CHAPTER 14

STRUCTURE OF SEA WAVE FREQUENCY SPECTRUM

By S.S. Strekalov [1], V.Ph. Tsyploukhin[2], S.T. Massel[3]

INTRODUCTION

A knowledge of the frequency spectrum is necessary for working a number of applied problems concerning wave parameter calculations, ship rolling, effects of wave loads upon sea structures and so on.

According to the data of work /1/ the frequency spectrum in deep sea may be represented as the following equation:

$$S(\mu) = A\mu^{-l}\exp(-B\,\mu^{-k}) \qquad (1)$$

The spectra obtained from the experiments on parameter values of l and k may be divided into three groups (Table 1).

Table 1

The parameter values of l and k on the data of various authors

Groups	Authors	l	k
I	W. Pierson and L. Moskowitz /3/	5	4
II	G. Neumann /2/, S. Strekalov /4/, B. Glukhovskii /5/	6	2
III	Yu. Krylov /5/	7	4

1) Scientific worker, M.Sc.Tech., USSR, Moscow, Sojuzmorniiproject.

2) Scientific worker, M.Sc.Tech., USSR, Moscow, Sojuzmorniiproject.

3) Scientific worker, M.Sc.Tech., Poland, Gdansk, The Institute for Water Economy of the Polish Academy of Sciencies.

At present, it is well-known that there is the Phillips' equilibrium range /6/ in the high-frequencies part of spectrum.

In this range the spectral density adheres to the equation

$$S(\mu) = \alpha g^2 \mu^{-5} \qquad (2)$$

In accordance with the Phillips' data $\alpha = 0{,}74 \cdot 10^{-2}$. In cosecquence of this the spectra of groups II-III (1= 6, 7) describe inaccurately the behavior of a spectral density in the high-frequency range in distinction of Pirson's and Moskowitz's (1 = 5), which is in agreement with the experimental data for this range. In the range of maximum the spectral density of power falls out of the equilibrium range. In this case a high degree of approximation to full-scale data from spectra groups I-III is given by the Krylov's spectrum /5/. And hence, considering the equation (1) it is impossible to describe the spectrum correctly in the whole range of frequencies.

BASIC ANALYSIS DATA

The authors have made an attempt to refine the structure of spectrum for deep sea conditions on the basis of measurements taken on various seas of the USSR. For this purpose the wave recordings were used with the wind generated waves in the range of 7-12 m/sec. wind velocities and 50-200 km fetches. The wind flow velocity along the fetch was a practically constant.

The electrocontact wave recorders of "TM-16"-type were also used. Wave recording data were handled by the computer "Razdan-3". Each calculated spectrum was normalized by the equation:

$$S(\mu)/\bar{\tau} D = S(\mu/\bar{\mu}) \qquad (3)$$

ANALYSIS DATA

In fig. 1 the generalized empirical spectrum is shown by a solid curve line (1), and the confidence limit of 90% probability is presented by dushed lines (2). The spectrum (curve 1) has two peaks: low-frequency ($\mu_1/\bar{\mu}$ = 0,8) and high-frequency ($\mu_2/\bar{\mu}$ = 1,1). Another important character of the spectrum is in keeping of condition (2) in the range of high-frequencies (fig. 1-b). Here the generalized spectrum (curve 1) is described by function of $0{,}22(\mu/\bar{\mu})^{-5}$ (curve 3). The spectral density deviates gradually from the equilibrium range when frequencies are $\mu/\bar{\mu}$ = 1,5 and lower. Availability of two peaks in the generalized empirical spectrum is typical for calculated individual spectra.

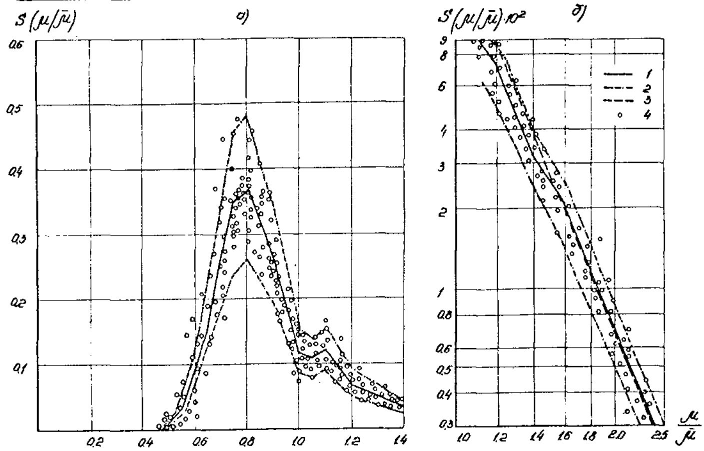

Fig.1. - Generalized empirical spectrum (curve 1) in the range of low (a) and high (б) frequencies. Confidence limit of 90% probability are showed by a dotted line (2), function (3) $0{,}22(\mu/\bar{\mu})^{-3}$ by a dushed one and individual spectra by spots (4)

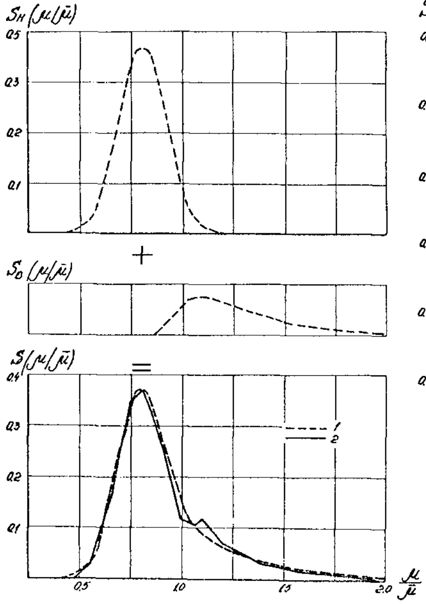

Fig.2. - Dimensionless spectrum $S(\mu/\bar{\mu})$ in form of sum $S_H(\mu/\bar{\mu})$ and $S_B(\mu/\bar{\mu})$. Curve 2 is a generalized empirical spectrum

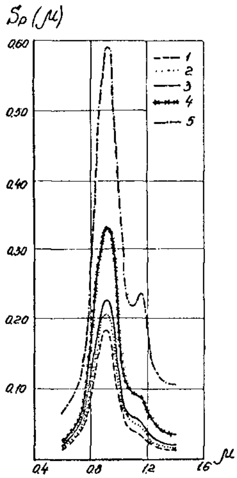

Fig.3. - Wave load spectra $S_p(\mu)$ at a pier of vertical type at levels: 11.5 m(1), 9.5 m(2), 7.3 m(3), 5.1 m(4), 0.6 m(5)

SPECTRUM FORM

The form of frequency spectrum is determined as a sum of a low-frequency $S_H(\mu/\bar{\mu})$ and a high-frequency $S_B(\mu/\bar{\mu})$ parts /7/

$$S(\mu/\bar{\mu}) = S_H(\mu/\bar{\mu}) + S_B(\mu/\bar{\mu}) \qquad (4)$$

The spectral density may be expressed by such a function:

$$S_B(\mu/\bar{\mu}) = a(\mu/\bar{\mu})^{-n} \exp\left[-b(\mu/\bar{\mu})^{-m}\right] \qquad (5)$$

where n=5, a=0,22; b, m - parameters which to be determined. By means of expression (5) the single-peak unsymmetric curve is described. Maximum of this curve conforms the frequency of $\mu_2/\bar{\mu}$ = 1,1. From this condition for the derivative

$$dS_B(\mu/\bar{\mu})/d(\mu/\bar{\mu}) = 0, \qquad (6)$$

"b" is determined

$$b = \frac{n}{m}(\mu_2/\bar{\mu})^m = \frac{5}{m}(1,1)^m$$

Assuming that $S_p(\mu/\bar{\mu})$ on the frequency μ_1/μ is a negligible quantity, and $\mu_1/\bar{\mu}$ = 0,8; $S(\mu/\bar{\mu})$ = 0,37 from fig.1, m=8, b = 1,34 are obtained from expressions (5-6). In the final analysis the high-frequency part of spectrum may be written as

$$S_B(\mu/\bar{\mu}) = 0,22(\mu/\bar{\mu})^{-5}\exp\left[-1,34(\mu/\bar{\mu})^{-8}\right] \qquad (7)$$

In the frequency range of $\mu/\bar{\mu} \geq 1,5$ the relation (7) with regard to the normalized expression (3) will be calculated as:

$$S_B(\mu) = 0,22(2\pi)^2(\bar{\delta}^*)^2 g^2 \mu^{-5}, \qquad (8)$$

where

$$\bar{h} = \sqrt{2\pi D}, \qquad (9)$$

$$\bar{\delta}^* = \bar{h} \Big/ \frac{g\bar{\tau}^2}{2\pi} \qquad (10)$$

and $\bar{\delta}^*$ - parameter characterizing a steepness of waves. When $\alpha = 0,22(2\pi^2).(\bar{\delta}^*)^2$ the equation (8) is equivalent to the Phillips' formula (2). Considering that the parameter $\bar{\delta}^*$ is the dimensionless fetch function gx/v^2 the following equation is obtained

$$\alpha = \alpha(gX/v^2) \qquad (11)$$

TABLE 2.-VARYING OF PARAMETER α ACCORDING TO THE DATA OF FULL-SCALE OBSERVATIONS /9/

Nos. of wave recordings	99	95	135	150	148	144	147	235	68	124	49	198	6	203	43
Wind velocity, v, in m/sec.	18,4	14,4	17,0	10,4	11,8	10,6	12,5	9,8	17,2	15,2	14,7	7,1	13,6	6,7	12,6
Fetch, X, in km	3,7	2,3	8,9	5,2	11,1	11,1	20,0	41,0	180	280	330	116	625	157	550
Average height, h, in m	0,65	0,47	0,74	0,40	0,56	0,49	0,74	0,64	2,25	2,00	2,09	0,60	2,52	0,59	2,05
Average period, τ, in sec.	3,2	2,8	3,8	2,7	3,2	3,1	3,9	3,9	7,0	6,6	7,2	3,8	7,7	3,9	7,0
$\frac{gX}{v^2} \cdot 10^{-3}$	0,11	0,11	0,30	0,47	0,78	0,97	1,25	4,19	5,93	11,9	15,0	22,6	33,1	34,3	37,5
$\alpha \cdot 10^{-2}$	1,46	1,25	0,94	1,06	1,06	0,94	0,83	0,63	0,73	0,73	0,59	0,63	0,63	0,54	0,63

It is pointed by S.Kitaigorodsky and O.Phillips that the parameter α is a variable. The authors estimated variable range of α from the formula (11) using actual measurement data. The values of α proved to be enclosed within $0{,}54 \cdot 10^{-2} \leq \alpha \leq 1{,}46 \cdot 10^{-2}$.

As the parameter of gX/v^2 increases the values of decreases. It is characteristic that the value of constant $\alpha = 0{,}74 \cdot 10^{-2}$ determined before by O.Phillips (6) is an approximately average value for the conditions of rough sea when $\delta^* = 0{,}025 - 0{,}030$.

The low-frequency part of spectrum $S_H(\mu/\bar{\mu})$ to be found in the form of difference /7/:

$$S_H(\mu/\bar{\mu}) = S_\vartheta(\mu/\bar{\mu}) - S_B(\mu/\bar{\mu}), \qquad (12)$$

where $S_\vartheta(\mu/\bar{\mu})$ - values of the generalized empirical spectrum, and
$S_B(\mu/\bar{\mu})$ - already known function (7).

The low-frequency part of spectrum (12) with sufficient accuracy may be presented as a symmetric Hauss-type curve with a maximum displaced on frequency scale from the origin by $\mu_1/\bar{\mu} = 0{,}8$;

$$S_H(\mu/\bar{\mu}) = \frac{p_1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{[1/\bar{\mu}(\mu-\mu_1)]^2}{2\sigma^2}\right\} \qquad (13)$$

According to the calculations when $p_1 = 0{,}11$ and $S(\mu_1/\bar{\mu}) = 0{,}37$

$$\sigma = \frac{p_1}{\sqrt{2\pi}\, S(\mu_1/\bar{\mu})} \simeq 0{,}12 \qquad (14)$$

The total spectrum $S(\mu/\bar{\mu})$ consisting of members (3) and (4) is given in fig.2. In the dimension form the spectrum $S(\mu)$ is written as

$$S(\mu) = D\bar{\tau}\left([p_1/\sqrt{2\pi}\sigma] \exp\left\{-\frac{[1/\bar{\mu}(\mu-\mu_1)]^2}{2\sigma^2}\right\} + \right.$$

$$\left. + a(\mu/\bar{\mu})^{-5} \exp\left[-b(\mu/\bar{\mu})^{-8}\right]\right) \qquad (15)$$

SPECTRUM UNIVERSAL PROPERTIES

The authors realized that the spectrum form (15) is not universal for all the stages of wave development. However, the spectrum describes specific wave properties at the particular stages. In fig.3 the full-scale wave load spectra $S_p(\mu)$, $(T/M^2)^2$ sec. at a pier of vertical type at various levels are shown /10/. Two maxima with the relation of frequencies close to the relation in spectrum (15) are followed at all the levels. The low-frequency part of spectrum may be described by a symmetric function of (13)-type, and the high-frequency part by an unsymmetric one of (7) - type.

CONCLUSIONS

1. Two maxima in frequency spectrum are followed.
2. The low-frequency part of spectrum may be approximately described by a Hauss symmetric type curve.
3. The high-frequency part is an unsymmetric owing to the presence of the equilibrium range.

APPENDIX I.-REFERENCES

1. Burling A. The spectrum of waves at a short fetches. "Dtsch.hydrogr.Z.", 1959, 12, 1, 3.
2. Neumann G., Pierson W. A detailed comparison of theoretical wave spectra and wave forecasting methods. "Dtsch.hydrogr.Z.", 1957, 10, 3.
3. Pierson W., Moskowitz L. A proposed spectral form for fully developed seas, based on the similarity theory of S.A. Kitaigorodskii. "J.Geophys. Res.", 1964, 69, №24.
4. Стрекалов С.С. К определению аналитического вида энергетического спектра развитого волнения. "Океанология", 1961, I, вып. 3
5. Крылов Ю.М. Спектральные методы исследования и расчёта ветровых волн. Л., Гидрометеоиздат, 1966
6. Phillips O. The equilibrium range in spectrum of wind generated waves. "J. Fluid. Mech.", 1958
7. Strekalov S.S., Massel S. Niektore zagdneenia widmowej analizy falowania wiatrowego. Apch. Hydrotechniki. T. XVII, Z 4, Poland, 1971.
8. Китайгородский С.А. Физика взаимодействия атмосферы и океана. Л., Гидрометеоиздат, 1970
9. Ржеплинский Г.В.,Крылов Ю.М., Матушевский Г.В., Стрекалов С.С., Назаретский Л.Н. Новый метод анализа и расчёта элементов ветровых волн. Труды Гос. Океанографического ского института, вып. 93, 1968
10. Крылов Ю.М. и др. Волноизмерительная станция в порту Шесхарис на Черном море. Информационные материалы по гидрологическим приборам и методам наблюдений. Сб., 1971, вып. 46

APPENDIX II.-NOTATION

A, B = dimensional parameters
a, b, n, m = dimensionless parameters of spectrum (5)
D = dispersion
g = acceleration of gravity
h = average height
l, k = dimensionless parameters
p = normalizing factor
$S(\mu)$ = spectral density of power $[L^2T]$
$S_p(\mu)$ = spectral density of wave pressure in $(T/M^2)^2$sec.
$S_э(\mu/\bar{\mu})$ = experimental value of spectral density
v = wind velocity in m/sec.
X = fetch
α = dimensionless Phillips' constant
μ = frequency in rad./sec.
$\bar{\mu}$ = $2\pi/\bar{\tau}$ - average frequenoy in rad./sec.
μ = frequency of low-frequency maximum (rad./sec.)
μ_2 = frequency of high-frequency maximum (rad./sec.)
$\bar{\tau}$ = average period
σ = parameter characterizing spectrum form (13)