CHAPTER 27

ENERGY TRANSFER MECHANISM FOR FINITE AMPLITUDE WAVES

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ABSTRACT.-

A basic mechanism is proposed to explain the growth of finite amplitude water waves due to the effect of normal stresses. The proposed mechanism can probably be applied to the whole range of wave's growth, starting from a small amplitude wave up to the case of a limiting and breaking wave condition. The transfer of energy from air flow to the water wave is explained by the existance of a circulation flow pattern above the water surface, which is responsable for a phase-lag of the normal stress distribution in relation to the wave's profile. Such a circulation flow pattern extends throughout the whole wave length and it is quite different from the classical concept of flow separation, as postulated by Jeffrey's. The difference becomes as a result of considering a different boundary condition at the interface. Experiments performed on wavy models with moving boundary conditions, for small amlitude waves and finite amplitude waves showed that the normal stress distribution is similar in both cases, and displays a noticible phase-lag with respect to the wave's profile. It was observed that for both cases a circulating flow pattern was present above the water surface, which indicated some relation between the flow vorticity above the wave and the normal stress distribution.

To prove the role of circulation in the energy transfer mechanism, a model was built with water and mercury as working fluids. In this model the interface was initially non disturbed, when both fluids move in opposite directions. However, when a forced circulation was applied by means of a variable speed rotor located above the interface, a wave would form. The wave would be inicially of small amplitude, but with an increase of circulation would become of finite amplitude, then reach the angular crest condition and finally would reproduce the breaking condition at the crest.

The obtained experimental results proved the importance of the circulation flow pattern present above the wave surface, and suggest that a mathematical model could be formulated based on vorticity analysis, which would be able to provide an explanation for the energy transfer mechanism due to normal stresses at all stages of wave's growth.

INTRODUCTION.-

It has been proven both theoreticly (ref. 1,2) and experimentally (ref. 3,4), that in the case of already formed small amplitude waves , the normal stress distribution displays a phase-lag with respect to the wave's profile, which accounts for the energy transfer due to this stresses from wind to the water waves. However, these linear theories fail when the wave reaches a finete amplitude, being the only available mechanism that of Jeffrey's (ref.5) based on flow separation. The experimental data used to estimate Jeffrey's sheltering coefficient where obtained from fixed boundary wavy models tested in wind tunnels or water channels. Such an empirical approach had not been succesufull to provide a good agreement between the predicted values and real observations, and hence the problem remain unsolved. Since most of the water waves observed in nature are of finite amplitude, it is important to understand the basic mechanism of their growth, and this paper intend to provide new basic ideas to develop a better understanding on the subject.

To properly analyse the air motion over a wave profile which is fixed in space, a coordinate system which moves with the wave has to be introduced. In this moving coordinate system the air flow (far away from the interface) will move at a speed U–C, while water will flow in opposite direction with a speed equal to the wave celerity C. At the interface the water particle velocity will vary with location along the wave, being larger than C at the trought and smaller than C at the crest. A graphical representation of the coordinate transformation is shown in fig. 1 .

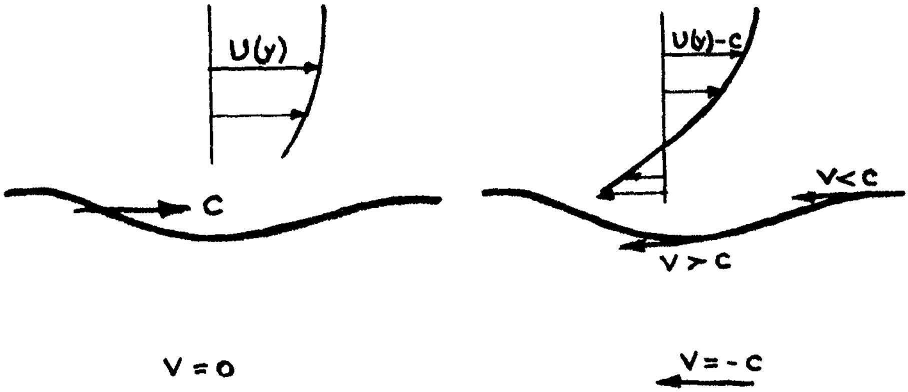
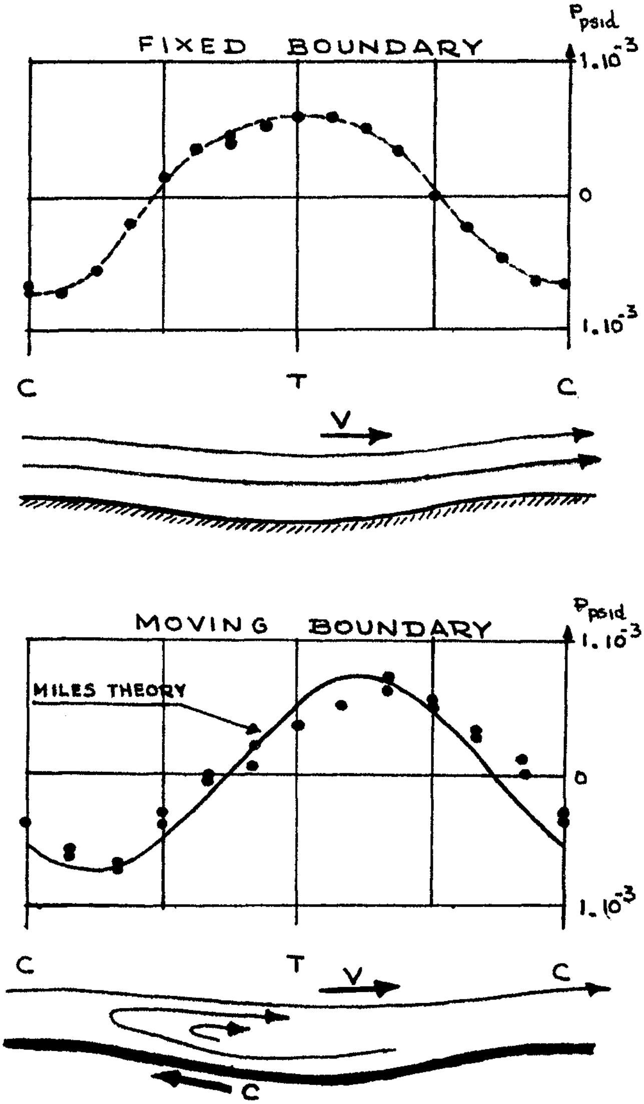


FIG. 1

Unsteady Flow Picture | Steady Flow Picture

When the wave's amplitude is small as compared with the wave's length the difference of particle velocities at the interface is small and can be neglected. Experimental results obtained in wavy models with constant velocity boundary conditions (ref.3,4) showed that the effect of the moving boundary is to produce a phase-lag in the normal stress distribution. In fig. 2 a comparison is presented between the obtained experimental results for normal stress distribution with fixed and movin boundary conditions. This phase-lag in the pressure distribution has been also obtained in laboratory measurements over wind generated waves (ref. 6), and in the field (ref. 7).

Experimental results for finite amplitude waves with the appropiate moving boundary conditions were reported by the author at the XIIth Conference on Coastal Engineering held in Washington in 1970 (Unfortunately was not published in the proceedings). The obtained results, which will be briefly described in this paper, showed that for finite amplitude waves the pressure distribution looks almost the same as in the case of small amplitude waves, and is quite different from the normal stress distribution



PRESSURE DISTRIBUTION OVER SMALL AMPLITUDE WAVES

FIG. 2

obtained from fixed boundary experiments as reported by Motzfeld (ref. 8) and others. A comparison between both results can be seen in fig. 5. Flow visualization showed that instead of a classical separated region as obtained in fixed models (fig. 6), the flow over the wave displaied a large circulation region which extend throughout the wave's length (fig. 7), as if the flow would separate from one crest to the another one. This flow picture coincided with that proposed by the author in 1966 (ref. 3) for a limiting wave profile.

The obtained experimental results for pressure distribution and flow visualization suggested that the circulation flow pattern above the wave surface is responsable for the phase-lag observed in the normal stress distribution, and hence, for the growth of the wave. If that is true, a theory could be developed based on vorticity which will cover the whole range of wave's growth. However, the role of circulation in the process of energy transfer is not obvious, and it is the purpose of this paper to provide a definite foundation for a future mathematical model based on vorticity, which would provide the answer to such important problem.

MOVING BOUNDARY EXPERIMENTS FOR FINITE AMPLITUDE WAVES.-

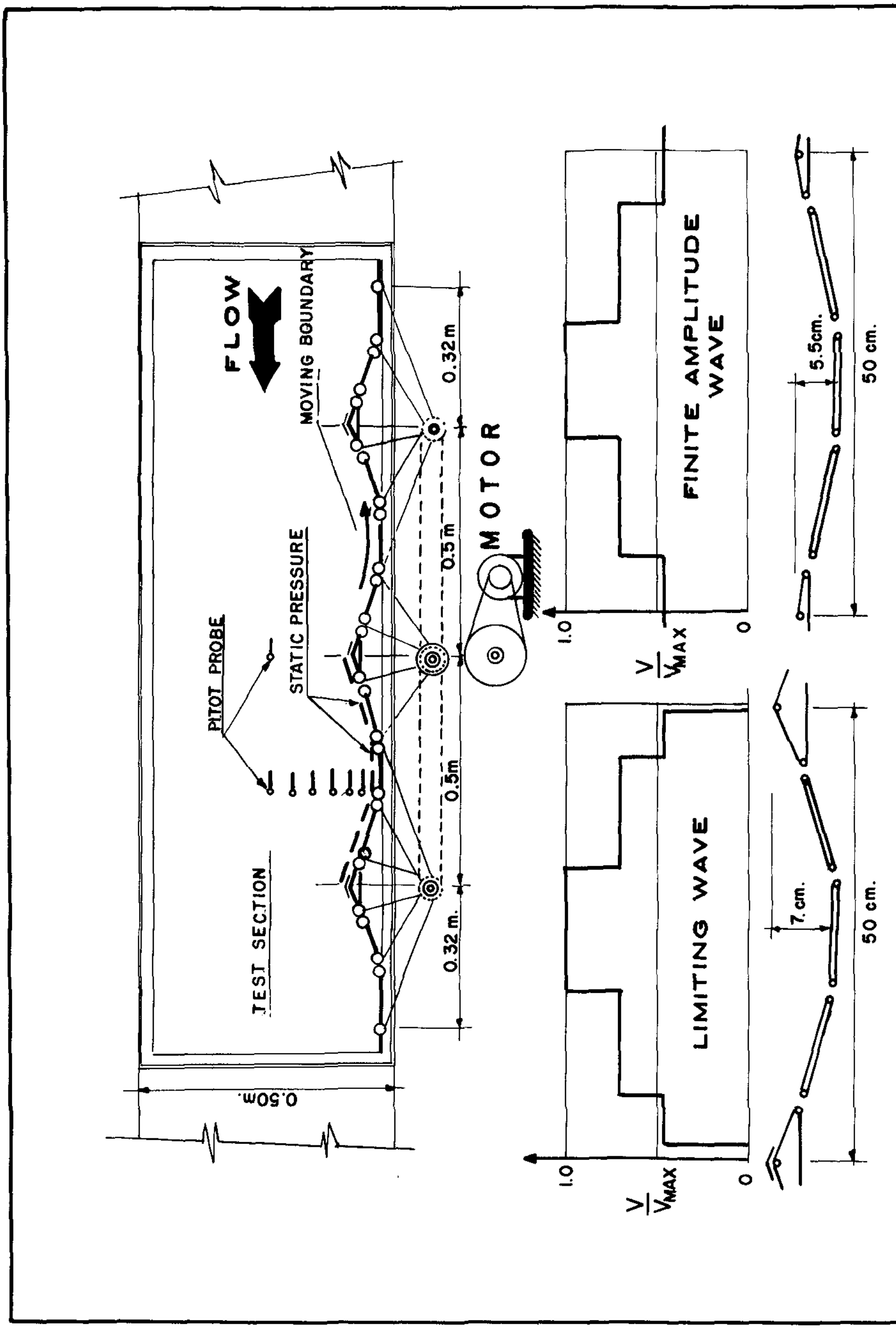
The result of this experimental work were presented at the XIIth. Conference on Coastal Engineering held in Washington in 1970, and therefore only a summary of the used procedure and the main results will be included in this paper.

Since in the case of a finite amplitude wave the boundary can not be considered as moving with a constant speed, an approximation to the real particle velocity distribution along the wave surface had to be introduced. Such an approximation was made by dividing the wave profile in several regions, each of them, moving at different constant velocities.

Such a moving boundary model was built in a water tunnel at the Hydraulic Laboratory of the Central University of Venezuela, where three waves with 50 cm. wave length and 5,5 cm heigth were represented. The total test section had the dimenssions of 1,8 x 0,5 x 0,1 mt. and the water flow velocity could be varied from 0 to 3 mt./sec. The moving boundary was set to move at a speed corresponding to the celerity of a gravity water wave of the same wave length, and the velocity distribution along the boundary was adopted in the form shown in fig. 3 . The motion of each section of the wave surface was achieved through a combination of wheels with different diameters connected by chains. A view of the installation can be seen in fig. 4.

A series of test were performed with several water flow velocities, for both cases, of fixed and moving boundary conditions. For each case pressure distribution was obtained by means of a series of static pitot tubes located just above the wave's profile, measuring the pressure difference with a highly sensitive pressure transducer. Also, flow visualization was obtained for each run by adding bouyant plastic particles to the water flow and taking pictures of the flow field above the wave's profile.

In fig. 5 a typical set of data is shown for the normal stress distribution for both cases os fixed and moving boundary conditions, being in the later case the ratio between the water velocity and the mean boundary speed equal to unity. In fig. 6 and 7 the corresponding flow visualization is presented. From these results it can be concluded that in the case of a fixed boundary model, the normal stress distribution and the flow visualization coincides with the known experimental data. However, when the boundary is moving , the normal stress distribution curve presents an astonishing re-



MOVING BOUNDARY INSTALLATION

FIG. N° 3

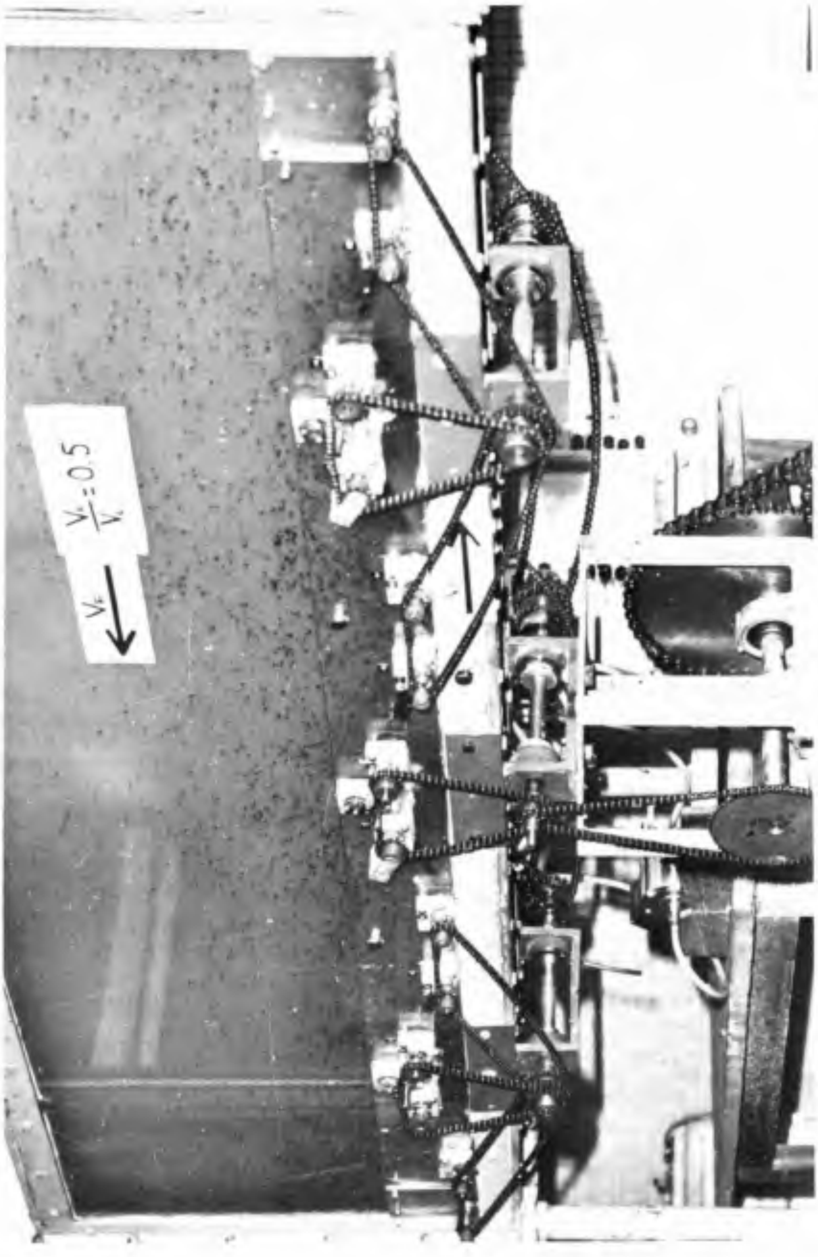


Fig. 4.- General View of the Moving Boundary Installation for Finite Amplitude Waves

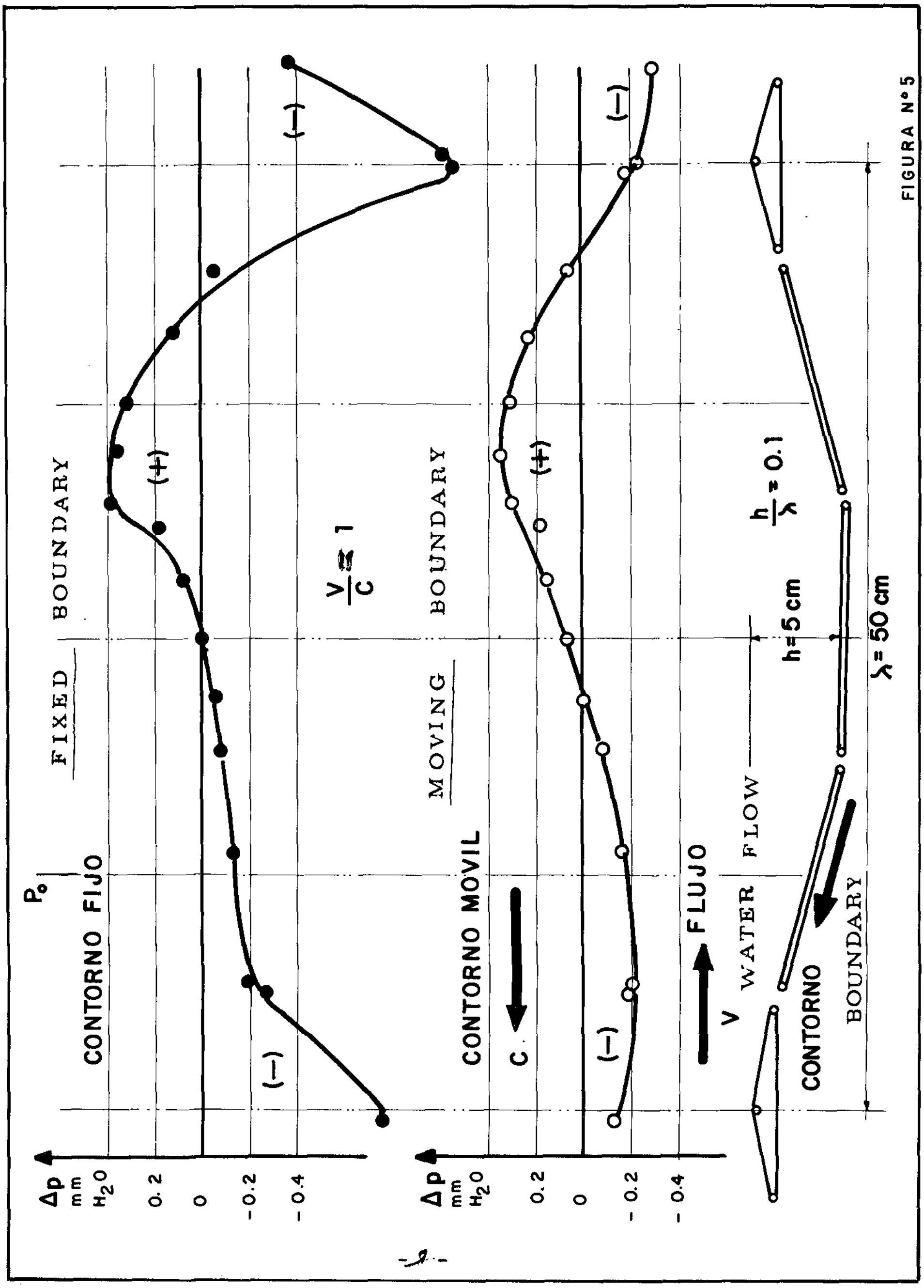

Δp mm H2O
0.2
0
-0.2
-0.4
P_o
CONTORNO FIJO
FIXED BOUNDARY
(+)
(—)
(—)
V/C ≅ 1
CONTORNO MOVIL
MOVING BOUNDARY
C
(+)
(—)
(—)
Δp mm H2O
0.2
0
-0.2
-0.4
V
FLUJO
WATER FLOW
CONTORNO
BOUNDARY
h=5 cm
h/λ = 0.1
λ = 50 cm

FIGURA N° 5

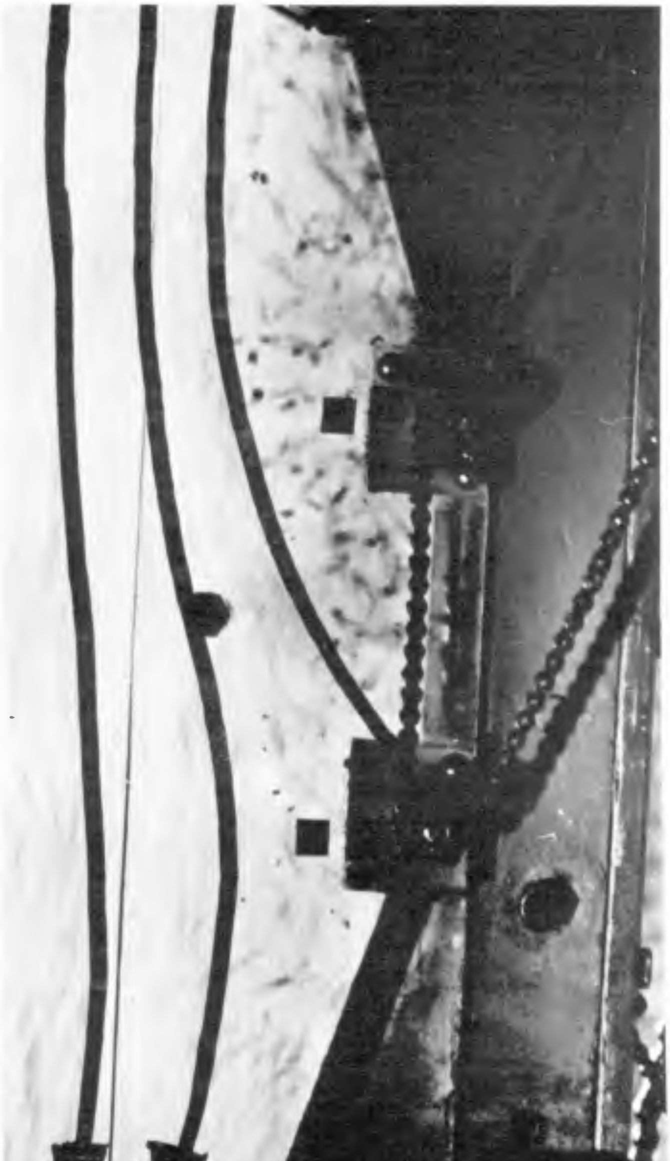


Fig. 6. Flow Characteristics Over a Fixed Wavy Boundary.

Fig. 7.- Flow Characteristics Over a Finite Amplitude Wave With Moving Bondaries

Fig. 8.- Flow Characteristics Over a Limiting Wave With Moving Boundaries.

gular form, and it is almost similar to the one shown in fig. 2 for the case of small amplitude waves. The variation of the pressure magnitude between both crests is due to the fact that the pressure difference was recorded in relation to a fixed point in the water tunnel. From fig. 7 it can be seen that the flow pattern above the wavy surface presents a large circulation which extends from one crest to the other one. This result coincides with the flow picture obtained by the author in 1966 for the case of a limiting wave profile (ref. 3), which is shown in the picture of fig. 8.

The fact that the pressure distribution curve is similar in shape and in phase-lag (for the same ratio of flow velocity to wave celerity), for small and finite amplitude waves, suggest that the mechanism that produces such effect must be similar in both cases. Since in small amplitude wave experiments a circulation flow pattern over the wave was also observed, it is almost certain that such phenomena has something to do with the phase-lag of the pressure distribution curve. To prove this statement, a different experimental installation was built, and its description follows.

STRATIFIED FLOW EXPERIMENTS WITH FORCED CIRCULATION.-

To prove the role of vorticity on the process of wave's growth, a model was built with two stratified flows moving in opposite directions with a forced circulation produced above the interface. If vorticity has something to do with the process of wave's growth, a wavy disturbance on the interface should occur when a forced circulation is applied above the interface.

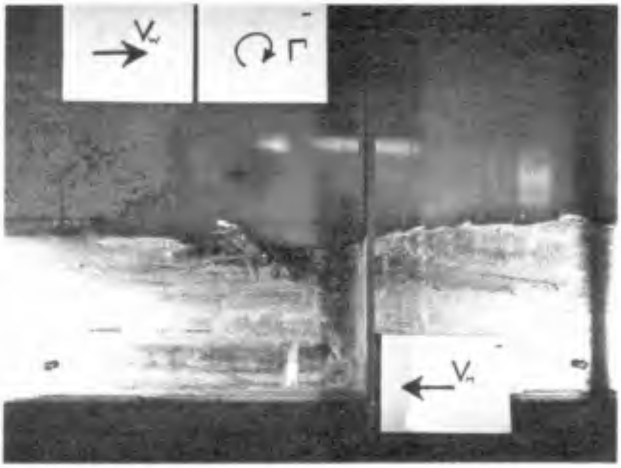
The working fluids for such a model were mercury and water, which were moved in opposite directions in a closed test sections with dimenssions 1,2 x 0,7 x 0,01 mt. The motion of both fluids was achieved by means of pumps and the mean velocity could be controlled by valves . Circulation was produced by mechanical means, using a cross made out of stainless steel which rotated in a clockwise direction with a speed that could be varied by an electrical motor.

When both fluids moved without any applied circulation the interface remained undirsturbed. However, as soon as circulation was applied a small amplitude wave would form at the interface. The wave location in relation to the vortex coincided with the expected position, if the pressure distribution would be similar to that obtained from the moving boundary experiments. With the increase of circulation the wave profile would grow, until it reaches the angular shape at the crest, any further increase in the magnitude of circulation produced a breaking effect at the wave's crest. In fig. 9 and 10 a general view of the apparatus is shown, along with a set of pictures of the water-mercury interface for different degrees of the applied circulation. It can be seen that the position of the vortex in relation to the formed wave remains the same regardless of the amplitude of the wave , and coincides with the expected location based on the pressure distribution along the wave's profile obtained from the moving boundary experiments. Also, this experiments showed that the wave length was a function of the mercury mean velocity, as it should be expected from the steady state analysis of a deep water gravity water wave.Therefore, for a given mercury mean velocity a set of waves could be generated by installing a series of rotors located at intervals equal to the wave's length.

General View of the Stratified Flow Installation.

Interface Disturbance when a Small Circulation is Applied

Fig.9



Wave at the Interface Reaching Almost the Limiting Condition

Breaking of the Crest Produced by too Much Applied Circulation

Fig. 10

CONCLUSIONS.-

The obtained experimental results proved that the circulating flow pattern prevailing over the wave surface of small or finite amplitude waves, plays an important role in the energy transfer process from air flow to the water waves, due to the effect of normal stresses. The role of this circulation is to produce a phase-lag in the pressure distribution curve, in relation to the wave's profile. The magnitud of the phase-lag is the same for small or finite amplitude waves, provided that the ratio of flow velocity to wave celerity remains constant.

The fact that the wave's growth can be reproduced by a variable strength vortex located above the interface of two stratified flows, suggest the possibility of developing a mathematical model based on vorticity considerations, which could be applied for the whole history of wave's growth.

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