CHAPTER 63

SCALE SELECTION FOR MOBILE BED WAVE MODELS

by

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ABSTRACT

A rational basis for the design of wave models with a mobile bed is presented. The discussion is of a preliminary nature and further model analysis, underway at present at Queen's University, should introduce a sounder basis for model design.

INTRODUCTION

The determination of scales for mobile bed wave models is an area of great uncertainty and a discussion at the 12th Coastal Engineering Conference in Washington in September 1970 resulted in the conclusion that in spite of a great deal of practical experimentation and model analysis relatively little is known about modelling with mobile beds.

An earlier report (1) has been written to serve as a framework for further study of modelling techniques presently under way at the Queen's University Coastal Engineering Research Laboratory. Here the derivation of scale relationships for fixed bed models as well as for mobile bed models is described in greater detail and the reader is referred to this work for further information.

FIXED BED MODEL SCALES

For wave models it is necessary to distinguish between short wave models and models of long waves and unidirectional current.

Short waves may be said to consist of two regions, the upper region, outside of the boundary layer, and the boundary layer region.

Upper region models are the normal type of fixed bed short wave

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model constructed to model refraction, diffraction, reflection, etc. Often these models are distorted and the scale relationships governing these models are

$$n_H = n_L = n_d = n_z = n_a = n$$
$$n_u = n_t = n_T = n^{1/2} \qquad (1)$$
$$n_x = n_y = Nn$$

Here n is the general model scale - prototype value divided by model value to result in integer model scales rather than fractions for the major scales. H is the wave height, L the wave length, d the depth of water, z the vertical co-ordinate, a the orbital amplitude, u the water particle velocity, t the time co-ordinate, T the wave period, x and y the space co-ordinates and N the model distortion $= n_x/n_z = n_y/n_z$.

Both N and n may be freely chosen, and each could be coined as a "degree of choice". n is usually a function of the accuracy of the field measurements of depth and wave height, model accuracies of the same quantities, minimum depths required in certain model areas and maximum slopes that can be used without causing additional effects such as separation and vortices; N is normally determined by the laboratory size and the area to be modelled. A model with two "degrees of choice" may appear to be an easier model to design, but it is based on a number of additional trade-offs, necessary in order to achieve this extra choice. These must be carefully evaluated for each model. As an example consider a refraction-diffraction model. If two "degrees of choice", i.e. a distorted model, are insisted upon, the total wave field, consisting of the model diffraction pattern and the model refraction pattern, does not correspond to the prototype wave field. Thus the effect of this discrepancy must be evaluated and, in the light of this, the number of "degrees of choice" must be determined - either one (undistorted), or two (distorted).

When the researcher is interested in the motion within the boundary layer, as he would be in the case of mobile bed models the situation becomes considerably more complex. Only models with a rough turbulent boundary layer are considered here. Details regarding other boundary layer models may be found in Kamphuis [1].

It is not unusual that the bottom roughness k_s cannot be reproduced to model scale, because the model bottom would be too smooth. Not modelling k_s to scale represents another "degree of choice" and again constitutes certain further limitations imposed on the model. The ratio of n_{k_s} / n may be defined as the bottom roughness distortion N_{k_s}. When a larger roughness is used (usual case), $N_{k_s} < 1$.

The required model scales for fully developed rough turbulent oscillatory boundary layer flow may be derived as:

$$n_H = n_L = n_d = n_z = n_Z = n_a = n$$
$$n_u = n_{U_\delta} = n_t = n_T = n^{1/2} \quad ; \quad n_U = n^{1/2} N_{k_s}^{-1/42} \simeq n^{1/2}$$
$$n_x = n_y = n_X = n_Y = Nn \quad ; \quad n_{k_s} = n N_{k_s} \qquad (2)$$
$$n_\delta \simeq n N_{k_s}^{1/7}$$
$$n_\tau \simeq n N_{k_s}^{2/7} \quad ; \quad n_{v_*} \simeq n^{1/2} N_{k_s}^{1/7}$$

Here Z refers to the vertical co-ordinate within the boundary layer, measured from the bottom up. U is the velocity within the boundary layer, δ refers to the top of the boundary layer, τ is the shear stress and v_* is the shear velocity = $\sqrt{\tau_o/\rho}$, where τ_o is the shear stress on the bottom.

For long waves and unidirectional flow models the following scale relationships apply.

$$n_d = n_Z = n$$
$$n_{\bar{U}} = n_U = n^{1/2} \quad ; \quad n_t = n_T = Nn^{1/2}$$
$$n_X = n_Y = n_L = Nn \qquad (3)$$
$$N_{k_s} = N^{-4} \quad ; \quad n_\tau = nN^{-1}$$
$$n_C = N^{1/2} \quad ; \quad n_f = N^{-1} \quad ; \quad n_S = N^{-1}$$

Here $\bar{U}$ is the average velocity. It is capitalized because the totality of the flow constitutes a boundary layer. C, f and S are the Chezy friction coefficient, the Darcy-Weisbach friction factor and the surface slope.

Equation 3 refers to the situation where all additional roughness required as a result of model distortion is supplied on the bottom. If vertical roughness elements are used, the bottom shear may be considered to result from k_s only and

$$n_\tau = nN_{k_s}^{1/4} \tag{4}$$

while C and f refer to the combination of bottom and vertical "roughness".

When a combined model is used, i.e. short waves, long waves and unidirectional currents it becomes evident that when all the additional roughness is placed in the bottom, the short wave boundary layer thickness becomes grossly exaggerated. Thus vertical roughness elements must be used. The resulting scale relationships for such a combination model are:

$$\begin{aligned}
& n_H = n_d = n_z = n_Z = n_a = n \\
& n_u = n_{U_\delta} = {}_\ell n_{\bar{U}} \simeq {}_s n_U = n^{1/2} \\
& n_x = n_y = n_X = n_Y = Nn \quad ; \quad n_{k_s} = nN_{k_s} \\
& {}_s n_L = n \quad ; \quad {}_\ell n_L = Nn \\
& {}_s n_t = {}_s n_T = n^{1/2} \quad ; \quad {}_\ell n_t = {}_\ell n_T = Nn^{1/2} \\
& {}_s n_\tau = nN_{k_s}^{2/7} \quad ; \quad {}_\ell n_\tau = nN_{k_s}^{1/4} \\
& {}_s n_\delta = nN_{k_s}^{1/7} \\
& n_C = N^{1/2} \quad ; \quad n_f = N^{-1} \quad ; \quad n_S = N^{-1}
\end{aligned} \tag{5}$$

where $_s$ refers to short waves and $_\ell$ refers to long waves and unidirectional flow.

EQUATIONS FOR A MOBILE BED

Sediment transport along the bottom may be described by the following parameters, Yalin [2,ch 6].

$$A = f(\rho, \mu, D, \rho_s, g, \ell, v_*) \tag{6}$$

where ρ and μ are the fluid density and viscosity, D and ρ_s are the particle diameter and density, g is the acceleration due to gravity, and ℓ is a typical length.

If initiation of motion is basically considered to be a lift phenomenon it is possible to replace g by the submerged unit weight

$$\gamma_s^{\bullet} = (\rho_s - \rho)g \tag{7}$$

This results in the following dimensionless relationship

$$\Pi_A = \phi\left(\frac{v_* D}{\nu}, \frac{\rho {v_*}^2}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{\ell}{D}\right) \tag{8}$$

The first two dimensionless variables are the grain size Reynolds Number and the Shields parameter, the X and Y axes of the Shields diagram.

For a flat bed and unidirectional flow v_* is readily determined and easily varied in a definable fashion since

$$v_* = (gRS)^{1/2} \tag{9}$$

where R is the hydraulic radius of the flow. If bed forms are present

$$S = S_{k_s} + S_f \tag{10}$$

where S_{k_s} is the slope caused by the friction on the actual grains and S_f is the slope caused by the bed form. Experimentally the following two expressions have been derived, Yalin [2, ch 6].

$$S_{k_s} = \frac{\mathbf{F}}{(2.5 \ln 11 \frac{d}{k_s})^2} \quad ; \quad S_f = \frac{1}{2}\frac{\Delta^2}{\Lambda d}\mathbf{F} \tag{11}$$

where $\mathbf{F}$ is the Froude number of the flow $= \bar{U}^2/gd$, Δ is the height of the bed form and Λ its length.

Under waves v_* is not an easy variable to use and it would be convenient to rewrite Eq. 8 as

$$_s\Pi_A = \phi_s \left(\frac{U_\delta D}{\nu}, \frac{\rho U_\delta^2}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{a_\delta}{D} \right) \quad (12)$$

$$_\ell\Pi_A = \phi_\ell \left(\frac{\bar{U} D}{\nu}, \frac{\rho \bar{U}^2}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{d}{D} \right) \quad (13)$$

for short waves and long waves respectively.

This is justified when deriving model scales, if

$$n_{U_\delta} \simeq {}_s n_{v_*} \quad \text{and} \quad n_{\bar{U}} \simeq {}_\ell n_{v_*}$$

and Eq. 5 indicates that this is not unreasonable except for the case when all roughness is bottom roughness (1,p49). The direct determination of τ below waves is at present also under investigation at Queen's (Riedel et al [14]).

SCALE SELECTION FOR SHORT WAVE MOBILE BED MODELS

General

In order to model the boundary layer and the wave motion both simultaneously and correctly, it was seen that the boundary layer motion must be fully developed rough turbulent in case of the prototype as well as the model.

The following model scales may be derived from Eq. 8 using a_δ as the typical length for models of short waves and assuming $n_\nu = n_\rho = 1$

$$n_{v_*} n_D = 1 \quad (14) \qquad n_D \, n_{\gamma_s} = n_{v_*}^2 \quad (15)$$

$$n_{\rho_s} = n_\rho = 1 \quad (16) \qquad n_D = n_{a_\delta} = n \quad (17)$$

Equations 14 and 15 ensure that both model and prototype fall on the same point on the Shields diagram, i.e. it ascertains that when motion occurs in the prototype, motion will also occur in the model.

Valembois [3] combines these into

$$n_{\gamma_s} = n_D^{-3}$$

Equation 16 states that the density scales for the fluid and the sediment must be the same. Since n_ρ is usually equal to unity, the only proper model material is the material found in the prototype, a very restricting concept. Yalin [2,p162] states that this is only of importance when considering the motion of individual grains. When mass movement of bed form and discharge of material is of interest, this very stringent scale law may sometimes be relaxed.

Without the assumption that Eq. 16 need not be satisfied, it is impossible to construct models using larger, light-weight sediment. If the effect of ρ_s/ρ is small, the assumption introduces a scale effect, where scale effect is defined as a discrepancy in the model results caused by not adhering to all scale laws. If the effect of ρ_s/ρ is large, the model is useless. When particle ballistics are involved, e.g. in the formation of ripples or the formation of equilibrium beaches, Mogridge and Kamphuis [4], Paul, Kamphuis and Brebner [5], it may well be impossible to use light-weight material.

If the mobile bed is flat, i.e. without bed forms such as ripples, then from Eq. 2, 14 and 15

$$n_D = n^{-1/2}\, N_{k_s}^{-1/7} \quad \text{and} \quad n_{\gamma_s} = n^{3/2}\, N_{k_s}^{3/7} \tag{18}$$

For mobile beds, it is at present usual to assume that

$$n_{k_s} = n_D \tag{19}$$

and therefore Eq. 18 becomes

$$n_D = n^{-5/16} \quad \text{and} \quad n_{\gamma_s} = n^{15/16} \tag{20}$$

For beds with bedform it can either be assumed that the bottom roughness is the same as the grain roughness (Assumption I) or that it is equal to the total roughness (Assumption II). In the latter case

$$n_D = n^{-1/2} N_k^{-1/7} \quad \text{and} \quad n_{\gamma_s} = n^{3/2} N_k^{3/7} \tag{21}$$

Both assumptions are incorrect, but certain situations, e.g. long ripples may warrant Assumption I, whereas other configurations, such as short ripples may warrant Assumption II.

Bijker [6] and others follow the example of Einstein and introduce a ripple factor, α , so that $\tau' = \alpha\tau$, where τ' is the effective shear stress, i.e. the shear stress that moves the sediment. In how far this applies to ripples under waves remains to be seen and in any case, it is unlikely that the other properties such as growth of bed form, bed form dimensions, etc. are dependent on α . Furthermore, it is found that in most cases $n_\alpha \simeq 1$. Therefore the ripple factor has only a small effect on the scaling problem and this approach becomes synonymous to Assumption II.

Equation 17 has not been discussed so far and it may be seen that since $n > 1$, Eq. 17 results in $n_D > 1$ while Eqs. 18 and 21 yield $n_D < 1$. This is a conflict which must be resolved for all mobile bed models.

Large Grain Sizes

From the Shields diagram it may be inferred that when

$$\frac{v_* D}{\nu} > 100 \tag{22}$$

the grain size Reynolds number effect, i.e. Eq. 14 may be neglected and thus Eqs. 15 and 17 yield

$$n_{\gamma_s} = \frac{n_{v_*}^2}{n_D} = N_k^{2/7} \quad ; \quad n_D = n_{a_\delta} = n \tag{23}$$

where it is understood that N_k refers to total roughness or grain size roughness, depending on the assumption made. For Assumption I, since k_s varies with D , N_{k_s} is likely very close to unity and $n_{\gamma_s} \simeq 1$.

The resulting model bed consists of prototype sand material, with its grain size scaled down by the model scale. Note that in this case Eq. 16 is also satisfied automatically. For Assumption II,

N_k is a function of the grain size, as well as model wave conditions which cause the bed form. At present little is known about the variation of bed form below waves and research is underway at Queen's University [4] to determine this. It is very likely that N_k is a function of n_{γ_s} , which means that an iterative procedure may need to be followed in order to select the correct n_{γ_s} even after expressions for N_k have been determined experimentally.

Preliminary test results indicate that both ripple height Δ and ripple length Λ are primarily functions of a_δ/D . Thus if Eq. 23 is satisfied, N_k could still have a value close to one, as long as a_δ/D does not change a great deal within the model and prototype. Therefore ripple sizes scale down approximately by the model scale and again $n_{\gamma_s} \simeq 1$.

There are obvious lower limits to this type of model. Problems arise when the model boundary layer becomes smooth and laminar, or when sand size particles are modelled by clay size particles. Under those circumstances the condition in Eq. 23 must be dropped.

Smaller Grain Sizes

In most cases the flow around the individual grains is not turbulent and a conflict between Eqs. 14 and 17 exists. Since it is almost impossible to satisfy Eq. 17, especially for the smaller grain sizes, as shown above, Eq. 14 is considered, leaving the misrepresentation of Eq. 17 to scale effect. This scale effect is in addition to the scale effect resulting from not adhering to Eq. 16, which now is an impossible condition to meet.

As an example the total bed roughness k , may be considered since it influences the value of N_k which, under Assumption II, may be used eventually for scale selection. This is one problem presently under investigation at Queen's University.

$$\Pi_k = \frac{k}{D} = \phi_k \left(\frac{v_* D}{\nu} , \frac{\rho v_*^2}{\gamma_s D} , \frac{\rho_s}{\rho} , \frac{a_\delta}{D} \right) \tag{24}$$

If the scale laws resulting from Eqs. 14 and 15 are satisfied, it may be stated in very simplified terms that

$$n_k = m_k\, n_D = \frac{m_k}{n^{1/2}\, N_{k_s}^{\;1/7}} = \frac{m_k}{n^{5/16}}\ ;\ n_k = m_k\, n_D = \frac{m_k^{\;7/8}}{n^{5/16}} \tag{25}$$

for Assumption I, and Assumption II respectively. Here m_k is the scale effect with respect to k, resulting from Eqs. 16 and 17 not being satisfied.

Yalin (2,p226) and present research at Queen's University indicate that in models, ripple height and length are indeed functions of a_δ/D and that therefore k/D must also be a function of this parameter (2,p227). But a_δ is also a function of water depth and therefore in a single model there are many values of m_k, each corresponding to a different depth. The use of a single value of m_k is dangerous since this means that the model is designed for one distortion of roughness with respect to D. The factor m_k must therefore be thoroughly investigated.

If Assumption II is made the scale effect m_k is found back in the other scale relationships which include N_k, e.g.

$$n_\tau = n\, N_k^{\;2/7} = n^{5/8}\, m_k^{\;1/4}\ ;\ n_\delta = n\, N_k^{\;1/7} = n^{13/16}\, m_k^{\;1/8}$$

$$n_D = \frac{1}{n^{5/16}\, m_k^{\;1/8}}\ ;\ n_{\gamma_s} = n^{15/16}\, m_k^{\;3/8} \tag{26}$$

The smaller powers of m_k indicate that the influence of the scale effect is not very serious for n_δ and n_D, but the actual value of m_k can conceivably be quite large since for small waves Δ and Λ are direct functions of a_δ/D, while for prototype waves this relationship may be decoupled and Δ and Λ may be independent of a_δ/D.

The sediment transport scale may be derived in a similar fashion

$$\frac{q}{v_* D} = \phi_q \left(\frac{v_* D}{\nu}, \frac{\rho v_*^{\,2}}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{a_\delta}{D} \right) \tag{27}$$

q is the solid volume of material transported per unit width, per unit time. By analogy to the above derivation

$$n_q = m_q\, n_{v_*}\, n_D = m_q \tag{28}$$

Thus if Eqs. 16 and 17 are not satisfied the sediment transport scale is not equal to unity. Also m_q is again a function of depth and therefore n_q is not constant throughout the model. Bijker and Svasek [7] discuss this in more detail. It is only possible to design a model for a single value of m_q . The proper choice is of extreme importance and worthy of additional study.

Model Distortion

Model distortion is a "degree of choice" for fixed bed models but must be chosen more carefully for mobile bed models. Since many models involve beaches, the model shoreline and slope must correspond to the prototype shoreline and slope so that they both represent the same conditions. If the equilibrium beach slope is denoted by Θ and the shoreline position is called x_s , then it may be stated in a simplified fashion that

$$N = F_N(\Theta, x_s) \tag{29}$$

where the function is determined by an experimental fit of model vs prototype. Equation 29 is at present under investigation at Queen's University [e.g. 5].

Summary

The scale relationships for short wave models with a mobile bed are therefore Eqs. 2 and 29, with for models with large grain size Reynolds number Eq. 23 and if N_{k_s} or N_k are near unity

$$n_{\gamma_s} \simeq 1 \quad ; \quad n_\rho \simeq n_{\rho_s} \tag{30}$$

Because of very small model grain sizes, these models quickly become physically impossible. For the more usual models with grain size Reynolds numbers below the fully turbulent range,

$$n_D = n^{-1/2} N_{k_s}^{-1/7} = n^{-5/16} \quad \text{or} \quad n_D = n^{-5/16} m_k^{-1/8} \tag{31}$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/7} = n^{15/16} \quad \text{or} \quad n_{\gamma_s} = n^{15/16} m_k^{3/8}$$

$$n_k = m_k\, n^{-1/2} N_{k_s}^{-1/7} = m_k\, n^{-5/16} \quad \text{or} \quad n_k = m_k^{7/8}\, n^{-5/16}$$

$$n_\tau = n\, N_{k_s}^{2/7} = n^{5/8} \quad \text{or} \quad n_\tau = n^{5/8}\, m_k^{1/4} \qquad (31)$$

$$n_\delta = n\, N_{k_s}^{1/7} = n^{13/16} \quad \text{or} \quad n_\delta = n^{13/16}\, m_k^{1/8} \qquad \text{(Cont'd)}$$

$$n_q = m_q$$

for Assumption I and Assumption II respectively.

Substitutions for v_*

Although v_* has been conveniently eliminated from the scaling problem, the actual experimentation with the model still depends on the measurement of v_* as expressed in Eq. 8. It was indicated earlier that for short waves, substitution of U_δ for v_* is not unreasonable. If the wave motion is sinusoidal, Eq. 12 may be further simplified to

$${}_s\Pi_A = \phi_s' \left(\frac{\hat{U}_\delta D}{\nu} , \frac{\rho \hat{U}_\delta^2}{\gamma_s D} , \frac{\rho_s}{\rho} , \frac{a_\delta}{D} \right) \qquad (32)$$

where ^ indicates the maximum value. Since for sinusoidal motion

$$\hat{U}_\delta = \text{cst} \left(\frac{a_\delta}{T} \right) \qquad (33)$$

$${}_s\Pi_A = \phi_s'' \left(\frac{a_\delta D}{\nu T} , \frac{\rho a_\delta^2}{\gamma_s T^2 D} , \frac{\rho_s}{\rho} , \frac{a_\delta}{D} \right) \qquad (34)$$

It must be recognized that above dimensionless quantities are not entirely constant, for instance:

$$n_{X_1} = \frac{n_{a_\delta}\, n_D}{n_\nu\, n_T} = \frac{n}{n^{1/2}} \cdot \frac{1}{n^{1/2} N_{k_s}^{1/7}} = N_{k_s}^{-1/7} = n^{3/16} \qquad (35)$$

or

$$n_{X_1} = \frac{n}{n^{1/2}\, n^{5/16}\, m_k^{1/8}} = n^{3/16}\, m_k^{-1/8} \qquad (36)$$

Distortion of H

At times it is suggested in the literature, e.g. Goddet and Jaffry [8] that the wave height may be distorted to force proper sediment transport conditions by waves, i.e. $n_H = N_H\, n$, since an increase in wave height has little influence on the wave refraction patterns. It must be borne in mind, however, that the position of the breaker is influenced by this distortion and therefore the effects on the model results of this "degree of choice" must be evaluated carefully. Goddet and Jaffry [8] suggest that $N_H = N^{1/4}$ is permissible.

SCALE SELECTION FOR LONG WAVE AND UNIDIRECTIONAL FLOW MOBILE BED MODELS

For long waves and unidirectional flow similar reasoning may be used. From Eq. 8, using the water depth d as a typical length, Eqs. 14, 15 and 16 are valid while Eq. 17 becomes

$$n_D = n_d = n \qquad (37)$$

If the mobile bed is flat, substitution of Eq. 3 into Eqs. 14 and 15 yields

$$n_D = n^{-1/2}\, N^{1/2} \quad ; \quad n_{\gamma_s} = n^{3/2}\, N^{-3/2} \qquad (38)$$

These equations are identical to those derived by Le Méhauté [9,p1091] for both short wave and long wave and unidirectional models! When multiplied together, Goddet and Jaffry's [8] expression may be derived as well as Bijker's expression [6,V3-3] assuming n_α, the ripple factor scale, to approximate one. Model distortion introduces a requirement for additional roughness and Eq. 38 presupposes that all this additional roughness is added in the form of bottom roughness. There is also again a conflict between Eqs. 37 and 38 with respect to n_D. Eq. 38a is the basis for a common expression for model distortion developed by Yalin [2,p235]. From experiment it has been found that

$$C \sim \left(\frac{d}{D} \right)^{1/5}$$

Using Eqs. 5 and 38a it is possible to derive

$$N = n^{1/2} \quad \text{or} \quad n_X = n_Z^{3/2} = n^{3/2} \tag{39}$$

Yalin (2,p236) has plotted values for model studies performed at Wallingford which substantiate Eq. 39. It may be noted in passing that Eq. 39 may be derived from Lacey's regime equations. These equations are based on erodible channels in an identical soil medium where the smaller channels form "models" of the larger channels. Le Méhauté (9) calls this the natural distortion and states it is only valid when prototype material is used in the model, i.e. when n_D and n_γ in Eq. 38 are equal to unity. The above development, however, indicates that this distortion can be generally accepted for all long wave models as long as all additional roughness is added to the bottom.

Most sediment transport problems do not present a flat bed and if the model is distorted, additional roughness must be added. Thus the total model roughness may be described as the sum of grain size roughness, bed form roughness and artificial roughness, i.e. $\tau = \tau_{k_s} + \tau_f + \tau_A$. The extra roughness may be added to the bottom or consist of vertical roughness elements. The use of the latter in sediment transport models may be open to question. Roughness strips will cause substantial scour in their immediate vicinity and also, roughness strips must be present in the original bottom before erosion has taken place. It is felt, however, that the addition of roughness to the bottom causes unacceptable conditions, since the additional roughness will greatly alter the bed forming process. Also if short waves are present, it causes exaggerated bottom boundary layers. Additional vertical roughness must undoubtedly consist of a close grid of small elements in order to bring the local scour problems to a minimum.

The shear acting on the bottom particles and causing sediment movement surely excludes the artificial roughness. Therefore it is a function of the actual bottom roughness distortion N_{k_s} or N_k depending on whether Assumption I or II is used. If Eq. 37 is not satisfied, Eqs. 4, 14, 15 and 19 yield

$$n_D = n^{-1/2} N_{k_s}^{-1/8} = n^{-1/3} \quad \text{or} \quad n_D = n^{-1/2} N_k^{-1/8} \tag{40}$$

$$n_k = m_k n_D = m_k n^{-1/3} \quad \text{or} \quad n_k = m_k^{8/9} n^{-1/3} \tag{41}$$

which leads in turn to

$$n_\tau = n N_{k_s}^{1/4} = n^{2/3} \quad \text{or} \quad n_\tau = n N_k^{1/4} = n^{2/3} m_k^{2/9} \tag{43}$$

$$n_D = n^{-1/2} N_{k_s}^{-1/8} = n^{-1/3} \quad \text{or} \quad n_D = n^{-1/2} N_k^{-1/8} = n^{-1/3} m_k^{-1/9} \tag{44}$$

$$n_{\gamma_s} = n^{3/2} N_{k_s}^{3/8} = n \quad \text{or} \quad n_{\gamma_s} = n^{3/2} N_k^{3/8} = n m_k^{1/3} \tag{45}$$

for Assumptions I and II respectively. Here m_k is the scale effect. It is again only possible to design a model for a single value of m_k, while in fact m_k is a function of the variation in the water depth d.

Some indication as to the value of N_k or m_k may be obtained from using Eqs. 11 which may be assumed to apply to both model and prototype. These yield

$$n_{\tau_{k_s}} \simeq m_{S_{k_s}} n_d = n N_{k_s}^{1/4} \tag{46}$$

$$n_{\tau_f} \simeq \frac{n_\Delta^2}{n_\Lambda} \tag{47}$$

Yalin (10,p242) indicates that Λ is a function of d and independent of D for dunes while ripples Λ is a function of D only. Also $\frac{\Delta}{\Lambda}$ depends slightly on excess shear but may be approximated as a constant for model and prototype with considerable sediment transport.

Similar to the case of short wave models, the scale for sediment transport may be derived as $n_q = m_q$.

MOBILE BED MODELS FOR COMBINED SHORT WAVES, LONG WAVES AND UNIDIRECTIONAL FLOW

From the foregoing discussion it appears that to combine short waves with the other two, vertical additional roughness elements are necessary in distorted models. In previous studies Bijker (6), Goddet and Jaffry (8) and Le Méhauté (9) inherently assume that all the shear necessary for the long wave model portion is supplied on the bottom, hence their scale relationships are variations of

$$n_\tau = n\,N^{-1} \quad ; \quad n_D = n^{-1/2}\,N^{1/2} \quad ; \quad n_{\gamma_s} = n^{3/2}\,N^{-3/2} \qquad (48)$$

This results, however, in an undue distortion of the short wave boundary layer.

Bijker (6) has performed his experiments without additional roughness. This is the method used for most wave-current models. For this case he correctly suggests that in order to achieve the correct wave-current interaction, the long wave or unidirectional current velocities must be exaggerated. The bottom shear stresses, resulting from waves and currents, must be the same. This results in

$$n_{\bar{U}} = N^{1/2}\,N_k^{1/7}\,n_{U_\delta} \qquad (49)$$

Bijker's relation (7-V3-2) is slightly more simplified and compares with Goddet and Jaffry (8).

The following scale relationships for combination models may be deduced from Eqs. 5,29,31 and 41 to 45.

$$\begin{aligned}
& n_H = n_d = n_z = n_Z = n_a = n_{a_\delta} = n \\
& n_x = n_y = N_X = N_Y = Nn \quad ; \quad N = f_N\,(\Theta, x_s) \\
& {}_s n_u = {}_s n_{U_\delta} = {}_\ell n_{\bar{U}} = {}_\ell n_U \approx {}_s n_U = n^{1/2} \\
& {}_s n_L = n \quad ; \quad {}_\ell n_L = Nn \\
& {}_s n_t = {}_s n_T = n^{1/2} \quad ; \quad {}_\ell n_t = {}_\ell n_T = Nn^{1/2} \\
& n_C = N^{1/2} \quad ; \quad n_f = n_S = N^{-1}
\end{aligned} \qquad (50)$$

These scales are based on the assumption that additional roughness is supplied to cause $n_C = N^{1/2}$. If no additional roughness is supplied, U must be exaggerated with respect to U_δ as outlined in Eq. 49.

For Assumption I, i.e. velocity distribution is a function of grain size

$$\begin{aligned}
{}_s n_D &= n^{-5/16} & {}_\ell n_D &= n^{-1/3} \\
{}_s n_{\gamma_s} &= n^{15/16} & {}_\ell n_{\gamma_s} &= n \\
{}_s n_k &= {}_s m_k n^{-5/16} & {}_\ell n_k &= {}_\ell m_k n^{-1/3} \\
{}_s n_\tau &= n^{5/8} & {}_\ell n_\tau &= n^{2/3} \\
{}_s n_\delta &= n^{13/16} & & \\
{}_s n_q &= {}_s m_q & {}_\ell n_q &= {}_\ell m_q
\end{aligned} \qquad (51)$$

For Assumption II, i.e. velocity distribution is a function of the bed form and grain size

$$\begin{aligned}
{}_s n_D &= n^{-5/16}\, {}_s m_k^{-1/8} & {}_\ell n_D &= n^{-1/3}\, {}_\ell m_k^{-1/9} \\
{}_s n_{\gamma_s} &= n^{15/16}\, {}_s m_k^{3/8} & {}_\ell n_{\gamma_s} &= n\, {}_\ell m_k^{1/3} \\
{}_s n_k &= {}_s m_k^{7/8}\, n^{-5/16} & {}_\ell n_k &= {}_\ell m_k^{8/9}\, n^{-1/3} \\
{}_s n_\tau &= n^{5/8}\, {}_s m_k^{1/4} & {}_\ell n_\tau &= n^{2/3}\, {}_\ell m_k^{2/9} \\
{}_s n_\delta &= n^{13/16}\, {}_s m_k^{1/8} & & \\
{}_s n_q &= {}_s m_q & {}_\ell n_q &= {}_\ell m_q
\end{aligned} \qquad (52)$$

It may be seen that although Eqs. 51 and 52 give slightly different scales, the short wave and long wave scales are quite close and relatively similar so that if either one is chosen, the other will not be very wrong, as long as the additional roughness is supplied by vertical roughness elements.

TIME

In Eq. 50, two time scales are given and these are the time scales used for the wave motion, the forcing mechanism in a mobile bed model. For the sediment transport, the list in Eq. 6 may be extended using the general time parameter t, resulting in a time scale for individual grain motion

$$_i n_t = \frac{n_D}{n_{v_*}}$$

which when using Eq. 14 becomes

$$_i n_t = n_D^{\,2} = \frac{1}{n_{v_*}^{\,2}} \tag{53}$$

where n_{v_*} is a function of Assumption I or II and whether or not vertical roughness elements are used. When dealing with erosion or deposition, it is the volume eroded or deposited which determines the time scale. Therefore, using Eq. 28, recognizing that q refers to transport of solids:

$$_e n_t = \frac{n_V n_{1-p}}{n_y n_q} = \frac{N^2 n^3}{N n m_q} n_{1-p} = \frac{N n^2}{m_q} n_{1-p} \tag{54}$$

where V is the total volume of material and p is the material porosity. m_q is different for the short wave and long wave portions of the model.

Similar time scales may be derived for other model transport phenomena such as movement of sand waves, transport of tracer materials, etc.

From Eqs. 50, 53 and 54 it may be noted that

$$_e n_t > {}_\ell n_t > {}_s n_t > {}_i n_t \tag{55}$$

This is very fortunate. Changes in bed formation, erosion and accretion, are very long term processes. Because $_e n_t$ is so large, it is possible to perform model studies on these phenomena within a reasonable time.

BREAKERS AND LONGSHORE CURRENTS

Since sediment motion in a coastal mobile bed model is brought about mainly by the agitation of sediment in the breaker zone, it is essential that conditions in this area are modelled correctly. The breaker position will be correct if $n_H = n_d$ and the refraction pattern will be correct if ${}_s n_L = n_d$ while the beach upon which the waves act will be modelled correctly if $N = f_N(\Theta, x_s)$. All these conditions are incorporated in Eq. 50. However, if a simplified littoral transport mechanism is envisioned, in which the waves stir up the material which is subsequently transported by the wave orbital motion and the longshore current, generated by the wave action, it is essential that

$$\frac{n_w}{{}_s n_u} = 1 \quad \text{and} \quad \frac{n_w}{n_{U_L}} = 1 \tag{56}$$

where U_L is the longshore current velocity and w is the fall velocity of the sediment particles.

Yalin (10,p69) shows $\frac{wD}{\nu}$ to be a function of $\frac{\gamma_s D^3}{\rho\nu^2}$. This is also stated by Valembois (3). Using Eqs. 51 and 52 it may be seen that $n_{\frac{\gamma_s D^3}{\rho\nu^2}} = 1$, i,e, $n_{\frac{wD}{\nu}} = 1$ and thus for proper reproduction of fall velocity

$$n_w = n_D^{-1} \tag{57}$$

This relation has also been derived by Bonnefille (11).

Yalin (10,p71) also demonstrates that if the X and Y parameters on the Shields diagram are the same for model and prototype, i.e. Eqs. 14 and 15 are satisfied, then $\frac{w}{v_*} = cst$ which is the same as Eq. 57.

The above argumentation is based on spherical particles but could be extended to particles of any shape, as long as the shape factors in model and prototype are similar.

The conditions expressed in Eq. 56 must now be checked. The longshore current velocity U_L is generated by the waves and many formulas are proposed for the generation of longshore currents, e.g. Fan and Le Méhauté (12,p22). Preliminary investigations into this area, Kamphuis (13) indicate that most of these formulations fit laboratory results adequately but do not describe field results. The single factor that appears to influence the longshore current velocity most is the wave height. Results indicate that

$$U_L = cst\ (H)^n \tag{58}$$

where n lies between 0.5 and 1, and from Eqs. 50, 51, 52 and 57 it may be seen that Eq. 56 is approximately satisfied. It is not very fruitful to pursue this line of thought any further until additional research has shown more clearly what drives the longshore current velocity.

FOOD FOR THOUGHT

Throughout this paper the problem of the proportion of total shear going into sediment transport has been touched upon as a basic criterion for similarity in sediment transport. It has been suggested that the prototype and model points must fall on the same location of the Shields diagram. It is obvious that if the model falls below the Shields curve for initiation of motion, while the prototype falls above, the model will be useless, but it is not entirely clear when Eqs. 14 and 15 are satisfied, that the model will represent the prototype correctly. If the bed form is identical, α , the proportion of shear going into sediment transport should be the same and the model results should represent the prototype. If the bed form is different, α must be taken into account and the following scale laws may be developed from

$$\frac{n_{\alpha \rho v_*}{}^2}{\gamma_s D} = \frac{n_\alpha{}^{1/2}\ v_* D}{\nu} = 1 \tag{59}$$

Present sediment transport relationships for unidirectional flow are usually presented in a form related to

$$\frac{q}{v_* D} = f\left(\frac{\rho v_*^2}{\gamma_s D}\right) \tag{60}$$

i.e. a simple version of Eq. 27. The relationships apply to the turbulent region of the Shields curve and it is understood that v_*

is not the total shear. Therefore, to extend this system using the present terminology, it is possible to write

$$\frac{q}{\alpha^{1/2} v_* D} = f\left(\frac{\alpha^{1/2} v_* D}{\nu}, \frac{\rho\alpha\, v_*^2}{\gamma_s D}, \frac{\rho_s}{\rho}, \frac{\ell}{D} \right) \qquad (61)$$

The relationship between Y and X_1 and X_2 in Eq. 61 may be represented by a modified Shields diagram with axes X_1 and X_2 projected into a third dimension thus presenting a surface as shown in Fig. 1.

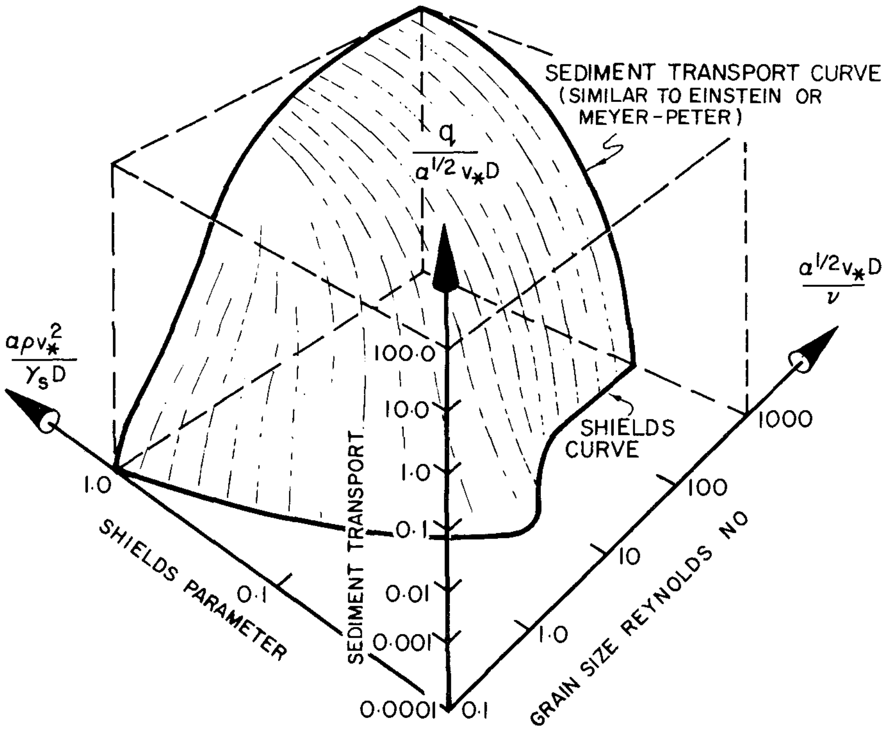

FIGURE 1 - SEDIMENT TRANSPORT SURFACE

This yields the more complete scaling laws

$$n_\alpha^{1/2}\, n_{v_*}\, n_D = 1 \quad ; \quad n_\alpha\, n_{v_*}^{2} = n_{\gamma_s}\, n_D \quad ; \quad n_q = n_\alpha^{1/2}\, n_{v_*}\, n_D$$

or

$$n_\alpha^{1/2}\, n_{v_*} = \frac{1}{n_D} \quad ; \quad n_{\gamma_s} = \frac{n_\alpha\, n_{v_*}^{2}}{n_D} = \frac{1}{n_D^{3}} \quad ; \quad n_q = 1 \qquad (62)$$

The above equations assume no scale effect in q and indicate that the derivations used in this paper are limited to $n_\alpha = 1$. The problem associated with the determination of α and n_α is of course the largest single problem in sediment transport study. It should be of prime concern, not only to the model builder, but also to the sediment transport student in general. Under oscillatory waves, the problem may be relatively simple and is very worthy of intensive investigation.

Finally it must be stated that Eqs. 27 and 61, concerned with sediment discharge are incomplete. These equations are essentially correct for unidirectional flow and long waves but for short oscillatory waves, sediment transport is highly dependent upon the asymmetry of particle accelerations, velocities and displacement. This asymmetry is a function of wave shape and relative depth d/L. The wave shape cannot be modelled in most cases; the relative depth in a propertly designed model is the same as in the prototype. Thus Eqs. 27 and 61 are adequate for scaling purposes, but the actual value of sediment transport under short waves lies considerably below the plane sketched in Fig. 1. Research underway at Queen's University indicates that under waves, sediment discharge is 1/2 - 10% of the value given by the Einstein function.

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