

CHAPTER 21

MAXIMUM BREAKER HEIGHT FOR DESIGN

J. Richard Weggel
Coastal Engineering Research Center
Washington, D. C. 20016

ABSTRACT

A re-evaluation of previously published breaking wave data is used to develop a relationship for the maximum breaker height in terms of the depth in which the wave breaks, breaker steepness and the local beach slope. This relationship is used with the breaker travel distance equation of Galvin (3)* to estimate the maximum breaker height to which a coastal structure might be subjected. In addition, the range of depths in which a wave of given height will break is found by examination of the upper bound of observed values of d_b/H_b . An example problem is presented to illustrate the use of the general maximum breaker height design curves.

INTRODUCTION

Coastal structures such as groins, jetties and breakwaters are usually subjected to waves breaking directly against them. The range of breaker heights to which such a structure is subjected depends critically on the range of depths at the structure site with the largest breaker occurring for the greatest depth at the site. It is necessary to determine this breaker height since it usually establishes the critical design condition for the structure. This maximum design breaker height, H_b , is a function of depth at the structure, d_s , wave period, T , and the post-construction beach slope, m , on which the structure is situated. The relationship between the above variables and breaker height must be based on empirical data since it is not at present possible to adequately describe breaking waves in mathematical terms. This paper presents a re-evaluation of some previously published breaker data in order to establish this maximum breaker height and to present the results in a form easily applied to engineering design calculations.

BREAKING WAVES IN SHALLOW WATER

In relatively deep water ($d/L > 0.5$) wave breaking is initiated when the wave steepness becomes greater than some limiting value. Michell (11) found that theoretically this limiting steepness is given by,

$$\left\{ \frac{H}{L} \right\}_{\text{limiting}} = \frac{H_b}{L_b} = 0.142 \approx \frac{1}{7} \quad (1)$$

For a finite amplitude wave in deep water having the above limiting

*Numbers in parentheses correspond with references listed in Appendix I.

steepness, the wave length, as a function of wave period, is given approximately by,

$$L_b = 1.2 \frac{gT^2}{2\pi} \quad (2)$$

and consequently breaking occurs when

$$\frac{H_b}{T^2} = 0.875 \quad (3)$$

As a wave moves into shoaling water the local water depth also influences the initiation of breaking. Miche (10) found theoretically that this dependence on depth could be approximated quite well by,

$$\frac{H_b}{L_b} = 0.142 \tanh\left(\frac{2\pi d_b}{L_b}\right) \quad (4)$$

For small values of d_b/L_b , $\tanh\left(\frac{2\pi d_b}{L_b}\right)$ approaches $\frac{2\pi d_b}{L_b}$ and equation 4 reduces to

$$H_b = 0.89 d_b \quad (5)$$

McCowan (9), using a solitary wave theory analysis, found that the breaker height in shallow water is given by

$$H_b = 0.78 d_b \quad (6)$$

Experimental studies by Collins and Wier (1), Galvin (2, 3) and Iversen (5, 6) have shown that the ratio H_b/d_b also depends on the beach slope in shallow water. For plunging breakers, Galvin (2) found

$$\frac{H_b}{d_b} = \frac{1}{\beta_b} \quad (7)$$

with $\beta_b=0.92$ for $m \geq 0.07$ and $\beta_b=1.40-6.85m$ for $m \leq 0.07$ where m is the beach slope (tangent of the angle the beach makes with the horizontal). Collins and Weir (1) give

$$\frac{H_b}{d_b} = 0.72 + 5.6 m \quad (8)$$

for the relationship between H_b/d_b and beach slope.

The present analysis takes into consideration the dependence of H_b/d_b on wave steepness in addition to its dependence on beach slope.

DATA ANALYSIS

In order to arrive at an expression for H_b/d_b in terms of beach slope, m , and the wave steepness parameter, H_b/T^2 , the data presented by Iversen (6), Galvin (2), Jen and Lin (8), Weggel and Maxwell (13) and Reid and Bretschneider (12) have been used. These

data are shown on Figure 1 along with McCowan's equation based on solitary wave theory (equation 6), Michell's limiting steepness in deep water (equation 1) and Miche's transition equation (equation 4). In spite of the considerable amount of scatter in the data shown on the figure, a general trend of decreasing H_b/d_b with increasing H_b/T^2 for each beach slope is apparent. An increase in H_b/d_b with increasing beach slope for a given steepness is also in evidence. Thus a given deepwater wave will have a higher H_b/d_b on a steep beach than it will have on a flat beach.

A series of straight line envelope curves were constructed to the data so that they would be tangent to Miche's transition curve at the higher values of H_b/T^2 . Consequently, the form of the relationship between H_b/d_b and H_b/T^2 is taken to be

$$\frac{H_b}{d_b} = b[m] - a[m] \frac{H_b}{T^2} \tag{9}$$

where $b[m]$ and $a[m]$ are functions of beach slope. The function, $b[m]$ is the value of H_b/d_b when H_b/T^2 equals zero and $-a[m]$ is the slope of the envelope curve. Values of $1/b[m]$ and $a[m]$ found from the empirical envelope curves are shown on Figure 2. Two sets of data for $1/b[m]$ and $-a[m]$ are shown for each function; one set (solid circles and squares) is based on an envelope curve over all the data for the given slope while the second set (open circles and squares) is based on an envelope curve to the data with the highest data point omitted. Approximating equations for $1/b[m]$ and $a[m]$ are also given on Figure 2. The forms of the approximating functions were chosen because of their asymptotic behavior for flat and steep slopes. As the beach slope approaches zero, $b[m]$ approaches 0.78 ($1/b[m] \rightarrow 1/0.78 = 1.28$) and $a[m]$ approaches zero so that H_b/d_b is independent of H_b/T^2 and equals McCowan's value of 0.78. As the slope approaches infinity (a vertical wall), the assumed upper limit on H_b/d_b was taken as twice the theoretical value of 0.78 or 1.56; consequently, the limiting value of $1/b[m]$ is $1/1.56$ or 0.64. The equations for $a[m]$ and $b[m]$ are shown on Figure 2 and are given by,

$$a[m] = 1.36(1.0 - e^{-19m}) \tag{10}$$

(sec²/ft)

and

$$b[m] = \frac{1.0}{0.64(1.0 + e^{-19.5m})} \tag{11}$$

Straight line envelope curves based on equations 9, 10 and 11 are shown on Figure 1 for beach slopes of 1:∞, 1:50, 1:20, 1:10, and 1:0. The relationship in terms of d_b/H_b as a function of m and H_b/T^2 is shown on Figure 3 along with an upper envelope to d_b/H_b defined by the data (corresponding to the lower H_b/d_b envelope on Figure 1).

For flat slopes, a transition may be defined by the intersections of the sheath of curves defined by equations 9, 10 and 11. This is obtained by solving for the intersections of equation 9 and the equation,

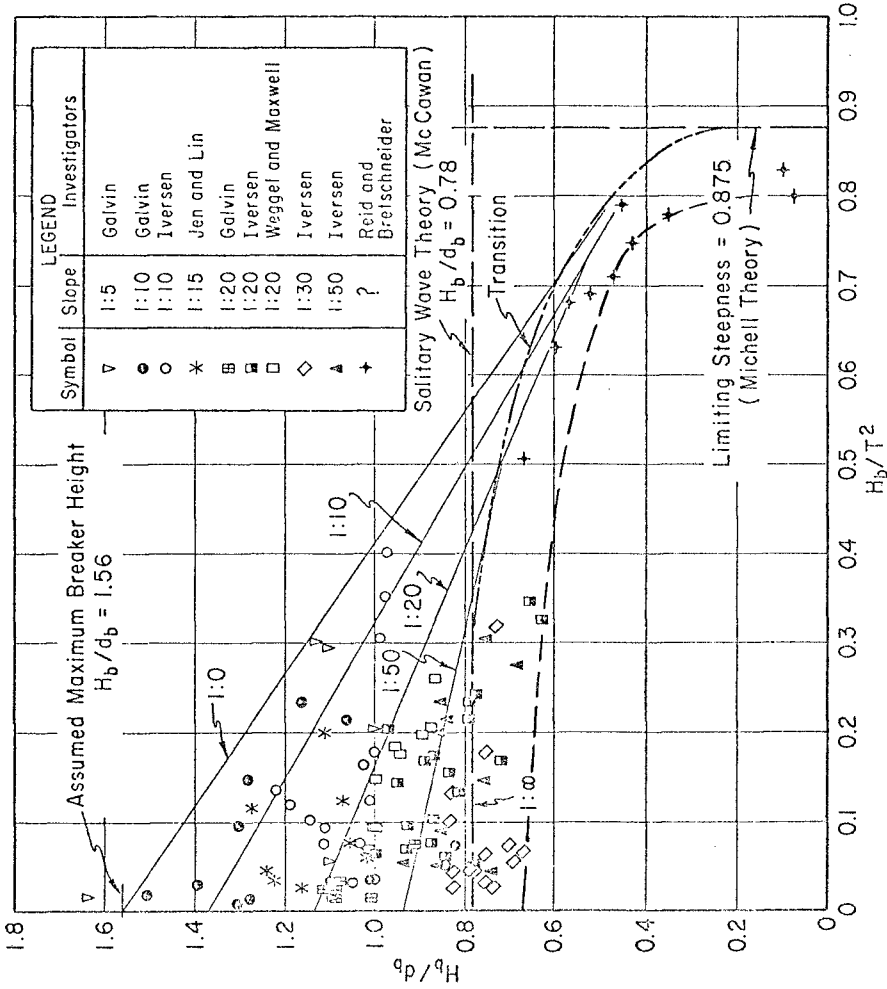


Figure 1. Experimental Observations of d_b/H_b vs. Breaker Steepness, H_b/T^2 .

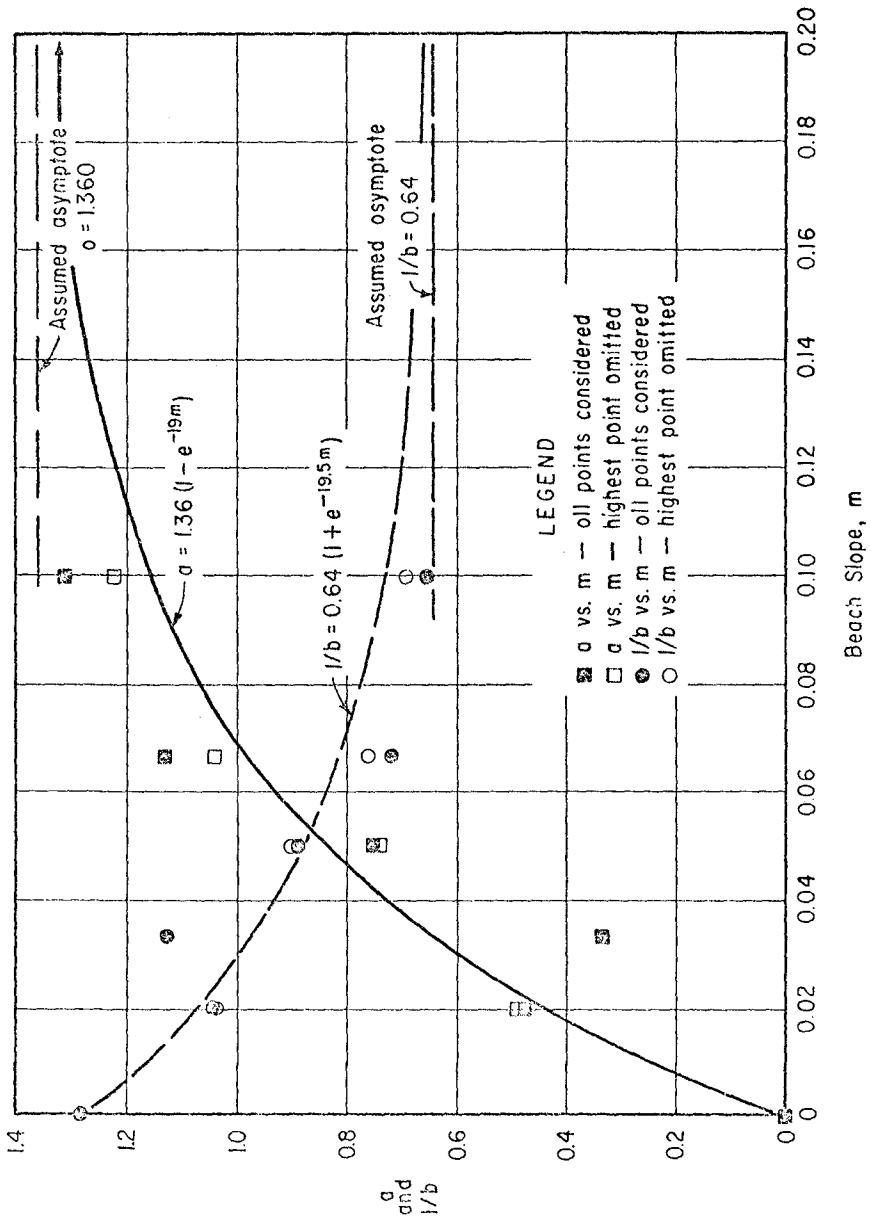


Figure 2. Variations of the Functions a and 1/b with Beach Slope.

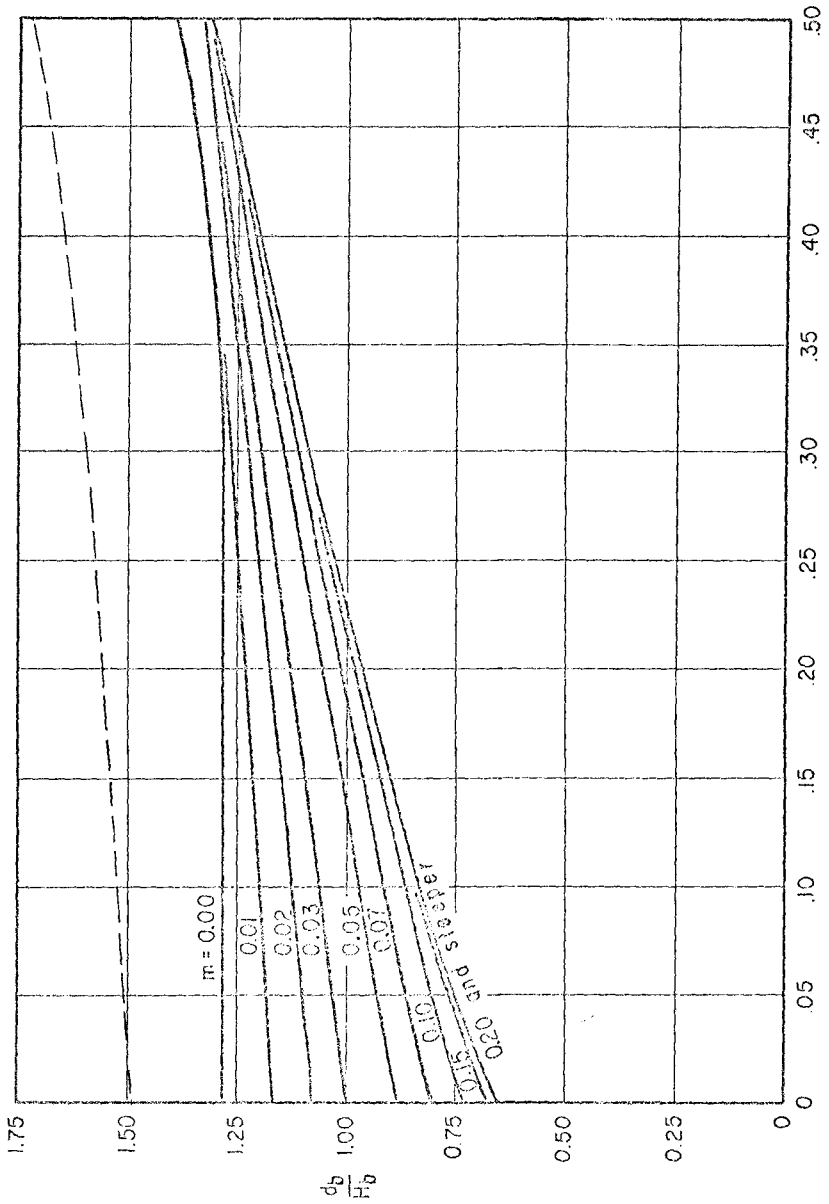


Figure 3. Empirical Equation of d_b/H_b vs. Breaker Steepness for Various Values of Beach Slope H_b/T^2 (ft./sec.²)

$$\frac{H_b}{d_b} = b[m+dm] - a[m+dm] \frac{H_b}{T^2} \tag{12}$$

This empirical transition is given in parametric form by,

$$\frac{H_b}{d_b} = b[m] - a[m] \left\{ \frac{db[m]}{dm} \right\} \left\{ \frac{dm}{da[m]} \right\} \tag{13a}$$

and

$$\frac{H_b}{T^2} = \left\{ \frac{db[m]}{dm} \right\} \left\{ \frac{dm}{da[m]} \right\} \tag{13b}$$

for $0 \leq m \leq 0.105$. The transition given by equation 13 is defined for $0.30 < H_b/T^2 < 0.875$. Evaluating the derivatives in equation 13 gives,

$$\frac{H_b}{d_b} = \frac{156 - 1.6e^{-0.5m} + 4.72e^{-19.5m} - 1.6e^{-20m} + 3.16e^{-39m}}{(1.0 + e^{-19.5m})^3} \tag{14a}$$

and

$$\frac{H_b}{T^2} = \frac{1.18e^{-0.5m}}{(1.0 + e^{-19.5m})^2} \tag{14b}$$

for the transition. Equation 14b also gives the largest value of H_b/T^2 for which equation 9 should be used. For values of H_b/T^2 greater than this maximum value, the value of m which satisfies equation 14b for the given H_b/T^2 should be used in equation 14a to determine H_b/d_b . Alternatively, equation 4 could be used.

MAXIMUM DESIGN BREAKER HEIGHT

The maximum breaker height that actually strikes a coastal structure can be somewhat higher than the value obtained by simply substituting the depth at the structure toe into equation 9. The breaker height data analyzed pertain to the point where breaking is initiated; consequently if breaking is initiated some distance seaward of the structure and the wave travels to the structure during the breaking process a larger breaker than that predicted by equation 9 will strike the structure. If the depth at the toe of the structure is denoted by d_s , geometrical considerations give a relationship between d_s and the critical breaking depth, d_b , which results in the maximum breaker height. This relationship is given by,

$$d_s = d_b - m X_b \tag{15}$$

where m is the beach slope and X_b is the distance traveled by the wave during breaking. If H_b is used to make equation 15 dimensionless, equation 16 results;

$$\frac{d_s}{H_b} = \frac{d_b}{H_b} - m \frac{X_b}{H_b} \tag{16}$$

Galvin (2, 3) investigated the distance traveled by plunging breakers and found,

$$\frac{X_b}{H_b} = 4.0 - 9.25m \quad (17)$$

Strictly, equation 17 should be applied only to plunging breakers; however, in the absence of breaker travel data for other breaker types, it will be assumed generally applicable. Combining equations 9, 10, 11, 15 and 16 results in a quadratic equation for H_b/d_s in terms of m and d_s/T^2 . The dimensionless variables H_b/d_s and d_s/T^2 are selected here because m , d_s and T are usually known at the outset of a design problem; hence, the unknown H_b can be found from known variables. The expression for H_b/d_s is given by,

$$\frac{H_b}{d_s} = \left\{ \frac{1.0}{ma(18.5m-8.0)} \right\} \left\{ a + \frac{1}{d_s/T^2} [1.0 + 9.25m^2b - 4.0mb] - \sqrt{\left[a + \frac{1}{d_s/T^2} (1.0 + 9.25m^2b - 4mb) \right]^2 - \frac{4mba}{d_s/T^2} (9.25m - 4.0)} \right\} \quad (18)$$

where a and b are given by equations 10 and 11. For ease of application equation 18 is presented in graphical form on Figure 4 for several specific beach slopes.

ILLUSTRATIVE EXAMPLE

Figure 4 may be used to obtain an estimate of the maximum breaker height a coastal structure could experience. For example, given a structure sited in water having a maximum design depth, $d_s = 20$ ft, and fronted by a beach having a slope of 1:20 ($m = 0.050$), determine the maximum breaker height for a 10 second design wave period. Calculate $d_s/T^2 = 20/(10)^2 = 0.20$ and enter Figure 4 to the curve for $m = 0.050$ and read $H_b/d_s = 0.83$. Therefore $H_b = 0.83 d_s$ or $H_b = 16.6$ ft. Breakers larger than 16.6 feet will break farther offshore from the structure and will have dissipated a sizable amount of their energy before reaching the structure. While smaller breakers may strike the structure, they will not establish a critical design condition. Thus the bathymetry in front of a structure can be thought of as a filter for the wave spectrum causing larger waves to break farther offshore and permitting only breakers with heights less than or equal to the maximum breaker height to reach the structure. Figure 3 (or equations 9, 10, and 11) may now be used to establish the critical depth in which breaking is initiated. Calculate $H_b/T^2 = 16.6/(10)^2 = 0.166$ and enter the figure using the curve for $m = 0.050$. Read $d_b/H_b = 0.64$; hence, $d_b = 0.64 H_b = 0.64 (16.6) = 10.6$ ft. The breaking depth for the given breaker height can be bracketed by using the upper envelope (dashed) curve to all the breaker data shown on Figure 3. For the given wave steepness $H_b/T^2 = 0.166$, the upper limit for d_b/H_b is 1.55 and therefore $\{d_b\}_{max} = 25.7$ ft with the critical breaking depth being $d_b = 10.6$ ft.

In order to establish the deepwater wave height that results in the maximum breaker height, refraction data for the site and the

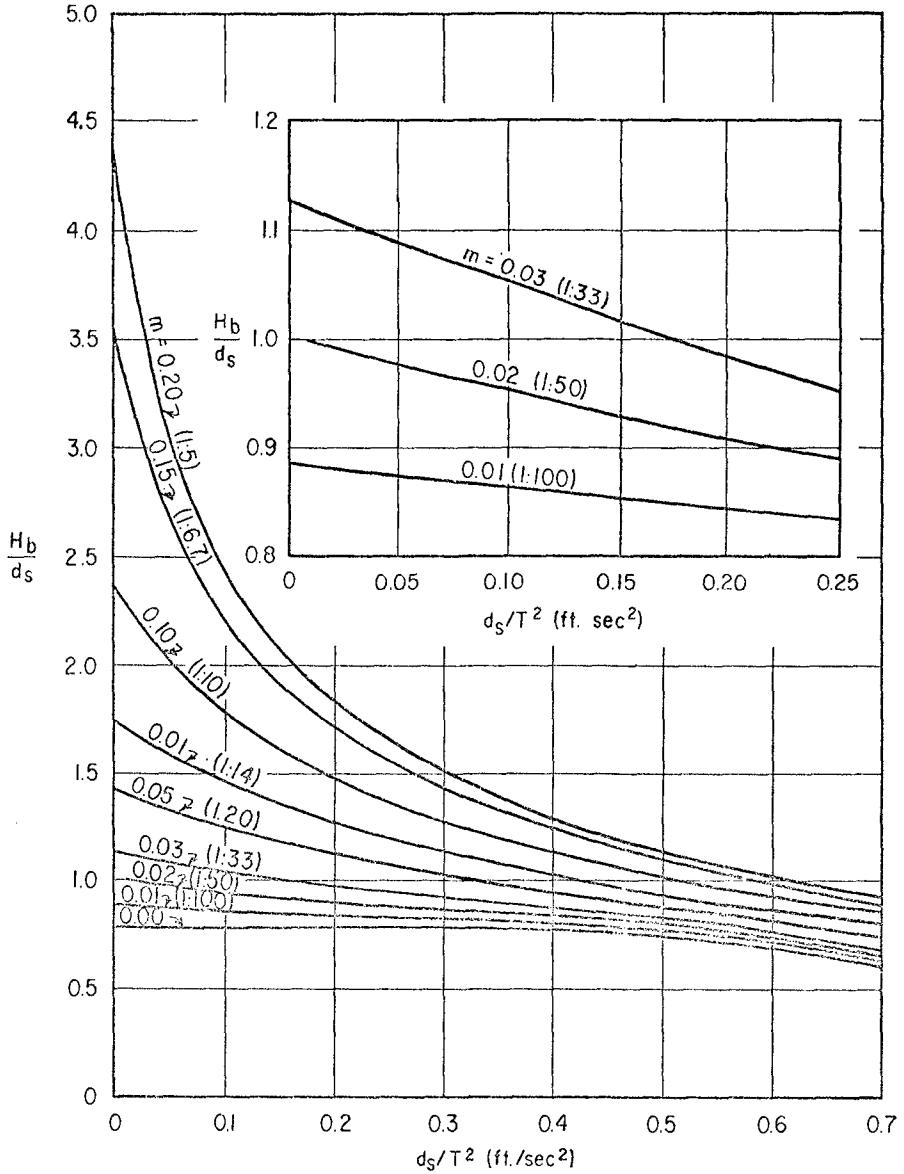


Figure 4. Dimensionless Maximum Breaker Height, H_b/d_s , vs. Depth at Structure, d_s/T^2 .

breaker height index, H_b/H_0' , is required. Figure 5, which presents the breaker height indices of Goda (4) in a modified form, can be used to find the desired deepwater wave height. If, for example, the refraction coefficient for waves having a period of 10 seconds approaching from a given direction is $K_R = H_0'/H_0 = 0.95$, where H_0' is the unrefracted deep water height and H_0 is the actual deep water height, enter Figure 5 with the calculated $H_b/T^2 = 0.166$ and from the curve for $m = 0.020$ read $H_b/H_0' = 1.12$. Then, $H_0' = H_b/1.12 = 16.6/1.12 = 14.8$ ft and $H_0 = H_0'/K_R = 14.8/0.95 = 15.6$. Consequently a 15.6 ft deepwater wave with $T = 10$ sec. approaching from the given direction will result in the maximum breaker height on the structure.

If the slope fronting the structure is irregular an average slope between the depth at the toe of the structure and the depth at breaking should be used in equations 16 and 17 and the slope seaward of db used in equations 9, 10 and 11. Equation 18 and Figure 4 were derived by assuming that a constant beach slope extends some distance seaward of the structure; that is, the slope on which the wave breaks is identical to the slope used in equation 16. This assumption is not valid for the case of a varying beach slope, hence the appropriate slopes should be used with the equations to establish the upper limit on breaker height.

DISCUSSION AND CONCLUSIONS

The applicability and validity of the methods presented here are limited by the experimental conditions under which the data were obtained. With the exception of a few of the data points shown on Figure 1, the data were obtained in laboratory wave tanks and are subject to the scale effects and reflections often inherent in such facilities. In addition, the studies were conducted on impermeable, smooth, uniform, unobstructed slopes with monochromatic waves. The effects of interaction between waves of different heights and periods were not present. Also the effects of reflections from structures on the slope were not considered. Jackson (7) presents some data on a rubble structure's influence on breaking conditions on a 1 on 10 beach slope. Figure 6 gives a comparison of Jackson's data with the data of Iversen (6) and Galvin (2) for the 1:10 slope. The equations of Galvin (2) and Miche (10) are also shown on Figure 6 along with an equation based on the present analysis and Airy wave theory (solid line).

The method proposed here for estimating maximum breaker height is believed reasonably conservative. For beach slopes steeper than 1:10, additional verification is recommended and additional research on the effects of reflecting structures on breaker conditions is indicated.

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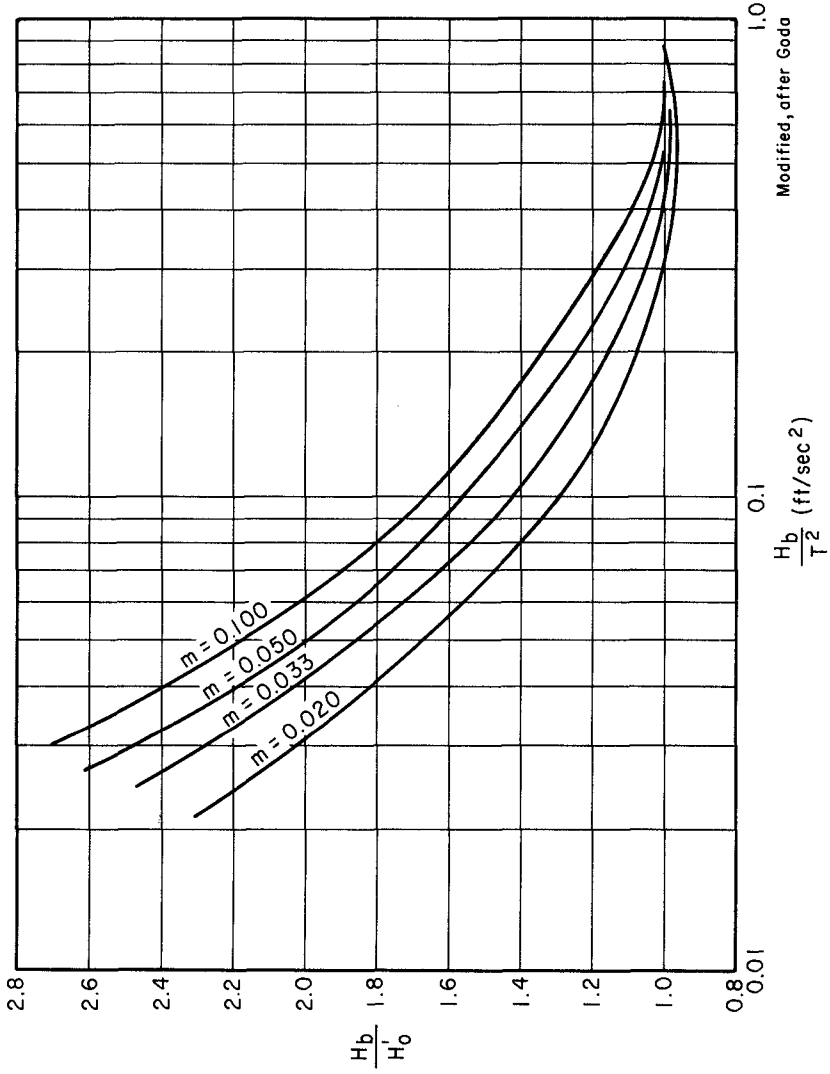


Figure 5. Breaker Height Index vs. Breaker Steepness, H_b/T^2

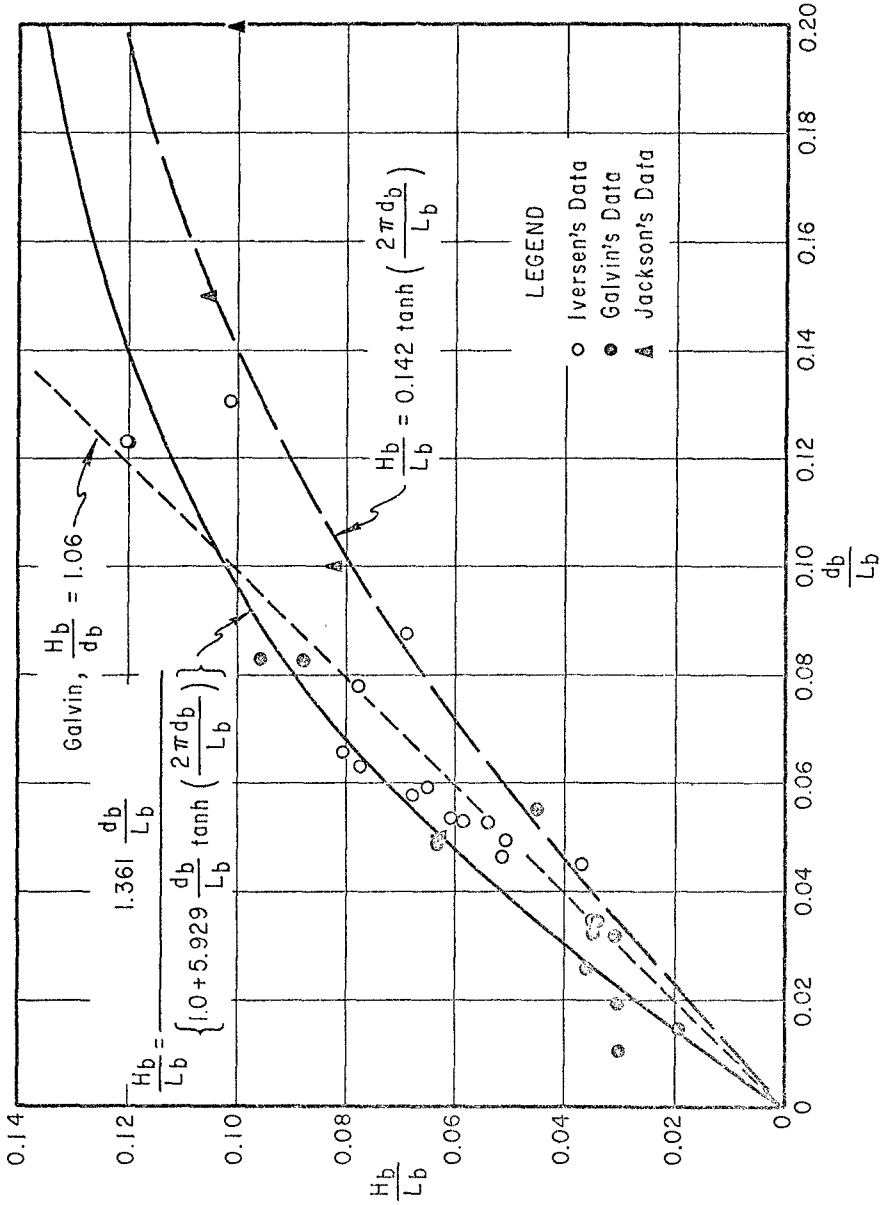


Figure 6. Breaker Steepness, H_b/L_b , vs. Breaker Depth, d_b/L_b , for a Beach Slope of 1:10.

APPENDIX I REFERENCES

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APPENDIX II - NOTATION

The following symbols are used in this paper;

$a[m]$	= slope of H_b/d_b vs H_b/T^2 line, a function of beach slope.
$b[m]$	= value of H_b/d_b for $H_b/T^2 = 0$, a function of beach slope.
d	= depth.
d_b	= depth at which breaking is initiated.
d_s	= depth at the toe of a structure.
g	= gravitational acceleration.
H	= wave height
H_b	= wave height at breaking (breaker height)
H_0'	= unrefracted deepwater wave height
m	= beach slope.
L	= wave length.
L_b	= wave length at breaking.
L_0	= wave length in deep water
T	= wave period.
x_b	= distance traveled by a plunging wave during breaking
β_b	= ratio of d_b to H_b , a function of beach slope.