CHAPTER 131

RESONANCE IN HARBORS OF ARBITRARY SHAPE

by Jiin-Jen Lee,* Visiting Assistant Professor of Civil Engineering,
University of Southern California Los Angeles, California, U S A
and
Fredric Raichlen Associate Professor of Civil Engineering,
W M Keck Laboratory of Hydraulics & Water Resources
California Institute of Technology Pasadena California U S A

ABSTRACT

A theory is presented for analyzing the wave induced oscillations in an arbitrary shape harbor with constant depth which is connected to the open-sea The solution is formulated as an integral equation which is then approximated by a matrix equation The final solution is obtained by equating, at the harbor entrance the wave amplitude and its normal derivative obtained from the solutions for the regions outside and inside the harbor

The results of experiments conducted using a harbor model of the East and West Basins of Long Beach Harbor (Long Beach, California) are presented and compared to the theory Good agreement has been found between the theory and experiments

INTRODUCTION

Experience has shown that a natural or an artificial harbor can be excited by incident waves from the open-sea in such a way that large water surface oscillations can result for certain wave periods This phenomena of resonance is similar to the dynamic response of mechanical or acoustical systems when exposed to time-varying forces, pressures, or displacements

Resonant oscillations in harbors (also termed seiche and harbor surging) can cause significant damage to moored ships and marine structures especially if the resonant period is close to that of the ship-mooring system In addition the currents induced by these oscillations can create navigation hazards Such resonant oscillations have occurred at many locations around the world and have damaged moored ships and dockside facilities, e g Table Bay Harbor Cape Town, South Africa In order to correct an existing resonance problem or design a harbor which is free of such problems one must be able to predict the resonant frequencies and the expected wave amplitude at various locations within the harbor as a function of the wave period

Many previous investigators have studied various aspect of the harbor resonance problem McNown (1952) studied seiches in circular harbors with small entrances by assuming an antinode occurred at the harbor entrance when the harbor was in resonance A similar method was applied to rectangular harbors by Kravtchenko and McNown (1955) Thus, due to this assumption the resonant periods were identical to those of the eigenvalues for the free oscillations of a completely closed circular (or rectangular) basin The problem of a rectangular harbor connected directly to the open-sea was investigated by Miles and Munk (1961) They included the effect of the wave radiation from the harbor

* Formerly Research Fellow in Civil Engineering W M Keck Labr of Hydr & Water Res , California Institute of Technology, Pasadena, Calif , U S A

mouth to the open-sea thereby limiting the maximum wave amplitude within the harbor for the inviscid case to a finite value even at resonance Ippen and Goda (1963) also studied the problem of a rectangular harbor connected to the open-sea by using the Fourier transformation method to evaluate the wave radiation from the entrance to the open-sea good agreement was found by them between the theory and experiments

In recent years studies on more complicated harbor geometries have been conducted Wilson Hendrickson and Kilmer (1965) studied the oscillations induced by long waves in a basin with a complicated shape and variable depth incorporating an assumption of a nodal line (a line of zero amplitude) at the harbor entrance Leendertse (1967) developed a finite-difference numerical scheme for the propagation of long-period waves in an arbitrary shape basin of variable depth given the water surface elevations at the open boundary Recently, Hwang and Tuck (1970) in a study independent of the authors' study investigated the wave induced oscillations in an arbitrary shape basin with constant depth connected to the open-sea Their approach was to superimpose scattered waves which were computed using a distribution of sources on the standing wave system In their analysis the calculation of the source strengths along the entire reflecting boundary must be terminated at some distance from the entrance the location of such a termination is not obvious unless trial solutions are made

The present theoretical analysis is developed by applying Weber's solution of the Helmholtz equation in both the regions inside and outside the harbor with the final solution obtained by matching the wave amplitudes and their normal derivatives at the harbor entrance In this way some of the problems of application which other investigators have experienced are eliminated since only the wave characteristics at infinity in the open-sea need be specified to obtain a complete solution Experiments were performed in the laboratory to verify the theoretical solutions

THEORETICAL ANALYSIS

For the theoretical analysis the flow is assumed irrotational and the fluid incompressible thus one can define a velocity potential Φ such that the fluid particle velocity can be expressed as a vector as $\vec{u} = \nabla\Phi$, where ∇ is the gradient

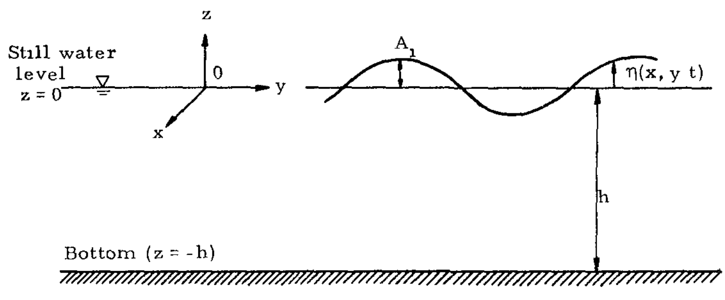

Fig 1 Definition sketch of the coordinate system

operator (The coordinate system is defined in Fig 1) The velocity potential Φ must satisfy Laplace's equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \qquad (1)$$

and a number of prescribed boundary conditions one of these is that the fluid does not penetrate the solid boundaries which define the limits of the domain of interest i e $\partial\Phi/\partial n = 0$ on solid boundaries (where n is normal to the boundary and directed outward)

If the water depth is considered constant the method of separation of variables can be used to obtain the function which represents the depthwise variation of the velocity potential Φ Thus, within the limitation of small amplitude water wave theory the following form of the velocity potential Φ is found

$$\Phi(x\ y\ z\ t) = \frac{1}{i\sigma}\ \frac{A_1\ g\ \cosh k(h+z)}{\cosh kh}\ f(x\ y)\ e^{-i\sigma t} \qquad (2)$$

where σ is the angular frequency defined as $2\pi/T$ (T is the wave period) $i = \sqrt{-1}$, A_1 is the wave amplitude at the crest of the incident wave h is the water depth (assumed constant), g is the acceleration of gravity k is the wave number defined as $2\pi/L$ (L is the wave length), and the function $f(x, y)$ termed the wave function describes the variation of Φ in the x-y directions The function $f(x\ y)$ must satisfy the Helmholtz equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0 \qquad (3)$$

To complete the expression for the velocity potential Φ the remaining problem is to determine the function $f(x\ y)$ which satisfies Eq 3 (known as the Helmholtz equation) and the following boundary conditions

(i) $\partial f/\partial n = 0$ along all fixed boundaries such as the coastline and the boundary of the harbor

(ii) as $\sqrt{x^2 + y^2} \to \infty$ there is no effect of the harbor on the wave system this is termed the radiation condition Mathematically the radiation condition is necessary to ensure a unique solution of the function $f(x\ y)$ in the unbounded domain

A method for solving the Helmholtz equation Eq 3 for an arbitrary shape harbor is presented in the following The domain of interest is divided at the harbor entrance into two regions as shown in Fig 2 the infinite ocean (Region I) and the region bounded by the limits of the harbor (Region II) For convenience the function f of Eq 3 in Region II is denoted as f_2 and in Region I it is denoted as f_1

In Region II the function f_2 that satisfies the Helmholtz equation at any position $\vec{x}$ inside the harbor can be expressed as the following line integral

$$f_2(\vec{x}) = -\frac{i}{4}\int_s \left\{ f_2(\vec{x}_o)\frac{\partial}{\partial n}[H_o^{(1)}(kr)] - H_o^{(1)}(kr)\frac{\partial}{\partial n} f_2(\vec{x}_o)\right\} ds(\vec{x}_o) \qquad (4)$$

where $\vec{x}_o$ is the position vector of the boundary point r is the distance $|\vec{x}-\vec{x}_o|$ and n is normal to the boundary and is directed outward The function

$H_o^{(1)}(kr) = J_o(kr) + iY_o(kr)$ and is termed the Hankel function of the first kind and zeroth order. The integration in Eq 4 is to be performed along the boundary of the harbor traveling in a counter-clockwise direction.

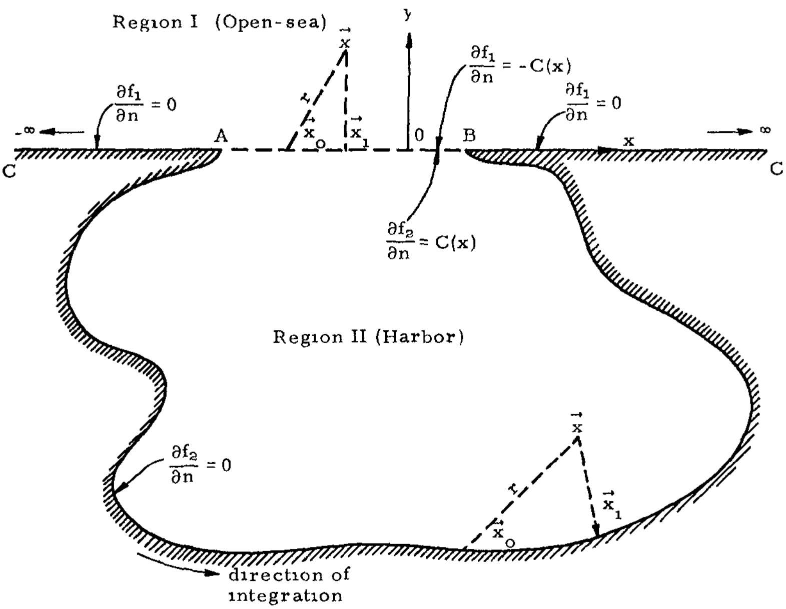

Fig 2 Definition sketch of an arbitrary shape harbor

Eq 4 indicates that if one knows the value of f_2 and $\partial f_2/\partial n$ at the harbor boundary the function f_2 at any position $\vec{x}$ inside the harbor can be obtained readily. However, the value of f_2 at the boundary (including the harbor entrance) is not known at this stage of the development; the value of $\partial f_2/\partial n$ at the harbor entrance is also not known although it is known that $\partial f_2/\partial n$ is zero on all solid boundaries within the harbor. In order to evaluate $f_2(\vec{x}_o)$ at the boundary as a function of the value of $\partial f_2/\partial n$ at the harbor entrance the field point $\vec{x}$ is allowed to approach a boundary point $\vec{x}_1$. If the boundary is sectionally smooth an integral equation is obtained as follows

$$f_2(\vec{x}_1) = -\frac{i}{2}\int_s [f_2(\vec{x}_o)\frac{\partial}{\partial n}H_o^{(1)}(kr) - H_o^{(1)}(kr)\frac{\partial f_2}{\partial n}(\vec{x}_o)]\, ds(\vec{x}_o) \qquad (5)$$

where $r = |\vec{x}_o - \vec{x}_1|$. Although the exact solution of the integral equation (Eq 5) is difficult to obtain it is possible to obtain an approximate solution of Eq 5 by converting this integral equation into a matrix equation This is accomplished by dividing the boundary into a sufficiently large number of segments (N) and the value of f_2 (or $\partial f_2/\partial n$) at each boundary segment is considered constant and equal to the value at the mid-point of each segment Thus Eq 5 can be approximated by the following matrix equation

$$(F)_i = -\frac{i}{2}\left[\sum_{j=1}^{N} (G_n)_{ij}(F)_j - \sum_{j=1}^{N} (G)_{ij}(F_n)_j\right], \qquad (6)$$

$$\text{for } i = 1\ 2 \qquad N$$

using the notation

$$(F)_i = f_2(\vec{x}_i)\ , \qquad i = 1, 2 \qquad N \qquad (7a)$$

$$(G_n)_{ij} = -kH_1^{(1)}(kr_{ij})\frac{\partial r_{ij}}{\partial n}\Delta s_j$$

$$= -k[J_1(kr_{ij}) + iY_1(kr_{ij})]\left[-\frac{x_i - x_j}{r_{ij}}\left(\frac{\partial y}{\partial s}\right)_j + \frac{y_i - y_j}{r_{ij}}\left(\frac{\partial x}{\partial s}\right)_j\right]\Delta s_j\ ,$$

$$i, j = 1, 2 \qquad N \quad i \neq j \qquad (7b)$$

$$(G_n)_{ii} = 2\int_0^{\frac{1}{2}\Delta s_i}\left(-kH_1^{(1)}(kr)\frac{\partial r}{\partial n}\right)dr$$

$$\approx \frac{i}{\pi}\left(\frac{\partial x}{\partial s}\frac{\partial^2 y}{\partial s^2} - \frac{\partial^2 x}{\partial s^2}\frac{\partial y}{\partial s}\right)_i \Delta s_i\ , \qquad i = 1, 2 \qquad N \qquad (7c)$$

$$(G)_{ij} = H_0^{(1)}(kr_{ij})\Delta s_j\ , \qquad i\ j = 1, 2, \qquad N \quad i \neq j \qquad (7d)$$

$$(F_n)_i = \partial f_2/\partial n(\vec{x}_i), \qquad i = 1\ 2, \qquad N \qquad (7e)$$

$$(G)_{ii} = 2\int_0^{\frac{1}{2}\Delta s_i} H_0^{(1)}(kr)\, dr$$

$$\approx \left[1 + i\frac{2}{\pi}\left[\log\frac{k\Delta s_i}{4} - 0\ 42278\right]\right]\Delta s_i \qquad i = 1\ 2 \qquad N \qquad (7f)$$

The vector F_n in Eq 6 involves the unknown value of $\partial f_2/\partial n$ at the harbor entrance as well as the value of $\partial f_2/\partial n$ at the solid boundaries (these latter values are zero) Thus the vector F_n can be represented as

$$F_n \quad \sum_{j=1}^{p} \delta_{ij} C_j = U\underline{C} \qquad (8)$$

where p is the total number of segments into which the harbor entrance is divided the vector $\underline{C}$ represents the p unknown values of $\partial f_2/\partial n$ at the mid-point of each entrance segment, and the matrix

$U = \delta_{ij} = \begin{cases} 0 \text{ for } i \neq j \\ 1 \text{ for } i = j \end{cases}$ (with the index i=1 2 N and the index j=1, 2 p)

Substituting Eq 8 into Eq 6 and rearranging one obtains

$$f_2(\vec{x}_i) - \sum_{j=1}^{p} M_{ij} C_j = M\underline{C} \qquad (\text{for } i=1\ 2 \qquad N) , \tag{9}$$

in which the matrix $M = (\lambda/2G_n + I)^{-1}(\lambda/2GU)$ is a N x p matrix and can be computed directly (I is an identity matrix) Eq 9 shows that the function $f_2(\vec{x}_i)$ on the harbor boundary can be expressed as a function of the unknown value of $\partial f_2/\partial n$ at the entrance Eq 19 can also be interpreted as the contribution to the value of f_2 on the harbor boundary from the superposition of the effect of p small harbor openings

In order to evaluate the unknown vector $\underline{C}$ defined in Eq 9, i e the value of $\partial f_2/\partial n$ at the entrance the function f_1 in Region I at the harbor entrance must be expressed as a function of $\partial f_1/\partial n$ Thus using the "matching condition" at the entrance $f_1 = f_2$, $\partial f_1/\partial n = -\partial f_2/\partial n$ one is able to solve for the vector $\underline{C}$ and the complete solution to the response problem can be obtained

For the region outside the harbor (Region I) the function f_1 may be expressed as follows

$$f_1(x, y) = f_i(x\ y) + f_r(x\ y) + f_3(x, y) \tag{10}$$

where f_i represents an incident wave function f_r represents a reflected wave function considered to occur as if the harbor entrance were closed and f_3 (termed the radiated wave function) represents a correction to f_r due to the presence of the harbor For the case of a periodic incident wave with the wave ray perpendicular to the coastline the function $f_i(x\ y)$ can be represented by $\frac{1}{2}e^{\lambda ky}$ (the factor $\frac{1}{2}$ is taken for convenience) Thus the reflected wave function f_r can be represented by $f_r(x\ y) = f_i(x, y) = \frac{1}{2}e^{-\lambda ky}$

The function $f_3(x\ y)$ can be obtained by the same procedure as used in Eq 4 for determining the function f_2 the reader is referred to Lee (1969) for its development as well as other details of the analysis

For the condition of a periodic incident wave propagating in a direction perpendicular to the coastline (x-axis) the function f_1 at the harbor entrance can be expressed as

$$f_1(x\ 0) = 1 + \left(-\frac{\lambda}{2}\right) \int_{\overline{AB}} H_0^{(1)}(k|x-x_0|)\left[\frac{\partial}{\partial n} f_2(x_0\ 0)\right] dx_0 \tag{11}$$

The first term on the right-hand side of Eq 11 represents the incident wave plus the reflected wave if the harbor entrance is closed, the second term represents the contribution from the radiated wave function f_3 It is noted that in deriving Eq 11 the relation $\partial f_1/\partial n = \partial f_3/\partial n = -\partial f_2/\partial n$ has been used at the harbor entrance

Eq 11 can be approximated by a matrix equation as

$$f_1(\vec{x}_i) = 1 + \left(-\frac{i}{2}\right)\sum_{j=1}^{p} H_{ij}C_j\ , \qquad (\text{for } i=1\ 2 \qquad p) \tag{12}$$

where

$$H_{ij} = H_o^{(1)}(kr_{ij})\Delta s_j \qquad \text{for } i\ j=1\ 2 \qquad p\ i\neq j$$

$$H_{ii} = \left[1+i\frac{2}{\pi}\left(\log\left(\frac{k\Delta s_i}{4}\right)-0\ 42278\right)\right]\Delta s_i \quad \text{for } i=1\ 2 \qquad p\ ,$$

and the vector $\underline{C}$ is the derivative $\partial f_2/\partial n$ at the harbor entrance as defined in Eq 8

Eqs 9 and 12 can now be equated to solve for the unknown value of $\partial f_2/\partial n$ at the harbor entrance i e the vector $\underline{C}$ This is done by first taking the first p equations from Eq 9

$$f_2(x_i) = \sum_{j=1}^{p} M_{ij}C_j = M_p \cdot \underline{C} \tag{13}$$

in which the index i=1 2, p and the matrix M_p is a p x p matrix obtained from the first p rows of the matrix M Then by equating Eq 13 to Eq 12 and solving for the unknown vector $\underline{C}$ one obtains

$$\underline{C} = \left(M_p + \frac{i}{2}H\right)^{-1} \underline{1} \tag{14}$$

where $\left(M_p+\frac{i}{2}H\right)^{-1}$ is the inverse of the matrix $\left(M_p+\frac{i}{2}H\right)$, and $\underline{1}$ is the vector with each p element equal to unity

With the value of $\partial f_2/\partial n$ at the entrance i e the vector $\underline{C}$ determined from Eq 14 the value of $f_2(\vec{x}_i)$ at the harbor boundary can be evaluated from Eq 9 and the value of $f_2(\vec{x})$ at any position inside the harbor can be obtained from the following discrete form of Eq 4

$$f_2(\vec{x}) = -\frac{i}{4}\left\{\sum_{j=1}^{N} f_2(\vec{x}_j)\left[-kH_1^{(1)}(kr)\frac{\partial r}{\partial n}\right]\Delta s_j - \sum_{j=1}^{p} H_o^{(1)}(kr)\ C_j\Delta s_j\right\} \tag{15}$$

where $\vec{x}_j$ is at the midpoint of the j^{th} boundary segment and $r = |\vec{x}-\vec{x}_j|$

The response of a harbor to incident waves is described by a parameter called the "amplification factor" which is defined as the ratio of the wave amplitude at any position (x y) inside the harbor to the sum of the incident and the reflected wave amplitude at the coastline (with the harbor entrance closed)

$$R = \frac{|\eta_2(x\ y\ t)|}{|A_i(f_i+f_r)e^{-i\sigma t}|} = \frac{|A_i f_2(x,y)e^{-i\sigma t}|}{|A_i \cdot 1 \cdot e^{-i\sigma t}|} = |f_2(x,y)| \tag{16}$$

EXPERIMENTAL EQUIPMENT

Experiments were conducted in the laboratory in a wave basin 1 ft 9 in deep 15 ft 5 in wide, and 31 ft 5 in long An overall view of the wave basin is presented in Fig 3 The wave generator was a pendulum type 11 ft 8 in long, 2 ft high located at one end of the basin and it was designed to operate either as a paddle- or piston-type wave generator with a maximun stroke of ±6 in (For detailed description and design considerations the reader is referred to Raichlen (1965)) Wave periods ranging from 0 34 sec to 3 8 sec can be obtained

The wave period was determined by a pulse counting technique the pulses are generated by interrupting a light beam which was directed at a photo cell by a disc with 360 evenly spaced holes arranged in a circle around its outer edge The voltage pulses which are produced in this manner are counted by an industrial counter over an interval of 10 sec Thus the wave period measured was an average over 10 sec and throughout an experiment this period varied at most by ±0 05%

Wave amplitudes were measured electronically using resistance wave gages and an oscillograph recorder The wave gage was calibrated before and after an experiment (approximately one hour apart) Even though these calibrations showed very little difference a calibration curve representing an average over the duration of an experiment was used in reducing the experimental data (see Lee (1969))

In order to simulate the open-sea in the laboratory wave basin two types of wave energy dissipators were employed a wave filter placed in front of the wave generator and wave absorbers located along the side-walls of the wave basin these can be seen in Fig 3 The wave filter was 11 ft 9 in long, 1 ft 4 in high and 5 ft thick in the direction of wave propagation and was constructed of 70 sheets of galvanized iron wire screen with each sheet spaced 0 8 in apart The wire diameter of the screens was 0 011 in with 18 wires per inch in one direction and 14 wires per inch in the other The wave absorbers placed along the sidewalls of the basin were each 1 ft 6 in high 1 ft 10 in thick normal to the sidewall, and 30 ft long and consisted of 50 equally spaced layers of the same galvanized iron screen as used in the wave filter The majority of waves used in the experiments were reduced in amplitude by at least 80% as the result of passing through the wave filter (or absorber) reflecting from the wave machine (or wall) and passing through the wave filter (or absorber) again Such wave energy dissipating materials were necessary in order to simulate the open-sea condition satisfactorily Without this wave filter and these absorbers waves radiated from the harbor entrance would be reflected from the wave paddle and the sidewalls of the basin creating a wave system which is quite different from the open-sea This problem was described by Raichlen and Ippen (1965) in which it was shown that, due to coupling between the harbor and the wave basin the response of a rectangular harbor in a highly reflective basin was radically different from that of a similar harbor connected to the open sea

In order to fully test the theory developed it was decided to use a model of Long Beach Harbor which in the past experienced problems from long period waves This model shown in Fig 4 was constructed from ¼ in thick lucite

Fig. 3 Over-all view of the wave basin and wave generator with wave filter and absorbers in place

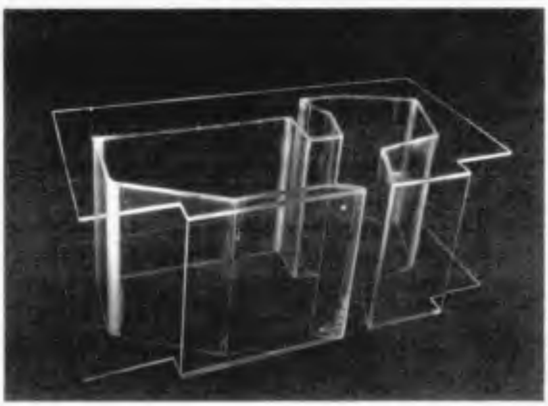

Fig. 4 Model of the East and West Basins of Long Beach Harbor (Long Beach, California, U. S. A.)

plate with a planform which was simplified from the prototype East and West Basins of Long Beach Harbor The harbor model built to a horizontal scale equal to 1 4700 was designed so that it would fit into an opening at the center of a false wall simulating a perfectly reflecting coastline which was installed 27 ft 6 in from and parallel to the wave paddle

PRESENTATION AND DISCUSSION OF RESULTS

Prior to applying this theory to a complicated harbor the theory was applied to harbors of simpler shapes circular and rectangular The circular harbor represents an extreme where the harbor boundary is curved and the tangent to the boundary is continuously changing direction The rectangular shape harbor represents the other extreme since the harbor boundary is composed of straight lines, along each line the direction of the tangent to the boundary remains the same It has been found (see Lee 1969 1970) that the results of this theory applied to circular and rectangular harbors agree well with corresponding experiments

As mentioned previously in order to verify the theory for a harbor of complicated shape Long Beach Harbor was studied theoretically and experimentally in the laboratory A sketch of the harbor model which was used is presented in Fig 5 which shows the width of the entrance as 0 2 ft and the characteristic dimension of the harbor a equal to 1 44 ft The water depth was constant in both the harbor and the "open-sea" and equal to 1 ft

Response curves at four different locations inside the harbor are presented in Figs 6 to 9 The four points are designated as A B, C, D and their relative positions in the model are shown in Fig 5 as A (0 30 ft, -0 525 ft), B (0 30 ft, -0 96 ft) C (1 32 ft, -0 96 ft), and D (-0 45 ft, -1 245 ft), where the first number inside the parenthesis is the x-coordinate and the second number is the y-coordinate For all the response curves the abscissa is the wave number parameter ka (where k is the wave number, and "a" is shown in Fig 5) the ordinate is the amplification factor R, defined as the wave amplitude at point A (or B, C, D) divided by the average standing wave amplitude at the harbor entrance when the entrance is closed (see Eq 16)

The theoretical results obtained are shown as solid lines in the response curves while the experimental results are shown as circles In applying the theory the boundary of the harbor is divided into 75 unequal straight-line segments including two segments for the harbor entrance The segments are numbered counter-clockwise starting from the right-hand limit of the harbor entrance and this numbering system is also shown in Fig 5 For accurate results the length of these boundary segments should be less than approximately one-tenth of the smallest wave length investigated (see Lee 1969)

From Figs 6 7 8, and 9 it is seen that the theoretical results agree well with the experimental data at all four locations which were investigated The complicated shape of the response curves are due to the irregular shape of the harbor and the fact that this harbor really consists of two coupled basins One common feature of the four response curves is that while the theory has prediced the frequency of every resonant mode of oscillation correctly the theoretical amplification factor at resonance is slightly larger than the experimental data especially for the resonant modes at larger values of ka There are two possible reasons for this First in applying

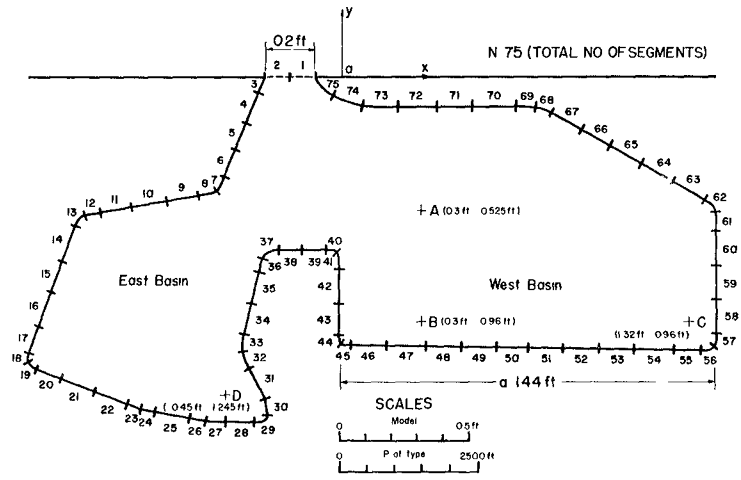

Fig 5 Dimensions sketch of the Long Beach Harbor model

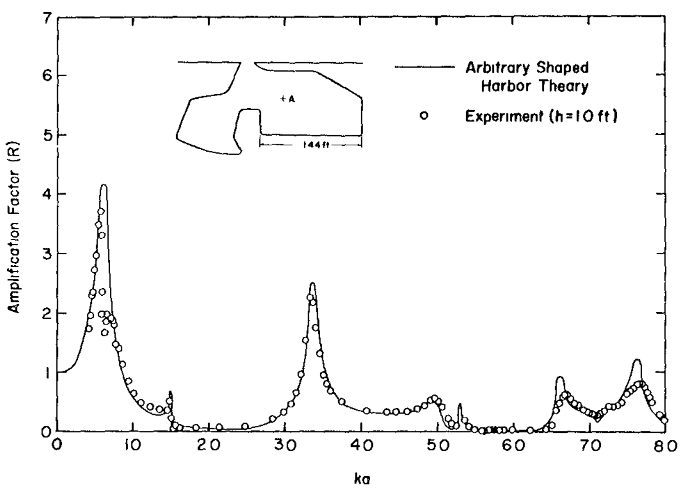

Fig 6 Response curve at point A of the Long Beach Harbor model

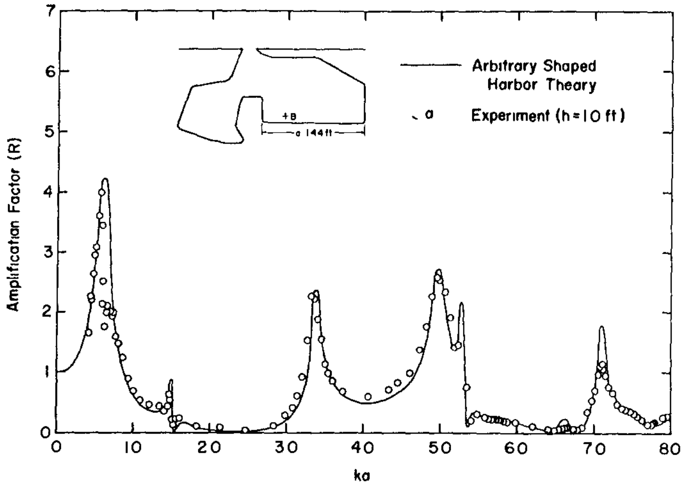

Fig 7 Response curve at point B of the Long Beach Harbor model

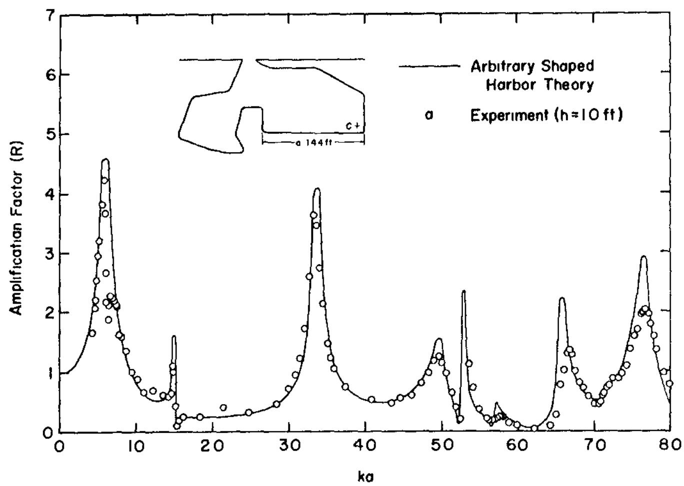

Fig 8 Response curve at point C of the Long Beach Harbor model

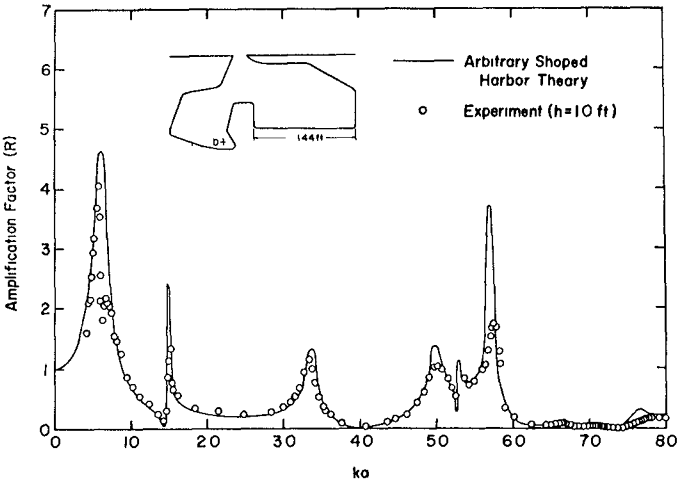

Fig 9 Response curve at point D of the Long Beach Harbor model

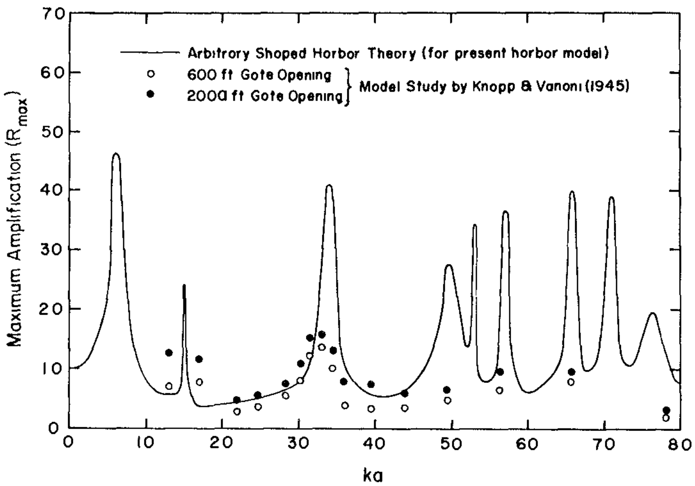

Fig 10 Response curve of the maximum amplification for the model of Long Beach Harbor compared with the data of the model study by Knapp and Vanoni (1945)

the theory the harbor boundary was divided into the same number of segments for all wave numbers thus the ratio of the segment length Δs to wave length L is smaller for the smaller values of ka than for larger values of ka Therefore the theoretical results for small ka are more accurate than the results which correspond to large ka and thus better agreement between the theory and experiments is to be expected Second energy dissipation is larger near resonance for large values of ka which also tends to decrease the experimental amplification factors compared to those determined theoretically

It can also be seen from the response curves that the agreement between the theoretical solution and the experimental data is uniformly good at each of the locations This suggests that the theory has also accurately predicted the wave amplitude distribution inside the harbor for each mode of resonant oscillation

It is interesting to note that for larger values of ka, the shape of the mode of oscillation inside the harbor becomes more complex For example for the first resonant mode (ka 0 61) the amplification factors at the four positions (A B C and D) are approximately the same In fact for this mode of oscillation the wave amplitude is fairly uniform throughout the harbor and either positive or negative water surface displacements occur simultaneously throughout the harbor Thus the first resonant mode (ka = 0 61) is usually termed the "pumping mode" However for the mode corresponding to ka = 7 62 the amplification factors at the four locations differ considerably indicating that this is a much more complicated mode of oscillation than the "pumping mode"

The variation of the maximum amplification within the entire basin plotted as a function of ka is presented in the respone curve of Fig 10 The ordinate is the ratio of the maximum wave amplitude within the harbor regardless of location to the standing wave amplitude with the entrance closed This curve shows every possible mode of resonant oscillation for the range of ka that has been investigated, as well as the maximum amplification for each mode Experimental data from a model study conducted by Knapp and Vanoni (1945) are included in Fig 10 for comparison (The gate or entrance width used in the present model corresponds to a prototype width of 940 ft) The original data of Knapp and Vanoni were presented by them as the maximum amplification factor as a function of prototype wave period In order to convert their wave period to the wave number parameter (ka) used herein an average prototype water depth of 40 ft was used throughout the harbor along with a characteristic dimension of the harbor a = 6768 ft These hydraulic model data and the present theoretical curve show qualitative agreement of the wave periods of resonant oscillations especially the mode of oscillation at ka = 3 38 However there is a considerable difference between the predicted maximum amplification and the measured There are two factors that may contribute to such differences First the maximum amplification factor used by Knapp and Vanoni was defined as the ratio of the maximum wave amplitude inside the harbor model to the maximum wave amplitude measured outside the harbor thus it differs from the definition used in the present theory Second, the water depth in the model used by Knapp and Vanoni was small increasing the importance of viscous effects in their model compared either to the inviscid theory or the experiments of this study

Fig 11 shows the distribution of wave amplitude inside the harbor for the resonant mode at ka = 3 38 determined from the present theory The wave amplitude has been normalized with respect to the wave amplitude at point C (the coordinates of this position are shown in Fig 8) Two nodal lines (lines with zero amplitude) occur one in the East Basin and one in the West Basin with maxima at the ends of each basin and a minimum near the confluence of the two Data are presented in Fig 12 from Knapp and Vanoni (1945) on the wave amplitude distribution for a prototype wave period of 6 minutes (ka = 3 30) By comparing Figs 11 and 12 it is seen that the general shape of wave amplitude are similar for the two models (e g , the location of the two nodes and the maximum) even though the boundary of the model used for present study is simplified compared to the hydraulic model

Fig 13 shows the average maximum total velocity at the harbor entrance (maximum in time averaged across the harbor entrance) as a function of the wave number parameter ka The ordinate has been normalized with respect to the maximum horizontal water particle velocity for a small amplitude shallow water wave $\sqrt{gh}\, A_1/h$ From this figure it is seen that there are nine maxima in the range of ka presented which correspond to the nine resonant modes shown in Fig 10 demonstrating that each maximum of the total entrance velocity is assiciated with a mode of resonant oscillation inside the harbor Fig 13 also shows that the velocity at the entrance for the pumping mode (ka = 0 61) is significantly larger than that of any other mode of oscillation Using the prototype dimensions mentioned previously the wave period of this mode is about 33 minutes and could possibly be excited by tsunamis If A_1 = 0 5 ft and h = 40 ft, Fig 12 indicates that the maximum entrance velocity for the pumping mode would be 10 fps and for other modes it would be of the order of 2 fps such velocities may cause damage to structures located near the entrance as well as creating navigation problems

CONCLUSIONS

The following major conclusions may be drawn from this study

1 The present linear-inviscid-theory predicts the response to periodic incident waves of an arbitrary shape harbor with constant depth connected to the open-sea quite well even near resonance

2 The theoretical prediction of the resonant wave numbers (or resonant frequencies) agree well with the experimental data The theoretical amplification factor at resonance is generally somewhat larger than the experimental data especially for the resonance modes at larger values of ka

3 The average total velocity across the harbor entrance reaches a maximum when a resonant oscillation develops inside the harbor the magnitude of such entrance velocities may be much larger than the corresponding water particle velocity of the incident wave

4 The present theoretical results also agree qualitatively with the experimental data obtained from a model study conducted by Knapp and Vanoni (1945) although the planform of the model investigated by them was more complicated and their study included depthwise variations

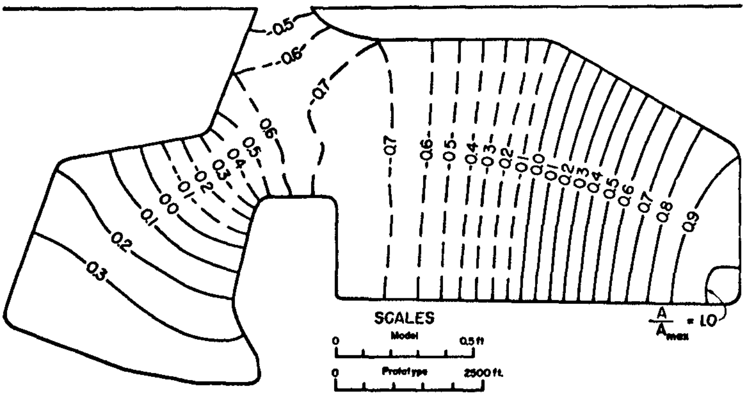

Fig 11 The theoretical wave amplitude distribution in the Long Beach Harbor model (ka = 3 38)

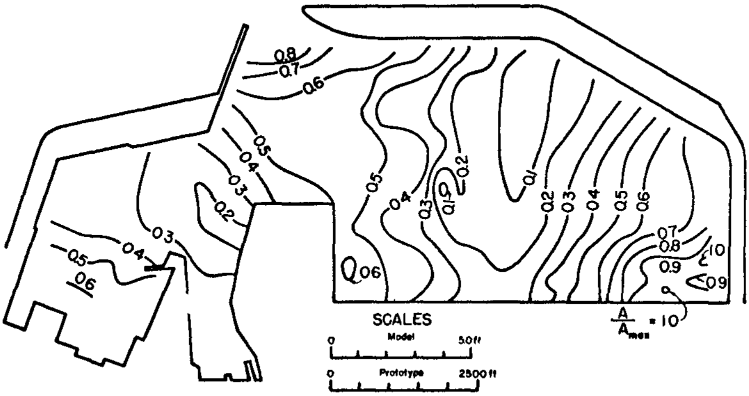

Fig 12 Wave amplitude distribution inside the harbor model of Knapp and Vanoni (1945) for six minute waves (ka = 3 30) (see Knapp and Vanoni (1945) p 133)

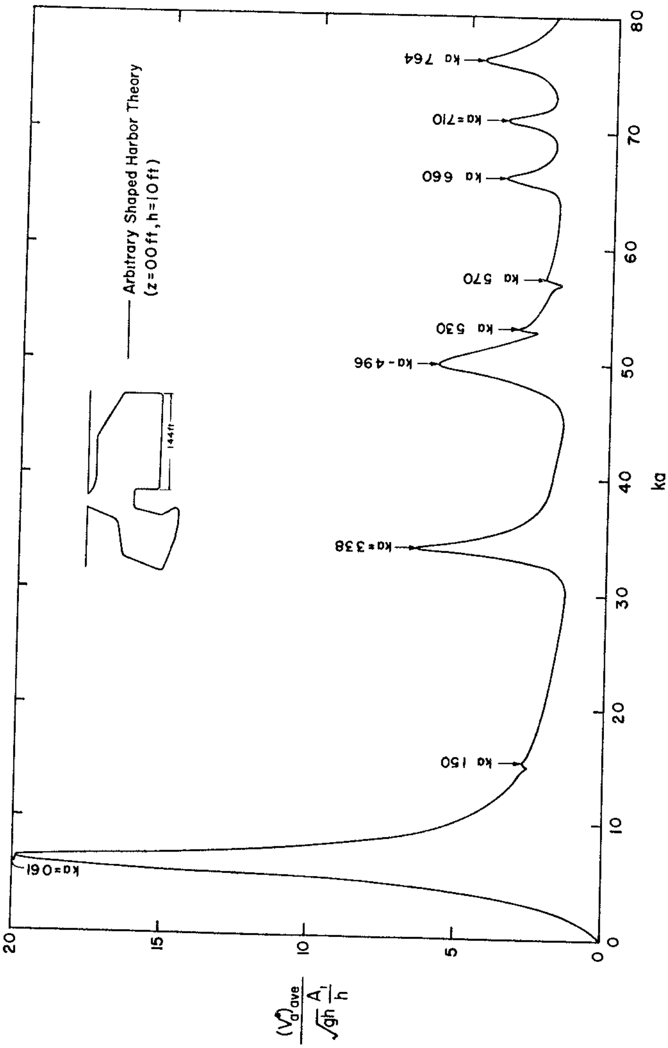

Fig 13 Average maximum total velocity on the water surface at the harbor entrance as a function of ka for the Long Beach Harbor model

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