CHAPTER 5

WAVE STATISTICS ALONG THE NORTHERN COAST OF EGYPT

By

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ABSTRACT : As a forerunner of a comprehensive study of wave and energy climate of the Nile Delta coast covering a period of more than 50 years, wave measurements taken in 3 locations along the coast in the year 1972 were statistically analysed. Shortcut methods based on statistical approach were used to analyse the wave records enabling quicker analysis. Spectral analysis of the wave records indicates narrow spectral band similar to Raleigh distribution. The histograms and frequency distribution curves of significant wave heights, and the relationships between various statistical parameters such as H_{max}, $H_{1/10}$, $H_{1/3}$ and H_{rms} also agree closely with Raleigh distribution curves and parameters enabling the use of Raleigh distribution function in subsequent studies. Finally since the wave characteristics and wave energy climates are most important in the analysis of coastline changes, they are drawn from the data obtained from the analysis.

INTRODUCTION : Statistical characteristics of the ocean surface are of interest when the dynamics of coastal accretion and erosion is considered. When the coast and beaches are composed of loose sediment such as along the Nile Delta coast, this assumes more importance since energy even of smaller magnitude affects the coast.

THEORY : Waves are the primary force operative on the beach and the most common ones are generated by winds. Ocean waves are complex in character due to variations in wind pressure which generate them. The period, celerity and wave length of the waves are exceedingly irregular and therefore, statistical methods must be used to describe their properties. Though wave heights are nearly as irregular, it is easier to define the wave height since it rises from ready reference level. Thus if a satisfactory statistical distribution of wave height is possible, the expected wave height can be reasonably predicted.

The statistical characteristics of ocean surface are related to the wave spectra which in turn may be used to describe the processes of wave generation and wave decay.

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The theoretical distribution function for wave height variability based on the assumption of random phase and a narrow spectrum is the Raleigh distribution function (3). According to Longuet Higgins, if the statistics of wave amplitude can be approximated by the statistics of wave envelope function, and if the wave heights are considered proportional to the wave amplitude, wave heights can be expected to follow Raleigh distribution. Subsequently, Cartwright and Longuet Higgins (1) reviewed Rice's (5) derivation of a frequency function for a sum of a number of functions of a random phase with a wide frequency band. Rice's frequency function includes a parameter ε which is a measure of width of the energy spectrum. Raleigh's distribution is a special case of Rice's more general frequency function.

Raleigh's frequency function is of the form $p(H) = \frac{\pi}{2}\frac{H}{\mu} e^{(-\frac{\pi}{4})\frac{H^2}{\mu^2}}$ (1)

where H= wave height and μ= mean wave height. The nth moment about the origin is given by

$$M_n = \int_0^\infty H^n \, p(H) \, dH \qquad (2)$$

or by certain transformations

$$M_n = \left(\frac{2\mu}{\sqrt{\pi}}\right)^n \Gamma\left(\frac{n+2}{2}\right) \qquad (3)$$

For n=0 to 3, equations (1) and (2) give

$$\begin{aligned} M_o &= 1 \\ M_1 &= \mu \\ M_2 &= \frac{4}{\pi}\mu^2 \\ M_3 &= \frac{6}{\pi}\mu^2 \end{aligned} \qquad (4)$$

Generally μ estimated from $\overline{H} = H_{mean}$ = average wave height and $\sqrt{M_2} = \mu$ = maximum probable estimates of both H_{avg} and H_{rms} may be found from $\sqrt{\overline{H}^2} = H_{rms}$.

The probability distribution function of H may be written as

$$p(H) = \int_0^H p(H) \, dH = 1 - e^{-H^2/M_2} = 1 - e^{(-\frac{\pi H^2}{4\mu^2})} \qquad (5)$$

According to Longuet Higgins, $H_{1/n}$, the average of the highest 1/n waves in a group of N waves can be expressed as

$$H_{1/n} = M_2 \left| \sqrt{\ln n} + \frac{n\sqrt{\pi}}{2} \operatorname{erf} \sqrt{\ln n} \right| \qquad (6)$$

From (6), ratio of $\frac{H_{1/3}}{H_{rms}} = 1.416$ and $\frac{H_{1/3}}{\mu} = 1.598$.

Similar calculations may be made for all values of n from 1 to infinity.

Other relationships obtained by Longuet Higgins (3) and Putz (4) (from statistical analyses of 20 minute records) and spectrum analysis are given below.

	Longuet Higgins	Putz	Spectrum Analysis
H_{avg}	0.625 $H_{1/3}$	0.624 $H_{1/3}$	1.772 $\sqrt{E}$
$H_{1/10}$	1.27 $H_{1/3}$	1.29 $H_{1/3}$	3.600 $\sqrt{E}$
H_{max}	1.77 $H_{1/3}$ (from record of 300 waves)	1.87 $H_{1/3}$ (average of 25 twenty minute records)	$H_{1/3} = 3.60 \sqrt{E}$ where E= wave energy

Longuet Higgins (3) also showed that the most probable value of the maximum wave height depends upon the length of records or number of waves N. The table below gives values of $H_{max} / H_{1/3}$

N	$H_{max} / H_{1/3}$
10	1.11
20	1.25
50	1.42
100	1.53
200	1.64
500	1.77
1000	1.86

Whereas the wave spectra describes the characteristics of ocean surface, and the processes of wave generation and decay, the significant wave height, distribution of wave heights, distribution of wave periods, and marginal and joint distribution of wave heights and periods will describe the wave variability. Similarly, just how the significant wave height is related to the probability distribution and also the wave spectrum, can best be described by a histogram (that is, percent occurrence of waves in each wave height range), period spectrum diagram (sum of energy or square of wave heights for each range of period or frequency) and cumulative frequency distribution curve (that is, percent less or equal to occurrences for each significant wave height range).

The ultimate objective of these statistical studies along the Nile Delta coast is the determination of the wave height and wave energy (or energy flux or power H^2T) climate along the entire coast. These can be drawn in the form of their respective roses in relation to their directions.

FIELD DATA : Field data were obtained by means of OSPOS (offshore pressure operated-suspended) wave recorders anchored near the bottom in 3 different locations in Abuquir, Burullus and Ras el Bar in 6-8 m depths in the sea facing the 245 km long coastline (fig. 1). These recorders are essentially pressure meters which when placed in certain depths under water, measure and record the variations in pressure caused by the waves. In the instrument, the pressure variations are converted into straight horizontal displacements by a recording pen. Since the recording periods of the instruments used in the field were 20 minutes once every four hours, 6 wave records were obtained for each day.

DATA ANALYSIS : In the analysis of wave records, initially the crest to trough heights of all waves in the 20 minute record were scaled manually and the number of waves (equal to the zero up-crossings) was counted. Since the above procedure was found to be time consuming and laborious, some shortcuts were made as follows: (a) from the 20 minute record, a representative 10 minute length of record was chosen, (b) a mean water line (zero line was drawn by the eye),(c) number of crests was then counted. A crest is defined as a point where the water level is momentarily constant, falling to either side. Some crests may be below the mean water level, (d) number of times, the record crosses the zero line moving in an upward direction was then counted. These also represent the zero up-crossings and the number of waves in the interval, (e) the maximum one-third of the highest waves was then measured,& (f) from such data, $H_{1/3}$, $H_{1/10}$, H_{max}, period T (also equal to T_z = period of zero crossings), period T_c = period of crests and the spectral width parameter calculated from the relationship

$$\varepsilon^2 = 1 - \left(\frac{T_c}{T_z}\right)^2$$

Even this method was found to be time consuming and therefore another shortcut method namely the one proposed by Draper (2) in 1966 was used. In that method $H_{1/3}$ was calculated from the most probable value of the height H_1 of the highest wave in the specific interval of time (4 hours in these cases). H_1 was obtained from the records by adding the height 'A' of the highest crest and depth C of the lowest trough both measured as positive from the mean water line. Then the calculation of $H_{1/3}$ and H_1 was a simple process and depended only on the number of waves and ratio of $H_{1/3}$ / H_1.

The above simpler and fast procedure was possible because relationships between $H_{1/3}$ and H_1, number of zero up-crossings and H_{max}, $H_{1/3}$ and $H_{1/10}$, $H_{1/3}$ and H_{avg}, etc. could be obtained from the two previously described procedures.

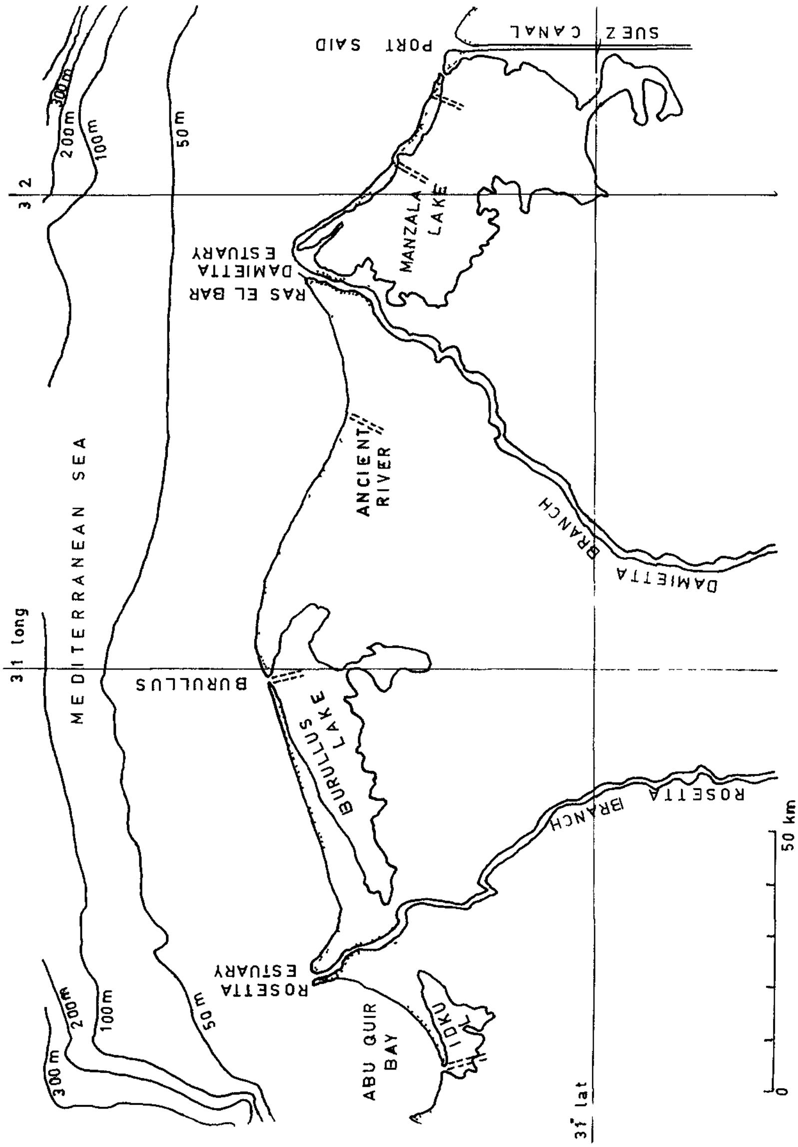

Fig 1· Nile Delta Coast

RESULTS FROM ANALYSIS : Figs. 2 and 3 are typical wave period spectra diagrams in which all waves were grouped around an average frequency of 0.050 sec^{-1}. Whereas in fig. 2, the frequency f and $\frac{H^2}{f}$ represent the axes, fig. 3 gives ΣH^2 on the ordinate and T on the abscissa. Both show narrow spectral width and insignificant contribution of energy at the two lower ends of the curves. Fig. 4 is a typical histogram of the frequency of occurrence of significant wave heights for Burullus area for August-December 1972. Figs. 5 and 6 represent significant wave height cumulative frequency distribution curves partially and collectively for the Burullus area. Figs. 7 and 8 show the relationship of the statistical parameters $H_{1/3}$, $H_{1/10}$ and H_{max} again for the Burullus area. The theoretical curves from the Raleigh distribution analysis are also indicated in the figures. Similar analysis was made for the two other areas also and they also show good agreement between theory and field data.

It is evident that a distribution similar to Raleigh distribution could describe the wave spectra, frequency distribution, and relationships between various statistical parameters (such as $H_{1/3}$, $H_{1/10}$, H_{max}, etc.).

With the knowledge of the wave heights from the above analysis and their directions from the synoptic weather maps wave height roses and wave energy flux/roses were calculated for the coast. Figs. 9 and 10 indicate the wave height and energy flux roses respectively for the Burullus area. Fig. 11 was drawn to determine the most predominant wave period for various classes of waves and it is interesting to find that a small range of periods namely 7-9 sec predominates over other periods.

Similar curves were drawn for other areas also so as to get wave height and wave energy climate along the entire coast. Since the period under study was only 8 months, conclusions regarding the climates could not be given in this paper. A more detailed study using hindcasting procedures to get the wave heights of previous years to discern any cyclic behaviour in the climates is under way. The present study is only a forerunner of such study and has given promising analysis for further use.

SUMMARY AND CONCLUSIONS : In order to study the wave climate along the Nile Delta coast, waves were measured in 3 places by offshore pressure operated recorders placed in 6-8 m depths. Since analysis of the records was time consuming, some short-cut methods were employed.

Period spectrum and energy spectrum of waves show narrow spectral band similar to Raleigh distribution. Relationships between various statistical parameters such as H_{max}, $H_{1/10}$, $H_{1/3}$ and H_{rms}, the histograms and frequency distribution curves drawn by using the wave data also indicate very close agreement with Raleigh distribution parameters and curves. With the use of synoptic charts for wave directions, wave height roses and wave energy roses were drawn for the coast to determine the wave height and wave energy climate along the coast.

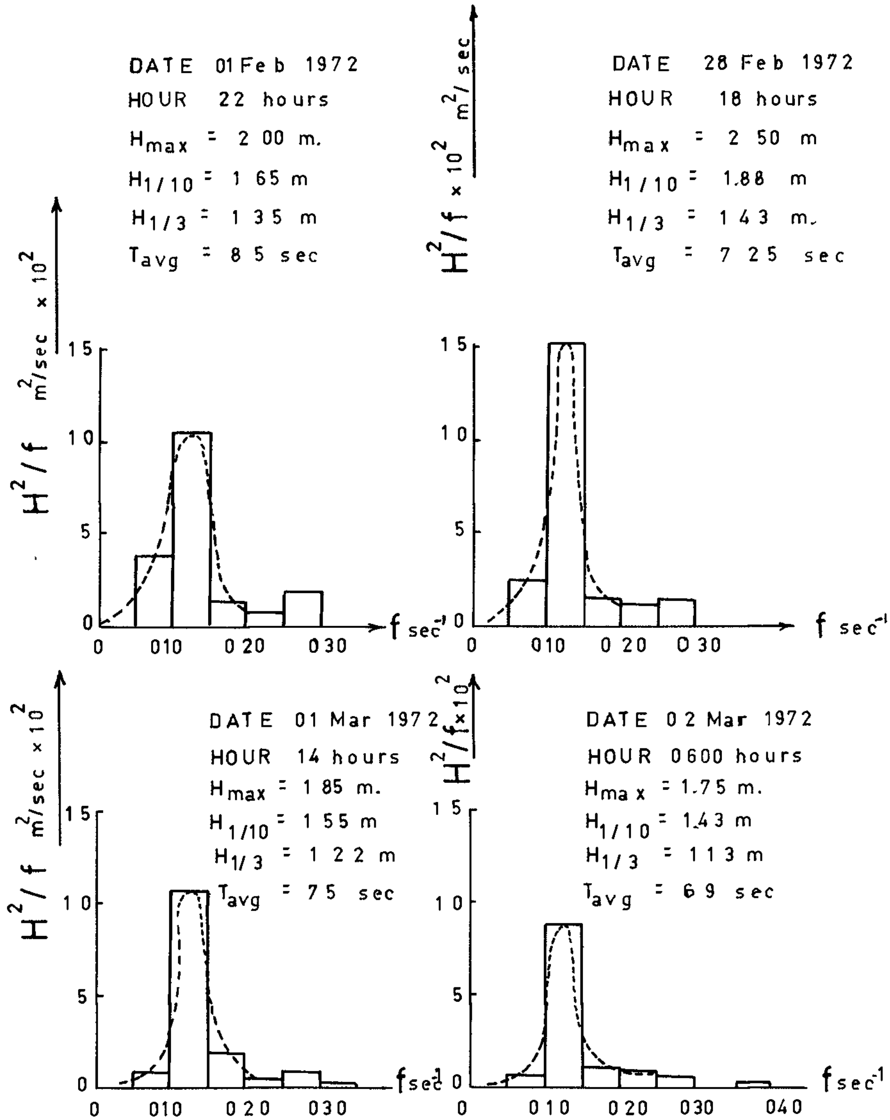

Fig. 2: Wave Period Spectrum (ABU QUIR)

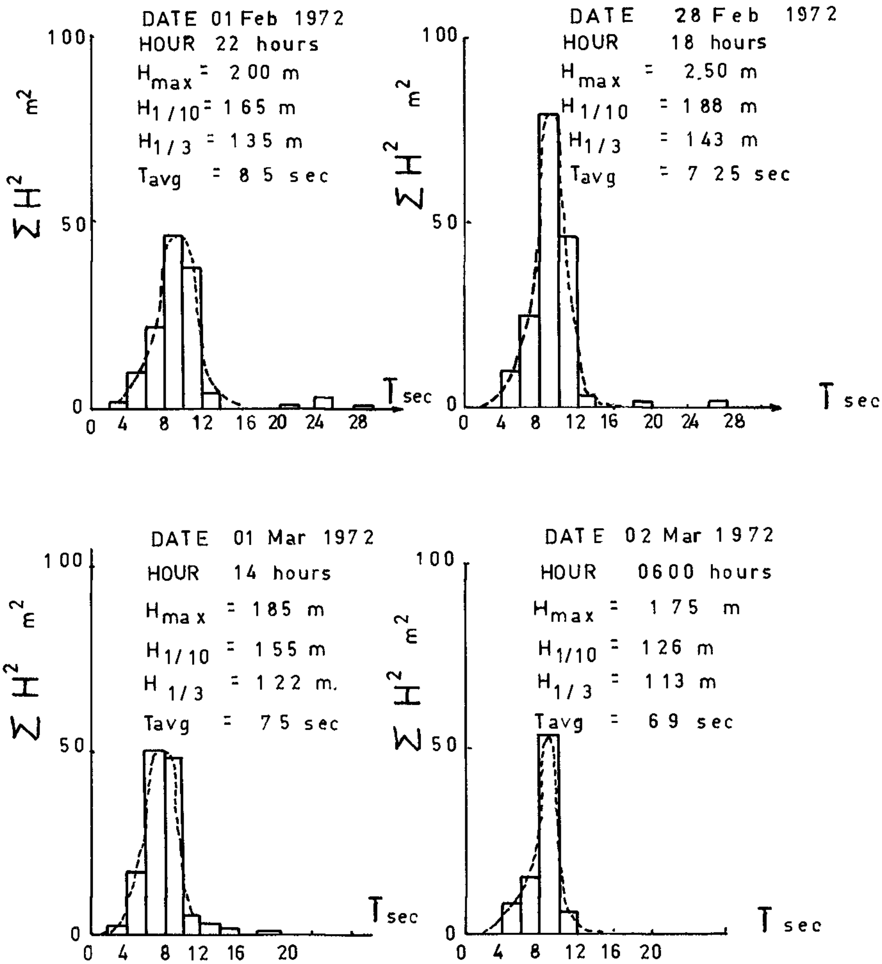

Fig. 3: Wave Spectrum - ABU-QUIR

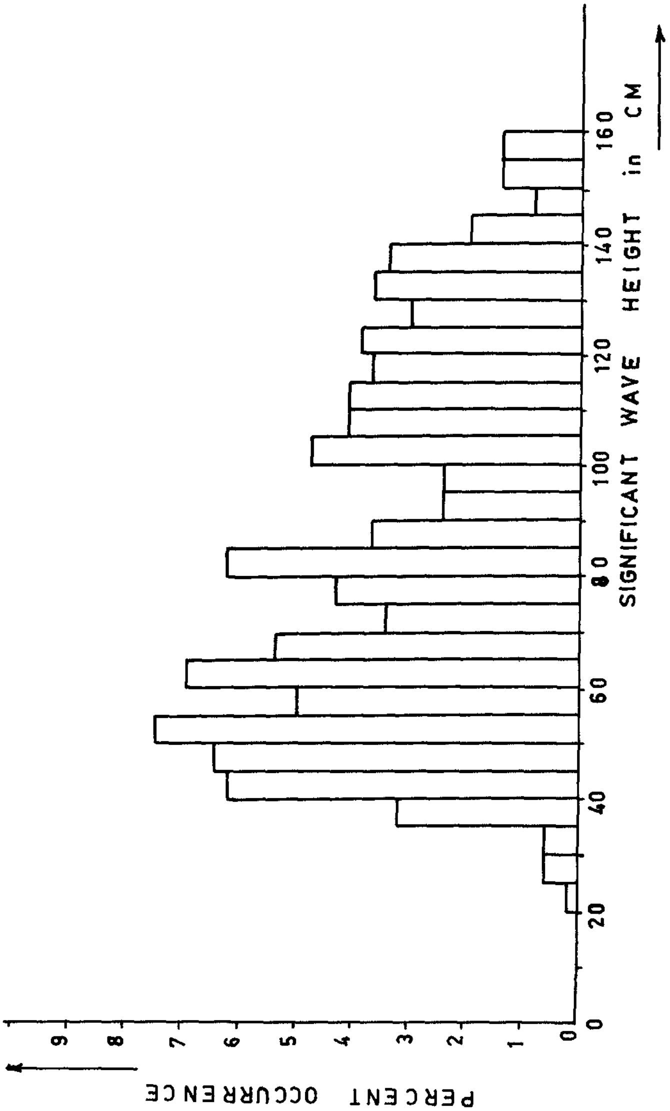

Fig. 4 Frequency of Occurrence - Burullus

(AUG TO DEC 1972)

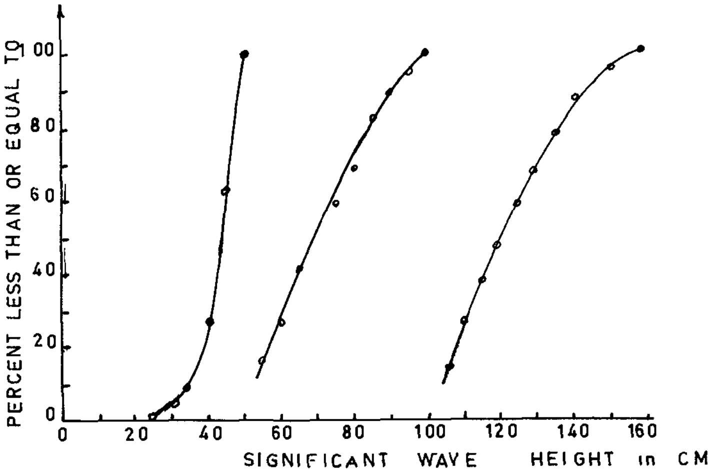

Fig. 5. Partial Cumulative Distribution of $H_{1/3}$

(BURULLUS)

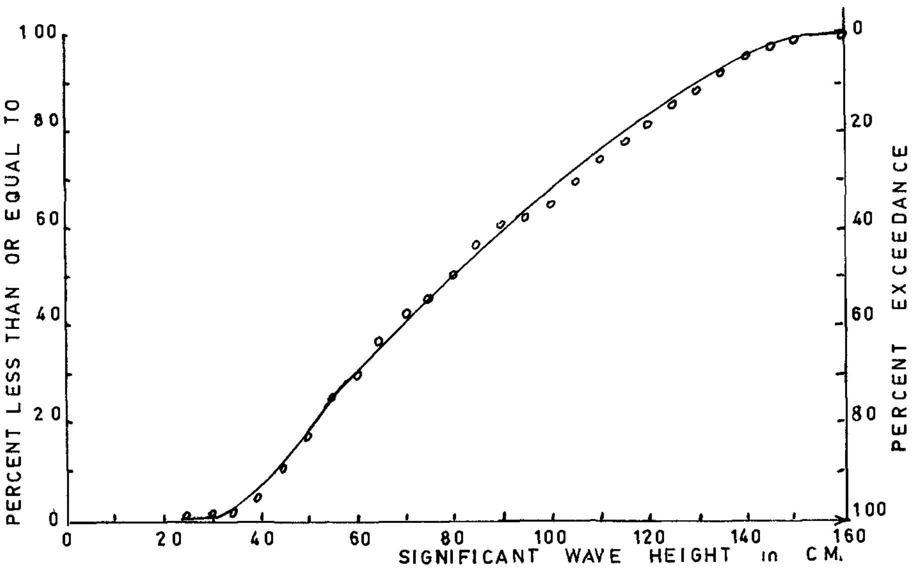

Fig. 6: Cumulative Frequency Distribution of $H_{1/3}$

ABU QUIR (APRIL TO AUGUST 1972)

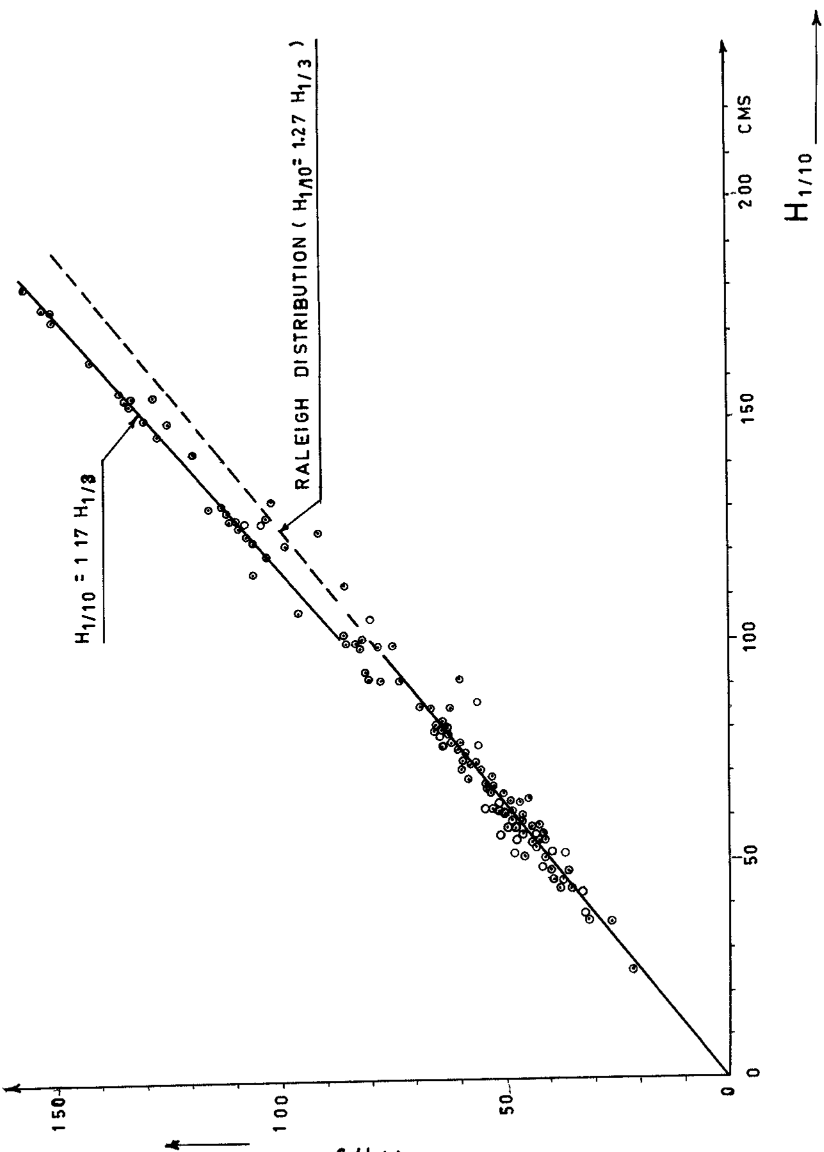

Fig. 7: $H_{1/10}$ Versus $H_{1/3}$ — Burullus

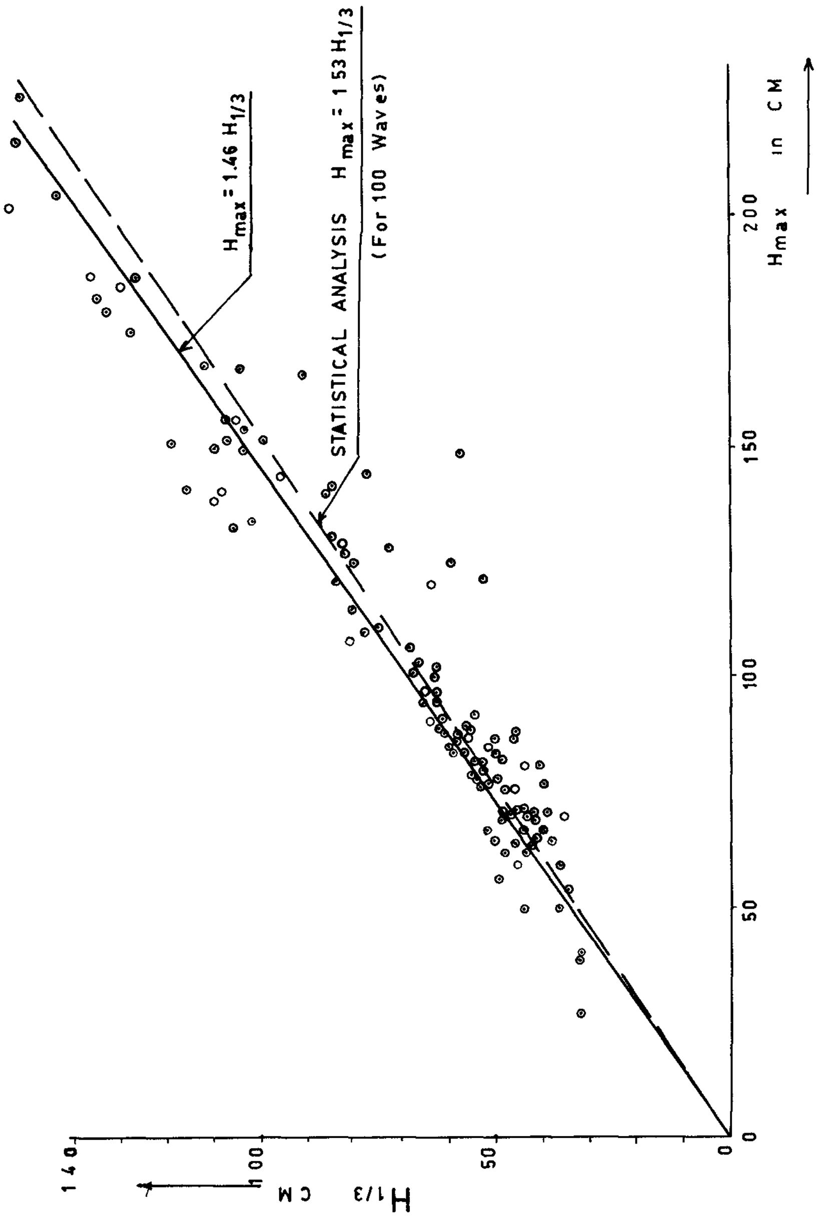

Fig.8: H_{max} Versus $H_{1/3}$ - Burullus.

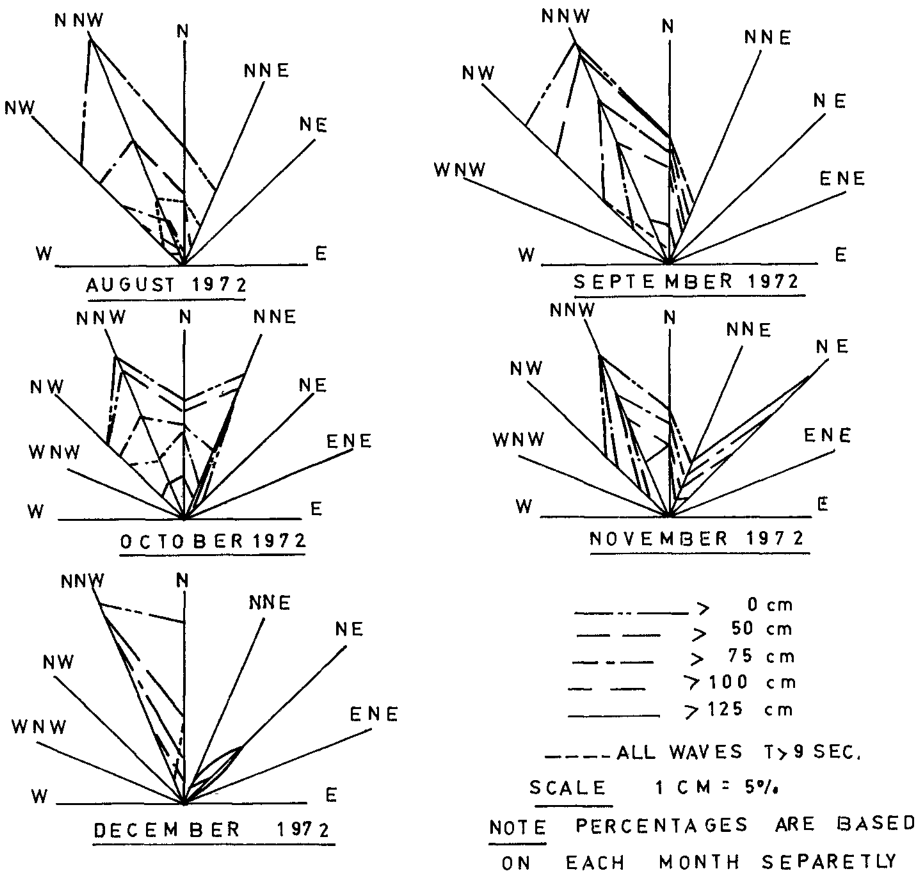

Fig. 9: Monthly Wave Rose Burullus

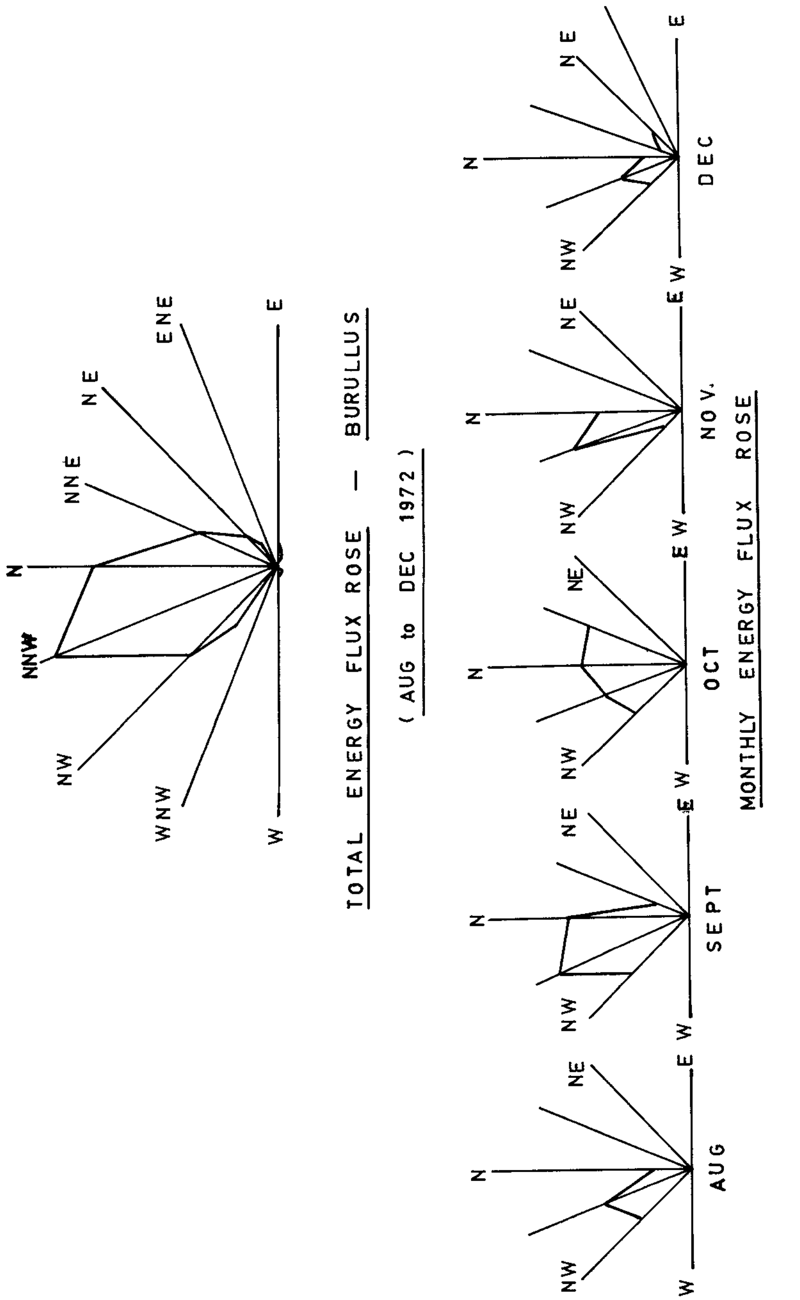

Fig.10 Monthly & Total Energy Flux Roses Burullus

SCALE 1 CM = 200 m^2.sec.

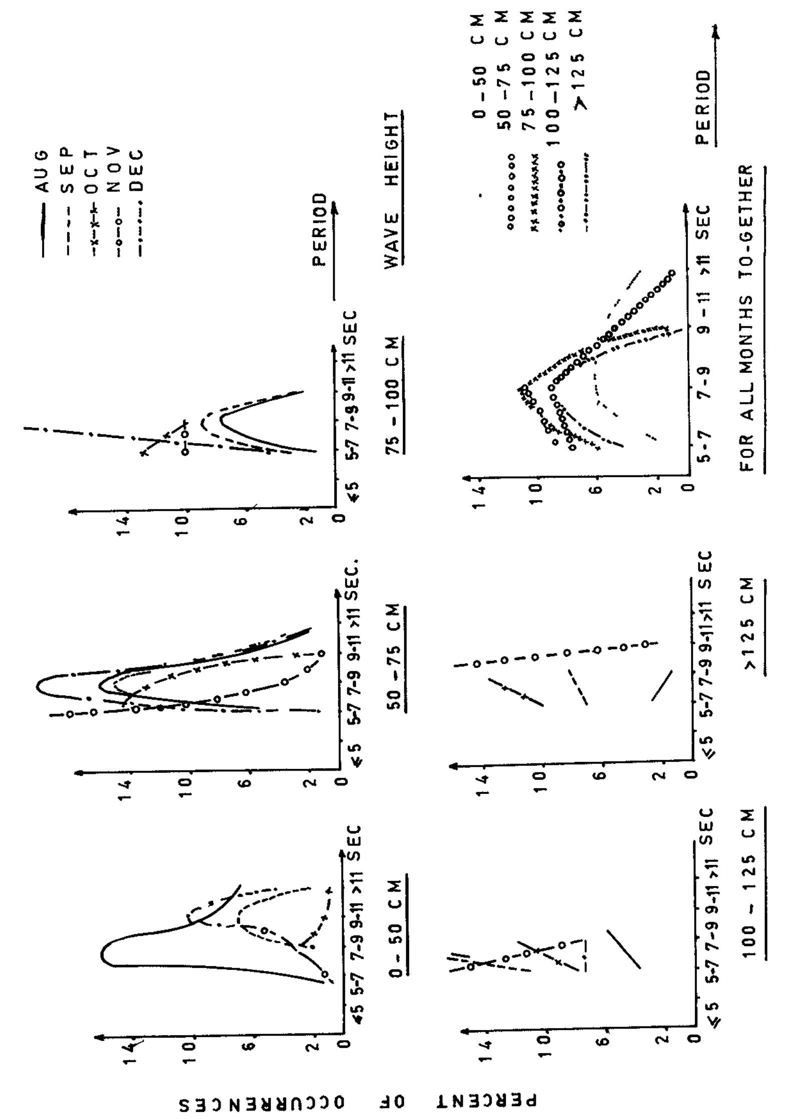

Fig 11. Percentage of Occurrences of Periods Burullus

(AUG to DEC 1972)

The above study is a forerunner of a comprehensive study using hindcasting procedures to determine the wave climate for a large period of years (about 60 years).

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