area The ratio of the reformed to incident equivalent wave height is shown to relate to the ratio of incident wave height to water depth over the bar The predominant periods of the reformed waves are found to be the same as for the incident waves but the presence of energy at higher frequencies is also observed The cause of these higher frequency waves is discussed

INTRODUCTION

During parts of the year, generally the winter months, intense storms at sea generate waves that are quite steep These waves, upon reaching the shore, will erode material from the beaches and deposit this material some distance from the shoreline Continued deposition of material results in the formation of a longshore bar A typical bar formation will have a rather gently sloping face toward the sea, a rounded crest, and a steep slope downward into a trough in the lee of the crest These bar formations are generally of transient nature in shape and location

After a wave breaks over the bar and reforms, questions arise concerning the characteristics and energy of the resulting wave form, particularly if expensive structures are to be placed where the wave can act on them or if the shore is composed of material which can be carried away by the waves The object of the research described herein was to define some of the characteristics, particularly the heights and periods, of waves which have broken while passing a longshore bar and have reformed in the lee of the bar

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REVIEW OF THE LITERATURE

Investigations concerning breaking waves have been conducted by Iverson (7), Nakamura (9), and others (4, 5) However, the beach slopes used in those studies were such that the water depths never increased shoreward of the breaking point of the waves The work of Iverson showed that if the beach slope and the deepwater wave steepness are known, several characteristics of the breaking wave, including the height of the wave and the water depth at the breaking point, can be predicted

Investigations have also been performed with submerged rectangular breakwaters where waves approach the breakwater in fairly deep water, peak up and break over the relatively shallow structure, and then reform in the deeper water in the lee of the structure Tests conducted by Dick (1) with such breakwaters indicated that as a wave passed over the structure and broke, it was reduced to a fundamental wave whose period was the same as the incident wave but whose height was lower than the incident wave He also found lesser waves of higher frequency superimposed on the fundamental wave

Nakamura, Shiraishi, and Sasaki (10) also conducted experiments with submerged breakwaters Their test results indicated alterations of wave length and period as the waves broke on the submerged dike For instance, their data showed that for a depth in the lee of the breakwater equal to 10 percent of the deepwater wave length, the wave length in the lee of the breakwater was reduced to approximately 30 percent of the deepwater length and the wave period was reduced to approximately 60 percent of the deepwater period

Diephuis (2) has reported on a hydraulic model study conducted at the Hydraulics Laboratory, Delft, Netherlands, to study the transmission of wave energy past submerged bars Although much of his data were taken in the range of wave lengths and heights where surface tension and viscous effects play a large role in wave attenuation, his tests indicate that the wave heights in the lee of the bar where water depths are constant are some function of the deepwater wave steepness and that the heights of the reformed waves are more-or-less constant

THE LABORATORY STUDY

A laboratory experiment was devised to study the characteristics of gravity water waves after the waves had encountered a two-dimensional long-shore bar and had broken while passing the bar A model simulating a long-shore bar formation was designed and installed in a two-dimensional wave channel in the Hydromechanics Laboratory of Texas A&M University Monochromatic, gravity water waves were generated toward this bar formation Most waves broke as they passed over the bar and then reformed as they entered the deeper water in the lee of the bar A limit on the waves studied was those waves that were on the verge of breaking but failed to do so These waves did however distort and develop shorter period components Recordings of the water-surface time-histories were obtained before and after the bar From the insight into the wave characteristics gained from these recordings, those characteristics which the wave possessed before reaching the bar were compared with the characteristics of the wave after it had reformed in the lee of the bar

The laboratory model simulating the bar formation was fabricated by fashioning wood ribs to a predetermined shape and affixing a stiff aluminum covering to the ribs to form the bar surface The shape of the model, as shown in Fig 1, was chosen arbitrarily, but is believed to have a counterpart in nature The height of the model crest above the channel floor was 0 75 ft and the total length of the model was 10 ft

The model was attached to the floor of the channel by drilling and tapping threaded ways into the floor and passing bolts through the model into the ways In this manner the model was securely fastened to the flume and showed little tendency to shift under the oscillating forces created by the passing waves

The model was constructed with small tolerances between the aluminum covering and the walls of the channel A caulking clay was forced into the spaces which did exist, to inhibit any leakage around or under the model

In nature, a longshore bar formation is a steeper discontinuity in an already sloping beach However, the sloping beach was not reproduced in this study The channel floor, at constant elevation, served as the ocean floor so that the effects of bottom slope were not studied

The wave channel in which this study was conducted is 2-ft wide by 3-ft deep by 120-ft long The walls of the channel are glass so that tests may be observed visually A pendulum-type wave generator was used to generate the monochromatic incident waves The displacement of the wave generator was determined by the eccentricity of the rod connecting the generator to a motor-driven flywheel The period of the generator was controlled by regulating the angular speed of the flywheel with the electric motor whose speed could be adjusted by the setting of a potentiometer A wire-mesh wave filter was installed near the generator to attenuate the higher frequency noise waves superimposed on the incident wave At the end of the flume, a permeable wave absorber minimized reflection of the reformed waves back into the testing area

Water-surface time-histories were measured by a pair of capacitance-type wave probes The circuitry of this system includes a capacitance bridge of which the probe forms one leg Operation of the probe requires balancing the bridge by immersing the probe to its proper depth in the water and then balancing the bridge with an adjustable capacitor which forms another of the legs of the bridge With the circuit so balanced, any displacement of the water surface from the mean water level will cause an imbalance of the circuit The magnitude and sense of the imbalance is monitored and this signal is amplified The amplified signal is then recorded by a dual channel strip-chart recorder to yield a record of water-surface position versus time at each probe location

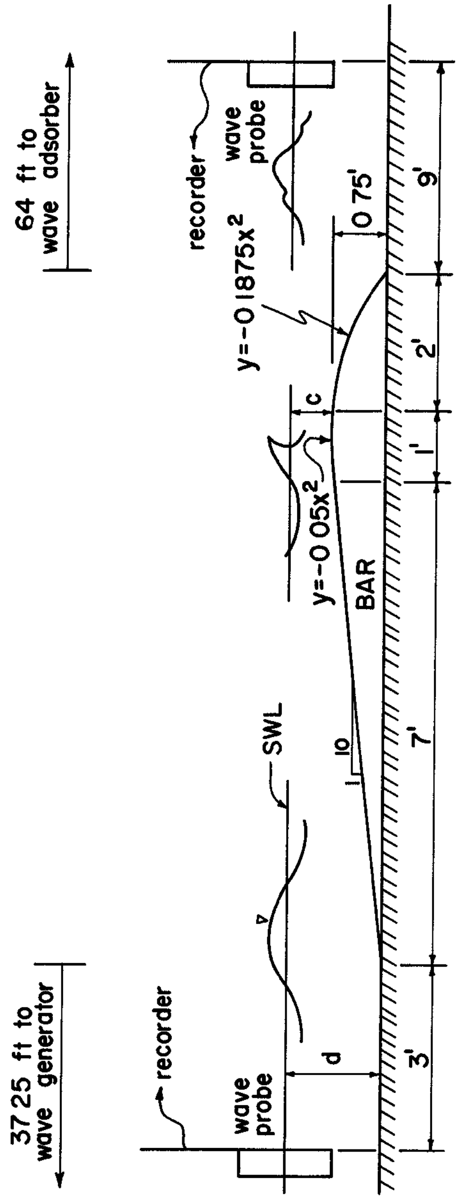


FIG I EXPERIMENTAL ARRANGEMENT

EXPERIMENTAL PROCEDURE

Several experimental runs with a range of wave characteristics were conducted for each of three water depths over the bar These depths over the bar crest, C, were 1 5 in , 3 0 in , and 5 0 in giving total water depths, d, of 10 5 in , 12 0 in , and 14 0 in

Two wave probes were installed in the channel at the locations shown in Fig 1 and calibrated before each set of test runs The calibration was checked quite often during the study

After each test run the wave channel was allowed to sit for several minutes to allow any ripples on the water surface or any oscillation of the water in the channel to dissipate

With the water completely still, the wave probes and the recorder were switched on and the wave generator was started Records of approximately one minute in length of the water-surface time-histories at the two probe locations were taken The wave generator and recorder were then stopped, the water in the channel was allowed several minutes to still, and the test was repeated After this repeat test, the amplitude and/or period of the wave generated were changed and the entire procedure was repeated for the new wave train

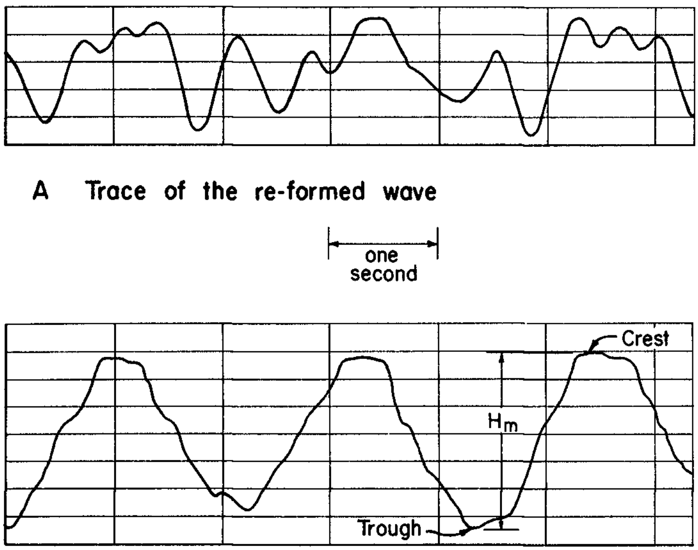
In addition, a wave probe was attached to a movable carriage atop the wave channel and attempts were made to measure any reflected waves from the bar formation by the envelope method (3) Although the procedure was repeated for waves of various steepnesses, in no instance were reflected waves detected

EXPERIMENTAL RESULTS

The laboratory data were obtained in the form of recorder traces of the water-surface time-histories for the two wave probe locations shown in Fig 1 Samples of the recorder trace from each probe are shown in Fig 2

Wave heights were obtained from the wave records by measuring the vertical distance between the peaks and troughs of several successive waves and averaging these values This procedure was easily followed for the relatively uniform incident waves, but the irregular wave forms which developed in the lee of the bar presented some difficulty For those waves, the position of the crest and trough could not always be determined exactly, thus, there was some question concerning the heights obtained Therefore, to provide a basis for comparison of test results as free as possible from subjectivity, a procedure was used in which an "equivalent" wave height was determined

In order to evaluate the equivalent wave height, it was necessary to assume that the recorder trace of the water-surface time-history is periodic and may be expressed as a Fourier series



A Trace of the re-formed wave

B Trace of the incident wave

FIG 2 A SAMPLE OF THE RECORDER TRACE

$$\eta(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos n\sigma t + b_n \sin n\sigma t) \tag{1}$$

where $\eta(t)$ is the displacement of the water surface from mean water level at time t, a_o is the mean displacement of the recorder trace from mean water level, a_n and b_n are Fourier coefficients, n is an integer coefficient, and σ is the angular frequency The Parseval theorem for periodic functions (6) gives

$$\frac{\rho g}{\bar{T}} \int_0^{\bar{T}} \eta(t)^2 \, dt = \frac{\rho g}{8} \sum_{j=1}^{\infty} H_j^2 \tag{2}$$

where ρ is the fluid density, g is the acceleration due to gravity, $\bar{T}$ is the time length of the wave record, j is an integer, and H_j is the height of each constituent wave comprising a complex wave form The right-hand side of Eq 2 may be recognized as the total energy per unit surface area in the wave form obtained by adding the energy of each constituent of the wave The incident equivalent height, H_{eq}, may now be introduced and defined by

$$\frac{1}{8} \rho g \, H_{eq}^{\;2} = \frac{\rho g}{\bar{T}} \int_0^{\bar{T}} \eta(t)^2 \, dt$$

or

$$H_{eq} \doteq 2\sqrt{2} \left[\frac{1}{\bar{T}} \int_0^{\bar{T}} \eta(t)^2 \, dt \right]^{1/2} \tag{3}$$

The values of $\eta(t)$ were measured for all tests at time increments of 0 05 sec for record lengths, $\bar{T}$, ranging from 20 to 25 sec These values were punched onto cards and the equivalent wave heights, as defined by Eq 3, were evaluated numerically by a digital computer

Although the equivalent wave heights need not have physical counterparts, they do provide an approximation of the measured wave heights The plots in Fig 3 and 4 for C = 3 0 in show this to be the case, particularly for the monochromatic incident waves The measured heights, H_m and h_m, were obtained by averaging the heights of several successive waves from the recorder traces H_m is the measured height of the incident wave and h_m is the measured height of the reformed wave heq is the equivalent height of the reformed wave, defined in the same manner as Heq Table 1 lists pertinent data on the waves studied in these experiments

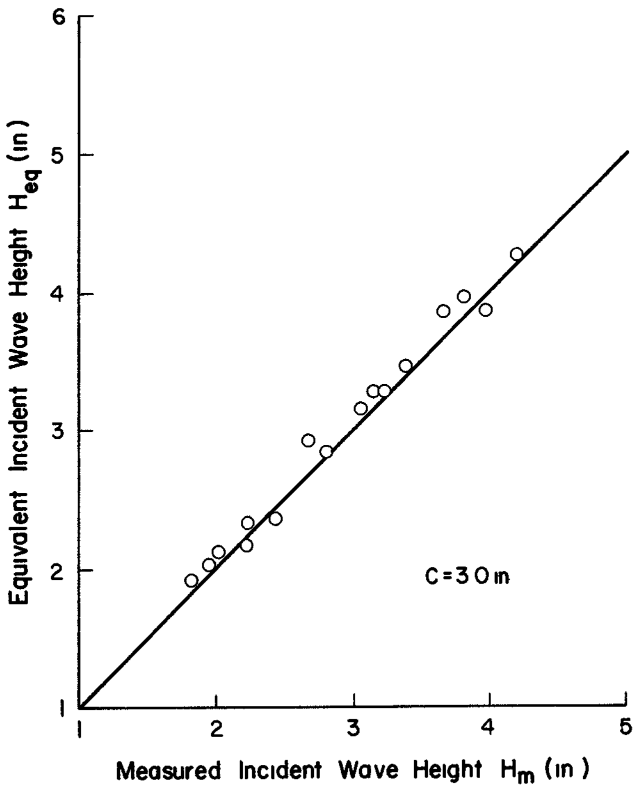


FIG 3—EQUIVALENT VERSUS MEASURED WAVE HEIGHT—INCIDENT WAVES

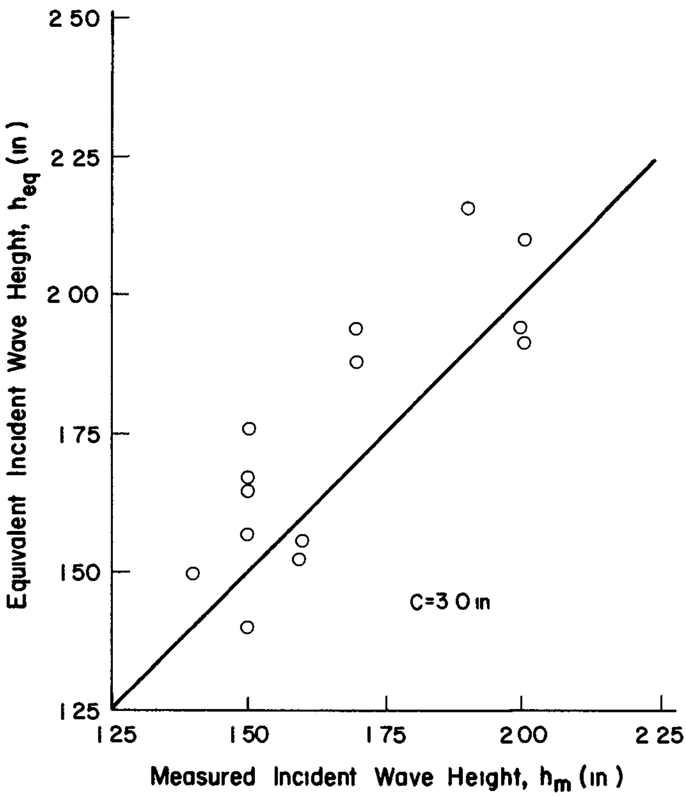


FIG 4—EQUIVALENT VERSUS MEASURED WAVE HEIGHT—REFORMED WAVES

TABLE I - WAVE DATA

Run Number	Depth Over Bar C (in)	Wave Period T (sec)	Heq (in)	heq (in)
1	1 5	1 22	1 75	0 76
2	1 5	1 00	1 90	0 64
3	1 5	0 85	1 84	0 61
4	1 5	0 73	1 62	0 51
5	1 5	0 84	1 56	0 52
6	1 5	1 22	1 54	0 53
7	1 5	1 21	1 14	0 47
8	1 5	0 92	1 33	0 52
9	1 5	1 20	2 29	0 81
10	1 5	1 69	2 04	0 63
11	1 5	1 71	2 17	0 71
12	1 5	1 21	2 52	0 78
13	1 5	1 23	0 99	0 51
14	3 0	2 24	3 04	1 93
15	3 0	2 17	3 79	2 14
16	3 0	2 19	2 03	1 49
17	3 0	1 94	1 90	1 58
18	3 0	1 93	2 68	1 90
19	3 0	1 81	3 67	2 08
20	3 0	1 57	2 36	1 40
21	3 0	1 54	3 96	1 87
22	3 0	1 58	3 14	1 57
23	3 0	1 32	4 19	1 92
24	3 0	1 33	3 38	1 76
25	3 0	1 35	2 14	1 49
26	3 0	1 10	2 02	1 40
27	3 0	1 14	2 41	1 54
28	3 0	1 10	3 20	1 67
29	3 0	0 90	2 79	1 64
30	5 0	1 26	3 45	2 65
31	5 0	1 54	3 20	2 71
32	5 0	1 98	5 21	2 78
33	5 0	2 53	4 54	2 96
34	5 0	1 92	4 34	2 42
35	5 0	1 47	5 64	2 74

Visual observations of the wave forms in the lee of the bar indicated that the water-surface history repeated itself in a time interval approximately equal to the period of the incident waves However, a more rigorous determination of the period of the waves in the lee of the bar may be made by conducting a spectral analysis of the wave form The assumption is made that the frequency at which the peak energy is located corresponds to the period of the wave The spectral analysis will show the frequency band in which the peak energy is located in both the incident wave and the reformed wave A comparison of the spectra of the incident wave and the spectra of the reformed wave will show if a shift in the location of the peak energy has occurred If a shift has occurred, then the period of the wave will have changed during the breaking and reforming process

The spectral analysis may be carried out using the data which were gathered earlier for the determination of the equivalent wave heights A method for determining the wave spectra is presented by Kinsman (8) and is outlined here with some slight changes in constants

With N values of $\eta(t_i)$ spaced at time intervals $\Delta\tau$ and with m as the lag, the sample autocorrelation function, r, may be computed

$$r(\nu\Delta\tau) = \frac{1}{N - \nu} \sum_{i=1}^{N-\nu} \eta(t_i)\ \eta(t_i + \nu\Delta\tau) \tag{7}$$

where $\nu = 0, 1, 2, \quad , m$ The lag, m, is some fraction of the length of the record, usually less than 0 1N

The Fourier transform of the autocorrelation function is the energy spectrum, which is required This transform may be effected by calculating the unsmoothed estimates of the spectrum as follows

$$L_{\tau=0} = \frac{1}{m}\left[\frac{1}{2}[r(0) + r(m\Delta\tau)] + \sum_{k=1}^{m-1} r(k\Delta\tau)\right] \tag{8}$$

$$L_{\tau=\nu} = \frac{2}{m}\left[\frac{1}{2}[r(0) + (-1)^{\nu}\ r(m\Delta\tau)] + \sum_{k=1}^{m-1} r(k\Delta\tau)\cos\frac{\pi k\nu}{m}\right] \tag{9}$$

$\nu = 1, 2, \quad , m - 1$, and

$$L_{\tau=m} = \frac{1}{m}\left[\frac{1}{2}[r(0) + (-1)^{m}\ r(m\Delta\tau)] + \sum_{k=1}^{m-1} (-1)^{k}\ r(k\Delta\tau)\right] \tag{10}$$

where L is the unsmoothed spectral estimates at time $t = \tau$ The final smoothed estimates of the spectrum, Φ, are formed by the moving, weighted average

$$\Phi(\nu\Delta\tau) = \sum_{i=0}^{m} a_{\nu,i} L_{\nu} \tag{11}$$

where $\nu = 1, 2, \quad , m - 1$, and

$$\left.\begin{aligned} a_{\nu,\nu-1} &= 0\ 25 \\ a_{\nu,\nu} &= 0\ 50 \\ a_{\nu,\nu+1} &= 0\ 25 \\ a_{\nu,i} &= 0 \end{aligned}\right\} \tag{12}$$

for $i \neq \nu - 1, \nu,$ or $\nu + 1$ For the end points,

$$\left.\begin{aligned} \Phi(0) &= (L_0 + L_1)/2 \\ \Phi(m) &= (L_{m-1} + L_m)/2 \end{aligned}\right\} \tag{13}$$

Typical results of the spectral analysis are presented in Figs 5 through 10 These plots show that the peak energy of the incident wave and the peak energy of the reformed wave generally occur at the same frequency Thus, the indication is that the basic period of a wave does not change as the wave passes over the bar, breaks, and reforms in the lee of the bar

In all reformed wave records a second energy peak occurs at a higher frequency that is consistently twice the frequency of the incident wave In a few cases, the peak value of this higher frequency energy is equal to or slightly exceeds the reformed energy of the incident wave frequency (see Fig 8 for example) In most cases, the peak energy at this higher frequency is only 20 to 40 percent of the main peak of the reformed energy spectra

Also, in most of the reformed wave records analyzed, a very low level of energy with a frequency of approximately triple the incident wave frequency can be detected

The ratio of reformed to incident equivalent wave height, heq/Heq, has been plotted against the deep water wave steepness, Heq/T^2, (Fig 11) and against the relative depth over the bar crest, C/T^2, (Fig 12) As can be seen, no correlation is apparent in either of these figures

Fig 13 shows heq/Heq plotted against the ratio of incident equivalent wave height to water depth over the bar crest, Heq/C The data for the 3 in and 5 in water depths plot together and produce a relatively well defined relationship between Heq/C and heq/Heq Some of the scatter may be caused by the fact that some waves break over the bar crest while others break ahead of the crest

The data for the 1 5 in depth fall below the other data, particularly for the low incident waves This is possibly due to scale effects in the breaking of these low amplitude waves

CONCLUSIONS

The objective of this investigation was to gain some understanding of the characteristics of monochromatic waves after they break over an offshore bar Generally, most of the energy in the reformed waves is at the same period as in the incident wave but a large portion of the energy shifts to higher frequencies The ratio of equivalent wave heights (square root of wave energy per unit surface area) before and after the bar depends primarily on the ratio of the incident wave height to water depth over the bar crest

Further tests are planned for 5 in and greater (e g 7 in and 9 in) depths over the bar with waves that break and waves that do not break but are disturbed enough by the bar to cause the development of higher frequency components in the lee of the bar These tests should better define the relationship between heq/Heq and Heq/C They should also lead to a greater understanding of the reformed wave energy spectrum Also, a closer look at the nature of the waves as they break as well as the point on the bar at which breaking occurs is needed

ACKNOWLEDGEMENT

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REFERENCES

1 Dick, T M , "On Solid and Permeable Submerged Breakwaters", C E Research Reoort 59, Queen's University, Kingston, Ontario, Nov 1968, 144 p

2 Diephuis, J G H R , "Scale Effects Involving the Breaking of Waves", Proceedings, 6th Conference on Coastal Engineering, Gainesville, Palm Beach, and Miami, Fla , 1957, Chapter 11

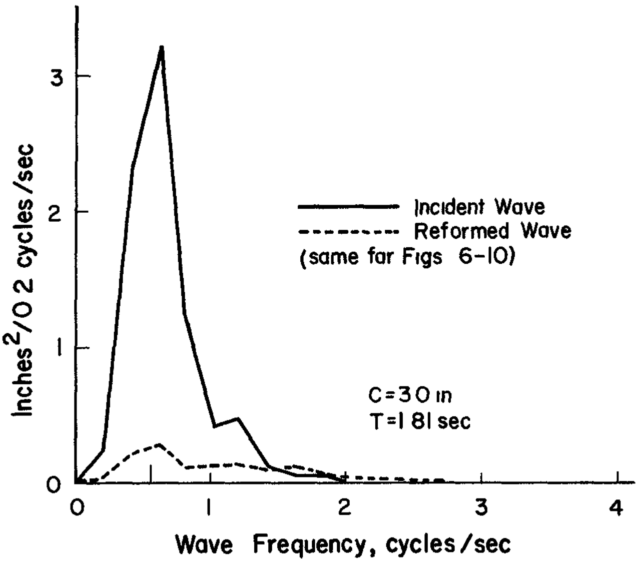

FIG 5 WAVE SPECTRA, RUN NO. 19

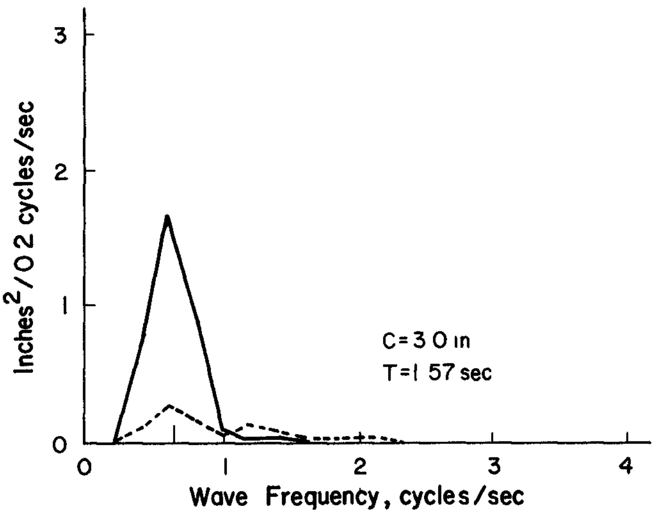

FIG 6 WAVE SPECTRA, RUN NO 20

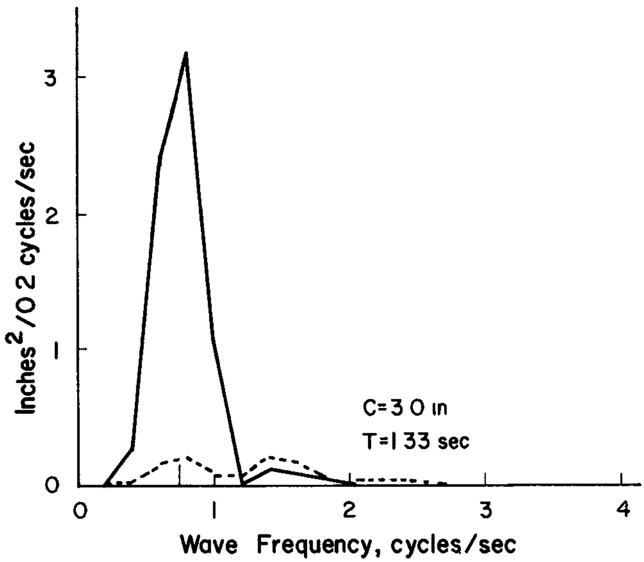

FIG 7 WAVE SPECTRA, RUN NO 24

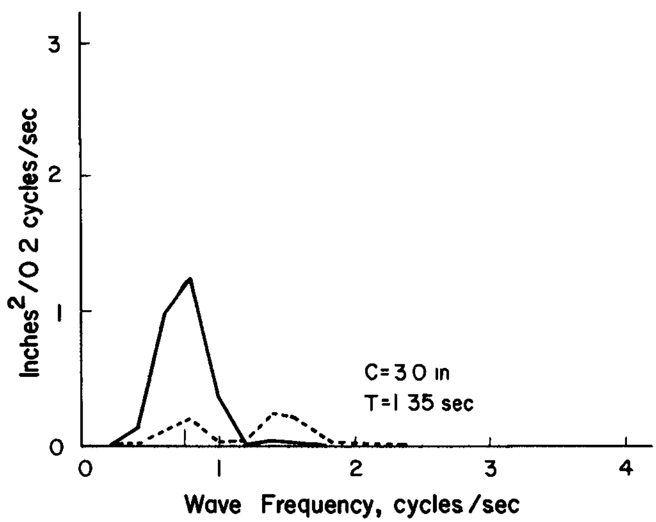

FIG 8 WAVE SPECTRA, RUN NO. 25

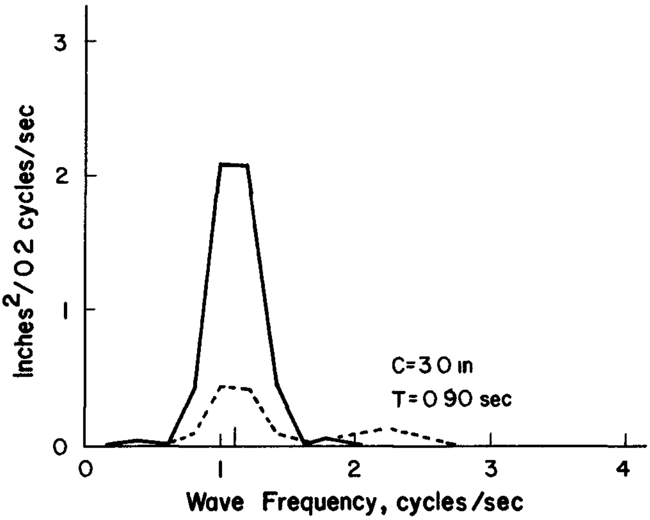

FIG 9 WAVE SPECTRA, RUN NO 29

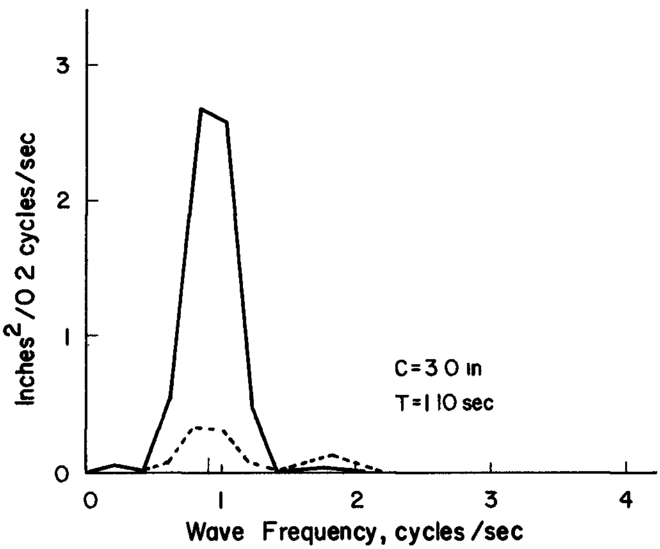

FIG 10 WAVE SPECTRA, RUN NO 28

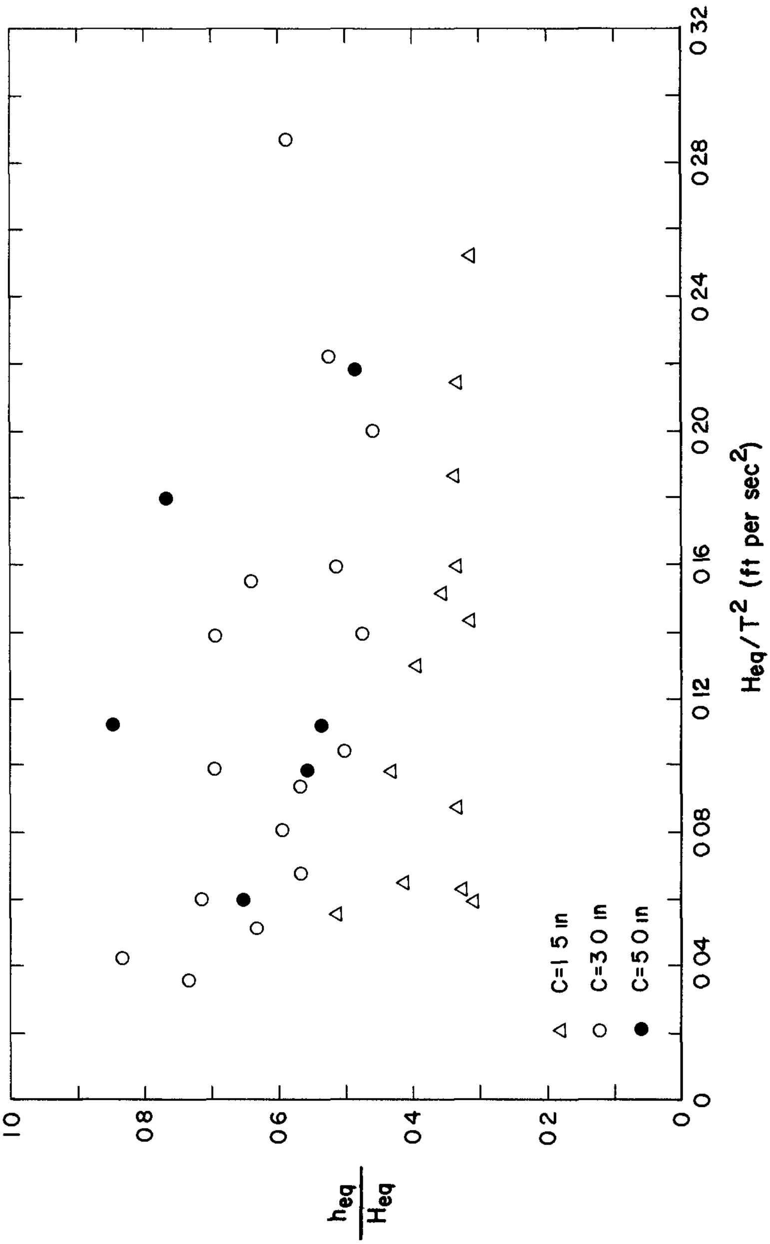

FIG 11—h_{eq}/H_{eq} VERSUS H_{eq}/T^2 FOR ALL DEPTHS

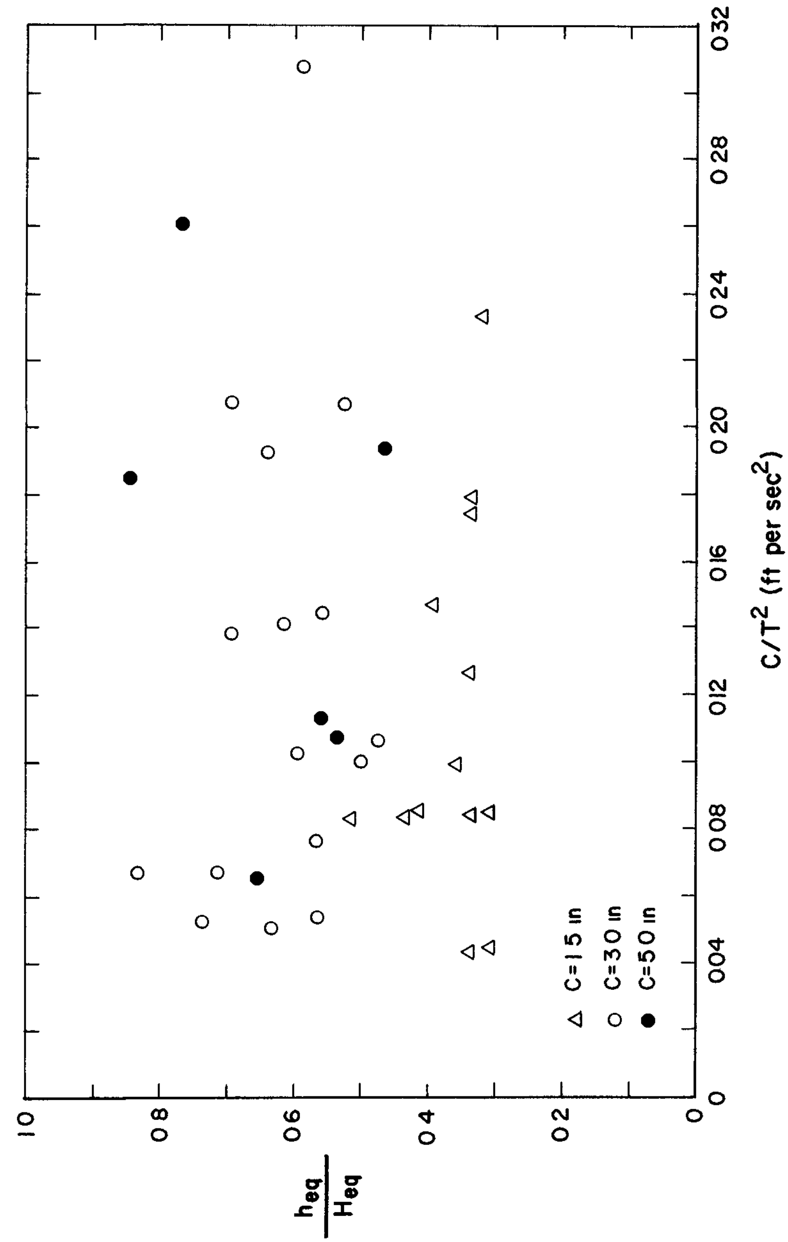

FIG 12 — h_{eq}/H_{eq} VERSUS C/T^2 FOR ALL DEPTHS

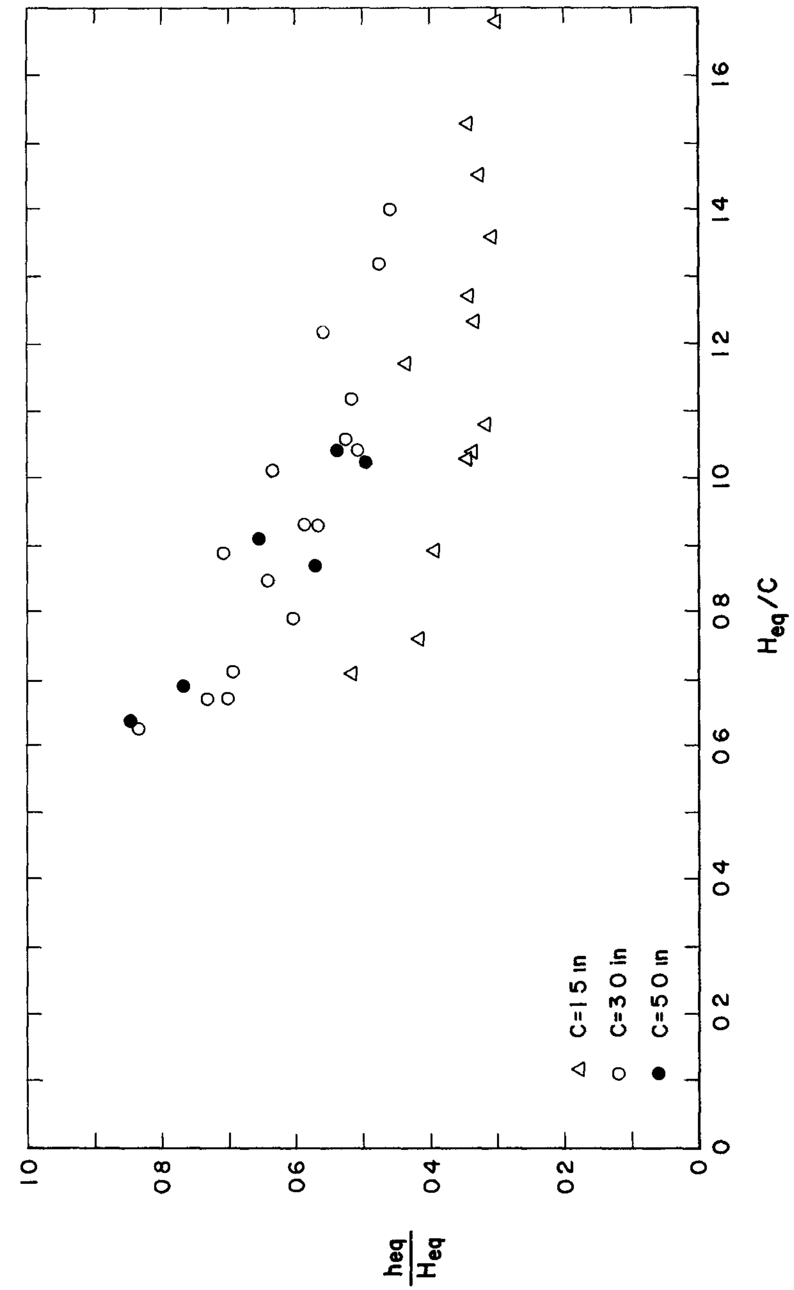

FIG 13—h_{eq}/H_{eq} VERSUS H_{eq}/C FOR ALL DEPTHS

3 Herbich, J B, "Experimental Studies of Wave Filters and Absorbers", Project Report No 44, University of Minnesota, St Anthony Falls Hydraulic Laboratory, St Paul, Minn, Jan, 1956

4 Hom-ma, M, and Horikawa, K, "Wave Forces Against Sea Wall", Proceedings, 9th Conference on Coastal Engineering, Lisbon, Portugal, 1954, Chapter 31

5 Horikawa, K, and Juo, C, "A Study on Wave Transformation Inside Surf Zone", Proceedings, 10th Conference on Coastal Engineering, Vol 1, Tokyo, Japan, 1966, Chapter 15

6 Ippen, A T, ed, Estuary and Coastline Hydrodynamics, McGraw-Hill Book Co, New York, 1966

7 Iverson, H W, "Laboratory Study of Breakers", National Bureau of Standards Circular 521, Gravity Waves Symposium, 1952

8 Kinsman, Blair, Wind Waves, Their Generation and Propagation on the Ocean Surface, Prentice-Hall, Inc, Englewood Cliffs, N J, 1965, Chapter 9

9 Nakamura, M, Shiraishi, H, and Sasaki, Y, "Wave Decaying Due to Breaking", Proceedings, 10th Conference on Coastal Engineering, Vol 1, Tokyo, Japan, 1966, Chapter 16

10 ________, "Wave Damping Effect of Submerged Dike", Proceedings, 10th Conference on Coastal Engineering, Vol 1, Tokyo, Japan, 1966, Chapter 17