CHAPTER 16

SHALLOW WATER WAVE CHARACTERISTICS

by

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Abstract

Prototype data from 24 wave stations on and around the tidal flats south of the Elbe estuary enable us to elaborate special shallow water wave characteristics, concerning the variations and correlations of heights, periods, lengths and velocities. This paper deals with some interesting aspects from the engineer's point of view. It turns out that the steepness factor $\frac{\overline{H}}{\overline{L}}$ or $\frac{\overline{H}}{g \cdot \overline{T}^2}$ of breaking waves is much smaller than of non-breaking waves and that steepness is no suitable parameter to describe a natural wave spectrum in shallow waters.

On the tidal flats the maximum wave heights only depend on the depth of water, not on the steepness. Moreover the possible wave height proves to become much higher than theoretically predicted, especially in depths of water less than 2 m.

Introduction

In 1962 the Coastal Engineering Research Group in Cuxhaven, a branch of the City of Hamburg Harbour Authority, was established to carry out a general programme of hy-

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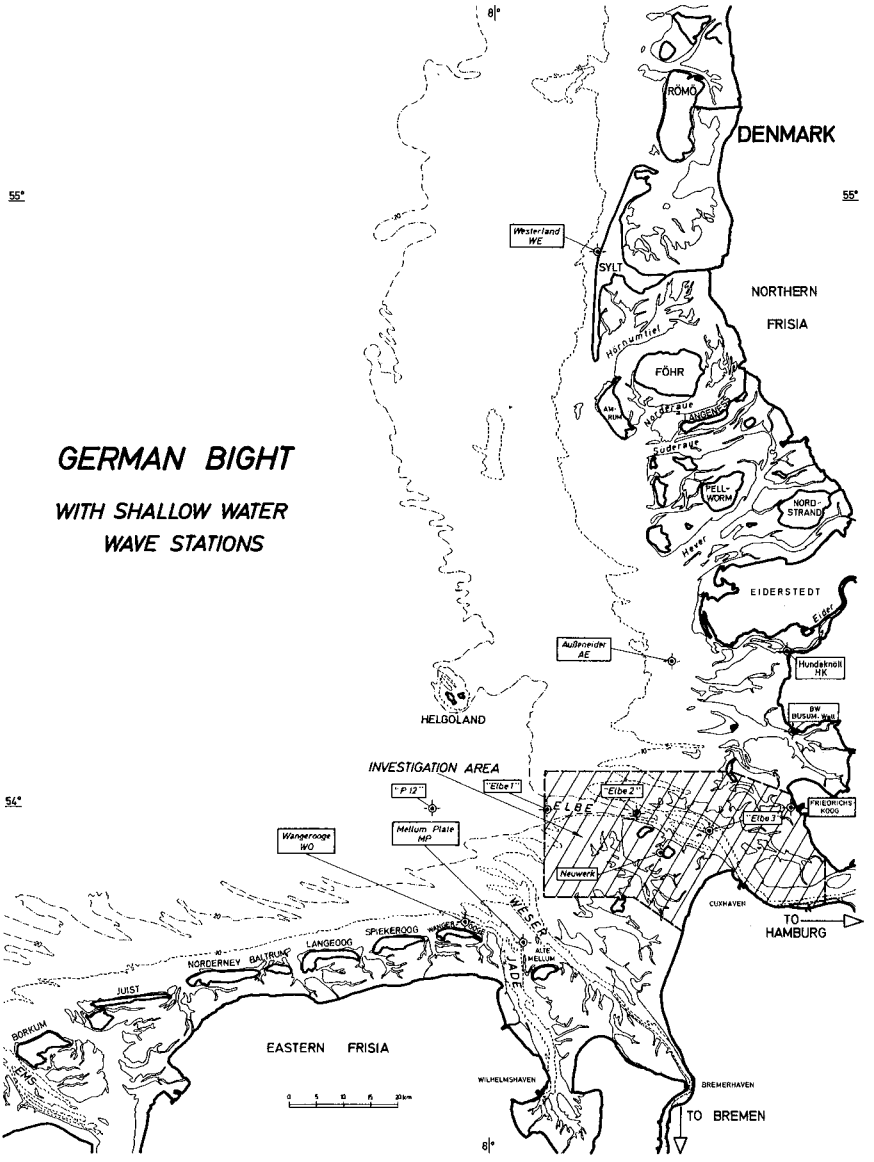

Fig. 1
Coast of the German Bight

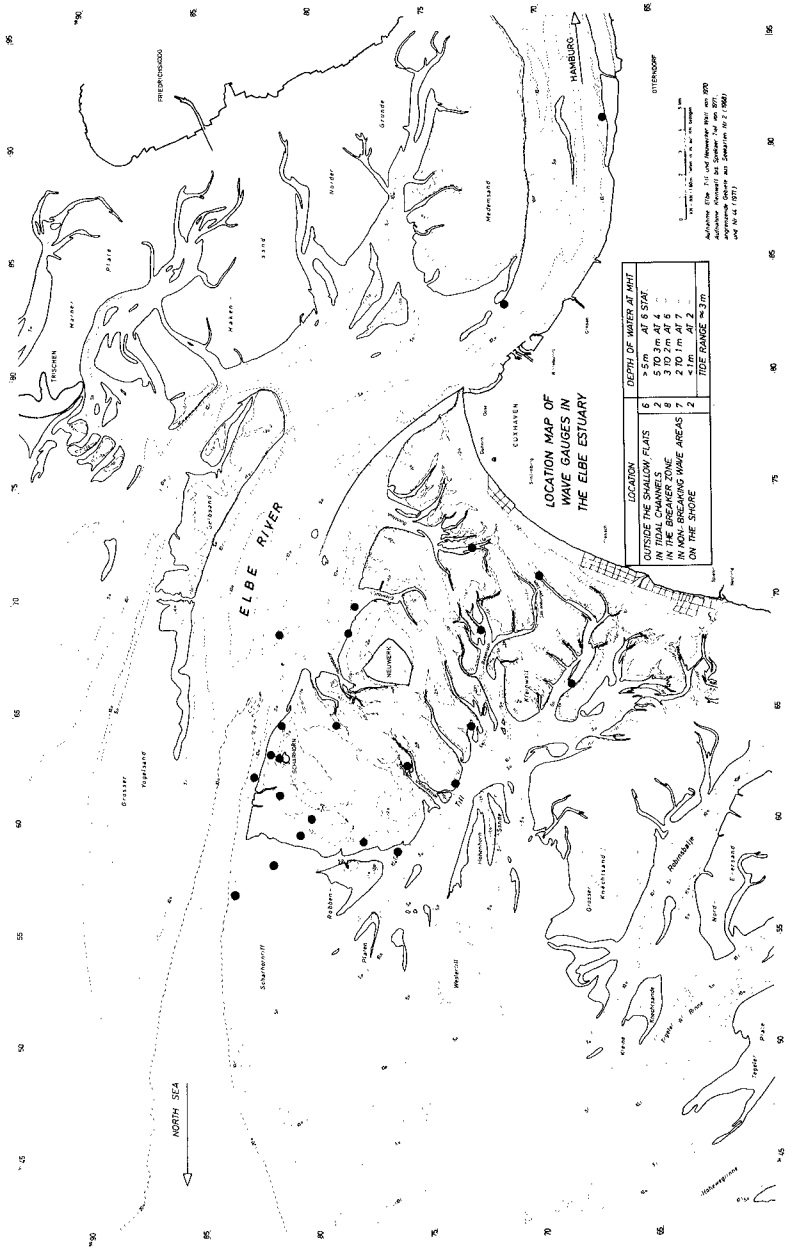

Fig. 2
Wave Investigations in the Elbe Estuary

drologic, hydrodynamic and morphologic investigations in the Elbe estuary. One part of these are wave measurements in an area that is morphologically very complicated. The location map (fig. 1) shows the structure of extended tidal flats, interrupted by tidal channels of variing width and depth with 2 small islands on the higher part of the flats.

To get a sufficient survey of the wave conditions in an area with strongly changing depths and under the influence of mean tide ranges of about 3 m, detailed research is necessary. Till 1972 the number of measuring stations amounts to 24 in water depths from 30 cm to 10 m (fig. 2). To give an impression of one special investigation programme, fig. 3 shows two of these stations on the beach of the dune island of Scharhörn with a distance of 20 m between them.

Fig. 3

Previous evaluations gave strong connections between mean wave height and wave height distribution, morphologic structure and wave period distribution, $\overline{H}$ and $\overline{T}$ and wind conditions, and others (Siefert, 1971). The following examples shall indicate the variation of some simple para-

meters, as $\overline{H}$, $H_{1/3}$, $\overline{T}$, $\overline{L}$ and $\overline{c}$, due to changing water depths and wave character, especially in depths below 1 m, whereof prototype data have not yet been available.

Waves in Very Shallow Water ($d < 1$ m)

Fig. 4 contains data from the two above mentioned beach stations in water depths not over 1 m on a beach slope of about 1:70. Waves are breaking here for the second time, as most of them already broke in the flat area in front of the beach. In a few words the results can be resumed as follows:

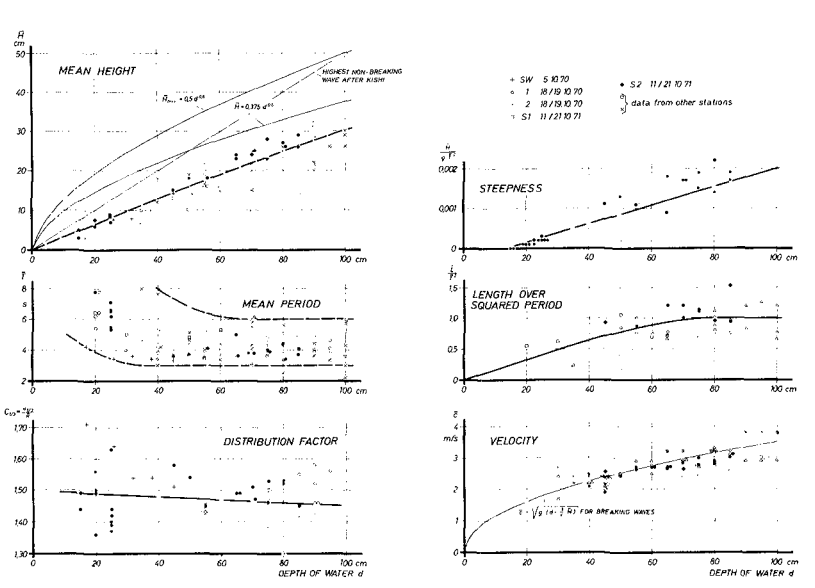

Fig. 4

1. $\overline{H}$ increases nearly proportional to depth, and the highest mean values are lower than in the first breaker zone (see last chapter) and even than the highest non-breaking waves after Kishi (1959).

2. $\overline{T}$ decreases with increasing depth, until for $d > 0.5$ m $\overline{T}$ reaches the range from 3 to 6 s that is typical for

waves on tidal flats in the southern part of the North Sea.

3. The weak decrease of the wave height distribution factor $C_{1/3}$ with increasing depth indicates that the width of the wave height distribution becomes smaller with deeper water and consequently higher waves. Besides this the distribution is obviously narrower than theoretically expected with $C_{1/3} = 1.60$.

4. Concerning the wave steepness factor $\frac{\overline{H}}{g \cdot \overline{T}^2}$, it may be remembered that it is generally used only in connection with sinusoidal waves and waves of constant profile in a wave tank. As a ratio of mean prototype data it receives a different valuation. This fact will be discussed later.

   Fig. 4 indicates that the steepness factor increases with increasing depth and increasing wave height, though it remains very small with $\frac{\overline{H}}{g \cdot \overline{T}^2} = 0.002$ for d = 1 m.

5. The relation $\frac{\overline{L}}{\overline{T}^2}$ in very shallow water increases with decreasing period and becomes constant about 1 at depths over 0.7 m. So usually $\overline{L}$ equals $\overline{T}^2$, as was determined in deeper water areas by earlier investigations. Consequently the steepness factor $\frac{\overline{H}}{g \cdot \overline{T}^2}$ really equals $\frac{\overline{H}}{\overline{L}}$ except the constant acceleration due to gravity.

6. The theoretical relation between mean wave velocity $\overline{c}$ and depth of water in very shallow water

$$c = \sqrt{g \cdot d}$$

   is valid only for non-breaking waves. Concerning breaking waves, it must be regarded that the crests of these waves lie $\frac{3}{4}$ H above still-water-level (Wiegel, 1964) and that the velocity-distribution in the wave is different, thus

getting

$$c = g(d + \frac{3}{4} H).$$

Prototype data on fig. 4 state that they represent breaking waves and - what is likely more important - that the theoretical formula for single waves can be used to describe the mean velocity of waves of a natural spectrum by using the mean wave height under the root.

Waves on Tidal Flats

At the southern and eastern coast of the North Sea, beaches with those wave characteristics are usually surrounded by shallow flats, where also special, but of course different wave characteristics are dominating. Fig. 5 gives a comprehensive impression of these. It is relatively easy to divide

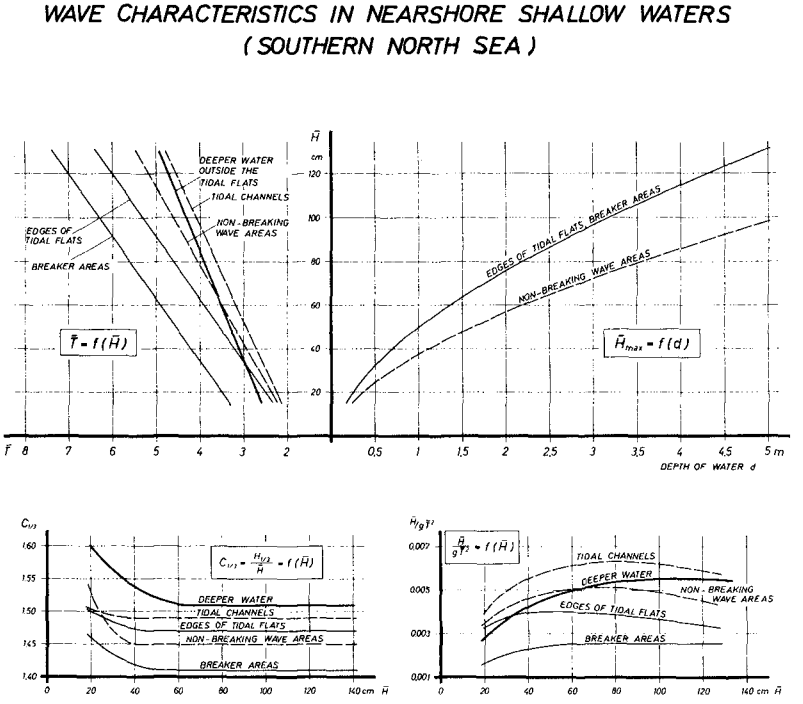

Fig. 5

such an area into some parts with constant characteristics. The first criterion is the depth of water, that separates the tidal flats from the deeper water area in front of them and from the tidal channels. The exposure to deeper water areas determines, whether mainly breaking or non-breaking waves are to be expected. This is of greatest importance for the determination of the highest possible mean wave heights, $\overline{H}_{max}$, as a function of the depth of water (Siefert, 1973).

The connections of $\overline{H}$ and d are given on the upper right side of fig. 5. Later on we will look at them again. The round about linear connections between $\overline{H}$ and $\overline{T}$ are plotted on the upper left side. For each area of special wave characteristics there is a different line, the result being somewhat surprising but reasonable:

Waves of a certain height obviously proceed with different periods. Mean waves of 60 cm height for example have a period of 3.5 s in deeper water outside the flats. In tidal channels waves of the same height proceed with a 10% shorter period, due to strong wave deformation by refraction, the period being even shorter than in the neighbouring non-breaking wave areas on the tidal flats. Waves of this height within the breaker areas have much longer periods, up to 5 s (fig. 5). The consequence for engineering projection is that the type of coastal area and its special wave characteristics are of high interest for the determination of design waves.

The variation of wave characteristics during the transition of two wave height groups from the deep Elbe fairway on to the shallow flats is generalized on fig. 6 for waves with (left) and without breaking, resuming the results as before mentioned:

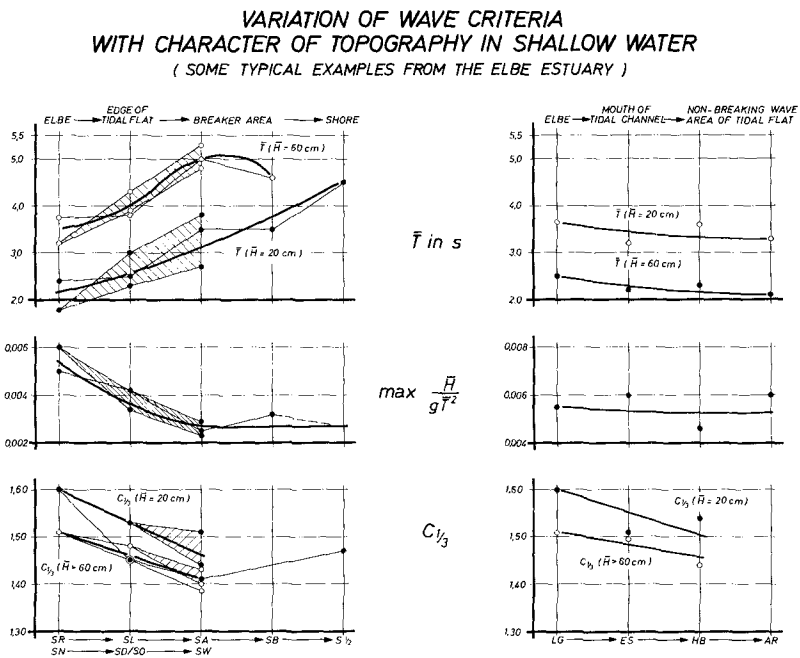

Fig. 6

1. Increase of period for breaking waves of constant height, slow decrease of period for non-breaking waves,

2. Decrease of maximum steepness factor for breaking waves (from 0.0055 to 0.0027), nearly constant steepness for non-breaking waves at about 0.0055,

3. Narrowing of the wave height distribution factor for all waves.

The increase of period with decreasing depth at MHW for waves of the same height can clearly be indicated on fig. 7. In water depths deeper than 4 m the period range is about

INFLUENCE OF TOPOGRAPHY ON SHALLOW WATER WAVE CHARACTERISTICS

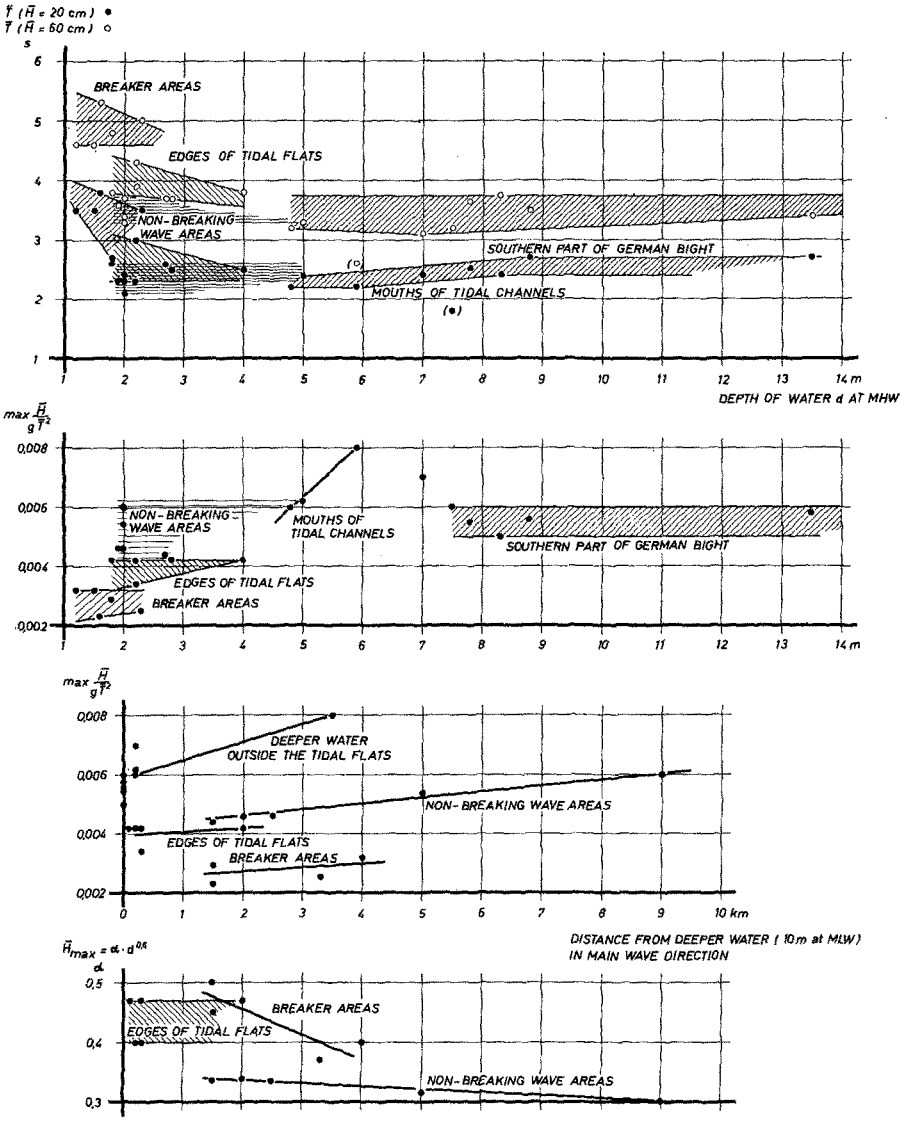

Fig. 7

constant. With the following transition into the tidal flats, going from right to left on the 2 upper graphs of fig. 7, the period range changes concerning to the wave character, as shown on fig. 6.

Wave Height Distributions

Prototype data on the tidal flats in the Elbe estuary proved wave heights and periods to follow Rayleigh-distributions (Siefert, 1971). The distribution functions of the heights are represented by the distribution factor $C_{1/3}$, i.e. significant over mean wave height (fig. 8). Now the data indicate, that combined with the increase of wave height up to $\overline{H}$ = 60 cm there is a clear narrowing of the wave height distribution, as represented by the factor $C_{1/3}$ on fig. 5 and 6. In the breaker zone on tidal flats this value becomes 1.41 instead of 1.60 in deep water. It can be seen than the determination of a realistic distribution is of remarkable importance for the choise of a design wave height. For example, the distribution of Longuet-Higgins with $C_{1/3}$ = 1.60 on fig. 8 delivers a value for the 10% highest waves of 2.03 $\overline{H}$ instead of 1.72 $\overline{H}$, as can be expected in the breaker zone on the tidal flats.

Steepness of Waves in Shallow Water

The periods of waves of constant height growing longer on the way into the breaker zone consequently reduce the steepness factor (fig. 5 and 6). Though these graphs do not represent simultanous waves in different areas, the tendency can be generalized: Waves running from deeper water into the tidal flats loose up to 50% of their height, while the period remains constant or gradually increases, the result always being a distinct decrease of the so-called steepness factor.

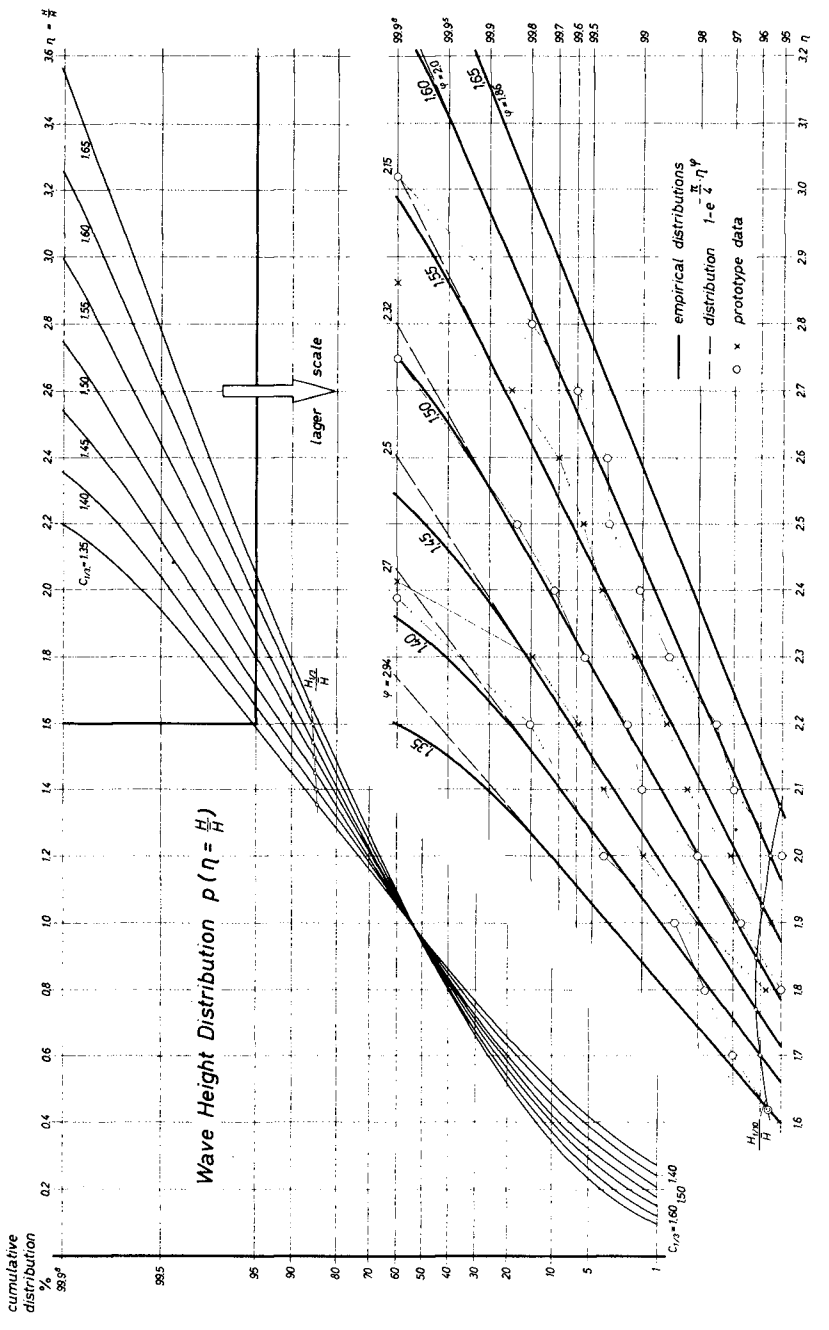
Wave Height Distribution $p(\eta = \frac{H}{\bar{H}})$
cumulative distribution %
larger scale
empirical distributions
distribution $1-e^{-\frac{\pi}{4}\eta^2}$
prototype data
$\frac{H_{1/3}}{\bar{H}}$
$\frac{H_{1/10}}{\bar{H}}$
$\eta = \frac{H}{\bar{H}}$
$C_{1/3}$=1.35
$C_{1/3}$=1.60
φ=2.34
φ=2.20
φ=1.86

Fig. 8

Comparing maximum steepness of breaking and non-breaking waves, fig. 5 to 7 show, that the steepness-factor of breaking waves on tidal flats is clearly smaller than of non-breaking waves. This is also indicated by the upper limits of the scattering areas for two stations with non-breaking and breaking waves respectively (fig. 9).

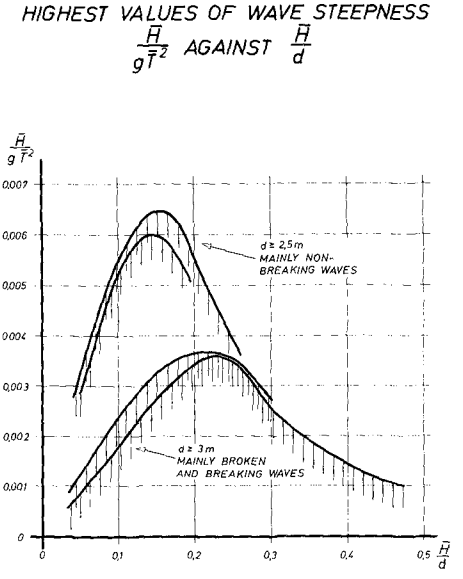

Fig. 9

Apparently the steepness factor is very small for small relative wave heights, i.e. small wind waves on the flats, and again becomes small for great relative heights, i.e. breaking waves in shallow water. The steepness factor does not increase continuously with relative wave height, indicating that not steepness but water depth is the criterion for the highest wave heights in shallow water. Neither does $\frac{\bar{H}}{\bar{L}}$ sufficiently represent the steepness of breakers with long troughs, and steep slopes only near

the crests. So the ratio $\frac{\overline{H}}{g \cdot \overline{T}^2}$ seems to be unsuitable as an item to describe wave characteristics in shallow water. A factor would be much more valuable that represents the asymmetric profile of a wave in shallow water. But as there is no correlation between heights and periods of single waves in a natural spectrum, the asymmetry must become smaller the higher the mean waves are. This again leads to a somewhat peculiar conclusion.

Highest Waves in Shallow Water

The height of the highest possible waves in shallow water has to be discussed with special respect to the conclusions for design waves. Data from 8 stations in breaking areas on shallow flats delivered a very simple empirical formula for the highest possible mean wave heights (Siefert, 1973)

$$\overline{H}_{max} = 0.5 \cdot d^{0.6}$$

indicating higher waves than usually are thought to be possible. In areas with mainly non-breaking waves the highest mean waves reach three quarters of these values, i.e.

$$\overline{H}_{max} = 0.375\ d^{0.6}$$

(fig. 4 and 5).

The values for the extreme waves in the wave spectrum with a probability of occurence 1:5000, i.e. once during a tide, are much higher than usually recognized, especially in water depths below 2 m. They can be evaluated as functions of d and $C_{1/3}$ as shown in fig. 5:

$$C_{1/3} = f(\overline{H}) \text{ with } \overline{H} = f(d)$$

Fig. 8 then delivers

$$H(1:5000) = H_{max}(\overline{H}_{max}) = f(C_{1/3})$$

as the highest possible single wave in a spectrum. It is

$$H_{max}(\overline{H}_{max}) = 1.2\ d^{0.6}$$

in breaker areas and

$$H_{max}(\overline{H}_{max}) = 0.95\ d^{0.6}$$

in areas with mainly non-breaking waves (fig. 10). Ratios H over d higher than 1.0 and even 1.5 are realistic. In 1 m deep water the extreme wave measures 1.2 m, and in half a meter deep water the highest possible wave is double as high as the often used 0.78 d (fig. 10).

With respect to a recent publication concerning breaker travel and breaking processes (Galvin, 1969), the maximum breaker heights on slopes were determined for three height groups and different depths of water (fig. 11).

Galvin's breaker travel distance X_b reads

$$X_b = 2 \cdot (4.0 - 9.25\ m) \cdot H_b$$

with m = slope $\tan\alpha$

H_b = breaker height

Galvin recommends to calculate the design wave height as the height at a distance of $\frac{1}{2} X_b$ before the structure. The safest way is of course to regard X_b completely, as is done on fig. 11. Further investigation has to be done on the problem of the definition of H_b. Unter the supposition that H_b can be replaced by any mean height of a breaking wave group of the natural spectrum, the curves on fig. 11 can

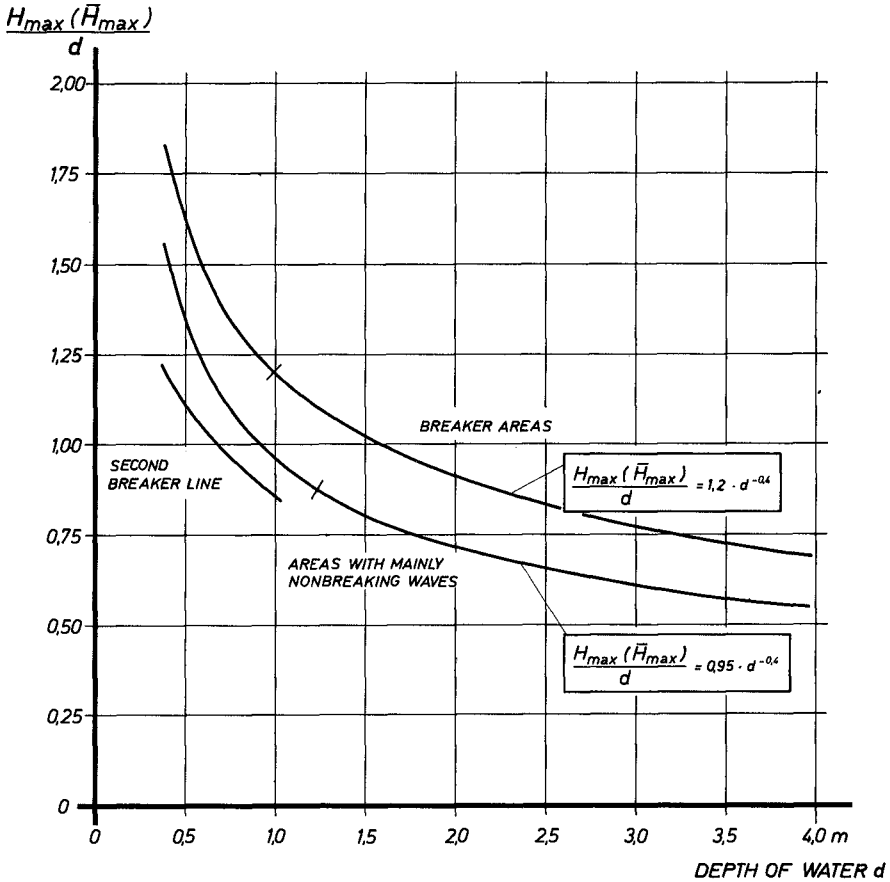

Fig. 10
Highest Possible Single Waves in Shallow Water
(Prototype Data from Elbe Estuary)

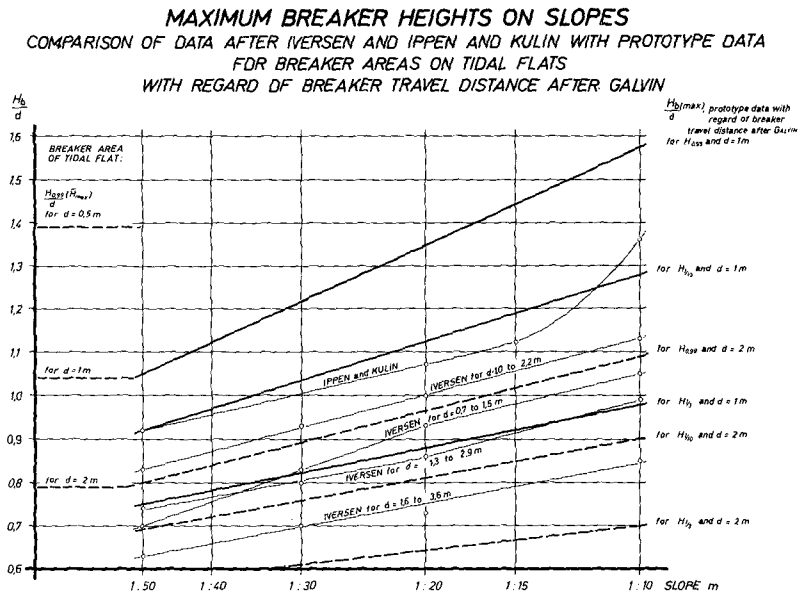

Fig. 11

be constructed, i.e.

$$X_b\ (H_{1/3}) = 2 \cdot (4.0 - 9.25\ \text{m}) \cdot H_{1/3}$$

If treated in this way, it turns out that even the highest heights of Ippen and Kulin do not represent the highest possible values. They seem to be sufficient for waves in water depths no less than 2 m.

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