CHAPTER 34

ANALYTICAL APPROACH ON WAVE OVERTOPPING ON LEVEES

Hiroyoshi Shi-igai, Dr Eng ,M Eng ,M of JSCE.*
Tsugio Kono, B Eng ,M of JSCE**

1 Analytical Approach

An analytical approach to evaluate the amount of overtopping for given conditions is studied in this paper Here, the wave overtopping is considered as a similar phenomenon to the flow over a weir changing the depth with respect to time

The theoretical approaches used by many scholars have been mostly based on the dimensional analysis In this paper, however, another kind of approach is tried, which is rather deterministic, in order to get a general view of wave overtopping mechanism

The well-known formula which expresses the discharge over a sharp-edged weir is as follows

$$q = \frac{2}{3} m \sqrt{2g}\, y^{\frac{3}{2}} \qquad (1)$$

where q is the discharge per unit width, m is the discharge coefficient and y is the overflow depth It is usually admitted that Eq (1) is valid only for steady flow However, if we assume that y does not change very rapidly with respect to time, Eq (1) may be used for the analysis of wave overtopping Writing that

$$y = z(t) - z_o \qquad (2)$$

* Associate Professor, Asian Institute of Technology, Bangkok, Thailand
** Research Associate, Department of Civil Engineering, Tokyo Institute of Technology, Tokyo, Japan

where $z(t)$ is the surface elevation of waves over the levee measured from SWL, and z_o is the elevation of the top of the levee, we can obtain the quasi-steady equation for the overtopping discharge

$$q(t) = \frac{2}{3} m \sqrt{2g} \, (z(t) - z_o)^{\frac{3}{2}} \tag{3}$$

Further, we write $z(t)$ as

$$z(t) = z_m F(t) \tag{4}$$

where $F(t)$ is a non-dimensional function of time which expresses the wave profile at the levee z_m is approximately equal to the wave run-up height but not the same one, because normally wave run-up height R is measured without any overtopping If there is overtopping, the reflection rate must change and R and z_m cannot be of the same value

Using Eq (2), Eq.(3) and Eq (4), we obtain·

$$q = \frac{2}{3} m \sqrt{2g} \, (kH_o)^{\frac{3}{2}} \{F(t) - \frac{z_o}{kH_o}\}^{\frac{3}{2}} \qquad \text{for } F(t) \geq \frac{z_o}{z_m} \tag{5}$$

$$q = 0 \qquad \text{for } F(t) < \frac{z_o}{z_m}$$

where k is defined as:

$$k = z_m / H_o \tag{6}$$

If m and k are constant for a wave period, we integrate q with respect to time

$$Q = \int_{t_1}^{t_2} q dt$$

$$= \frac{2}{3} m \sqrt{2g} \, (kH_o)^{\frac{3}{2}} \int_{t_1}^{t_2} \{F(t) - \frac{z_o}{kH_o}\}^{\frac{3}{2}} dt \tag{7}$$

where $F(t) > \frac{z_o}{kH_o}$ for $t_1 < t < t_2$ Q is the total discharge of overtopping for a period per unit width of the weir Eq (7) can be expressed as a non-dimensional form, which is

$$\frac{Q}{TH_o\sqrt{2gH_o}} = \frac{2}{3}\frac{mk^{\frac{3}{2}}}{T}\int_{t_1}^{t_2}\{F(t) - \frac{z_o}{kH_o}\}^{\frac{3}{2}} dt \quad (8)$$

where T is the wave period The left hand side is a kind of Froude Number Fig 1 and Fig 2 show the schematic diagrams for the definitions Eq (8) may be simplified if the wave profile can be approximated by triangular waves

$$\frac{Q}{TH_o\sqrt{2gH_o}} = \frac{2}{15} mk^{\frac{3}{2}} (1 - \frac{z_o}{kH_o})^{\frac{5}{2}} \quad (9)$$

For sinous waves we obtain Eq.(10)

$$\frac{Q}{TH_o\sqrt{2gH_o}} = \frac{4}{3} mk^{\frac{3}{2}} \frac{1}{T}\int_{t_1}^{\frac{T}{4}} \{\sin \frac{2\pi}{T} t - \frac{z_o}{kH_o}\}^{\frac{3}{2}} dt \quad (10)$$

where $t_1 = \frac{T}{2\pi} \sin^{-1} \frac{z_o}{kH_o}$

Though Eq (9) looks very simple, it gives us fairly good results as is shown Fig 3, if we choose the value of k carefully, while the value of k is affected by so many factors Fig 4 shows the variation of k with respect to H_o/L and α, the slope of levees The value of k is much less than the value of R/H_o, which is for the case without overtopping, while both of them take the maximum value at $\tan \alpha = 0\ 35$

Eq (9) may be arranged to the conventional form obtained through dimensional analysis as follows

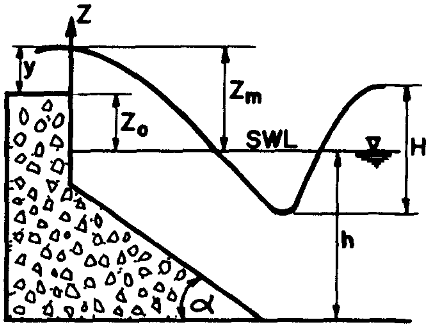

Fig 1 Definition sketch (1)

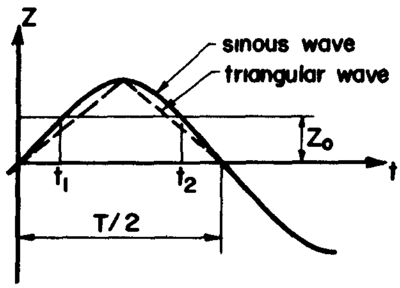

Fig 2 Definition sketch (2)

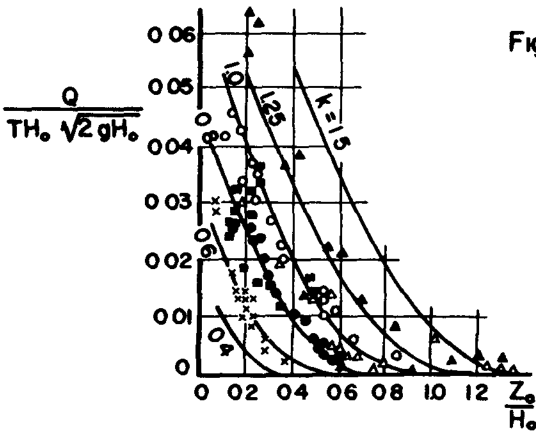

Fig 3 Eq (9) and experimental data

	RUN	α, S	Note
×	1	90°	
○	2	30°	
●	3	30°, 90°	
▲	4	1/10, 1/3	*, wind
△	5	1/10, 1/6	*, wind
■	6	1/6	**

* done by B E B

** done by W E S

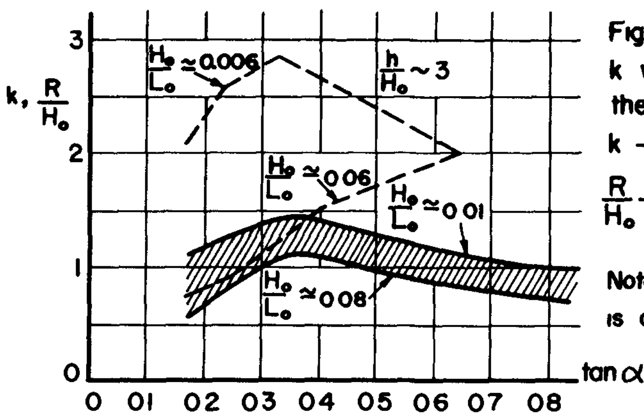

Fig 4 Variation of k with respect to α, the levee slope

k ————

$\frac{R}{H_o}$ - - - - -

Note Value of R/H_o is after Iwagaki[8]

$$\frac{2\pi Q}{H_o L_o} = \frac{4\pi}{15}\sqrt{2gH_o} \quad \frac{Tm}{L_o}\left(1 - \frac{z_o}{kH_o}\right)^{\frac{5}{2}} \tag{11}$$

Normally, Eq (10) gives us the better results but Eq (9) also gives us a satisfactory result The effect of wind can also be expressed by the change of k, while the breakers' effect may not

2 Computer Experiment

For engineering purposes, what we need is the "prediction" Wave-overtopping phenomenon is a combination of statistical and deterministic factors, since wave period and wave height, for example, are statistical variables, while over-topping caused by a given wave can be a deterministic phenomenon In this section the authors treat each over-topping as a completely deterministic process for the given wave conditions, which may include some statistical characteristics The frequency response function being known, we can estimate the total amount of wave overtopping if all the informations of wave characteristics are available However, we face the following problem· Can we estimate the total amount of wave over-topping by the statistical parameters only ? Already this attempt has been done by Tsuruta and Goda by using authors' formula, Eq (9)[7] They estimated the expected amount of overtopping for irregular waves by using $H_{1/3}$ However, the authors would like to point out that the amount of over-topping depends upon the wave period as well as the wave height, and also that though Eq (9) is dimensionless, it is not very convenient to use it directly as the response function, since it includes H_o in both sides

Another important point which is related with the prediction problem is that the response function is highly non-linear Therefore the usual statistical values, $H_{1/3}$ for example, may not be a suitable parameter at all In order to study this point, some numerical experiments were carried out at AIT in Bangkok

The following assumptions are made

1 For simplicity Equation (9) is used instead of sinuous wave formula in the modified form

2 Wave height H_0 and Wave period T follow Rayleigh distribution

3 Cross correlation of H_0 and T is very small

The modification of Eq (9) is as follows·

$$\left.\begin{aligned} \frac{Q}{Z_o^2} &= C' \frac{(\zeta-1)^{5/2}}{\zeta} \tau & (\zeta > 1) \\ &= 0 & (\zeta \leqq 0) \end{aligned}\right\} \tag{12}$$

where

$$C' = \frac{2}{15}\sqrt{2}\, m \tag{13}$$

$$\tau = T\sqrt{\frac{g}{Z_o}} \tag{14}$$

and

$$\zeta = k\frac{H_o}{Z_o} \tag{15}$$

Eq (12) is convenient for prediction, since normally Z_o is given and the variable of left hand side is only Q $H_o/\bar{H}_o$ and $T/\bar{T}$ may follow the following Rayleigh distributions

$$p(\frac{H_o}{\bar{H}_o}) = \frac{\pi}{2}\frac{H_o}{\bar{H}_o} \exp\left[-\frac{\pi}{4}(\frac{H_o}{\bar{H}_o})^2\right] \tag{16}$$

$$p(\frac{T}{\bar{T}}) = 2\ 7(\frac{T}{\bar{T}})^3 \exp\left[-0.675(\frac{T}{\bar{T}})^4\right] \tag{17}$$

where p( ) means the probability density functions, and

$\bar{T}$ and $\bar{H}_o$ mean the ensemble averages

Eq (16) and Eq (17) are only examples, since in fully developed seas $p(\frac{H_o}{\bar{H}_o})$ is Gaussian Anyhow the numerical experiment is always possible if $p(\frac{H_o}{\bar{H}_o})$ and $p(\frac{T}{\bar{T}})$ are given

The procedure of numerical experiment is as follows

1 Prepare the necessary values for the numerical experiments They are Z_o, $\bar{T}$, $\bar{H}_o$ and k If the levee slope is given, k may be estimated from Fig 4 m can be 0 6

2 Compute $T/\sqrt{Z_o/g}$ and $k\bar{H}_o/Z_o$

3 Generate $H_o/\bar{H}_o$ and $T/\bar{T}$ which are statistically random but follow the given probability density Normally, if they are random, the cross correlation is rather small However, we should compute the cross correlation always

4 Compute the values of τ and ζ, which are expressed by Eq (14) and Eq (15)

5 Compute Q/Z_o^2 by the use of Eq (12)

6 Repeat the above procedure for a certain number of times, say one hundred

7 Compute the summation of Q/Z_o^2 Also compute $H_{1/3}$ and $T_{1/3}$ statistically

The above procedure is enough for design purposes, since we can estimate the total amount of overtopping Through this information, we can determine, for instance, the capacity of the pump to drain the water exerted by overtopping

The numerical experiments were carried out for the case that $\bar{H}_o$, $\bar{T}$, $\bar{T}/\sqrt{Z_o/g}$ and $k\bar{H}_o/Z_o$ are all equal to unity The computer used for the experiment is IBM 1130 The results are shown in Table 1 Each run contains one hundred of waves and $H_{o1/3}$, $T_{1/3}$, $\bar{H}_o$ and $\bar{T}$ are statistically obtained For comparison, $H_{1/3}$ and $T_{1/3}$ are computed by Eq (18) and (19)

$$\text{'Computed' } H_{o1/3} = 1.60\, \bar{H}_o \tag{18}$$

$$\text{'Computed' } T_{1/3} = \bar{T} \tag{19}$$

For each wave the value of Q/Z_o^2 is computed and its mean value is obtained through Eq (20)

$$\frac{\sum_{i=1}^{100} \left(\frac{Q}{Z_o^2}\right)_i}{100} = \bar{q}_m \tag{20}$$

Again for comparison, $\bar{q}_{computed}$ is obtained, which is defined by

$$C_o'\left(\frac{T}{\bar{T}}\right)_{\frac{1}{3}} \frac{\{(H_o/\bar{H}_o)_{1/3} - 1\}^{\frac{5}{2}}}{(H_o/\bar{H}_o)_{1/3}} = \bar{q}_c \tag{21}$$

From Table 1 the values of $\bar{q}_c$ are always greater than those of $\bar{q}_m$, which means $\bar{q}_c$ is in safety side for design purposes Among seven runs of experiment, which contain 700 waves, there is a case that $\bar{q}_m = 0.945\ \bar{q}_c$ This fact shows us that for a quick estimation, the use of $H_{o1/3}$ and $T_{1/3}$ may be a good approximation However, the numerical experiments are more desirable for the actual design purposes

3 Conclusions

As conclusions we may say the followings

1 Eq (9) gives us a fairly good prediction for the discharge of wave overtopping, though it contains two empirical factors

2 The above empirical factors, however, do not change vigorously at least for the existing experimental data

3 Theoretically speaking, the use of $H_{o1/3}$ and $T_{1/3}$ for the estimation of overtopping are doubtful

4 According to the limited number of computer experiments, the estimated values of over topping obtained by the use of $H_{o1/3}$ and $T_{1/3}$ are always greater than the statistically obtained values

5 However, this formula may be useful by the use of $H_{o1/3}$ and $T_{1/3}$ in order to get the first approximation of wave overtopping caused by a set of irregular waves

6 For the design purposes the computer experiment is highly recommended since already we obtained the response function which is expressed by Eq (9) or Eq (12)

Table 1 - Results of Numerical Experiment

Run	$H_{1/3}$		$T_{1/3}$		$\bar{q}$		(1)/(2) factor
	Measured	Computed	Measured	Computed	(1) Measured	(2) Computed	
1	1 35	1 45	1 10	1 12	3 83x10^{-3}	12 1x10^{-3}	0 317
2	1 44	1 35	1 10	1 05	4 43	6 36	0 695
3	1 49	1 49	1 10	1 10	7 35	14.1	0 521
4	1 43	1 39	1 05	1 00	6 85	7 20	0 945
5	1 33	1 31	1 04	1 00	4 79	8 14	0 590
6	1 39	1 39	1 04	1 00	5 46	7 13	0 765
7	1 43	1.42	1 00	1.08	5 69	9 80	0 581

Note: All are non-dimensional variables
"Measured" means statistically obtained values
"Computed" means the values obtained by mean values
(for example $H_{1/3}$ = 1 6 $\bar{H}$)
Each run includes 100 of waves

Bibliography

1 Carrier, G F and Greenspan, H P Water waves of finite amplitude on a sloping beach, Jour of F M , Vol 4, 1958

2 Gei, C Report on the model test of sea-dike cross-section of Hsin-Chu Tidal Land, The Tainan Hyd. Lab Bulletin No 6, Taiwan, China

3 Kikkawa, H , Shi-igai, H and Kono, T On the experimental study of over-topping over the vertical wall, The Report of Annual Conf of JSCE, 1966 (in Japanese)

4 Kikkawa, H , Shi-igai, H and Kono, T Fundamental Study of wave over-topping on levees, Coastal Engineering in Japan, Vol 11, 1968

5 Saville, Jr , T : Wave run-up on shore structures, Proc ASCE, Vol 82, 1956

6 Sibul, O J and Tickner, E G Model study of over-topping of wind generated waves on levees with slopes of 1 3 and 6, Beach Erosion Board Tech Memo No 80, 1956

7 Tsuruta, S and Goda, Y.. Expected discharge of irregular wave over-topping, Proc of Coastal Engineering, Vol II, 1968, London

8 Iwagaki, Y , Shima, A and Inoue, M Effects of wave height and sea water level on wave overtopping and wave run up, Coastal Engineering in Japan, Vol 8, 1965