CHAPTER 110

RECIRCULATION IN SHALLOW BAYS AND RIVERS

by

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ABSTRACT

A theoretical model is presented to provide an approximation of the water and temperature recirculation in a shallow bay, lake or river, between the outlet and inlet canals of the cooling system for a power plant In particular, a temperature recirculation factor relating the outlet and inlet temperatures is derived

INTRODUCTION

Since the efficiency of a power plant depends critically on a low intake water temperature, an injudicious siting of the discharge canal terminus may cause recirculation of the discharge water, with a consequent increase in the temperature of the intake water Thus a prior knowledge of the effect of particular locations for the terminus of the discharge canal is important

The quantitative evaluation of the siting of the outlet and inlet canals, taking into account tides, meteorological conditions, the diffusion and convection of momentum and temperature, and the stratification due to temperature and salinity, is a difficult problem The complexity of the boundary conditions and governing equations for the system requires the use of numerical modeling The recent work of Wada (1966, 1967, 1968) has clarified many of the problems involved and given some solutions to particular models

The following work presupposes that steady conditions prevail and that, to the first order, temperature variations have no effect on the flow This rather crude assumption can be partially strengthened by considering the receiving water system to be shallow and by assuming that the turbulence in the flow is sufficient to ensure that the mixing of the water is complete throughout the water column These assumptions and conditions thus avoid the more difficult problems of variations in time, and vertical stratification of temperature

To simplify the analysis, the equations governing the flow in the bay are linearized This procedure is reasonable except near the outlet for the plant cooling system where the non-linear convection of momentum terms may well be important We assume that the flow in the neighborhood of the outlet can be treated as a source rather than as a jet, whence the convection terms can be neglected

For steady flow, the equations of momentum and continuity can be written as

$$-fv = -g\frac{\partial\eta}{\partial x} + \frac{\tau^{sx}}{\rho h} - \frac{\kappa\tilde{U}}{h}u \qquad (1)$$

$$fu = -g\frac{\partial\eta}{\partial y} + \frac{\tau^{sy}}{\rho h} - \frac{\kappa\tilde{U}}{h}v \qquad (2)$$

and

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \qquad (3)$$

Here u and v are the water particle velocities in the x and y directions respectively, η is the displacement of the free surface from its mean level, τ^{sx} and τ^{sy} are the stresses exerted by the wind on the flow, h is the mean depth, f is the Coriolis parameter, κ is related to the Chezy bottom stress coefficient, C, by $\kappa = g/C^2$, $\tilde{U}$ is a scale velocity and g is the acceleration due to gravity

The linearization of equations (1) and (2) involved firstly the neglect of the non-linear inertia terms, as commented above, and also the adoption of a linear form for the bottom stress terms The linearization of these latter terms has been shown to be reasonable for oscillating flows such as tides in estuaries However, the use of the linear form for steady flows has not been so readily confirmed It is used here because the basic equations can be written in a particularly useful form and also, the resulting flows appear to give a good representation of the physical situation Equations (1), (2) and (3) also require $\eta \ll h$

From equation (3), a stream function ψ can be defined such that

$$u = -\frac{1}{h}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = \frac{1}{h}\frac{\partial\psi}{\partial x} \qquad (4)$$

By eliminating η between (1) and (2),

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = \frac{2}{h}\left(\frac{\partial\psi}{\partial x}\frac{\partial h}{\partial x} + \frac{\partial\psi}{\partial y}\frac{\partial h}{\partial y}\right) + \frac{f}{\kappa\tilde{U}}\left(\frac{\partial\psi}{\partial x}\frac{\partial h}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial h}{\partial x}\right) - \frac{h^2}{\rho\kappa\tilde{U}}\left\{\frac{\partial}{\partial y}\left(\frac{\tau^{sx}}{h}\right) - \frac{\partial}{\partial x}\left(\frac{\tau^{sy}}{h}\right)\right\} \qquad (5)$$

Equation (5) takes on a particularly simple form if h is uniform and $\tau^{sx} = \tau^{sy} = 0$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = 0 \qquad (6)$$

In this case the streamlines are given by potential flow theory, whence sources and sinks can readily be modeled This method is used as the basis for the study of recirculation of the water between the outlet and inlet canals The extension of this type of model in the case of a variable depth and a non-zero wind stress requires the use of numerical solutions of (5)

WATER RECIRCULATION

Consider inlet and outlet canals embedded in a straight coastline bordering a receiving water system which covers the half-plane and has a uniform depth If the distance between the outlet and inlet canals is 2a, then the complex velocity potential for the flow in the receiving system is

$$w \equiv \phi + i\psi = - \mu \log \frac{z - a}{z + a} - \bar{U}z \tag{7}$$

where μ is the strength of the source - sink flow, $z \equiv x + iy$, the source is at $z = a$ and the sink at $z = -a$, and $\bar{U}$ is the speed of a uniform steady current along the coast, cf Figures 1 and 2 This current can be regarded as a steady river flow, or a "steady state" tidal current In either case, $\bar{U}$ may be a function of time t, subject to the condition that the time scale T for changes in $\bar{U}$ are large compared with $h/\kappa\bar{U}$ and $2a/\tilde{U}$ If $\tilde{U}$ = 1 0 ft/sec , the (varying) tidal amplitude is 1 5 ft and κ = 002, then for T $\approx$ 12 hrs , it can be shown that h must be less than approximately 14 ft Further, a must be less than 33,000 ft These are fairly strict conditions on h and a The conditions may be realistic only for a few situations

From equation (7), the non-dimensional form for the streamlines is

$$x'^2 = 1 - y'^2 - 2y' \cot(\bar{U}'y' + \psi') \tag{8}$$

where $\bar{U}' \equiv \bar{U}a/\mu$, $\psi' \equiv \psi/\mu$ is the dimensionless value for the streamline, and $x' = x/a$, $y' = y/a$ It can be shown from equation (8) that the percentage recirculation of water is 100% for a zero current ($\bar{U}' = 0$) or for a current aided recirculation ($\bar{U}' < 0$) However, for an opposing current ($\bar{U}' > 0$), the percentage circulation is reduced Figure 3 shows the percentage recirculation of water for aiding and opposing non-dimensionalized currents It can be seen that for an opposing current $\bar{U}'$ greater than 2 0, there will be no recirculating water

Other elementary shaped coastlines can be treated by the technique of comformal mapping In the general situation, equation (5) can readily be solved numerically Further details on this technique will be discussed by Price (1971) Figure 4 gives the streamlines in an idealized model of a bay connected to the ocean This model includes variations in the depth and coastline configuration and permits an inflow of water across two of the boundaries

It is of interest to point out that uniform depth flows can be modeled experimentally using a Hele-Shaw apparatus A Hele-Shaw model has been used at the University of Florida to determine the streamline patterns for flow into a bay

THERMAL RECIRCULATION

The temperature difference between the outlet and inlet canal of a cooling system depends, in part, on the decay of temperature in the recirculating fluid with distance from the outlet It should be noted that mixing effects are not included in this treatment Under the assumption that there is no heat flow through the bottom or the fixed side boundaries, heat can only be lost or gained by the recirculating water in exchange with the surrounding receiving water or with the atmosphere We assume that the diffusion of temperature in the receiving system is negligible compared with the convection of temperature Thus the primary mechanism for extracting heat from the water is assumed to be via the atmosphere

The components of heat loss (or gain) from the surface of a body of water include

Q_s solar radiation by the sun

Q_b back radiation from the water to the atmosphere

Q_c heat loss due to convection

Q_e heat loss due to evaporation

Q_r heat advection by rain or other inflows

The " "s refer to differentiation with respect to time t Consider the anomalous heat flux due to the heated water, that is

$$Q_{net} = Q_E + Q'$$

$$T_w = T_{w_E} + T_w'$$

where the primes denote incremental heat fluxes due to the increased water temperature T_w' The quantities Q_E and T_{w_E} are defined such that the water body is in equilibrium (in the average sense) for $T_w = T_{w_E}$ This implies that $Q_E = 0$

The heat flux budget can be written as

$$Q' = Q_s - Q_b - Q_c - Q_e + Q_r \tag{9}$$

where the various components of heat flux are defined in the following equations

$$\begin{aligned} Q_s &= (1 - r)\, Q_m \\ \dot{Q}_b &= 0\ 97\ \sigma\ (\theta^4{}_w - \beta\theta_a{}^4) \\ Q_c &= 0\ 00407\ PW\ (T_w - T_a) \\ Q_e &= 12\ W(e_w - e_a) \end{aligned} \tag{10}$$

$$Q_r = M_r C_{w_r} (T_r - T_w)$$

here

Q_m = the solar radiation on a horizontal surface,

r = the reflectivity, assumed to be 0 05,

σ = the Stefan-Boltzmann radiation constant = 1 714 x 10^{-9} BTU/hr/ft^2/$^\circ$R^4 ,

θ_w = absolute water temperature in $^\circ$R (T($^\circ$F) + 459 69),

θ_a = absolute air temperature,

β = radiation factor, depending on cloud cover, vapor pressure, etc An average value of β is 0 85,

W = wind speed in knots,

P = atmospheric pressure in inches of mercury,

e_w = vapor pressure of water in saturated air at the temperature of water, in inches of mercury (For temperature ranges of interest, $e_w \approx 0\ 045T_w - 2\ 6$),

e_a = vapor pressure of water in air, in inches of mercury ($e_a \approx e_w$x relative humidity),

M_r = mass rate of rainfall,

c_r = specific heat of rainwater,

T_r = temperature of the rainwater,

and

T_a = temperature of the air

We assume for the purposes of this paper, that $Q_r = 0$ By utilizing the various definitions for the Q's from (10) and substituting in equation (9), then the incremental heat flux Q', due to the water temperature increased an amount T_w' above the equilibrium temperature, is approximately

$$Q' \text{ (BTU/ft}^2\text{/hr)} = \{1\ 04 + 0\ 66\ W\}\ T_w' \qquad (11)$$

Thus if Q is the amount of heat above ambient in a volume of water with unit surface area

$$\frac{dQ}{dt} = Q' = -KT'_w \qquad (12)$$

where $K = 1.04 + 0.66W$. K can be called the heat transfer coefficient, and as such includes the effects of radiation, convection and evaporation as outlined above

$$Q = \gamma h c_p T_w' \tag{13}$$

where γ is the specific weight of water with heat capacity c_p. Thus from equation (13)

$$\frac{dQ}{dt} = \gamma h c_p \frac{dT_w'}{dt} = -KT_w' \tag{14}$$

To examine the relation between the flow and temperature fields, consider Figure 5. If the flow is divided into N streamtubes of equal flow Δq, then it can be seen that for the flow between two particular streamlines is

$$\Delta q = h\ell \; ds/dt \tag{15}$$

where ℓ is the spacing normal to the streamlines, and ds/dt is the local water particle velocity parallel to the local streamline. Using equations (14) and (15)

$$\frac{dT_w'}{ds} = -\frac{K\ell T_w'}{\gamma c_p \Delta q} \tag{16}$$

The integration of equation (16) gives

$$\ell n \; T_{w_i} = -\frac{KA_j}{\gamma c_p \Delta q} + \ell n \; T_{w_o}$$

where T_{w_i} and T_{w_o} are the inlet and outlet temperatures respectively for the flow between the $(j-1)^{th}$ and the j^{th} streamlines, and A_j is the surface area between the two streamlines. By summing over the N streamlines and averaging the thermal recirculation is expressed by

$$C_R = \frac{\overline{T_{w_i}}}{T_{w_o}} = \frac{1}{N} \sum_{j=1}^{N} e^{-\beta A_j' N} \tag{17}$$

where

$$\beta = \frac{Ka^2}{\gamma c_p q} \tag{18}$$

The A_j''s are the non-dimensional areas between the streamlines (i e $A_j' = A_j/a^2$), where 2a is the distance between the outlet and inlet), and q is the total volume recirculating ($q = \pi\mu h$) C_R can be termed the temperature recirculation factor It can be seen that C_R depends on the sum of exponential terms, and of course must be less than unity

A numerical evaluation of C_R in the case when the equations of the streamlines are known can be carried out on a computer In the case of the source and sink embedded in the boundary of the half plane, with β = 0 78 and $\bar{U}'$ = - 3 75,(an aiding current and 100% water recirculation), C_R = 43%, with only 50% recirculation of the outlet water ($\bar{U}'$ = 0 326) and β = 0 78, C_R = 12% In this example N = 10, cf Figure 6

The extension of the technique to more complicated flows as considered briefly in Figure 4 is straightforward For example, one can divide the flow from the outlet using 9 interior streamlines, whence N = 10 The areas A_j can readily be found from graph paper when the streamlines have been plotted For a representative value of K and the proper dimensions and discharge, β can be calculated (equation 18), and the temperature recirculation determined from equation (17)

CONCLUSION

The analysis presented here can readily be used to obtain an estimate of the temperature circulation for particular locations of the inlet and outlet canals for a power plant located by a shallow bay, lake or river which is being used in the cooling system A numerical solution of equation (5) can be used for particular cases, including the effects of prevailing wind stresses as well as an arbitrary bottom topography

The limitations of the above technique are obvious due to neglecting temperature gradients, diffusion, and any unsteady feature of the flow Also there may be some doubt about the validity of the linear form for the bottom stress in the case of large variations in the velocity of the water in the receiving system However, despite these limitations, the authors have found that the technique provides a realistic approximation of the temperature recirculation

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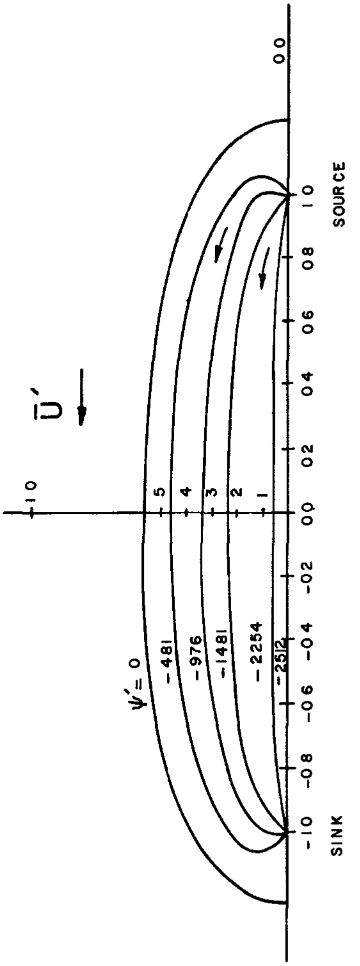

FIGURE 1· 100% RECIRCULATION WITH $\overline{U}'=-3\ 75$

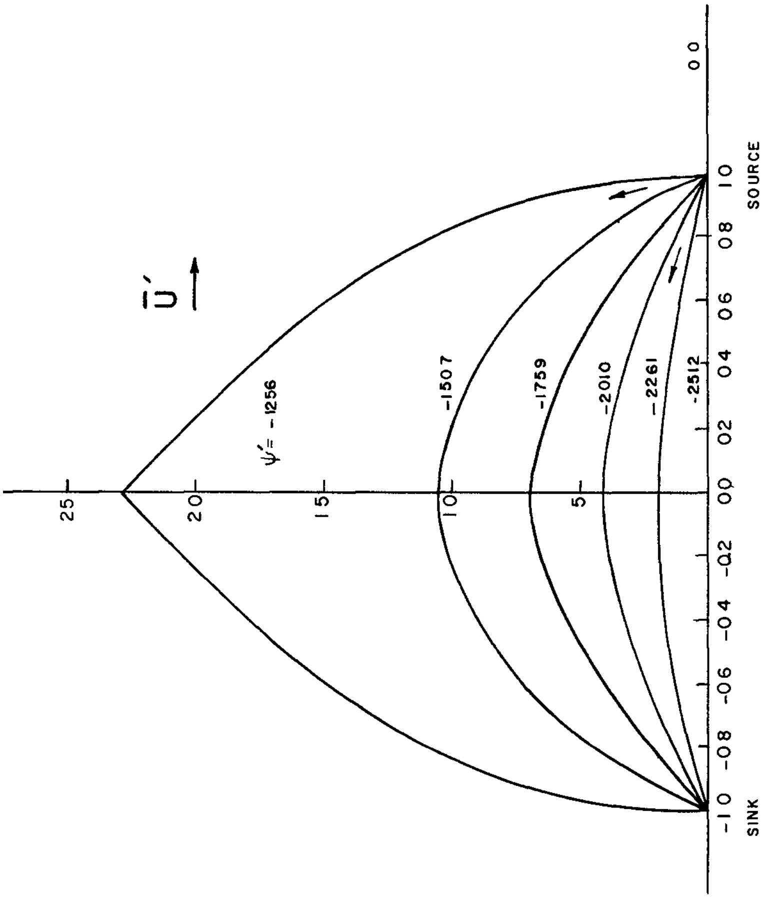

FIGURE 2 50% RECIRCULATION WITH $\bar{U}' = 0.325$

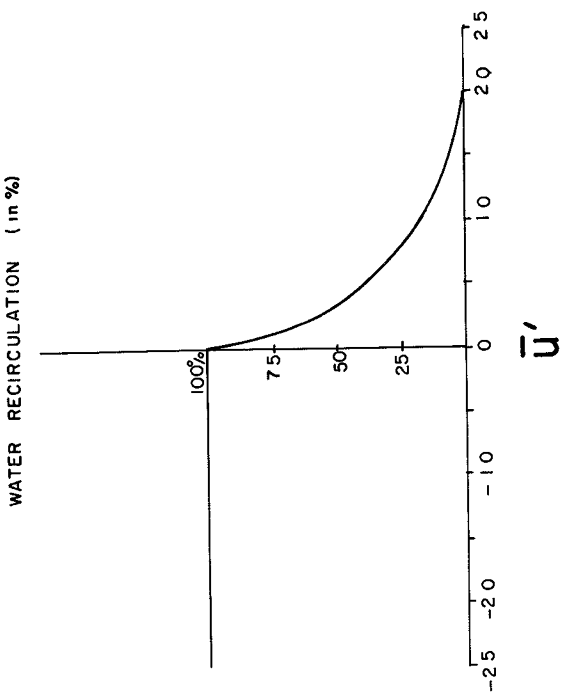

FIGURE 3 PERCENTAGE OF RECIRCULATED WATER FOR AIDING AND OPPOSING NONDIMENSIONAL UNIFORM CURRENTS

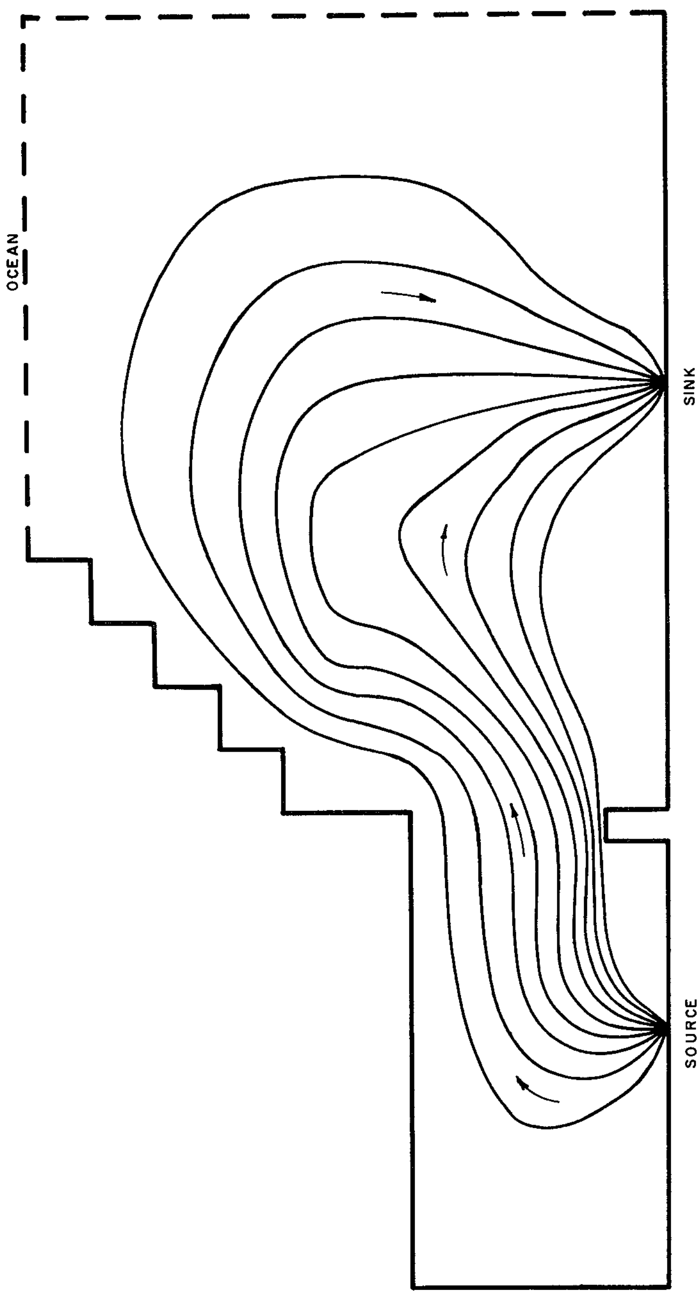

FIGURE 4 STREAMLINES IN AN IDEALIZED BAY

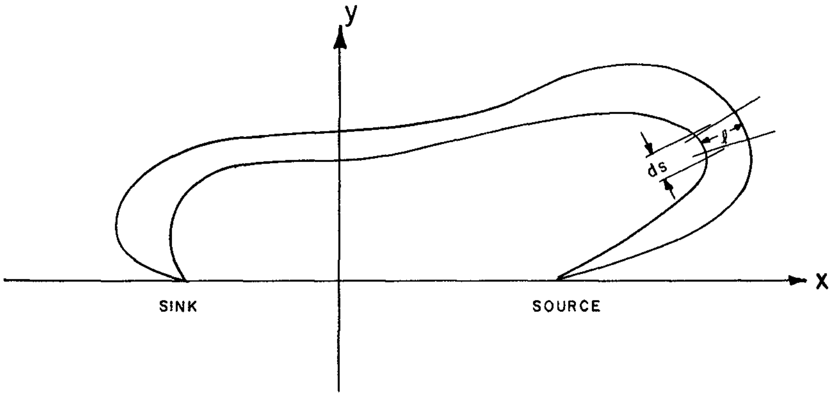

FIGURE 5 DEFINITION SKETCH

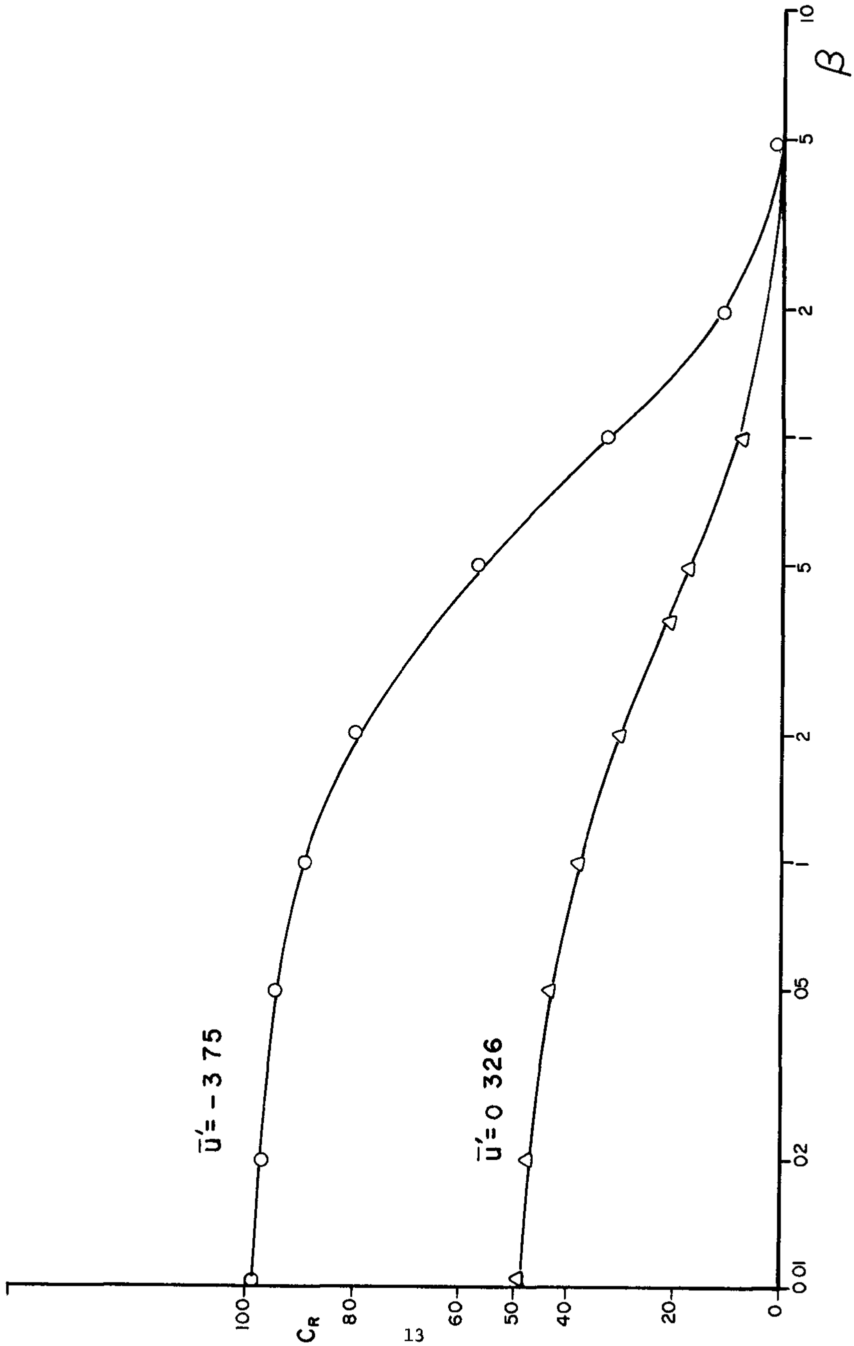

FIGURE 6 C_R VS β FOR TWO CASES OF FIGS 1 & 2