CHAPTER 64

THE DYNAMICS OF A COAST WITH A GROYNE SYSTEM

by W.T. Bakker, E.H J. Klein Breteler, and A. Roos

Coastal Research Department, Rijkswaterstaat
The Hague, Netherlands

ABSTRACT

This paper is a continuation of the paper with the same name, presented on the XIth Conference on Coastal Engineering by the first author [1] , in which a mathematical theory was given about the behaviour of a coast after the construction of a groyne system. Now this paper extends the former paper theoretically and practically.

1. Theoretically a computer program has been made in which the influence of diffraction behind the groyne has been taken into account.
2. Practically the coastal constants used in the theoretical model of the coast will be expressed in terms of wave height and wave direction, based on the theory of SVASEK [2] .

Results are given of computations with a coastal model in which the coast is schematized to one line (one-line theory) and a model in which the coast is schematized to a beach line and on inshore-line (two-line theory).
The influence of changing wave conditions is investigated.

INTRODUCTION

The construction of a groyne has the following effects (fig. 1)

1. Prevention of the littoral sanddrift in the area between the coastline and the head of the groyne.
2. Prevention of the longshore current in the same area.
3. Formation of a sheltered area at the lee-side of the groyne, caused by the diffraction.
4. Changing the wave height by reflection

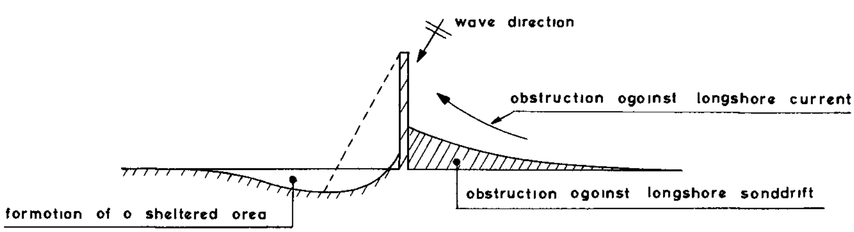


Fig 1 The effects of the construction of a groyne

The former paper dealt only about the first aspect, now we shall pay attention to the first and the third aspect. The second and fourth one will be investigated in the future.

ONE-LINE THEORY

The theory given here is an extension of the theory of PELNARD-CONSIDERE [3] .

PELNARD-CONSIDERE assumes, that the profile of the coast always remains the equilibrium profile, so that he only needs to consider one coastline, being one of the contourlines. He assumes no currents, constant wave direction, small angle of wave incidence and a linear relation between angle of wave incidence and the littoral drift.

The derivation of his theory is summarized in [1] .

For the littoral drift he finds

$$Q = Q_o - q \frac{\partial y}{\partial x} \quad \ldots\ldots (1)$$

in which Q = littoral drift.

Q_o = littoral drift at the point, where $\frac{\partial y}{\partial x} = 0$.

$q = \frac{\partial Q}{\partial \phi}$ = the derivate of the littoral drift Q to the angle of wave incidence φ.

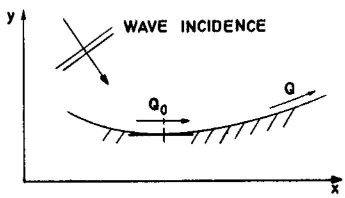


Fig 2 Littoral drift along the coast

He finds, that the accretion is proportional to the curvature of the coast

$$\frac{\partial y}{\partial t} = \frac{q}{D_{tot}} \frac{\partial^2 y}{\partial x^2} \quad \ldots\ldots\ldots\ldots\ldots (2)$$

From this equation the coastline y as a function of x and t can be found for many boundary conditions. Pelnard-Considère gives analytical solutions of his equations. The interrupted line in fig. 5 shows the accretion and erosion near a groyne according to his theory. He assumes that wave height and wave direction are constants along the coast. At the lee-side of the groyne however, the wave height is less and the waves have an other direction, as a result of the diffraction.

We introduce diffraction in the theory of Pelnard-Considère . The equations become more complicated, that's why we have to give numerical solutions. The derivation of the one-line theory including diffraction is given in appendix A1.

For the littoral drift the same formula of Pelnard-Considère remains of value

$$Q = Q_o^* - q^* . \frac{\partial y}{\partial x} \quad \ldots\ldots\ldots\ldots (3)$$

but now Q^* and q^* are functions of x.

The effect of the diffraction can be splitted in a stationary effect and an instationary effect. This can be made clear in the following way (fig. 4).

If everywhere wave height and wave direction are the same, a straight coastline is stable, the transport is everywhere the same. If the wave height and the wave direction change in x-direction, the transport will change also and therefore the coastal shape has to adapt itself in order to make the transport everywhere the same again and give a stable coastline. In appendix A1 a mathematical formulation of this problem is given. The transport has been taken proportional to the square of the wave height and to the angle of wave incidence. A possible stable coastline y_o as a function of x is found (appendix A1), ruled by the differential equation

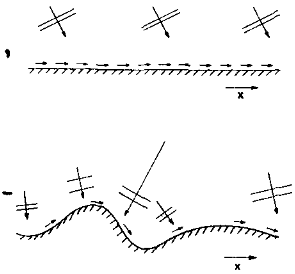


Fig 4

$$\frac{dy_o}{dx} = \varphi_x - \frac{\varphi_\infty}{h^2} \qquad (4)$$

in which φ_x is the angle of the waves with the x-axis, φ_∞ the angle of wave incidence far from the groyne and h is the ratio between the wave height at an arbitrary point (x,0) to the wave height at $x = \infty$, h is a function of x.

A short analysis of (4) learns, that if the wave height should be everywhere the same (h=1) this would give $\frac{dy_o}{dx} = \varphi_x - \varphi_\infty$, thus the changing of the coastal direction is equal to the changing of the wave direction.

However, the problem of diffraction near a harbour mole is more intricate.

As the groyne stops all the transport, and as at $x = \infty$ the transport remains Q_o, a stable coastline can never be achieved.

We split the coastline y into two parts, y_o being a stationary effect of the diffraction[1)] and y', being an unstationary effect

$$y\ (x,t) = y_o\ (x) + y'\ (x,t) \qquad (5)$$

As shown in appendix A1, the equation for the unstationary part y' becomes about (2), but with an additional term, because q* is a function of x

$$\frac{\partial y'}{\partial t} = \frac{q^*}{D_{tot}} \frac{\partial^2 y'}{\partial x^2} + \frac{1}{D_{tot}} \frac{dq^*}{dx} \cdot \frac{\partial y'}{\partial x} \qquad (6)$$

in which $q^* = Ah^2 \qquad (7)$

1) y_o is the stable coastline, which would develop, if an artificial nourishment Q_o would be administered on the lee-side of the groyne.

A is a proportionality constant, being investigated in the chapter "coastal constants".
The amount of h and φ_x in (4) and (7) in the diffraction case is found from the simplified theory of PUTNAM and ARTHUR [4] .
The unstationary part y' can be found by numerical integration of equation (6).
Superposition of y_0 and y', according to (4) gives the coastline y (x,t).
For the calculation of the coastlines a computerprogram has been made. Fig. 5 shows the calculated development of a coast with one groyne. Comparison of the interrupted and the solid line gives an impression of the influence of diffraction.

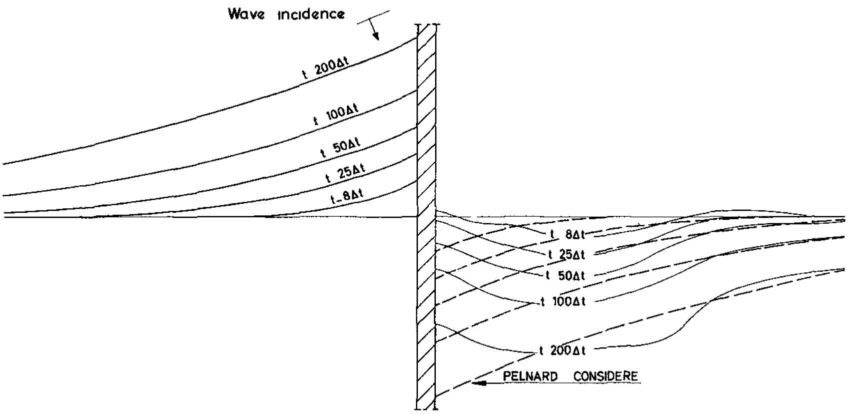


Fig 5 Accretion and erosion near a groyne numerical solution with diffraction (one line theory) The dotted lines at the right hand gives erosion according to Pelnard - Considere

With the computerprogram we calculated the behaviour of the coastline between two groynes with the influence of diffraction. The result is shown in fig. 6.

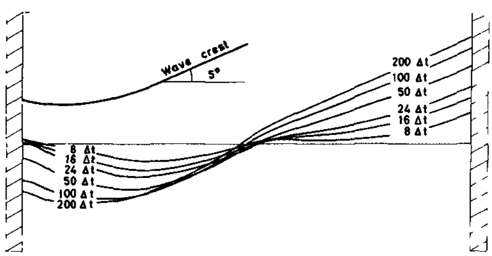


Fig 6 Behavior of the coastline between two groynes (one-line theory)

TWO-LINE THEORY

Now we schematize the coastal profile to two lines, the beach (y_1) and the inshore (y_2'). This gives the possibility to take the off- and onshore transport into account.

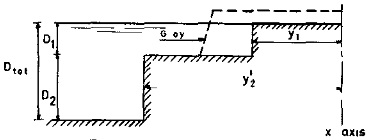


Fig 7 Schematized profile

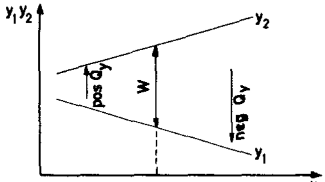


Fig 8 On- and offshore transport

In top view one sees two lines at a distance y_1 and y_2' from the x-axis. The "equilibrium distance" is the distance y_2' - y' between beach and inshore, when the profile is an equilibrium profile.

The following dynamic equations are assumed.

If the distance y_2' - y' is equal to the equilibrium distance W, no interaction is assumed. If the distance $y_2' - y_1$ is less than W, the profile is too steep and an offshore transport will be the result. An onshore transport will occur in the opposite case.

We linearize this relation and take for the offshore transport Q_y per unit length

$$Q_y = q_y \left\{ y_1 - (y_2' - W) \right\} \quad \dots . (9)$$

in which q_y is a proportional constant with the dimension $[1/t]$. It is a function of x. For a simpler notation, we denote

$$y_2 = y_2' - W \quad \dots\dots\dots (10)$$

Then (9) becomes

$$Q_y = q_y (y_1 - y_2) \quad \dots . (11)$$

With respect to the littoral drift, the assumption of PELNARD-CONSIDERE is applied, both for beach and inshore, the transport is linearized

$$Q_1 = Q^*_{o1} - q^*_1 \frac{\partial y_1}{\partial x} \quad \dots\dots (12a)$$

$$Q_2 = Q_{o2} - q_2 \frac{\partial y_2}{\partial x} \quad \dots\dots\dots (12b)$$

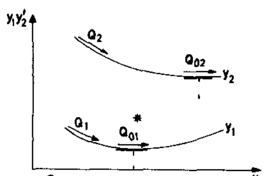


Fig 9 Littoral drift along beach and inshore

in which Q_{o1} and Q_{o2} are respectively the transports where $\frac{\partial y_1}{\partial x} = 0$.

Q^*_{o1} ‹ unction of x, Q_{o2} is a constant. q^*_1 and q_2 are proportionallity factors, q^* is a function of x and q_2 is a constant.

In appendix A2 the derivation of the two-line theory is given. The beach line y_1 is splitted into two parts, analogue to (4).

$$y_1 (x,t) = y_o (x) + y_1' (x,t) \quad \dots\dots (13)$$

in which $y_o(x)$ is the same function as given in (8).
For the accretion along the beach and the inshore we find

$$\frac{\partial y_1'}{\partial t} = \frac{q_1^*}{D_1}\frac{\partial^2 y_1'}{\partial x^2} + \frac{1}{D_1}\frac{\partial q_1^*}{\partial x}\cdot\frac{\partial y_1'}{\partial x} - \frac{q_y}{D_1}(y_1' + y_o - y_2) \quad ..(14)$$

$$\frac{\partial y_2}{\partial t} = \frac{q_2}{D_2}\frac{\partial^2 y_2}{\partial x^2} + \frac{q_y}{D_2}(y_1' + y_o - y_2) \quad .... (15)$$

These are two simultaneous partial differential equations. For the calculation of the beach line y_1 and the inshore line y_2 we made a computer program in which the equations are solved numerically (appendix A2). In fig. 10 the development of a coast with one groyne is shown.

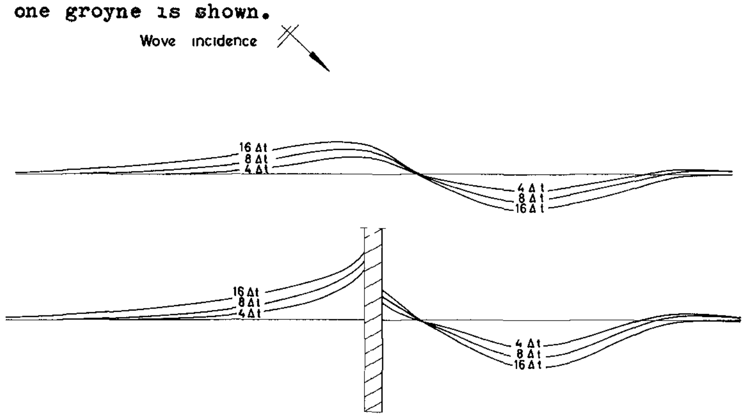


Fig 10 Accretion and erosion near a groyne numerical solution with diffraction (two-line theory)

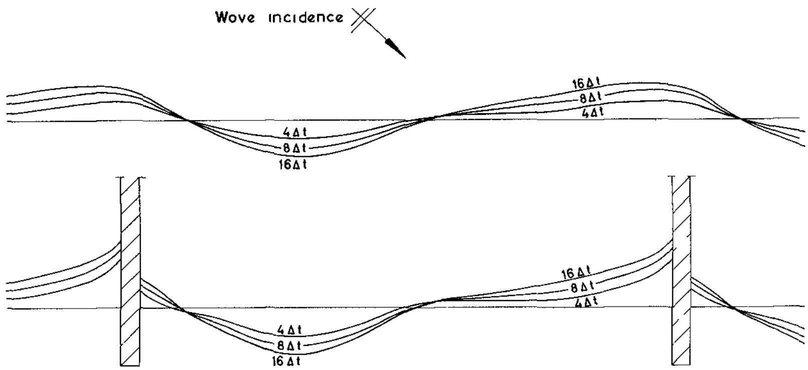


Fig 11 Behaviour of beach and inshore between two groynes (two-line theory)

From a coast with an infinite row of groynes, we calculated the development of the coastal shape between two groynes. This is shown in fig. 11.

COASTAL CONSTANTS

In this chapter some expressions will be derived for the coastal constants, respectively using the one-line and the two-line theory. For the one-line theory the CERC-formula will be used, for the two-line theory the SVASEK-variation of this [2] .
It is assumed, that the transport is confined to the breaker-zone.

$$D_{tot} = D_{br} \qquad \dots\dots\dots\dots\dots (16)$$

Considering the longshore theory of BOWEN [6] it may be expected that the transport takes place over a distance 1 to 1,5 times the breaker zone, and that most of the transport is confined to the breaker zone. Probably it is better to assume for D_{tot} the depth occurring at a distance $1\frac{1}{2}$ times the width of the breaker zone. In this case the factor q becomes less, for a concave profile about 100% to 80% of the computed value.

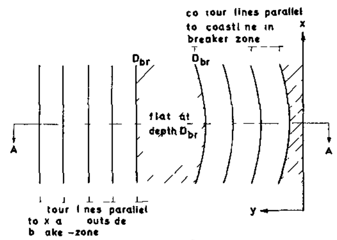


Fig 12ᵃ

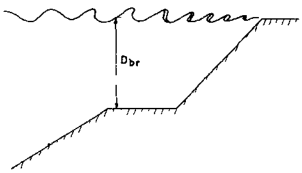


Fig 12ᵇ profile A-A

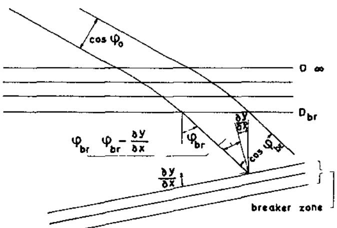


Fig 12ᶜ

One-line theory

We assume the topography and notation according to fig. 12
The CERC-formula relates the longshore transport Q to the longshore

component of the wave energy flux

$$Q = 1.4 \times 10^{-2} H_o^2 C_o K^2 \sin \varphi_{br} \cos \varphi_{br} \quad \ldots\ldots (17)$$

in which H_o = wave height in deep water
C_o = phase velocity in deep water
K = refraction coefficient
φ_{br} = angle of wave incidence in the breaker zone

From fig. 12 can be derived

$$\varphi_{br} = \varphi_{br}' - \frac{\partial y}{\partial x} \quad (\frac{\partial y}{\partial x} \text{ small}) \quad \ldots\ldots (18)$$

$$Q_o = 1.4 \times 10^2 H_o^2 C_o K^2 \sin\varphi_{br}' \cos\varphi_{br}' \quad \ldots\ldots (19)$$

$$q = \frac{dQ}{d(\frac{\partial y}{\partial x})} = 1.4 \times 10^{-2} H_o^2 C_o K^2 \cos 2\varphi_{br}' \quad \ldots (20)$$

One can write

$$H_{br} = A_2 D_{br} \quad \ldots (21) \quad \text{and} \quad C_{br} \sqrt{g D_{br}} \quad \ldots (22)$$

in which A_2 and A_3 can be taken from any wave theory or measurements (for instance, solitary wave theory [7]

$A_2 = 0.78$ and $A_3 = 2 \times 0.78$).

Conservation of wave energy between wave rays gives

$$H_o^2 C_o K^2 = H_{br}^2 C_{br} = A_2^2 A_3 g^{\frac{1}{2}} D_{br}^{5/2}$$

This makes

$$Q_o = A_1 A_2^2 A_3 g^{\frac{1}{2}} D_{br}^{5/2} \sin\varphi_{br}' \cos\varphi_{br}' \quad \ldots (23)$$

$$q = A_1 A_2^2 A_3 g^{\frac{1}{2}} D_{br}^{5/2} \cos 2\varphi_{br}' \quad \ldots\ldots (24)$$

in which $A_1 = 1.4 \times 10^2$

Often cos $_{br}$ can be taken equal to 1.

Now it is easy to give numerical values to the proportionality constants, used elsewhere in this paper, for instance, in (7)

$$A = 1.4 \times 10^{-2} H_o^2 C_o K^2$$

and in appendix A1, (A10)

$$\Delta t = \frac{D_{tot}}{A h^2_{max}} \quad \frac{(\Delta x)^2}{2}$$

$$\Delta t = \frac{D_{br}}{A_1 A_2^2 \; A_3 g^{\frac{1}{2}} \; D_{br}^{5/2} \; h^2_{max}} \quad \frac{(\Delta x)^2}{2}$$

$$\Delta t = \frac{1}{2A_1 \; A_2^2 \; A_3 h^2_{max}} \quad \frac{(\Delta x)^2}{D_{br}\sqrt{g \, D_{br}}}$$

Two-line theory

In the two-line theory, the coastal constants mentioned in fig. 13 are of importance. The exact definitions are giver in (9) and (12). The constant q_y, which defines, how the off-shore transport changes, when the profile changes, will be treated in a separate paper in the future. Some investigation about this constant has already been done 8 .

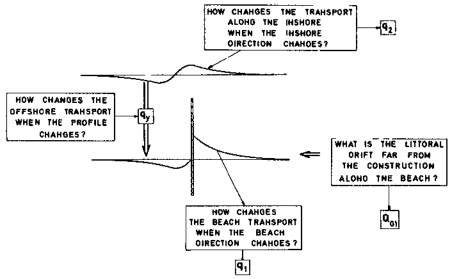


Fig 13

The coefficients Q_{o1}, q_1, Q_{o2} and q_2 will be computed with the SVASEK-theory 2 which only treats the longshore transport. SVASEK neglects the longshore transport outside the breaker-zone We assume, that the profile outside the breaker-zone has reached already its equilibrium profile, and that the on- and offshore transport can be neglected there.
The assumed profile topography can be like given in fig. 14 (see next page), more natural than shown in fig. 7.
SVASEK assumes, that the littoral drift between two depth contours is proportional to the longshore component of the loss energy between these contourlines (2 , formula 5 - 7)

$$\Delta Q = A_1' \cdot \Delta\left(\frac{H^2 C}{K^2}\right) K_m^2 \sin\varphi_m \quad \cos\varphi_2 \qquad \ldots . (25)$$

in which Q = littoral drift between two depth contours $D - \frac{1}{2}\Delta D$

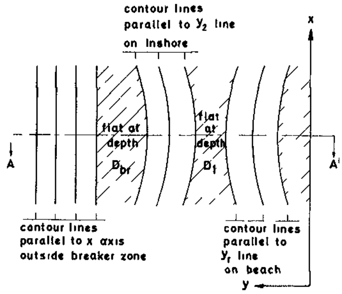


Fig 14ª Upper view

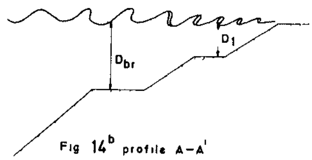


Fig 14ᵇ profile A–A'

and $D + \frac{1}{2}\Delta D$, $\Delta(\frac{H^2C}{K})$ = difference between $\frac{H^2C}{K}$ on both dept contours

K_m, φ_m = value of refraction coefficient K and angle of wave incidence φ in the midst between the depth contours. It appears (appendix A3), that ΔQ can be written

$$\Delta Q = 3A_1A_2^2\, A_3 g^{\frac{1}{2}} D^{1\frac{1}{2}}\, \Delta D \sin\varphi_m \cos\varphi_m \quad ..(26)$$

and after some simplifications, treated in appendix A3, the following constants are found for small angle of wave incidence

$$Q_{o1} = A_1A_2^2\, A_3 g^{\frac{1}{2}} D_1^{\ 3}\, D_{br}^{-\frac{1}{2}} \sin\varphi_{br}'$$

$$q_1 = A_1A_2^2\, A_3 g^{\frac{1}{2}}\, D_1^{2\frac{1}{2}} \quad \ldots\ldots (27)$$

$$Q_{o2} = A_1A_2^2\, A_3 g^{\frac{1}{2}}\, (D_{br}^{\ 3} - D_1^{\ 3})\, D_{br}^{-\frac{1}{2}} \sin\varphi_{br}'$$

$$q_2 = A_1A_2^2\, A_3 g^{\frac{1}{2}}\, (D_{br}^{\ 3} - D_1^{\ 3})\, D_{br}^{-\frac{1}{2}} \quad \ldots\ldots (28)$$

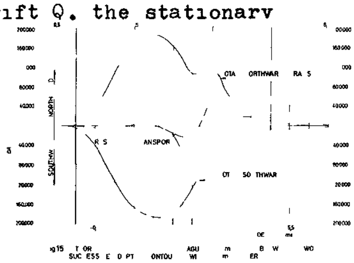
The factor $A_1A_2^2\, A_3 g^{\frac{1}{2}}$ varies between 2.37×10^{-2} and $3.85 \times 10^{-2}\ \sqrt{m/sec}$, dependent of the kind of waves (harmonic or random).

VARIABLE WAVE CONDITIONS

There has to be distinguished the influence of variable wave conditions on the coastal constants and the influence of the boundary conditions.

Influence variable wave conditions on coastal constants

The derivation used for the PELNARD-CONSIDÈRE-formula (2) keeps its validity when the littoral drift Q, the stationary transport Q_o and the constant q are averages over a year instead of instantaneous values. However, it will not be directly clear, which value has to be taken for D_{tot}. In order to estimate D_{tot} it is useful to compute first the distribution of the littoral drift perpendicular to the coast. An example of such a distribution gives fig. 15. The yearly littoral drift between two



depth contours $D - \frac{1}{2}\Delta D$ and $D + \frac{1}{2}\Delta D$ is computed with the aid of (26), which becomes in case of variable wave conditions ($\cos\varphi_b$ has been taken 1 and Snell's law has been applied)

$$\Delta Q = 3A_1A_2^2\,A_3 g D^2 \Delta D \sum_{\substack{\text{all wave classes} \\ \text{for which } D < D_{br}}} \frac{\sin\varphi_o}{C_o}\, fr(H,T,\varphi) \quad \ldots \quad (29)$$

in which $fr(H,T,\varphi)$ denotes the frequency of occurrence of a wave class for which H, T and φ lie between certain values (for instance $\frac{1}{2}$ m < H < 1 m, 5 sec < T < 6 sec, 30° < φ < 60°). More details about the computation are given in [9]

From the distribution of the transport Q_{o1} and Q_{o2} are found (fig. 16)

$$Q_{o1} = \sum_{0 < D < D_1} \overline{\Delta Q} \qquad Q_{o2} = \sum_{D > D_1} \overline{\Delta Q} \quad \ldots . \quad (30)$$

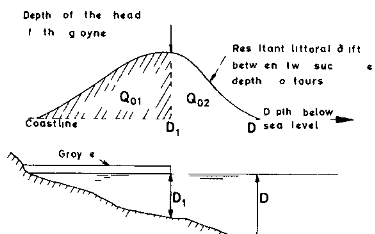


Fig 16

After that, q_1 and q_2 can be found by repeating the computation according to (29), but with a "wrong" coastal direction, which has been turned over an angle $\Delta\varphi$, say 15°. This gives the interrupted line in fig. 17, instead of the solid line, which represents the transport distribution for the original coastal direction.

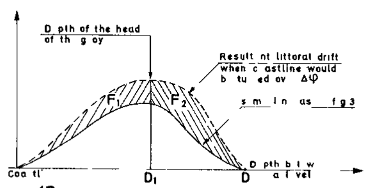


Fig 17

Now q_1 equals $F_1/\Delta\varphi$ (F_1 is the left-handed hatched area in fig. 17) and $q_2 = F_2/\Delta\varphi$.
From the transport distribution, also a reasonable guess about D_{tot} can be made.

Influence variable wave conditions on the boundary

This paper is concentrated on two effects of a groyne prevention of the littoral sand drift and formation of a sheltered area We shall investigate these two effects in case of changing wave conditions.
When the wave direction changes periodically for instance according to

$$\varphi = \hat{\varphi} \sin \omega_{\varphi} t \quad \ldots \quad (31)$$

this generates a sandwave near a groyne (fig. 15, [10])

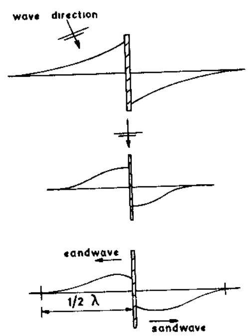


Fig 18 Generation of sandwaves by variable wave direction

$$y = \frac{\hat{\varphi}\lambda_\varphi}{2\pi\sqrt{2}} \cdot e^{-k_\varphi x} \quad \cos(\omega_\varphi t - k_\varphi x) \quad \ldots . \quad (32)$$

in which

$$\lambda_\varphi = \frac{2\pi}{k_\varphi} = 2\sqrt{\pi\frac{q}{D_{tot}}\ T_\varphi} \quad \ldots\ldots\ldots\ldots \quad (33)$$

T_φ being the period of the fluctuations of φ and λ_φ being the length of the sandwave.
Using (24) and taking $A_1 = 1.4 \times 10^{-2}$, $A_2 = 0.4$, $A_3 = \sqrt{1.4}$, this gives

$$\lambda_\varphi = .183\ D_{br}^{3/4}\ (g\ T^2)^{1/4} \quad \ldots\ldots\ldots\ldots \quad (34)$$

Taking as an example T_φ = 1 week, this makes λ_φ = 324 m.
Now the decay of this sandwave is very strong within $\frac{1}{2}\lambda_\varphi$ it is attenuated to 4%. Thus, outside this area, no influence of the stopping of the littoral drift by the groyne will be observed.
In case of the two-line theory D_{br} in (34) probably can be replaced by D_1, $\hat{\varphi}$ has to be replaced by φ_{01}, according to (A 36)
The second influence of the groyne is the wave-shelter. We shall assume first, that the sheltered area is large with respect to $\frac{1}{2}\lambda_\varphi$. As in (12a), Q_0 and q_1 become functions of x, called Q_{01}* and q_1*. Consider fig. 19. The influence of diffraction will be neglected with respect to the influence of changing wave conditions.

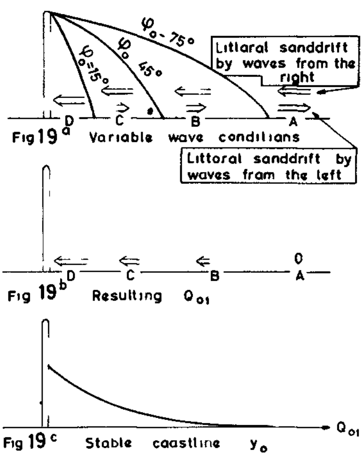


Fig 19a Variable wave conditions

Fig 19b Resulting Q_{01}

Fig 19c Stable coastline y_0

The computation of Q_{01} and q_1 in area A can be performed according to (29), (30) and fig. 14. But applying (44) to area B, all wave classes with $\varphi_0 \geqslant 75°$ must be excluded in the summation, in area C all wave classes with $\varphi_0 \geqslant 45°$, and so on.
When, for instance, the resulting transport in area A would be zero for a coastline parallel to the x-axis (fig. 19a), the transport

Q_{01}* $(\frac{\partial y_1}{\partial x} = 0)$ (started because it

changes in x-direction) will be larger and larger (in negative direction) in the areas B, C and D_1, and also q_1* will change
Now we have returned to the normal computer program, treated in (5) to (7) and in appendix 3, only with other values for y_0 and y' then in the diffraction cases.
The stable coastline y_0 can be found from continuity for y_0 the transport is everywhere the same

$$\left[Q_{o1}\right]_{x=\infty} = Q_1 \longrightarrow \left[Q_{o1}\right]_{x=\infty} =$$

$$= Q_{o1}^* - q_1^* \frac{dy_o}{dx} \longrightarrow \frac{dy_o}{dx} = \frac{Q_{o1}^* - \left[Q_{o1}\right]_{x=\infty}}{q_1^*} \quad . \quad . \ (35)$$

From (35) the stable coastline y_o can be found, which gives the initial value of the unstationary part y_1'

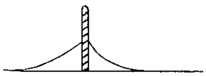


Fig 20a

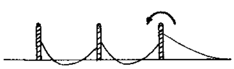


Fig 20b

In case of changing wave conditions, and no resultant drift, the final coastal shape near one single groyne will become just the stable shape y_o, because then everywhere the resultant drift is zero. This will give accretion on both sides of the groyne, which will be withdrawn from a very long stretch of coast (fig. 20a). In case of a row of groynes, the sand for the accretion near the groyne is withdrawn from the area in the midst between the groynes, and only near the boundary of the groyne system some real accretion can be expected (fig. 20b). However, after some time this sand will move to the areas between the groynes, and so this shelter effect may give some accretion (in case of no resultant drift), starting from the boundaries of the groyne system

In case $\frac{1}{2}\lambda_\varphi$ is not small with respect to the sheltered area, the best way of computation is a kind of "hindcasting", using the one-line or two-line computer program described before, and changing the wave conditions during the program.
This has been done at the Coastal Research Department of Rijkswaterstaat.

N B The vertical scale of fig 5 and 6 is 5 times and of fig. 10 and 11 is 10 times exaggerated.

APPENDIX

A1. One-line theory Assumptions and formulae (diffraction)

Assumptions littoral drift proportional to the angle of wave incidence and to the square of the wave height (fig. A1)

$$Q = Ah^2 \left(\varphi_x - \frac{\partial y}{\partial x}\right) \qquad \text{(A1)}$$

in which A is a proportionality constant and h is the ratio between the wave height at x = x and the wave height at $x = \infty$ Eq (A1) is a special case of (3)

Continuity $\frac{\partial y}{\partial t} = - \frac{q}{D_{tot}} \frac{\partial Q}{\partial x}$ $\qquad$ (A2)

Using (3) $\frac{\partial y}{\partial t} = \frac{1}{D_{tot}} \left(q^* \frac{\partial^2 y}{\partial x^2} + \frac{\partial q^*}{\partial x} \cdot \frac{\partial y}{\partial x}\right) - \frac{1}{D_{tot}} \frac{dQ_o}{dx}$ $\qquad$ (A3)

The stable coastline y_o from (5) is a solution of (A2), or (A3) Continuity gives Q is constant for y_o. The amount of Q can be derived from the condition at infinity

$h = 1$, $\varphi_x = \varphi_\infty$, $\frac{\partial y}{\partial x} = 0$, from (A1) follows (4)

$$A\,\varphi_\infty = Ah^2 \left(\varphi_x - \frac{dy_o}{dx}\right)$$

Eq (6) can be derived from (A3) by substituting y_o in (A3) and subtracting this equation from (A3).

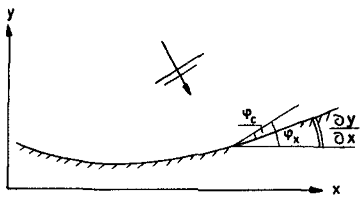


Fig A1

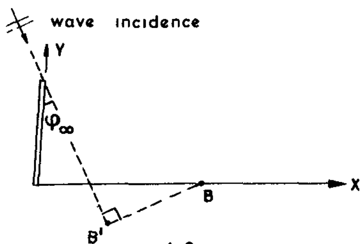


Fig A2

For the determination of the values of φ_x and h the simplified diffraction theory of PUTNAM and ARTHUR [4] is used.
It can be shown [11] that φ_x is about equal to

$$\varphi_x = \varphi_\infty - \frac{\lambda}{2\pi} \frac{d\theta}{dx} \qquad \text{(A4)}$$

in which λ - wave length diffracted wave and θ = phase difference of the waves between B and B' (fig. A2). B = point on x-axis, for which y_o is computed.

Substitution of this result in (4) and integration gives the relation between y_0 and the basic data of diffraction h and Θ

$$y_0 = -\frac{\lambda}{2\pi}\Theta + \varphi_\infty \int \frac{h^2 - 1}{h}\,dx \quad \dots\dots (A6)$$

The first term of the right hand side of (A 6) is the influence of the turning of the waves, the second term gives the influence of changing the wave height by diffraction.

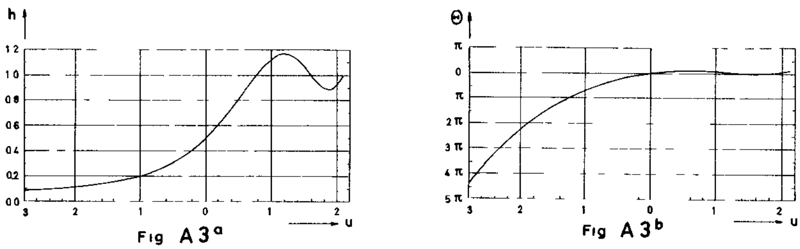


Fig A3 WAVE DIFFRACTION ACCORDING TO PUTNAM AND ARTHUR [6]

Θ and h as a function of $u = 2\sqrt{\frac{AB-AB'}{\lambda}}$ according to PUTNAM AND ARTHUR [4] are shown in fig. A5.

Unstationary part y'
(6) has been taken as a difference equation, taking

$$q_{max} = Ah^2_{max} \quad \dots\dots (A7)$$

$$\Delta t = D_{tot}.(\Delta x)^2/2q_{max} \quad \dots\dots (A10)$$

Substituting (A7) and (A10) in (6), from three adjacent points of a curve at time t one point of the curve at point t + Δt can be found (explicite method)

$$\begin{aligned} y'[x,t+\Delta t] = \; & (\tilde{q}[x+\Delta x]/8 + \tilde{q}[x]/2 - \tilde{q}[x-\Delta x]/8).y'[x+\Delta x,t] \; + \\ & (-\tilde{q}[x] + 1).y'[x,t] \; + \\ & (-\tilde{q}[x+\Delta x]/8 + \tilde{q}[x]/2 + \tilde{q}[x-\Delta x]/8).y'[x-\Delta x,t] \end{aligned} \quad \dots (A11)$$

in which $\tilde{q} = q^*/q_{max}$ has been introduced to avoid instability.

Boundary conditions at x = o no transport and therefore

$$\frac{\partial y}{\partial x} = \frac{dy}{dx} + \frac{\partial y'}{\partial x} = \left[\varphi_x\right]_{x=o} \quad \dots\dots (A12)$$

Substitution of (4) in (A12) gives

$$\frac{\partial y'}{\partial x} = \varphi_x - (\varphi_x - \frac{\varphi_\infty}{h^2}) = \frac{\varphi_\infty}{h^2} \quad \ldots\ldots\ldots \quad (A13)$$

We can write this equation into differences and express y'[- x] in y'[Δx] This gives for the boundary point at the lee-side

$$y'[0, t + \Delta t] = - (- \tilde{q}[\Delta x/8 + \tilde{q}[0]/2 + q[-\Delta x]/8).2\Delta x.\frac{\varphi}{[h]^2_{x=0}} + (- q[o] + 1).\ y'[0,t] + \tilde{q}[0].y'[\Delta x,t]$$

$$\ldots\ldots\ldots\ldots \quad (A14)$$

The expression for the luffside can be found by changing everywhere in (A14) Δx by -Δx.

A2 Two-line theory

(14) and (15) can be derived by substitution of the dynamic equations ($12_{a,\ b}$) in the continuity equations

$$\left.\begin{aligned} \frac{\partial Q_1}{\partial x} + Q_y + D_1\frac{\partial y_1}{\partial t} = 0 \\ \frac{\partial Q_2}{\partial x_2} - Q_y + D_2\frac{\partial y_2}{\partial t} = 0 \end{aligned}\right\} \quad \ldots\ldots \quad (A17)$$

We state $\Delta t = cD(\Delta x)^2/q_{1max}$ and call $q_y.(\Delta x)^2/q_{1max} = q'_y$

in which c is a coefficient to get a stable numerical process. Then the following difference equations are derived

$$y'_1[x,\ t + \Delta t] =$$

$$c.\frac{D_1}{D}\Big\{(\tilde{q}_1[x + \Delta x]/4 + \tilde{q}_1[x] - \tilde{q}_1[x - \Delta x]/4).y^1_1[x + \Delta x,t] +$$

$$+ (D/CD_1 - 2q_1[x] - q^1_y).\ y'_1[x,t] +$$

$$+(-\tilde{q}_1[x + \Delta x]/4 + q_1[x] + q_1[x - \Delta x]/4).\ y'_1[x - \Delta x,t]$$

$$- q'_y\ (y_0[x] - y_2[x,t])\Big\}. \ldots\ldots\ldots \quad (A21)$$

$$y_2[x, t+\Delta t] -$$

$$c.\frac{D}{D_2}\{(y_2[x+\Delta x,t] + y_2[x-\Delta x,t]).q_2/q_{1max} +$$

$$+(D_2/cD - 2q_2/q_{1max} - q'_y)\ y_2[x,t] +$$

$$q'_y\ (y'_1\ [x,t] + y_0[x])\}\ldots \quad \ldots\ldots \quad .(A22)$$

The boundary conditions for y', can be found by substituting x = o and (lee-side)

$$y'_1\ [-\Delta x] = y'_1[\Delta x] \quad \Delta x.\frac{\varphi_\infty}{[h^2]_{x=0}} \quad \ldots\ldots\ldots \ (A23)$$

A3. Coastal constants according to the adapted SVASEK-theory

SVASEK assumes, that the littoral drift between two depth contours is proportional to the longshore component of the loss of energy between these contour lines ((25))

We assume that in the breaker zone cos y = cos for ' and we neglect the influence of the refraction factor K inside the breaker zone

$$Q = A'_1\ .\Delta(H^2C)\ \sin\varphi\cos\varphi \quad \ldots\ldots \quad .(A25)$$

/e assume, that the relations between H and D and between C and according to (21) and (22) on the boundary of the breaker zone remain their validity inside the breakerzone (spilling breaker)

$$H = A_2D \ . \quad ..(A26) \qquad\qquad C = A_3\ \ gD \ .. \quad .(A27)$$

Thus the difference in H^2C between two adjacent depth contours equals

$$\Delta(H^2C) = \frac{d}{dD}\ (A_2^2\ A_3\ g^{\frac{1}{2}}\ D^{2\frac{1}{2}})\ \Delta D = \frac{5}{2}\ \ A_2^2\ A_3\ g^{\frac{1}{2}}\ D^{1\frac{1}{2}}\Delta D \ \ldots (A28)$$

Now first the stationary transport Qo will be computed, according to SVASEK's theory. In this case all contour lines are parallel and Snell's law is valid

$$\frac{\sin \varphi}{\sin \varphi_{br}} = \frac{C}{C_{br}} = \sqrt{\frac{D}{D_{br}}} \quad \ldots \ldots \ldots \ldots \text{(A29)}$$

Substitution of (28) and (29) in (25) gives ΔQ, expressed in D

$$\Delta Q = \frac{5}{2} A_1 \, A_2^2 \, A_3 \, g \quad D^2 \, D_{br}^{-\frac{1}{2}} \Delta D \sin \varphi_{br} \cos \varphi_{br} \quad \ldots \text{(A30)}$$

We find the total transport by integration over the depth. Again we assume $\cos \varphi = \cos \varphi$.

$$Q_o = \int_0^{D_{br}} \Delta Q \; dD = \frac{5}{6} A_1 \, A_2^2 \, A_3 \, g^{\frac{1}{2}} \, D_{br}^{2\frac{1}{2}} \sin \varphi_{br} \cos \varphi_{br} \quad \ldots \text{(A31)}$$

Comparison of (A31) and (23) leads to the conclusion, that for parallel depth contours the relation should exist

$$A_1' = \frac{6}{5} A_1 \quad \ldots \ldots \ldots \ldots \ldots \ldots \text{(A32)}$$

The reason is, that SVASEK multiplies the component of the wave energy with $\sin \varphi$ instead of $\sin \varphi_{br}$ and in the breakerzone $\sin \varphi$ is less than $\sin \varphi_{br}$.
Thus the transport between two depth contours will be, in general, using (A25) (A29) and (A32)

$$\Delta Q = 3 \, A_1 \, A_2^2 \, A_3 \, g^{\frac{1}{2}} \, D^{1\frac{1}{2}} D \sin \varphi_m \cos \varphi_m \quad \ldots \ldots \text{(A33)}$$

In 12 has been considered in detail how the littoral drift changes when the beach and inshore direction change in case of the topography at fig. A5 (cf fig. 14).

Using SVASEK's assumptions and a proper use of Snell's law, for the littoral drift along the inshore is found

$$Q_2 = A_1 \, A_2^2 \, A_3 \, g^{\frac{1}{2}} \left(D_{br}^3 - D_1^3 \right) D_{br}^{-\frac{1}{2}} \sin \varphi_{br} \cos \varphi_{br} \quad \ldots \text{(A34)}$$

and for the transport along the beach

$$Q_1 = A_1 \, A_2^2 \, A_3 \, g^{\frac{1}{2}} \, D_1^{2\frac{1}{2}} \quad \sin \varphi_{10} \cos \varphi_{10} - \frac{\partial y_1}{\partial x} \cos 2\varphi_{10} + \frac{\partial y_2}{\partial x} \left(1 - \tfrac{1}{2} \sin 2\varphi_{10} \cos \varphi_{br}\right) \frac{\cos^2 2\varphi_{10}}{\cos^2 \varphi_{10}} \quad \text{(A35)}$$

in which φ_{10} is the angle of incidence of the wave on the beach (fig. A5), which occurs, when the inshore is parallel with the x-exis

$$\varphi_{10} = \arcsin\left(\sqrt{\frac{D_1}{D_{br}}} \sin \varphi_{br}' \right) \quad \ldots \ldots \ldots \text{(A36)}$$

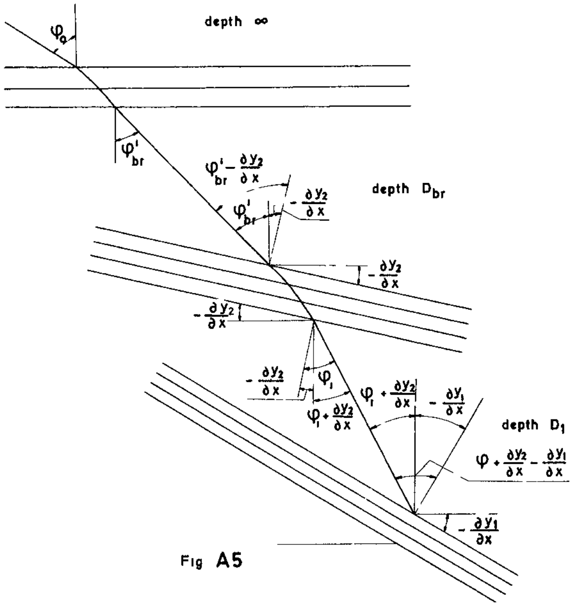


Fig A5

in which φ'_{br} is the breaker angle, if the inshore would be parallel to the x-axis.

For small angle of wave incidence, (A35) can be written

$$Q_1 = A_1 A_2^2 A_3 g^{\frac{1}{2}} D^{2\frac{1}{2}} \left[\sin \varphi_{10} - \frac{\partial y_1}{\partial x} + \frac{\partial y_2}{\partial x} \left(1 - \sqrt{\frac{D_1}{D_{br}}} \right) \right. \quad \text{(A37)}$$

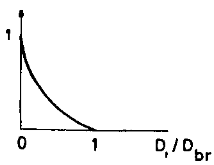


Fig A6
Influence direction inshore / Influence direction beach versus D_1/D_{br}

As would be expected, there is some influence of refraction on the inshore on the transport on the beach. The dynamic equations ($12_{a,b}$) do not account for that. With (A37) we are able to estimate the inaccuracy caused by this neglection. (without taking the curvature of the inshore into account) When the beach and the inshore turn over the same angle, the influence of the direction of the inshore on the transport on the beach is $(1-\sqrt{D_1/D_{br}})$ times the influence of the direction of the beach. This function is shown in fig. A6.

The formulae for q_1 and q_2 can be derived by differentiation of (A34) and (A35) to $\frac{\partial y_2}{\partial x}$ and $\frac{\partial y_1}{\partial x}$ respectively.

For φ_{br} equals

$$\varphi_{br} = \varphi'_{br} - \frac{\partial y_2}{\partial x}$$

Thus the derivative to $\frac{\partial y_2}{\partial x}$ is minus $d\varphi_{br}$

The influence of the inshore on the beach has been neglected.

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