CHAPTER 102

RESONANCE OF MOORED OBJECTS IN WAVE TRAINS

by

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Abstract

In this paper the resonance of a moored object in wave trains is discussed and demonstrated with the help of some results obtained from model tests with a right angled barge
The measured results correspond well with a simple theory which calculates the slowly varying drifting force of regular wave trains.

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1. Introduction

Due to the enormous increase of tanker size during the last years, a lot of harbours could not be adapted quick enough to receive these big ships.
That was, besides other reasons, why it became necessary to load and discharge large tankers in open sea near the coast
This caused a lot of problems, because now, the loading or unloading ship was fully exposed to wind, waves and current
In the course of time many different mooring systems have been developed and tested, such as.

- single buoy mooring
- spread mooring
- turret mooring
- ships moored to piers both rotating and fixed

Of these systems the single buoy mooring is probably the most used.
With this system the ship is moored by means of a bow hawser to a floating buoy.
The buoy itself is anchored to the sea bottom by means of one or more anchor legs.
At the Netherlands Ship Model Basin, extensive studies and model test programs have been executed concerning mooring of ships at sea.
A range of systems have been investigated and tested both as feasibility and as design studies
Normally model tests with moored vessels are conducted in irregular waves
From the results of this type of tests it was found that high peak forces can occur also in light weather conditions.
It appeared that these high forces often occurred after a group of higher waves had passed the ship

These waves gave the moored vessel a horizontal excursion which induced a long periodical oscillating motion.
The period of this oscillating excursion equalled the natural period of the horizontal motion of the moored vessel in still water.
Superimposed on the long periodical excursion, the moored object normally performed a high frequency oscillating sway, surge and yaw motion.
The periods of these motions corresponded to the period of the waves.
Especially when the relationship between the elongation and the tension of a mooring line is progressively non-linear, the resultant horizontal motion could induce rather high peak forces in the mooring lines.
A lot of literature is available concerning the vessel motions due to the higher frequency oscillating wave motion
However, only little information is available about the long periodical phenomena, which are important in studying mooring problems.
Therefore this paper is concerned with that problem

2. The wave drifting force

Considering the moored vessel as a mass spring system with a natural period T_n, it is known that big excursions will be induced by oscillating forces with a period near that natural period.
For the large ships under consideration the natural period of the horizontal motion will be 50 to 200 seconds or more
This indicates that the high frequency exciting forces due to the wave cannot induce important oscillating motions
So there has to be another source of forces
From the theory of Maruo [1] and the experiments of Ogawa [2] the phenomenon is known of the steady drifting force on bodies in regular waves

This steady drifting force per unit length equals:

$$\frac{1}{2} \rho g a^2$$

in which:

ρ = specific mass density of water
g = acceleration due to gravity
a = amplitude of the wave reflected and scattered by the body

For the case that the body is restrained, no waves can be generated by the motions of the body, which means that there is only wave reflection

When the direction of the reflected waves is just opposite to the direction of the incident waves the amplitude of the reflected wave can be written, according to Haskind [3] , for deep water as:

$$a = \zeta_a \quad R$$

in which:

ζ_a = amplitude of incident wave
R = reflection coefficient depending on $k\ T$ (see Figure 5)
k = wave number = $\frac{2\pi}{\lambda}$
λ = wave length
T = draft of body

This expression is exact for deep water and a vertical barrier of infinite length extending a distance T (= draft) below the still water surface

From this it will be clear that when the wave height is not constant but varies slowly with a certain period, also the drifting force will vary with that period.

In other words, when in an irregular sea wave groups are present which encounter a moored body with a frequency in the neighbourhood of the natural period of the mooring system, resonance phenomena may induce large slowly oscillating horizontal motions

In order to check this conception some model tests have been carried out with a simplified single point mooring system

3 Description of the tests

A vessel was moored by means of a single bow hawser to a fixed point

Generally the external conditions may be current, wind and waves, while the ship may have propulsion and steering capacity

However, from the point of simplicity only waves have been taken into consideration

The model represented a right angled barge of 107,000 metric tons displacement to a scale 1 80 (see Figure 1), in which also the main particulars are given for the full size barge

The bow hawser consisted of a linear spring with spring constant C

To avoid that the bow hawser became slack, a counter mass M was used as can be seen in Figure 1

The tests were executed in the Wave and Current Laboratory of the N S.M B which has a length of 60 m and a breadth of 40 m

The water depth was 0 915 m, corresponding to 73 20 meters for the full scale

The barge was positioned with its centre line parallel to the direction of propagation of the waves

In Figure 2 an example is given of the wave trains generated

The period of the wave groups was kept constant during all tests and amounted to about 100 sec

By changing the spring constant of the bow hawser, the natural period of the surge motion of the moored barge could be varied

Tests have been executed with 5 different springs, inducing natural surge periods of 63, 88, 103, 113 and 134 seconds, in a number of wave trains different with regard to the mean period and the significant wave height

The wave train depicted in Figure 2 has a mean period of about 9 sec. and a significant wave height (double amplitude) of about 5 m.

The wave spectrum and the distribution of the wave elevation is given in Figure 4.

During the tests the force in the bow hawser was measured and recorded on magnetic tape and paper chart,

In Figure 2 an example is given of the measured force in the bow hawser for two different springs

From this it will be clear that the forces are much higher for the spring which gives the system a natural period T_n which almost equals the period of the wave groups

It is also easy to distinguish the slowly varying force component on which a high frequency component has been superimposed

The amplitude of the low frequency force has been plotted for each wave train as a function of the ratio

$$\Lambda = \frac{\text{natural period of vessel's surge motion}}{\text{period of wave groups} = 100 \text{ sec}}$$

An example is given in Figure 3 for two wave trains with a mean period of 9 seconds and a significant height of 5 and 8 meters

The same type of resonance curves were found for wave trains with mean periods of 7, 11, 13 and 15 seconds

4. Theoretical considerations

The generated wave trains can be considered to be built-up by two regular waves with different amplitudes and with only a small difference $\Delta\omega$ in frequency
Then the wave elevation ζ can be expressed as·

$$\zeta = b_1 \cos (kx - \omega t) + b_2 \cos (kx + \Delta kx - (\omega + \Delta\omega) t + \Delta\varepsilon)$$

This can be written in a slowly varying form.

$$\zeta = a \cos (kx - \omega t)$$

with $a = b_1 + b_2 e^{i (\Delta kx - \Delta\omega t + \Delta\varepsilon)}$

In which a is the envelope of the wave elevation or the slowly varying amplitude
It was already shown that the drifting force is a function of the square of the wave amplitude a

$$a^2 = b_1{}^2 + b_2{}^2 + 2 b_1 b_2 \cos (\Delta kx - \Delta\omega t + \Delta\varepsilon)$$

Also notice that taking the square of the wave motion ζ , one finds

$$\zeta^2 = \tfrac{1}{2} (b_1{}^2 + b_2{}^2) + b_1 b_2 \cos (\Delta kx - \Delta\omega t + \Delta\varepsilon) + \\ + \text{high frequency components}$$

Assuming that the heave and pitch motion of the barge can be neglected and that the waves will be reflected by the flat bow in a direction parallel to the direction of the incident waves, the drifting force F_d on the barge, according to Haskind [3] , can be expressed as

$$F_d = \tfrac{1}{2}\ \rho\, g\, a^2\, R^2\, B$$

$$B = \text{breadth of barge}$$

in which:

$$a^2 = b_1^{\,2} + b_2^{\,2} + 2\, b_1\, b_2 \cos\,(\Delta kx - \Delta\omega t + \Delta\varepsilon)$$

From this expression it follows that there is a slowly oscillating drift force component with frequency $\Delta\omega$ and amplitude F_{da}.

$$F_{da} = \rho\, g\, R^2\, b_1\, b_2\, B$$

The equation of motion of the mass spring system representing the moored ship can be written as

$$M_x\ x + N_x\ x + Cx = F_d$$

in which

$$M_x\,(\Delta\omega) = \text{total mass of system}$$
$$= M_{ship} + M_{added} + M_{weight}$$

$$N_x\,(\Delta\omega) = \text{damping coefficient}$$
$$C = \text{spring constant}$$
$$F_d = \text{drift force}$$

M_x and N_x are functions of the frequency $\Delta\omega$ of the surge motion and have been determined from extinction tests

Because the force in the bow hawser F_b equals C x the amplitude F_{ba} of the bow hawser force due to the slowly oscillating drifting force component F_{da} cos $(\Delta kx - \Delta\omega t + \Delta\varepsilon)$ can be calculated from the solution of the equation of motion

$$F_{ba} = \frac{F_{da} \quad C}{\sqrt{(C - M_x \quad \Delta\omega^2)^2 + B^2_x \, \Delta\omega^2}}$$

In Figure 3 the results of the calculations for two wave trains, with mean periods of about 9 seconds, have been plotted as a function of the non-dimensional frequency Λ

$$\Lambda = \frac{2\pi \sqrt{\dfrac{M_x}{C_x}}}{T_g}$$

T_g = period of wave groups = 100 sec

The results of the calculations appear to be in a good agreement with the measured ones

For the wave trains with periods of 11 and 13 seconds the difference between measurement and calculation appeared to be larger

5 Discussion of the results

From the results of the tests it is clear that wave groups can induce resonance phenomena at moored ships
It has been illustrated by some simple calculations that the slowly varying drifting force can be determined from the square of the wave motion

Also for irregular waves the square of the wave motion gives information about the square of the irregular slowly varying wave envelop, which is a measure for the drifting force
Therefore, to obtain information about the occurrence of slowly varying drifting forces, one has to calculate the spectral density of the record representing the square of the wave motion
From the distribution of this spectral density in the low frequency range it can be seen whether there will be important drifting force components near the natural period of the moored vessel
This means that describing the sea state only by the spectral density distribution of the wave motion is not sufficient to predict what may happen with a moored vessel at a particular location
What one really needs is the wave motion record itself or the spectral density distribution of the square of the wave motion.

What has been mentioned up to now are the results of a not yet finished study
There are still a lot of items which have to be taken into consideration, like·

- vessel's pitch and heave motions
- influence of bow form
- influence of restricted water depth

However, at this stage of the study two important conclusions have been obtained

- Wave groups can induce resonance phenomena on moored ships
- The normal wave height spectrum does not provide sufficient information to predict the behaviour of the moored vessel.

Wageningen, October 1970

Literature

[1] Maruo, H , "The drift of a body floating on waves"
Journal of Ship Research, Vol 4
Number 3, December 1960

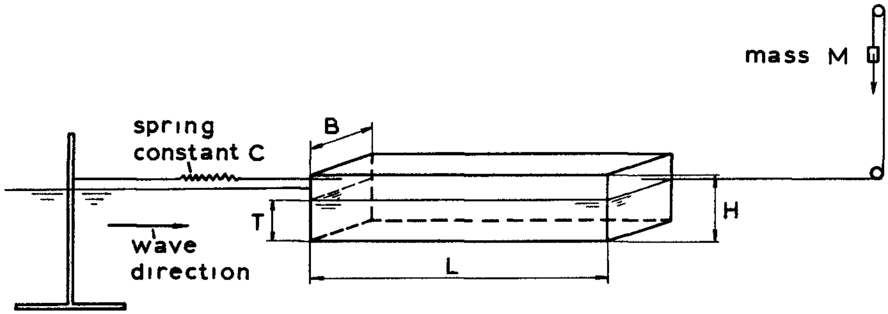
[2] Ogawa, A , "The drifting force and moment on a ship
in oblique regular waves"
Publication no 31 of the Delft Shipbuilding
Laboratory, Holland

[3] Haskind, M D , "The pressure of waves on a barrier"
Inzhen S b. 4, no 2, 147-160 (1948)
see also "Encyclopedia of Physics"
Springer Verlag, Berlin 1960
"Surface waves" of Wehausen and Laitone
sect 17

Test set-up and main particulars of barge

scale 1 80

water depth = 73 20 m



length	L = 182 40	m
breadth	B = 48 96	m
draft	T = 12 00	m
depth	H = 19 20	m
displacement	∇ = 107,163	m^3
counter mass	M = 819	ton

natural period of heave motion ≈ 11 00 sec

natural period of pitch motion ≈ 13 00 sec

natural period of roll motion ≈ 11 35 sec

FIG 1

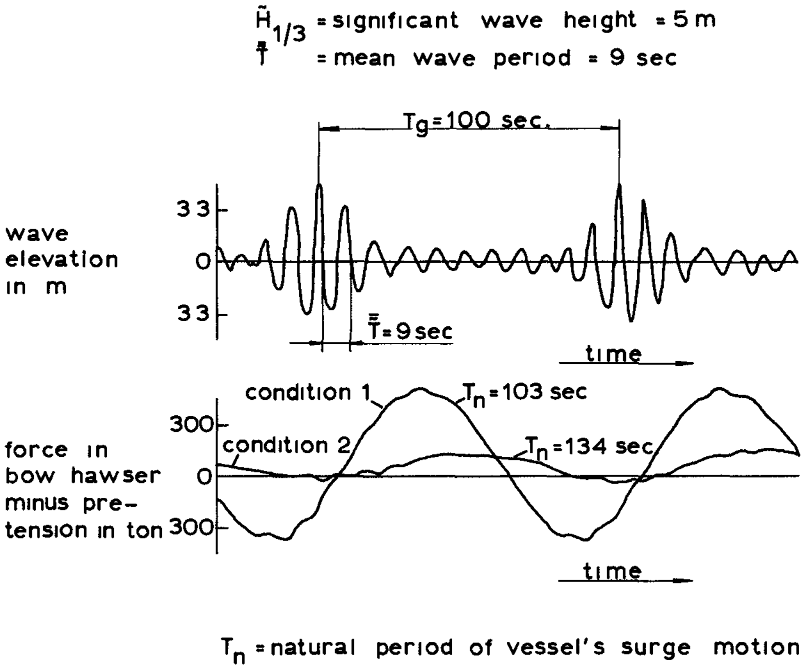
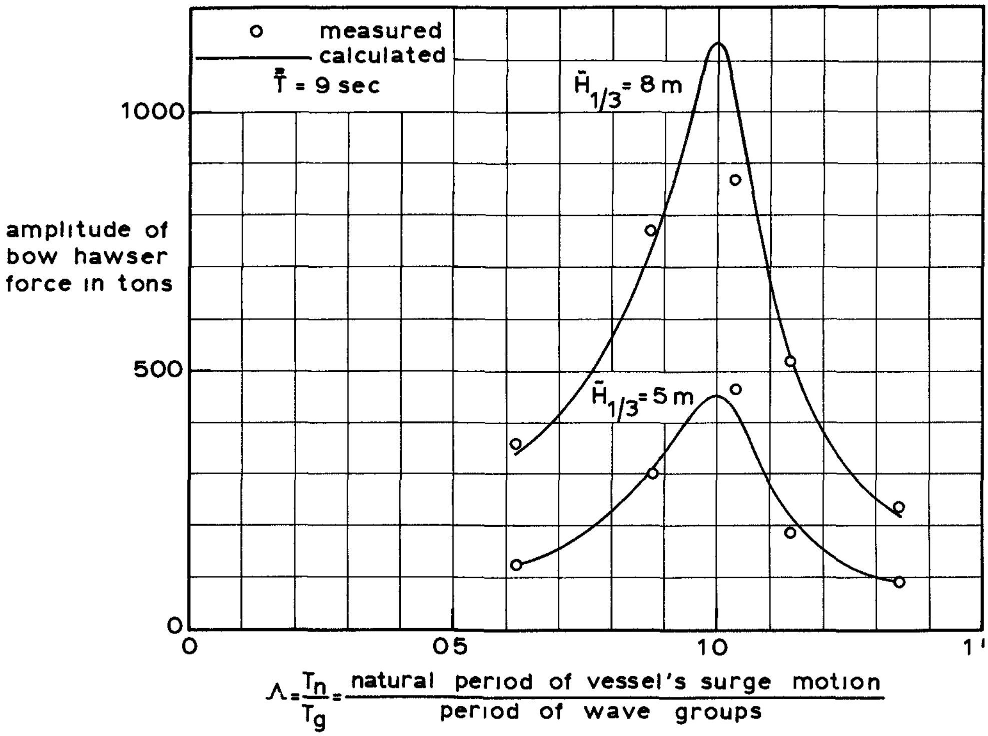

$\tilde{H}_{1/3}$ = significant wave height = 5 m
$\bar{\bar{T}}$ = mean wave period = 9 sec
Tg = 100 sec.
wave
elevation
in m
3.3
0
3.3
$\bar{\bar{T}}$ = 9 sec
time
condition 1
T_n = 103 sec
300
condition 2
T_n = 134 sec
force in
bow hawser
minus pre-
tension in ton
0
300
time
T_n = natural period of vessel's surge motion

FIG 2



o measured
calculated
$\bar{\bar{T}}$ = 9 sec
$\tilde{H}_{1/3}$ = 8 m
$\tilde{H}_{1/3}$ = 5 m
1000
500
0
amplitude of bow hawser force in tons
0
0 5
1 0
1 '
$\Lambda = \frac{T_n}{T_g} = \frac{\text{natural period of vessel's surge motion}}{\text{period of wave groups}}$

FIG 3

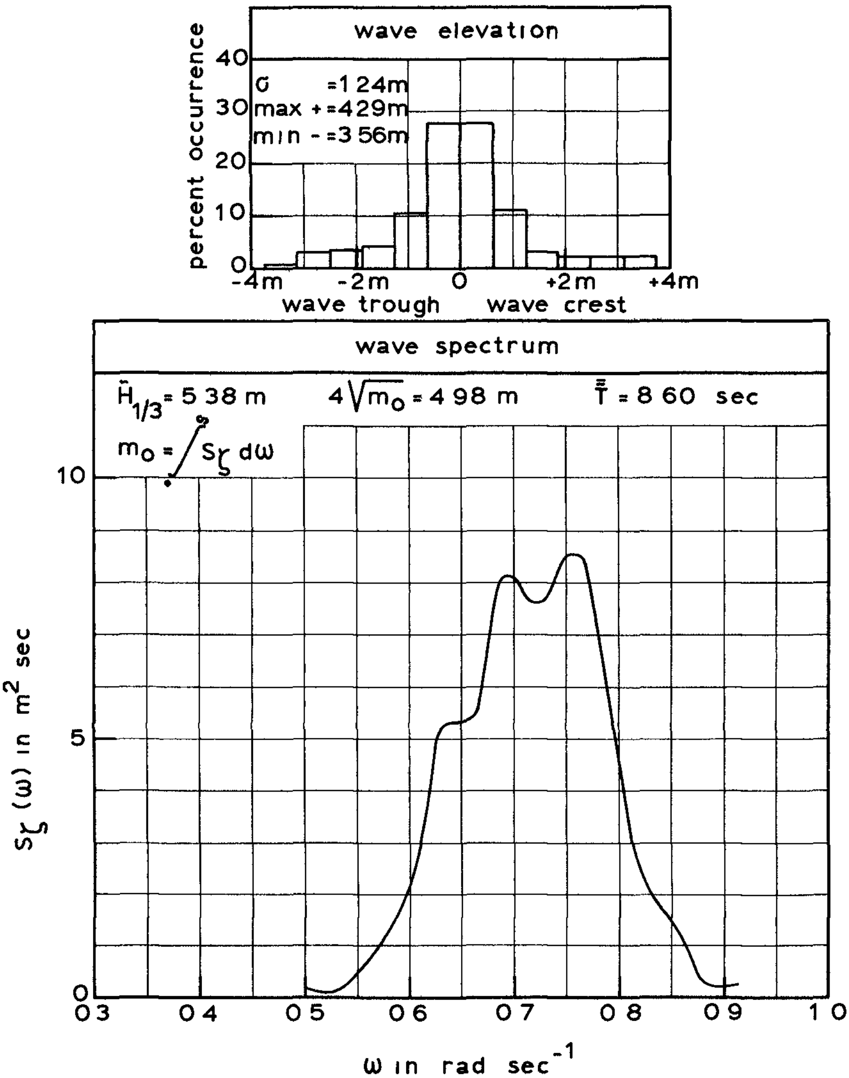

wave elevation
percent occurrence
40
30
20
10
0
σ =1 24m
max + =429m
min - =3 56m
-4m
-2m
0
+2m
+4m
wave trough
wave crest
wave spectrum
$\bar{H}_{1/3}$ = 5 38 m
$4\sqrt{m_0}$ = 4 98 m
$\bar{\bar{T}}$ = 8 60 sec
$m_0 = \int_0^\infty S_\zeta \, d\omega$
$S_\zeta(\omega)$ in m^2 sec
10
5
0
0 3
0 4
0 5
0 6
0 7
0 8
0 9
1 0
ω in rad sec^{-1}

FIG 4

Reflection coefficient R as a function of kT

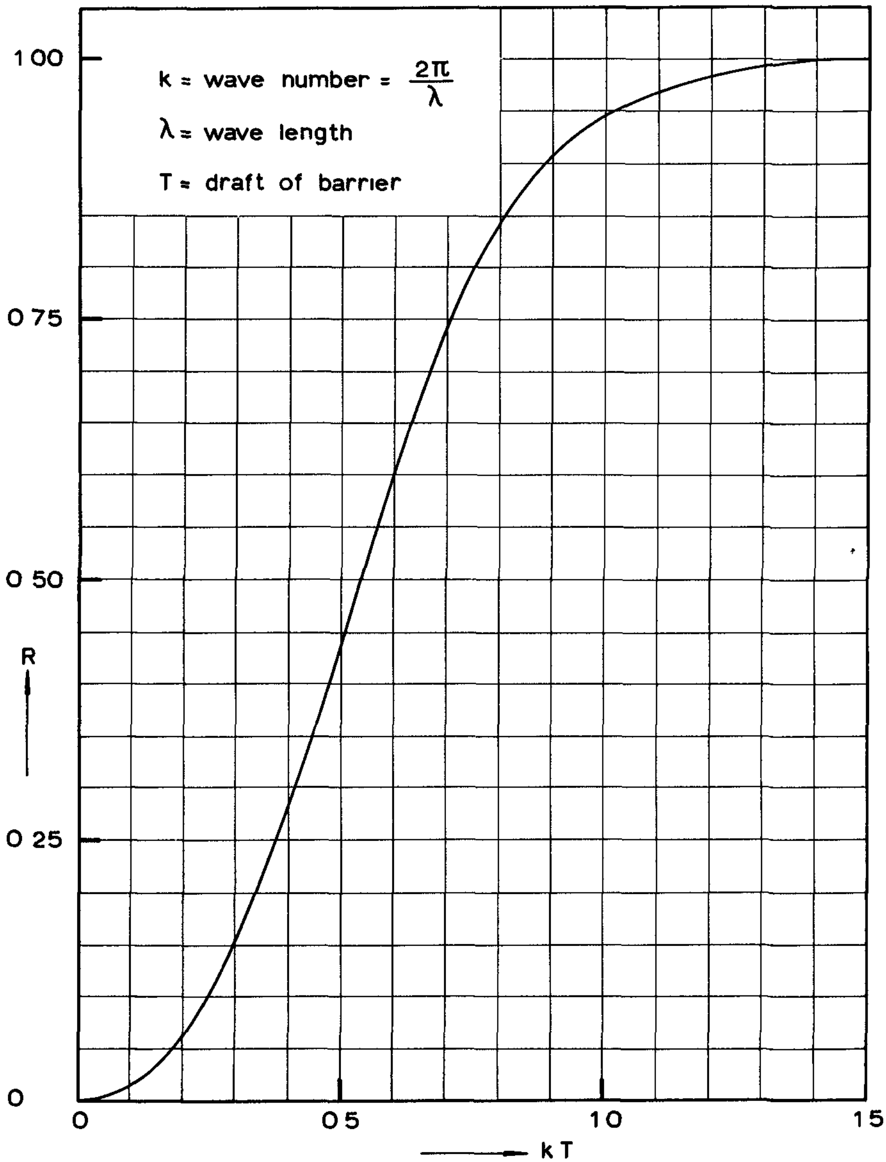


FIG 5