CHAPTER 33

TURBULENT CURRENTS IN THE PRESENCE OF WAVES

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1. ABSTRACT

This paper presents an approximate theory for the reduction of the velocity of a current due to the presence of sinusoidal waves.

For a given slope, S, in water of constant depth, d, the current velocity profile is

$$U(z) = U_f \,(2.5 \ln \frac{z}{z_o} - A) \tag{1}$$

as a function of the height, z, above the bed. Eq. 1 is valid only above the thin wave boundary layer near the bed, the roughness of which is $k = 30\ z_o$. U_f is the current friction velocity defined by

$$\rho\ U_f^2 = \gamma\ d\ S = \tau_{cw} \tag{2}$$

where τ_{cw} is the current shear stress in the presence of waves.

Values of A can be found from:

Fig. 2 where A_1 applies when the direction of wave propagation is parallel to the current direction, and

Fig. 3 where A_2 applies when the direction of wave propagation is perpendicular to the current direction,

cf. Notation in Sec. 2.

The theory is based upon a number of assumptions (see Sec. 4).

2. NOTATION

A_1 Constant of velocity defect for a current parallel to the direction of propagation of the waves.

A_2 Constant of velocity defect for a current perpendicular to the direction of propagation of the waves.

a_b Amplitude of particle orbit at bed for sinusoidal wave according to potential theory.

d Depth of water.

f_w Characteristic friction coefficient in wave boundary layer.

k Roughness of bed (Nikuradse).

n_c Turbulent (kinematic) viscosity in a pure current.

n_{cw} Turbulent viscosity in combination of current and waves.

n_w Average turbulent viscosity in pure wave motion.

r Wave friction velocity ratio u_f/u_b.

S_1 Slope of still water level for a current in the direction of wave propagation (for constant depth).

S_2 Slope of still water level for a current perpendicular to the direction of wave propagation (for constant depth).

U_1 Current velocity in the direction of wave propagation.

U_2 Current velocity perpendicular to the direction of wave propagation.

U_f Current friction velocity.

u Velocity in wave boundary layer.

u_b Maximum orbital velocity at bed for sinusoidal wave according to potential theory.

u_f Maximum wave friction velocity.

v Velocity.

z Height above bed.

z_o Roughness parameter = $k/30$.

γ Specific gravity.

δ Characteristic thickness for wave viscosity decreasing exponentially with height above bed.

ρ Specific density.

τ_c Shear stress in a current.

τ_{cw} Current shear stress at the bed in the presence of waves.

τ_w Shear stress in the wave boundary layer.

3. INTRODUCTION

In recent years there has been much progress with respect to the theory of interaction of currents and waves. Most of this progress, however, has taken place within the realm of potential flow, i.e. without consideration of bed friction and turbulence.

Much less has been done with a view to the influence of the bed friction on the combination of currents and waves, for the simple reason that the turbulence problems of the wave boundary layer and of the combined flow are very complicated. In some cases it has been attempted to approach the turbulence problem of the combination by simple assumptions taken from the turbulence theory of steady flow, for example that the Prandtl mixing length be known. Indeed, to-day no theory of the combined bed friction would be possible without a number of simplifying and, perhaps, quite hypothetical assumptions.

In the potential theory of the interaction of waves and currents, the main problem is the influence of varying depths and current velocities, and most interesting results have been obtained. However, the very solution of the problems in nature cannot be obtained without knowledge of the bed friction since a natural problem will normally be posed in the following manner: "For a region exposed to a given wave motion the mean water levels are known along the boundary, for example as a result of tides or storm surges; it is then desired to determine the current and wave fields within the region, with due consideration of the interaction effects and the bed friction hampering the current."

In general, no methods are available for the rational solution of such problem because of the influence of wave breaking. For problems outside the breaker zone, however, the approximate theory in this paper, which assumes sinusoidal waves, might be of some value.

Since no experiments, against which the theory could be tested, are available, the present results may be taken only as an indication of the order of magnitude of the reduction of the current velocities due to the influence of waves.

In addition, it must be stressed that the theory is valid for fixed beds only. If the sediments of the bed are brought into motion by the waves, they are expected to act as a "lubricating" agent, with the result that the current is less hampered or, perhaps, for heavy suspension of sediments, even enhanced by the waves.

For the turbulent interaction of currents with sinusoidal waves over a fixed bed, one may distinguish between 4 zones from the surface to the bed:

Z_1 Zone of Low Current Frequencies: In this zone the dominant frequencies of the turbulent stresses are much lower than the wave frequency, so that turbulent production, diffusion and dissipation take place independently of the waves.

Z_2 Zone of Intermediate Current Frequencies: In this zone the non-linear terms may give some interaction between the turbulent stresses and the wave frequency. The wave-produced bed turbulence has no influence.

Z_3 Transition Zone: In this zone the turbulent viscosity produced by the wave boundary layer near the bed is of the same order of magnitude as the viscosity pertaining to the current turbulence. The dominant frequencies of the turbulence become much larger than the wave frequency towards the lower boundary of this zone.

Z_4 Zone of Dominant Wave Viscosity: In this zone the wave-produced viscosity is much larger than the current-produced viscosity. If the waves are low and the current strong, zone Z_4 may vanish, with Z_3 extending all way to the bed.

In order to give a picture of the extent of these zones, the following situation may be considered: Water depth $d = 10$ m, bed roughness $k = 0.1$ m, current velocity $U = 0.5$ m/s, wave period $T = 10$ s, wave height $H = 3$ m. Then the transition between the zones takes place approximately at the following elevations above the bed: $z_{1-2} \sim 5$ m, $z_{2-3} \sim 0.5$ m, $z_{3-4} \sim 0.1$ m.

The reason for the reduction of the current velocity by the presence of the waves lies in the zones Z_3 and Z_4 where the resulting viscosity is larger, respectively, much larger than the normal current-produced viscosity. When the viscosity is increased the current velocity gradient pertaining to a given current bed shear stress is reduced. This reduction of gradient is largest near the bed where the velocity gradients are particularly large for a pure current.

4. ASSUMPTIONS

The main assumptions of the theory presented below are:

(a) The bed is fixed. It has the equivalent sand roughness $k = 30\, z_o$, where z_o is the roughness parameter used in the following.

(b) The water depth, d, is constant, the still water level and the bed having the same slope, S.

(c) The wave motion at the bed is sinusoidal.

(d) The current velocity, U, is so small compared with the wave celerity, c, that the relative wave motion - as seen by an observer moving with the current - can be calculated from Stokes' first-order theory, cf. Sec. 5.

(e) The interaction between the turbulent stresses of the current and the wave frequency in the zone Z_2 (see Sec. 3) can be neglected.

(f) In the zones Z_3 and Z_4 the pulsations of the current with the wave frequency are not considered, but only the average of the current velocity over the wave period.

(g) The turbulence is everywhere fully developed with negligible molecular viscosity.

(h) The gradient, dU/dz, in the vertical direction, z, of the average current velocity, U(z), can be derived from the current shear stress, τ_{cw}, in the presence of waves and from the combined turbulent viscosity, n_{cw}, by the same formula as for a pure current, cf. Sec. 10.

(i) In the zone Z_3 the combined viscosity, n_{cw}, may be calculated from the current-produced viscosity, n_c, and the wave produced viscosity, n_w, by means of the formula

$$n_{cw} = n_c + n_w \tag{3}$$

for a current in the direction of propagation of the waves, and the formula

$$n_{cw}^2 = n_c^2 + n_w^2 \quad (4)$$

for a current perpendicular to the wave direction, cf. Sec. 9.

(j) At any level above the bed, the current viscosity, n_c, is related to the current velocity gradient, dU/dz, by the same formula as for a pure current, cf. Sec. 6.

(k) Whereas the viscosity in the boundary layer of a pure wave motion actually varies with a frequency twice the wave frequency, it suffices to use an average value, n_w, over the wave period, cf. assumption (f).

(l) The average wave viscosity, n_w, may be calculated from average (numerical) values of the shear stresses and the velocity gradients, as found from measurements in the wave boundary layer, cf. Sec. 7.

5. POTENTIAL WAVE MOTION

For a sinusoidal wave of height H, wave length L, and period T relative to the current, the amplitude of the orbital motion at the bed according to potential theory is

$$a_b = \frac{H}{2} \frac{1}{\sinh (2\pi d/L)} \quad (5)$$

The maximum orbital velocity at the bed is

$$u_b = \frac{2\pi}{T} a_b \quad (6)$$

With good approximation the wave boundary layer at the bed is uniquely determined by a_b and u_b together with the roughness parameter z_o, independent of L and d.

6. CURRENT VISCOSITY

For steady flow the general relation between a shear stress τ and the velocity gradient dv/dz is

$$\tau = \rho\, n\, dv/dz \quad (7)$$

where n is the turbulent (kinematic) viscosity. If this formula is applied to the usual velocity profile of a steady current

$$U(z) = 2.5\, U_f \ln (z/z_o) \quad (8)$$

where U_f is the current friction velocity, the viscosity is found to be

$$n_c = 0.4\, U_f\, z\, (1 - \frac{z}{d}) \quad (9)$$

the coefficient 0.4 being von Kármán's constant.

Since the velocity gradient is

$$dU/dz = 2.5\ U_f/z \tag{10}$$

U_f can be eliminated from Eq. 9 giving

$$n_c = 0.16\ z^2\ (1 - \frac{z}{d})\ \frac{dU}{dz} \tag{11}$$

where the coefficient 0.16 is the square of von Kármán's constant.

Because of assumption (e) in Sec. 4, these formulae apply directly above the wave boundary layer, i.e. above the layer where the wave viscosity has any influence.

In the wave boundary layer z is so small compared with d that Eq. 11 may be written

$$n_c = 0.16\ z^2\ \frac{dU}{dz} \tag{12}$$

Thus the wave boundary layer is within the constant stress layer of the current.

The constant stress layer of a pure current is characterized by the equilibrium between local production and local dissipation of turbulence, i.e. the current viscosity is locally produced, with no essential contribution from diffusion of turbulence. The physical basis of Eq. 12 is the similarity within the constant stress layer, with the vertical mixing length, 0.4 z, as length scale and the velocity gradient, dU/dz, as frequency scale.

When a wave motion is superimposed upon the current, the viscosity is increased within the wave boundary layer because of the wave turbulence, resulting in a reduction of the current velocity gradient, dU/dz, for a given current shear stress, τ_{cw}. It will be assumed, however, that the viscosity, n_c, originating from the current, is still locally produced according to Eq. 12, whereby the mixing length is supposed to play the same role as in a pure current.

Within the wave boundary layer Eq. 9 could not be used, because this formula involves a velocity scale U_f of the current-produced turbulence, and the current-produced vertical velocity fluctuations will be strongly influenced by the wave-produced turbulence. There is much better hope that the mixing-length of the current-produced turbulence is largely independent of the wave-produced turbulence.

7. WAVE VISCOSITY

In the present context the wave viscosity is of interest inasmuch as it influences the current velocity gradient, but not as a factor governing the development of the wave boundary layer, because the latter must anyway be studied by means of experiments.

It would be of little or no use to define the instantaneous value of the wave viscosity by means of Eq. 7, because τ and dv/dz do not vanish at the same time. Hence, the values of n thus calculated would now be zero, now infinite, whereas the wave viscosity that participates in controlling the current velocity gradient must always be a positive, finite quantity.

Since it is not intended to study the pulsations of the current with the pulsating wave viscosity, cf. assumption (f), it is natural to average the wave viscosity over the period. In analogy to Eq. 7, the average wave viscosity, n_w, can most simply be introduced as a function of z by the following formula

$$n_w(z) = \frac{\int |\tau_w(z)/\rho| \, dt}{\int |\partial u(z)/\partial z| \, dt} \tag{13}$$

where the integrals are taken over the wave period. It is necessary to use the numerical values of the shear stresses and the velocity gradients because both change sign every half period.

In the oscillating water tunnel of the Technical University of Denmark two tests have been run with measurements in the wave boundary layer, and the main results published in Refs. 1 and 2, respectively. The characteristic data of these tests are:

	TEST 1	TEST 2
a_b	2.85 m	1.79 m
u_b	2.11 m/s	1.53 m/s
k	23 mm	63 mm
u_f	0.208 m/s	0.210 m/s
r	0.0984	0.1371
f_w	0.0194	0.0376
δ	42 mm	52 mm

The quantities u_f, r, f_w and δ will be explained in Sec. 8.

The basic data from the tests have been introduced in Eq. 13, and the resulting empirical wave viscosities plotted in Fig. 1 with dimensionless coordinates.

8. WAVE VISCOSITY FORMULA

In order to generalize the test results to other values of the parameters a_b, u_b and k, the following formula has been constructed

$$n_w = \frac{0.4 \, u_f \, z}{1 + 1.34 \sqrt{r} \, \frac{z}{\delta} \exp \frac{z}{\delta}} \tag{14}$$

The determination of u_f, r and δ will be explained below. This formula satisfies two basic requirements: (a) Near the bed n_w is proportional to z. (b) In the upper part of the wave boundary layer n_w decreases exponentially with z.

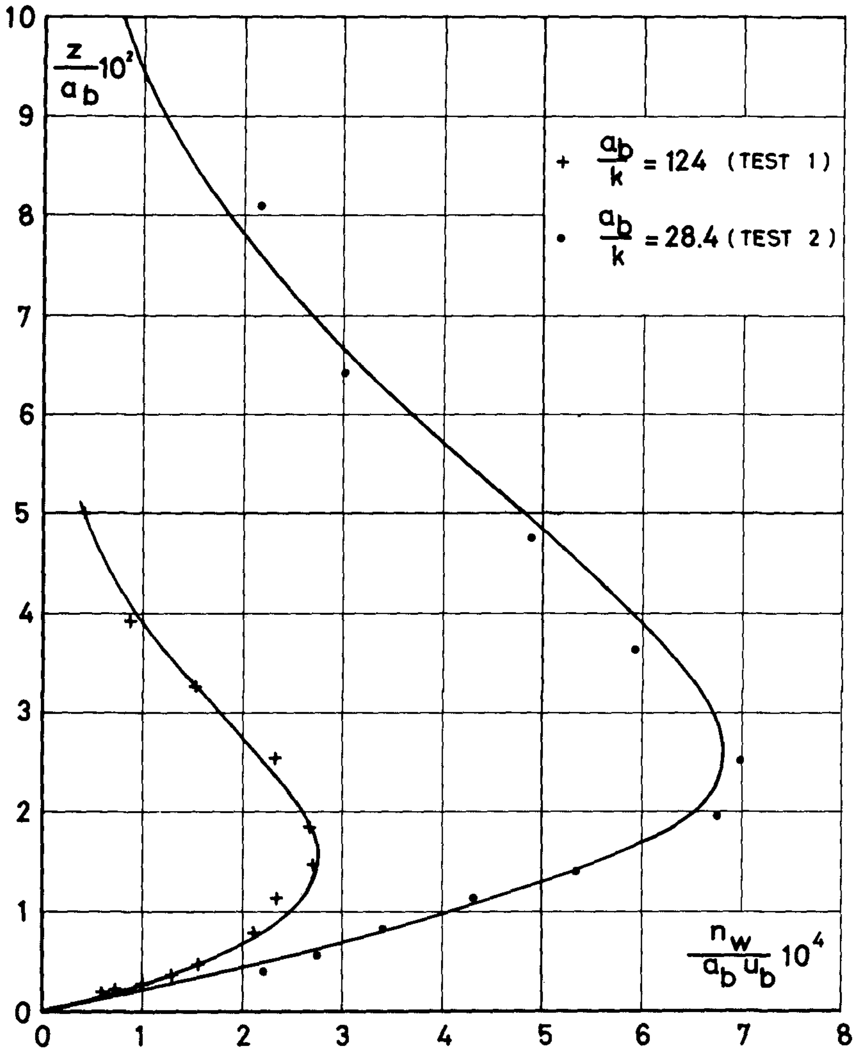

Fig. 1 Wave viscosity, n_w, as a function of z

For Tests 1 and 2 the full curves in Fig. 1 show the variation of the wave viscosity according to Eq. 14.

In a wave boundary layer the maximum bed shear stress, $\tau_{w,max}$, occurs before the maximum velocity, u_b, in the potential flow, the phase shift being about 30°. The maximum wave friction velocity, u_f, and the characteristic wave friction coefficient, f_w, are defined by

$$\tau_{w,max} = \rho u_f^2 = \frac{1}{2} f_w \rho u_b^2 \tag{15}$$

The wave friction velocity ratio, r, is defined as

$$r = u_f/u_b = \sqrt{\tfrac{1}{2}\, f_w} \tag{16}$$

and can be determined from the following equation

$$\frac{0.174}{r} + \log \frac{0.174}{r} = -\,0.077 + \log \frac{a_b}{k} \tag{17}$$

According to Eq. 14, δ is a characteristic thickness for the upper part of the wave boundary layer, where the viscosity decreases exponentially with z. δ can be calculated from

$$\frac{2.74\ \delta}{k} \log \frac{2.74\ \delta}{k} = 0.0282\ \frac{a_b}{k} \tag{18}$$

The compositions of Eqs. 17-18 have been derived by very simplified considerations of the growth of the wave boundary layer in accelerated potential flow (Ref. 3).

The procedure for calculating the wave viscosities is then:

(a) Solve Eq. 17 for $0.174/r$ and find r.

(b) Solve Eq. 18 for $2.74\,\delta/k$ and find δ.

(c) Calculate $u_f = r\ u_b$, cf. Eq. 16.

(d) Calculate n_w from Eq. 14.

The validity of Eq. 14 has been tested partly by Fig. 1, partly by the following method: For Tests 1 and 2, a semilogarithmic diagram was plotted of two dimensionless quantities: $n_w/z \cdot u_b$ (logarithmic scale) versus $z/f_w \cdot a_b$ (linear scale), whereby the curves for the two tests came very close to each other over a large range. This diagram was used for an independent extrapolation of the viscosities to the ratio $a_b/k = 540$. The extrapolated curve fitted Eq. 14 very well.

9. COMBINED VISCOSITY

For the combination of a current with waves the total viscosity is n_{cw}.

Above the wave boundary layer n_w is negligible, and $n_{cw} = n_c$ can be determined from Eq. 9. Here the current friction velocity U_f is defined by

$$\tau_{cw} = \rho\ U_f^2 = \gamma\ d\ S \tag{19}$$

where τ_{cw} is the current shear stress at the bed.

Within the wave boundary layer the situation is very complicated because both n_c and n_w contribute to n_{cw}. It was decided to find n_{cw} by adding a vector n_c in the current direction to a vector n_w in the wave direction, because such vectorial combination would be correct in the case of two currents U' and U" in arbitrary directions, cf. Eqs. 8-9.

The vectorial combination of n_c and n_w is probably the most questionable of the assumptions of the present theory, because the structures of the current-produced turbulence and the wave-produced turbulence are widely different. There are two zones, however, where the error of n_{cw} is believed to be small: The zone where n_w varies linearly with z, and the zone where n_c is essentially larger than n_w. Between these zones the error may be larger but will probably have a rather limited influence on the resulting velocity profile.

The vectorial combination has been applied only when the angle between the current and wave directions is 0° or 90°, resulting in the Eqs. 3 and 4.

10. CURRENT VELOCITY PROFILES

Throughout the wave boundary layer, where the current shear stress is constant, the velocity profile U(z) has been found by numerical integration of

$$\tau_{cw} = \rho \, n_{cw} \frac{dU}{dz} \tag{20}$$

Near the bed the velocity gradient dU/dz is essentially reduced because of the influence of the wave viscosity.

The numerical integration has been carried out by Mr. Roger B. Wallace of the Danish Hydraulic Institute, for all combinations of the values

$$\frac{a_b}{k} = 28.4 - 124 - 540 \tag{21}$$

and

$$\frac{\tau_{cw}}{\rho \, u_b^2} = 10^{-5} - 10^{-4} - 10^{-3} - 10^{-2} \tag{22}$$

both for parallel and perpendicular directions of the current and the waves.

Above the wave boundary layer the velocity profile becomes

$$U(z) = U_f \, (2.5 \ln \frac{z}{z_o} - A) \tag{23}$$

It has the normal logarithmic shape but is reduced by the constant velocity $A \, U_f$. The constant, A, of velocity defect can be found from Figs. 2 and 3, for parallel and perpendicular directions, respectively. The values of A_1 and A_2 must be seen in relation to the value of $2.5 \ln (d/z_o)$, which is normally of the order of magnitude of 20.

For waves propagating at an arbitrary angle with the current, the friction velocity U_f is determined from the total slope S by means of Eq. 19 as usual. It is suggested to calculate the current velocity profile without waves and then deduct A_1 or A_2, respectively, from the velocity components parallel and perpendicular to the wave direction.

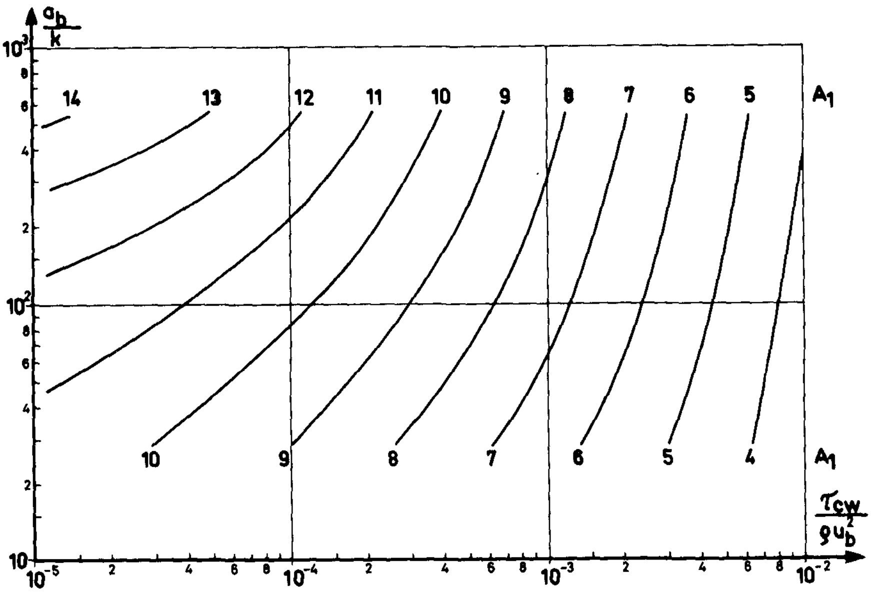

Fig. 2 Constant, A_1, of velocity defect for current $\parallel$ wave direction

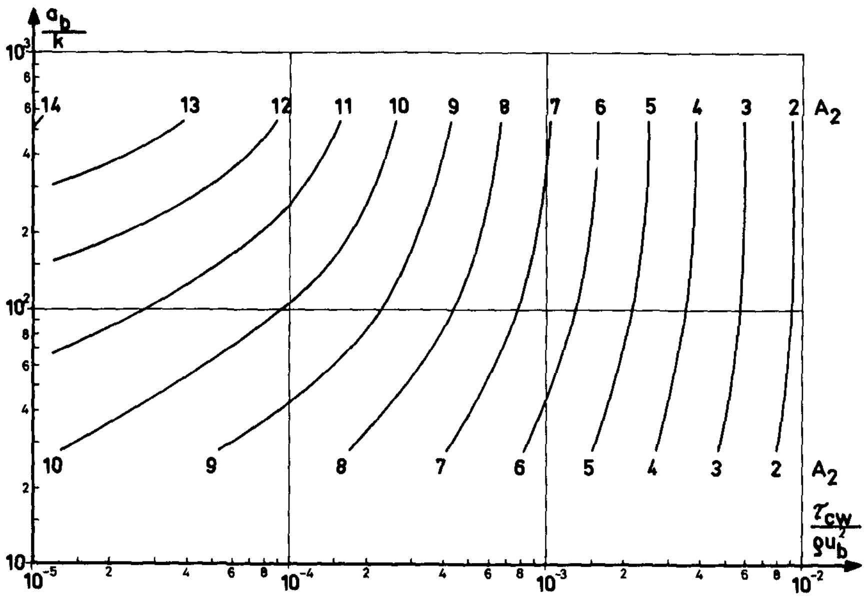

Fig. 3 Constant, A_2, of velocity defect for current $\perp$ wave direction

11. REFERENCES

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