CHAPTER 21

MEASUREMENT OF INCIDENT WAVE HEIGHT IN COMPOSITE WAVE TRAINS

by

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ABSTRACT

A method is proposed for measurement of the incident wave height in a composite wave train. The composite wave train is assumed to consist of a superposition of regular incident and reflected waves with the same wave period. An approximate value of the incident wave height is obtained as the arithmetic mean of the wave heights measured by two gauges separated a quarter of a wave length. The accuracy of the method in relation to the location of the gauges and the wave parameters is investigated using linear and second order wave theory. Results of the calculations are presented in diagrams.

INTRODUCTION

At model tests with regular waves in a wave flume it is often difficult to determine the height of the incident waves which attack a model in the flume. The model causes reflected waves which move backwards towards the wave generator and by superposition give rise to a composite wave train. In many cases, the distance from the wave generator to the model is too short to allow the incident wave height to be measured without disturbing influence by reflected waves. Within a distance of some wave lengths from the wave generator, disturbances from the wave generation make accurate recording impossible. In the remaining part of the flume up to the model, the situation in flumes of limited length often is that there is no point where the waves have grown to the full height before reflected waves of considerable height

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arrive.

In many cases, it is therefore necessary to record the composite waves and in some way separate the incident wave from the recordings. A method, which has often been used, involves location of the nodes and anti-nodes. This is achieved by slowly moving a wave gauge horizontally along the wave train. The distance between nodes and anti-nodes is a quarter of a wave length. The wave heights at a node and anti-node are measured and the incident wave height is calculated as the arithmetic mean of the two values. The drawback of this method is that it may be difficult to locate the nodes if the reflected wave is small.

PROPOSED METHOD OF MEASUREMENT

A couple of two wave gauges assembled at a fixed distance of a quarter of a wave length is used. The couple is placed in an arbitrary position along the wave train. An approximate value of the incident wave height is obtained as the arithmetic mean of the two recorded heights. If one of the gauges happens to be located at a node, according to linear wave theory, the correct incident wave height is received. If the location of the gauges deviates from this ideal location, the obtained wave height will differ slightly from the correct value.

ACCURACY OF THE METHOD

To be able to calculate the occurring errors, the envelopes of the composite wave train are studied. Using linear wave theory an expression for the envelopes can be deduced as

$$y = \pm \frac{H}{2} \sqrt{(1 - K_r)^2 \cos^2 kx + (1 + K_r)^2 \sin^2 kx} \qquad (1)$$

where H is the incident wave height and $K_r \cdot H$ is the height of the reflected waves (Fig. 1).

The values of the wave heights H_1 and H_2, which are registered by the two gauges, can be calculated from equation (1). The approximation for for the incident wave height is

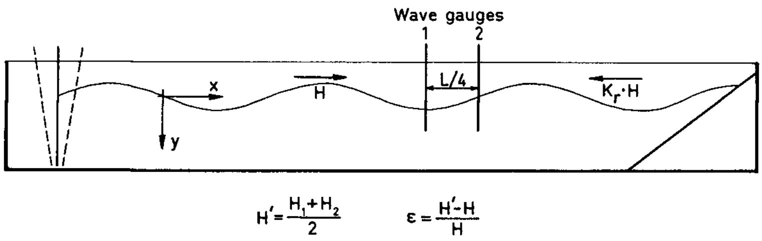

Fig. 1. Notation.

$$H' = \frac{1}{2}(H_1 + H_2) \tag{2}$$

and the relative error is

$$\varepsilon = \frac{H' - H}{H} \tag{3}$$

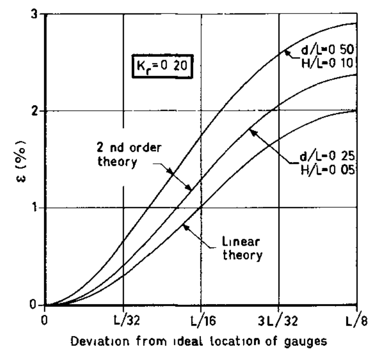

Fig. 2. Error vs. location of the gauges.

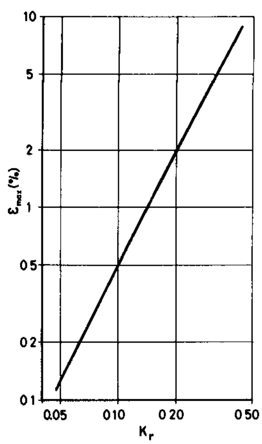

Fig. 3. Maximum error vs. reflection coefficient according to linear theory.

The error is shown in Fig. 2 (linear theory) for $K_r = 0.20$ vs. the deviation from the ideal location for the couple of gauges. The maximum error ε_{max} occurs when the gauges are situated half way between nodes and anti-nodes. The error grows with the height of the reflected waves - approximately as the square of K_r (Fig. 3).

The linear wave theory implies small steepness of the waves and gives a sinusoidal water surface. Waves of finite amplitude have narrow crests and broad shallow troughs. In order to study the influence on the accuracy of the method of the unsymmetrical wave profile of steep waves a second order wave theory is used. According to Rundgren (1) the wave profile of a composite wave train, correct to the second order of H, is given by the following equation

$$y = \frac{H}{2}\left\{(1 + K_r)\sin kx\cos\sigma t - (1 - K_r)\cos kx\sin\sigma t - \right.$$
$$- \frac{kH}{4}\coth kd\left[(1 + K_r)^2\left(\cos^2\sigma t + \frac{3\cos 2\sigma t - \tanh^2 kd}{4\sinh^2 kd}\right)\cdot\right.$$
$$\cdot\cos 2kx - (1 - K_r)^2\left(\sin^2\sigma t - \frac{3\cos 2\sigma t + \tanh^2 kd}{4\sinh^2 kd}\right)\cos 2kx +$$
$$\left.\left. + (1 - K_r)^2\left(1 + \frac{3}{2\sinh^2 kd}\right)\sin 2\sigma t\sin 2kx\right]\right\} \qquad (4)$$

where H = incident wave height
$K_r \cdot H$ = reflected wave height
$k = 2\pi/L$
$\sigma = 2\pi/T$

Fig. 4 shows examples of wave profiles for successive times (time step T/16) during one wave period calculated with equation (4). Because of interaction between incident and reflected waves of finite amplitude the amplitude at the nodes is not equal to zero for $K_r = 1.00$, as can be seen in Fig. 4 c. This means that the error ε is not zero even with the gauges placed at the ideal location with one gauge at a node and the other at an anti-node. Goda and Abe (2) have calculated corrections which should be applied to reflection coefficients and incident wave

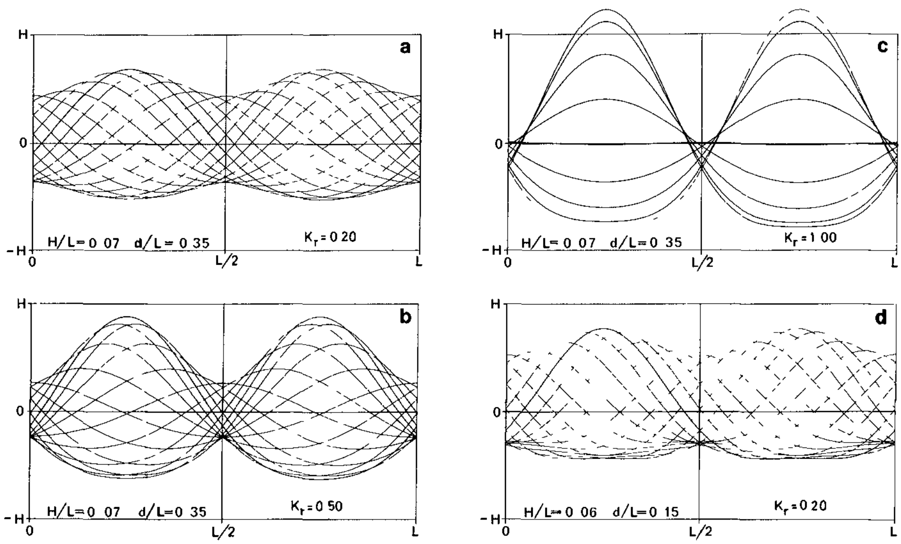

Fig. 4. Calculated wave profiles.

heights calculated from measured amplitudes at nodes and anti-nodes. For small ratios of depth to wave height the second order theory gives profiles with humps on the troughs, Fig. 4 d, which is an indication that the theory is not satisfactory for those waves.

The envelopes of some wave trains and the corresponding values of ε were calculated with equation (4). In Fig. 2 examples of results are shown for a steep deepwater wave and a wave with ordinary steepness and intermediate depth of water. The error ε is somewhat greater than according to the linear theory. In Fig. 5, the maximum error ε_{max} is shown for K_r = 0.10 and 0.20 vs. the steepness of the incident wave (H/L) and the relative depth of water (d/L). Above the dashed lines humps occur on the wave troughs, cf. Fig. 4 d.

The method described is very useful for the measurement of incident wave height when the superposed reflected waves are not too high (K_r less than about 0.20). When K_r is greater it is, however, easy to locate the nodes approximately, so that the couple of gauges can be placed near enough the ideal location to give acceptable accuracy.

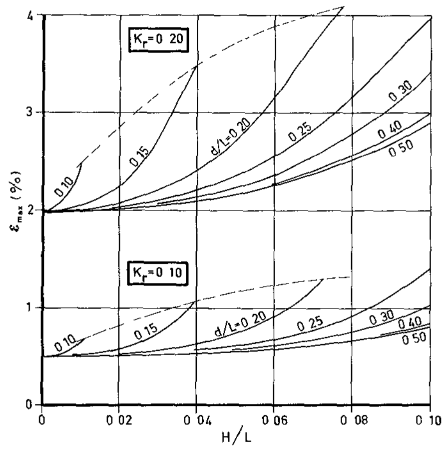

Fig. 5. Maximum error according to second order wave theory.

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2. Goda, Y. and Abe, Y. 1968. Apparent Coefficient of Partial Reflection of Finite Amplitude Waves. Report of the Port and Harbour Research Institute, Vol. 7, No. 3, 1968.